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Chapter 4. Probability.

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Definitions

- Experiment
A process of observation that leads to a single outcome that cannot be predicted with certainty.
- Sample Point
The most basic outcome of an experiment.
- Sample Space
The collection of all the sample points.
- Event
A specific collection of sample points.

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Example No. 1:

Two coins are tossed, and their up faces are recorded.

Sample Space

$$S : \{HH, HT, TH, TT\}$$

Events

$$A : \{\text{Observe exactly one head}\} = \{HT, TH\}$$

$$B : \{\text{Observe at least one head}\} = \{HH, HT, TH\}$$

$$C : \{\text{Observe at least one tail}\} = \{HT, TH, TT\}$$

$$B \cap C = \{HT, TH\} = A$$

$$B \cup C = \{HH, HT, TH, TT\} = S$$

$$B' = \{TT\}$$

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Probability.

The probability of a sample point is a number between 0 and 1 inclusive that measures the likelihood that the outcome will occur when the experiment is performed.

Probability Rules for Sample Points

- a) All sample point probabilities must lie between 0 and 1 inclusive.
- b) The probabilities of all sample points within a sample space must sum 1.

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How do we assign probabilities to sample points?

Classical Method

Relative Frequency of Occurrence

Subjective

How do we assign probabilities to events?

By adding the probabilities of the sample points included in the event.

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Example:

Suppose we have an unbalanced die with six faces and that the probabilities for each sample point are as follows:

Outcome	Probability
1	0.2
2	0.1
3	0.4
4	0.1
5	0.1
6	0.1

$$A = \{\text{The up face is an even number}\} = \{2, 4, 6\}$$

$$B = \{\text{The up face is less than 3}\} = \{1, 2\}$$

$$C = \{\text{The up face is at least 5}\} = \{5, 6\}$$

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$$P[A] = 0.3 \quad P[B] = 0.3 \quad P[C] = 0.2$$
$$P[A \cap B] = ? \quad P[A \cup C] = ? \quad P[B \cup A'] = ?$$

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Probability Rules:

- $P[A] + P[A'] = 1$
- If $A \cap B = \emptyset$ then $P[A \cup B] = P[A] + P[B]$
- $P[A \cup B] = P[A] + P[B] - P[A \cap B]$

Conditional Probability:

- $P[A|B] = \frac{P[A \cap B]}{P[B]}$

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Multiplicative Rule:

$$P[A \cap B] = P[A] \times P[B|A]$$

Independent events:

The events A and B are independent iff

$$P[A \cap B] = P[A] \times P[B]$$

or, equivalently,

$$P[B|A] = P[B]$$

which is also equivalent to

$$P[A|B] = P[A]$$

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Random Sample.

If n elements are selected from a population in such a way that every set of n elements in the population has an equal probability of being selected, the n elements are said to be a **random sample**.

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Problem 4.36

U.S. PHYSICIANS IN A RECENT YEAR						
Age (years)						
Gender	<35	35-44	45-54	55-64	>65	
<i>Male</i>	.11	.20	.19	.12	.16	.78
<i>Female</i>	.07	.08	.04	.02	.01	.22
	.18	.28	.23	.14	.17	1.00

$$A = [< 35] \quad B = [35 - 44] \quad C = [45 - 54] \quad D = [55 - 64] \quad E = [> 65]$$

$$U = [Male] \quad V = [Female]$$

a) $P[B] = .28$.

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Problem 4.36

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Age (years)						
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	.18	.28	.23	.14	.17	1.00

$$A = [< 35] \quad B = [35 - 44] \quad C = [45 - 54] \quad D = [55 - 64] \quad E = [> 65]$$

$$U = [Male] \quad V = [Female]$$

b) $P[V \cap C] = 0.04$

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Problem 4.36

U.S. PHYSICIANS IN A RECENT YEAR						
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	.18	.28	.23	.14	.17	1.00

$$A = [< 35] \quad B = [35 - 44] \quad C = [45 - 54] \quad D = [55 - 64] \quad E = [> 65]$$

$$U = [Male] \quad V = [Female]$$

$$c) P[U \cup B] = .78 + .28 - .20 = 0.86.$$

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Problem 4.36

U.S. PHYSICIANS IN A RECENT YEAR						
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Gender	<35	35-44	45-54	55-64	>65	
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	.18	.28	.23	.14	.17	1.00

$$A = [< 35] \quad B = [35 - 44] \quad C = [45 - 54] \quad D = [55 - 64] \quad E = [> 65]$$

$$U = [Male] \quad V = [Female]$$

$$d) P[A \cup D] = .18 + .14 = 0.32.$$

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Problem 4.36

U.S. PHYSICIANS IN A RECENT YEAR						
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	.18	.28	.23	.14	.17	1.00

$$A = [< 35] \quad B = [35 - 44] \quad C = [45 - 54] \quad D = [55 - 64] \quad E = [> 65]$$

$$U = [Male] \quad V = [Female]$$

$$e) P[V|C] = P[V \cap C]/P[C] = .04/.23 = 0.174$$

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Problem 4.36

U.S. PHYSICIANS IN A RECENT YEAR						
Age (years)						
Gender	<35	35-44	45-54	55-64	>65	
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<i>Female</i>	.07	.08	.04	.02	.01	.22
	.18	.28	.23	.14	.17	1.00

$$A = [< 35] \quad B = [35 - 44] \quad C = [45 - 54] \quad D = [55 - 64] \quad E = [> 65]$$

$$U = [Male] \quad V = [Female]$$

$$f) P[V' \cap D'] = P[(V \cup D)'] = 1 - P[V \cup D] = 1 - (.14 + .22 - .02) = 0.66$$

$$A = [< 35] \quad B = [35 - 44] \quad C = [45 - 54] \quad D = [55 - 64] \quad E = [> 65]$$

$$U = [Male] \quad V = [Female]$$

a) $P[B] = .28.$

b) $P[V \cap C] = 0.04.$

c) $P[U \cup B] = .78 + .28 - .20 = 0.86.$

d) $P[A \cup D] = .18 + .14 = 0.32.$

e) $P[V|C] = P[V \cap C]/P[C] = .04/.23 = 0.174$

f) $P[V' \cap D'] = P[(V \cup D)'] = 1 - P[V \cup D] = 1 - (.14 + .22 - .02) = 0.66$

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