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Entrepreneurial finance: Banks versus venture capital ☆

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Abstract

We analyze how entrepreneurial firms choose between two funding institution: banks, which monitor less intensively and face liquidity demands from their own investors, and venture capitalists, who can monitor more intensively but face a higher cost of capital because of the liquidity constraints that they impose on their own investors. Because the firm's manager prefers continuing the firm over liquidating it and aggressive (risky) continuation strategies over conservative (safe) continuation strategies, the institution must monitor the firm and exercise some control over its decisions. Bank finance takes the form of debt, whereas venture capital finance often resembles convertible debt. Venture capital finance is optimal only when the aggressive continuation strategy is not too profitable, ex ante; the uncertainty associated with the risky continuation strategy (strategic uncertainty) is high; and the firm's cash flow distribution is highly risky and positively skewed, with low probability of success, low liquidation value, and high returns if successful. A decrease in venture capitalists' cost of capital encourages firms to switch from safe strategies and bank finance to riskier strategies and venture capital finance, increasing the average risk of firms in the economy.
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1. Introduction

Although start-ups and venture capital finance are often linked in the public eye, bank loans are a more common source of finance for entrepreneurial firms.[1] Both sources share some common features. Because entrepreneurial firms are usually small and have high risk of failure, both venture capital and bank loans

[1]Using data from the National Survey of Small Business Finances, Berger and Udell (1998) find that commercial bank loans provide 19% of all financing for small businesses, finance company loans provide 5%, and venture capital investments provide 2%. Other major sources include principal owner's equity (31%) and trade credit (16%).

require careful monitoring of borrowers. Both types of finance use covenants to restrict the borrower's behavior and provide additional levers of control in the event that the firm performs poorly. These covenants often restrict the ability of the firm to seek financing elsewhere, which ties to yet another common feature: the use of capital rationing through staged financing and credit limits as means of controlling borrowers' ability to continue and grow their business.

Despite these similarities, significant differences exist between these two types of financing. Whereas banks lend to a wide variety of firms, firms with venture capital finance tend to have very risky and positively skewed return distributions, with a high probability of weak or even negative returns and a small probability of extremely high returns (see Sahlman, 1990; Fenn, Liang, and Prowse, 1995). Whereas bank loans usually take the form of pure debt, venture capitalists almost always employ convertible securities or a combination of debt and equity (see Kaplan and Stromberg, 2001). Banks' monitoring and control rights are typically far less intensive than those of venture capitalists and focus on avoiding or minimizing bad outcomes. Banks mostly monitor for covenant violations, deteriorating performance, or worsening collateral quality that might jeopardize their loan. They exercise control by threatening to force default and possible liquidation. By contrast, as shown by Sahlman (1990) and Kaplan and Stromberg (2001), venture capitalists often hold seats on the borrowing firm's board and voting rights far in excess of their cash flow rights, and they could have the contractual right to replace the entrepreneur with a new manager if covenants are violated. Along with these rights, venture capitalists monitor borrowers more frequently than banks do and play an active role in most of the firm's major decisions.[2] Finally, the funding structures of the two types of institutions are very different. Banks offer their investors relatively liquid investments, which in turn subjects banks to possible liquidity shocks. Venture capital funds impose restrictions on their investors, insulating the funds from liquidity shocks but forcing them to pay investors a premium for lack of liquidity.

In this paper, we develop a simple theoretical model that captures these differences. An entrepreneurial firm seeks financing from one of two institutions, a bank and a venture capital fund. Once funded, two possible conflicts arise between the entrepreneur and the institution. The first is the well-known tension between the entrepreneur's desire to keep the firm going to maintain her control benefits and the institution's desire to liquidate poorly performing investments. The second conflict is more novel: Even if it is optimal to keep the entrepreneur's firm going, there could be additional choices to be made. Should the firm expand conservatively or aggressively? Should the firm attempt an initial public offering (IPO) or settle for sale to another corporation? Again, a tension exists between the entrepreneur and the institution, because the entrepreneur could excessively prefer aggressive or risky decisions that maintain or expand her control benefits even when such decisions do not always maximize contractable cash flows. We model this phenomenon as a choice between a safe and a risky continuation strategy if the firm is not liquidated early.

If the firm is to be financed, the institution must be given incentive to monitor the firm's situation to reduce these conflicts. We assume that banks are less skilled at monitoring than venture capitalists. Banks can determine only whether or not the firm should be liquidated, whereas venture capitalists can also learn (at added cost) whether a safe or risky continuation strategy is best. The underlying idea is that the firm's optimal continuation strategy is affected by subtle details of the firm's situation that the institution can identify only through more intensive monitoring. Intuitively, venture capital funds are better at assessing the firm's strategic situation because they are more specialized and have more expertise at running firms than banks do. In our model, passive monitoring by a bank reveals only whether the firm is in a good or bad state, while active monitoring by a venture capitalist also reveals whether a firm in the good state is in a high substate, in which the risky strategy is the better choice, or a medium substate, in which the safe strategy is the better choice. It follows that the value of learning the substate so as to choose the best strategy is higher as the variance of the cash flow from the risky strategy across the two substates is higher. We refer to this variance as the firm's strategic uncertainty.

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[2] Gorman and Sahlman (1989) survey venture capitalists and find that lead venture capitalists visit their portfolio companies an average of 18.7 times per year. By contrast, Blackwell and Winters (1997) find that most bank loans to smaller firms are monitored once or twice a year. The most risky loans are monitored at least quarterly and sometimes more frequently. Similarly, Hellmann and Puri (2002) find that venture capital-backed firms are more likely to have higher measures of professionalization than start-up firms that rely on other types of financing, which is consistent with venture capitalists playing a more active role in the firm's management.

Venture capital's expertise comes at a cost, however. As shown by Lerner and Schoar (2004), venture capital funds impose liquidity restrictions on their investors so as to shield themselves from liquidity shocks. Such shocks would be especially problematic for these funds because of their lack of diversification and their highly information-intensive assets. To compensate for the illiquidity, fund investors demand a higher return, which in turn causes venture capital funds to require a high return from the firm. By contrast, banks' funding structure leaves them open to liquidity shocks, but with a lower average required rate of return.

For many firms, the level of strategic uncertainty might not be high, or the cost of intensive active monitoring could be so high that the entrepreneur is unwilling to reimburse the institution for this cost. For these firms, bank finance and passive monitoring are optimal. Moreover, the optimal contract for bank finance is debt. As shown by Winton (2003), debt is less risky than equity, and so the institution's assets are less affected by its private information about the firm, reducing adverse selection problems when the institution itself needs additional funding. Because these costs are passed on to the entrepreneur in her cost of funds, she shares this preference for debt, all else equal.

If instead strategic uncertainty is high, so that the impact of the choice between risky and safe strategies varies greatly with the firm's precise situation, venture capital finance and active monitoring are optimal. For the venture capital fund to have incentive to monitor actively, it must gain greatly from having the firm pursue a risky strategy when conditions are favorable and otherwise gain greatly from a conservative strategy. In general, debt does not accomplish this in a cost-effective manner. Although the venture capital fund's promised payment can be set so high that it bears most of the firm's cash flow risk, this implies that the fund effectively buys much of the firm initially. To the extent that this is more than the firm's required investment, this needlessly increases the firm's reliance on costly venture capital. A position that combines debt with equity (either a convertible security or joint holdings of debt and equity securities) can give the venture capital fund the necessary exposure to the firm's strategic decision with a lower overall investment.

The firm must have several characteristics if venture capital is to be optimal. First, strategic uncertainty must be high. Second, expected profits from the risky continuation strategy cannot be too high. Otherwise, the institution can recoup its investment even if the firm unconditionally pursues the risky strategy, which is the manager's preference. Third, the firm's cash flow distribution must be sufficiently positively skewed; i.e., the probability of success must be low, the value of the firm in liquidation low, and the firm's cash flows in success high. Greater skewness means that the institution can recoup its investment only by taking high payments when the firm is successful, so that it gains more from active monitoring of the firm's strategic decisions. It also implies that, if a bank financed the firm, its liquidity costs would be extremely high.

Venture capital is also more likely to be optimal as venture capital funds' cost of capital decreases or the severity of bank liquidity shocks increases. Moreover, a decrease in venture capitalists' cost of capital not only increases the number of firms that receive venture capital funding, but it also increases the number of firms pursuing risky strategies at least part of the time, as firms that would have opted for bank finance and conservative strategies switch to venture capital finance. This has both positive and negative consequences. On the one hand, there is an increase in risky activity, including the adoption or aggressive expansion of innovative products. On the other hand, even though the venture capital funds permit the risky strategy only when conditions seem good, the risky strategy still has the potential to perform badly. Thus a boom in venture capital finance could sometimes be followed by a bust when risky strategies do not pan out. This could help account for the 1990s boom in venture capital financing followed by the bust of the early 2000s.

Although several papers consider various aspects of venture capital financing, few researchers address the issue of what determines the choice between venture capital and bank financing. In Landier (2003), an economy's entrepreneurs choose safe projects backed by bank debt and low monitoring if the stigma associated with failure is high and risky projects backed by venture capital finance and high monitoring if the stigma associated with failure is low. In Ueda (2004), the choice between bank and venture capital financing depends on the relative importance of more accurate screening and the level of intellectual property rights protection. By contrast, our model makes cross-sectional predictions on the relative use of bank loans and venture capital based on differences in the risk and returns of firms' cash flows, and it explains the differences between banks and venture capitalists in terms of financial securities employed and exercise of control.

A paper that is closer to our model is that of Schmidt (2003), with a model in which both entrepreneur and venture capitalist can invest effort to improve the performance of the firm in different states of the world. Convertible debt gives all cash flows in the bad state to the venture capitalist and splits cash flows between both parties in the good state, which is optimal for getting both parties to exert effort in the states where their contribution is assumed to matter most. Although Schmidt provides a motivation for the use of convertible debt in venture capital, he does not model the choice between bank and venture capital finance, which is the key focus of our paper. Also, whereas Schmidt simply assumes that venture capital targets have certain features, we model the pros and cons of venture capital in terms of primitives, such as the underlying characteristics of the firm's cash flow distribution, the importance of strategic choices that follow directly from these characteristics, and the monitoring activity of the financing institution.[3]

Another related paper is Hellmann (2006). In his model, the start-up firm faces a choice between an acquisition and an IPO. It is assumed that both the entrepreneur and the venture capitalist contribute to firm value after an IPO, whereas they both retreat from the firm following an acquisition. Convertible securities that automatically convert to common equity upon completion of an IPO provide the best incentives to the entrepreneur and the venture capitalist when their effort is required the most. Like Schmidt (2003), Hellmann assumes that venture targets have certain specialized features, and he does not model the choice between bank and venture capital finance. In our concluding section, we discuss the contrasts between our results and those of Schmidt and Hellmann in more detail.

Whereas we focus on the choice between banks and venture capital, two recent papers focus on the choice between venture capital and angels (wealthy individuals who invest directly in entrepreneurial firms). In Chemmanur and Chen (2003), venture capitalists can add value to some of the firms they finance, but angels cannot. In Bernhardt and Krasa (2004), venture capitalists are informed investors who have control rights, angels are informed investors who do not have control rights, and banks are uninformed investors who do not have control rights. In reality, banks have some private information and control rights, as we assume.

The rest of the paper is organized as follows. Section 2 outlines our model and basic assumptions. Section 3 examines how exercise of control by the institution at date 1 depends on the type of monitoring that it engages in. Section 4 describes the bank's liquidity costs as a function of its information and liquidity needs. Section 5 examines equilibrium behavior for the bank and for the venture capital fund. Section 6 examines the firm's optimal choice between the bank and the venture capital fund as a function of the underlying characteristics of the firm. Section 7 discusses empirical implications and concludes.

2. The model

The firm: A firm operates over three dates: 0, 1, and 2. At date 0, the manager of the firm makes an investment I. Because the manager has no investable funds of her own, she must obtain the necessary funds by issuing claims to an institution. The firm yields verifiable cash flows at date 2, which we denote by X. At date 1, the firm can be in one of two states, good or bad; also, the good state has two substates, which we refer to as medium (substate $i = m$) and high (substate $i = h$). We refer to a firm in the bad state as a bad firm. Good, medium, and high firms are defined similarly. A bad firm yields $X = 0$ with certainty. By contrast, for a good firm, X depends in part on its substate i and in part on an action a it takes at date 1.

There are three choices of action a available to the firm at date 1, which we refer to as liquidation (a_L), safe continuation strategy (a_S), and risky continuation strategy (a_R). If the firm is liquidated at date 1, its assets have a liquidation value $L < I$, and it yields no cash flow at date 2. If instead the firm is continued, it must choose either the safe strategy a_S or the risky strategy a_R. If a good firm chooses a_S, it yields a date-2 cash flow of $X = X_S > I$ with certainty, regardless of the substate i. If it chooses a_R, it yields a cash flow $X = X_R > X_S$ with probability p_i and $X = 0$ otherwise. We assume that $p_m < p_h$, i.e., a high firm generates a higher expected cash flow than a medium firm under the risky strategy.

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[3]Other motivations for the use of convertible debt as a financial contract include reducing entrepreneurial risk-shifting incentives (Green, 1984), obtaining indirect equity financing when a conventional equity issue is unattractive (Stein, 1992), reducing an entrepreneur's incentives to engage in short-term window-dressing (Cornelli and Yosha, 2003), and solving renegotiation problems that arise with debt financing (Dewatripont, Legros, and Matthews, 2002). Instead of demonstrating the virtues of convertible debt, our work focuses on predicting where convertible debt or a debt-equity mix is most useful as opposed to pure debt finance.

Table 1
Cash flows and private benefits

Action	Cash flows	Control benefits
Liquidation at date 1	L at date 1. $X = 0$.	0
Safe strategy	$X = X_S$ if firm is good, and zero otherwise.	C_S
Risky strategy	$X = \begin{cases} X_R & \text{with probability } p_i, \ i \in \{m,h\} \\ & \text{if firm is good,} \\ 0 & \text{otherwise.} \end{cases}$	C_R

In addition to cash flows, the manager gets non pecuniary or otherwise non contractable private benefits of control from operating the firm (see Diamond, 1993).[4] If the firm is liquidated at date 1, the manager does not get any control benefits. If the firm continues to operate through date 2, the manager receives control benefits that are valued at $C_S > 0$ if the safe strategy is chosen and $C_R > C_S$ if the risky strategy is chosen, regardless of the firm's state or substate. For example, in the context of an expansion decision, a larger, more aggressive firm could offer greater managerial perquisites or prestige than a smaller, more conservative firm. In the context of choosing whether to do an IPO or to sell the firm to another firm, after an IPO the entrepreneur would still be the chief executive officer (CEO) of an independent firm, whereas after a sale she would either be replaced or would end up as a division manager of the acquiring firm.

The value of the firm at date 2 is equal to the cash flow generated plus the value of control benefits. Table 1 summarizes the cash flows and private benefits arising from the different decisions.

Information structure: At date 0, it is common knowledge that the firm is in the good state with probability θ and the bad state with probability $1 - \theta$. It is also common knowledge that a good firm is in the high substate $i = h$ with probability ϕ and the medium substate $i = m$ with probability $1 - \phi$. We define the average probability that a good firm that undertakes the risky strategy a_R produces the cash flow X_R as

$$q \equiv \Pr(X = X_R | a = a_R, \text{ state} = \text{good}) = \phi p_h + (1 - \phi) p_m. \tag{1}$$

At intermediate date $\frac{1}{2}$, the manager of the firm freely observes what the date-1 state and substate of the firm will be. By contrast, the institution can learn the firm's state and substate only by engaging in costly monitoring. Even though the state and substate are observable by the agents, they cannot be verified by an outside agency such as a court of law.

Institutions, monitoring, and control: There are two levels of monitoring that an institution can engage in, which we call passive and active monitoring. An institution that monitors passively learns the firm's state (good or bad) at date $\frac{1}{2}$ but not its substate (whether a good firm is medium or high). In this sense, passive monitoring is imperfect. By contrast, an institution that monitors actively not only learns the firm's state but can also learn a good firm's substate by incurring an additional cost m_g at date $\frac{1}{2}$. Intuitively, the substate reflects subtle details of the firm's situation that affect its optimal strategy, details that only intensive monitoring reveals. We assume that passive monitoring costs m_P, while active monitoring costs $m_A > m_P$ at date 0.

There are two institutions in our model: a bank (denoted by $j = b$) and a venture capital fund (denoted by $j = vc$). The bank and the venture capital fund differ in three crucial respects. First, the bank can monitor passively but not actively, whereas the venture capital fund can monitor either passively or actively. Active monitoring is more intensive than passive monitoring and requires a clear understanding of the firm's business and the environment that the firm operates in. Venture capital funds acquire the expertise required for active monitoring by specializing in a small group of industries and providing fund managers with high-powered incentives. Banks, on the other hand, lend to a wide variety of firms, so they lack the specialized expertise to be active monitors. Also, because banks typically have many lending officers, providing high-powered contracts contingent on each lender's performance is difficult.

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[4]For evidence that entrepreneurs do, in fact, derive high non pecuniary benefits from having control, see Moskowitz and Vissing-Jorgensen (2002).

The second difference is that the bank is subject to random liquidity shocks whereas the venture capital fund is not. These liquidity shocks are described in greater detail in Section 2.1. Banks face liquidity shocks because they offer demandable deposits and other deposits with short-term maturities and because they offer liquidity insurance to borrowers in the form of lines of credit. By contrast, venture capital funds restrict the redemption and resale rights of their investors so as to insulate the fund from liquidity shocks (see Lerner and Schoar, 2004). They demand liquidity insurance from their investors in the form of commitments to provide additional funds. Failure to honor commitments leads to forfeiture of existing investments.

This in turn leads to the third difference between the institutions: The bank's net cost of capital, r_b, is lower than that of the venture capital fund, r_{vc}. The restrictions on redemption and resale rights for venture capital fund investors make investments in venture capital funds less liquid than other investments, and so fund investors demand a higher return than they would if they invested in a bank. Without any loss of generality, we set $r_b = 0$ and $r_{vc} = r > 0$. Moreover, for simplicity, we assume that there is no discounting of cash flows between dates 1 and 2. Thus, cash flows from either date are discounted once to arrive at date 0 present values.

The firm decides to approach either a bank or a venture capital fund at date 0. If the firm approaches the venture capital fund, the fund decides whether to monitor actively or passively at date 0. This monitoring choice is observed by the manager. Monitoring and the information it reveals cannot be verified by an outside agency, so monitoring cannot be contracted upon directly. It follows that at date 0 it is not possible to write a complete contract specifying action as a function of the firm's date-1 state and substate. By contrast, because actions and cash flows are verifiable, the contract can specify payments as a function of actions and cash flows.

Parametric restrictions: To focus attention on circumstances that are most relevant to entrepreneurial firms, we impose several parametric restrictions. We also impose restrictions that simplify the number of cases we must deal with without altering the basic thrust of our results.

Assumption 1. $p_m X_R < X_S < p_h X_R$.

In other words, when a good firm is in the medium substate, the expected date-2 cash flow is maximized by taking the safe action a_S. When it is in the high substate, the expected date-2 cash flow is maximized by taking the risky action a_R. This assumption focuses our attention on the case in which the firm's substate has a meaningful impact on the choice between safe and risky actions.

Our next set of assumptions focuses on the nature of the agency problem between the institution and the manager.

Assumption 2. The following conditions hold.

(a) $\max\{L, \theta X_S, \theta q X_R\} < I$,
(b) $(1 - \theta)L + \theta(1 - \phi)(X_S - (C_R - C_S)) + \theta \phi p_h X_R < I + m_P$,
(c) $(1 - \theta)(L - C_S) + \max\{\theta X_S, \theta q X_R\} < I$.

Assumption 2(a) states that neither unconditional liquidation nor unconditional continuation allows the institution to break even. This assumption guarantees that if the firm is to be funded, it faces a non trivial operating decision.

Assumption 2(b) states that the institution cannot break even if it monitors passively and bribes the manager of a medium firm to choose the safe action by compensating her for her loss of control benefits, $C_R - C_S$.

Assumption 2(c) states that the institution cannot break even if it does not monitor and bribes the manager to liquidate by compensating her for her loss of control benefits C_S.

The upshot of Assumptions 2(a) and (c) is that the agency problem between the institution and the manager cannot be solved by giving the manager simple cash flow incentives. The institution must monitor and exercise some control over action choices if the firm is to be feasibly funded. Moreover, Assumption 2(b) guarantees that the firm's financial slack is too small to allow the institution to construct cash incentives that get the manager to voluntarily maximize cash flows for medium and high firms. This implies that only active monitoring can maximize the firm's contractable cash flows.

Initial contracting: The date-0 contract cannot specify actions depending on the firm's state or substate because these are unverifiable. However, it can specify the ex post payment S to be received by the institution. S is subject to limited liability constraints. Because the firm can generate only four possible cash flows (0, L, X_S, and X_R), the ex post payment S can take on only three free values, S_L, S_S, and S_R, which represent the payment to the institution when the cash flow is L, X_S, and X_R, respectively. (When the cash flow is 0, limited liability implies that the cash flow to the institution is also 0.) Limited liability implies that the institution's cash flows satisfy $0 \leqslant S_L \leqslant L$, $0 \leqslant S_S \leqslant X_S$, and $0 \leqslant S_R \leqslant X_R$. We use the triple (S_L, S_S, S_R) to represent the initial contract.

If the institution is to break even, it must have the ability to force liquidation if the firm proves to be bad and possibly some ability to control the firm's choice between safe and risky strategies. One way to do this is to have the contract require a payment from the entrepreneur at date 1, with transfer of control if the payment is not made. Because the firm has no cash inflows at this date, it cannot make such a payment. (One can also show that uninformed investors would not be willing to refinance the firm. Essentially, asking for refinancing is at best a neutral signal, and the firm is not worth funding on an uninformed basis.) For debt, this can be accomplished by having the debt mature at date 1. For preferred stock, this can be accomplished by requiring a dividend at date 1 along with a covenant granting the institution control if the dividend is not paid.

In reality, the timing of decisions about continuation strategy could be uncertain. One way that the structure could reflect this is to also include a covenant that gives the institution veto power over any change in business strategy. In fact, covenants aimed at this (e.g., forbidding a change in business, a sale of assets, or seeking additional financing) are common. Also, in reality, continuation strategies could require an additional investment at date 1. For example, this would usually be the case for an expansion strategy. In this case, the institution can exercise control by refusing to fund the additional investment. We return to this alternative structure in the conclusion.

For simplicity, we assume that the manager gets to propose a contract to the institution, which the institution can either accept or reject. This gives all initial bargaining power to the manager, which simplifies analysis without affecting our qualitative results.

Renegotiation: At date 1, if the institution decides to exercise control, the manager can try to renegotiate the contract and thus the action choice. To simplify analysis, we again assume that the manager makes an offer to the institution, which the institution can either accept or reject. If the institution rejects the offer, then the initial contract remains valid. This gives the manager all bargaining power in renegotiation.

Timing of events: At date 0, the manager signs a contract with either a bank or a venture capital fund in exchange for cash. The manager then makes an investment I. The institution also chooses its level of monitoring. At date $\frac{1}{2}$, the manager freely observes the firm's state and substate, and the institution observes the result of its monitoring. At date 1, the institution either forces liquidation or allows the firm to continue. In the event of continuation, the institution can exercise control to force the safe action a_S. The manager can try to renegotiate with the institution. At date 2, cash flow X is realized.

2.1. Bank liquidity costs

A bank faces ongoing needs for funds (liquidity needs) in the course of business. Such needs might involve meeting demands for additional loans, takedowns under existing credit lines and loan commitments, or meeting higher than usual demands for repayment from depositors and other investors. In all of these cases, failure to meet liquidity needs hurts the bank's business, creating costs.

Following Winton (2003), we assume that the bank is subject to a random liquidity need that is large enough that the bank can meet it only by selling its claim on the firm or by issuing securities backed by this claim. Specifically, at some interim time $\frac{1}{2}$, with probability λ, the bank has a sudden need for funds. When this occurs, the value of one dollar of date $\frac{1}{2}$ cash flows increases to $1 + \beta$ dollars, where $\beta > 0$ measures the severity of the liquidity need. Winton (2003) shows that, because the bank has private information about the value of its claims, this leads to adverse selection. On average, the bank cannot access the full value of its assets, so some liquidity needs go unmet, creating liquidity costs.

The precise nature of these liquidity costs depends on the equilibrium behavior of the bank as a function of whether it has liquidity needs and what its private information about the firm is. Before we can describe this

behavior, however, we need to specify the bank's payments as a function of its information about the firm. Before turning to that task, we impose one more assumption.

Assumption 3. $\lambda\beta < r$.

Assumption 3 implies that the bank's cost of capital, including expected liquidity costs, is lower than the venture capital fund's cost of capital. This holds so long as the bank can reduce the frequency of large liquidity needs (large enough to force the use of its investment in the firm as collateral) to a reasonable level. In practice, banks can do this by being well-diversified, holding pools of liquid securities such as Treasuries, and issuing some long-term securities for funding. See Winton (2003) for further discussion.

At this point, one might wish for an institution that combines the monitoring expertise of a venture capital fund with the redemption rights of a bank. Such a hybrid is unlikely to work well in practice. Venture capital funds are typically relatively undiversified, with a heavy focus on a few related industries so as to maintain and leverage their specialized expertise. Such lack of diversification tends to increase the frequency λ and severity β of liquidity needs that affect any given asset, so a venture capital fund that allowed bank-like redemption rights might face an expected liquidity cost $\lambda\beta$ in excess of the cost of capital r it faces when redemption rights are restricted. This would argue for the fund to restrict its investors' redemption rights, which is what is seen in practice.

Our paper's results suggest another reason that venture capital funds with bank-like redemption rights would face high liquidity costs. Liquidity needs can represent funding requests from existing borrowers, and failure to meet these needs can in some cases lead to the borrower's default and liquidation. As we show in this paper, venture capital targets tend to be riskier and have lower collateral than bank borrowers. Thus, if venture capital funds faced withdrawals when their target firms needed more financing, the funds' expected losses from not meeting these needs would be higher than those of banks, increasing β.

3. Equilibrium exercise of control at date 1

In this section, we begin by defining equilibrium in our setting. We then analyze the institution's exercise of control and possible renegotiation with the manager at date 1, taking the institution's level of monitoring and contractual payments as given. This allows us to state the expected value of the firm (cash flows plus control benefits) and the expected payments to the institution as a function of monitoring level and contract structure. We show that under any contract that lets the institution break even on its investment and monitoring costs, the manager always seeks to implement the risky action. She is successful whenever the firm's expected cash flows under the risky action are high enough to compensate the institution for what it has been promised under the safe action.

Before formally defining equilibrium, we make some preliminary observations. Define $E_{P,j}(V)$ and $E_{P,j}(S)$, respectively, as the expected value of the firm (including control benefits) and the expected discounted payments to the institution of type $j \in \{b, vc\}$ under passive monitoring. The expectation assumes that both the institution and the manager behave optimally at date 1, when actions and renegotiation take place. Also, payments to the institution are discounted at the institution's cost of capital, r_j. Similarly, define $E_{A,j}(V)$ and $E_{A,j}(S)$, respectively, as the expected value of the firm and the expected discounted payments to the institution under active monitoring. Because the manager is the residual claimant, her expected payoff at date 0 is $E_j(V) - (1 + r_j)E_j(S)$, where the level of monitoring is implicit. In our model, the manager does not discount payments to the institution. (The manager might have her own subjective discount rate, but she would apply this equally to $E_j(V)$ and to $(1 + r_j)E_j(S)$. All that is critical for our model is that venture capitalists have a higher required return than banks, making venture capital finance relatively more expensive.)

Next, note that the institution's expected date-0 payoff after it has invested is its expected discounted payments less any expected monitoring costs and liquidity costs, $E_j(S) - E_j(m) - \Lambda_j(S)$, where $\Lambda_j(S)$ denotes the institution's liquidity costs. [To be specific, $E_j(S) - E_j(m)$ equals $E_{P,j}(S) - m_P$ if the institution monitors passively and $E_{A,j}(S) - m_A - \theta m_g/(1 + r_j)$ if it monitors actively. Also, recall that $\Lambda_{vc}(S) = 0$.]

Equilibrium requires that, given the contractual payments it receives, the institution must prefer its monitoring choice at date 0 over alternative choices. In principle, the institution could randomize its choice of monitoring strategy, monitoring actively with probability α_A, monitoring passively with probability α_P, and

not monitoring at all with probability $1 - \alpha_A - \alpha_P$. Nevertheless, because Assumption 2(a) implies that not monitoring does not allow the institution to break even, the case in which the institution does not monitor with positive probability $(1 - \alpha_A - \alpha_P > 0)$ cannot arise in equilibrium.[5]

In fact, given our assumptions regarding banks and venture capital funds, mixed strategy equilibria do not occur. By assumption, banks cannot monitor actively, so mixed strategy equilibria cannot arise in bank financing. If instead the venture capitalist prefers to mix active and passive monitoring, it must be indifferent between the two. But because of its lower required return, the bank can always provide passive monitoring more cheaply than the venture capitalist. The only possibility for mixing is if the manager prefers to randomize between passive bank finance and active venture capital finance. In general, this is dominated by choosing either bank finance all the time or venture capital finance all the time. Thus, we can restrict our attention to equilibria in which the manager either chooses active venture capital monitoring or passive bank monitoring; i.e., either $\alpha_A = 1$ or $\alpha_P = 1 - \alpha_A = 1$.

Also, because we assume that the manager makes a take-it-or-leave-it offer to the institution, the manager receives all surplus initially as well as subsequently. If the institution's expected payments net of monitoring costs and liquidity costs are $E_j(S) - E_j(m) - \Lambda_j(S)$, then the manager's initial offer requires that the institution invest this amount up front, so that the institution breaks even on average. It follows that if the firm is to be funded initially, $E_j(S) - E_j(m) - \Lambda_j(S)$ must weakly exceed I.

We assume that if the manager initially receives more cash than the required investment I, she consumes the excess. Thus, the manager's date-0 expected utility is the sum of her expected net payoffs as of date 1 and any initial excess cash. Because the manager's expected payoff at date 1 is given by $E_j(V) - (1 + r_j)E_j(S)$, and her initial excess cash is given by $E_j(S) - E_j(m) - \Lambda_j(S) - I$, it follows that, in equilibrium, the manager's expected utility at date 0 is $E_j(V) - E_j(m) - r_j E_j(S) - \Lambda_j(S) - I$.

Given these observations, an equilibrium in this game consists of a choice of financing institution $\hat{\alpha}_A \in \{0, 1\}$, and a contract (S_L, S_S, S_R) offered by the manager such that

- If venture capital finance $(\hat{\alpha}_A = 1)$ is chosen, the venture capital fund has incentives to monitor actively given (S_L, S_S, S_R); i.e.,

$$E_{A,vc}(S) - m_A - \frac{\theta m_g}{1 + r} \geqslant E_{P,vc}(S) - m_P, \tag{2}$$

where once more the expectations assume optimal exercise of control subject to renegotiation at date 1; and
- (S_L, S_S, S_R) maximizes the manager's ex ante utility subject to the financing constraint given $\hat{\alpha}_A$; i.e., (S_L, S_S, S_R) is a solution to

$$\max_{S_L, S_S, S_R} U_M = \hat{\alpha}_A \left(E_{A,vc}(V) - m_A - \frac{\theta m_g}{1 + r} - r E_{A,vc}(S) \right)$$
$$+ (1 - \hat{\alpha}_A)(E_{P,b}(V) - m_P - \Lambda_b(S)) - I \tag{3}$$

subject to the financing constraint

$$\hat{\alpha}_A \left(E_{A,vc}(S) - m_A - \frac{\theta m_g}{1 + r} \right) + (1 - \hat{\alpha}_A)\big(E_{P,b}(S) - m_P - \Lambda_b(S) \big) \geqslant I. \tag{4}$$

To analyze the equilibrium, we begin with behavior at date 1. We can assume without loss of generality that $S_S \leqslant S_R$. Otherwise, in any situation in which the manager wanted to choose the risky action, the institution could always use its control rights to force the safe action, guaranteeing itself a higher return. Similarly, we can assume that $S_L \leqslant S_S$, because otherwise the institution could always threaten to force liquidation. Because $L < X_S < X_R$, neither condition is ever binding on the set of feasible contracts.

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[5] If the institution does not monitor, it can choose only a completely unconditional strategy for the firm. Assumption 2(a) immediately shows that the institution could not possibly break even on the required investment I. But for the institution to be willing to randomize between monitoring and not monitoring, it must be indifferent between its payoffs under the different choices. Thus, its expected payoff from monitoring would have to be the same as that from not monitoring, so it would never break even.

First, suppose that at date 1 the firm is in the bad state. The institution observes this regardless of whether it has monitored passively or actively. We have the following result (unless otherwise noted, all proofs of this and subsequent results are given in the appendix).

Lemma 1 (*liquidation at date 1*). *Suppose that at date* 1 *the firm is in the bad state. Under any contract that meets the financing constraint Eq.* (4), *the institution exercises control and forces liquidation.*

Intuitively, if the institution lets the bad firm continue, it get zero for sure. Liquidation allows the institution to get a positive payment S_L.

Now suppose that the firm is in the good state. If the institution has monitored passively, this is all that it knows. If the institution has monitored actively, it can learn the firm's substate by incurring an additional cost m_g. If it chooses not to incur m_g, the active institution has the same information as a passive institution. Because $m_A > m_P$, this strategy is strictly dominated by passive monitoring, so the institution would not choose active monitoring in the first place. Therefore, we can focus on the case in which an active institution also incurs m_g at date 1 if the firm is good and learns whether the firm's substate is medium or high. This leads to our next result.

Lemma 2 (*continuation at date 1*). *Suppose that at date* 1 *the firm is in the good state. Under any contract that meets the financing constraint Eq.* (4), *the outcome is as follows.*

1. *If the institution does not exercise control, then the manager chooses the risky strategy.*
2. *The institution exercises control to force the safe strategy if, and only if, $S_S > bS_R$, where $b = q$ if the institution has monitored passively and $b = p_i$ if the institution has monitored actively and the substate is $i \in \{m, h\}$.*
3. *If the institution exercises control, the manager successfully renegotiates to choose the risky strategy if, and only if, $S_S \leqslant bX_R$, where b is defined in* 2 *above. Such renegotiation gives the institution an expected payment of S_S. Renegotiation is always successful when the institution monitors actively and the substate is high ($b = p_h$).*

By Assumption 2(b), a contract that lets the institution break even cannot give the manager the incentive to voluntarily pick the safe strategy. Thus, the institution knows that if it does not exercise control, the manager chooses the risky strategy. In this case, the institution believes that it receives S_R with probability b, where b reflects the information that the institution has gained from monitoring. The institution prefers to exercise control to force the safe strategy if, and only if, its payment S_S exceeds bS_R.

It follows that if the institution decides to exercise control, the manager always wants to renegotiate to allow the risky strategy. Such renegotiation is successful whenever the manager can offer the institution a higher payment $S_R' = S_S/b$, which gives the institution the same expected payoff from the risky strategy that it would get from the safe strategy. (Because the manager has all the bargaining power, she claims all the gains from successful renegotiation.) This higher payment is feasible only if bX_R exceeds S_S.

Because $p_m < q < p_h$, Lemma 2 implies that the institution is most likely to exercise control and least likely to be susceptible to renegotiation when the institution has monitored actively and the firm is in the medium substate. Here, expected payments and cash flows from the risky action are lowest. When the institution has monitored actively and the firm is in the high substate, the institution is least likely to exercise control. Moreover, because $S_S \leqslant X_S < p_h X_R$ by Assumption 1, renegotiation is always successful in this case.

With the results of Lemmas 1 and 2, we can now write down the expected value of the firm $E_j(V)$ and expected payments to the institution $E_j(S)$ under the two possible monitoring levels. Let $V_S \equiv X_S + C_S$ denote the value of a good firm if it chooses the safe action at date 1. Let $V_{hR} \equiv p_h X_R + C_R$ denote the value of a high firm if it chooses the risky action at date 1. Finally, let $V_R \equiv q X_R + C_R$ denote the value of a good firm if it chooses the risky action at date 1 in both substates.

First, if the institution has monitored passively, then

$$E_{P,j}(V) = \begin{cases} (1-\theta)L + \theta V_R & \text{if } qX_R \geqslant S_S, \\ (1-\theta)L + \theta V_S & \text{otherwise,} \end{cases} \tag{5}$$

and $(1 + r_j)E_{P,j}(S) = (1-\theta)S_L + \theta S_{g,P}$,

where $S_{g,P} = \max\{qS_R, S_S\}$. $\tag{6}$

If instead the institution is a venture capital fund and has monitored actively, then

$$E_{A,vc}(V) = \begin{cases} (1-\theta)L + \theta V_R & \text{if } p_m X_R \geqslant S_S, \\ (1-\theta)L + \theta((1-\phi)V_S + \phi V_{hR}) & \text{otherwise,} \end{cases} \tag{7}$$

and $(1 + r)E_{A,vc}(S) = (1-\theta)S_L + \theta S_{g,A}$, $\tag{8}$

where $S_{g,A} = (1-\phi)\max\{p_m S_R, S_S\} + \phi\max\{p_h S_R, S_S\}$. $\tag{9}$

The variables $S_{g,P}$ and $S_{g,A}$ denote the expected payoffs to the institution in the good state under passive monitoring and under active monitoring, respectively.

4. Expected liquidity costs

With the results of Section 3 in hand, we can now specify the banks's liquidity costs $\Lambda_b(S)$. We begin with a simple formulation and then discuss which features generalize to other possible liquidity settings. The critical feature is that these costs tend to fall as the bank's claim on the firm becomes flatter and safer.

We have already defined the frequency λ and severity β of the bank's liquidity needs. Suppose that there is also a small chance δ that the bank is erroneously thought to have liquidity needs when it does not. Then, as shown in Winton (2003) and the Appendix, so long as β is sufficiently large, there is a pooling equilibrium in which the bank seeks as much funds as possible whenever it has liquidity needs or whenever it is erroneously thought to have liquidity needs and it knows that the firm's state is bad. If the institution knows that the firm's state is good, it is unwilling to seek funds when it has no liquidity needs. Thus, the price the bank can get for issuing securities backed by its claim is an average of the expected value of its claim, $E_{P,b}(S)$, and the value of its claim in the bad state, S_L:

$$P_{\text{pool}} = \frac{\lambda E_{P,b}(S) + \delta(1-\theta)S_L}{\lambda + \delta(1-\theta)}. \tag{10}$$

Because the price is less than $E_{P,b}(S)$, there are liquidity costs $\Lambda_b(S)$. As discussed in the Appendix, these are given by

$$\begin{aligned} \Lambda_b(S) &= \frac{\lambda\beta}{1+\lambda\beta}[E_{P,b}(S) - P_{\text{pool}}] \\ &= \frac{\lambda\beta}{1+\lambda\beta} \cdot \frac{\delta(1-\theta)}{\lambda + \delta(1-\theta)}[E_{P,b}(S) - S_L] \equiv \xi[E_{P,b}(S) - S_L]. \end{aligned} \tag{11}$$

Note that $\xi < \lambda\beta/(1+\lambda\beta) < r/(1+r)$, where the last inequality follows from Assumption 3.

Again, the only requirement for this equilibrium is that the bank's liquidity needs must be sufficiently severe (β is large), so that the bank always wishes to seek as much funding as possible when it has liquidity needs. If the bank's liquidity needs are less severe, the bank becomes unwilling to pool when it has liquidity needs but knows that the firm is in the good state. In this case, a separating equilibrium of the sort analyzed in Winton (2003) results, in which the bank issues fewer securities the more valuable its claim on the firm is. This separating equilibrium has higher liquidity costs than the pooling equilibrium, but otherwise it has similar comparative statics. In particular, making the bank's claim flatter (reducing the spread between S_R or S_S and S_L) reduces liquidity costs and also makes it more likely that the cheaper pooling equilibrium is feasible.

The main points of our analysis require that liquidity costs are decreasing in the bank's overall exposure to the firm and are increasing in the spread between good-state and bad-state values of the bank's claim. It is easy to verify that other possible equilibria and liquidity costs have these properties. Because the pooling equilibrium's liquidity costs have a particularly simple form, we focus on this in what follows.

5. Equilibrium monitoring

In this section, we analyze the equilibrium outcomes under bank finance and venture capital finance. We examine the conditions under which bank finance and venture capital finance are feasible. Finally, we characterize optimal contract structures for both forms of financing. For bank finance, the optimal contract is debt; and for venture capital finance, the optimal contract usually has some equity-like features as well. Intuitively, active monitoring requires a minimum level of exposure to the firm's choice of continuation strategy, which typically involves an equity-like component to the venture capitalist's payments. These results form the basis for the manager's overall choice between bank finance and venture capital finance, which we deal with in Section 6. There we show that whenever venture capital finance is preferred, convertible debt is an optimal contract.

5.1. Equilibria with bank finance

We begin with passively monitored finance from the bank. The bank does not observe a good firm's substate, so it can enforce only a continuation strategy that does not depend on this substate. It follows that under passive monitoring there are two possible outcomes. Let P1 be the outcome in which the bank (possibly after renegotiation) allows the good firm's manager to choose the risky strategy, and let P2 be the outcome in which the bank forces the good firm's manager to choose the safe strategy. We denote the manager's utility under outcome P1 as $U_{M,P1}(S)$, and the manager's utility under outcome P2 as $U_{M,P2}(S)$. These outcomes can occur only when the following constraint is satisfied:

$$E_{P,b}(S) - \Lambda_b(S) - m_P = (1 - \xi)E_{P,b}(S) + \xi S_L - m_P \geq I. \tag{12}$$

Eq. (12) is the bank's financing constraint. Passive monitoring must let the bank break even on the required investment I, its monitoring cost m_P, and its liquidity cost $\Lambda_b(S)$.

Lemma 2 provides the key to which outcome occurs. If a contract (S_L, S_S, S_R) satisfies Eq. (12) and

$$qX_R \geq S_S, \tag{13}$$

then the outcome is P1. In this case, the manager can and will successfully renegotiate if the bank tries to force the safe strategy. By contrast, when Eq. (13) does not hold, such renegotiation is impossible. Because $qS_R \leq qX_R < S_S$, the bank in fact forces the safe strategy, and the outcome is P2.

Lemma 5 in the Appendix details when these outcomes are feasible. Intuitively, outcome P1 requires that letting the good firm choose the risky strategy allows the bank to break even. Outcome P2 requires two conditions: a break-even condition like that for P1 and the condition that $qX_R < X_S$. Without this second condition, any feasible payment S_S satisfies the renegotiation constraint Eq. (13), so the safe strategy cannot be forced in equilibrium.

Out of the contracts that can implement outcome P1, the manager chooses a contract that maximizes her expected utility, $U_{M,P1}(S)$. We use the symbol $U^*_{M,P1}$ to denote the manager's optimal expected utility under this outcome. Similarly, we define the optimal expected utility for outcome P2 as $U^*_{M,P2}$. Our next result characterizes the optimal contracts that implement each outcome, and the optimal utilities under each of these outcomes.

Define

$$\hat{D} \equiv \frac{I + m_P - L}{\theta(1 - \xi)} + L. \tag{14}$$

Proposition 1 (*optimal contracts for implementing outcomes P1 and P2*).

1. *Suppose that $qX_R \geqslant \hat{D}$. Then outcome P1 is feasible. The optimal value of $U_{M,P1}(S)$ is*

$$U^*_{M,P1} = (1 - \theta)L + \theta V_R - \xi \cdot \frac{I + m_P - L}{1 - \xi} - (I + m_P). \tag{15}$$

   (a) *If $\hat{D} \leqslant X_S$, an optimal contract that implements this outcome is debt with face value $\hat{D}$: $S^*_L = L$, $S^*_S = S^*_R = \hat{D}$.*
   (b) *If $\hat{D} > X_S$, an optimal contract that implements this outcome is debt with face value $\hat{D}/q$: $S^*_L = L$, $S^*_S = X_S$, $S^*_R = \hat{D}/q$.*
2. *Suppose $X_S \geqslant \hat{D}$ and $qX_R < X_S$. Then outcome P2 is feasible.*
   (a) *If $\hat{D} > qX_R$, an optimal contract that implements outcome P2 is debt with face value $\hat{D}$: $S^*_L = L$, $S^*_S = S^*_R = \hat{D}$. The optimal value of $U_{M,P2}(S)$ is*

$$U^*_{M,P2} = (1 - \theta)L + \theta V_S - \xi \cdot \frac{I + m_P - L}{1 - \xi} - (I + m_P). \tag{16}$$

   (b) *If $\hat{D} \leqslant qX_R$, then there is no optimal contract that implements outcome P2. Also, outcome P1 is feasible and strictly dominates outcome P2.*

The feasibility conditions are derived from Lemma 5 in the Appendix. The optimal values for the manager's utility come about as follows. The manager's expected utility is $E_{P,b}(V) - \Lambda_b(S) - I - m_P$. For a given outcome P1 or P2, $E_{P,b}(V)$ is fixed. Thus, given the outcome, the manager's utility is maximized by minimizing the expected liquidity cost, $\Lambda_b(S)$, subject to feasibility. It follows that the optimal contract that can implement outcome P1 is one under which the bank's expected payment in the good state is $\hat{D}$ and its payment in the bad state is L. Under this contract, the financing constraint is binding, so the bank does not provide excess funds to the manager. Moreover, this contract maximizes the claim's worst-case value S_L and minimizes its best-case value $\max\{qS_R, S_S\}$. Because $\Lambda_b(S) = \xi(E_{P,b}(S) - S_L)$, this minimizes expected liquidity costs. If $\hat{D} \leqslant X_S$, an optimal contract is $(L, \hat{D}, \hat{D})$, which corresponds to a standard debt contract with face value $\hat{D}$. If $\hat{D} > X_S$, then an optimal contract is $(L, X_S, \hat{D}/q)$, which corresponds to a standard debt contract with face value $\hat{D}/q$. This accounts for part 1 of Proposition 1.

In part 2, the argument is the same, with one crucial difference: the debt face value must also exceed qX_R so as to prevent renegotiation. If $qX_R < \hat{D}$, this requirement is not a problem. An optimal contract is $(L, \hat{D}, \hat{D})$, the standard debt contract with face value $\hat{D}$. However, if $qX_R \geqslant \hat{D}$ as in part 2(b), then under any contract that satisfies the no-renegotiation constraint ($S_S > qX_R$), the financing constraint does not bind. This implies that there is no optimal contract, because for every S^*_S, it is possible to construct an alternative $S'_S = S^*_S - \varepsilon$, where $\varepsilon > 0$ is chosen such that the financing constraint is satisfied. Note that S'_S is feasible and strictly dominates S^*_S by having lower liquidity costs.

More important, however, if the situation in part 2(b) arises, then outcome P1 is feasible and strictly dominates outcome P2. The feasibility of P1 is immediate from $\hat{D} \leqslant qX_R$. To see why the manager prefers P1 to P2, recall that the manager's control benefits are higher under P1. Because P1 is feasible, the manager would choose P2 if, and only if, the cash flow under P2 was higher than the cash flow and incremental control benefits under P1. But then it would be possible to have the institution monitor passively and structure payments so that the manager voluntarily chooses the safe action in the medium substate, violating Assumption 2(b). Thus, P1 is chosen over P2.

5.2. Equilibria with venture capital finance

We now turn to equilibria with actively monitored finance from the venture capital fund. Under active monitoring, the venture capital fund knows not only the firm's state but also its substate. However, active monitoring is costlier than passive monitoring. For convenience, we define the incremental expected

discounted cost involved in active monitoring:

$$\Delta \equiv m_A + \frac{\theta m_g}{1+r} - m_P. \tag{17}$$

Note that Δ depends on θ and that $\Delta > 0$.

From Lemma 2 and Eq. (7), we can see that there are two possible outcomes with active monitoring. The manager always pursues the risky strategy in the high substate, but her actions in the medium substate can vary. Let A1 be the outcome in which the venture capital fund (possibly after renegotiation) allows the good firm's manager to choose the risky strategy in the medium substate. Let A2 be the outcome in which the venture capital fund forces the safe strategy in the medium substate. These outcomes can occur only when the venture capital fund's financing constraint

$$E_{A,vc}(S) - m_A - \frac{\theta m_g}{1+r} \geqslant I \tag{18}$$

and incentive compatibility constraint

$$E_{A,vc}(S) - E_{P,vc}(S) \geqslant \Delta \Rightarrow \theta(S_{g,A} - S_{g,P}) \geqslant (1+r)\Delta \tag{19}$$

are satisfied.

In Lemma 6 in the Appendix, we show that the incentive compatibility constraint Eq. (19) can be satisfied only if $p_m S_R < S_S < p_h S_R$. Otherwise, the venture capital fund either always forces the safe strategy on the good firm (if $S_S \geqslant p_h S_R$) or always allows a good firm's manager to choose the risky action (if $S_S \leqslant p_m S_R$), regardless of the substate. In either case, the expected payments from active monitoring equal the expected payments from passive monitoring, so the venture capital fund has no incentive to incur the added cost of active monitoring. Thus, we can rewrite $S_{g,A}$ as

$$S_{g,A} = (1 - \phi)S_S + \phi p_h S_R. \tag{20}$$

We now turn to detailed examination of the two outcomes. Outcome A1 is implemented by any contract (S_L, S_S, S_R) that satisfies Eqs. (18) and (19), along with the following renegotiation constraint:

$$p_m X_R \geqslant S_S. \tag{21}$$

Constraint Eq. (21) follows from Lemma 2(3). The manager of a medium firm must be able to get the institution to agree to the risky strategy.

Lemma 3 (*outcome A*1 *is never optimal*). *Under outcome A*1, *the manager's expected utility is* $U_{M,A1}(S) < U_{M,P1}(S) - \Delta$ *for every feasible contract S. Whenever outcome A*1 *is feasible, outcome P*1 *is feasible. Thus, outcome A*1 *is never chosen.*

Intuitively, outcome A1 achieves the same continuation strategy as outcome P1 (the risky action is always chosen), but it is more costly because it requires active instead of passive monitoring, and because, by Assumption 3, the venture capital fund's cost of capital is higher than the bank's liquidity cost. Because of these added costs, whenever A1 is feasible, P1 is feasible and strictly dominates A1. This implies that the manager never chooses a contract that implements outcome A1.

Turning to outcome A2, this outcome is implemented by any contract (S_L, S_S, S_R) that satisfies Eqs. (18) and (19), but does not meet Eq. (21). In other words, the manager of a medium firm cannot get the institution to accept the risky strategy. It follows that, under outcome A2, the manager's expected utility is given by

$$U_{M,A2}(S) = (1 - \theta)L + \theta[(1 - \phi)V_S + \phi V_{hR}] - rE_{A,vc}(S) - I - \frac{\theta m_g}{1+r} - m_A. \tag{22}$$

Define

$$\hat{X}_S = \min\left\{ X_S, p_h X_R - \frac{(1+r)\Delta}{\theta\phi} \right\} \tag{23}$$

and

$$\hat{X}_R = \min\left\{ X_R, \frac{1}{p_m}\left(X_S - \frac{(1+r)\Delta}{\theta(1-\phi)} \right) \right\}. \tag{24}$$

We have the following result on when outcome A2 is feasible.

Lemma 4 (*feasibility of outcome A2*). *Outcome A2 is feasible if, and only if, the following conditions are satisfied.*

1. $\frac{\theta\phi(1-\phi)(p_h-p_m)}{q(1+r)}\min\{X_S, qX_R\} \geqslant \Delta.$
2. $(1-\theta)L + \theta(1-\phi)\hat{X}_S + \theta\phi p_h\hat{X}_R \geqslant (1+r)(I+m_A) + \theta m_g.$

Condition 1 is a consequence of incentive compatibility. Condition 2 is needed to guarantee that the financing constraint is satisfied. $\hat{X}_S$ and $\hat{X}_R$ are bounds on payments under safe and risky actions that are caused by incentive compatibility. If S_S is too high relative to S_R, or vice versa, then passive monitoring and an unconditional choice of continuation strategy dominates active monitoring.

We can now derive optimal contracts for implementing outcome A2. Define

$$\hat{S}_L \equiv \min\left\{\frac{(1+r)(I+m_P) - \theta\min\{qX_R, X_S\}}{1-\theta}, L\right\} \tag{25}$$

and

$$\hat{S}_S = \frac{(1+r)(I+m_P) - (1-\theta)\hat{S}_L}{\theta}. \tag{26}$$

Note that Assumption 2(a), which states that I exceeds θqX_R and L, implies that $\widehat{S}_L$ and $\widehat{S}_S$ are positive.

Proposition 2 (*optimal contract for implementing outcome A2*). *Suppose outcome A2 is feasible. Then,*

1. *In any optimal contract that implements outcome A2, the financing constraint Eq.* (18) *strictly binds. An optimal contract that implements outcome A2 is* $S_L^* = \hat{S}_L$, $S_S^* = \min\{\hat{S}_S, X_S\}$ *and* $S_R^* = \frac{1}{\phi p_h}(\hat{S}_S + \frac{(1+r)\Delta}{\theta} - (1-\phi)S_S^*)$.
2. *The optimal value of $U_{M,A2}(S)$ is*

$$U_{M,A2}^* = (1-\theta)L + \theta((1-\phi)V_S + \phi V_{hR}) - (1+r)\left(I + m_A + \frac{\theta m_g}{1+r}\right). \tag{27}$$

3. *Pure debt can be an optimal contract for implementing outcome A2 only if* $(1-\theta)L + \theta X_S \leqslant (1+r)(I+m_P).$

An examination of Eq. (22) shows that an optimal contract is one that minimizes the venture capital fund's cost of capital, $rE_{A,vc}(S)$, subject to feasibility. So it follows that in any optimal contract, the financing constraint Eq. (18) must strictly bind. Otherwise, by decreasing S_L if $S_L > 0$, or by decreasing S_S and S_R, it is possible to lower $E_{A,vc}(S)$ without violating either the financing constraint or the incentive compatibility constraint.

Using this result, we construct an optimal contract in Proposition 2(1). If the financing constraint is to bind, the venture capital fund's payoff in the good state, $S_{g,A}$, must equal $((1+r)(I+m_A) + \theta m_g - (1-\theta)S_L)/\theta$. However, incentive compatibility requires that $S_{g,A}$ be sufficiently high to justify the higher cost of active monitoring. Setting $S_L = \hat{S}_L$, defined in Eq. (25), is sufficient to ensure that both the requirements (incentive compatibility and a strictly binding financing constraint) can be met.

If $\hat{S}_L = L$, then the optimal contract resembles convertible debt. The venture capital fund has strict seniority and receives all cash flows in liquidation but receives an additional payment that increases in the firm's final payoff, X. If $\hat{S}_L < L$, i.e., if $(1-\theta)L + \theta\min\{qX_R, X_S\} > (1+r)(I+m_P)$, then it is no longer true that the venture capital fund receives all the cash flows in liquidation. In this case, however, as we show in Section 6, outcome P1 is feasible and strictly dominates A2.

Can pure debt implement outcome A2? From Lemma 6 in the Appendix, we know that active monitoring requires that $S_R > S_S$. Therefore, the only way pure debt can induce active monitoring is if the face value of debt exceeds X_S. In the Appendix, we show that such a contract can be optimal only if

$(1 - \theta)L + \theta X_S \leqslant (1 + r)(I + m_P)$. Otherwise, the financing constraint does not strictly bind under any feasible debt contract.

It must be noted that these optimal contracts are not unique. They are the optimal contracts with the lowest risk (flattest payment profile) that is possible. Others with more risk are possible. Because the goal of our analysis is to show that venture capital contracts typically entail more exposure to the firm's risk, focusing on the lowest risk contracts that are feasible makes sense. Moreover, if the venture capital fund faces even a small chance of liquidity needs, then it (like the bank) prefers flatter contracts, all else equal.

6. Choice between bank and venture capital finance

Thus far, we have shown that there are three possible monitoring and contracting outcomes to consider. Two outcomes involve bank finance, and one involves venture capital finance. We now determine which of these three outcomes is in fact chosen by the manager, given the underlying parameters that govern the firm's contractable cash flows, the manager's control benefits, and the probabilities of the various states and substates. We then examine how the manager's choice varies with changes in these parameters.

We show that the manager generally prefers bank finance. Venture capital finance is only preferred when the average profitability of the firm's risky continuation strategy is not too high, the incremental cost of active monitoring is not too high, and the firm's strategic uncertainty is high. When venture capital is preferred, convertible debt or very risky debt is an optimal contract. Moreover, the conditions for venture capital finance to be optimal are most likely to hold when the risk and skewness of the firm's cash flow distribution are high or when the firm's liquidation value is low. These circumstances are consistent with those of firms that receive venture capital finance.

We also examine the impact of changes in the venture capital fund's cost of capital or the bank's liquidity needs. Not surprisingly, increases in the fund's cost of capital make venture capital finance less attractive. By contrast, more severe liquidity needs make bank interest rates higher, making bank finance less attractive. Because different actions are chosen under the two financing sources, these changes have implications for overall levels of risk and innovation in the economy.

6.1. Optimal outcomes

We now examine the circumstances under which each outcome is the optimal choice. Our first result is that outcome P1 is in fact optimal whenever it is feasible.

Proposition 3. *The manager chooses P1 whenever it is feasible, i.e., when* $qX_R \geqslant \hat{D}$.

In Proposition 1, part 2(b), we show that when P1 is feasible, the manager strictly prefers it over P2. A similar argument shows that P1 is also preferred over A2. Because the manager's control benefits are higher under P1, the manager would choose A2 if, and only if, the cash flow under A2 was higher than the cash flow and incremental control benefits under P1. But then it would be possible to have the institution monitor passively and structure payments so that the manager voluntarily chooses the safe strategy in the medium substate, violating Assumption 2(b). Thus, if P1 is feasible, it is always preferred to A2.

Proposition 3 implies that the manager chooses P2 or A2 if, and only if P1, is not feasible. We now show that A2 is the optimal outcome only in restricted circumstances.

Proposition 4 (*optimality of outcome A2*).

1. *Outcome A2 is optimal if, and only if, the following conditions hold.*
    (a) *Outcome P1 is not feasible*:

$$qX_R < \hat{D}. \tag{28}$$

(b) *Outcome A2 is feasible*:

$$\Delta \leqslant \frac{\theta\phi(1-\phi)(p_h - p_m)}{(1+r)q}\min\{X_S, qX_R\}, \tag{29}$$

and

$$(1-\theta)L + \theta(1-\phi)\hat{X}_S + \theta\phi p_h\hat{X}_R \geqslant (1+r)(I+m_A) + \theta m_g. \tag{30}$$

(c) *Either P2 is not feasible*,

$$X_S < \hat{D}, \tag{31}$$

or the manager prefers outcome A2 to outcome P2,

$$(1+r)\Delta + r(I+m_P) - \xi \cdot \frac{I+m_P - L}{1-\xi} < \theta\phi(V_{hR} - V_S). \tag{32}$$

2. *Whenever outcome A2 is optimal*, $\hat{S}_L = L$, *i.e., the venture capital fund has strict seniority and receives all the cash flows under liquidation*.

An immediate corollary of Propositions 3 and 4 is as follows.

Corollary 1 (*optimality of outcome P2*). *Outcome P2 is optimal if, and only if*,

1. *Condition* (28) *holds*,
2. $X_S \geqslant \hat{D}$, *and*
3. *At least one of Eqs.* (29), (30), *or* (32) *is violated*.

Returning to Proposition 4, it is clear that outcome A2 is optimal only when several restrictive conditions are satisfied. Eq. (28) limits the average profitability of the risky strategy for a good firm. If instead this strategy is sufficiently profitable, then outcome P1 is feasible and the manager strictly prefers this over outcome A2. Similarly, Eq. (31) limits the profitability of the safe strategy so as to rule out outcome P2. Eq. (30) requires that outcome A2 must be sufficiently profitable.

Now consider Eq. (29). The left-hand side is the increase in monitoring costs from monitoring actively, not passively. The right-hand side is the gain in cash flows from monitoring actively. This gain is directly related to the uncertainty of the firm's risky strategy. Conditional on information revealed by active monitoring, the variance of cash flows under this strategy is $\phi(1-\phi)(p_h - p_m)^2 X_R^2$. Intuitively, the value of active monitoring is the value of the option to use better information to choose between the safe and risky strategies. As the conditional variance of risky cash flows increases, the value of this option increases, making active monitoring more attractive. This is consistent with the stylized fact that firms that receive venture capital finance typically exhibit higher strategic uncertainty than those that rely on bank finance.

Finally, Eq. (32) contrasts the relative costs and benefits of bank and venture capital finance. The left-hand side of Eq. (32) is the net cost of venture capital finance: higher monitoring costs (first term), and higher cost of capital compared with bank liquidity costs (second and third terms). On the right-hand side, we have the benefit of active monitoring. In the high substate, the manager is able to pursue the risky strategy instead of the safe strategy.

These conditions are in part 1 of the proposition. Part 2 shows that when venture capital finance is preferred, it is optimal to give the fund all cash flows in liquidation. Combined with the equity-like features of the contract, this means that convertible debt is optimal.

6.2. Comparative statics

Having established the conditions under which the three outcomes are optimal, we now turn to the question of how changes in the underlying parameters affect optimality. Our analysis focuses on several firm characteristics: the risk and skewness of cash flows, the level of collateral, and the strategic uncertainty of the firm's continuation strategy. In all cases, the circumstances under which venture capital is preferred correspond to the stylized facts describing actual venture capital targets. We also examine how changes in the bank's liquidity needs or the venture capitalist's required return affect optimal financing choice.

A well-known feature of venture capital targets is their highly skewed returns, with low chances of success and low liquidation values, but extremely high payoffs if the firm is successful. We can parameterize this type of skewness as follows.

Definition 1. For $\gamma > 1$, consider the transformation $(\theta, X_S, X_R, C_S, C_R, L) \rightarrow (\gamma\theta, X_S/\gamma, X_R/\gamma, C_S/\gamma, C_R/\gamma, ((1-\theta)/(1-\gamma\theta))L)$. We define this transformation as a mean-preserving decrease in the firm's risk and positive skewness.

Under this transformation, the expected value of the firm's cash flows and control benefits under any monitoring and liquidation or continuation strategy is unchanged, but success is more likely, liquidation value is higher, and cash flows and control benefits under continuation are lower. Thus, an increase in γ reduces the risk of the firm's returns. The spread between good and bad outcomes decreases. It also decreases the positive skewness of the firm's returns. Success is more likely but has a lower value, whereas bad outcomes are less likely but have a higher value. In other words, the upper tail of returns shrinks and receives higher probability weight.

Definition 2. Suppose p_h increases and p_m decreases so that $q = (1-\phi)p_m + \phi p_h$ is unchanged. We define this transformation as a mean-preserving increase in the firm's strategic uncertainty.

Again, this transformation does not alter the overall expected value of the risky continuation strategy, but it increases the conditional variance of this strategy across the high and medium substates.

Proposition 5 (*comparative statics*).

1. *A decrease in the firm's risk and positive skewness makes it less likely that venture capital finance is optimal.*
2. *An increase in the firm's strategic uncertainty makes it more likely that venture capital finance is optimal.*
3. *An increase in the venture capital fund's cost of capital r makes it less likely that venture capital is optimal.*
4. *An increase in the severity of the bank's liquidity needs β makes it more likely that venture capital is optimal.*
5. *An increase in the firm's liquidation value L makes all outcomes more likely to be feasible. Eqs.* (28) *and* (31) *are less likely to hold, whereas Eq.* (29) *is more likely to hold. It also makes it more likely that bank finance (outcome P2) is preferred to venture capital finance. Eq.* (32) *is less likely to hold.*

The intuition for part 1 of the proposition is as follows. A decrease in risk and positive skewness raises liquidation proceeds and the chance of success, lowering the face value of (passive) bank debt, $\hat{D}$. This makes it more likely that either outcome P1 or outcome P2 is feasible. The transformation leaves the firm's expected value under active or passive monitoring unchanged, but it reduces the bank's liquidity costs. This makes outcome P2 more attractive relative to outcome A2. Finally, because success is more likely, the firm is more likely to be allowed to continue. This makes active monitoring more costly. The institution is more likely to have to make the decision between safe and risky continuation strategies, which is when the additional monitoring cost m_g must be incurred. With higher monitoring costs, outcome A2 is less attractive and less likely to be feasible.

Part 2 of the proposition follows from the intuition given at the end of Section 6.1. Increasing the uncertainty of the risky strategy increases the option value of using better information to decide between the safe and risky strategies. This makes active monitoring and venture capital finance more attractive. Parts 3 and 4 of the proposition are straightforward. Increasing the expected cost of capital of either financing option makes the alternative financing option more attractive.

Part 5 focuses on the impact of the firm's liquidation value: as this is higher, any form of finance is more feasible, but the bank's liquidity costs drop because the firm's downside risk is lower. On net, higher

liquidation or collateral value tends to favor bank finance. This is consistent with the stylized fact that banks tend to focus on firms with more collateral. By contrast, venture capital funds focus on firms with low collateral and highly positively skewed cash flows, tying back to part 1 of the proposition as well.

These results also have implications for the overall risk of firms as the supply of capital shifts. Suppose that the venture capital fund's required return r decreases as a result of an increase in the supply of funds to venture capital. More firms choose venture capital finance, and they come from two sources: firms that would otherwise have not received any financing (none of the outcomes was feasible, but now outcome A2 is), and firms that would otherwise have received bank financing with the safe strategy (outcome P2). Note that firms that would otherwise have chosen bank finance and the risky strategy (outcome P1) continue to choose that outcome, because it is still feasible and thus preferred to outcome A2. The upshot is that more firms are financed, more firms choose venture capital finance over safe bank finance (outcome P2), and more firms choose the risky strategy at least part of the time.

6.3. Equilibrium map

In Proposition 5, we analyze the impact on the optimal outcome of changing an underlying parameter, holding all other parameters constant. To further illustrate these results, we turn to graphical analysis using a numerical example. Consider the following parameter values.

- $I = 1$, $r = 5\%$, $\xi = 0.03$, $m_P = 0.25$, $m_A = 0.3$, and $m_g = 0.3$.
- $p_h = 0.8\kappa$ and $p_m = 0.3 + 0.8\phi(1 - \kappa)/(1 - \phi)$, so $q = \phi p_h + (1 - \phi)p_m = 0.45$ for all κ. Parameter κ is a proxy for strategic uncertainty (see Definition 2).
- $\theta = 0.3\gamma$, $\phi = 0.3$, $X_S = 3/\gamma$, $X_R = 5/\gamma$, $C_S = 1/\gamma$, $C_R = 2.5/\gamma$, and $L = 0.56/(1 - 0.3\gamma)$, where $1/\gamma$ is a proxy for skewness (see Definition 1).

We first analyze the impact of varying the firm's skewness and strategic uncertainty on the optimal outcome, holding the average profitability of the risky action constant (i.e., holding q constant). To do this, we vary $1/\gamma$ in the interval [0.5, 2] and κ in the interval [0.75, 1] in discrete increments and solve for the equilibrium outcome for all the possible $(1/\gamma, \kappa)$ pairs. Note that κ cannot be lower than 0.75, because then $p_h X_R$ is lower than X_S, in violation of Assumption 1.

Fig. 1 shows the regions in which bank finance and venture capital finance are optimal. Consistent with our predictions, we find that venture capital finance (outcome A2) is optimal only if strategic uncertainty is high and the firm's cash flows are more positively skewed; otherwise, bank finance (outcome P2) is optimal. (We have deliberately chosen a low q such that outcome P1 is infeasible, because this allows us to focus on the
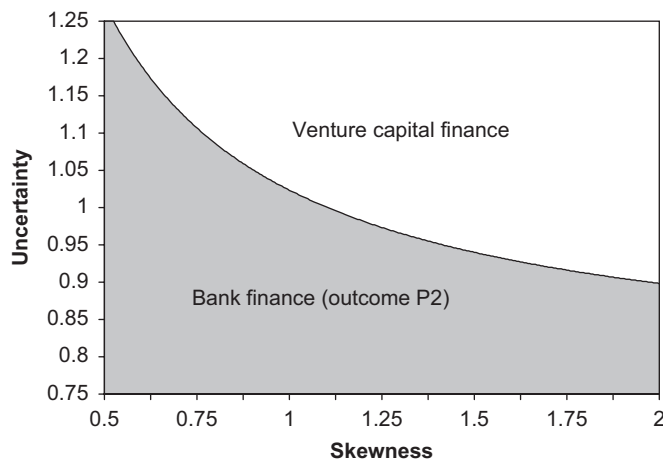

Fig. 1. Impact of skewness ($1/\gamma$) and strategic uncertainty (κ) on the equilibrium outcome.
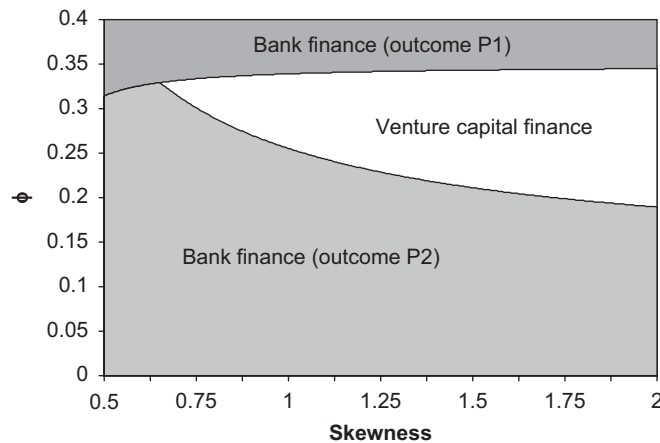
Fig. 2. Impact of skewness ($1/\gamma$) and ϕ on the equilibrium outcome.

interesting case of the firm's choice between outcomes P2 and A2. As we have shown in Proposition 3, P1 is optimal whenever it is feasible.)

Next, we analyze the impact of the firm's positive skewness ($1/\gamma$) and the parameter ϕ on the equilibrium outcome, holding κ constant at $\kappa = 1$. Recall that ϕ denotes the probability of a good firm being in the high substate and that the average profitability of the risky action, qX_R, is increasing in ϕ. As above, we vary $1/\gamma$ in the interval $[0.5, 2]$ and ϕ over the interval $[0, 0.4]$. Note that ϕ cannot exceed 0.4 if Assumption 2(b) is to met. Our findings are plotted in Fig. 2.

Consistent with our predictions, we find that when the risky action is highly profitable on average (high ϕ leading to high q), it is optimal for the bank to monitor passively and allow the manager of a good firm to choose the risky action (outcome P1), regardless of skewness. Similarly, if ϕ is very low, then, regardless of skewness, it is optimal for the bank to monitor passively and force the manager of a good firm to choose the safe action (outcome P2). Venture capital finance is optimal only when positive skewness is high, and ϕ is neither too high nor too low.

Thus, in this section we have shown that venture capital finance is only optimal under a restricted set of circumstances. The firm's average profitability cannot be too high, and its strategic uncertainty and the risk and positive skewness of its returns must be sufficiently high. Otherwise, bank finance is optimal. These conditions are consistent with the circumstances under which firms choose venture capital over bank finance, and vice versa.

7. Concluding remarks: implications and extensions

We suggest that venture capital differs from bank finance by greater use of equity features and by more active monitoring, particularly when the firm is choosing continuation strategies. We now discuss our model's implications for the choice between these two sources of finance, how our results compare with those of two related papers, and some extensions of our analysis.

One point that our model emphasizes is that, from the manager's viewpoint, monitoring is often a necessary evil. Thus, if the good firm's risky continuation strategy is lucrative enough on average (i.e., qX_R is high enough), the manager prefers passively monitored bank finance over actively monitored venture capital finance. In this case, liquidating only when the firm is an out-and-out failure is enough to allow the institution to break even on its investment. Venture capital only worsens matters. It is more costly both in terms of monitoring effort and cost of capital, and it limits the manager's benefits of control. Thus, firms with higher financial slack are less likely to prefer venture capital.

Even when the risky strategy is less profitable, on an ex ante basis the manager could prefer being forced to hew to the safe continuation strategy regardless, even though ex post she would prefer to be allowed to opt

sometimes for the risky strategy. Once again, debt finance from a bank does the job at low cost. It follows that venture capital is feasible and preferred only in a limited range of circumstances.

First, strategic uncertainty $\phi(1 - \phi)(p_h - p_m)X_R$ must be high. Intuitively, active monitoring adds most value when the possible outcomes of the risky strategy are most uncertain, because this increases the option value of choosing between risky and safe strategies. As evidence for this, Hellmann and Puri (2000) find that innovator firms are more likely to obtain venture capital financing than are imitator firms. Because strategic uncertainty is higher for newer products, this is consistent with our prediction.

Second, even if the firm is successful, the average profitability of the risky continuation strategy cannot be too high. If this strategy is very profitable, then if the firm pursues this strategy whenever it is allowed to continue, it has more than enough cash to allow the institution to recoup its investment. Because the manager gets higher control benefits from the risky strategy than from the safe strategy, she prefers this outcome, and she can implement it by going to the bank and obtaining debt finance.

A third point is borne out by our discussion of the impact of decreasing the firm's risk and positive skewness by simultaneously increasing the initial chance of success θ, reducing values in continuation strategies, and increasing value under liquidation. Such a transformation makes it more likely that passive monitoring is preferred. Conversely, it follows that active monitoring is most preferred for firms that are long shots; that is, firms with low liquidation values and low chances of success, but high values if and when success occurs. This long-shot aspect is increased by our result that q and thus ϕ, the chance that the successful firm does best under a risky continuation strategy, cannot be too high. These results accord well with the stylized facts of venture capital targets discussed in the Introduction.

Fourth, the firm's collateral value as proxied by its value in liquidation cannot be too high. This effect holds independently of the skewness result just mentioned. Increasing collateral value makes passive outcomes more feasible and reduces the bank's potential liquidity costs. Thus, venture capital targets are likely to have low collateral values.

Further implications follow from the effects of the venture capitalist's required return r and the severity of the bank's liquidity needs β. Consider the boom in entrepreneurial activity and interest in venture capital that occurred in the late 1990s. This lowered r, as investors were more willing to invest in venture capital despite its liquidity restrictions. At the same time, because of investors' increased focus on equity markets, banks had more trouble attracting funds at the same time as entrepreneurial demand for loans increased. These changes would tend to increase β, because banks would have increased needs for funds. Our model predicts that such changes should lead to an increase in venture capital finance relative to bank finance, which certainly seems to have been the case in the late 1990s.

A decrease in r also has implications for the overall economy. As discussed in Section 6, this not only increases the number of firms that are funded and the number of firms that receive venture capital finance, but also increases the number of firms in the economy that choose risky strategies at least part of the time. This can be beneficial, in that risky strategies could correspond to greater expansion of new products or more product innovation. There is also a downside to increased risk-taking. Firms could be in the high substate and yet subsequently find themselves with zero returns from the risky strategy. To the extent signals and outcomes are correlated across firms, this can lead to greater boom-bust behavior in the economy. Something of this sort could have happened in the 1990s, when initial prospects seemed good for risky expansion plans and IPOs, only to be followed by evidence that the good prospects had not materialized.

The two papers most closely related to ours are Schmidt (2003) and Hellmann (2006). Although neither addresses the choice between venture capital and bank finance per se, their results on venture capital contracting make implicit statements about when venture capital is most useful. In both papers, the driving force behind the equity feature of venture capital is the need to provide the manager and the venture capitalist with incentives to work hard in the highest states of the world. The more likely such states are, the more value is created by using venture capital finance.

By contrast, our paper focuses on ex ante incentives (that is, effort that is chosen before the firm's situation is known) as well as effort that takes place in relatively good states. Also, the financing institution's effort is directed at monitoring firm choices instead of improving outcomes per se. Although the perspective of Schmidt and Hellmann undoubtedly has some validity, venture capitalists spend time and effort throughout the financing process, not merely after a firm's true situation is known. Moreover, our perspective leads

directly to the prediction that firms with a higher chance of good outcomes will use less informationally intensive financing from banks or other intermediaries (outcome P1) instead of more informationally intensive financing from venture capital funds (outcome A2). Venture capital finance is used only when the firm's returns are highly risky and skewed, with good outcomes being unlikely, which is in direct contrast with the prediction in Schmidt (2003) and Hellmann (2006). This matches the situations in which venture capital is used.

A further distinction between our models concerns the role of banks. Neither Schmidt nor Hellmann models banks explicitly. We do and note that the potential liquidity shocks that banks face make them provide debt finance to relatively safe or profitable firms, particularly those with high collateral. Thus, banks complement venture capital funds.

A final contrast between their models and ours is that we explicitly incorporate venture capital funds' higher cost of capital in relation to banks. This allows us to make predictions about how changes in the supply of funds to venture capital affect the prevalence of venture capital and how this in turn affects the risk of entrepreneurial firms.

In our analysis, we have assumed that continuation strategies do not require additional investments. If this is not the case, then the institution can exercise control by threatening to refuse to fund the continuation. It is easy to show that our qualitative results go through even in this more complex setting. Debt is optimal for bank finance, greater cash flow risk favors venture capital finance, and so forth. Nevertheless, one interesting change appears. Suppose we parameterize additional investments and cash flows so that the net expected cash flow to the safe and risky strategies is unchanged from our basic setting. It is easy to show that venture capital finance is now more likely to be optimal. Intuitively, the additional required investment increases the operating leverage of the risky strategy, making its return variance higher. This increases the value of the option to choose between the safe and risky strategies, making venture capital finance more attractive.

Looking to future research, several possible extensions suggest themselves. The first is the link between our model and recent work suggesting that entrepreneurs are irrationally optimistic (see, for example, Manove and Padilla, 1999). If entrepreneurs put too high a probability on states with higher cash flows but financial institutions do not, our basic results should be reinforced. All else equal, the entrepreneur prefers to give cash flows in bad states of the world to the institution, reserving more of the cash flows in good states for herself. Thus, debt should still be optimal unless it is necessary to give the institution incentive to monitor actively, in which case a share in the firm's upside should be necessary. Still, incorporating irrational optimism might lead to additional predictions on the circumstances under which different contracting structures are preferred. For example, Landier and Thesmar (2003) examine entrepreneurs' choice between short- and long-term debt and find evidence consistent with the theoretical effects of excessive optimism.

Second, although thus far we have interpreted passive monitoring and debt as commercial bank debt, in reality, other financial institutions often make loans to privately held firms. Finance companies often extend shorter-term loans on a heavily collateralized basis. The firms they lend to are riskier than those that banks lend to, and there is anecdotal evidence that finance companies monitor more intensively than banks (see Carey, Post, and Sharpe, 1998). Life insurers invest in privately placed bonds and monitor somewhat less intensively than banks (see Carey, Prowse, Rea, and Udell, 1993). In all cases, these institutions focus their monitoring on avoiding bad outcomes, but the intensity varies. Extending our model to allow for such different types of monitoring should let us examine the circumstances under which these different lending arrangements are optimal.

Appendix

Proof of Lemma 1. If the institution does not force liquidation in the bad state, it gets zero in that state, so the most it can receive in expected payments is $\theta \max\{qX_R, X_S\}/(1+r_j) < I$ by Assumption 2(a). This means the financing constraint Eq. (4) cannot hold. Thus, it must be the case that the institution gets $S_L > 0$, and so it forces liquidation in the bad state.   □

Proof of Lemma 2. (1) We first show, by contradiction, that if the institution does not exercise control, the manager chooses action a_R. If in substate i the manager chooses a_S, then we must have

$C_R + p_i(X_R - S_R) \leqslant C_S + X_S - S_S$. Because $p_m < p_h$, this holds for $i = m$ if it holds at all. Accordingly, suppose the manager chooses a_S for $i = m$.

Consider the case in which the institution monitors actively with probability α. For the financing constraint Eq. (4) to hold, we must have $I + \alpha(m_A + \theta m_g/(1 + r_j)) + (1 - \alpha)m_P \leqslant ((1 - \theta)S_L + \theta(1 - \phi)S_S + \theta\phi A)/(1 + r_j)$, where A equals S_S if the manager chooses a_S when $i = h$ and $p_h S_R$ otherwise. (Because the manager has all bargaining power in renegotiation, the institution never gets more than its promised payment under a given action, even if it agrees to a different action.) Because $S_L \leqslant L$, $S_S \leqslant X_S < p_h X_R$ (by Assumption 1), $S_R \leqslant X_R$, $m_A + \theta m_g/(1 + r_j) > m_P$, and $r_j \geqslant 0$, it follows that $I + m_P \leqslant (1 - \theta)L + \theta(1 - \phi)S_S + \theta\phi p_h X_R$. Combining this with Assumption 2(b) and canceling common terms, we have $S_S > X_S - (C_R - C_S) \Rightarrow C_R > C_S + X_S - S_S$. Adding $p_m(X_R - S_R)$ to the left-hand side gives $C_R + p_m(X_R - S_R) > C_S + X_S - S_S$, which contradicts the assumption that the manager is willing to choose a_S for $i = m$.

(2) It follows from (1) that if the institution does not exercise control, its payoff is bS_R. If it exercises control to choose action a_S, its payoff is S_S. So the institution exercises control if, and only if, $S_S > bS_R$.

(3) We next show that if the institution exercises control, the manager renegotiates if, and only if, $S_S \leqslant bX_R$. *Proof of necessity*: The manager can renegotiate with the institution to choose action a_R by offering a payment S_S/b to the institution when output X_R is realized. The expected payoff to the institution if it accepts this offer is S_S. Because the institution is indifferent between accepting and rejecting the offer, we assume that it accepts the offer. Limited liability requires that $S_S/b \leqslant X_R$, so $S_S \leqslant bX_R$.

Proof of sufficiency: Suppose $S_S \leqslant bX_R$. We have already shown in (1) that $C_R > C_S + X_S - S_S$ if Eq. (4) holds, so $C_R + p_i(X_R - \frac{S_S}{b}) > C_S + X_S - S_S$, i.e., the manager is strictly better off renegotiating.

Finally, when $b = p_h$, $bX_R = p_h X_R > X_S$ by Assumption 1. Because $X_S \geqslant S_S$, renegotiation is always successful in this case.   □

Proof of liquidity cost formulation.   First, we conjecture the following equilibrium behavior. If the bank has a liquidity need, it issues 100% equity financing in its claim on the firm. If the bank is erroneously thought to have a liquidity need, it issues such financing if it knows that the firm is in the bad state and does not issue any financing if the firm is in the good state. If this equilibrium holds, then it easily follows that the price of the equity is P_{pool} as given in the text.[6]

For this to be an equilibrium, the bank must prefer to act in this way. Consider the case of a bank with liquidity needs that knows the firm is good. If the bank issues equity now, it gets a total value of $(1 + \beta)P_{\text{pool}}$. If it does not issue equity, it gets an expected value of $\max\{qS_R, S_S\}$. So long as β is sufficiently high, it prefers to issue equity. The same is also true if the bank has liquidity needs and its information about the firm's value is less favorable. Because $\max\{qS_R, S_S\} > P_{\text{pool}} > S_L$, when the bank is erroneously thought to have liquidity needs, it wishes to issue equity when it knows that the firm is in the bad state, but it does not wish to issue equity when it knows that the firm is in the good state. (One must also show that out of equilibrium beliefs support this equilibrium. The proof follows that of Winton, 2003, and is omitted.)

The derivation of expected liquidity costs follows along the lines given in Winton (2003). For the institution to be willing to invest a dollar today, it must receive an expected return of $1 + \lambda\beta$ so as to compensate for expected liquidity needs. Thus, the present value of future expected cash flows of one dollar is $(1 + \lambda\beta)^{-1}$. Calculating expected cash flows (including liquidity benefits) under the conjectured equilibrium and discounting gives the formula for $\Lambda_b(S)$ given in the text.   □

Lemma 5 (*feasibility of bank financing*).   *Consider equilibria in which the firm obtains financing from a bank.*

1. *An equilibrium in which the good firm is allowed to choose the risky strategy (outcome $P1$) can be implemented if, and only if, $(1 - \xi)((1 - \theta)L + \theta qX_R) + \xi L \geqslant I + m_P$. In this equilibrium, the manager's expected utility is given by*

$$U_{M,P1}(S) = (1 - \theta)L + \theta V_R - \Lambda_b(S) - m_P - I. \tag{33}$$

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[6]As shown in Winton (2003), if the bank is not thought to have liquidity needs, then any equity issue is viewed as coming from a bank that knows the firm is in the bad state.

2. *An equilibrium in which the good firm is forced to follow the safe strategy (outcome P2) can be implemented if, and only if, $qX_R < X_S$ and $(1 - \xi)((1 - \theta)L + \theta X_S) + \xi L \geq I + m_P$. In this equilibrium, the manager's expected utility is given by*

$$U_{M,P2}(S) = (1 - \theta)L + \theta V_S - \Lambda_b(S) - m_P - I. \tag{34}$$

Proof of Lemma 5. In both parts 1 and 2, the formula for the manager's expected utility follows directly from Eq. (5).

(1) *Proof of necessity*: Suppose outcome P1 is feasible. The feasibility condition in the lemma is obtained by substituting $S_L \leq L$, $S_S \leq qX_R$ (by the renegotiation constraint), and $S_R \leq X_R$ in the financing constraint Eq. (12).

Proof of sufficiency: Suppose $(1 - \xi)((1 - \theta)L + \theta qX_R) + \xi L \geq I + m_P$. Then a contract with $S_L = L$ and $\max\{S_S, qS_R\} = qX_R$ implements outcome P1. (If $qX_R \leq X_S$, $\max\{S_S, qS_R\} = qX_R$ can be achieved by setting $S_S = S_R = qX_R$. Otherwise, if $qX_R > X_S$, this can be achieved by setting $S_S = X_S$ and $S_R = X_R$.) Such a contract satisfies limited liability, the renegotiation constraint (because $S_S \leq qX_R$), and the financing constraint (by construction).

(2) *Proof of necessity*: If the safe outcome is to be forced, we must have $qX_R < S_S \Rightarrow qX_R < X_S$. The proof of necessity for the other condition is similar to that in part 1 above.

Proof of sufficiency: Suppose the conditions in part 2 of the lemma hold. Then following the same lines as the proof of part 1 above, it can be shown that the contract (L, X_S, X_S) implements outcome P2. $\square$

Proof of Proposition 1. An optimal contract is one that is feasible and maximizes the manager's expected utility.

(1) Substituting for $\hat{D}$ in the feasibility condition in Lemma 5(1) and rearranging yields the feasibility condition in Lemma 1(1). Under outcome P1, the manager's utility, after substituting for $\Lambda_b(S)$, is

$$U_{M,P1}(S) = (1 - \theta)L + \theta V_R - \xi\theta(\max\{qS_R, S_S\} - S_L) - m_P - I. \tag{35}$$

Because $U_{M,P1}$ is increasing in S_L and decreasing in $\max\{qS_R, S_S\}$, the optimal contract has $S_L^* = L$ and the lowest possible $\max\{qS_R^*, S_S^*\}$ such that the financing constraint binds. Substituting $S_L = L$ into the financing constraint Eq. (12), and forcing the constraint to bind, we obtain $\max\{qS_R^*, S_S^*\} = \hat{D}$ as defined in Eq. (14).

If $\hat{D} \leq X_S$, $\max\{qS_R^*, S_S^*\} = \hat{D}$ can be achieved by setting $S_S^* = S_R^* = \hat{D}$. If $\hat{D} > X_S$, limited liability does not allow $S_S = \hat{D}$. In this case, $\max\{qS_R^*, S_S^*\} = \hat{D}$ can be achieved by setting $S_S^* = X_S$ and $S_R^* = \hat{D}/q$.

Finally, substituting $\max\{qS_R^*, S_S^*\} = \hat{D}$ and $S_L^* = L$ into the formula for $U_{M,P1}$ yields Eq. (15) in the text.

(2) Substituting for $\hat{D}$ in the feasibility condition in Lemma 5(2) and rearranging yields the feasibility condition in Lemma 1(2). The manager's utility is

$$U_{M,P2}(S) = (1 - \theta)L + \theta V_S - \xi\theta(\max\{qS_R, S_S\} - S_L) - m_P - I. \tag{36}$$

(a) $qX_R < \hat{D}$: It is straightforward to show that $S_L^* = L$ and $S_S^* = S_R^* = \hat{D}$ maximizes utility subject to the financing constraint and meets the renegotiation constraint $S_S > qX_R$. Substituting these payments into the formula for $U_{M,P2}$ yields Eq. (16) in the text.

(b) $qX_R \geq \hat{D}$: Because $S_S > qX_R$ must be satisfied to rule out renegotiation, the financing constraint is not binding. Thus, there is no optimal contract that implements outcome P2. For every S_S^* that is feasible, there exists a $S_S' = S_S^* - \varepsilon$ (for a sufficiently small $\varepsilon > 0$) that is feasible and strictly dominates S_S^*.

Because $qX_R \geq \hat{D}$, P1 is feasible. To prove that P1 also strictly dominates P2, we first note that $(1 - \theta)L + \theta qX_R > (1 - \xi)((1 - \theta)L + \theta qX_R) + \xi L$, which in turn exceeds $I + m_P$ by Lemma 5(1). Combining this with Assumption 2(b) and canceling common terms, we obtain $p_m X_R + C_R > X_S + C_S$. Because $p_m X_R < qX_R$, it follows that $qX_R + C_R > X_S + C_S$; i.e., $V_R > V_S$, so $U_{M,P1}^* > U_{M,P2}^*$. $\square$

Lemma 6. *Outcomes A1 and A2 can be implemented only by contracts satisfying $p_m S_R < S_S < p_h S_R$.*

Proof of Lemma 6. If $S_S \geq p_h S_R$ or if $S_S \leq p_m S_R$, the incentive compatibility constraint Eq. (19) reduces to $\Delta \leq 0$, which is false. Thus, $p_m S_R < S_S < p_h S_R$. $\square$

Proof of Lemma 3. (i) First, note that, in outcome A1, the manager's utility is

$$U_{M,A1}(S) = (1-\theta)L + \theta V_R - m_A - \frac{\theta m_g}{1+r} - rE_{A,vc}(S) - I. \tag{37}$$

Comparing Eqs. (33) and (37), and substituting $\Lambda_b(S) = \xi(E_{P,b}(S) - S_L)$, we obtain

$$U_{M,P1}(S) - U_{M,A1}(S) = \Delta + (rE_{A,vc}(S) - \xi E_{P,b}(S)) + \xi S_L. \tag{38}$$

Now, because $r > \lambda\beta \Rightarrow \frac{r}{1+r} > \xi$, and $S_{g,A} - S_{g,P} > 0$ (by incentive compatibility), it follows that

$$rE_{A,vc}(S) - \xi E_{P,b}(S) = \frac{r}{1+r}((1-\theta)L + \theta S_{g,A}) - \xi((1-\theta)L + \theta S_{g,P})$$
$$> \xi\theta(S_{g,A} - S_{g,P}) > 0. \tag{39}$$

Substituting this in Eq. (38), we obtain that $U_{M,P1}(S) - U_{M,A1}(S) > \Delta$.

(ii) Suppose outcome A1 is feasible. Substituting $S_L \leqslant L$, $S_R \leqslant X_R$, and $S_S \leqslant p_m X_R$ (by the renegotiation constraint) into the financing constraint Eq. (18) yields

$$\frac{(1-\theta)L + \theta q X_R}{1+r} \geqslant (I + m_A) + \frac{\theta m_g}{1+r} = (I + m_P) + \Delta. \tag{40}$$

Next, Lemma 5 states that P1 is feasible if $(1-\xi)((1-\theta)L + \theta q X_R) + \xi L \geqslant I + m_P$. Now, $\lambda\beta < r$ (Assumption 3) implies that $\xi < r/(1+r) \Rightarrow 1 - \xi > (1+r)^{-1}$. Combining this with Eq. (40),

$$(1-\xi)((1-\theta)L + \theta q X_R) > \frac{(1-\theta)L + \theta q X_R}{1+r} \geqslant I + m_P + \Delta > I + m_P. \tag{41}$$

Thus, outcome P1 is feasible whenever outcome A1 is feasible. By (i) above, $U_{M,P1}(S) > U_{M,A1}(S)$. The rest of the lemma follows immediately. $\square$

Proof of Lemma 4. *Proof of necessity*: Suppose outcome A2 is feasible.

(1) The IC constraint Eq. (19) implies that

$$\theta\phi(p_h S_R - S_S) \geqslant (1+r)\Delta, \text{ and} \tag{42}$$

$$\theta(1-\phi)(S_S - p_m S_R) \geqslant (1+r)\Delta. \tag{43}$$

Now, $(1-\phi)p_m * $ Eq. (42) $+\phi p_h * $ Eq. (43) implies that

$$(1+r)\Delta \leqslant \theta\phi(1-\phi)(p_h - p_m)\frac{S_S}{q} \leqslant \theta\phi(1-\phi)(p_h - p_m)\frac{X_S}{q}. \tag{44}$$

Similarly, $(1-\phi) * $ Eq. (43) $+ \phi * $ Eq. (42) implies that

$$(1+r)\Delta \leqslant \theta\phi(1-\phi)(p_h - p_m)S_R \leqslant \theta\phi(1-\phi)(p_h - p_m)X_R. \tag{45}$$

Combining the above two inequalities, we obtain 1 of the lemma.

(2) Rearranging Eq. (42), we obtain

$$S_S \leqslant p_h S_R - \frac{(1+r)\Delta}{\theta\phi} \leqslant p_h X_R - \frac{(1+r)\Delta}{\theta\phi}$$

$$\text{i.e., } S_S \leqslant \hat{X}_S = \min\left\{X_S, p_h X_R - \frac{(1+r)\Delta}{\theta\phi}\right\}, \text{ because } S_S \leqslant X_S. \tag{46}$$

By a similar logic, rearranging Eq. (43), we obtain

$$S_R \leqslant \hat{X}_R = \min\left\{X_R, \frac{1}{p_m}\left(X_S - \frac{(1+r)\Delta}{\theta(1-\phi)}\right)\right\}. \tag{47}$$

Condition 2 of the lemma is obtained by substituting $S_L \leqslant L$, $S_S \leqslant \hat{X}_S$, and $S_R \leqslant \hat{X}_R$ into the financing constraint Eq. (18).

Proof of sufficiency: Suppose conditions 1 and 2 hold. Consider the following two cases.

(a) Suppose $X_S > qX_R$. We show that the contract $(S_L = L, S_S = \hat{X}_S, S_R = X_R)$ implements outcome A2. Condition 2 of the lemma guarantees that the financing constraint Eq. (18) is satisfied.[7] Condition 1 of the lemma guarantees that $p_h X_R - (1+r)\Delta/\theta\phi > p_m X_R$. Combining this with the fact that $X_S > p_m X_R$, we obtain that $S_S = \hat{X}_S > p_m X_R$. So, Eq. (21) is violated as it should be.

Finally, we prove that the incentive compatibility constraint Eq. (19) is also met, i.e., that $\theta((1-\phi)\hat{X}_S + \phi p_h X_R - \max\{qX_R, \hat{X}_S\}) \geqslant (1+r)\Delta$. Condition 1 implies that $qX_R \leqslant p_h X_R - \Delta(1+r)/\theta\phi$ [otherwise, $(1+r)\Delta > \theta\phi(1-\phi)(p_h - p_m)X_R$, contradicting condition 1 of the lemma]. Combining this with our assumption that $qX_R < X_S$, we obtain $qX_R \leqslant \hat{X}_S$. Therefore, we need only show that $\theta\phi(p_h X_R - \hat{X}_S) \geqslant (1+r)\Delta$, i.e., that $\hat{X}_S \leqslant p_h X_R - \Delta(1+r)/\theta\phi$, which is true by the definition of $\hat{X}_S$.

(b) Suppose $X_S \leqslant qX_R$. We show that the contract $(S_L = L, S_S = X_S, S_R = \hat{X}_R)$ implements outcome A2. Condition 2 of the lemma guarantees that the financing constraint Eq. (18) holds. Condition 1 implies that $\frac{q}{p_m}(X_S - \Delta(1+r)/\theta(1-\phi)) \geqslant X_S$. Combining this with our assumption that $qX_R \geqslant X_S$, we obtain $q\hat{X}_R \geqslant X_S$. Therefore, to show that the IC constraint holds, we need only show that $\theta(1-\phi)(X_S - p_m\hat{X}_R) \geqslant (1+r)\Delta$, i.e., that $p_m\hat{X}_R \leqslant X_S - \Delta(1+r)/\theta(1-\phi)$, which is true by our definition of $\hat{X}_R$. $\square$

Proof of Proposition 2. Suppose outcome A2 is feasible. From the expression for $U_{M,A2}(S)$, it is evident that the problem is to minimize $E_{A,vc}(S)$, subject to the incentive compatibility constraint, the financing constraint

$$E_{A,vc}(S) \geqslant I + m_A + \frac{\theta m_g}{1+r} \qquad (48)$$

and the condition $S_S > p_m X_R$.

Step I: We show that the financing constraint Eq. (48) must strictly bind in any optimal contract (S_L^*, S_S^*, S_R^*). Suppose not, i.e., suppose $E_{A,vc}(S) > I + m_A + \theta m_g/(1+r)$. Then it is possible to construct an alternative contract (S_L', S_S', S_R') that has no impact on the IC constraint but tightens the financing constraint, thus dominating the original contract because $E_{A,vc}(S') < E_{A,vc}(S^*) \Rightarrow U_{M,A2}(S') > U_{M,A2}(S^*)$.

If $S_L^* > 0$, the alternative contract is $(S_L', S_S', S_R') = (S_L^* - z, S_S^*, S_R^*)$ for a sufficiently small $z > 0$. If $S_L^* = 0$, the construction of (S_L', S_S', S_R') depends on whether $S_S^* < qS_R^*$ or not.

(a) Suppose $S_S^* < qS_R^*$. The IC constraint simplifies to $\theta(1-\xi)(1-\phi)(S_S^* - p_m S_R^*) \geqslant \Delta$. So, the alternative contract is given by $S_L' = S_L^*$, $S_S' = S_S^* - p_m z$ and $S_R' = S_R^* - z$ for a sufficiently small $z > 0$.

(b) Suppose $S_S^* \geqslant qS_R^*$. The IC constraint simplifies to $\theta(1-\xi)\phi(p_h S_R^* - S_S^*) \geqslant \Delta$. So, the alternative contract is given by $S_L' = S_L^*$, $S_S' = S_S^* - p_h z$ and $S_R' = S_R^* - z$ for a sufficiently small $z > 0$.

Step II: We characterize an optimal contract for implementing outcome A2.

We show that the contract (S_L^*, S_S^*, S_R^*) given by $S_L^* = \hat{S}_L$, $S_S^* = \min\{\hat{S}_S, X_S\}$, and $S_R^* = (1/\phi p_h)(\hat{S}_S + (1+r)\Delta/\theta - (1-\phi)S_S^*)$ is an optimal contract. By construction, the financing constraint strictly binds. So, all we need to show is that this contract is feasible. It would then follow that it is an optimal contract. We now prove all the required conditions for feasibility of (S_L^*, S_S^*, S_R^*).

1. Proving that $S_S^* > p_m X_R$: Substituting $\hat{S}_L \leqslant [(1+r)(I + m_P) - \theta\min\{qX_R, X_S\}]/(1-\theta)$ in the definition of $\hat{S}_S$, we obtain that $\hat{S}_S \geqslant \min\{qX_R, X_S\} > p_m X_R$. Because $X_S > p_m X_R$, it follows that $S_S^* = \min\{\hat{S}_S, X_S\} > p_m X_R$.

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[7]Because S_L must not exceed S_S, we also need to show that $L \leqslant \hat{X}_S$. We prove, by contradiction, that this requirement is met. Suppose not, i.e, suppose $L > \hat{X}_S$. Because $L < X_S$, it must be that $L > p_h X_R - \Delta(1+r)/\theta\phi$. But consider the left-hand side of Condition 2 of the Lemma. Substituting $\hat{X}_S < L$ and $p_h X_R < L + \Delta(1+r)/\theta\phi$, and simplifying, we obtain $(1-\theta)L + \theta(1-\phi)\hat{X}_S + \theta\phi p_h X_R < L + \Delta(1+r) < (1+r)(I + m_A) + \theta m_g$. This contradicts Condition 2.

2. Proving that limited liability holds: It is obvious that $S_L^* \leqslant L$ and $S_S^* \leqslant X_S$. We prove, by contradiction, that $S_R^* \leqslant X_R$. Suppose $S_R^* > X_R$, then it must be that

$$(1 - \theta)\hat{S}_L + \theta(1 - \phi)S_S^* + \theta\phi p_h X_R < (1 + r)(I + m_A) + \theta m_g. \tag{49}$$

Consider the following cases.

(a) Suppose $\hat{S}_L = L$. Combining Eq. (49) with condition 2 in Lemma 4, we obtain

$$\theta(1 - \phi)S_S^* + \theta\phi p_h X_R < \theta(1 - \phi)\hat{X}_S + \theta\phi p_h \hat{X}_R, \tag{50}$$

which is easily shown to be a contradiction given that $\hat{X}_S \leqslant X_S$ and $\hat{X}_R \leqslant X_R$. If $S_S^* = X_S$, it would violate Eq. (50) because $\hat{X}_S \leqslant X_S$ and $\hat{X}_R \leqslant X_R$. If instead $S_S^* = \hat{S}_S \leqslant X_S$, then $S_R^* = \frac{1}{\phi p_h}(\phi S_S^* + (1 + r)\Delta/\theta)$. So $S_R^* > X_R \Rightarrow S_S^* > p_h X_R - (1 + r)\Delta/\theta\phi \geqslant \hat{X}_S$, because $\hat{X}_S = \min\{X_S, p_h X_R - (1 + r)\Delta/\theta\phi\}$. So once again, Eq. (50) is violated.

(b) Suppose $\hat{S}_L < L$. By the definition of $\hat{S}_L$, if $\hat{S}_L < L$, it must be that

$$(1 - \theta)\hat{S}_L + \theta\min\{qX_R, X_S\} = (1 + r)(I + m_P). \tag{51}$$

Subtracting Eq. (51) from Eq. (49), we obtain

$$(1 + r)\Delta > \theta((1 - \phi)S_S^* + \phi p_h X_R - \min\{qX_R, X_S\}). \tag{52}$$

Now, $\hat{S}_S \geqslant \min\{qX_R, X_S\}$ [as shown in (1) above] and $X_S \geqslant \min\{qX_R, X_S\}$ imply that $S_S^* = \min\{\hat{S}_S, X_S\} \geqslant \min\{qX_R, X_S\}$. Substituting this in the above inequality and simplifying, we obtain

$$\begin{aligned}(1 + r)\Delta &> \theta\phi(p_h X_R - \min\{qX_R, X_S\}) \\ &\geqslant \theta\phi(p_h X_R - qX_R) = \theta\phi(1 - \phi)(p_h - p_m)X_R,\end{aligned} \tag{53}$$

which violates condition 1 of Lemma 4.

From (a) and (b), it follows that $S_R^* \leqslant X_R$.

3. Proving that incentive compatibility holds: We need to show that $S_{g,A}^* - \max\{S_S^*, qS_R^*\} \geqslant (1 + r)\Delta/\theta$. Note that $S_{g,A}^* = (1 - \phi)S_S^* + \phi p_h S_R^* = \hat{S}_S + (1 + r)\Delta/\theta$, by construction. Because $S_S^* = \min\{\hat{S}_S, X_S\} \leqslant \hat{S}_S$, it follows that $S_{g,A}^* \geqslant S_S^* + (1 + r)\Delta/\theta$. So it only remains to be shown that $S_{g,A}^* - qS_R^* \geqslant (1 + r)\Delta/\theta$. Because $S_{g,A}^* = \hat{S}_S + (1 + r)\Delta/\theta$, this is equivalent to showing that $\hat{S}_S - qS_R^* \geqslant 0$. Substituting for S_R^*, and simplifying, we obtain

$$\begin{aligned}\hat{S}_S - qS_R^* &= \frac{(\phi p_h - q)\hat{S}_S + q(1 - \phi)S_S^*}{\phi p_h} - \frac{q(1 + r)\Delta}{\theta\phi p_h} \\ &\geqslant \frac{\phi(1 - \phi)(p_h - p_m)S_S^*}{\phi p_h} - \frac{q(1 + r)\Delta}{\theta\phi p_h}, \quad \text{because } \hat{S}_S \geqslant S_S^*.\end{aligned} \tag{54}$$

Now $S_S^* \geqslant \min\{X_S, qX_R\}$ as shown in (1) above. So $\phi(1 - \phi)(p_h - p_m)S_S^*/\phi p_h \geqslant \phi(1 - \phi)(p_h - p_m)\min\{X_S, qX_R\}/\phi p_h \geqslant q(1 + r)\Delta/\theta\phi p_h$, by condition 1 in Lemma 4. Substituting this in the above inequality, we obtain that $\hat{S}_S - qS_R^* \geqslant 0$.

From points 1, 2, and 3 above, it follows that the contract (S_L^*, S_S^*, S_R^*) is feasible. Therefore, it is an optimal contract for implementing outcome A2.

Step III. Because the financing constraint binds in an optimal contract, the expression for $U_{M,A2}^* = U_{M,A2}(S^*)$ is obtained by substituting $E_{A,vc}(S^*) = I + m_A + \theta m_g/(1 + r)$ in the expression for $U_{M,A2}(S)$.

Step IV. We show that pure debt can be an optimal contract for implementing outcome A2 only if $(1 - \theta)L + \theta X_S \leqslant (1 + r)(I + m_P)$.

A pure debt contract that implements outcome A2 must be of the form (L, X_S, S_R), where $X_S < S_R \leqslant X_R$.[8] With $S_S = X_S$, to satisfy the IC constraint, we must have $S_R \geqslant \frac{1}{p_h}((1 + r)\Delta/\theta\phi + X_S)$.

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[8] Pure debt with face value less than or equal to X_S, i.e., a contract of the form (L, D, D), where $D \leqslant X_S$, cannot implement outcome A2, as it would violate the constraint $p_h S_R > S_S$ that must be satisfied if incentive compatibility is to be met.

Recall that in any optimal contract, the financing constraint must strictly bind. So substituting $S_L = L$, $S_S = X_S$, and $S_R \geqslant (1/p_h)((1 + r)\Delta/\theta\phi + X_S)$ into the financing constraint, and simplifying, we must have $(1 - \theta)L + \theta X_S \leqslant (1 + r)(I + m_P)$. $\square$

Proof of Proposition 3. We show in the proof of Proposition 1(2.b) that, when P1 is feasible, it strictly dominates P2. Next, we show that, when P1 is feasible, $U^*_{M,P1} > U^*_{M,A2}$, i.e., the manager prefers outcome P1 to A2. Substituting for $U^*_{M,P1}$ and $U^*_{M,A2}$, we obtain

$$U^*_{M,P1} - U^*_{M,A2} = \theta(1 - \phi)(p_m X_R + C_R - X_S - C_S) + (1 + r)\Delta$$
$$+ (1 + r)(I + m_P) - \frac{I + m_P - \xi L}{1 - \xi}. \tag{55}$$

In the proof of Proposition 1(2.b), we show that, whenever P1 is feasible, $C_R + p_m X_R - X_S - C_S \geqslant 0$. Also, by Assumption 3, $1 + r > (1 - \xi)^{-1}$, so $(1 + r)(I + m_P) - \frac{I + m_P - \xi L}{1 - \xi} \geqslant 0$. Because $\Delta > 0$, it follows that $U^*_{M,P1} > U^*_{M,A2}$ whenever P1 is feasible. $\square$

Proof of Proposition 4. (1) (a) Proposition 3 reveals that A2 cannot be the optimal outcome if P1 is feasible. So P1 must be infeasible. The condition under which this happens is obtained from Lemma 5. (b) A2 can be the optimal outcome only if it is feasible, i.e., only when these two conditions (obtained from Lemma 4) are met. (c) Finally, A2 must strictly dominate P2. This requires either that P2 is infeasible [condition obtained from Lemma 5] or that $U^*_{M,A2} > U^*_{M,P2}$. Now, because $qX_R < \hat{D}$ [from Condition (i) of the proposition], $U^*_{M,P2} = (1 - \theta)L + \theta V_S - (I + m_P - \xi L)/(1 - \xi)$. Therefore, $U^*_{M,A2} > U^*_{M,P2}$ is equivalent to the following condition:

$$\theta\phi(V_{hR} - V_S) > (1 + r)\left(I + m_A + \frac{\theta m_g}{1 + r}\right) - \frac{I + m_P - \xi L}{1 - \xi}$$
$$= (1 + r)\Delta + (1 + r)(I + m_P) - \frac{I + m_P - \xi L}{1 - \xi}. \tag{56}$$

Simplification yields Eq. (32) in the text.

(2) Suppose A2 is the optimal outcome. Proposition 4(1) reveals that P1 must be infeasible, i.e., $(1 - \xi)((1 - \theta)L + \theta q X_R) + \xi L < I + m_P$. Because $1 - \xi > 1/(1 + r)$, we have $(1 - \theta)L + \theta q X_R < (1 + r)(I + m_P)$, i.e., $L < ((1 + r)(I + m_P) - \theta q X_R)/(1 - \theta)$. But then, from the definition of $\hat{S}_L$ in Eq. (25), it follows that $\hat{S}_L = L$. $\square$

Proof of Proposition 5. In the proof, we use the superscript 't' to denote the transformed parameters. So $\theta^t = \gamma\theta$, $X^t_S = X_S/\gamma$, and so on.

(1) Change in skewness, γ: (i) Consider Eq. (28), which can be rewritten as $\theta^t q X^t_R \leqslant \theta^t \hat{D}^t$. Now, $\theta^t q X^t_R = \theta q X_R$ remains unchanged as γ increases. However, $\theta^t \hat{D}^t = (I + m_P - \xi L^t)/(1 - \xi) - (1 - \theta^t)L^t$ decreases as γ increases, because $(1 - \theta^t)L^t = (1 - \theta)L$ remains unchanged, while L^t increases. Therefore, Eq. (28) is less likely to be met as γ increases.

(ii) By a similar reasoning as in (i), Eq. (31) is also less likely to be met as γ increases.

(iii) Next, note that $\Delta^t = m_A + \theta^t m_g/(1 + r) - m_P$ increases as γ increases (because θ^t increases). However, $\theta^t \hat{X}^t_S = \min\{\theta^t X^t_S, \theta^t p_h X^t_R - (1 + r)\Delta^t/\phi\}$ weakly decreases as γ increases because $\theta^t X^t_S = \theta X_S$ remains unchanged, and

$$\theta^t p_h X^t_R - \frac{(1 + r)\Delta^t}{\phi} = \theta p_h X_R - \frac{(1 + r)(m_A - m_P)}{\phi} - \frac{\gamma\theta m_g}{\phi} \tag{57}$$

decreases as γ increases. By a similar reasoning, $\theta^t \hat{X}^t_R = \min\{\theta^t X^t_R, (1/p_m)(\theta^t X^t_S - (1 + r)\Delta/(1 - \phi))\}$ weakly decreases as γ increases. Overall, this implies that Eqs. (29) and (30) are also less likely to be met as γ increases.

(iv) Finally, because Δ^t and L^t increase as γ increases, even Eq. (32) is less likely to be met as γ increases. So overall, A2 is less likely to be the optimal outcome as γ increases.

(2) An increase in strategic uncertainty has no impact on Eqs. (28) and (31), but it makes it more likely that Eqs. (29), (30) and (32) are met.

(3) An increase in r makes it less likely that Eqs. (29), (30) and (32) are met and has no impact on Eqs. (28) and (31).

(4) An increase in β makes it more likely that Eqs. (28), (31) and (32) are met and has no impact on Eqs. (29) and (30).

(5) The proof is obvious. □

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