# Moral hazard, hold-up, and the optimal allocation of control rights

Vijay Yerramilli*

*I examine the optimal allocation of control rights in a model with manager moral hazard, where the manager and investor may hold up each other* ex post. *The control allocation determines both the likelihood of hold-up and the agents' renegotiation payoffs. In equilibrium, only two control allocations are optimal: either exclusive investor control or a contingent control allocation that allows the manager to remain in control* if, and only if, *interim performance is good. Thus, my model explains why it may be optimal to link control to the firm's performance such that managers retain control only following good performance.*

## 1. Introduction

■ Financial contracts are inherently incomplete, and cannot specify every future investment decision that a firm must make. Given this, how should contracts allocate the right to make future decisions ("control right") between managers and investors? Aghion and Bolton (1992) address this question by arguing that the manager and the investor may have potentially conflicting objectives regarding the future decision, because the manager cares about both monetary returns and nonmonetary *private benefits* whereas the investor is only concerned about monetary returns. Therefore, control must be allocated such that the efficient decision plan is implemented *ex post*. One of their main results is that if neither monetary returns nor private benefits are comonotonic with total returns, then it may be optimal to specify a contingent control allocation in which control is assigned to the investor in states where maximizing monetary returns is efficient, and to the manager in states where private benefits are more important. Although this result is consistent with real-world contracts, it is based on very specific assumptions regarding the agents' utilities and cannot explain why investors take control only in bad states and control to managers in good states and never the other way a round.[1]

[1] As Hart (2000) notes, if we assume that a professional manager can run a successful start-up firm better than its founding entrepreneur, then as per Aghion and Bolton's analysis, it should be optimal to assign control to the investor in

In this article, I address the question of the optimal allocation of control rights in a simple setting with manager moral hazard and incomplete contracts, where the contract may be renegotiated after the manager has exerted costly effort and the investor has committed funds to the project. Renegotiation gives rise to a two-sided hold-up problem, in the sense that the party in control, investor or manager, can hold up the other party. The control allocation affects the manager's *ex ante* incentives because it affects both the likelihood of hold-up *ex post* and the renegotiation payoffs of the two agents. The optimal control allocation is the one that provides the strongest incentives to the manager but still satisfies the investor's participation constraint. The key contribution of my article is that it offers a theory of control rights based on first principles, without making any ad hoc assumptions about the manager's private benefits or future conflicts of interest. Moreover, it also explains why it may be optimal to link the control allocation to the firm's future performance, such that the investor takes control only following poor performance and leaves control to the manager if performance is good.

The basic set-up in my model is very similar to that in Rajan (1992). A manager with a project idea, but with no funds of her own, starts a firm by raising the necessary funds from an investor. The contract between the agents cannot specify the manager's effort and a key future investment decision, continuation versus liquidation, because the state of the firm ("good" or "bad") cannot be observed or verified by outside parties (Grossman and Hart, 1986; Hart and Moore, 1998). Given the firm's opacity and high *ex ante* probability of failure, new financing from outside investors is difficult to obtain if the original investor decides to withdraw from the firm. Thus, there is potential for hold-up once the investor has sunk in its funds and the manager has exerted costly effort. The investor may threaten liquidation even in the good state in order to extract a higher share of the continuation surplus, and the manager may refuse to liquidate the firm in the bad state unless she gets a share of the liquidation proceeds. A *key difference* from Rajan's model is that, in my model, the agents also observe a verifiable performance measure ("high" or "low") that is imperfectly correlated with the firm's state. Hence, the agents may choose to write contracts contingent on the noisy performance measure, as in Aghion and Bolton (1992).

The contract assigns the control right over the investment decision to either the manager or the investor, possibly contingent on the verifiable performance measure. The contract may specify exclusive manager control or exclusive investor control regardless of the firm's performance, or may specify a contingent control allocation under which control switches from one agent to another contingent on the realization of the performance measure. The contract may also specify any payoff rule, subject to the restriction that the manager is protected by limited liability if the venture fails (i.e., the lowest possible payoff to the manager in the event of liquidation is zero). I do not impose any limited liability or wealth constraints on the investor.

For the manager's incentives to be high, she must be rewarded when the venture succeeds and penalized when the venture fails. Therefore, any control allocation that allows the manager to remain in control following the low performance signal is strictly dominated by the contingent control allocation, under which control switches from the manager to the investor if the low performance signal is realized. Thus, in contrast with Rajan (1992) and Aghion and Bolton (1992), the manager control allocation (e.g., nonvoting equity, long-term debt) is never optimal in my model.

In equilibrium, only two control allocations can be optimal: either investor control or contingent control. The main advantage of contingent control over investor control is that it mitigates hold-up by the investor in the good state by allowing the manager to remain in control if the interim performance is high; its main drawback is that the manager may also obtain control in the bad state if the interim performance is high (which happens with positive probability) and can hold up the investor by refusing to liquidate the firm. I show that contingent control is more

---

the good state so that the investor can take the value-maximizing decision of firing the entrepreneur. However, we do not observe such contracts in practice.

likely to be optimal for the relatively safer firms, that is, firms with higher *ex ante* probability of success, higher success returns, and higher liquidation values.

Two key parameters of interest in my model are the correlation between the verifiable performance measure and the firm's state, denoted $\beta$, and the investor's renegotiation bargaining power, denoted $\mu$. Although these parameters do not affect the firm's cash flow directly, they nevertheless affect firm value through their impact on the manager's incentives. An increase in either $\beta$ or $\mu$ makes it more likely that the contingent control allocation is optimal by lowering the rents that the manager captures in the bad state. Thus, my model predicts that contingent control allocations are more likely to be used when verifiable performance measures are highly informative about the firm's true profitability, and when the manager's outside options are weak, causing the investor's renegotiation bargaining power to be high.

The predictions of my model are consistent with empirical evidence on venture capital contracts. As Kaplan and Stromberg (2003) document, a key feature of these contracts is the relationship between future firm performance and the allocation of control rights: contracts are structured so that the venture capitalist obtains full control if the firm performs poorly, whereas the entrepreneur retains/obtains control rights as firm performance improves. Contingent control is usually implemented in one of two ways: either through "adverse-state" provisions that transfer board control, voting control and/or liquidation rights to the venture capitalist if the firm's performance verifiably deteriorates; or through "milestone" provisions, which transfer control rights to the entrepreneur if the firm achieves some prespecified performance targets.[2] By focusing on the incentive properties of control allocations, I am able to explain the observed hierarchy in contingent control allocations.

My article builds on the analysis of Rajan (1992), who contrasts the incentive properties of short-term bank debt and arm's-length financing, which in my model are equivalent to investor control and manager control, respectively.[3] I extend Rajan's analysis by introducing the possibility of contingent contracts into his model along the lines proposed in Aghion and Bolton (1992). It is also important to emphasize that, unlike with some of the existing research on contingent contracts (e.g., Hellmann, 2006; Repullo and Suarez, 2004), my analysis does not rely on the assumption that the underlying state variable is verifiable. Instead, I focus on a more realistic setting where the interim performance measure is only imperfectly correlated with the firm's true state. Thus, in my model, contingent contracting does not eliminate renegotiation or hold-up problems. Although the contingent control allocation mitigates hold-up by the investor in the good state in comparison to the investor control allocation, it also gives rise to hold-up by the manager in the bad state.

My article is related to Dewatripont and Tirole (1994), who use a model with managerial moral hazard in an incomplete-contracts setting to explain why firms are financed by multiple outside investors holding a diversity of claims. In their model, the safer action (intervention) is optimal in the bad states of nature. However, the manager will never voluntarily choose intervention, because she obtains either private benefits or higher expected monetary rewards by pursuing the risky action (continuation). Hence, it is desirable to endow outsiders with control rights, and incentivize them to choose the efficient action plan. As intervention reduces the riskiness of the final value of the firm, an outsider with a concave (convex) claim on firm value will be biased toward intervention (continuation). As the incentive scheme assigned to the controlling outsider may not induce *ex post* maximization of firm value, it is necessary to have an additional outsider as a residual claimant to balance the accounts. Restricting attention to standard assets such as debt and equity, the model predicts debtholder control following poor performance,

---

[2] Consistent with the predictions of my model that contingent contracting is used to mitigate hold-up by the investor, Bienz and Hirsch (2006) find in a sample of German venture capital contracts that milestone staging is more likely to be used when the entrepreneur's outside options are limited.

[3] Other articles that point to the incentive role of short-term financing arrangements are Dewatripont and Maskin (1995) and von Thadden (1995).

shareholder control following good performance, and congruence of interests between managers and passive shareholders.

The key difference between my article and Dewatripont and Tirole (1994) is that I focus on *ex post* hold-up problems between the manager and the investor, which *can occur in both the good and bad states*, and their effect on the manager's *ex ante* incentives. In contrast, renegotiation plays a limited role in Dewatripont and Tirole's analysis, and is assumed to not affect the manager's utility. Thus, their theory is effectively based on the assumption that there is a conflict of interest between the manager and outside investors only in the bad states, where the safe action (intervention) is the efficient action choice.[4] Moreover, as Dewatripont and Tirole acknowledge, their theory does not necessarily imply contingent control rights because it is possible to induce the optimal action choice by picking a single controlling outsider and designing a complex incentive scheme (see footnote 14 in their article). In contrast, the hold-up problems that I model and the inefficiencies they engender cannot be dealt with by incentive schemes alone. The control allocation plays a central role because it affects both the likelihood of hold-up and the renegotiation payoffs of the two agents. Contingent control is optimal in my model only because it mitigates hold-up by the investor in the good state.

My article is also related to the literature that provides a theory of capital structure based on manager moral hazard, using the idea that the manager's incentives are strengthened by rewarding her in the good state and penalizing her in the bad state. Innes (1990) shows the optimality of debt financing for a risk-neutral entrepreneur, whereas Hermalin and Katz (1991) and Dewatripont, Legros, and Matthews (2003) show the optimality of riskless debt and risky debt, respectively, when the entrepreneur is risk averse but the investor is risk neutral. I use a similar idea to determine the optimal allocation of control rights between the manager and the investor. My results do not require the manager to have all the renegotiation bargaining power; in fact, the contingent control allocation is more likely to be optimal as the investor's bargaining power increases.

Another article that examines long-term versus short-term financing in an incomplete-contracts setting is Berglof and von Thadden (1994). They argue that having two (or more) classes of investors, with one holding only a secured short-term claim and another holding a long-term state-dependent claim, can deter strategic default by the borrower by strengthening the bargaining position of the short-term lender, who does not have to worry about the negative impact of liquidation on his long-term claims. Strategic default is not an issue in my model because I assume that the firm's cash flows are verifiable. My main focus is on agency conflicts surrounding the continuation versus liquidation decision, and incentivizing the manager for her effort provision.

The rest of the article is organized as follows. I describe the base assumptions of my model in Section 2 and provide a formal definition of the equilibrium in Section 3. I characterize the manager's effort choice in Section 4 and the optimal contract in Section 5. Section 6 concludes the article.

## 2. The model

■ There are three dates in the model; 0, 1, and 2. At date 0, an entrepreneur ("manager") with a project idea sets up a new firm by making an investment $I$. As the manager has no funds of her own, she raises the required funds from an investor. Both the manager and the investor are risk neutral.

At date 1, the firm is revealed to be in one of two states, "good" or "bad." After observing the state, the firm can choose to either continue operating as before ("continuation") or redeploy its assets in some alternative use ("liquidation"). If the firm is in the good state, then continuation yields a date-2 cash flow of $\tilde{R} = R$ with probability $p_g$, and $\tilde{R} = 0$ with probability $1 - p_g$. If the

---

[4] Reversing this assumption, and assuming that intervention increases the firm's risk, generates the opposite result about shareholder interventionism and creditor passivity (see Berkovitch and Israel, 1996).

YERRAMILLI / 709

firm is in the bad state, continuation yields $\tilde{R} = 0$ with certainty. On the other hand, liquidation yields a value of $L \in (0, I)$ at date 1, regardless of the firm's state. No cash flow is realized at date 2 if the firm is liquidated at date 1.

*Assumption 1.* $\Delta \equiv p_g R - L > 0.$

Assumption 1 states that continuation generates a higher value than liquidation in the good state. The variable $\Delta$ denotes the continuation surplus in the good state. In the bad state, it is clearly better to liquidate the firm because liquidation yields $L > 0$ at date 1 whereas continuation yields 0 at date 2.

The probability of the good state being realized depends on costly unobservable effort expended by the manager at date 0. Specifically, the state of the firm is good with probability $\theta e$ and bad with probability $(1 - \theta e)$, where $e \in [0, 1]$ is the manager's date-0 effort. The manager incurs a private cost of $\frac{\psi e^2}{2}$ for exerting effort $e$, where $\psi > 0$ is the manager's unit marginal cost of effort. The constant $\theta \le 1$ captures the impact of external factors, such as the demand for the firm's products and level of competition in the industry, that have a bearing on the firm's profitability. I refer to $\theta$ as firm quality, and assume that it is known to both the manager and the investor.

*Assumption 2.* $\psi > 2\theta\Delta.$

Assumption 2 ensures that the manager's effort in equilibrium is always an interior solution. Moreover, it also ensures that the probability of the good state being realized does not exceed $\frac{1}{2}$. The latter assumption, although not necessary, simplifies analysis by allowing me to focus on situations that are most relevant to start-up firms.

□    **Information structure.** There is no information asymmetry between the manager and the investor at any point of time regarding the state of the firm. Both agents observe the state of the firm only at date 1 (i.e., after the investor has invested $I$ and the manager has exerted effort $e$, but before the continuation decision is made). However, the state of the firm cannot be verified by outsiders.

Outsiders do observe a public signal on the firm's interim performance at date 1, denoted $\tilde{r}$, that is imperfectly correlated with the true state of the firm. Some real-life examples of $\tilde{r}$ are: interim sales or earnings, achievement of milestones such as U.S. Food and Drug Administration (FDA) approval of a new drug, and so forth. I model the correlation between $\tilde{r}$ and the firm's state by assuming that $\tilde{r}$ can be either high (denoted $r = h$) or low (denoted $r = l$) and that

$$\Pr(r = h \mid \text{good state}) = \Pr(r = l \mid \text{bad state}) = \beta,$$

where $\beta \in (0.5, 1)$ is a given constant. The above probability distribution implies that low performance is more likely in the bad state and high performance is more likely in the good state, although the correlation is not perfect. The parameter $\beta$ measures the informativeness of the public signal $\tilde{r}$.[5]

All the cash flows are verifiable.

---

[5] By Bayesian updating, a low performance indicates that the firm is more likely in the bad state, whereas a high performance indicates that the firm is more likely in the good state. Formally,

$$\Pr(\text{Good state} \mid r = h) = \frac{\beta\theta e}{\beta\theta e + (1 - \beta)(1 - \theta e)},$$

$$\text{and} \quad \Pr(\text{Bad state} \mid r = l) = \frac{\beta(1 - \theta e)}{(1 - \beta)\theta e + \beta(1 - \theta e)}.$$

It is easily verified that both the above expressions are increasing in $\beta$. As $\beta \to 1$, $\Pr(\text{Good state} \mid r = h) \to 1$ and $\Pr(\text{Bad state} \mid r = l) \to 1$ (i.e., the high (low) performance signal almost certainly indicates that the firm is in the good (bad) state). On the other hand, the signal $\tilde{r}$ becomes uninformative as $\beta \to 0.5$, because then $\Pr(\text{Good state} \mid r = h) \to \theta e$ and $\Pr(\text{Bad state} \mid r = l) \to 1 - \theta e$ (i.e., the posterior probabilities are almost the same as the prior probabilities).

□ **The contract.** The contract between the manager and the investor cannot be contingent on either the manager's effort or the state of the firm, because the manager's effort is unobservable and the firm's state is unverifiable. The contract can, however, assign control over the liquidation decision to either the manager or the investor, possibly contingent on the realization of the verifiable performance measure $\tilde{r}$.

Let $\Phi = \{\phi_l, \phi_h\}$ denote the control allocation specified in the initial contract, where $\phi_r \in \{inv, mgr\}$ denotes the identity of the agent who is assigned control following the interim performance $r \in \{l, h\}$. There are four possible control allocations that the contract may specify: "investor control"(denoted $\Phi = IC$), under which the investor has control over the liquidation decision regardless of $r$ (i.e., $\phi_l = \phi_h = inv$); "manager control" (denoted $\Phi = MC$), under which the manager has control regardless of $r$ (i.e., $\phi_l = \phi_h = mgr$); "contingent control" (denoted $\Phi = CC$), under which the manager has control if interim performance is high and the investor has control if interim performance is low (i.e., $\phi_h = mgr$, $\phi_l = inv$); and "inverse contingent control" (denoted $\Phi = ICC$), under which the manager has control if interim performance is low and the investor has control if interim performance is high (i.e., $\phi_h = inv$, $\phi_l = mgr$).

The contract also specifies a payoff rule, which describes how the realized cash flows are to be shared between the two agents and outlines other cash transfers among the agents. The contract may specify payments $(D_l, D_h)$, where $D_r$ denotes the payment to the investor if the project is allowed to continue operating following the interim performance $r \in \{l, h\}$, and the cash flow $R$ is realized at date 2; the manager being the residual claimant gets a payoff of $R - D_r$. Similarly, the contract may specify payments $(Y_l, Y_h)$, where $Y_r$ denotes the payment to the investor if the firm is liquidated at date 1 following the interim performance $r \in \{l, h\}$; the manager's payoff then is $L - Y_r$. Apart from specifying how the realized cash flows are to be shared, the contract may also specify an additional cash transfer of $T_r$ from the investor to the manager if the firm is allowed to continue operating following the interim performance $r \in \{l, h\}$; one interpretation of $T_r$ is that it denotes a bond posted by the investor up front to make a prespecified payment to the manager in the event of continuation.[6] As I allow for the cash transfers $(T_l, T_h)$, there is no loss of generality in assuming that both agents get a payoff of zero if $\tilde{R} = 0$ is realized at date 2. I refer to $\Omega = (Y_l, Y_h, D_l, D_h, T_l, T_h)$ as the payoff rule. Given the assumptions about verifiability, the payoff rule $\Omega$ is completely general, because it allows all payoff variations contingent on the three cash flows, $0$, $R$, and $L$, and on the interim signal $r$.

I impose two important restrictions on $\Omega$. First, the payoff to the manager in the event of liquidation must be nonnegative, that is, $L - Y_r \geq 0$. In other words, there is a limit to the punishment that can be imposed on the manager in the bad state. This is a reasonable restriction because managers are protected by limited liability in case the venture fails. Moreover, in my model, the manager has no money of her own. Second, $\Omega$ must satisfy the following "feasibility constraint": the investor's total expected payoff at date 0 must weakly exceed the amount it invests in the firm (i.e., the investor's participation constraint must be met), and the amount invested by the investor at date 0 must be sufficient to finance the initial investment $I$. Observe that I do not impose any limited liability restrictions on the investor in any state at date 1. So $D_r$ and $T_r$ can take any possible value as long as the contract satisfies the feasibility constraint.

□ **Renegotiation at date 1 and hold-up problems.** After observing the state of the firm at date 1, the manager and the investor may choose to renegotiate the initial contract. In the event of renegotiation, the manager and the investor split the surplus from renegotiation between them. One way to obtain this outcome is to employ the generalized Nash bargaining solution in which the investor gets its disagreement payment plus a fraction $\mu$ of the surplus from renegotiation, whereas the manager gets her disagreement payoff plus a fraction $(1 - \mu)$ of the surplus from renegotiation. Alternatively, I could assume that the investor gets to make a

---

take-it-or-leave-it offer with probability $\mu$, and the manager gets to make a take-it-or-leave-it offer with probability $(1 - \mu)$.[7] The parameter $\mu \in (0, 1)$ is a measure of the investor's bargaining power at date 1.

Renegotiation creates the possibility of hold-up problems at date 1. If the manager is in control of the firm in the bad state, she may refuse to liquidate the project unless the investor agrees to share some of the liquidation proceeds with her. Similarly, if the investor is in control in the good state, it may use the threat of liquidation to capture a share of the continuation surplus $\Delta$. The *crucial assumption* underlying hold-up by the investor is that the firm is constrained in obtaining refinancing from outside investors at date 1 if the initial investor refuses to continue to finance the firm. For tractability, as in Rajan (1992), I model this by assuming that the firm cannot obtain any refinancing from outside investors at date 1 if the original investor refuses to continue financing the firm; hence, liquidation becomes the disagreement outcome in the renegotiation game between the investor and the manager in the good state. This is a reasonable assumption in the case of entrepreneurial firms given their opacity and the high *ex ante* probability of failure.[8]

It is convenient, but not necessary, to assume that the manager captures all the initial surplus by offering a take-it-or-leave-it contract to the investor at date 0.[9]

## 3. Definition of equilibrium

■ Before formally defining the equilibrium in this section, I introduce some notation. For a given initial contract $(\Omega, \Phi)$, let $V_\Phi(\Omega, e)$ and $S_\Phi(\Omega, e)$ denote the expected total firm cash flow and the expected payoff to the investor, respectively, at date 0 as a function of the manager's effort, $e$. The expectations assume that both the manager and the investor behave optimally at date 1 when renegotiation takes place and a continuation decision is made.

As the manager is the residual claimant, her expected payoff from the project's cash flow is $V_\Phi(\Omega, e) - S_\Phi(\Omega, e)$. Therefore, in equilibrium, the manager's initial effort $e_\Phi(\Omega)$ must satisfy the following incentive compatibility condition:

$$e_\Phi(\Omega) = \arg \max_e V_\Phi(\Omega, e) - S_\Phi(\Omega, e) - \frac{\psi e^2}{2}. \tag{1}$$

In a rational expectations equilibrium, the investor will correctly conjecture the manager's effort $e_\Phi(\Omega)$ and compute the expected value of its claim at date 0 as $S_\Phi(\Omega, e_\Phi(\Omega))$. As the manager makes a take-it-or-leave-it offer to the investor, she raises an amount $S_\Phi(\Omega, e_\Phi(\Omega))$ from the investor at date 0. For the contract to be feasible, it must satisfy the following "feasibility" constraint:

$$S_\Phi(\Omega, e_\Phi(\Omega)) \geq I. \tag{2}$$

Condition (2) states that the manager must raise enough funds at date 0 to cover the initial investment $I$. If $S_\Phi(\Omega, e_\Phi(\Omega)) > I$, I assume that the manager simply consumes the excess funds at date 0. Therefore, the manager's total cash flow at date 0 is $V_\Phi(\Omega, e_\Phi(\Omega)) - I$, which is obtained by adding the manager's residual cash flow, $V_\Phi(\Omega, e_\Phi(\Omega)) - S_\Phi(\Omega, e_\Phi(\Omega))$, to her excess date-0 funds, $S_\Phi(\Omega, e_\Phi(\Omega)) - I$.

---

[7] I thank an anonymous referee for this suggestion.

[8] It may be that outside investors do not know the firm quality $\theta$ and, hence, cannot compute the correct posterior probability of the firm being in the good state conditional on the performance measure $\tilde{r}$. As $\tilde{r}$ is a noisy signal of the firm's state, refusal by the inside investor to refinance the project at date 1 may signal to outside uninformed investors that the project is in the bad state, causing them to stay away.

[9] It is easy to show that the manager's equilibrium effort choice is invariant to her bargaining power at date $-0$. Also, the optimal contract will not change qualitatively if the investor is allowed to capture a positive fraction of the initial surplus.

Subtracting the cost of effort from the manager's total cash flow at date 0 yields her net surplus at date 0, which I refer to as firm value. For a given contract $(\Omega, \Phi)$, the firm value is

$$NV_\Phi(\Omega) = V_\Phi(\Omega, e_\Phi(\Omega)) - I - \frac{\psi(e_\Phi(\Omega))^2}{2}. \tag{3}$$

Equilibrium in this game consists of choosing an initial contract $(\Omega^*, \Phi^*)$ that maximizes $NV_\Phi(\Omega)$ subject to the manager's incentive compatibility condition (1) and the feasibility constraint (2).

## 4. Characterizing the manager's effort

☐ **The hold-up problem at date 1.** I begin by analyzing the behavior of the manager and the investor at date 1 after they have observed the firm's state. I restrict attention to contracts with $Y_r \geq 0$, because otherwise the investor will not have any incentive to liquidate the firm in the bad state. I show in Lemma A3 that a contract with $Y_r < 0$ can never be optimal.

Define

$$S_g(r, \Omega) \equiv \max\{Y_r + \mu\Delta, \, p_g D_r - T_r\} \tag{4}$$

$$S_b(r, \Omega) \equiv \min\{Y_r, \, \mu L\}. \tag{5}$$

*Lemma 1 (Exercise of control at date 1).*

(i) If the manager is in control in the good state, she allows the firm to continue operating untill date 2; the payoffs to the investor and the manager are $p_g D_r - T_r$ and $p_g(R - D_r) + T_r$, respectively, for $r \in \{l, h\}$.

   If the investor is in control in the good state, it allows the firm to continue operating untill date 2, possibly after renegotiating the initial contract. Renegotiation occurs if, and only if,

$$Y_r + \mu\Delta > p_g D_r - T_r \text{ for } r \in \{l, h\}. \tag{6}$$

   The payoffs to the investor and the manager are $S_g(r, \Omega)$ and $p_g R - S_g(r, \Omega)$, respectively.

(ii) If the investor is in control in the bad state, it will liquidate the firm; the payoffs to the investor and the manager are $Y_r$ and $L - Y_r$, respectively.

   If the manager is in control in the bad state, she will liquidate the firm, possibly after renegotiating the initial contract. Renegotiation occurs if, and only if, $Y_r > \mu L$. The payoffs to the investor and the manager are $S_b(r, \Omega)$ and $L - S_b(r, \Omega)$, respectively.

As the manager and the investor observe the true state of the firm at date 1, it is not surprising that the efficient liquidation decision is made at date 1, possibly after renegotiation. That, however, does not mean that the initial contract $(\Omega, \Phi)$ is irrelevant, because $(\Omega, \Phi)$ determines when renegotiation occurs and what the renegotiation payoffs of the two agents are. Hold-up by the investor in the good state is more likely when its payoff under continuation is low (i.e., low $D_r$ or high $T_r$), its payoff under liquidation $Y_r$ is high, and the surplus from continuation $\Delta$ is high. Hold-up by the manager in the bad state is more likely when the investor's payoff under liquidation $Y_r$ is high.

For a given payoff rule $\Omega$, hold-up by the investor in the good state is more likely under the investor control allocation ($\Phi = IC$) than under the contingent control allocation ($\Phi = CC$), because the contingent control allocation allows the manager to retain control in the good state with probability $\beta > 1/2$. On the other hand, for a given $\Omega$, hold-up by the manager in the bad state is more likely under the contingent control allocation than under the investor control allocation, because the contingent control allocation allows the manager to retain control even in the bad state with a positive probability $1 - \beta$.

◻ **Manager's effort, $e_\Phi(\Omega)$.** As the efficient continuation decision is always made *ex post*, regardless of $\Omega$ and $\Phi$ (see Lemma 1), it follows that

$$V_\Phi(\Omega, e) = V(e) = \theta e \Delta + L. \tag{7}$$

Define

$$P_{IC}(\Omega) = \Delta - \beta S_g(h, \Omega) - (1 - \beta)S_g(l, \Omega) + \beta Y_l + (1 - \beta)Y_h \tag{8}$$

$$P_{MC}(\Omega) = \Delta - \beta p_g D_h - (1 - \beta)p_g D_l + \beta S_b(l, \Omega) + (1 - \beta)S_b(h, \Omega) \tag{9}$$

$$P_{CC}(\Omega) = \Delta - \beta p_g D_h - (1 - \beta)S_g(l, \Omega) + \beta Y_l + (1 - \beta)S_b(h, \Omega) \tag{10}$$

$$P_{ICC}(\Omega) = \Delta - \beta S_g(h, \Omega) - (1 - \beta)p_g D_l + \beta S_b(l, \Omega) + (1 - \beta)Y_h. \tag{11}$$

The expressions $P_\Phi(\Omega)$ characterized in equations (8)–(11) denote the incremental payoff to the manager in the good state over the bad state, under the contract $(\Omega, \Phi)$.[10] Therefore, the investor's incremental payoff in the good state is $\Delta - P_\Phi(\Omega)$.

*Lemma 2*. Under any feasible contract, $P_\Phi(\Omega) < \Delta$. Given an initial contract with the control allocation $\Phi$ and payoff rule $\Omega$, the manager chooses an effort

$$e_\Phi(\Omega) = \frac{\theta P_\Phi(\Omega)}{\psi}, \tag{12}$$

where $P_\Phi(\Omega)$ for $\Phi \in \{IC, \ MC, \ CC, \ ICC\}$ is defined in equations (8)–(11).

As the investor's incremental payoff in the good state is $\Delta - P_\Phi(\Omega)$, and its payoff in the bad state cannot exceed $L$, it follows that $S_\Phi(\Omega, e_\Phi(\Omega)) \leq \theta e_\Phi(\Omega) \cdot (\Delta - P_\Phi(\Omega)) + L$. Clearly, if the feasibility constraint, $S_\Phi(\Omega, e_\Phi(\Omega)) \geq I$, is to be satisfied, it is necessary that $P_\Phi(\Omega) < \Delta$. Hence, no feasible contract can allow the manager to capture the entire continuation surplus in the good state.

Note that the manager's marginal cost of effort is $\psi e$, and the marginal value is $\theta P_\Phi(\Omega)$. As $\psi > \theta \Delta$ (by Assumption 2) and $\Delta > P_\Phi(\Omega)$, it follows that there exists an $e \in (0, 1)$ at which $\psi e = \theta P_\Phi(\Omega)$. Solving for $e$ yields the expression for $e_\Phi(\Omega)$ in equation (12).

## 5. Characterizing the optimal contract

■ Combining equations (3), (7), and (12), the firm value can be rewritten as

$$NV_\Phi(\Omega) = \frac{\theta^2 P_\Phi(\Omega)}{2\psi}(2\Delta - P_\Phi(\Omega)) + L - I. \tag{13}$$

*Lemma 3*. For any payoff rule $\Omega$ with $T_r > 0$, there exists an alternative payoff rule $\hat{\Omega}$ with $T_r = 0$ that leads to the same effort and firm value as $\Omega$.

Suppose the payoff rule $\Omega$ has $T_r > 0$. Consider an alternative payoff rule $\hat{\Omega}$ such that $\hat{Y}_r = Y_r$, and $p_g \hat{D}_r = p_g D_r - T_r$ for $r \in \{l, h\}$ (i.e., $\hat{\Omega}$ and $\Omega$ provide the same expected payoff to the agents in both states). It is easy to verify that the manager's effort and firm value will be the same under both these payoff rules. Lemma 3 implies that I can restrict attention to contracts with $T_r = 0$ without any loss of generality.

---

[10] To see why, note that $P_{IC}(\Omega)$ may be rewritten as follows after substituting $\Delta = p_g R - L$:

$$P_{IC}(\Omega) = [p_g R - \beta S_g(h, \Omega) - (1 - \beta)S_g(l, \Omega)] - [L - \beta Y_l - (1 - \beta)Y_h].$$

In the above expression, $p_g R - \beta S_g(h, \Omega) - (1 - \beta)S_g(l, \Omega)$ is the manager's payoff in the good state, and $L - \beta Y_l - (1 - \beta)Y_h$ is the manager's payoff in the bad state.

*Lemma 4.* An optimal contract is one that maximizes $P_\Phi(\Omega)$, subject to the feasibility constraint $S_\Phi(\Omega, e_\Phi(\Omega)) \geq I$.

Notice that the initial contract affects firm value only through $P_\Phi(\Omega)$, that is, only through its impact on the manager's incentives. It is evident from equation (13) that $\frac{\partial NV_\Phi}{\partial P_\Phi} = \frac{\theta^2}{\psi}(\Delta - P_\Phi(\Omega)) > 0$ for any feasible contract because $P_\Phi(\Omega) < \Delta$ (by Lemma 2). Hence, an optimal contract must maximize $P_\Phi(\Omega)$, subject to the feasibility constraint.

*Proposition 1.* The inverse contingent control allocation ($\Phi = ICC$) or the manager control allocation ($\Phi = MC$) can never be optimal, because both these are strictly dominated by the contingent control allocation ($\Phi = CC$).

The manager's incentives are strengthened when she is rewarded in the good state and penalized in the bad state (i.e., when $P_\Phi(\Omega)$ is high). Clearly, then, the inverse contingent control allocation ($\Phi = ICC$) can never be optimal because it weakens the manager's incentives by punishing her for high performance and rewarding her for low performance.

The argument for why the manager control allocation ($\Phi = MC$) is always dominated by the contingent control allocation ($\Phi = CC$) is a bit more subtle. The main advantage of the manager control allocation is that it eliminates hold-up by the investor in the good state, whereas its main disadvantage is that it rewards the manager in the bad state by allowing her to extract liquidation rents of $(1 - \mu)L$ from the investor. The contingent control allocation lowers the rents that the manager can extract in the bad state by transferring control to the investor with probability $\beta > 0.5$, but it also exposes the manager to hold-up by the investor with positive probability of $(1 - \beta)$ in the good state. However, the key is to realize that even though contingent control does not eliminate hold-up by the investor in the good state, the contract can set the payment $D_h$ such that the investor gets a low payoff when the manager is in control in the good state, thus partially offsetting the effect of hold-up by the investor. So overall, the contingent control allocation dominates the manager control allocation because it lowers the rents that the manager extracts in the bad state.[11]

Proposition 1 highlights a key difference between my analysis and that in Aghion and Bolton (1992) and Rajan (1992). In Aghion and Bolton's model, manager control emerges as a possible optimal control allocation because they do not consider the manager's *ex ante* incentives to expend costly effort. On the other hand, although Rajan models the manager's effort problem, he does not consider the possibility of contingent contracts. I show that if it is possible to write contracts contingent on a noisy performance measure, then the contingent control allocation will strictly dominate the manager control allocation.

Proposition 1 implies that the optimal contract, if it exists, will specify either an investor control allocation ($\Phi = IC$) or a contingent control allocation ($\Phi = CC$). I now characterize the conditions under which either of these control allocations is feasible and optimal.

I solve for the optimal contract, denoted $(\Omega^*, \Phi^*)$, in two steps. First, I characterize the conditions under which each $\Phi \in \{CC, IC\}$ is feasible, and solve for the optimal payoff rule $\Omega_\Phi^*$ and the corresponding incentives $P_\Phi^* \equiv P_\Phi(\Omega_\Phi^*)$ for each $\Phi$. Then, I compare the $P_\Phi^*$ for $\Phi \in \{CC, IC\}$ to see which of them implements the highest effort and, hence, the highest firm value.

☐ **Optimal control allocation with simpler payoff rules.** Purely for ease of exposition, I begin my analysis by restricting attention to simpler contracts with $Y_l = Y_h = L$, so that the payoff rule simplifies to $\Omega = (D_l, D_h)$; these may be interpreted as debt contracts, where the repayment value $D$ depends on the realization of the interim performance measure $\tilde{r}$. I then show in the next subsection that the qualitative results in this subsection hold even in the general case when $Y_l$ and $Y_h$ are not constrained to equal $L$.

---

[11] Formally, I show in the proof of Proposition 1 that for every feasible contract with $\Phi = MC$, it is possible to design another contract with $\Phi = CC$ that is feasible and implements a higher effort.

*The contingent control allocation ($\Phi = CC$).* For a contract with $\Phi = CC$ to be feasible, it must be that $S_{CC}(\Omega, e_{CC}(\Omega)) \geq I$, where $e_{CC} = \frac{\theta P_{CC}}{\psi}$. The investor's incremental payoff in the good state is $\Delta - P_{CC}$, and its payoff in the bad state is $L - (1 - \beta)(1 - \mu)L$. Hence, the feasibility constraint can be rewritten as

$$\frac{\theta^2 P_{CC}}{\psi}(\Delta - P_{CC}) - (I - L + (1 - \beta)(1 - \mu)L) \geq 0. \tag{14}$$

The largest value of $P_{CC}$ at which the above inequality is satisfied is

$$P_{CC}^+ \equiv \frac{1}{2}\left(\Delta + \sqrt{\Delta^2 - \frac{4\psi(I - L + (1 - \beta)(1 - \mu)L)}{\theta^2}}\right), \tag{15}$$

which is well-defined only if

$$(\theta\Delta)^2 \geq 4\psi(I - L + (1 - \beta)(1 - \mu)L). \tag{16}$$

*Proposition 2.* A contract with a contingent control allocation ($\Phi = CC$) is feasible if, and only if, condition (16) is satisfied. If condition (16) is satisfied, then $P_{CC}^* = P_{CC}^+$ and the optimal payoff rule $\Omega_{CC}^*$ is as follows.

(i) If $(1 - \mu)(\Delta - (1 - \beta)L) \geq P_{CC}^+$, then $\Omega_{CC}^*$ is given by $D_l = D_h$ such that

$$p_g(R - D_h) = P_{CC}^+ + (1 - \beta)(1 - \mu)L. \tag{17}$$

Under this contract, $p_g D_l \geq L + \mu\Delta$. Hence, when the investor is in control in the good state, it does not hold up the manager.

(ii) If $(1 - \mu)(\Delta - (1 - \beta)L) < P_{CC}^+$, then $\Omega_{CC}^*$ is given by $D_l < \frac{L + \mu\Delta}{p_g}$ and a $D_h$ that satisfies

$$\beta p_g(R - D_h) = P_{CC}^+ - (1 - \mu)(1 - \beta)(\Delta - L). \tag{18}$$

Under this contract, $p_g D_l < L + \mu\Delta$. Hence, when the investor is in control in the good state, it will force renegotiation to increase its payoff to $L + \mu\Delta$.

The necessity of condition (16) follows from the discussion preceding the proposition. If this condition is met, then an optimal payoff rule must satisfy $P_{CC}(\Omega_{CC}^*) = P_{CC}^+$ because $P_{CC}^+$ is the largest value of $P_{CC}$ at which the feasibility constraint is satisfied. Setting $P_{CC}(\Omega_{CC}^*) = P_{CC}^+$ is equivalent to choosing payments $D_h$ and $D_l$ such that the manager's expected payoff in the good state equals $P_{CC}^+ + (1 - \beta)(1 - \mu)L$; the term $(1 - \beta)(1 - \mu)L$ represents the liquidation rents that the manager extracts in the bad state. For low values of $\mu$, this is achieved by the contract characterized in part (i) of the proposition, whereas for higher values of $\mu$, this is achieved by the contract characterized in part (ii) of the proposition.

It is easily verified from equation (15) that $P_{CC}^+$ is increasing in $\mu$. In other words, under the contingent control allocation, an increase in the investor's bargaining power strengthens the manager's incentives. This is because an increase in $\mu$ lowers the liquidation rents extracted by the manager in the bad state, $(1 - \beta)(1 - \mu)L$. Therefore, as $\mu$ increases, the feasibility condition (16) is more likely to be met and $P_{CC}^+$ increases.

*The investor control allocation ($\Phi = IC$).* In this case, the investor's incremental payoff in the good state is $\Delta - P_{IC}$, and its payoff in the bad state is $L$. So by a similar intuition as in the $\Phi = CC$ case above, the feasibility constraint can be rewritten as

$$\frac{\theta^2 P_{IC}}{\psi}(\Delta - P_{IC}) - (I - L) \geq 0. \tag{19}$$

The quadratic expression in the above inequality is nonnegative only for $P_{IC} \in [P_{IC}^-, P_{IC}^+]$, where $P_{IC}^-$ and $P_{IC}^+$ are as under

$$P_{IC}^{\pm} \equiv \frac{1}{2}\left(\Delta \pm \sqrt{\Delta^2 - \frac{4\psi(I-L)}{\theta^2}}\right). \tag{20}$$

For $P_{IC}^-$ and $P_{IC}^+$ to be well-defined, it is necessary that

$$(\theta\Delta)^2 \geq 4\psi(I-L). \tag{21}$$

Clearly, condition (21) is necessary for the feasibility of any contract with $\Phi = IC$, because otherwise condition (19) cannot be met for any $P_{IC}$. However, condition (21) is *not sufficient* to guarantee the feasibility of $\Phi = IC$. Feasibility also depends on the investor's bargaining power $\mu$, because of the restriction that $P_{IC} \leq (1-\mu)\Delta$. As $\mu$ increases, the manager's incentives are severely weakened because the investor captures most of the surplus from continuation.

If $(1-\mu)\Delta < P_{IC}^-$, then the feasibility constraint (19) cannot be satisfied by any $P_{IC} \leq (1-\mu)\Delta$. Therefore, for a contract with $\Phi = IC$ to be feasible, it is also necessary that

$$(1-\mu)\Delta \geq P_{IC}^- \iff \mu \leq \mu_{IC}^-, \tag{22}$$

where the threshold $\mu_{IC}^-$ is defined such that

$$(1-\mu_{IC}^-)\Delta = P_{IC}^-. \tag{23}$$

Similarly, define the threshold $\mu_{IC}^+$ such that $(1-\mu_{IC}^+)\Delta = P_{IC}^+$. It is easy to verify that if $(\theta\Delta)^2 > 4\psi(I-L)$, then $0 < \mu_{IC}^+ < \frac{1}{2} < \mu_{IC}^- < 1$.

*Proposition 3.* A contract with investor control allocation ($\Phi = IC$) is feasible if, and only if, conditions (21) and (22) are satisfied. If these conditions are satisfied, then the optimal payoff rule $\Omega_{IC}^*$ is as follows.

(i) If $\mu \leq \mu_{IC}^+$, then $\Omega_{IC}^*$ is given by $D_l = D_h = \frac{p_g R - P_{IC}^+}{p_g}$. The contract is renegotiation proof because the investor does not hold up the manager in the good state. In this case, $P_{IC}^* = P_{IC}^+$.

(ii) If $\mu_{IC}^+ < \mu \leq \mu_{IC}^-$, then $\Omega_{IC}^*$ will have $D_l < \frac{L+\mu\Delta}{p_g}$ and $D_h < \frac{L+\mu\Delta}{p_g}$. In this case, the investor will force renegotiation in the good state to increase its payoff to $L + \mu\Delta$, and $P_{IC}^* = (1-\mu)\Delta$.

The necessity of conditions (21) and (22) follows from the discussion preceding the proposition. If these conditions are met, then an optimal payoff rule $\Omega_{IC}^*$ must satisfy $P_{IC}(\Omega_{IC}^*) = \min\{P_{IC}^+, (1-\mu)\Delta\}$, because that is the largest value of $P_{IC}(\Omega) \leq (1-\mu)\Delta$ that also satisfies the feasibility constraint. If $\mu \leq \mu_{IC}^+$ (which is equivalent to $P_{IC}^+ \leq (1-\mu)\Delta$), then $\Omega_{IC}^*$ is characterized in part (i) of Proposition 3. For such low values of $\mu$, the investor does not hold up the manager in the good state; so $P_{IC}(\Omega_{IC}^*) = P_{IC}^+$. On the other hand, if $\mu_{IC}^+ < \mu \leq \mu_{IC}^-$ (which is equivalent to $P_{IC}^- \leq (1-\mu)\Delta < P_{IC}^+$), then $\Omega_{IC}^*$ is characterized in part (ii) of Proposition 3. In this case, the investor does hold up the manager in the good state, so that $P_{IC}(\Omega_{IC}^*) = (1-\mu)\Delta$.

*Contingent control allocation versus investor control allocation.* In Propositions 2 and 3, I characterized the optimal payoff rule, $\Omega_\Phi^*$, and the corresponding $P_\Phi(\Omega_\Phi^*)$ for $\Phi \in \{IC, CC\}$. I showed that $P_{IC}^* = \min\{P_{IC}^+, (1-\mu)\Delta\}$ when $\Phi = IC$ is feasible, and that $P_{CC}^* = P_{CC}^+$ when $\Phi = CC$ is feasible. The only remaining step in characterizing the optimal contract is to determine whether the contingent control allocation dominates the investor control allocation, or vice versa.

Of course, neither form of financing is feasible if $(\theta\Delta)^2 < 4\psi(I-L)$, and only $\Phi = IC$ is feasible if $(\theta\Delta)^2 = 4\psi(I-L)$. The more interesting case occurs when $(\theta\Delta)^2 > 4\psi(I-L)$. Intuitively, the contingent control allocation will dominate the investor control allocation if either the latter is infeasible or $P_{CC}^* > P_{IC}^*$. I formalize this intuition in Proposition 4.

*Proposition 4 (The optimal contract, $(\Omega^*, \Phi^*)$).* Suppose $(\theta\Delta)^2 > 4\psi(I-L)$.

(i) If $\mu \leq \mu_{IC}^+$, then the investor control allocation strictly dominates the contingent control allocation, regardless of $\beta$. In this case, the optimal contract has $\Phi^* = IC$ and the payoff rule $\Omega^*$ characterized in part (i) of Proposition 3.

(ii) If $\mu > \mu_{IC}^+$, then there exists a threshold $\hat{\beta} < 1$ such that:

    (a) if $\beta \geq \hat{\beta}$, then the optimal contract has the contingent control allocation ($\Phi^* = CC$) and the payoff rule $\Omega^*$ characterized in part (ii) of Proposition 2.

    (b) if $\beta < \hat{\beta}$ and $\mu \leq \mu_{IC}^-$, then the optimal contract has the investor control allocation ($\Phi^* = IC$) and the payoff rule $\Omega^*$ characterized in part (ii) of Proposition 3.

    (c) if $\beta < \hat{\beta}$ and $\mu > \mu_{IC}^-$, then neither form of financing is feasible.

    The threshold $\hat{\beta}$ decreases as $\theta$, $p_g R$, $L$, and $\mu$ increase.

When $\mu \leq \mu_{IC}^+$, there is no hold-up by the investor in the good state under a contract with an investor control allocation (see part (i) of Proposition 3). Therefore, in this case, the investor control allocation ($\Phi = IC$) strictly dominates the contingent control allocation ($\Phi = CC$) even if the latter is feasible, because $\Phi = IC$ does not reward the manager in the bad state. Formally, it is easily verified that $P_{IC}^* = P_{IC}^+ > P_{CC}^+$.

If $\mu > \mu_{IC}^+$, then a contract with $\Phi = IC$ leads to hold-up by the investor in the good state, which becomes more severe as $\mu$ increases. In this case, the main advantage of the contingent control allocation over the investor control allocation is that it mitigates hold-up by the investor in the good state by allowing the manager to be in charge of the continuation decision with probability $\beta > \frac{1}{2}$; the main drawback is that it also rewards the manager with liquidation rents of $(1 - \beta)(1 - \mu)L$ in the bad state. As the verifiable signal becomes more informative (i.e., as $\beta$ increases), hold-up by the investor in the good state as well as hold-up by the manager in the bad state become less likely. Similarly, as the investor's bargaining power $\mu$ increases, hold-up by the manager in the bad state becomes less severe because the investor captures most of the surplus from liquidation. Therefore, $\Phi = CC$ is more likely to be optimal for high values of $\beta$ and $\mu$ (in fact, if $\mu > \mu_{IC}^-$, then $\Phi = CC$ may be the only feasible allocation). Specifically, I show that there exists a threshold $\hat{\beta}$, such that $\Phi = CC$ strictly dominates $\Phi = IC$ if $\beta \geq \hat{\beta}$. The existence of this threshold follows by noting that $P_{CC}^+$ is increasing in $\beta$, and that $P_{CC}^+ \to P_{IC}^+$ as $\beta \to 1$.

As noted earlier, the main drawback of the investor control allocation is that it exposes the manager to the threat of hold-up by the investor in the good state. Intuitively, this should be a more serious concern for high-quality (i.e., high $\theta$) firms that are more likely to be in the good state *ex post*, and for firms with high success returns $p_g R$ that can be expropriated by the investor. Moreover, the renegotiation rents of the investor increase with its bargaining power $\mu$ and with the firm's liquidation value $L$. Therefore, the contingent control allocation is more likely to be optimal (i.e., the threshold $\hat{\beta}$ is lower) for high values of $\theta$, $p_g R$, $L$, and $\mu$.
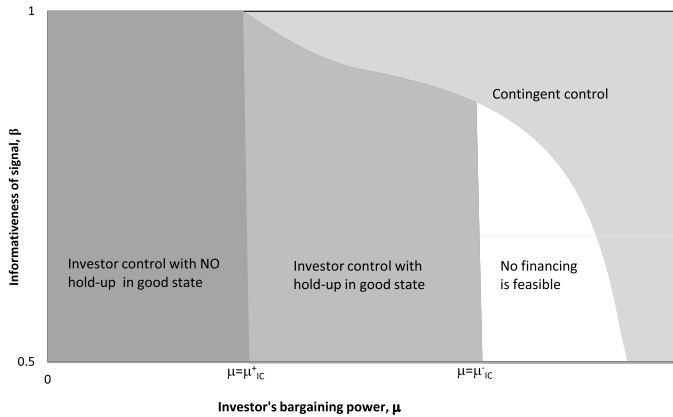
To further illustrate these results, I turn to graphical analysis using a numerical example. Consider the following parameter values: $I = 1$, $L = 0.8$, $p_g = 0.7$, $R = 5$, $\theta = 0.8$, and $\psi = 5$; therefore, $\Delta = p_g R - L = 2.7$. It is easily verified that these parameter values satisfy Assumptions 1 and 2, and condition (21), which is required for any form of financing to be feasible. Given the above parameter values, $\mu_{IC}^+ \simeq 0.311$ and $\mu_{IC}^- \simeq 0.689$.

Figure 1 provides an equilibrium map that characterizes the optimal control allocation for different values of $\beta$ and $\mu$. The leftmost region in the figure corresponds to $\mu \leq \mu_{IC}^+$. For such low values of $\mu$, the investor control allocation strictly dominates the contingent control allocation because there is no hold-up by the investor in the good state. If $\mu_{IC}^+ < \mu \leq \mu_{IC}^-$, then the contingent control allocation is optimal if $\beta > \hat{\beta}$, and the investor control allocation is optimal otherwise. The downward-sloping curve indicates that the threshold $\hat{\beta}$ decreases as $\mu$ increases. Finally, consider the region where $\mu > \mu_{IC}^-$. In this region, the investor control allocation is infeasible. Therefore, the contingent control allocation is feasible and optimal if $\beta > \hat{\beta}$; otherwise, no financing is feasible, as represented by the region in white. Observe that $\hat{\beta} \to 0.5$ as $\mu \to 1$.

The predictions in Proposition 4 are consistent with the key prediction in Aghion and Bolton (1992) that investor control is optimal only when the firm is highly financially constrained (i.e., the firm is not very profitable on average and the liquidation value is low) and contingent control cannot protect the investor's claims. Aghion and Bolton derive their results by focusing on *ex post*

FIGURE 1

IMPACT OF $\beta$ AND $\mu$ ON THE EQUILIBRIUM OUTCOME



conflicts of interest between the manager and the investor. In their model, contingent control is strictly optimal only when manager control is infeasible, and investor control cannot implement the first-best action plan (see Proposition 5 in their article). In the context of the continuation versus liquidation decision, this requires assuming that the manager is biased against liquidation in the bad state and *also that the investor is biased against continuation in the good state*; if the latter assumption is violated, then investor control does as well as contingent control. Thus, Aghion and Bolton require very specific exogenous assumptions on the agents' utilities for contingent control to be strictly optimal. They also leave open the possibility that if their assumptions about conflicts of interest are reversed, then the inverse contingent control allocation may be optimal. In contrast, I derive the optimality of the contingent control vis-à-vis investor control by focusing on the manager's *ex ante* incentives to expend costly effort. My analysis also explains why it is optimal to allow the manager to retain control over the firm following good performance and to have control switch to the investor *only* following poor performance.

*Characterizing the firm value.* Let $P^* \equiv P_{\Phi^*}(\Omega^*)$ denote the value of $P$ under the optimal contract $(\Phi^*, \Omega^*)$. Then, the optimal firm value is given by

$$NV^* = \frac{\theta^2 P^*}{2\psi}(2\Delta - P^*) + L - I. \tag{24}$$

I now characterize how $NV^*$ varies with the informativeness of the verifiable signal, $\beta$, and the investor's bargaining power, $\mu$. Note that $\beta$ and $\mu$ do not have any direct impact on firm value; they may only affect firm value through their impact on the manager's incentives (i.e., through $P^*$).

*Proposition 5 (Impact of $\beta$ and $\mu$ on firm value).*

(i) If $\mu \leq \mu_{IC}^+$, then firm value does not change with $\beta$. If $\mu > \mu_{IC}^+$, then firm value does not change with $\beta$ for $\beta < \hat{\beta}$, but increases with $\beta$ for $\beta \geq \hat{\beta}$.

(ii) For any $\beta \in (\frac{1}{2}, 1)$, there exists a threshold $\hat{\mu} \in (\mu_{IC}^+, 1)$ such that firm value does not change with $\mu$ for $\mu \leq \mu_{IC}^+$, decreases with $\mu$ for $\mu_{IC}^+ < \mu < \hat{\mu}$, and increases with $\mu$ for $\mu \geq \hat{\mu}$.

The key to Proposition 5 is to understand how $\beta$ and $\mu$ impact firm value under the investor control allocation and the contingent control allocation. Under the investor control allocation, it is clear that $\beta$ has no impact on firm value. The firm value is also invariant to $\mu$ if $\mu \leq \mu_{IC}^+$, because for such low values of $\mu$, the investor does not hold up the manager in the good state. On the other hand, if $\mu > \mu_{IC}^+$, then an increase in $\mu$ weakens the manager's incentives and lowers firm value.

Under the contingent control allocation, as $\beta$ increases, the manager is more likely to retain control in the good state and less likely to retain control in the bad state, which strengthens her incentives and improves firm value. Interestingly, and in sharp contrast to the investor control allocation, an increase in $\mu$ improves the manager's incentives under the contingent control allocation by lowering the manager's payoff in the bad state and increases firm value. Formally, it is easy to verify that $P_{CC}^+$ is increasing in $\mu$.

Overall, there is a "U-shaped" relationship between the investor's bargaining power and firm value. The investor control allocation is optimal for low values of $\mu$, in which case, firm value either does not vary with $\mu$ (if $\mu \leq \mu_{IC}^+$) or decreases with $\mu$ (if $\mu > \mu_{IC}^+$). As $\mu$ exceeds a threshold $\hat{\mu}$ which I characterize in the proof of Proposition 5, the contingent control allocation becomes optimal. In this region, firm value increases as $\mu$ increases.

An interesting implication of Proposition 5 is that firm value is maximized at the two extremes of $\mu$: either $\mu \in [0, \mu_{IC}^+]$ or $\mu = 1$. To see why, note that $[P_{CC}^+]_{\mu=1} = P_{IC}^+$, which implies that $[NV^*]_{\mu=0} = [NV^*]_{\mu=1}$. Therefore, when there is potential for hold-up, it would be best if one of the agents had all the bargaining power.

☐ **Optimal control allocation with a general $\Omega = (Y_l, Y_h, D_l, D_h)$.** In the previous subsection, I characterized the optimal control allocation after restricting attention to simpler contracts with $Y_l = Y_h = L$. I now revert to more general contracts where the investor can commit *ex ante* to share some of the liquidation proceeds with the manager (i.e., $Y_l$ and $Y_h$ are not constrained to equal $L$). Although the analysis of the investor control allocation is more complicated in this case, I show that the qualitative results from the previous subsection continue to hold.

*The contingent control allocation ($\Phi = CC$)*

*Lemma 5.* For a contract with the contingent control allocation ($\Phi = CC$), it is optimal to set $Y_l = Y_h = L$. Therefore, the feasibility conditions and the optimal payoff rule $\Omega_{CC}^*$ are the same as those characterized in Proposition 2.

An increase in $Y_l$ has the following countervailing effects on the manager's incentives (i.e., $P_{CC}(\Omega)$): on the one hand, it increases $P_{CC}$ by increasing the investor's payoff when it is in control in the bad state, which occurs with probability $\beta$; on the other hand, it decreases $P_{CC}$ by increasing the investor's renegotiation rents when it is in control in the good state, which occurs with probability $1 - \beta$. The former effect dominates because $\beta > 1 - \beta$ and, hence, $P_{CC}(\Omega)$ is increasing in $Y_l$. It is much easier to show that $P_{CC}(\Omega)$ is (weakly) increasing in $Y_h$. As the financing constraint is also more likely to be met for higher values of $Y_l$ and $S_b(h, \Omega) = \min\{Y_h, \mu L\}$, it is optimal to set $Y_l = L$ and $Y_h = L$, such that $S_b(h, \Omega) = \mu L$. (Actually, $Y_h$ is irrelevant as long as it is greater than $\mu L$, because $S_b(h, \Omega) = \min\{Y_h, \mu L\}$.) As $Y_l = Y_h = L$ arises endogenously, the feasibility conditions for $\Phi = CC$ and $\Omega_{CC}^*$ are the same as in Proposition 2.

*The investor control allocation ($\Phi = IC$).* Next, consider a contract with $\Phi = IC$. By the same intuition as in the case of $\Phi = CC$, it is optimal to set $Y_l = L$. However, the same logic does not apply to $Y_h$, because

$$\frac{dP_{IC}}{dY_h} = \begin{cases} 1 - \beta > 0, & \text{if } Y_h + \mu\Delta \leq p_g D_h \\ 1 - 2\beta < 0, & \text{if } Y_h + \mu\Delta > p_g D_h. \end{cases} \tag{25}$$

Note that $P_{IC}$ is increasing in $Y_h$ if there is no hold-up by the investor in the good state, and is decreasing in $Y_h$ otherwise. Therefore, the feasibility conditions for $\Phi = IC$ as well as the optimal contract are somewhat different from those characterized in Proposition 3.

Define the threshold $\mu_{IC}^h$ such that

$$(1 - \mu_{IC}^h)\Delta = \max\{P_{IC}^- - L, 0\}. \tag{26}$$

It is easy to show that if $\Delta^2 > \frac{4\psi(I-L)}{\theta^2}$, then $0 < \mu_{IC}^+ < \mu_{IC}^- < \mu_{IC}^h \leq 1$.

Also, define

$$P_\beta^+ = \frac{1}{2}\left(\Delta - \frac{\rho\psi}{\theta^2} + \sqrt{\left(\Delta + \frac{\rho\psi}{\theta^2}\right)^2 - \frac{4\psi}{\theta^2}[(I - L) + \mu\rho\Delta]}\right), \tag{27}$$

where $\rho = \frac{1-\beta}{2\beta-1}$.

*Proposition 6.*

(i) A contract with investor control allocation ($\Phi = IC$) is feasible *only if* condition (21) holds and $\mu < \mu_{IC}^h$. If these conditions are satisfied, then:

(a) a contract with $\Phi = IC$ is feasible if $\mu \le \mu_{IC}^-$.

(b) if $\mu_{IC}^- < \mu < \mu_{IC}^h$, then there exists a threshold $\beta_{IC}^h < 1$, such that a contract with $\Phi = IC$ is feasible if, and only if, $\beta \ge \beta_{IC}^h$. The threshold $\beta_{IC}^h$ is high when $\mu$ is high and $\frac{\theta^2\Delta^2}{4\psi} - (I - L)$ is low; $\beta_{IC}^h \to 1$ as $\frac{\theta^2\Delta^2}{4\psi} - (I - L) \to 0$.

(ii) When a contract with $\Phi = IC$ is feasible, the optimal payoff rule $\Omega_{IC}^*$ depends on whether $\mu \le \mu_{IC}^+$ or not.

(a) If $\mu \le \mu_{IC}^+$, then $\Omega_{IC}^*$ is the same as in part (i) of Proposition 3, and $P_{IC}^* = P_{IC}^+$.

(b) If $\mu > \mu_{IC}^+$, then $\Omega_{IC}^*$ is given by $Y_l = L$, $Y_h = \max\{0, L - \frac{P_\beta^+ - (1-\mu)\Delta}{2\beta-1}\}$, $D_l \le \frac{L+\mu\Delta}{p_g}$, and $D_h \le \frac{Y_h^* + \mu\Delta}{p_g}$. In this case,

$$\begin{aligned} P_{IC}^* &= (1 - \mu)\Delta + (2\beta - 1)(L - Y_h^*) \\ &= \min\{P_\beta^+, (1 - \mu)\Delta + (2\beta - 1)L\}. \end{aligned} \tag{28}$$

Observe that if $\mu \le \mu_{IC}^+$, then there is no difference in either the feasibility conditions or the optimal contract when compared with Proposition 3. For such low values of $\mu$, the investor does not hold up the manager in the good state. Therefore, it is optimal to set $Y_h = L$ because that imposes the maximum penalty on the manager in the bad state without hurting her in the good state. As $Y_l = Y_h = L$ arises endogenously in this case, the feasibility conditions and $\Omega_{IC}^*$ are exactly the same as in Proposition 6.

If $\mu > \mu_{IC}^+$, then an increase in $Y_h$ weakens the manager's incentives (i.e., lowers $P_{IC}$) by worsening the hold-up problem in the good state. In this region, it is not optimal to set $Y_h = L$; hence, $P_{IC}(\Omega) = (1 - \mu)\Delta + (2\beta - 1)(L - Y_h)$. Substituting $L - Y_h = \frac{P_{IC}(\Omega) - (1-\mu)\Delta}{2\beta-1}$ and solving for the highest $P_{IC}$ at which the feasibility constraint binds yields the expression for $P_\beta^+$ in equation (27). I show in the proof of Proposition 6 that $P_\beta^+$ is well-defined if $\mu \in [\mu_{IC}^+, \mu_{IC}^-]$ and does not exist if $\mu \ge \mu_{IC}^h$. In the region $\mu \in (\mu_{IC}^-, \mu_{IC}^h)$, $P_\beta^+$ is well-defined only if $\beta$ is sufficiently high.

Comparing Propositions 3 and 6, the key differences are as follows. First, relaxing the constraint $Y_h = L$ may *expand* the range of values of $\mu$ over which the investor control allocation is feasible; $\Phi = IC$ may now be feasible even if $\mu \in (\mu_{IC}^-, \mu_{IC}^h)$ if $\beta$ is sufficiently high. Second, $P_{IC}^*$ is slightly higher (and hence, so is firm value) when the constraint $Y_h = L$ is relaxed; $P_{IC}^* = (1 - \mu)\Delta + (2\beta - 1)(L - Y_h^*)$ as opposed to $P_{IC}^* = (1 - \mu)\Delta$ in part (ii) of Proposition 3.

*Contingent control allocation versus investor control allocation.* Define $\mu_{IC}^l$ to satisfy

$$(1 - \mu_{IC}^l)\, p_g R = P_{IC}^+. \tag{29}$$

As $p_g R = \Delta + L$, it follows that $\mu_{IC}^l > \mu_{IC}^+$.

*Proposition 7.* Suppose $(\theta\Delta)^2 > 4\psi(I - L)$.

(i) If $\mu \le \mu_{IC}^l$, then the investor control allocation strictly dominates the contingent control allocation, regardless of $\beta$.

(ii) If $\mu > \mu_{IC}^l$, then there exists a threshold $\hat{\beta} < 1$ such that the contingent control allocation dominates the investor control allocation only if $\beta \geq \hat{\beta}$.

Proposition 7 is qualitatively similar to Proposition 4, and the underlying intuition is the same: the contingent control allocation is more likely to be optimal when $\beta$ and $\mu$ are high.

## 6. Concluding remarks

■  I provide a theory for the optimal allocation of control rights using a simple model with manager moral hazard and incomplete contracts, where the contract may be renegotiated after the manager has exerted costly effort. The contract is incomplete because it cannot specify a future continuation versus liquidation decision that the firm must make. The incompleteness of the contract also creates the potential for hold-up problems once the investor has sunk in its funds and the manager has exerted costly effort. The contract must allocate the control right over the continuation decision to either the manager or the investor, possibly contingent on the realization of a noisy performance measure. The control allocation affects the manager's *ex ante* incentives because it affects both the likelihood of *ex post* renegotiation and the renegotiation payoffs of the two agents. The optimal control allocation must strengthen the manager's incentives by mitigating hold-up by the investor in profitable states but still punish the manager for bad outcomes.

In this setting, I show that any control allocation that allows the manager to remain in control following a poor performance is suboptimal. In equilibrium, only two control allocations are optimal: either exclusive investor control or a contingent control allocation that allows the manager to remain in control if the firm's performance is good but transfers control to the investor if performance is poor. An increase in the informativeness of the performance measure or an increase in the investor's renegotiation bargaining power makes it more likely that the contingent control allocation is optimal. Thus, this article provides a novel theoretical rationale for contingent control allocations, and explains why it is optimal for investors to take control only following poor performance while leaving control to managers if the firm's performance is good. Unlike existing theory models, I obtain this result without making any ad hoc assumptions regarding the manager's private benefits or future conflicts of interest between the manager and the investor.

### Appendix

This Appendix contains the proofs of all the results presented in the article.

*Proof of Lemma 1.* Suppose the firm is in the good state. If the manager is in control, she will allow the firm to continue operating as per the original contract. The investor, when in control, has two choices: (i) allow continuation under the terms of the original contract, and get a payoff of $p_g D_r$; or (ii) force renegotiation by threatening to liquidate the project. In the latter case, the payoffs to the investor and the manager are $Y_r + \mu\Delta$ and $L - Y_r + (1 - \mu)\Delta$, respectively (i.e., each agent gets his liquidation payoff plus a fraction of the surplus from continuation). Clearly, the investor forces renegotiation if, and only if, $Y_r + \mu\Delta > p_g D_r$. Overall, the investor's expected payoff in the good state is $S_g(r, \Omega) = \max\{Y_r + \mu\Delta, p_g D_r\}$. The manager, being a residual claimant, gets a payoff of $p_g R - S_g(r, \Omega)$.

Suppose the firm is in the bad state. It is obvious that the investor, when in control, will liquidate the project because $Y_r \geq 0$. The manager, when in control, can either liquidate the project and get a payoff of $L - Y_r$, or force renegotiation and get a payoff of $(1 - \mu)L$. Clearly, the manager will force renegotiation if, and only if, $(1 - \mu)L > L - Y_r \Rightarrow Y_r > \mu L$. So the investor's payoff now is $S_b(r, \Omega) = \min\{Y_r, \mu L\}$, whereas the manager's payoff is $L - S_b(r, \Omega)$.

*Proof of Lemma 2.* Lemma 1 allows me to write the following expressions for $S_\Phi(\Omega, e)$ corresponding to $\Phi \in \{IC, MC, CC, ICC\}$:

$$S_{IC}(\Omega, e) = \theta e(\beta S_g(h, \Omega) + (1 - \beta)S_g(l, \Omega)) + (1 - \theta e)(\beta Y_l + (1 - \beta)Y_h) \tag{A1}$$

$$S_{MC}(\Omega, e) = \theta e(\beta p_g D_h + (1 - \beta)p_g D_l) + (1 - \theta e)(\beta S_b(l, \Omega) + (1 - \beta)S_b(h, \Omega)) \tag{A2}$$

$$S_{CC}(\Omega, e) = \theta e(\beta p_g D_h + (1 - \beta)S_g(l, \Omega)) + (1 - \theta e)(\beta Y_l + (1 - \beta)S_b(h, \Omega)) \tag{A3}$$

$$S_{ICC}(\Omega, e) = \theta e(\beta S_g(h, \Omega) + (1 - \beta)p_g D_l) + (1 - \theta e)(\beta S_b(l, \Omega) + (1 - \beta)Y_h). \tag{A4}$$

It follows from equation (7) and equations (A1)–(A4) that $\frac{\partial(V(e)-S_\Phi(\Omega,e))}{\partial e} = \theta P_\Phi(\Omega)$, where $P_\Phi(\Omega)$ for $\Phi \in \{IC, MC, CC, ICC\}$ is defined in equations (8)–(11).

(i) *I prove that under any feasible contract, $P_\Phi(\Omega) < \Delta$.*

I prove this by contradiction for $\Phi = IC$; similar logic applies to $\Phi \in \{MC, CC, ICC\}$. Note that the expression for $S_{IC}(\Omega, e_{IC}(\Omega))$ can be written as

$$S_{IC}(\Omega, e_{IC}(\Omega)) = \theta e_{IC}(\Omega) \cdot (\Delta - P_{IC}(\Omega)) + \beta Y_l + (1 - \beta)Y_h. \tag{A5}$$

Suppose there exists a feasible contract for which $P_{IC}(\Omega) \geq \Delta$. Then, it must be that $S_{IC}(\Omega, e_{IC}(\Omega)) \leq \beta Y_l + (1 - \beta)Y_h \leq L < I$, which violates the feasibility constraint (2). Hence, it follows that, under any feasible contract, $P_{IC}(\Omega) < \Delta$.

(ii) *Characterizing $e_\Phi(\Omega)$.*

The manager's marginal cost of effort is $\psi e$, and her marginal value of effort is $\frac{\partial(V(e)-S_\Phi(\Omega,e))}{\partial e} = \theta P_\Phi(\Omega)$. As $\psi > \theta \Delta$ (Assumption 2) and $\theta \Delta > \theta P_\Phi(\Omega)$, it follows that there exists an $e_\Phi(\Omega) \in (0, 1)$ such that $\psi e_\Phi(\Omega) = \theta P_\Phi(\Omega)$. Hence, the manager's equilibrium effort is $e_\Phi(\Omega) = \frac{\theta P_\Phi(\Omega)}{\psi}$.

*Proof of Proposition 1.* I prove the proposition by showing that for every feasible contract with $\Phi = ICC$ (or $\Phi = MC$), there exists another feasible contract with $\Phi = CC$ that implements a higher effort.

Suppose the contract with the payoff rule $\Omega$ and $\Phi = ICC$ is feasible, that is, $S_{ICC}(\Omega, e_{ICC}(\Omega)) \geq I$. Let $S_g = \beta \max\{Y_h + \mu\Delta, p_g D_h\} + (1 - \beta)p_g D_l$ denote the investor's expected payoff in the good state. Consider an alternative contract with $\Phi = CC$ and the payoff rule $\hat{\Omega}$, where $\hat{Y}_l = \hat{Y}_h = L$, and $\hat{D}_l$ and $\hat{D}_h$ that are chosen such that $\beta p_g \hat{D}_h + (1 - \beta)\max\{L + \mu\Delta, p_g \hat{D}_l\} = S_g$ (i.e., the investor gets the same expected payoff in the good state).

As the manager gets the same expected payoff in the good state under both contracts, it follows that

$$P_{CC}(\hat{\Omega}) - P_{ICC}(\Omega) = \beta L + (1 - \beta)\mu L - \beta \min\{\mu L, Y_l\} - (1 - \beta)Y_h. \tag{A6}$$

Substituting $\min\{\mu L, Y_l\} \leq \mu L$ and $Y_h \leq L$ in the above equation, and simplifying, yields $P_{CC}(\hat{\Omega}) - P_{ICC}(\Omega) \geq (2\beta - 1)(1 - \mu)L > 0$, because $\beta > 0.5$. Hence, it must be that $e_{CC}(\hat{\Omega}) > e_{ICC}(\Omega)$, which in turn, implies that $S_{CC}(\hat{\Omega}, e_{CC}(\hat{\Omega})) > S_{CC}(\hat{\Omega}, e_{ICC}(\Omega))$ because $S_{CC}(., e)$ is increasing in $e$. But,

$$\begin{aligned}
S_{CC}(\hat{\Omega}, e_{ICC}(\Omega)) &= \theta e_{ICC} S_g + (1 - \theta e_{ICC})(\beta L + (1 - \beta)\mu L) \\
&> \theta e_{ICC} S_g + (1 - \theta e_{ICC})(\beta \min\{\mu L, Y_l\} + (1 - \beta)Y_h) \\
&= S_{ICC}(\Omega, e_{ICC}(\Omega)).
\end{aligned} \tag{A7}$$

where the inequality in the second line follows because $\beta L + (1 - \beta)\mu L > \beta \min\{\mu L, Y_l\} + (1 - \beta)Y_h$ (as I showed earlier in the proof). Therefore, $S_{CC}(\hat{\Omega}, e_{CC}(\hat{\Omega})) > S_{ICC}(\Omega, e_{ICC}(\Omega)) \geq I$. Hence, the alternative contract with $\Phi = CC$ is also feasible. As it also implements a higher effort, it strictly dominates the original contract with $\Phi = ICC$.

By a similar logic, it can be shown that $\Phi = MC$ is strictly dominated by $\Phi = CC$. $\qquad$ Q.E.D.

*Proof of Proposition 2.* The necessity of condition (16) follows from the discussion preceding the proposition. Now, I prove the existence of an optimal payoff rule, $\Omega_{CC}^*$, when condition (16) is satisfied (this will also prove the sufficiency of condition (16)). As $P_{CC}^+$ is the largest value of $P_{CC}$ at the which the feasibility constraint binds, an optimal contract must satisfy $P_{CC}(\Omega_{CC}^*) = P_{CC}^+$. Substituting $Y_l = Y_h = L$ and $\Delta = p_g R - L$ into the expression for $P_{CC}$, this is equivalent to

$$\beta p_g(R - D_h) + (1 - \beta)\min\{p_g(R - D_l), (1 - \mu)\Delta\} = P_{CC}^+ + (1 - \beta)(1 - \mu)L. \tag{A8}$$

Consider the following two cases separately:

(i) *Suppose $P_{CC}^+ + (1 - \beta)(1 - \mu)L \leq (1 - \mu)\Delta$.* In this case, equation (A8) can be satisfied by setting $D_h = D_l$, such that $p_g(R - D_l) = P_{CC}^+ + (1 - \beta)(1 - \mu)L$. As $p_g(R - D_l) \leq (1 - \mu)L$, the investor is better off allowing continuation under the original contract.

(ii) *Suppose $P_{CC}^+ + (1 - \beta)(1 - \mu)L > (1 - \mu)\Delta$.* In this case, equation (A8) can be satisfied by choosing any $D_l < \frac{L + \mu\Delta}{p_g}$ (so the investor will hold up the manager when it is in control in the good state), and a $D_h$ that satisfies

$$\beta p_g(R - D_h) + (1 - \beta)(1 - \mu)\Delta = P_{CC}^+ + (1 - \beta)(1 - \mu)L. \tag{A9}$$

As $P_{CC}^+ + (1 - \beta)(1 - \mu)L > (1 - \mu)\Delta$, it follows that $p_g(R - D_h) > (1 - \mu)\Delta$. $\qquad$ Q.E.D.

*Proof of Proposition 3.* The necessity of condition (21) follows from the discussion preceding the proposition.

(i) *Proving the necessity of the condition $\mu \leq \mu_{IC}^-$.* I prove this by contradiction. Suppose there exists a feasible contract when $\mu > \mu_{IC}^-$. As $(1 - \mu_{IC}^-)\Delta = P_{IC}^-$, it follows that $(1 - \mu)\Delta < P_{IC}^-$.

Now, if $Y_l = Y_h = L$, the expression for $P_{IC}$ can be rewritten as

$$P_{IC} = \beta \min\{p_g(R - D_h), (1 - \mu)\Delta\} + (1 - \beta)\min\{p_g(R - D_l), (1 - \mu)\Delta\}$$

$$\leq (1 - \mu)\Delta. \tag{A10}$$

So if $(1 - \mu)\Delta < P_{IC}^-$, it must be that $P_{IC} < P_{IC}^-$. But then, by the definition of $P_{IC}^-$, it must be that $\frac{\theta^2 P_{IC}}{\psi}(\Delta - P_{IC}) - (I - L) < 0$, which contradicts the feasibility of the contract. Therefore, a contract with $\Phi = IC$ is feasible only if $\mu \leq \mu_{IC}^-$.

(ii) *Proving the sufficiency of the conditions in the proposition.*
   I prove the existence of an optimal contract when $\mu \leq \mu_{IC}^-$ and $\Delta^2 \geq \frac{4\psi(I-L)}{\theta^2}$ (which also proves the sufficiency of these conditions). Any feasible $\Omega$ must have $P_{IC}(\Omega) \in [P_{IC}^-, P_{IC}^+]$. Moreover, it must be that $P_{IC}(\Omega) \leq (1 - \mu)\Delta$. Consider the following two cases separately.

(i) *Suppose* $\mu \leq \mu_{IC}^+$ , *which is equivalent to* $(1 - \mu)\Delta \geq P_{IC}^+$. In this case, the requirement that $P_{IC}(\Omega) \leq (1 - \mu)\Delta$ is satisfied by every $P_{IC} \in [P_{IC}^-, P_{IC}^+]$. So an optimal contract will have $P_{IC}^* = P_{IC}^+$. This can be achieved by setting $D_l^* = D_h^* = \frac{p_g R - P_{IC}^+}{p_g}$. Under this contract $p_g(R - D_l^*) = p_g(R - D_h^*) = P_{IC}^+ \leq (1 - \mu)L$, so the investor will not force renegotiation in the good state.

(ii) *Suppose* $\mu_{IC}^+ < \mu \leq \mu_{IC}^-$ , *which is equivalent to* $P_{IC}^- \leq (1 - \mu)\Delta < P_{IC}^+$. In this case, an optimal contract will have $P_{IC}^* = (1 - \mu)\Delta$, which can be achieved by setting $D_l^* < \frac{L + \mu\Delta}{p_g}$ and $D_h^* < \frac{L + \mu\Delta}{p_g}$. Under this contract, the investor will always force renegotiation in the good state to increase its payoff to $L + \mu\Delta$. The feasibility constraint is satisfied because $P_{IC}^* = (1 - \mu)\Delta \in [P_{IC}^-, P_{IC}^+]$. Q.E.D.

*Proof of Proposition 4.* As $\beta L + (1 - \beta)\mu L < L$, it follows that: (a) $P_{IC}^+ > P_{CC}^+$; and (b) condition (21) is satisfied whenever condition (16) is satisfied. Consider the following cases separately.

(i) *Suppose* $\mu \leq \mu_{IC}^+$ .Then, $\Phi = IC$ is feasible and $P_{IC}^* = P_{IC}^+$ (Proposition 3). As $P_{IC}^* = P_{IC}^+ > P_{CC}^+$ , $\Phi = IC$ implements a higher effort than $\Phi = CC$ when the latter is feasible. Therefore, in this case, $\Phi = IC$ strictly dominates $\Phi = CC$ , regardless of $\beta$. The optimal contract will have $\Phi = IC$ and the payoff rule $\Omega^*$ characterized in part (i) of Proposition 3.

(ii) *Suppose* $\mu > \mu_{IC}^+$. Then, consider the following subcases.

   (a) *Suppose* $\mu_{IC}^+ < \mu \leq \frac{1}{2}$. In this region, $\Phi = IC$ is feasible and $P_{IC}^* = (1 - \mu)\Delta$ (Proposition 3). So $\Phi = CC$ can be optimal if, and only if, $P_{CC}^+ > (1 - \mu)\Delta$ (this condition is also sufficient because it implies that $P_{CC}^+$ exists (i.e., $\Phi = CC$ is feasible). Now, $P_{CC}^+$ is increasing in $\beta$, and $\lim_{\beta \to 1} P_{CC}^+ \to P_{IC}^+ > (1 - \mu)\Delta$, where the last inequality follows because $\mu > \mu_{IC}^+$. So there must exist a threshold $\hat{\beta} \in [0.5, 1)$ such that $P_{CC}^+ \geq (1 - \mu)\Delta$ (i.e., $\Phi = CC$ is optimal) if, and only if, $\beta \geq \hat{\beta}$.
   Now, a little algebra shows that if $\mu_{IC}^+ < \mu \leq \frac{1}{2}$, then the requirement $P_{CC}^+ > (1 - \mu)\Delta$ is equivalent to

   $$\frac{\mu(1 - \mu)\theta^2\Delta^2}{\psi} - (I - L + (1 - \beta)(1 - \mu)L) > 0. \tag{A11}$$

   So either there exists a $\hat{\beta} > 0.5$ such that

   $$\frac{\mu(1 - \mu)\theta^2\Delta^2}{\psi} - (I - L + (1 - \hat{\beta})(1 - \mu)L) = 0, \tag{A12}$$

   or condition (A11) holds for all $\beta \in (0.5, 1)$, in which case, $\hat{\beta} = 0.5$.

   (b) *Suppose* $\mu > \frac{1}{2}$. Then, $P_{IC}^* = (1 - \mu)\Delta < \frac{\Delta}{2}$ when $\Phi = IC$ is feasible. As $P_{CC}^+ \geq \frac{\Delta}{2}$, in this case, $\Phi = CC$ is optimal whenever it is feasible. Consider the feasibility condition (16) for $\Phi = CC$. Condition (16) is more likely to be met as $\beta$ increases, and is met as $\beta \to 1$ because $(\theta\Delta)^2 > 4\psi(I - L)$. By the same logic as in ii(a), it follows that there exists a $\hat{\beta} \in [0.5, 1)$ such that condition (16) is satisfied (i.e., $\Phi = CC$ is optimal) if, and only if, $\beta \geq \hat{\beta}$. Either there exists a $\hat{\beta} > 0.5$ such that

   $$(\theta\Delta)^2 - 4\psi(I - L + (1 - \hat{\beta})(1 - \mu)L) = 0, \tag{A13}$$

   or condition (16) is met for all $\beta \in (0.5, 1)$, in which case, $\hat{\beta} = 0.5$.

(iii) *Proving the comparative statics on* $\hat{\beta}$.

   I prove this for the case where $\mu_{IC}^+ < \mu \leq \frac{1}{2}$ (the same logic holds when $\mu > \frac{1}{2}$). As I showed in ii(a), $\hat{\beta}$ is the smallest value of $\beta \in [0.5, 1)$ at which condition (A11) is satisfied. Let $LHS$ denote the expression on the left-hand side of condition (A11). It is evident that $LHS$ is increasing in $\theta$ and $p_g R$. Moreover,

   $$\frac{dLHS}{d\mu} = \frac{(1 - 2\mu)\theta^2\Delta^2}{\psi} + (1 - \beta)L > 0, \tag{A14}$$

because $\mu < 0.5$, and

$$\frac{dLHS}{dL} = 1 - (1-\beta)(1-\mu) - \frac{2\mu(1-\mu)\theta^2\Delta}{\psi} > 0, \tag{A15}$$

because $\mu(1-\mu) < \frac{1}{4}$, $\frac{\theta^2\Delta}{2\psi} < 0.5$, and $(1-\beta)(1-\mu) \le (1-\beta) < 0.5$.

As $LHS$ is increasing in $\theta$, $p_g R$, $\mu$, and $L$, it follows that $\hat{\beta}$ decreases as $\theta$, $p_g R$, $\mu$, and $L$ increase.   *Q.E.D.*

*Proof of Proposition 5.*

(i) Differentiating equation (24) with respect to $\beta$ yields

$$\frac{dNV^*}{d\beta} = \frac{\partial NV^*}{\partial P^*} \cdot \frac{dP^*}{d\beta} = \frac{\theta^2(\Delta - P^*)}{\psi} \cdot \frac{dP^*}{d\beta}. \tag{A16}$$

As $P^* < \Delta$ (by Lemma 2), $\frac{dNV^*}{d\beta}$ will have the same sign as $\frac{dP^*}{d\beta}$. As I showed in Proposition 4, $P^* = P_{CC}^+$ if, and only if, $\mu > \mu_{IC}^+$ and $\beta > \hat{\beta}$. In this region, $\frac{dNV^*}{d\beta} > 0$ because $\frac{dP_{CC}^+}{d\beta} > 0$. Otherwise, $\frac{dNV^*}{d\beta} = 0$ because $P^*$ equals either $P_{IC}^+$ or $(1-\mu)\Delta$, both of which are invariant to $\beta$.

(ii) By the same logic as in (i), $\frac{dP^*}{d\mu}$ will have the same sign as $\frac{dP^*}{d\mu}$. If $\mu \le \mu_{IC}^+$, then $P^* = P_{IC}^+$, which is invariant to $\mu$; in this region, $\frac{dNV^*}{d\mu} = 0$.

Suppose $\mu > \mu_{IC}^+$. I showed in the proof of Proposition 4 that the threshold $\hat{\beta}$ (beyond which $\Phi = CC$ becomes optimal) decreases with $\mu$, and that $\hat{\beta} \to \frac{1}{2}$ as $\mu \to 1$. By the same logic, given a $\beta$, it follows that there exists an $\hat{\mu} < 1$ such that $\Phi = CC$ is optimal for $\mu \ge \hat{\mu}$, and either $\Phi = IC$ is optimal or no financing is feasible if $\mu < \hat{\mu}$. If $\mu < \hat{\mu}$, then $P^* = (1-\mu)\Delta$, which is decreasing in $\beta$; in this region, $\frac{dNV^*}{d\mu} < 0$. If $\mu > \hat{\mu}$, then $P^* = P_{IC}^+$, which is increasing in $\mu$; therefore, in this region, $\frac{dNV^*}{d\mu} > 0$.

*Proof of Lemma 5.* It is evident from equation (10) that $\frac{dP_{CC}}{dY_h} = (1-\beta)\frac{d\min\{Y_h,\mu L\}}{dY_h} \ge 0$ and $\frac{dP_{CC}}{dY_l} = \beta - (1-\beta)\frac{dS_g(l,\Omega)}{dY_l} > 0$ because $\frac{dS_g(l,\Omega)}{dY_l} \le 1$ and $\beta > 0.5$. As the financing constraint is also more likely to be met for higher values of $Y_l$ and $S_b(h,\Omega) = \min\{Y_h, \mu L\}$, it is optimal to set $Y_l$ and $Y_h$ as high as possible. Hence, $Y_l = Y_h = L$ (although $Y_h$ is irrelevant if $Y_h \ge \mu L$). As $Y_l = Y_h = L$ arises endogenously, the feasibility conditions for $\Phi = CC$ and $\Omega_{CC}^*$ are the same as in Proposition 2.   *Q.E.D.*

*Proof of Proposition 6.*

(i) *Proving the necessity of condition (21).*

Here again, $\frac{dP_{IC}}{dY_l} \ge 2\beta - 1 > 0$. So it is optimal to set $Y_l = L$, because the feasibility constraint is also more likely to be met for higher values of $Y_l$. On the other hand, as demonstrated in equation (25), $P_{IC}$ is increasing in $Y_h$ if there is no hold-up by the investor in the good state, and is decreasing in $Y_h$ otherwise.

As the investor's expected payoff in the bad state is $L - (1-\beta)(L - Y_h)$, the feasibility constraint, $S_{IC}(\Omega, e_{IC}(\Omega)) \ge I$, can be rewritten as

$$\frac{\theta^2 P_{IC}}{\psi}(\Delta - P_{IC}) - (I - L) \ge (1-\beta)(L - Y_h). \tag{A17}$$

As discussed before, the left-hand side of condition (A17) is nonnegative only if $P_{IC}^+$ and $P_{IC}^-$ are well-defined, that is, only if $\frac{\theta^2\Delta^2}{4\psi} \ge (I - L)$, which explains the necessity of condition (21).

(ii) *I show that if $\mu \le \mu_{IC}^-$, then condition (21) is also sufficient.*

If $\mu \le \mu_{IC}^+$, the sufficiency of condition (21) follows from the same logic as in the proof of Proposition 3.

Suppose $\mu_{IC}^+ < \mu \le \mu_{IC}^-$ (which is equivalent to $P_{IC}^- \le (1-\mu)\Delta < P_{IC}^+$). In this case, I construct a contract such that there is always hold-up by the investor in the good state (i.e., $S_g(h,\Omega) = Y_h + \mu\Delta$ and $S_g(l,\Omega) = L + \mu\Delta$). Making these substitutions in the expression for $P_{IC}(\Omega)$ yields

$$P_{IC} = (1-\mu)\Delta + (2\beta - 1)(L - Y_h). \tag{A18}$$

Substituting $L - Y_h = \frac{P_{IC} - (1-\mu)L}{2\beta - 1}$ from equation (A18) into the feasibility constraint yields

$$\frac{\theta^2 P_{IC}}{\psi}(\Delta - P_{IC}) - (I - L) \ge \rho(P_{IC} - (1-\mu)\Delta), \tag{A19}$$

where $\rho = \frac{1-\beta}{2\beta-1}$. An optimal contract will choose the highest $P_{IC}$, subject to feasibility constraint (A19), and the requirement that $Y_h \geq 0$ (or, equivalently, the requirement that $P_{IC} \leq (1-\mu)\Delta + (2\beta-1)L$.)

In the remainder of this proof, I refer to the expression on the right-hand side of condition (A19) as the "RHS line," and the expression on the left-hand side as the "LHS curve." The RHS line is positive and increasing in $P_{IC}$ for all $P_{IC} > (1-\mu)\Delta$, whereas the LHS curve is a quadratic expression in $P_{IC}$ that is non negative and has an "inverted U shape" for $P_{IC} \in [P_{IC}^-, P_{IC}^+]$. So if $P_{IC}^- \leq (1-\mu)\Delta < P_{IC}^+$, it follows that there exists a unique $P_\beta^+ \in ((1-\mu)\Delta, P_{IC}^+)$ at which the RHS line intersects the LHS curve (i.e., condition (A19) binds with equality).

Equating the two sides of condition (A19), and solving the resulting quadratic equation, yields the expression for $P_\beta^+$ in equation (27). As $Y_h$ cannot be negative, it follows that an optimal contract will have $\hat{P}_{IC} = \min\{P_\beta^+, (1-\mu)\Delta + (2\beta-1)L\}$.

(iii) *I characterize the feasibility conditions when* $\mu_{IC}^- < \mu < \mu_{IC}^h$ *(which is equivalent to* $\max\{P_{IC}^- - L, 0\} < (1-\mu)\Delta < P_{IC}^-$).

   (a) In this case, additional conditions are required to guarantee that the RHS line intersects the LHS curve. First, it is necessary that $\frac{\theta^2\Delta^2}{4\psi} > (I - L)$, because otherwise the LHS curve is never positive, in which case the RHS line cannot intersect the LHS curve for any $\beta < 1$.

   (b) Suppose $\frac{\theta^2\Delta^2}{4\psi} > (I - L)$. Then, there exists a $\rho^{low} > 0$ such that the line, $\rho^{low}(P_{IC} - (1-\mu)\Delta)$, is tangential to the LHS curve. Clearly, the RHS line will intersect the LHS curve if, and only if, $\rho \leq \rho^{low}$. Suppose $\rho \leq \rho^{low}$. Then, let $P_\beta^-$ and $P_\beta^+$ denote the values of $P_{IC}$ at which the RHS line intersects the LHS curve, where $P_\beta^- \leq P_\beta^+$. For a contract with $\Phi = IC$ to be feasible, it is also necessary that $P_\beta^- \leq (1-\mu)\Delta + (2\beta-1)L$, because, as I explained in ii(b) above, $P_{IC}$ cannot exceed $\min\{P_{IC}^+, (1-\mu)\Delta + (2\beta-1)L\}$.

   It is easy to show that $P_\beta^-$ decreases as $\rho$ decreases. [12] In particular, as $\rho \to 0$, $P_\beta^- \to P_{IC}^- < (1-\mu)\Delta + L$, where the last inequality follows because $\mu > \mu_{IC}^h$. Therefore, there exists a $\rho_{IC}^h \in (0, \rho^{low}]$ such that $P_\beta^- \leq (1-\mu)\Delta + (2\beta-1)L$ if, and only if, $\rho \leq \rho_{IC}^h$. As $\rho$ is decreasing in $\beta$, and $\rho \to 0$ as $\beta \to 1$, this is equivalent to saying that there exists a threshold $\beta_{IC}^h < 1$ such that a contract with $\Phi = IC$ is feasible only if $\beta \geq \beta_{IC}^h$.

(iv) *I show that if* $\mu \geq \mu_{IC}^h$ , *then a contract with* $\Phi = IC$ *is infeasible.*

   (Note that, as per the definition of $\mu_{IC}^h$, this situation can arise only if $P_{IC}^- - L > 0$, because if $P_{IC}^- - L \leq 0$, then $\mu_{IC}^h = 1$.) If $\mu \geq \mu_{IC}^h$, then it must be that $P_{IC}^- \geq (1-\mu)\Delta + L$. As $P_\beta^- > P_{IC}^-$ for any $\beta < 1$, it follows that in this case, $P_\beta^- > (1-\mu)\Delta + (2\beta-1)L$ for all $\beta < 1$. Therefore, a contract with $\Phi = IC$ is infeasible in this case. Q.E.D.

*Proof of Proposition 7* (In this proof, I make use of Lemmas A1 and A2, which are stated and proved below.)

Consider the following cases separately.

(i) *Suppose* $\mu \leq \mu_{IC}^+$. By the same argument as in part (i) of the proof of Proposition 4, it follows that $\Phi = IC$ strictly dominates $\Phi = CC$ in this case, even if the latter is feasible.

(ii) *Suppose* $\mu_{IC}^+ < \mu \leq \mu_{low}$. In this case, again $\Phi = IC$ is feasible. Moreover, as I show in Lemma A1 below, $P_{IC}^* = P_\beta^+ > P_{CC}^+$ for all $\beta$. Therefore, in this case too, $\Phi = IC$ strictly dominates $\Phi = CC$. The optimal contract will have $\Phi = IC$ and the payoff rule $\Omega^*$ characterized in ii(b) of Proposition 6.

(iii) *Suppose* $\mu_{low} \leq \mu < \mu_{IC}^h$. In this case, even if $\Phi = IC$ is feasible, there exists a threshold $\hat{\beta} < 1$ such that $P_{CC}^+ > P_{IC}^*$ if $\beta \geq \hat{\beta}$ (by Lemma A1).

(iv) *Suppose* $\mu \geq \mu_{IC}^h$. In this case, $\Phi = IC$ is infeasible (by Proposition 6). Therefore, $\Phi = CC$ is optimal if it is feasible. Then, by the same argument as in part (iii) of the proof of Proposition 4, it follows that there exists a threshold $\hat{\beta} < 1$ such that $\Phi = CC$ is feasible and, hence, optimal only if $\beta \geq \hat{\beta}$. Q.E.D.

*Lemma A1.* If $\mu \leq \mu_{low}$, then $P_{IC}^* > P_{CC}^+$ regardless of $\beta$. If $\mu > \mu_{low}$, then there exists a $\hat{\beta} < 1$ such that $P_{CC}^+ > P_{IC}^*$ if, and only if, $\beta > \hat{\beta}$.

*Proof of Lemma A1.*

(i) *I characterize the conditions under which* $P_{CC}^+ > P_\beta^+$.

   Recall that $P_{CC}^+$ is the highest value of $P_{CC}$ at which $\frac{\theta^2 P_{CC}}{\psi}(\Delta - P_{CC}) = I - L + (1-\beta)(1-\mu)L$. So if $P_{CC}^+ > P_\beta^+$,

---

[12] As $P_\beta^-$ is the lower root of the quadratic equation (A25), by implicit differentiation, it follows that

$$\frac{dP_\beta^-}{d\rho} = \frac{(P_\beta^- - (1-\mu)\Delta)}{\frac{\theta^2}{\psi}(\Delta - 2P_\beta^-) - \rho} > 0,$$

because in this case $P_\beta^- > P_{IC}^- > (1-\mu)\Delta$ and $2P_\beta^- \leq \frac{\theta^2\Delta}{\psi} - \rho$.

then it must be that

$$\frac{\theta^2 P_\beta^+}{\psi}(\Delta - P_\beta^+) > I - L + (1 - \beta)(1 - \mu)L. \tag{A20}$$

But, by the definition of $P_\beta^+$,

$$\frac{\theta^2 P_\beta^+}{\psi}(\Delta - P_\beta^+) - (I - L) - (1 - \beta)X_\beta^+ = 0, \tag{A21}$$

where

$$X_\beta^+ \equiv \frac{P_\beta^+ - (1 - \mu)\Delta}{2\beta - 1}. \tag{A22}$$

Notice that if $X_\beta^+ < L$, then $P_{IC}^* = P_\beta^+$; otherwise, $P_{IC}^* = (1 - \mu)\Delta + (2\beta - 1)L$.
Combining condition (A20) and equation (A21), it follows that

$$P_{CC}^+ > P_\beta^+ \iff X_\beta^+ > (1 - \mu)L. \tag{A23}$$

As $X_\beta^+$ is increasing in $\beta$ (by Lemma A2), the above condition is more likely to be met as $\beta$ increases. Notice that

$$[X_\beta^+]_{\beta=1} = P_{IC}^+ - (1 - \mu)\Delta, \tag{A24}$$

because $[P_\beta^+]_{\beta=1} = P_{IC}^+$. Consider the following cases separately.

(a) *Suppose* $\mu \leq \mu_{low}$. By the definition of $\mu_{low}$, $\mu \leq \mu_{low} \to (1 - \mu)p_g R \geq P_{IC}^+$. Substituting $p_g R = \Delta + L$, this is equivalent to $P_{IC}^+ - (1 - \mu)\Delta \leq (1 - \mu)L$. Hence, in this region, $[X_\beta^+]_{\beta=1} \leq (1 - \mu)L$, which implies that $X_\beta^+ < (1 - \mu)L$ for all $\beta < 1$. This has the following implications: (i) $X_\beta^+ < (1 - \mu)L \to P_{CC}^+ < P_\beta^+$ (by (A23)); and (ii) $X_\beta^+ < (1 - \mu)L < L$ also implies that $P_{IC}^* = P_\beta^+$. Therefore, in this region, $P_{IC}^* > P_{CC}^+$ for all $\beta$.
(b) *Suppose* $\mu > \mu_{low}$. In this region, $[X_\beta^+]_{\beta=1} > (1 - \mu)L$. Hence, there must be some threshold value of $\beta$ above which $X_\beta^+ > (1 - \mu)L \to P_{CC}^+ > P_\beta^+$. As $P_{IC}^* \leq P_\beta^+$, it follows that there exists a $\hat{\beta} < 1$ such that $P_{CC}^+ > P_{IC}^*$ if, and only if, $\beta > \hat{\beta}$. *Q.E.D.*

*Lemma A2.* $P_\beta^+$ and $X_\beta^+$ are increasing in $\beta$.

*Proof of Lemma A2.*

(i) *Proving that* $P_\beta^+$ *is increasing in* $\beta$. Recall that $P_\beta^+$ is the larger root of the following quadratic equation:

$$\frac{\theta^2 P}{\psi}(\Delta - P) - (I - L) - \rho(P - (1 - \mu)\Delta) = 0, \tag{A25}$$

where $\rho = \frac{1 - \beta}{2\beta - 1}$. Implicitly differentiating with respect to $\rho$ yields

$$\frac{dP_\beta^+}{d\rho} = -\left[\frac{-(P_\beta^+ - (1 - \mu)\Delta)}{\frac{\theta^2}{\psi}(\Delta - 2P_\beta^+) - \rho}\right] < 0, \tag{A26}$$

because $\frac{\theta^2}{\psi}(\Delta - 2P_\beta^+) - \rho < 0$ (see equation (27)) and $P_\beta^+ \geq (1 - \mu)\Delta$. As $\rho$ is decreasing in $\beta$, it follows that $P_\beta^+$ is increasing in $\beta$.
(ii) *Proving that* $X_\beta^+$ *is increasing in* $\beta$. Recall that $X_\beta^+$ is a root of the equation (A21) after substituting $P_\beta^+ = (1 - \mu)\Delta + (2\beta - 1)X_\beta^+$. Let $LHS$ denote the expression on the left-hand side of this equation. Differentiating $LHS$ with respect to $X_\beta^+$, and substituting $(1 - \beta) = \rho(2\beta - 1)$, yields

$$\frac{\partial LHS}{\partial X_\beta^+} = (2\beta - 1)\left(\frac{\theta^2}{\psi}(\Delta - 2P_\beta^+) - \rho\right) \tag{A27}$$

$$< 0,$$

because $2P_\beta^+ > \Delta - \frac{\rho\psi}{\theta^2}$ (by equation (27)). Also,

$$\frac{\partial LHS}{\partial \beta} = X_\beta^+ - \frac{2X_\beta^+ \theta^2}{\psi}(2P_\beta^+ - \Delta) \tag{A28}$$

$$> X_\beta^+ - \frac{2X_\beta^+ \theta^2 \Delta}{\psi}, \quad \text{because} \quad P_\beta^+ < \Delta$$

$$> 0,$$

because $\frac{2\theta^2 \Delta}{\psi} < 1$ (by assumption 2). Therefore, by implicit differentiation, $\frac{dX_\beta^+}{d\beta} = -[\frac{\partial LHS/\partial \beta}{\partial LHS/\partial X_\beta^+}] > 0$. *Q.E.D.*

*Lemma A3.* A contract with $Y_r < 0$ can never be optimal.

*Proof of Lemma A3.* As I showed in Section 5, it is always optimal to set $Y_l = L$ regardless of the control allocation $\Phi$. Moreover, if $\Phi = CC$, then it is also optimal to set $Y_h = L$. Therefore, I only need to show that a contract with investor control ($\Phi = IC$) and $Y_h < 0$ can never be optimal. I prove this by contradiction.

Suppose a contract with $\Phi = IC$ and $Y_h < 0$ is optimal. Under such a contract, when $\tilde{r} = h$ is realized in the bad state (which happens with probability $1 - \beta$), the investor will not liquidate the firm even though liquidation is efficient; both the agents get a payoff of 0. Let $e_\Phi(\Omega) = \frac{\theta P_{IC}(\Omega)}{\psi}$ be the effort induced by this contract. Therefore, firm value is

$$V(\Phi, \Omega) = \theta e_\Phi(\Omega) p_g R + (1 - \theta e_\Phi(\Omega)) \beta L \tag{A29}$$

and

$$P_{IC}(\Omega) = p_g R - \beta S_g(h, \Omega) - (1 - \beta) S_g(l, \Omega). \tag{A30}$$

Consider an alternative contract $(\hat{\Phi}, \hat{\Omega})$ with $\hat{\Phi} = CC$ and a payoff scheme $\hat{\Omega}$ which is identical to $\Omega$ except that $\hat{Y}_h = L$ and $\hat{D}_h = \frac{\beta S_g - (1-\beta)(1-\mu)L}{\beta}$. Under this alternative contract, when $\tilde{r} = h$ is realized in the bad state, the manager will allow the firm to be liquidated after renegotiating the contract; the payoffs to the investor and the manager are $\mu L$ and $(1 - \mu)L$, respectively. Therefore,

$$P_{CC}(\hat{\Omega}) = p_g R - \beta p_g \hat{D}_h - (1 - \beta) S_g(l, \Omega) - (1 - \beta)(1 - \mu)L$$
$$= P_{IC}(\Omega), \quad \text{by construction of } \hat{\Omega}. \tag{A31}$$

Therefore, the alternative contract will implement the same effort as the original contract. However, firm value under the alternative contract is

$$V(\hat{\Phi}, \hat{\Omega}) = \theta e_\Phi(\Omega) p_g R + (1 - \theta e_\Phi(\Omega))L$$
$$> V(\Phi, \Omega), \quad \text{because} \quad \beta < 1, \tag{A32}$$

which contradicts the optimality of the original contract. *Q.E.D.*

# References

AGHION, P. AND BOLTON, P. "An Incomplete Contracts Approach to Financial Contracting." *Review of Economic Studies,* Vol. 59 (1992), pp. 473–494.

BERGLOF, E. AND VON THADDEN, E.-L. "Short-Term versus Long-Term Interests: Capital Structure with Multiple Investors." *Quarterly Journal of Economics,* Vol. 109 (1994), pp. 1055–1084.

BERKOVITCH, E. AND ISRAEL, R. "The Design of Internal Control and Capital Structure." *Review of Financial Studies*, Vol. 9 (1996), pp. 209–240.

BIENZ, C. AND HIRSCH, J. "The Dynamics of Venture Capital Contracts." Working Paper no. 2006/11, Center for Financial Studies, Universität Frankfurt, 2006.

DEWATRIPONT, M. AND MASKIN, E. "Credit and Efficiency in Centralized and Decentralized Economies." *Review of Economic Studies,* Vol. 62 (1995), pp. 541–555.

——— AND TIROLE, J. "A Theory of Debt and Equity: Diversity of Securities and Manager-Shareholder Congruence." *Quarterly Journal of Economics,* Vol. 109 (1994), pp. 1027–1054.

———, LEGROS, P., AND MATTHEWS, S. "Moral Hazard and Capital Structure Dynamics." *Journal of the European Economic Association,* Vol. 1 (2003), pp. 890–930.

GROSSMAN, S. AND HART, O. "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration." *Journal of Political Economy,* Vol. 94 (1986), pp. 691–719.

HART, O. "Financial Contracting." Nancy L. Schwartz Lecture, Northwestern University, 2000.

——— AND MOORE, J. "Default and Renegotiation: A Dynamic Model of Debt." *Quarterly Journal of Economics*, Vol. 113 (1998), pp. 1–41.

HELLMANN, T. "IPOs, Acquisitions, and the Use of Convertible Securities in Venture Capital." *Journal of Financial Economics,* Vol. 81 (2006), pp. 649–679.

HERMALIN, B. AND KATZ, M. "Moral Hazard and Verifiability: The Effects of Renegotiation in Agency." *Econometrica,* Vol. 59 (1991), pp. 1735–1754.

INNES, R. "Limited Liability and Incentive Contracting with *Ex Ante* Action Choices." *Journal of Economic Theory,* Vol. 52 (1990), pp. 45–67.

KAPLAN, S. AND STROMBERG, P. "Financial Contracting Theory Meets the Real World: An Empirical Analysis of Venture Capital Contracts." *Review of Economic Studies,* Vol. 70 (2003), pp. 281–315.

Rajan, R. "Insiders and Outsiders: The Choice between Informed and Arm's-Length Debt." *Journal of Finance,* Vol. 47 (1992), pp. 1367–1400.

Repullo, R. and Suarez, J. "Venture Capital Finance: A Security Design Approach." *Review of Finance,* Vol. 8 (2004), pp. 75–108.

von Thadden, E.-L. "Long-Term Contracts, Short-Term Investment and Monitoring." *Review of Economic Studies,* Vol. 62 (1995), pp. 557–575.