Internet Appendix

for

"Monitoring in Originate-to-Distribute Lending: Reputation versus Skin in the Game"

This Internet Appendix contains the following details which are not reported in the paper due to space constraints: In Section IA.1 we offer a more detailed structure of the borrowing game and monitoring that is consistent with the reduced-form employed in the paper. In Section IA.2 we characterize reputational monitoring equilibria in which market participants condition their beliefs about the bank's monitoring based on the number of defaults d incurred by the bank in the previous 2 periods (i.e., $N = 2$). In Section IA.3 we illustrate how the reputational monitoring equilibrium characterized in Proposition 1 in the paper differs from alternative punishment equilibria supported by grim-trigger punishments for defaults. In Section IA.4 we characterize the "low monitoring" (LM) equilibrium with retention in which the bank retains a portion of the loan and monitors with certainty in the high reputation state (i.e., $q(0) = 1$ and $\alpha(0) > 0$) but sells off the loan entirely and does not monitor in the low reputation state (i.e., $q(1) = 0$ and $\alpha(1) = 0$). We also analyze how our results for the HM equilibrium are affected if the low-reputation bank can be forced to retain $\alpha(1) > \alpha_{pr}$. In Section IA.5 we analyze an alternative reputational equilibrium with lending booms in which the bank would not lose or gain reputation if its loan volume is γ, and reputation depends on defaults only if loan volume is 1. In Section IA.6 we use the *maximal score method* of Fudenberg and Levine (1994) to solve for the maximum bank value attainable in any perfect public equilibrium for the various settings in our paper. Section IA.7 contains the proofs of all results stated in the Internet Appendix.

IA.1 Simple Model of Gains from Monitoring

In this section we offer a more detailed structure of the borrowing game and monitoring that is consistent with the reduced-form model employed in the paper. There are three dates in the borrowing game: the *funding date* at which the borrower raises a dollar of financing to undertake a risky project; an *intermediate date* at which the bank observes a signal on project quality and must decide whether to liquidate the project or to allow it to continue operating; and a *final date* at which the project cash flow is realized. If the project is liquidated at the intermediate date, it yields a liquidation value of $C < 1$ and a zero cash flow at the final date. On the other hand, if the project is allowed to continue operating, it yields a random pledgable cash flow at the final date, denoted $\tilde{X}$, which varies with the state of nature $s \in \{g, b\}$ as follows: $\tilde{X} = X > 1$ if $s = g$ ("good" project), and $\tilde{X} = 0$ if $s = b$ ("bad" project). Given the focus of our paper, we restrict attention to simple debt contracts in which the bank exercises control over the liquidation decision and captures the entire liquidation value C in the event of liquidation.

The state of the project is not known to either the borrower or the bank at the time of financing. It is commonly believed that the good and bad states occur with probability θ and $1-\theta$, respectively. At the funding date, the bank must also decide whether to monitor the borrower at a cost $m > 0$. Monitoring allows the bank to observe the state of the project $s \in \{g, b\}$ perfectly at the intermediate date, thus allowing it to make an informed continuation decision. On the other hand, if the bank doesn't monitor, it only observes a noisy signal $\sigma \in \{g, b\}$ that is imperfectly correlated with the true state s. For tractability, we assume the following conditional distribution for the signal σ:

$$\Pr(\sigma = b | s = b) = 1 \text{ and } \Pr(\sigma = b | s = g) = \varepsilon,$$

where $\varepsilon \in (0, 1)$. In other words, the bad signal is realized with certainty in the bad state, but may also be realized in the good state with a positive probability ε. For example, $\sigma = b$ could denote a deterioration in the borrower's financial condition or a covenant violation, which is more likely when the project is in the bad state but can also happen in the good state. The assumption that the bad signal is realized with certainty in the bad state is not crucial, but simplifies analysis considerably. The variable ε is a measure of signal noise, because the informativeness of the bad signal decreases as ε increases.

Let $p_g \equiv \Pr(s = g | \sigma = g)$ denote the posterior probability of the project being in the good state, conditional on the good signal ($\sigma = g$) being realized at the intermediate date; similarly define $p_b \equiv \Pr(s = g | \sigma = b)$. Applying the Bayes' rule, it follows that $p_g = 1$ and

$$p_b = \frac{\varepsilon\theta}{1 - \theta(1 - \varepsilon)}. \tag{IA.1}$$

It is easily verified that p_b is increasing in ε, and that $p_b \to \theta$ as $\varepsilon \to 1$.

Assumption: $p_b X < C \iff \theta\varepsilon(X - C) < (1 - \theta)C.$

This assumption implies that if the bank hasn't monitored, then it will liquidate the project following the realization of the bad signal. Therefore, the key problem in this set up is that there is "excessive" liquidation if the bank does not monitor.

The cash flow from the project varies with bank monitoring as follows. If the bank does not monitor, it allows the project to continue operating only if the good signal ($\sigma = g$) is realized at the intermediate date, which occurs with probability $\theta(1-\varepsilon)$, and liquidates the project if the bad signal ($\sigma = b$) is realized. Hence, the expected cash flow from the project if the bank does not monitor is $\theta(1-\varepsilon)(X-C)+C$. On the other hand, if the bank monitors, it allows the project to continue operating in the good state, which is realized with probability θ, and liquidates the project only in the bad state. Hence, the expected cash flow from the project if the bank monitors is $\theta(X-C)+C$. Thus, monitoring increases the odds of the project succeeding by $\theta\varepsilon$.

In the reduced-form model employed in the paper, we assumed that the project succeeds with probability p is the bank does not monitor, and with probability $p+\Delta$ if it does, where $\Delta > 0$ denotes the impact of monitoring. This maps into the model described above if we substitute $p = \theta(1-\varepsilon)$ and $\Delta = \theta\varepsilon$. More generally, if the bank monitors with probability q, then the project succeeds with probability $\theta(1-\varepsilon)+q\theta\varepsilon$, which corresponds to $p+q\Delta$ in the reduced-form model employed in the paper.

IA.2 Reputation Based on Performance in the Past Two Periods

In Section 3 of the paper we characterized a reputational monitoring equilibrium in which borrowers and investors condition their beliefs about the bank's monitoring intensity based on whether the bank's loan in the most recent period defaulted or not. In this section we characterize reputational monitoring equilibria in which market participants condition their beliefs about the bank's monitoring based on the number of defaults d incurred by the bank in the previous 2 periods (i.e., $N = 2$). Because the bank can experience two outcomes every period (default or no default), its past performance profile, x, can take on $2^2 = 4$ possible combinations. Denoting default and no default by 1 and 0, respectively, the four possible combinations are: 00, 01, 10 and 11, where the *left-most digit denotes the outcome in the most recent period.* Observe that in the binary system, these performance profiles correspond to $x = 0$, 1, 2 and 3, respectively. The number of defaults, d, is obtained by summing the two digits in the performance profile; i.e.,

$$d(x) = \begin{cases} 2 & \text{if} \quad x = 3 \\ 1 & \text{if} \quad x = 1,2 \\ 0 & \text{if} \quad x = 0 \end{cases} \tag{IA.2}$$

Observe that while both the performance profiles $x = 1$ and $x = 2$ have the same reputation today (because $d(1) = d(2) = 1$), the default is more recent in the $x = 2$ profile compared with the $x = 1$ profile. As we show below, this affects the transition in the bank's reputation over the next period.

The bank's next period performance profile and reputation will depend on whether or not its current period loan defaults. Given the current performance profile x, let $x^-(x)$ and $x^+(x)$ denote its performance profile next period following a default and no default, respectively. It is easily verified that

$$x^-(x) = \begin{cases} 2 & \text{if} \quad x = 0,1 \\ 3 & \text{if} \quad x = 2,3 \end{cases} \tag{IA.3}$$

and

$$x^+(x) = \begin{cases} 0 & \text{if} \quad x = 0,1 \\ 1 & \text{if} \quad x = 2,3 \end{cases} \tag{IA.4}$$

Let $V(x)$ denote the expected discounted value of the bank's profits in equilibrium, given the performance profile x. By the same intuition as in the $N = 1$ case, monitoring is incentive compatible for a bank with the performance profile x only if $V(x^+) - V(x^-) \geq \frac{m}{\delta\Delta}$. Using the performance profile transitions in equations (IA.3) and (IA.4), we obtain the following incentive compatibility conditions:

$$V(0) - V(2) \geq \frac{m}{\delta\Delta}, \tag{IA.5a}$$

$$V(1) - V(3) \geq \frac{m}{\delta\Delta} \tag{IA.5b}$$

By the same logic as in Section 2, the Bellman equation can be written as

$$V(x) = q(d(x)) \cdot (A - m) + B + \delta(p + \Delta q(d(x))) \cdot (V(x^+) - V(x^-)) + \delta V(x^-) \tag{IA.6}$$

Lemma IA.1 *In any monitoring equilibrium, the incentive compatibility conditions (IA.5a) and (IA.5b) bind with equality. Moreover, $q(0) > q(1) > q(2)$; the probability that a bank monitors in the current period is strictly decreasing in the number of defaults it has caused in the previous two periods.*

Although the proof of Lemma IA.1 is more involved than that of Lemma 1 in the paper, the underlying intuition is very similar. For the incentive compatibility condition (IA.5a) to hold, it is necessary that a bank with no past defaults monitor more intensively than a bank that has experienced one default in the past two period. Similarly, for condition (IA.5b) to hold, it is necessary that a bank with only one past default monitor more intensively than one with two past defaults. These conditions can be met only if the two incentive compatibility conditions bind with equality.

As in Section 2 of the paper, we now solve for a full monitoring equilibrium in which the bank fully monitors the loan in the highest reputation state ($q(0) = 1$), but monitors with lower probability in the lower reputation states such that the probability of monitoring is strictly decreasing in the number of past defaults. Specifically, let $q(d)$ be of the form $q(d) = 1 - d\theta$,where $\theta > 0$

is a constant that needs to be characterized. For such an equilibrium to exist, there must exist a $0 < \theta < \frac{1}{N}$ to ensure that the bank monitors with positive probability in all states.

Our next result describes the conditions under which the full monitoring equilibrium is feasible, and characterizes θ and the value function $V(x)$ for $x \in \{0, 1, 2, 3\}$.

Proposition IA.1 *The full monitoring equilibrium described above is feasible if, and only if,*

$$m \leq \frac{\delta}{2}(1+\delta)\Delta^2(1+\beta)(X-C). \tag{IA.7}$$

If condition (IA.7) is satisfied, then the equilibrium is characterized by

$$\theta = \frac{m}{\delta(1+\delta)\Delta^2(1+\beta)(X-C)} \tag{IA.8}$$

and the value function given by: $V(0) = V^*$, $V(1) = V^* - \frac{m}{\delta(1+\delta)\Delta}$, $V(2) = V^* - \frac{m}{\delta\Delta}$, *and* $V(3) = V^* - (\frac{2+\delta}{1+\delta})\frac{m}{\delta\Delta}$.

Using the Bellman Equation (IA.6), and the fact that the incentive compatibility conditions bind with equality (Lemma IA.1), it is easy to show that $V(0) - V(2) = (1+\delta)\theta A$. But incentive compatibility requires that $V(0) - V(2) = \frac{m}{\delta\Delta}$. Equating these two expressions and solving for θ yields the expression in equation (IA.8). For the full monitoring equilibrium to be feasible, it must be that $\theta < \frac{1}{2}$, because otherwise $q(2) = 1 - 2\theta \leq 0$. Setting $\theta < \frac{1}{2}$ yields the feasibility condition in (IA.7).

Note that condition (IA.7) is more likely to be met when the monitoring cost m is low, when the impact of monitoring Δ is high, when the value of liquidity β is high, and when the bank's discount factor δ is high. Also note that, because $\frac{1+\delta}{2} < 1$, condition (IA.7) is more stringent than the equivalent condition for the $N = 1$ case. Substituting $d = 0$ and $V(0) - V(2) = \frac{m}{\delta\Delta}$ in equation (IA.6), and solving the equation for $V(0)$ yields $V(0) = V^*$, where V^* is as defined in equation (8) in the paper. The expressions for $V(1)$, $V(2)$ and $V(3)$ are obtained using the incentive compatibility conditions and the Bellman equation.

Thus, we have shown that using two-period reputation histories do not make full-monitoring equilibria more likely to be feasible, and do not increase the maximum value achieved by the bank.

IA.3 Randomized Grim-Trigger Strategies

In this section we illustrate how the reputational monitoring equilibrium characterized in Proposition 1 in the paper differs from two alternative punishment equilibria which feature grim-trigger punishments for defaults: (a) equilibria in which the bank is punished with certainty for a fixed number of periods following a default (Section IA.3.1); and (b) equilibria which feature coordinated randomized punishments for defaults (Section IA.3.2).

IA.3.1 Grim-trigger punishment for N periods

In this section we consider equilibria in which a bank that experiences a default receives the "unmonitored" loan price (i.e., loan price based on the belief that the bank hasn't monitored the loan) and is not allowed to rebuild its reputation for the next N periods. Otherwise, the market prices the bank's loan under the belief that it has monitored with some positive probability q. The parameters N and q are endogenous. We refer to these as P-equilibria.

Let $V(q, \bar{q}, N)$ denote the "continuation value" (i.e., the expected value of current and future profits) of the bank, given N, the market's conjecture of the bank's monitoring $\bar{q}$, and the bank's actual monitoring q. Let V_{punish} denote the bank's value if it receives the N-period punishment following a default. Recall that the bank's current period surplus under the belief that it hasn't monitored the loan is $B = (1+\beta)(p(X-C) + C - 1 - u)$. Hence,

$$V_{punish} = \frac{\left(1-\delta^N\right)B}{1-\delta} + \delta^N V(q, \bar{q}, N),$$

In the expression above, $\frac{\left(1-\delta^N\right)B}{1-\delta}$ is the present value of an N-period annuity due with a per-period surplus of B, and $\delta^N V(q, \bar{q}, N)$ denotes the present value of the bank's reputation which is restored after N periods.

If the bank monitors with intensity q, then the probability that it will receive the punishment next period is $1 - p - q\Delta$. Therefore, $V(q, \bar{q}, N)$ must satisfy the following equation:

$$V(q, \bar{q}, N) = A\bar{q} + B - mq + \delta\left[(p + q\Delta)\, V(q, \bar{q}, N) + (1 - p - q\Delta)\, V_{punish}\right] \tag{IA.9}$$

As the current period surplus, $A\bar{q} + B$, is sunk at the time the bank chooses its monitoring q, monitoring is incentive-compatible only if

$$\begin{aligned} V(q, \bar{q}, N) - V_{punish} &\geq \frac{m}{\delta\Delta}, \\ \text{i.e., } \left(1-\delta^N\right)\cdot\left(V(q, \bar{q}, N) - \frac{B}{1-\delta}\right) &\geq \frac{m}{\delta\Delta} \end{aligned} \tag{IA.10}$$

Our focus is focus on a full monitoring equilibrium in which the bank fully monitors the loan (i.e., $q = 1$) till it receives the punishment described above. We have the following result.

Proposition IA.2 *A full monitoring P-equilibrium in which the bank always monitors (i.e. $q = 1$) till it receives the N-period punishment described above is feasible if and only if*

$$m \leq \frac{\delta\Delta^2(1+\beta)(X-C)}{1-\delta p} \tag{IA.11}$$

The highest possible continuation value that can be sustained under a full monitoring P-equilibrium *is*

$$V^*_{P-eqm} \leq \frac{1}{(1-\delta)}\left(v - \frac{m(1-p)}{\Delta}\right), \tag{IA.12}$$

where the inequality is generally strict except under a limited set of parameter values.

Note that because $1 - \delta p < 1$, condition (IA.15) is less stringent than the equivalent feasibility condition (9) for the one-period reputational monitoring equilibrium characterized in Proposition 1 of the paper. However, the bank's continuation value V^*_{P-eqm} is generally strictly lower than the V^* under the reputational monitoring equilibrium characterized in Proposition 1 of the paper. This inefficiency arises because N is constrained to be an integer, as a result of which the IC constraint (IA.10) will hold strictly in equilibrium except under a limited set of parameter values.

IA.3.2 Randomized grim-trigger punishments

In this section we consider equilibria with the following randomized punishment if the bank experiences a default: with some probability $\pi > 0$, the bank receives the "unmonitored" loan price of (i.e., loan price based on the belief that the bank hasn't monitored the loan) for the rest of its life. Otherwise, the market prices the bank's loan under the belief that it has monitored with some positive probability q. The parameters π and q are endogenous. Note that, because setting monitoring at zero forever corresponds to an infinite repetition of the one-shot game's Nash equilibrium (no monitoring), it follows that the punishment strategy is in fact subgame perfect.

Let $V(q, \bar{q}, \pi)$ denote the "continuation value" (i.e., the expected value of current and future profits) of the bank, given π, the market's conjecture of the bank's monitoring , and the bank's actual monitoring q. Let V_{punish} denote the bank's value if it receives the randomized punishment. Recall that the bank's current period surplus under the belief that it hasn't monitored the loan is $B = (1+\beta)(p(X-C)+C-1-u)$. Hence, $V_{punish} = (1-\delta)^{-1}B$. If the bank monitors with intensity q, then the probability that it will receive the randomized punishment next period is $\pi(1-p-q\Delta)$. Therefore, $V(q, \bar{q}, \pi)$ must satisfy the following equation:

$$\begin{aligned} V(q, \bar{q}, \pi) \quad &= \quad A\bar{q} + B - mq + \delta[1 - \pi(1-p-q\Delta)] \cdot V(q, \bar{q}, \pi) \\ &\quad\; +\delta\pi(1-p-q\Delta) \cdot (1-\delta)^{-1}B. \end{aligned} \tag{IA.13}$$

As the current period surplus, $A\bar{q} + B$, is sunk at the time the bank chooses its monitoring q, monitoring is incentive-compatible only if

$$\pi[V(q, \bar{q}, \pi) - (1-\delta)^{-1}B] \geq \frac{m}{\delta\Delta}. \tag{IA.14}$$

Our focus is focus on a full monitoring equilibrium in which the bank fully monitors the loan (i.e., $q = 1$) till it receives the randomized punishment described above. We have the following result.

Proposition IA.3 *A full monitoring equilibrium in which the bank always monitors till it receives the randomized punishment (i.e. $q = 1$) is feasible if and only if*

$$m \leq \frac{\delta\Delta^2(1+\beta)(X-C)}{1-\delta p} \tag{IA.15}$$

The highest possible continuation value that can be sustained under a full monitoring equilibrium is

$$V^* = \frac{1}{(1-\delta)}\left(v - \frac{m(1-p)}{\Delta}\right). \tag{IA.16}$$

Note that condition (IA.15) is identical to the feasibility condition for the P-equilibrium characterized above, and is less stringent than the equivalent feasibility condition (9) for the one-period reputational monitoring equilibrium characterized in Proposition 1 of the paper. Furthermore, if condition (IA.15) is satisfied, then the bank's continuation value V^* is that same as that as in the high reputation state under the reputational monitoring equilibrium characterized in Proposition 1 of the paper. This establishes that the full-monitoring equilibrium is easier to achieve if the many investors in the loan market can somehow coordinate on their punishment strategies.

IA.4 Additional Equilibria with Retention

In this section we characterize the "low monitoring" (LM) equilibrium with retention in which the bank retains a portion of the loan and monitors with certainty in the high reputation state (i.e., $q(0) = 1$ and $\alpha(0) > 0$) but sells off the loan entirely and does not monitor in the low reputation state (i.e., $q(1) = 0$ and $\alpha(1) = 0$). We then analyze extension of the HM equilibrium from the text.

IA.4.1 LM Equilibrium with Retention

Under an LM equilibrium, we can write the Bellman equation for $V(0)$ and $V(1)$ as follows:

$$\begin{aligned} V(0) &= (1+\beta-\beta\alpha(0)) \cdot P(1) - (1+\beta) - m + \delta(p+\Delta)\Lambda + \delta V(1) \\ &= A + B - \beta\alpha(0) \cdot P(1) - m + \delta(p+\Delta)\Lambda + \delta V(1), \end{aligned} \tag{IA.17}$$

and

$$\begin{aligned} V(1) &= (1+\beta) \cdot P(0) - (1+\beta) + \delta p\Lambda + \delta V(1) \\ &= B + \delta p\Lambda + \delta V(1). \end{aligned} \tag{IA.18}$$

Differencing the above equations yields $\Lambda = A - \beta\alpha(0) P(1) - m + \delta\Delta\Lambda$, which implies that

$$\Lambda = (1-\delta\Delta)^{-1} \cdot [A - \beta\alpha(0) P(1) - m]. \tag{IA.19}$$

The following conditions need to be satisfied for the LM equilibrium to be feasible: (LM-1) $\delta\Delta\Lambda < m$ so that the low-reputation bank does not monitor; (LM-2) $\delta\Delta\Lambda + \alpha(0) \cdot \Delta(R(1) - C) \geq m$ so that the high-reputation bank has incentives to monitor; (LM-3) the high-reputation bank should not deviate to $\alpha'(0) = 0$ and $q'(0, \alpha'(0)) = 0$ after the loan rate has been set at $R(1)$; and (LM-4) the low-reputation bank should not deviate to some $\alpha'(1) > 0$ and $q(1, \alpha'(1)) = 1$ after its loan rate has been set at $R(0)$.

Assuming these four conditions are met, bank value in the high reputation state is

$$V_{LM}(0) = (1-\delta)^{-1} \cdot \left[B + \frac{(1-\delta+\delta p)(A - \beta\alpha(0)P(1) - m)}{(1-\delta\Delta)}\right] \tag{IA.20}$$

We obtain this expression by substituting $V(1) = V(0) - \Lambda$ along with the expression for Λ from equation (IA.19) into equation (IA.17), and solving the resulting equation for $V(0)$.

We can now compare welfare under the HM equilibria and the LM equilibria. We have the following result.

Proposition IA.4 *If the HM equilibrium is feasible, then either it strictly dominates the LM equilibrium or the LM equilibrium is not feasible.*

As we showed in Proposition 3 and Corollary 1 in the paper, the HM equilibrium achieves the constrained first-best outcome: the value of the high-reputation bank, $V_{HM}(0) = V^*$. If $\delta\Delta A \geq m$, then it is easily verified that $V^* > V_{LM}(0)$; hence in this case, the HM equilibrium strictly dominates the LM equilibrium even if the latter is feasible. On the other hand, if $\delta\Delta A < m$, then we show that LM equilibrium is infeasible whenever the HM equilibrium is feasible. This is because the IC constraint for the high-reputation bank under the LM equilibrium cannot be met if $\delta\Delta A < m$ and $\delta\Delta\alpha_{pr}\beta P(1) \geq m$, and the latter condition holds if the HM equilibrium is feasible.

The upshot of Proposition IA.4 is that the LM equilibrium is the dominant equilibrium only if it is feasible and the HM equilibrium is infeasible. Our next result shows that this is impossible when $u = 0$.

Proposition IA.5 *If $u = 0$, then the LM equilibrium is infeasible whenever the HM equilibrium is infeasible.*

We fully characterize the equilibrium conditions (LM-1) through (LM-4) in the proof of Proposition IA.5. These conditions require contradictory requirements on $\alpha(0)$: condition (LM-3) requires that $\alpha(0)$ not be too high (i.e., $\alpha(0) \leq \hat{\alpha}_{ND,high}$), whereas condition (LM-4) requires that $\alpha(0)$ not be too low (i.e., $\alpha(0) \geq \hat{\alpha}_{ND,low}$). Condition (LM-2) is more complicated and requires an upper bound on $\alpha(0)$ if $\delta\Delta A \geq m$, but requires a lower bound on $\alpha(0)$ if $\delta\Delta A < m$.

In general it is hard to fully characterize the parameter set under which the LM equilibrium is feasible. However if $u = 0$, then we show that conditions (LM-2), (LM-3) and (LM-4) cannot hold simultaneously if the HM equilibrium is infeasible. In other words, if $u = 0$, then the LM equilibrium is infeasible whenever the HM equilibrium is infeasible.

IA.4.2 HM Equilibrium with Higher Retention by Low-Reputation Bank

Unlike the main text, suppose investor beliefs allow HM equilibria with $\alpha(1) > \alpha_{pr}$. (Again, this requires that market participants punish any deviation to lower retention by assigning the bank to d_{low} the following period.)

We now outline how an equilibrium with $\alpha(1) > \alpha_{pr}$ would work. We must still have $\alpha(0) < \alpha_{pr}$; otherwise, the bank is always holding more than is needed to ensure full monitoring even without reputation benefits. Again, let $\alpha(0) = (1 - \rho)\alpha_{pr}$, where ρ serves to index equilibria by the high-reputation bank's (lack of) retention. The same analysis as in Proposition 3 shows that any equilibrium in which the high-reputation bank's IC does not bind is dominated by one in which it binds. Focusing on the binding case, we have $\Lambda = [\alpha(1) - \alpha(0)]\beta P(1) = \rho m/\delta\Delta$, and

$$\alpha(1) = \alpha(0) + \frac{\Lambda}{\beta P(1)} = (1 - \rho)\alpha_{pr} + \frac{\rho m}{\delta\Delta\beta P(1)}.$$

Note that $d\alpha(1)/d\rho \gtrless 0$ if and only if $m/\delta\Delta\beta P(1) \gtrless \alpha_{pr}$. It follows that an HM equilibrium can have $\alpha(1) > \alpha_{pr}$ if and only if $m/\delta\Delta\beta P(1) > \alpha_{pr}$, which in turn means that increasing the index ρ (decreasing retention by the high-reputation bank) *increases* the amount the low-reputation bank must retain.

Since $\alpha(1)$ is largest for $\rho = 1$ (so that $\alpha(0) = 0$), a sufficient condition for $\alpha(1) < \overline{\alpha}$ is $m/\delta\Delta\beta P(1) < \overline{\alpha}$, which is equivalent to $\delta\beta P(1) > R(0) - C$. For simplicity, we will focus on equilibria where $\alpha(1) < \overline{\alpha}$. (This means we don't need to consider downward deviations to $\overline{\alpha}$ instead of 0.) It is easy to show that this means Condition (16), the non-deviation condition in Proposition 3, is unchanged and that the value functions of the high- and low-reputation banks have the same form as in that proposition. We have

Proposition IA.6 *Suppose $\alpha_{pr}\beta P(1) < m/\delta\Delta < \overline{\alpha}\beta P(1)$. For any $\rho \in (0, 1]$, an HM-type equilibrium with $\alpha(1) > \alpha_{pr}$ exists if and only if Condition (16) from Proposition 3 in the main text holds. If this condition is satisfied, $\alpha(0) = (1-\rho)\alpha_{pr}$, $\alpha(1) = \alpha(0) + [\rho m/\delta\Delta\beta P(1)]$, and $q(0) = q(1) = 1$. Moreover, the high-reputation bank's value function $V_{HM}(0)$ is the same as in Condition (17) of the main text, and the low-reputation's bank's value function $V_{HM}(1)$ is also the same as in Proposition 3: $V_{HM}(1) = V_{HM}(0) - \rho m/\delta\Delta$.*

We now turn to the question of whether welfare (measured by the high-reputation bank's value function $V(0)$) and the feasibility condition (Condition (16) in the main text) are increasing or decreasing in the retention index ρ. We have

Corollary 1 *Suppose $\alpha_{pr}\beta P(1) < m/\delta\Delta < \overline{\alpha}\beta P(1)$.*
(i) $V_{HM}(0)$ is inceasing in ρ if and only if $(1+\beta)P(1) > R(1)$, which is equivalent to $\delta\beta P(1) > \delta(1-p-\Delta)(R(1)-C)$.
(ii) Consider Condition (16) in the main text. (a) If $A-(1-\rho)m > \beta\overline{\alpha}\overline{P}(1, R(0))$, then that condition is looser as ρ increases if and only if

$$\delta\beta P(1) > \frac{1+\delta(1-p-\Delta)}{1+\delta} \cdot (R(1)-C).$$

(b) If $A-(1-\rho)m \leq \beta\overline{\alpha}\overline{P}(1, R(0))$, then Condition (16) in the main text is looser if and only if

$$\delta\beta P(1) > \left[1-\delta\Delta-\frac{\delta p}{1+\delta}\right] \cdot (R(1)-C).$$

(iii) In both (ii.a) and (ii.b), the terms multiplying $R(1)-C$ are strictly between $\delta(1-p-\Delta)$ and 1. It follows that whenever $V_{HM}(0)$ is deceasing in ρ, Condition (16) in the main text is looser as ρ decreases.

The upshot is that when capital or liquidity costs are low or risk is high (so that $(1+\beta)P(1) < R(1)$), the optimal HM equilibrium involves $\rho = 0$—i.e. the bank always retains α_{pr} regardless of reputation—and the feasibility condition (16) in the main text is most likely to be feasible for $\rho = 0$, too. In this case, the cost of foregone capital or liquidity is low relative to the direct monitoring incentive effects of retaining a larger share of the loan, so higher retention is a cheaper form of incentive than reputation. This is precisely the opposite of the case in the main text. (In that case $\delta\beta P(1) > R(1)-C$, which guarantees that both $V_{HM}(0)$ and Condition (16) are inceasing in ρ.) Note that an increase in loan risk $X-C$ that leaves expected loan value unchanged increases $R(1)$ while not affecting $P(1)$ makes it more likely that $(1+\beta)P(1) < R(1)$.

Again, as discussed in the text, the beliefs required to support an HM equilibrium where the low-reputation bank retains more than α_{pr} seem implausible. As we have just shown, even if such beliefs can occur, sufficiently low β or sufficiently high loan risk make retention superior to reputation.

IA.5 Alternative Reputational Equilibrium with Lending Booms

In Section 5 of the paper ("Reputation and Lending Booms") we examined reputational equilibrium in which the bank's reputation next period depends only on whether its current loan defaulted or not, regardless of the lending volume. In this section we consider an alternative equilibrium in which the bank would not lose or gain reputation if its loan volume is γ, and reputation only works if loan volume is 1. Given that there are only two reputation states ($d \in 0, 1$) in this equilibrium, the analysis in Section 5.3 remains unchanged, and Lemma 4 continues to hold. That is, in any equilibrium in which monitoring occurs with positive probability, either $IC(I)$ bonds and $IC(\gamma)$ fails to hold, or else $IC(\gamma)$ binds and $IC(1)$ holds strictly.

As in Section 5.4 of the paper, we will examine equilibria in which $IC\,(1)$ binds but $IC\,(\gamma)$ fails to hold, so that the bank monitors with positive probability if it lends 1 unit but will not monitor if it lends γ units in a boom (i.e., $q\,(d,\gamma) = \overline{q}(d,\gamma) = 0$ for $d \in \{0,1\}$). Under this equilibria, if the bank lends γ units in a boom, it obtains a value of

$$V\,(d,\gamma) = \gamma B + \delta V\,(d)\,.$$

On the other hand, if the bank lends 1 unit, it obtains a value of

$$V_n\,(d) = Aq\,(d,1) + B + (pm/\Delta) + \delta V(1).$$

Note that the expression for $V\,(d,\gamma)$ is different from $V_{\gamma,nm}$ in the proof of Propositions 4 through 7 in the paper. Indeed, in an equilibrium in which $\Lambda = \frac{m}{\delta\Delta}$ (i.e., $IC\,(1)$ binds), it is easily seen that $V\,(1,\gamma) < V_{\gamma,nm} < V\,(0,\gamma)$. Hence, compared to the equilibria examined in the paper, the bank is now more (less) likely to prefer lending γ in the boom state when it is in the high (low) reputation state.

Proposition IA.7 *A monitoring equilibrium in which a low-reputation bank lends 1 unit during booms whereas a high-reputation bank lends γ units is infeasible.*

The intuition behind this result is similar to that behind Proposition 4 in the paper. A low-reputation bank has strictly higher incentive than a high-reputation bank to lend γ units in a boom and shirk on monitoring because it has less to lose from any resultant loss of reputation next period. Hence, equilibria in which a high-reputation bank lends γ in a boom whereas a low-reputation bank lends 1 unit are infeasible.

Proposition IA.8 *A tight credit ("TC") monitoring equilibrium in which the bank lends 1 unit in both economic states regardless of its reputation is feasible if and only if $m \leq \frac{\delta\Delta\cdot[A-(\gamma-1)B]}{1-\delta p}$. In this equilibrium, the bank monitors with probability 1 in the high reputation state, with probability $\widehat{q}_{tc} = 1 - m/\delta\Delta A$ in the low reputation state, and its value in the high reputation state is $V_{tc}\,(0) = \frac{1}{1-\delta}\left(A + B - \frac{m(1-p)}{\Delta}\right)$.*

The TC equilibrium above is more likely to be feasible compared to the similar equilibrium in Proposition 5 of the paper, although the expressions for $\hat{q}_{tc}$ and $V_{tc}\,(0)$ are identical. The intuition for this is as follows: the feasibility condition is mainly tied to the non-deviation constraint of the low-reputation bank (see the proof for details). Because the low-reputation bank is now less likely to prefer lending γ in the boom state compared to the equilibria examined in the paper, this explains why the TC equilibrium above is more likely to be feasible compared to that in the paper.

Proposition IA.9 *A partially tight credit ("PTC") monitoring equilibrium in which a high-reputation bank lends 1 unit during booms whereas a low-reputation bank lends γ units is feasible if and only if*

$$m \leq \delta\Delta \cdot \min\left\{\frac{A-(1-\phi)(\gamma-1)B}{(1-\delta p+\delta p\phi)}, \frac{A-(\gamma-1)B}{\delta-\delta p}\right\}. \tag{IA.21}$$

In this equilibrium, a high-reputation bank monitors with probability $\hat{q}_{0,ptc} \equiv \min\left\{1, \frac{(\gamma-1)B}{A}+\frac{m(1-\delta p)}{\delta\Delta A}\right\}$ *in both economic states. By contrast, a low-reputation bank monitors with probability* $\hat{q}_{1,ptc} \equiv \frac{1}{\phi}\left[\hat{q}_{0,ptc}-\frac{(1-\phi)(\gamma-1)B}{A}-\frac{m(1-\delta p+\delta p\phi)}{\delta\Delta A}\right]$ *in a normal economy and does not monitor at all in a boom. Bank value in the high reputation state is*

$$V_{ptc}(0) = (1-\delta)^{-1} \cdot \left(A\hat{q}_{0,ptc} + B - \frac{m(1-p)}{\Delta}\right)$$

The PTC equilibrium above delivers a (weakly) lower $\hat{q}_{0,ptc}$, and hence, (weakly) lower $V_{ptc}(0)$ than the corresponding PTC equilibrium in Proposition 6 of the paper. The effect on feasibility is more complicated: one of the conditions (see F1 in proof of proposition) is looser above compared to the paper, whereas the other (F2) is stricter above compared to that in the paper.

Proposition IA.10 *A loose credit ("LC") monitoring equilibrium in which the bank always lends γ units during booms regardless of reputation is feasible if and only if*

$$m \leq \phi\delta\Delta \cdot \min\left\{\frac{A}{1-\delta+\delta\phi}, \frac{(\gamma-1)B}{1-\delta+\delta\phi p}\right\}. \tag{IA.22}$$

In this equilibrium, the bank never monitors in a boom. In a normal economy, the high-reputation bank monitors with probability $\hat{q}_{0,lc} \equiv \min\left\{1, \frac{(\gamma-1)B}{A}+\frac{m(1-p)}{\Delta A}\right\}$ *and the low-reputation bank monitors with probability* $\hat{q}_{1,lc} \equiv \hat{q}_{0,lc} - \left(\frac{1-\delta+\delta\phi}{\phi}\right)\frac{m}{\delta\Delta A}$. *Bank value in the high-reputation state is given by*

$$V_{lc}(0) \equiv (1-\delta)^{-1} \cdot \left(\phi A\hat{q}_{0,lc} + \phi B + (1-\phi)\gamma B - \frac{\phi m(1-p)}{\Delta}\right)$$

The LC equilibrium above delivers a (weakly) higher $\hat{q}_{0,lc}$ and a strictly higher $V_{lc}(0)$ than the corresponding LC equilibrium in Proposition 7 of the paper; the reason $V_{lc}(0)$ is strictly higher is because the high-reputation bank does not suffer loss of reputation by lending γ in the boom state.

IA.6 Characterizing the Maximum Attainable Bank Value

In this section we use the *maximal score method* of Fudenberg and Levine (1994) to solve for the maximum bank value attainable in any perfect public equilibrium for the various settings in our paper. Because we have only one long-run player, our setting falls under the category of games with a product structure for which the Fudenberg and Levine method takes a simple form.

IA.6.1 Maximum bank value in baseline model

Consider our baseline setting in which the bank lends 1 unit each period. We use the following notation:

- Let V^* denote the maximum bank value.
- Let $K \in \{Y, N\}$ denote the pure strategy set for bank monitoring, where Y denotes that the bank monitors and N denotes that it doesn't.
- Let $q \in [0,1]$ denote the mixed strategy played by short-lived players (i.e., borrowers and investors in the credit market) to price the bank's loan, and denotes the market's assessment of the probability that the bank has monitored.
- Let U_+ and U_- denote the PV of bank payoffs in the stage game following no default and default, respectively.

Because the transition matrix has full rank, it follows that V^* is the solution to the following linear programming problem:

$$\max_{q,U_+,U_-} \; V,$$

subject to the following conditions:

$$\begin{aligned} V &= Aq + B - m + \delta \left[(p + \Delta) U_+ + (1 - p - \Delta) U_- \right] \text{ for } q > 0, && \text{(IA.23)} \\ &\geq Aq + B - m + \delta \left[(p + \Delta) U_+ + (1 - p - \Delta) U_- \right] \text{ for } q = 0, && \text{(IA.24)} \end{aligned}$$

$$\begin{aligned} V &= Aq + B + \delta \left[pU_+ + (1 - p) U_- \right] \text{ for } q < 1, && \text{(IA.25)} \\ &\geq Aq + B + \delta \left[pU_+ + (1 - p) U_- \right] \text{ for } q = 1, && \text{(IA.26)} \end{aligned}$$

and

$$V \geq U_+,\, V \geq U_-,\, U_+ \geq 0,\, U_- \geq 0.$$

Conditions (IA.23) through (IA.26) capture the idea that V equals the payoff from a pure strategy $K \in \{Y, N\}$ if investors attach a positive probability to that action, but V exceeds the payoff from a pure strategy K if investors attach a zero probability to that action.

Proposition IA.11 *The linear programming problem above has the following solution:* $q = 1$*,*

$$V^* = (1 - \delta)^{-1} \cdot \left(A + B - \frac{(1 - p)\, m}{\Delta} \right) \tag{IA.27}$$

$U_+ = V^*$ *and* $U_- = V^* - \frac{m}{\delta \Delta}$*. This solution exists if* $m \leq \frac{\delta \Delta (A+B)}{1 - \delta p}$*.*

Note that the high-reputation bank's value under the full-monitoring equilibrium (Proposition 1) and the optimal high monitoring equilibrium with retention (Proposition 2) equal the maximum attainable bank value, V^*.

IA.6.2 Maximum bank value with loan retention and stochastic loan demand

We now solve for the maximum bank value in the setting with stochastic lending volume when loan retention is credible. We modify the notation from Section IA.6.1 as follows:

- Let V^* denote the maximum bank value, and let V_n and V_b denote the bank value in the normal state and the boom state, respectively. Because we are interested in characterizing the maximal bank value, we will focus on equilibria where the bank always meets the boom demand because this leads to higher surplus.
- Let $K_1 \in \{Y, N\}$ and $K_\gamma \in \{Y, N\}$ denote the pure strategy set for bank monitoring for $\ell = 1$ and $\ell = \gamma$, respectively, where Y denotes that the bank monitors and N denotes that it doesn't.
- Let $q_1 \in [0, 1]$ and $q_\gamma \in [0, 1]$ denote the mixed strategy played by short-lived players (i.e., borrowers and investors in the credit market) to price the bank's loan for $\ell = 1$ and $\ell = \gamma$, respectively, where q denotes the market's assessment of the probability that the bank has monitored.
- Let α_1 and α_γ denote the loan retentions corresponding to $\ell = 1$ and $\ell = \gamma$, respectively.
- Let U_+ and U_- denote the PV of bank payoffs in the stage game following no default and default, respectively.
- Also let $P(1, R(q)) = (p + \Delta)(X - C) + C - \frac{u(p+\Delta)}{p+q\Delta}$ and $P(0, R(q)) = p(X - C) + C - \frac{up}{p+q\Delta}$ denote the expected value of a loan with face value $R(q)$ under full monitoring and no monitoring, respectively.

As per the Fudenberg and Levine approach,V^* is the solution to the following linear programming problem:

$$\max_{q_1, q_\gamma, \alpha_1, \alpha_\gamma, U_+, U_-} V = \phi V_n + (1 - \phi) V_b,$$

subject to the following conditions:

$$\begin{aligned} V_n &= Aq_1 + B - \beta\alpha_1 P(1, R(q_1)) - m + \delta[(p + \Delta)(U_+ - U_-) + U_-] \text{ if } q_1 > 0 &\text{(IA.28)} \\ &\geq Aq_1 + B - \beta\alpha_1 P(1, R(q_1)) - m + \delta[(p + \Delta)(U_+ - U_-) + U_-] \text{ if } q_1 = 0 &\text{(IA.29)} \end{aligned}$$

$$V_n = \begin{bmatrix} Aq_1 + B - \beta\alpha_1 P(1, R(q_1)) - \alpha_1 \Delta (R(q_1) - C) \\ +\delta (p(U_+ - U_-) + U_-) \end{bmatrix} \text{ if } q_1 < 1 \tag{IA.30}$$

$$\geq \begin{bmatrix} Aq_1 + B - \beta\alpha_1 P(1, R(q_1)) - \alpha_1 \Delta (R(q_1) - C) \\ +\delta (p(U_+ - U_-) + U_-) \end{bmatrix} \text{ if } q_1 = 1 \tag{IA.31}$$

$$V_b = \begin{bmatrix} \gamma \cdot [Aq_\gamma + B - \beta\alpha_\gamma P(1, R(q_\gamma)) - m] \\ +\delta [(p + \Delta)(U_+ - U_-) + U_-] \end{bmatrix} \text{ if } q_\gamma > 0 \tag{IA.32}$$

$$\geq \begin{bmatrix} \gamma \cdot [Aq_\gamma + B - \beta\alpha_\gamma P(1, R(q_\gamma)) - m] \\ +\delta [(p + \Delta)(U_+ - U_-) + U_-] \end{bmatrix} \text{ if } q_\gamma = 0 \tag{IA.33}$$

$$V_b = \begin{bmatrix} \gamma \cdot [Aq_\gamma + B - \beta\alpha_\gamma P(1, R(q_\gamma)) - \alpha_\gamma \Delta (R(q_\gamma) - C)] \\ +\delta [p(U_+ - U_-) + U_-] \end{bmatrix} \text{ if } q_\gamma < 1 \tag{IA.34}$$

$$\geq \begin{bmatrix} \gamma \cdot [Aq_\gamma + B - \beta\alpha_\gamma P(1, R(q_\gamma)) - \alpha_\gamma \Delta (R(q_\gamma) - C)] \\ +\delta [p(U_+ - U_-) + U_-] \end{bmatrix} \text{ if } q_\gamma = 1 \tag{IA.35}$$

$$V \geq U_+,\ V \geq U_-,\ U_+ \geq 0,\ U_- \geq 0.$$

Conditions (IA.28) through (IA.31) capture the idea that V_n equals the payoff from a pure strategy $K_1 \in \{Y, N\}$ if investors attach a positive probability to that action, but V_n exceeds the payoff from a pure strategy K_1 if investors attach a zero probability to that action; a similar logic underlies conditions (IA.32) through (IA.35).

Proposition IA.12 *The linear programming problem above has the following solution:* $q_1 = 1$, $q_\gamma = 1$, $\alpha_1 = 0$, $\alpha_\gamma = \frac{(\gamma-1)\alpha_{pr}}{\gamma}$,

$$V^* = (1-\delta)^{-1} \cdot \left[\gamma_\phi \cdot (A + B - m) - (1-\phi)(\gamma - 1)\beta\alpha_{pr} P(1) - \frac{m(1-p-\Delta)}{\Delta}\right], \tag{IA.36}$$

$U_+ = V^*$ *and* $U_- = V^* - \frac{m}{\delta\Delta}$. *This solution exists if*

$$m \leq \frac{\delta\Delta\gamma_\phi \cdot (A+B)}{\delta\Delta\gamma_\phi + (1 - \delta(p+\Delta)) + \delta(\gamma_\phi - 1)\beta(p+\Delta)\left(1 + \frac{C}{((p+\Delta)(X-C)-u)}\right)} \tag{IA.37}$$

Comparing the expression for V^* in equation (IA.36) with that for $V_{opt}(0)$ from Proposition 9 in the paper, and noting that $\gamma_\phi - 1 = (1-\phi)(\gamma - 1)$, it is clear that $V^* = V_{opt}(0)$.

IA.7 Proofs of Results in the Internet Appendix

Proof of Lemma IA.1: The proof utilizes the following expressions that are obtained by using the Bellman equation (IA.6) in conjunction with the transition equations (IA.3) and (IA.4):

$$V(0) = q(0)\cdot(A-m) + B + \delta(p+\Delta q(0))\cdot(V(0)-V(2)) + \delta V(2), \tag{IA.38a}$$

$$V(1) = q(1)\cdot(A-m) + B + \delta(p+\Delta q(1))\cdot(V(0)-V(2)) + \delta V(2), \tag{IA.38b}$$

$$V(2) = q(1)\cdot(A-m) + B + \delta(p+\Delta q(1))\cdot(V(1)-V(3)) + \delta V(3), \tag{IA.38c}$$

$$\text{and } V(3) = q(2)\cdot(A-m) + B + \delta(p+\Delta q(2))\cdot(V(1)-V(3)) + \delta V(3), \tag{IA.38d}$$

(1) Proving that $V(0) - V(2) = \frac{m}{\delta\Delta}$.

We will prove this by contradiction. Suppose $V(0) - V(2) > \frac{m}{\delta\Delta}$. Then, banks with types $x = 0$ and $x = 1$ will strictly prefer to monitor, so that $q(0) = q(1) = 1$, which in turn implies that $V(0) - V(1) = 0$. But if $V(0) = V(1)$, then it must be that $V(1) - V(2) = V(0) - V(2) > \frac{m}{\delta\Delta}$.

Next, subtracting equation (IA.38c) from equation (IA.38b), and using the fact that $V(0) - V(1) = 0$ yields $V(1) - V(2) = \delta[1 - p - \Delta q(1)] \cdot [V(2) - V(3)]$. As $V(1) - V(2) > \frac{m}{\delta\Delta} > 0$ and $1-p-\Delta q(1) > 0$, it follows that $V(2)-V(3) > 0$. Combining $V(1)-V(2) > \frac{m}{\delta\Delta}$ and $V(2)-V(3) > 0$ yields that $V(1) - V(3) > \frac{m}{\delta\Delta}$.

Next, if $V(1) - V(3) > \frac{m}{\delta\Delta}$, then it follows that banks with types $x = 2$ and $x = 3$ will strictly prefer to monitor, so that $q(1) = q(2) = 1$. However, $q(1) = q(2)$ implies that $V(2)-V(3) = 0$, which contradicts our earlier finding that $V(2) - V(3) > 0$. Therefore, it must be that $V(0) - V(2) = \frac{m}{\delta\Delta}$. By a similar logic, it can be argued that $V(1) - V(3) = \frac{m}{\delta\Delta}$.

(2) Proving that $q(0) > q(1) > q(2)$.

After substituting $V(0)-V(2) = V(1)-V(3) = \frac{m}{\delta\Delta}$, it is easy to see that $V(0)-V(1) = (q(0)-q(1))A$, and $V(2) - V(3) = (q(1) - q(2))A$. Therefore, it is sufficient to show that $V(0) - V(1) > 0$ and $V(2) - V(3) > 0$.

Note that $V(0) - V(2) = V(1) - V(3)$ implies that $V(0) - V(1) = V(2) - V(3)$. Therefore, it is sufficient to show that $V(2) - V(3) > 0$.

Subtracting equation (IA.38c) from equation (IA.38b), and substituting $V(0) - V(2) = V(1) - V(3) = \frac{m}{\delta\Delta}$, yields $V(1) - V(2) = \delta \cdot (V(2) - V(3))$. Therefore,

$$\begin{aligned} V(1) - V(3) &= V(1) - V(2) + V(2) - V(3) \\ &= (1+\delta)\cdot(V(2) - V(3)) \end{aligned} \tag{IA.39}$$

which proves that $V(2) - V(3) > 0$ because $V(1) - V(3) = \frac{m}{\delta\Delta} > 0$. ■

Proof of Proposition IA.1: In this proof, we make use of equations (IA.38a) through (IA.38d) from above, after substituting $V(0) - V(2) = V(1) - V(3) = \frac{m}{\delta\Delta}$.

Step I: Solving for θ.

Substituting $q(0) = 1$, $q(1) = 1 - \theta$, $q(2) = 1 - 2\theta$, and $V(0) - V(2) = V(1) - V(3) = \frac{m}{\delta\Delta}$ in equations (IA.38a) through (IA.38d) that we used in the proof of Lemma IA.1, it follows that

$$V(0) - V(1) = \theta A, \tag{IA.40}$$

$$V(2) - V(3) = \theta A, \tag{IA.41}$$

and

$$V(0) - V(2) = \theta A + \delta(V(2) - V(3)) = (1 + \delta)\theta A, \tag{IA.42}$$

where the second equation above is obtained using equations (IA.40) and (IA.41).

But $V(0) - V(2) = \frac{m}{\delta\Delta}$ by the IC constraint (IA.5a). Setting $(1 + \delta)\theta A = \frac{m}{\delta\Delta}$, and solving for θ yields the expression for θ in the proposition. For the equilibrium to be well defined, it must be that $\theta \leq \frac{1}{N} = \frac{1}{2}$, which is equivalent to condition (IA.7).

Step II: Solving the value function $V(x)$ for $x \in \{0, 1, 2, 3\}$.

We begin by solving for $V(0)$. Substituting $q(0) = 1$, $V(0) - V(2) = \frac{m}{\delta\Delta}$, and $\delta V(2) = \delta V(0) - \frac{m}{\Delta}$ in equation (IA.38a) yields $V(0) = A + B + \delta V(0) - \frac{m(1-p)}{\Delta}$. Solving for $V(0)$ yields $V(0) = V^*$. Once we have solved for $V(0)$, it is fairly straightforward to obtain $V(1)$, $V(2)$ and $V(3)$ using equations (IA.40), (IA.5a), and (IA.5b), respectively. ■

Proof of Proposition IA.2: Let V^* denote the highest possible continuation value that can be achieved under the full monitoring equilibrium with $q = \bar{q} = 1$. It is easily shown that $V(1, 1, N)$ is decreasing in N. Hence, it is efficient to choose the lowest N at which the IC constraint (IA.14) is satisfied; we denote this as N^*. That is, N^* is the lowest integer for which

$$\left(1 - \delta^N\right) \cdot \left(V^* - \frac{B}{1 - \delta}\right) \geq \frac{m}{\delta\Delta} \tag{IA.43}$$

Note that because N^* is constrained to be an integer, the IC constraint (IA.14) will generally hold strictly in the full monitoring P-equilibrium, except under a limited set of parameter values.

Substituting $V_{punish} \leq V^* - \frac{m}{\delta\Delta}$ in the Bellman equation (IA.13), and rearranging terms, yields

$$V^* \leq (1 - \delta)^{-1} \cdot \left(A + B - \frac{(1 - p)\, m}{\Delta}\right) \tag{IA.44}$$

Let LHS denote the expression on the left-hand side of condition (IA.43). Because LHS is increasing in N, it follows that

$$\begin{aligned} LHS &\leq [LHS]_{N\to\infty} = V^* - \frac{B}{1-\delta} \\ &\leq (1-\delta)^{-1} \cdot \left(A - \frac{(1-p)\,m}{\Delta}\right), \end{aligned}$$

where the last inequality follows from condition (IA.44). Hence, it follows from condition (IA.43) that N^* is well-defined only if

$$(1-\delta)^{-1} \cdot \left(A - \frac{(1-p)\,m}{\Delta}\right) \geq \frac{m}{\delta\Delta}.$$

Rearranging this condition yields the feasibility condition (IA.11) in the statement of the proposition. ■

Proof of the Proposition IA.3: Let V^* denote the highest possible continuation value that can be achieved under the full monitoring equilibrium. As $V(q,\pi)$ is decreasing in π, it is efficient to choose the lowest π at which the IC constraint (IA.14) binds, i.e., $\pi^* = \frac{m}{\delta\Delta(V^*-(1-\delta)^{-1}B)}$. Substituting $q = 1$ and the expression for π^* in the Bellman equation (IA.13) yields $V^* = A-m+B+\delta V^* - \frac{m(1-p-\Delta)}{\Delta}$. Solving for V^* yields the expression for V^* in Proposition IA.3. For the equilibrium to be feasible, it is necessary that $\pi^* \leq 1$, or equivalently, that $m \leq \delta\Delta(V^*-(1-\delta)^{-1}B)$. Substituting for V^* and simplifying yields the feasibility condition (IA.15). ■

Proof of Proposition IA.4: Suppose both the HM and LM equilibria are feasible. As per Proposition 3 and Corollary 1 in the paper, the high-reputation bank's value under the HM equilibrium is

$$V_{HM}(0) = (1-\delta)^{-1} \cdot \left[A + B - \frac{m\,(1-p)}{\Delta}\right].$$

The HM equilibrium will strictly dominate the LM equilibrium if $V_{HM}(0) > V_{LM}(0)$, which after substituting for $V_{HM}(0)$ and $V_{LM}(0)$. and simplifying, is equivalent to the following condition:

$$(1-p-\Delta)\,(\delta\Delta A - m) > -\,(1-\delta+\delta p)\cdot\Delta\beta\alpha\,(0)\,P\,(1) \tag{IA.45}$$

We consider the following cases separately:

(a) Suppose $\delta\Delta A \geq m$. Then, it is clear that condition (IA.45) is met because the expression on the right-hand side is negative.

(b) Suppose $\delta\Delta A < m$. We know that $\alpha(0)$ must satisfy the IC constraint for the high-reputation bank, which after substituting for Λ from equation (IA.19), can be rewritten as

$$
\begin{aligned}
\delta\Delta A - m &\geq \alpha(0)\cdot\left[\delta\Delta\beta P(1) - (1-\delta\Delta)\,\Delta\,(R(1)-C)\right] \\
&= \frac{\alpha(0)}{\alpha_{pr}}\cdot\left[\delta\Delta\alpha_{pr}\beta P(1) - (1-\delta\Delta)\,m\right], \qquad \text{(IA.46)}
\end{aligned}
$$

where the equality follows after substituting $\Delta(R(1)-C) = m/\alpha_{pr}$.

Now, as per Proposition 3 in the paper, $\delta\Delta\alpha_{pr}\beta P(1) \geq m$ if the HM equilibrium is feasible. But this means that condition (IA.46) cannot hold because the expression on the right-hand side is positive, whereas $\delta\Delta A - m < 0$, thus contradicting the feasibility of the LM equilibrium.

Overall, we have proved that if the HM equilibrium is feasible, then either $V_{HM}(0) > V_{LM}(0)$ or the LM equilibrium is infeasible. ∎

Proof of Proposition IA.5: Part I. Characterizing the equilibrium conditions (LM-1) through (LM-4). We do this for the general case of $u \geq 0$.

(LM-1) $\delta\Delta\Lambda < m$ so that the *low-reputation bank does not monitor.*

(LM-2) $\delta\Delta\Lambda + \alpha(0)\cdot\Delta(R(1)-C) \geq m$ so that the *high-reputation bank has incentives to monitor.* As we showed in the proof of Proposition IA.4, this requirement is equivalent to condition (IA.46). Let's define

$$
\hat{\alpha}_{IC} \equiv \frac{\delta\Delta A - m}{\Delta\left[\delta\beta P(1) - (1-\delta\Delta)(R(1)-C)\right]}.
$$

If $\delta\Delta A \geq m$, then it is clear that condition (IA.46) is satisfied if EITHER (a) $\delta\beta P(1) \leq (1-\delta\Delta)(R(1)-C)$; OR (b) $\delta\beta P(1) > (1-\delta\Delta)(R(1)-C)$ and $\alpha(0) \leq \hat{\alpha}_{IC}$. On the other hand, if $\delta\Delta A < m$, then condition (IA.46) is satisfied only if $\delta\beta P(1) < (1-\delta\Delta)(R(1)-C)$ and $\alpha(0) \geq \hat{\alpha}_{IC}$.

(LM-3) *Non-deviation constraint for the high-reputation bank* to ensure that it will not deviate to $\alpha'(0) = 0$ and $q'(0,\alpha'(0)) = 0$ after the loan rate has been set at $R(1)$. Its value from such a deviation is

$$
\begin{aligned}
V'(0) &= (1+\beta)\cdot\overline{P}(0,R(1)) - (1+\beta) + \delta V(0_{low}) \\
&= (1+\beta)\cdot\overline{P}(0,R(1)) - (1+\beta) + \delta V(1) + \delta\bar{A},
\end{aligned}
$$

where the second equation follows by noting that $V(0_{low}) = V(1) + \bar{A}$ (compare equation (12) in the paper with equation (IA.18) above). In the above expression, $\overline{P}(0,R(1)) = p\left(X - C - \frac{u}{p+\Delta}\right) + C$.

For the high-reputation bank to not prefer such a deviation, we require that $V(0) \geq V'(0)$, which is equivalent to the following condition (after some algebra):

$$
A - \frac{u\Delta(1+\beta)}{p+\Delta} - \beta\alpha(0)\cdot P(1) - m + \delta(p+\Delta)\Lambda - \delta\bar{A} \geq 0, \qquad \text{(IA.47)}
$$

where we have the following expression for $\bar{A}$ after substituting for Λ and simplifying:

$$
\begin{aligned}
\bar{A} &= \max\left(0, A - \beta\bar{\alpha}\overline{P}(1, R(0)) + \delta\Delta\Lambda - m\right) \\
&= \max\left(0, \frac{A - m - (1-\delta\Delta)\cdot\beta\bar{\alpha}\overline{P}(1, R(0)) - \delta\Delta\beta\alpha(0)P(1)}{1-\delta\Delta}\right)
\end{aligned}
$$

Because $\bar{A}$ is weakly decreasing in $\alpha(0)$, it follows that there exists an $\hat{\alpha}_{ND,high}$ such that condition (IA.47) is met only if $\alpha(0) \leq \hat{\alpha}_{ND,high}$. Moreover, because $\bar{A} \geq 0$, it follows from condition (IA.47) that

$$
\hat{\alpha}_{ND,high} \leq \frac{1}{\beta P(1)}\left[A - m - \left(\frac{1-\delta\Delta}{1+\delta p}\right)\cdot\frac{u\Delta(1+\beta)}{(p+\Delta)}\right], \tag{IA.48}
$$

where the above condition will hold with equality if $\bar{A} = 0$ in equilibrium.

(LM-4) *Non-deviation constraint for the low-reputation bank* to ensure that it will not deviate to some $\alpha'(1) > 0$ and $q(1, \alpha'(1)) = 1$ after its loan rate has been set at $R(0)$. The lowest value of $\alpha'(1)$ at the which monitoring become incentive-compatible is $\alpha'(1) = \frac{m-\delta\Delta\Lambda}{\Delta(R(0)-C)}$, and the bank's value from such a deviation is

$$
V'(1) = \left(1+\beta-\beta\alpha'(1)\right)\cdot\overline{P}(1, R(0)) - (1+\beta) - m + \delta(p+\Delta)\Lambda + \delta V(1),
$$

where $\overline{P}(1, R(0)) = (p+\Delta)(R(0)-C) + C$. For it to not prefer such a deviation, we require $V(1) \geq V'(1)$, which is equivalent to

$$
(1+\beta)\cdot P(0) \geq \left(1+\beta-\beta\alpha'(1)\right)\cdot\overline{P}(1, R(0)) - m + \delta\Delta\Lambda.
$$

After substituting for $P(0)$, $\overline{P}(1, R(0))$, and $\alpha'(1)$, and rearranging terms, we obtain

$$
m - \delta\Delta\Lambda \geq Z \equiv \frac{(1+\beta)\cdot\Delta^2(R(0)-C)^2}{\beta\overline{P}(1, R(0)) + \Delta(R(0)-C)}. \tag{IA.49}
$$

Because Z is positive, condition (IA.50) is stricter than the requirement that $m > \delta\Delta\Lambda$ (condition (1) to ensure that the low-reputation bank does not monitor). It is also easily verified that Z is decreasing in β, and increasing in Δ and $(R(0)-C)$. Substituting for Λ in the above condition, and simplifying, yields the following condition on $\alpha(0)$:

$$
\alpha(0) \geq \hat{\alpha}_{ND,low} \equiv \frac{[\delta\Delta A - m + (1-\delta\Delta)Z]}{\delta\Delta\beta P(1)} \tag{IA.50}
$$

Part II: We show that if $u = 0$, then the LM equilibrium is infeasible whenever the HM equilibrium is infeasible.

Note that if $u = 0$, then $R(1) = R(0) = X$, $\alpha_{pr} = \bar{\alpha} = \frac{m}{\Delta(X-C)}$, $\overline{P}(1, R(0)) = P(1)$, and

$Z = \frac{A\Delta(X-C)}{\beta P(1)+\Delta(X-C)}$. Also, as per Proposition 3 and Corollary 1in the paper, the HM equilibrium is infeasible if either $\delta\beta P(1) < (X - C)$ or

$$A - \frac{m(1+\delta-\delta p)}{\delta\Delta(1+\delta)} < \frac{\delta\max[0, A - \beta\bar{\alpha}P(1)]}{(1+\delta)}. \tag{IA.51}$$

We consider the following three cases separately, and will prove, by contradiction, that the LM equilibrium is infeasible under all these scenarios.

Case 1: $\delta\beta P(1) < (1-\delta\Delta)(X-C)$.

Suppose an LM equilibrium exists. For (LM-3) and (LM-4) to hold simultaneously, we need to check that $\hat{\alpha}_{ND,low}$ is lower than the upper-bound for $\hat{\alpha}_{ND,high}$. Comparing conditions (IA.48) and (IA.50), and setting $u = 0$, this is equivalent (after some algebra) to

$$Z \leq m \iff A \leq m + \frac{m\beta P(1)}{\Delta(X-C)}. \tag{IA.52}$$

But if $\delta\beta P(1) < (1-\delta\Delta)(X-C)/\delta$, then it follows from condition (IA.52) that $\delta\Delta A < m$.

Now if $\delta\Delta A < m$ and $\delta\beta P(1) < (1-\delta\Delta)(X-C)$, we have shown that (LM-2) is satisfied only if $\alpha(0) \geq \hat{\alpha}_{IC}$. For (LM-2) and (LM-3) to hold simultaneously, we require that

$$\frac{m - \delta\Delta A}{\Delta[(1-\delta\Delta)(X-C) - \delta\beta P(1)]} \leq \frac{A-m}{\beta P(1)}.$$

After some algebra, this condition simplifies to

$$m\beta P(1) \leq (A-m)\Delta(X-C),$$

which contradicts condition (IA.52) (except in the unlikely event that $A = m + \frac{m\beta P(1)}{\Delta(X-C)}$).

Case 2: $\delta\beta P(1) \geq (1-\delta\Delta)(X-C)$ *and* $\delta\beta P(1) < (X-C)$.

Suppose an LM equilibrium exists. In this case we need $\delta\Delta A \geq m$ and $\alpha(0) \leq \hat{\alpha}_{IC}$ for (LM-2) to hold.

As in part (II), condition (IA.52) is necessary for the LM equilibrium to be well-defined. Additionally, we need to verify that $\hat{\alpha}_{ND,low} \leq \hat{\alpha}_{IC}$, which after some algebra, is equivalent to the following condition:

$$Z \cdot [\delta\Delta\beta P(1) - (1-\delta\Delta)\Delta(X-C)] \leq (\delta\Delta A - m)\Delta(X-C).$$

Substituting $Z = \frac{A\Delta(X-C)}{\beta P(1)+\Delta(X-C)}$ in the above inequality, and simplifying, yields

$$m\beta P(1) \leq (A-m)\Delta(X-C),$$

which contradicts condition (IA.52) (except in the unlikely event that $A = m + \frac{m\beta P(1)}{\Delta(X-C)}$).

Case 3: $\delta\beta P(1) \geq (X-C)$ and condition (IA.51) holds.

We now consider two possibilities: (i) Suppose $A - \beta\bar{\alpha}P(1) \leq 0$. In this case, condition (IA.51) implies that

$$\delta\Delta A < \frac{m(1+\delta-\delta p)}{(1+\delta)} < m.$$

But if $\delta\Delta A < m$ and $\delta\beta P(1) \geq (X-C)$, then (LM-2) cannot be satisfied, and the LM equilibrium is infeasible.

(ii) Suppose $A > \beta\bar{\alpha}P(1) = \frac{\beta m P(1)}{\Delta(X-C)}$. Then, condition (IA.51) implies that

$$\begin{aligned}\delta\Delta A &< m(1+\delta-\delta p) - \frac{\delta^2\beta m P(1)}{(X-C)}. \\ &\leq m(1+\delta-\delta p) - \delta m = m(1-\delta p),\end{aligned}$$

where the second inequality follows because $\frac{\delta\beta P(1)}{(X-C)} \geq 1$. But if $\delta\Delta A < m(1-\delta p) < m$ and $\delta\beta P(1) \geq (X-C)$, then (LM-2) cannot be satisfied, and the LM equilibrium is infeasible. ■

Proof of Proposition IA.6: The proof is contained in the text perceding the proposition. ■

Proof of Corollary 1: (i) As shown in the proof of Corollary 1 in the main text,

$$\frac{dV(0)}{d\rho} = (1-\delta)^{-1} \cdot \left[\alpha_{pr} \cdot \beta P(1) - \frac{m(1-p-\Delta)}{\Delta}\right].$$

Making use of $\alpha_{pr} = m/[\Delta(R(1)-C)]$ it follows that this derivative is positive if and only if $\beta P(1) > (1-p-\Delta)(R(1)-C)$. Using the definition of $P(1)$ and rearranging yields $(1+\beta)P(1) > R(1)$.

(ii) The derivative of the LHS of Condition (16) in the main text with respect to ρ is

$$\frac{d(LHS(16))}{d\rho} = \alpha_{pr}\beta P(1) - \frac{m}{\delta\Delta} \cdot \frac{(1+\delta)(1-\delta\Delta)-\delta p}{1+\delta},$$

(a) If $A-(1-\rho)m > \beta\overline{\alpha}\overline{P}(1, R(0))$, then the derivative of the RHS of Condition (16) in the main text with respect to ρ equals $\delta m/(1+\delta)$. It follows that Condition (16) becomes looser as ρ increases if and only if $d(LHS(16))/d\rho - d(RHS(16))/d\rho > 0$. Making use of the definition of α_{pr} and rearranging yields the condition in the corollary.

(b) If $A-(1-\rho)m < \beta\overline{\alpha}\overline{P}(1, R(0))$, the LHS of Condition (16) in the main text is unchanges, but thhe derivative of the RHS with respect to ρ equals 0. The condition becomes looser as ρ increases if and only if $d(LHS(16))/d\rho > 0$. Again, using the definition of α_{pr} and rearranging yields the condition in the corollary.

(iii) That the terms that multiply $R(1)-C$ in (ii.a) and (ii.b) are both less than 1 is immediate. To show that they are both greater than $\delta(1-p-\Delta)$ we take them in turn. For (ii.a), the multiplier is

$$\frac{1+\delta(1-p-\Delta)}{1+\delta},$$

which is greater than $\delta(1-p-\Delta)$ if and only if $\delta(1+\delta)(1-p-\Delta) < 1+\delta(1-p-\Delta)$. Rearranging, this is the same as $\delta^2 < 1$, which is true since $\delta < 1$.

For (ii.b), the multiplier is$1 - \delta\Delta - \frac{\delta p}{1+\delta}$, which is greater than $\delta(1-p-\Delta)$ if and only if $\delta(1-p) < 1 - \frac{\delta p}{1+\delta}$. Since $\delta < 1$ and $\delta p > \frac{\delta p}{1+\delta}$, this is always true. ■

Proof of Propositions IA.7 through IA.10: There are four possible equilibria to consider, based on how the bank chooses its lending volume $\ell \in \{1, \gamma\}$ in a boom, in the high and low reputation states. For each equilibrium, we must check three necessary conditions: (a) IC(1) must bind; (b) in a boom, a high-reputation bank must not want to switch its loan volume choice, $\ell_b(0)$; and (c) in a boom, a low-reputation bank must not want to switch its loan volume choice, $\ell_b(1)$.

Case (1): "Tight credit equilibrium" with $\ell_b(0) = \ell_b(1) = 1$. Condition (a) is $\Lambda = V(0) - V(1) = m/\delta\Delta$. First, it is easy to see that, because loan volumes and thus monitoring choices are the same in normal and boom states, we will have $V_b(d) = V_n(d) = V(d)$. Therefore, $\Lambda = [q(0,1) - q(1,1)]A$. Setting $\Lambda = m/\delta\Delta$ (condition (a)), we obtain

$$q(1,1) = q(0,1) - \frac{m}{\delta\Delta A} \tag{IA.53}$$

To ensure that the high-reputation bank does not switch to $\ell = \gamma$ in the boom state (condition (b)), it must be that $V_n(0) \geq V(0,\gamma)$, which is equivalent to

$$\begin{aligned} Aq(0,1) + (pm/\Delta) &\geq (\gamma-1)B + \delta\Lambda \\ Aq(0,1) &\geq (\gamma-1)B + \frac{m(1-p)}{\Delta}, \end{aligned} \tag{IA.54}$$

where the second inequality is obtained by substituting $\Lambda = m/\delta\Delta$.

Similarly, to ensure that the low-reputation bank does not switch to $\ell = \gamma$ in the boom state (condition (c)), it must be that $V_n(1) \geq V(1,\gamma)$, which is equivalent to $Aq(1,1) \geq (\gamma-1)B - pm/\Delta$. Substituting for $q(1,1)$ from equation (IA.53), this is equivalent to

$$Aq(0,1) \geq (\gamma-1)B + \frac{m(1-\delta p)}{\delta\Delta} \tag{IA.55}$$

It is clear that condition (IA.55) is stricter than condition (IA.54). Because $q(0,1) \leq 1$, condition (IA.55) can only if satisfied if $A \geq (\gamma-1)B + \frac{m(1-\delta p)}{\delta\Delta}$, or equivalently, $m \leq \frac{\delta\Delta[A-(\gamma-1)B]}{1-\delta p}$, which proves the necessity of this condition. Sufficiency follows by noting that when this condition is satisfied, then the probabilities $q\left(0,1\right)=1$ and $q\left(1,1\right)=\hat{q}=1-\frac{m}{\delta\Delta A}$ satisfy all the equilibrium conditions above.

Case (2): "Partially tight credit" equilibrium with $\ell_b(0)=1,\ \ell_b(1)=\gamma$. In this case, $V_b(0)=V_n(0)$ and $V_b(1)=V\left(1,\gamma\right)$. Then,

$$\Lambda=Aq\left(0,1\right)-\phi q\left(1,1\right)A+\left(1-\phi\right)\left(pm/\Delta\right)-\left(1-\phi\right)\left(\gamma-1\right)B$$

Setting $\Lambda=m/\delta\Delta$ (condition (a)) and rearranging terms, we obtain that

$$q\left(1,1\right)=\frac{1}{\phi}\left[q\left(0,1\right)-\frac{\left(1-\phi\right)\left(\gamma-1\right)B}{A}-\frac{m\left(1-\delta p+\delta p\phi\right)}{\delta\Delta A}\right] \tag{IA.56}$$

As in case (1) above, condition (b) requires that $V_n\left(0\right)\geq V\left(0,\gamma\right)$, which is equivalent to

$$Aq\left(0,1\right)\geq\left(\gamma-1\right)B+\frac{m\left(1-p\right)}{\Delta}. \tag{IA.57}$$

On the other hand, to ensure that the low-reputation bank does not switch to $\ell=1$ in the boom state (condition (c)), it must be that $V_n\left(1\right)\leq V\left(1,\gamma\right)$, which is equivalent to $Aq\left(1,1\right)\leq\left(\gamma-1\right)B-pm/\Delta$. Substituting for $q\left(1,1\right)$ from equation (IA.56), this is equivalent to

$$Aq\left(0,1\right)\leq\left(\gamma-1\right)B+\frac{m\left(1-\delta p\right)}{\delta\Delta} \tag{IA.58}$$

Clearly, for condition (a) to be satisfied, it is necessary that $A\geq(1-\phi)(\gamma-1)B+\frac{m(1-\delta p+\delta p\phi)}{\delta\Delta}$ (condition "F1") Also, condition (b) requires that $A\geq(\gamma-1)B+m\left(1-p\right)/\Delta$ (condition "F2"). This proves the necessity of these two feasibility conditions, which can be combined into condition (IA.21).

To prove sufficiency, suppose conditions F1 and F2 are satisfied. Then, consider the monitoring probabilities, $q\left(0,1\right)=\hat{q}_{0,ptc}\equiv\min\left\{1,\frac{(\gamma-1)B}{A}+\frac{m(1-\delta p)}{\delta\Delta A}\right\}$ and $q\left(1,1\right)=\hat{q}_{1,ptc}\equiv\frac{1}{\phi}\left[\hat{q}_{0,ptc}-\frac{(1-\phi)(\gamma-1)B}{A}-\frac{m(1-\delta p+\delta p\phi)}{\delta\Delta A}\right]$. which are constructed so that conditions (IA.58) and (IA.56), respectively are satisfied. It only remains to be shown that $\hat{q}_{1,ptc}\geq0$ and that condition (IA.57) are satisfied. Consider the following two subcases:

(i) Suppose $A\geq\left(\gamma-1\right)B+\frac{m(1-\delta p)}{\delta\Delta}$, so that $\hat{q}_{0,ptc}\equiv\frac{(\gamma-1)B}{A}+\frac{m(1-\delta p)}{\delta\Delta A}\leq1$. Then $\hat{q}_{1,ptc}\equiv\frac{(\gamma-1)B}{A}+\frac{mp}{\Delta A}\geq0$. Moreover,

$$A\hat{q}_{0,ptc}-\left(\gamma-1\right)B=\frac{m\left(1-\delta p\right)}{\delta\Delta}>\frac{m\left(1-p\right)}{\Delta},$$

so that condition (IA.57) is satisfied.

(ii) Suppose $A<\left(\gamma-1\right)B+\frac{m(1-\delta p)}{\delta\Delta}$. Then, $\hat{q}_{0,ptc}=1$ and condition (IA.58) is satisfied. Also, condition F2 guarantees that $\hat{q}_{1,ptc}\geq0$.

Case (3): "Loose credit equilibrium" with $\ell_b(0) = \ell_b(1) = \gamma$. In this case, $V_n(d)$ is the same as in Case (1) above, but $V_b(d) = V(d, \gamma) = \gamma B + \delta V(d)$. Then,

$$\begin{aligned} \Lambda &= \phi[q(0,1) - q(1,1)]A + (1-\phi)\delta\Lambda \\ &\Rightarrow \quad \Lambda = \frac{\phi[q(0,1) - q(1,1)]A}{[1-\delta(1-\phi)]} \end{aligned}$$

Setting $\Lambda = m/\delta\Delta$ (condition (a)) and rearranging terms, we obtain that

$$q(1,1) = q(0,1) - \left(\frac{1-\delta+\delta\phi}{\phi}\right)\frac{m}{\delta\Delta A} \tag{IA.59}$$

To ensure that the high-reputation bank does not switch to $\ell = 1$ in the boom state (condition (b)), we need $V_n(0) \leq V(0, \gamma)$, which is equivalent to

$$Aq(0,1) - (\gamma - 1)B \leq \frac{m(1-p)}{\Delta}. \tag{IA.60}$$

To ensure that the low-reputation bank does not switch to $\ell = 1$ in the boom state (condition (c)), we need $V_n(1) \leq V(1, \gamma)$, which is equivalent to $Aq(1,1) \leq (\gamma - 1)B - pm/\Delta$. Substituting for $q(1,1)$ from equation (IA.59), we obtain

$$Aq(0,1) - (\gamma - 1)B \leq \left(\frac{1-\delta+\delta\phi}{\phi}\right)\frac{m}{\delta\Delta} - \frac{pm}{\Delta} \tag{IA.61}$$

It is easily verified that condition (IA.60) is stricter because the RHS of this condition is smaller than the RHS of condition (IA.61). Hence, we only need to verify that condition (IA.60) is satisfied.

Because $q(0,1) \leq 1$, it is clear from equation (IA.59) that we need $m \leq \frac{\phi\delta\Delta A}{1-\delta+\delta\phi}$ to ensure that $q(1,1) \geq 0$. Also, because equation (IA.59) imposes a lower bound on $q(0,1)$, whereas condition (IA.60) imposes an upper bound on $q(0,1)$, we need that

$$\begin{aligned} \left(\frac{1-\delta+\delta\phi}{\phi}\right)\frac{m}{\delta\Delta} &\leq (\gamma-1)B + \frac{m(1-p)}{\Delta} \\ \text{i.e.,} m &\leq \frac{\phi\delta\Delta(\gamma-1)B}{1-\delta+\delta\phi p} \end{aligned}$$

The two necessary conditions above can be combined into the single feasibility condition (IA.22).

To prove sufficiency, suppose condition (IA.22) is satisfied. Then, consider the monitoring probabilities $q(0,1) = \hat{q}_{0,lc}$ and $q(1,1) = \hat{q}_{1,lc}$, which have been constructed so that conditions (a) and (b) are satisfied; condition (c) is also satisfied because, as shown above, it is weaker than condition (b). The feasibility conditions guarantee that the probabilities $q(1,1)$ and $q(0,1)$ are well defined.

Case (4): $\ell_b(0) = \gamma$, $\ell_b(1) = 1$. In this case, $V_b(0) = V(0, \gamma)$ and $V_b(1) = V_n(1)$. To ensure that the high-reputation bank does not switch to $\ell = 1$ in the boom state (condition (b)), it must be that $V_n(0) \leq V(0, \gamma)$, which is equivalent to

$$Aq(0,1) \leq (\gamma - 1) B + \frac{m(1-p)}{\Delta}.$$

Similarly, to ensure that the low-reputation bank does not switch to $\ell = \gamma$ in the boom state (condition (c)), it must be that $V_n(1) \geq V(1, \gamma)$, which is equivalent to

$$Aq(1,1) \geq (\gamma - 1) B - pm/\Delta. \tag{IA.62}$$

The two inequalities above imply that

$$A[q(0,0) - q(1,1)] \leq \frac{m}{\Delta} \tag{IA.63}$$

Then,

$$\begin{aligned}
\Lambda &= \phi A[q(0,1) - q(1,1)] + (1-\phi)[(\gamma-1)B - (pm/\Delta) - Aq(1,1) + V(0) - \delta V(1)] \\
&\leq \frac{\phi m}{\Delta} + (1-\phi)[V(0) - \delta V(1)] \\
&< \frac{\phi m}{\Delta} + (1-\phi)\Lambda,
\end{aligned}$$

where the first inequality is obtained by substituting from conditions (IA.62) and (IA.63) into the expression for Λ, and the second inequality is obtained by noting that $V(0) - \delta V(1) < V(0) - V(1) = \Lambda$. But then, the above inequality implies that $\Lambda < m/\Delta$, which contradicts $\Lambda = \frac{m}{\delta\Delta}$.. ■

Proof of Proposition IA.11: Suppose we conjecture that $q = 1$. For conditions (IA.23) and (IA.26) to hold simultaneously, we need that $U_+ - U_- \geq \frac{m}{\delta\Delta}$. Since V is increasing in U_-, we want U_- to be as high as possible, which yields $U_- = U_+ - \frac{m}{\delta\Delta}$. Substituting this in equation (IA.23) along with $q = 1$ yields

$$V = A + B + \delta\left[U_+ - \frac{(1-p)m}{\delta\Delta}\right]$$

It is clearly optimal to set $U_+ = V$. Substituting in the above equation and solving for V yields the expression for V^*in equation (IA.27).

We still need to verify that $U_- \geq 0$. After some algebra, we obtain that $U_- \geq 0$ if and only if the following condition is satisfied:

$$m \leq \frac{\delta\Delta(A+B)}{1-\delta p}.$$

If this condition is not satisfied, then the solution to the linear maximization problem is $q = 0$, $U_+ = U_- = 0$. ■

Proof of Proposition IA.12: Suppose we conjecture that $q_1 = 1$ and $q_\gamma = 1$. Then for conditions (IA.28) and (IA.31) to hold simultaneously, we need

$$\alpha_1 \Delta \left(X - C - \frac{u}{p + \Delta} \right) + \delta\Delta \left(U_+ - U_- \right) \geq m \tag{IA.64}$$

Similarly, for conditions (IA.32) and (IA.35) to hold simultaneously, we need

$$\gamma \alpha_\gamma \Delta \left(X - C - \frac{u}{p + \Delta} \right) + \delta\Delta \left(U_+ - U_- \right) \geq \gamma m \tag{IA.65}$$

Because V is increasing in U_- and decreasing in α_1 and α_γ, it is optimal to choose the highest U_- and the smallest $\{\alpha_1, \alpha_\gamma\}$ at which the IC constraints (IA.64) and (IA.65) bind with equality. This can be achieved by setting $\alpha_1 = 0$, $U_- = U_+ - \frac{m}{\delta\Delta}$, and

$$\alpha_\gamma = \frac{(\gamma - 1)\, m}{\gamma \Delta \left(X - C - \frac{u}{p+\Delta} \right)} = \frac{(\gamma - 1)\, \alpha_{pr}}{\gamma}.$$

Then, we have the following expressions for V_n, V_b, and V:

$$V_n = A + B - m + \delta \left[U_+ - \frac{m\,(1 - p - \Delta)}{\delta\Delta} \right],$$

$$V_b = \gamma\,(A + B - m) - (\gamma - 1)\, \beta \alpha_{pr} P\,(1) + \delta \left[U_+ - \frac{m\,(1 - p - \Delta)}{\delta\Delta} \right],$$

and

$$V = \gamma_\phi \cdot (A + B - m) - (1 - \phi)\,(\gamma - 1)\, \beta \alpha_{pr} P\,(1) + \delta \left[U_+ - \frac{m\,(1 - p - \Delta)}{\delta\Delta} \right]$$

Clearly, it is optimal to set $U_+ = V$. Making this substitution and solving for V yields the expression for V^* in equation (IA.36).

We still need to verify that $U_- \geq 0 \iff U_+ \geq \frac{m}{\delta\Delta}$. After some algebra, this is equivalent to condition (IA.37) in the statement of the proposition.. ■

References

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