

The Determinants of Asymmetric Volatility

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Volatility in equity markets is asymmetric: contemporaneous return and conditional return volatility are negatively correlated. In this article I develop an asymmetric volatility model where dividend growth and dividend volatility are the two state variables of the economy. The model allows both the leverage effect and the volatility feedback effect, the two popular explanations of asymmetry. The model is estimated by the simulated method of moments. I find that both the leverage effect and volatility feedback are important determinants of asymmetric volatility, and volatility feedback is significant both statistically and economically.

The relationship between stock price and its volatility has long interested financial researchers. Empirically, returns and conditional variance of next period's returns are negatively correlated. That is, negative (positive) returns are generally associated with upward (downward) revisions of the conditional volatility. This empirical phenomenon is often referred to as asymmetric volatility in the literature [see Engle and Ng (1993), Zakoian (1994), and Wu and Xiao (1999)].

The presence of asymmetric volatility is most apparent during stock market crashes when a large decline in stock price is associated with a significant increase in market volatility. Formal econometric models have been developed by researchers to capture asymmetric volatility. For example, asymmetric ARCH models of Nelson (1991) and Glosten, Jagannathan, and Runkle (1993) have been found to significantly outperform their counterparts that do not accommodate the asymmetry. Moreover, continuous-time stochastic volatility models generally produce estimates of a negative correlation between return and return volatility [Bakshi, Cao, and Chen (1997) and Bates (1997); for a review of the literature on asymmetric volatility, see Bekaert and Wu (2000)].

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It is the purpose of this article to provide a formal explanation to the observed negative correlation between return and return volatility and to analyze its economic significance. Black (1976) and Christie (1982) were among the first to document and explain the asymmetric volatility property of individual stock returns in the U.S. equity markets. The explanation they put forth is the leverage effect hypothesis: A drop in the value of the stock (negative return) increases financial leverage, which makes the stock riskier and increases its volatility. Although, to many, “leverage effects” have become synonymous with asymmetric volatility, the asymmetric nature of the volatility response to return shocks could simply reflect the existence of time-varying risk premiums [Pindyck, (1984) and French, Schwert and Stambaugh (1987)]. If volatility is priced, an anticipated increase in volatility raises the required return on equity, leading to an immediate stock price decline. This is often referred to as the “volatility feedback effect.”

Certainly the leverage effect and the volatility feedback effect could both be at work. Suppose an event such as foreign market turmoil has raised traders’ expectation of volatility in the domestic market. The effect of such volatility shock is often reflected in traders’ reluctance to buy and willingness to sell in anticipation of a volatile market. As a result, stock prices have to drop to balance the buying and selling volume. Thus an anticipated increase in volatility leads to an immediate price drop, as predicted by the volatility feedback hypothesis. This drop in stock price raises the leverage ratio, which by the leverage effect hypothesis brings about a further increase in volatility and therefore a further drop in price. This process can go on indefinitely. Bekaert and Wu (2000) examined asymmetric volatility in the Japanese equity market. Using a general empirical framework based on a multivariate GARCH-in-mean model, they also tried to differentiate between the two main explanations for the asymmetry. They concluded that volatility feedback was the dominant cause of the asymmetry for the Japanese stock market.

One of the most important contributions toward a better understanding of volatility feedback is Campbell and Hentschel (1992). Despite much research on the subject, they were the first to present a fully worked-out model of the feedback mechanism. They modeled dividend process as a quadratic GARCH (QGARCH) process and linked dividend volatility to return by assuming a linear relation between the two. The key feature of their model is that return is positively linear in dividend shock and negatively linear in the square of the dividend shock. The model is able to produce asymmetric volatility and explain the negative skewness and excess kurtosis of the data.

To fully understand the impact of volatility feedback, we need to model dividend volatility as a separate factor. Even though Campbell and Hentschel discussed “news about dividends” and “news about volatility,” the latter is just the square of the former in their model. As acknowledged by the authors, the feedback parameter is constrained to be small, since otherwise the model

would not be able to account for rebounds following large stock market crashes. Thus it is possible that the feedback effect is more pronounced than that captured by their model. My model estimation suggests this is indeed the case.

In this article I develop a volatility feedback model where the growth of a firm's dividend follows a stochastic volatility process, that is, dividend shock and dividend volatility shock are two separate sources of uncertainty. Note that the changing volatility of dividends is a plausible cause of asymmetric volatility. When the volatility of dividends increases, the riskiness of the firm increases since it is the claim to the stream of dividends. A drop in stock price occurs immediately due to the higher expected return required to compensate for the added risk. Since the volatility of the stock increases as well, return and volatility are negatively correlated. By assuming dividend growth to be a stochastic volatility process, we are able to examine the individual impact of dividend news and dividend volatility news on return and return variance. An important implication of the model is that the changing uncertainty regarding dividends drives both the stock return and the volatility of the stock return. When the uncertainty increases, stock volatility increases and the stock price drops. Thus volatility asymmetry is generated independent of the leverage effect. Another feature of my model is that I allow innovations of dividend growth and dividend volatility to be correlated. This captures the leverage effect. Finally, I note that in this model a piece of good (bad) news regarding dividends is always reflected positively (negatively) in the stock return and there is no constraint on the size of the feedback parameter.

This article makes the following contributions to the research on asymmetric volatility. First, I develop an asymmetric volatility model from the basic pricing relation which holds in an arbitrage-free economy. The model nests the two popular explanations of the asymmetry: the leverage effect and the volatility feedback effect. The empirical analyses suggest the leverage effect is an important source of asymmetric volatility. In addition, volatility feedback is also a main determinant of asymmetric volatility. Second, since I specify a stochastic volatility dividend process, there are two state variables in the model, which extends the classical Campbell and Hentschel (1992) one-factor volatility feedback framework. Third, I examine and establish the economic significance of volatility feedback. The estimated structural model shows that the volatility feedback effect is stronger than previously documented.

The article proceeds as follows. Section 1 develops a model of asymmetric volatility in an equilibrium asset pricing framework. Section 2 describes the estimation method and the data, conducts the empirical exercise, and discusses the economic significance of volatility feedback. Section 3 contains concluding remarks. Some technical details are included in the appendix.

1. A Model of Asymmetric Volatility

1.1 The model

Similar to Turnbull and Milne (1991) and Constantinides (1992), my theoretical framework begins with the pricing kernel: the stochastic process for state-contingent claims prices. I do so because the documented discrepancies between representative agent theories and observed asset prices have been linked to the variability of the pricing kernel. Hansen and Jagannathan (1991) found that observed asset returns imply substantially larger standard deviations of the pricing kernel than we get from representative agent theory with power utility and a reasonable level of risk aversion. Starting with a suitable pricing kernel makes it possible for us to derive an empirically interesting model that allows close examination of the dynamics that generates asymmetric volatility. Moreover, due to the presence of stochastic volatility in the dividend growth process, we need to have a simple structure that provides an interpretable solution to the model.

From Duffie (1996, Chapter 1) we know that, in the absence of arbitrage, the state pricing density exists,

$$1 = E_t[m_{t+1}R_{t+1}], \tag{1}$$

where $R_{t+1} = (P_{t+1} + D_{t+1})/P_t$, and m_{t+1} is the state pricing density at time $t + 1$. I assume the pricing kernel to be

$$m_{t+1} = \exp\left(-r^f - \frac{1}{2}\sigma_{m,t}^2 + \epsilon_{m,t+1}\right), \quad \epsilon_{m,t+1}|I_t \sim N(0, \sigma_{m,t}^2). \tag{2}$$

This particular functional form has been used for asset pricing purposes by researchers. Amin and Ng (1993), for example, used this pricing density to derive an option pricing formula for ARCH processes.¹ The variance $\sigma_{m,t}^2$ is an exogenous process. For simplicity, I assume the risk-free rate to be constant. Let $g_t = \ln(D_{t+1}/D_t)$ be the growth rate of the dividend, which follows a stochastic volatility process,

$$\begin{cases} g_{t+1} = \alpha_0 + \alpha_1 g_t + \epsilon_{d,t+1}, & \epsilon_{d,t+1}|I_t \sim N(0, \sigma_{d,t}^2), \\ \sigma_{d,t+1}^2 = \beta_0 + \beta_1 \sigma_{d,t}^2 + \sigma_{d,t} \nu_{t+1}, & \nu_{t+1} \sim N(0, \eta_v^2). \end{cases} \tag{3}$$

I allow the shocks to the dividend and its volatility to be correlated: $\text{corr}(\epsilon_{d,t+1}, \nu_{t+1}) = \rho_l$, which captures the leverage effect. This is the basic

¹This pricing density is related to that of Constantinides (1992), whose focus is on capturing interest rate term structure dynamics. My interest is in stochastic volatility in the pricing kernel and the asset payoff structure. I do not have those “N” terms in his specification, but allow the volatility term to be time varying and stochastic, which he assumed to be constant. As in Constantinides (1992) and Backus and Zin (1994), my pricing kernel does not follow directly from a representative agent economy with power utility. A more general, multivariate version of this pricing kernel is explored in Backus et al. (1997). See also Bekaert and Grenadier (1997) for possible extensions to my model.

model setup. Note that the variance follows a square-root process which guarantees positivity as the time interval shrinks to zero.

Now I derive the return on firm value and its variance as functions of dividend growth and dividend variance. I allow the firm to have systematic risk by specifying the covariance between dividend growth and the pricing density to be $\text{cov}_t(\epsilon_{d,t+1}, \epsilon_{m,t+1}) = \rho_m \sigma_{d,t}^2$. Note that if $\rho_m < 0$, dividend growth is slower in states where dividends are more valuable due to the negative correlation. This specification implies that the variance risk of the firm is systematic and must be priced.

To solve the model, I apply the Campbell and Shiller (1988) approximation. Let $p_t = \ln P_t$ and $d_t = \ln D_t$,

$$\begin{aligned} r_{t+1} &= \ln R_{t+1} \\ &= k + \rho p_{t+1} + (1 - \rho)d_{t+1} - p_t \end{aligned} \tag{4}$$

$$= k + \rho(p_{t+1} - d_{t+1}) + g_{t+1} - (p_t - d_t), \tag{5}$$

where parameter ρ is the average ratio of the stock price to the sum of the stock price and the dividend, a number slightly smaller than one, and k is a constant related to ρ .

Substituting Equations (2) and (5) into Equation (1), we obtain

$$\begin{aligned} 1 = E_t \exp \left[-r^f - \frac{1}{2} \sigma_{m,t}^2 + \epsilon_{m,t+1} + k + \rho(p_{t+1} - d_{t+1}) \right. \\ \left. + g_{t+1} - (p_t - d_t) \right]. \end{aligned} \tag{6}$$

This equation is solved in the following proposition. The proof is provided in the appendix.

Proposition 1. *The linear solution to the log price:dividend ratio is*

$$p_t - d_t = c_0 + c_1 g_t + c_2 \sigma_{d,t}^2, \tag{7}$$

where the parameters are

$$c_1 = \frac{\alpha_1}{1 - \rho\alpha_1} \tag{8}$$

$$\begin{aligned} c_2 = \frac{(1 - \rho\alpha_1)(1 - \rho\beta_1) - \rho\eta_v\rho_l}{(1 - \rho\alpha_1)\rho^2\eta_v^2} \pm \\ \frac{\sqrt{[(1 - \rho\alpha_1)(\rho\beta_1 - 1) + \rho\eta_v\rho_l]^2 - \rho^2\eta_v^2[1 + 2\rho_m(1 - \rho\alpha_1)]}}{(1 - \rho\alpha_1)\rho^2\eta_v^2} \end{aligned} \tag{9}$$

$$c_0 = \frac{(-r^f + k)(1 - \rho\alpha_1) + \alpha_0}{(1 - \rho\alpha_1)(1 - \rho)} + \frac{\rho\beta_0}{(1 - \rho)} c_2. \tag{10}$$

Furthermore, the linear solution to the return process is

$$r_{t+1} = r^f + \lambda_1 \sigma_{d,t}^2 + \lambda_2 \epsilon_{d,t+1} - \lambda_3 \sigma_{d,t} v_{t+1}, \tag{11}$$

where the parameters are

$$\lambda_1 = -(1 - \rho\beta_1)c_2 \tag{12}$$

$$\lambda_2 = \frac{1}{1 - \rho_2} \tag{13}$$

$$\lambda_3 = -\rho c_2. \tag{14}$$

Note that for c_2 , there are two roots. For there to be volatility feedback in the model, only the negative root is feasible. The first term is positive since it has a positive denominator and the numerator is positive when $\rho_l < 0$. Recall that the negative correlation captures the leverage effect. It is straightforward to compute the conditional variance of returns,

$$\sigma_{r,t}^2 = (\lambda_2^2 + \lambda_3^2 \eta_v^2 - 2\lambda_2 \lambda_3 \eta_v \rho_l) \sigma_{d,t}^2. \tag{15}$$

With $\rho_l < 0$, the term in the parenthesis is positive. Therefore $\sigma_{r,t+1}^2$ is proportional to $\sigma_{d,t+1}^2$. I would like to point out here that λ_3 measures the feedback effect. If it is positive, then a positive (negative) realization of v_{t+1} , which corresponds to a larger (smaller) than expected conditional variance of the return $\sigma_{r,t+1}^2$, has a negative (positive) impact on the return. Moreover, since $\epsilon_{d,t+1}$ is negatively correlated to v_{t+1} due to the leverage effect, a positive (negative) realization of v_{t+1} is more likely to be associated with a negative (positive) $\epsilon_{d,t+1}$. Thus the volatility feedback effect is reinforced by the leverage effect. I summarize some interesting results in the following proposition. The proof is provided in the appendix.

Proposition 2. *If dividend growth and its variance are stationary ($-1 < \alpha_1 < 1, -1 < \beta_1 < 1$), and the firm's systematic risk parameter ρ_m is sufficiently large in size such that $\rho_m < -\frac{1}{2(1-\rho\alpha_1)}$, then volatility feedback exists. Specifically,*

- (i) $E_t [r_{t+1}]$ is positively related to dividend variance $\sigma_{d,t}^2$, that is, $\lambda_1 > 0$;
- (ii) r_{t+1} is positively related to dividend growth shock $\epsilon_{d,t+1}$, that is, $\lambda_2 > 0$;
- (iii) r_{t+1} is negatively related to v_{t+1} , $\sigma_{d,t+1}^2$ and $\sigma_{r,t+1}^2$, that is, $\lambda_3 > 0$.

In this model, return variance is driven by dividend variance. By (i), expected return is positively related to dividend variance, a feature which is similar to the key assumption of the Campbell and Hentschel (1992) model: expected return of the stock is a linear function of the variance of dividend news. This assumption, although reasonable as shown by the authors,

seems to be unusual at the outset since the conventional assumption is that expected return is a linear function of the return variance instead of the dividend variance. The proposition shows that they are actually consistent with each other. Moreover, we see that a positive risk-return relation exists in this model, that is, expected return is positively related to the conditional variance of the return. I would like to note that an appropriate measure of risk premium is the expected log return plus one-half the conditional variance of log returns, that is, we need to compute the expected returns adjusted for Jensen's inequality:

$$\text{Risk premium} = \left(\lambda_1 - \lambda_2 \lambda_3 \eta_v \rho_l + \frac{\lambda_2^2 + \lambda_3^2 \eta_v^2}{2} \right) \sigma_{d,t}^2. \quad (16)$$

By (ii), good news in dividend growth is immediately reflected in a price increase, and vice versa for bad news. News regarding dividend growth itself, however, does not affect return variance directly. However, since $\epsilon_{d,t+1}$ and v_{t+1} are correlated, a shock to dividend growth is likely to be associated with a shock in variance. When this correlation is negative, it captures the leverage effect. By (iii), r_{t+1} and $\sigma_{d,t+1}^2$ are negatively correlated. Thus volatility asymmetry is also generated via dividend variance shocks. Note that since r_{t+1} is also affected by dividend growth shocks, this negative correlation is not perfect.

1.2 The asymmetry property of the model

In this section I further explore the asymmetry property of the volatility feedback model,

$$r_{t+1} = E_t(r_{t+1}) + \lambda_2 \epsilon_{d,t+1} - \lambda_3 \sigma_{d,t} v_{t+1}, \quad (17)$$

where $E_t(r_{t+1}) = r^f + \lambda_1 \sigma_{d,t}^2$. The first term in the equation is the conditional mean. It is a linear function of the dividend variance. The second term reflects the impact of dividend news. The last term, which is the focus of our interest, shows the impact of a shock to dividend volatility. I prove below that it captures the volatility feedback effect.

Rearranging Equation (4) to relate p_t to p_{t+1} and d_{t+1} , we solve forward and impose the "no bubble" transversality condition to obtain

$$p_t = \frac{k}{1 - \rho} + (1 - \rho) E_t \sum_{j=0}^{\infty} \rho^j d_{t+1+j} - E_t \sum_{j=0}^{\infty} \rho^j r_{t+1+j}.$$

Using this equation to substitute out p_t and p_{t+1} in Equation (4) we get

$$r_{t+1} - E_t r_{t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j g_{t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+j}. \quad (18)$$

Campbell and Hentschel call the first term on the right-hand side of the above equation “news about dividends” and the second term “news about future returns.” Note that Equation (18) is not an economic model, but rather is derived from an accurate approximation of an identity. Therefore any return shock can be decomposed into two parts: changing expected future dividend growth rates and changing expected future returns. In models (1)–(3), the state variables of the economy are the dividend growth rate and its variance. The following proposition establishes the dividend news effect and the volatility feedback effect. The proof is provided in the appendix.

Proposition 3. *In the asymmetric volatility model [Equation (17)], news about future dividends is captured by $\lambda_2 \epsilon_{d,t+1}$, that is,*

$$(E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j g_{t+1+j} = \lambda_2 \epsilon_{d,t+1}. \quad (19)$$

More importantly, news about future returns is captured by $\lambda_3 \sigma_{d,t} v_{t+1}$, that is, changes in expected future returns are effected through a shock to dividend volatility, or the volatility feedback,

$$(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+j} = \lambda_3 \sigma_{d,t} v_{t+1}. \quad (20)$$

This simple proposition makes a strong statement: An unexpected change in dividend volatility changes expected future returns, which in turn has an immediate impact on the current stock price. To be more specific, if $\beta_1 > 0$, so volatility is positively autocorrelated, then an unexpected increase (decrease) in dividend volatility will increase expected future returns, which in turn will increase (decrease) the risk premia for future periods and lead to a price decline (increase). This is an impact on stock return separate from the dividend shock itself, although these two shocks are correlated. Just as in Campbell and Hentschel (1992), the shock described as “dividend news” may well contain news about future discount rates that are not being driven by volatility. As the empirical results will show below, the “dividend news” term has an estimated volatility close to the volatility of returns themselves.

The conditional correlation between return and return variance is computed to be

$$\text{Corr}_t(r_{t+1}, \sigma_{r,t+1}^2) = \frac{\lambda_2 \rho_l}{\sqrt{\lambda_2^2 + \lambda_3^2 \eta_v^2 - 2\lambda_2 \lambda_3 \eta_v \rho_l}} - \frac{\lambda_3 \eta_v}{\sqrt{\lambda_2^2 + \lambda_3^2 \eta_v^2 - 2\lambda_2 \lambda_3 \eta_v \rho_l}}. \quad (21)$$

This correlation consists of two components. The first is due to the leverage effect, which is negative if $\rho_l < 0$. The second term is due to the volatility feedback, which is negative if $\lambda_3 > 0$. Higher negative correlation is associated with a larger feedback parameter λ_3 and a larger leverage effect parameter ρ_l . If there is no volatility feedback effect, $\lambda_3 = 0$, this term becomes zero. The negative correlation between return and variance is generated only by the leverage effect, and $\text{Corr}_t(r_{t+1}, \sigma_{r,t+1}^2) = \rho_l$. If there is no leverage effect, $\rho_l = 0$, then the volatility feedback effect is the sole source of asymmetric volatility, $\text{Corr}_t(r_{t+1}, \sigma_{r,t+1}^2) = -\lambda_3 \eta_v / \sqrt{\lambda_2^2 + \lambda_3^2 \eta_v^2}$.

A concept closely related to volatility asymmetry is distributional skewness. For the standard continuous-time stochastic volatility model with mean-reverting square-root volatility process, Das and Sundaram (1997) found that the sign of skewness is determined by that of the coefficient of correlation between stock price and volatility, and the size of skewness is proportional to the coefficient [see also Bakshi, Cao, and Chen (1997) and Bates (1997)]. My discrete-time model is conditionally normally distributed. However, it converges in continuous-time to the case discussed by Das and Sundaram. Thus my model could explain the cause of empirical regularities such as the “smile” of implied volatilities in high-frequency data [Rubinstein (1994) and Jackwerth and Rubinstein (1996)].

My derived volatility feedback model [Equation (17)] is similar in form to the key Equation (12) in Campbell and Hentschel (1992), which is

$$r_{t+1} = \bar{\lambda}_0 + \bar{\lambda}_1 \sigma_{d,t}^2 + \bar{\lambda}_2 \epsilon_{d,t+1} - \bar{\lambda}_3 (\epsilon_{d,t+1}^2 - \sigma_{d,t}^2), \quad (22)$$

where $\bar{\lambda}_i > 0, i = 0, 1, 2, 3$, are the parameters. The dividend innovation $\epsilon_{d,t+1}$ is distributed as

$$\epsilon_{d,t+1} \sim N(0, \sigma_{d,t}^2), \quad (23)$$

$$\sigma_{d,t}^2 = a_0 + a_1 (\epsilon_{d,t} - b)^2 + a_2 \sigma_{d,t-1}^2. \quad (24)$$

The main difference between their model and mine is that they assume a quadratic GARCH (Q-GARCH) process for the variance of dividend while I let the variance be stochastic. Campbell and Hentschel’s main focus is to generate conditional skewness, which motivates their Q-GARCH specification of the variance process. My objective is to understand the mechanism that causes asymmetric volatility. I therefore model dividend volatility as a separate process.

2. Estimation and Empirical Results

2.1 Data and estimation methodology

In this section I apply the volatility feedback model to the U.S. stock market data. The log return series are the monthly returns on the value-weighted CRSP index from January 1926 to December 1997, and the weekly returns from July 1962 to December 1997 constructed from the daily series. I do not use the daily series since market microstructure issues such as nonsynchronous trading and transaction costs are likely to induce spurious autocorrelations in the daily returns. It is difficult, if possible, to separately identify the induced autocorrelations from the autocorrelations implied by my structural model. The risk-free rate is proxied by the 1 month Treasury bill yields. Table 1 reports some summary statistics of the monthly and weekly data, each with two subsamples.

To estimate my Equations (3) and (11), I encounter the problem of not being able to observe the dividend volatility.² Yet the data-generating process of the model is well specified by the model. This constitutes a natural setting for the simulated method of moments estimation [Duffie and Singleton (1993)]. I apply the method developed by Gallant and Tauchen (1996, 1998a, b) which they named efficient method of moments (EMM). An added benefit of the EMM approach is that it provides a systematic way to test the specification of the structural model. The tests are able to show if the model dynamics fit the observed returns data well.

I implement the following strategies to estimate the model parameters φ (or structural parameters) using EMM. I first select a set of moments of returns to be used in the estimation and choose an initial value of the model parameters. Following Gallant and Tauchen (1996, 1998a), I use the scores

² In an earlier version of the article, the author used implied volatility from the S&P 500 index options data as a proxy for conditional volatility. Yet that approach only allows a very short sample period due to a lack of options data. Since volatility feedback effect works through expected returns, which are hard to measure with a short sample, I opt to use more than 70 years of data on the CRSP value-weighted index.

Table 1
Summary statistics of the data

Sample period	No. of observations	Mean ($\times 10^3$)	Variance ($\times 10^3$)	Skewness	Excess kurtosis
Monthly data					
260130-971231	864	9.8791	3.0160	0.2336	8.1192
620703-791231	312	8.7241	5.3603	0.4302	5.8478
800102-971231	552	10.5320	1.6971	-0.3937	2.3503
Weekly data					
620706-971231	1853	2.3894	0.3669	-0.2751	4.0244
620706-791228	913	1.6993	0.3707	-0.0284	3.7643
800104-971231	940	3.0596	0.3626	-0.5215	4.3690

This table lists moments of the monthly and weekly returns on the value-weighted CRSP index. Sample mean, variance, skewness, and excess kurtosis are reported. Mean and variance have been multiplied by 1000.

of the likelihood function from the seminonparametric (SNP) framework as the moment conditions for the EMM estimator. A long time series of returns is then simulated using the chosen value of φ . I compute the sample versions of the selected moments using the historical time series and the simulated series, and then compute the difference between the two. The EMM estimate of φ is the set of values that minimizes the difference. I now discuss the estimation procedure in detail.

2.2 Efficient method of moments estimation

I would like to estimate the following equations,

$$\begin{cases} r_{t+1} = r^f + \lambda_1 \sigma_{d,t}^2 + \lambda_2 \epsilon_{d,t+1} - \lambda_3 \sigma_{d,t} v_{t+1}, \\ g_{t+1} = \alpha_0 + \alpha_1 g_t + \epsilon_{d,t+1}, & \epsilon_{d,t+1} | I_t \sim N(0, \sigma_{d,t}^2), \\ \sigma_{d,t+1}^2 = \beta_0 + \beta_1 \sigma_{d,t}^2 + \sigma_{d,t} v_{t+1}, & v_{t+1} \sim N(0, \eta_v^2), \end{cases} \quad (25)$$

where $\lambda_i (i = 1, 2, 3)$ is given in Equations (12)–(14). The original set of parameters for the model is $\varphi = (\rho, \alpha_0, \alpha_1, \beta_0, \beta_1, \rho_l, \rho_m, \eta_v)'$. However, not all these parameters are well identified in the model. For example, the long-run mean of dividend variance ($\beta_0/(1-\beta_1)$) is not identified from the standard deviation of the volatility innovation η_v . To estimate the model efficiently, I fix ρ at the sample average ratio of the index to the sum of the index and the dividend (0.9965 and 0.9992 for the monthly and weekly data, respectively). The long-run means of the log dividend growth ($\alpha_0/(1-\alpha_1)$) are fixed at the sample mean of the log dividend growth (0.0048 and 0.0011, respectively, or 5.91% annual rate). I fix the annualized long-run mean of dividend volatility ($\beta_0/(1-\beta_1)$) to be 18%. In the volatility feedback model the volatility of dividend growth and that of stock return are scaled versions of each other. In the estimated monthly model, this implies an annualized stock return volatility of about 20%. To check the robustness of the results I varied the long-run mean from 12% to 25% and found the results remain the same qualitatively. All my conclusions are valid with slightly different long-run volatility values. The set of parameters to be estimated is now $\varphi = (\alpha_1, \beta_1, \rho_l, \rho_m, \eta_v)'$. Note that based on estimates of the above model, all claims of the propositions can be verified, the volatility feedback mechanism can be examined, and the asymmetry correlation [Equation (21)] can be computed. These are the key parameters for my analysis.

Instead of the common practice of selecting a few low-order moments on an *ad hoc* basis, EMM presents a systematic approach to generating moment conditions, that is, using the expectation under the structural model of the score from an auxiliary model as the vector of moment conditions. The score is the derivative of the log density of the auxiliary model with respect to the parameters of the auxiliary model. Formally, let $f(y_t | x_{t-1}, \omega_0)$ denote the conditional density of y associated with the auxiliary description of the data. The maximum likelihood estimator of the parameter vector ω_0 with sample

size T (ω_T) sets the score to zero,

$$\frac{1}{T} \sum_{t=1}^T \frac{\partial}{\partial \omega} \log f(y_t | x_{t-1}, \omega_T) = 0. \tag{26}$$

The sample mean on the left-hand side converges to $E[\partial \log f(y_t | x_{t-1}, \omega_0) / \partial \omega]$ under certain regularity conditions. It follows that if the structural model, which was developed in Sections 1.1 and 1.2, is correctly specified, then the sample mean of the score evaluated at y 's simulated from the model (\hat{y}_s),

$$m_T(\varphi, \omega_T) = \frac{1}{T_s} \sum_{s=1}^{T_s} \frac{\partial}{\partial \omega} \log f(\hat{y}_s | \hat{x}_{s-1}, \omega_T), \tag{27}$$

where T_s is the simulation size, should also be approximately zero. The GMM estimator of the structural model parameter vector is

$$\varphi_T = \arg \min_{\varphi \in R} m'_T(\varphi, \omega_T) (I_T)^{-1} m_T(\varphi, \omega_T), \tag{28}$$

where $(I_T)^{-1}$ is the weighting matrix. Certainly ω must have at least as many elements as φ , and overidentifying restrictions may be used to test for the overall specification of the model.

The auxiliary model I use is the seminonparametric (SNP) framework proposed by Gallant and Tauchen (1998b). The method employs a Hermite polynomial series expansion to approximate the conditional density of a process, and nests directly the Gaussian VAR model, the semiparametric VAR model, the Gaussian ARCH model, and the semiparametric GARCH model. Therefore it should be able to fit well the distribution of stock returns. The auxiliary model need not nest my structural model for the EMM estimator to be consistent for the structural parameters. If it does, then the estimator is as efficient as the maximum likelihood estimator. Gallant and Long (1997) show that if my auxiliary model closely approximates the actual distribution of the data but does not nest it, the EMM estimator of the structural model is nearly fully efficient.

Gallant and Tauchen (1996) showed that under correct specification of the structural model, the optimized objective function is asymptotically chi-squared with degrees of freedom equal to the difference of the lengths of ω and φ . This allows a formal testing of the overall fitting of the structural model. Another added benefit of the SNP-EMM framework is the ability to test individually how well the structural model fits each score. The simulated SNP scores, evaluated at the estimated SNP-EMM parameters, are asymptotically normally distributed with zero mean. Thus standard t -statistics can be formed that have a standard normal asymptotic distribution. An indication of failure to fit a particular score can often be traced to the inability of the structural model to fit a certain aspect of the observed dynamics, such as the degree of the ARCH effect.

I will first search for an SNP auxiliary model that adequately describes the return process of the CRSP index based on the Schwarz Bayes BIC information criterion [Schwarz (1978)]. Experiences with the SNP-EMM framework tell me that I should find a parsimonious SNP model that adequately captures the dynamics of the historical data. I then use EMM to estimate the structural parameters φ . The structural model specification is tested using the overidentifying restrictions and individual scores. The implication for volatility asymmetry is analyzed and discussed.

The general form of the SNP conditional density is constructed as follows. Let the return data be $y_t, x'_t = (y'_t, y'_{t-1}, \dots, y'_{t-L_\mu})$ for some lag length L_μ , and z be a standard normal random variable. Define the location-scale shift $y_t = R_{x,t-1}z_t + \mu_{x,t-1}$, where $\Sigma_{x,t-1} = R_{x,t-1}^2$ is the variance and $\mu_{x,t-1}$ is the linear function $\mu_{x,t-1} = b_0 + Bx_{t-1}$. Let $\mathcal{P}(z)$ be a Hermite polynomial of order K_z . The SNP conditional density has the form

$$f(y_t|x_{t-1}, \omega_0) = c(x_{t-1}) [\mathcal{P}(z_t|x_{t-1})]^2 n(z_t), \tag{29}$$

where $c(x_{t-1}) = 1/\int[\mathcal{P}(z|x_{t-1})]^2\phi(z)dz$ is a constant of proportionality which makes Equation (29) integrate to one, and $n(z_t)$ is the density function of the standard normal distribution.

The SNP model I have chosen is a “non-Gaussian, AR(1), GARCH(1,1)” model. Its parameters are $\omega = (A_2, A_3, A_4, \psi_1, \psi_2, \tau_1, \tau_2, \tau_3)'$. The conditional mean is modeled as AR(1), $\mu_{x,t-1} = \psi_1 + \psi_2x_{t-1}$. The conditional variance is GARCH (1,1),³ $R_{x,t-1} = \tau_1 + \tau_2|y_{t-1} - \mu_{x,t-1}| + \tau_3R_{x,t-2}$. The “non-Gaussian” term refers to the fact that the conditional density is a product of a Gaussian density with a polynomial term as shown in Equation (29). The polynomial term is a third-order Hermite polynomial, squared to ensure positivity. A_1 is normalized to 1, while (A_2, A_3, A_4) are parameters for the first- to third-order polynomials. The total number of parameters in the SNP model, or the length of ω , is 8. Since there are five parameters to estimate in the structural model, there are three overidentifying restrictions in the GMM objective function.

Table 2 reports the estimated parameters of the SNP auxiliary model for both the monthly and weekly datasets. The model captures strong GARCH effects present in the data, as well as an autoregressive component in the mean. There is also evidence of non-Gaussian dynamics as represented by the statistically significant A_3 and A_4 for the monthly data and A_2, A_3 , and A_4 for the weekly data.

2.3 Basic estimation results

Table 3 lists the parameter estimates of the structural model, their standard deviations, and the t -ratios. All original parameter estimates are statistically significant at the 95% level, except α_1 for the monthly data.

³ The SNP version of the GARCH model is different from that by Bollerslev (1986). It works with absolute errors and standard deviations.

Table 2
SNP parameter estimates

Parameters	Estimates	Standard errors	<i>t</i> -statistics
Monthly data (January 1926–December 1997)			
A_2	-0.0665	0.0421	-1.581
A_3	-0.0964	0.0294	-3.269
A_4	-0.0309	0.0076	-4.039
ψ_1	0.2189	0.0372	5.876
ψ_2	0.0502	0.0306	1.641
τ_1	0.0813	0.0118	6.842
τ_2	0.2095	0.0203	10.320
τ_3	0.7756	0.0158	49.041
Weekly data (July 1962–December 1997)			
A_2	-0.0737	0.0253	-2.916
A_3	-0.0924	0.0222	-4.166
A_4	-0.0322	0.0056	-5.764
ψ_1	0.2416	0.0206	11.735
ψ_2	0.0216	0.0205	1.051
τ_1	0.1126	0.0108	10.400
τ_2	0.2262	0.0154	14.698
τ_3	0.7350	0.0132	55.856

This table lists the estimated parameters of the SNP model used in the EMM estimation. A_i 's are the parameters for the Hermite polynomials; ψ_i 's are the parameters for the autoregressive conditional mean; and τ_i 's are the parameters for the GARCH conditional variance. Standard errors and *t*-statistics are also listed. The upper panel reports the results for the monthly dataset and the lower panel for the weekly dataset.

Table 3
EMM model parameter estimates

Parameters	Estimates	Standard errors	<i>t</i> -statistics
Monthly data (January 1926–December 1997)			
α_0	0.0058	0.0007	8.169
α_1	-0.2147	0.1486	-1.445
β_0	1.287E-4	1.257E-4	1.024
β_1	0.9531	0.0458	20.812
ρ_l	-0.8679	0.1402	-6.192
ρ_m	-3.9057	1.1614	-3.363
η_v	0.0058	0.0026	2.264
λ_1	2.1087	0.5191	4.062
λ_2	0.8238	0.2405	3.425
λ_3	41.732	20.371	2.049
Weekly data (July 1962–December 1997)			
α_0	0.0013	3.530E-5	37.875
α_1	-0.2567	0.0332	-7.737
β_0	2.160E-5	6.948E-6	3.109
β_1	0.9640	0.0116	83.247
ρ_l	-0.5265	0.1699	-3.098
ρ_m	-2.2964	0.8969	-2.561
η_v	0.0033	0.0008	4.413
λ_1	1.0918	0.4523	2.414
λ_2	0.7956	0.0913	8.714
λ_3	29.667	14.718	2.016

This table lists the estimated parameters of the volatility feedback model using EMM. Standard errors and *t*-statistics are also listed. The upper panel reports the results for the monthly dataset and the lower panel for the weekly dataset.

Table 4 lists the chi-squared test statistic of the overidentifying restrictions and the SNP scores for the structural model. Associated with each element of the SNP parameter vector is an element of the sample score. Each element should be close to zero if the structural model adequately captures the dynamics that are present in the observed data. In the upper panel of Table 4, note that all sample scores for the monthly data are not statistically significantly different from zero. So are all the t -tests for the weekly data, except A_4 , which has a t -statistic of 1.891. The estimated volatility feedback model is able to fit monthly and weekly return series very well. Nearly all moment conditions are satisfied.

The chi-squared test statistic (with three degrees of freedom) for the monthly data is 8.426, which has a p -value of 0.076. Hence the structural model is not rejected at the 95% level. The chi-squared test statistic for the weekly data is 25.885, which is rejected at the 95% level. However, my experience with EMM estimation is that the overall chi-squared test tends to overreject models, as compared to tests on individual moments. Of course, it is possible that my structural model is not rich enough to capture all return dynamics in the weekly data. In summary, the testing results show that my volatility feedback model, developed from a set of simplifying assumptions and being conditionally normal, is overall capable of generating a time series that is “close” in terms of moments to the observed monthly data and weekly data.

Table 4
Fitted SNP scores and chi-squared statistics

Parameters	Score	Standard errors	t -statistics
Monthly data (January 1926–December 1997)			
A_2	-1.6111	2.5126	-0.641
A_3	-4.8283	4.6100	-1.047
A_4	-8.1962	11.4757	-0.714
ψ_1	-1.4732	1.9442	-0.758
ψ_2	1.4645	1.2198	1.201
τ_1	-17.6075	10.9982	-1.601
τ_2	-5.9363	7.0789	-0.839
τ_3	-16.4238	11.2574	-1.459
$\chi^2(3)$		8.426	
Weekly data (July 1962–December 1997)			
A_2	-0.4795	2.8213	-0.170
A_3	-8.7901	5.4114	-1.624
A_4	-24.3445	12.8761	-1.891
ψ_1	0.0599	2.0104	0.030
ψ_2	1.5647	1.2672	1.235
τ_1	-15.732	9.2363	-1.703
τ_2	-6.6937	6.8910	-0.971
τ_3	-16.9402	9.9728	-1.699
$\chi^2(3)$		25.885	

This table lists the fitted SNP scores, their standard errors and t -statistics, using estimated SNP and EMM parameters. The t -statistics are used to test if the individual scores are significantly different from zero. The chi-squared statistics test the overall specification of the volatility feedback model. The upper panel reports the results for the monthly dataset and the lower panel for the weekly dataset.

Substitute values for the λ_i 's, the estimated models for the monthly and weekly data are

$$\text{Monthly model: } \begin{cases} r_{t+1} = 0.003 + 2.1087\sigma_{d,t}^2 + 0.8238\epsilon_{d,t+1} - 41.732\sigma_{d,t}v_{t+1}, \\ g_{t+1} = 0.0058 - 0.2147g_t + \epsilon_{d,t+1}, \quad \epsilon_{d,t+1}|I_t \sim N(0, \sigma_{d,t}^2), \\ \sigma_{d,t+1}^2 = 0.00013 + 0.9530\sigma_{d,t}^2 + \sigma_{d,t}v_{t+1}, \quad v_{t+1} \sim N(0, 0.0058^2). \end{cases} \quad (30)$$

$$\text{Weekly model: } \begin{cases} r_{t+1} = 0.0005 + 1.0918\sigma_{d,t}^2 + 0.7956\epsilon_{d,t+1} - 29.667\sigma_{d,t}v_{t+1}, \\ g_{t+1} = 0.00134 - 0.2567g_t + \epsilon_{d,t+1}, \quad \epsilon_{d,t+1}|I_t \sim N(0, \sigma_{d,t}^2), \\ \sigma_{d,t+1}^2 = 0.0000216 + 0.964\sigma_{d,t}^2 + \sigma_{d,t}v_{t+1}, \quad v_{t+1} \sim N(0, 0.0033^2). \end{cases} \quad (31)$$

All estimated model parameters for both the weekly and monthly data are statistically significant at the 95% level, except α_1 and β_0 for the monthly data. Standard errors and the t -statistics for all parameters and coefficients are reported in Table 3. The estimated models [Equations (30) and (31)] show that returns are positively linear in the dividend variance. Therefore, volatility risk is priced in the model and returns are proportional to the amount of risk taken. Returns are also positively linearly related to shocks in dividend growth. Any positive dividend growth shock is reflected immediately as a positive return shock, regardless of the size of the shock. Therefore, “good news about dividends” is always good news for the stock itself. Finally I observe that the dividend variance process is persistent, which is necessary to capture the GARCH effect in the return data.

I now turn my attention to the most important aspect of the model: its implications for volatility asymmetry. Equation (15) shows that if there is a positive shock to the variance of dividend growth, that is, the uncertainty regarding dividend growth increases unexpectedly, it leads to an increase in the return variance immediately ($\sigma_{r,t} = 1.0408\sigma_{d,t}$ for the monthly data). Simultaneously, it has an immediate negative impact on the stock return, generating volatility asymmetry. This is clearly shown in the estimated Equations (30) and (31). The opposite happens if there is a negative shock to the variance of dividend growth. Thus volatility asymmetry, which is the existence of a negative correlation between return and return variance, is generated via shocks to the dividend variance.

2.4 The economic significance of volatility feedback

In this section I explore further the economic significance of volatility feedback for stock return itself. I would like to note that the estimated model obviously has the property that risk premium is positively related to the conditional return variance $\sigma_{r,t}^2$, as is clear from Equation (16). Higher risk is therefore compensated by higher expected return. In Figure 1 I plot the estimated risk premium for the monthly and weekly data. The conditional return variance is estimated by the simple sample variance approach, with a rolling

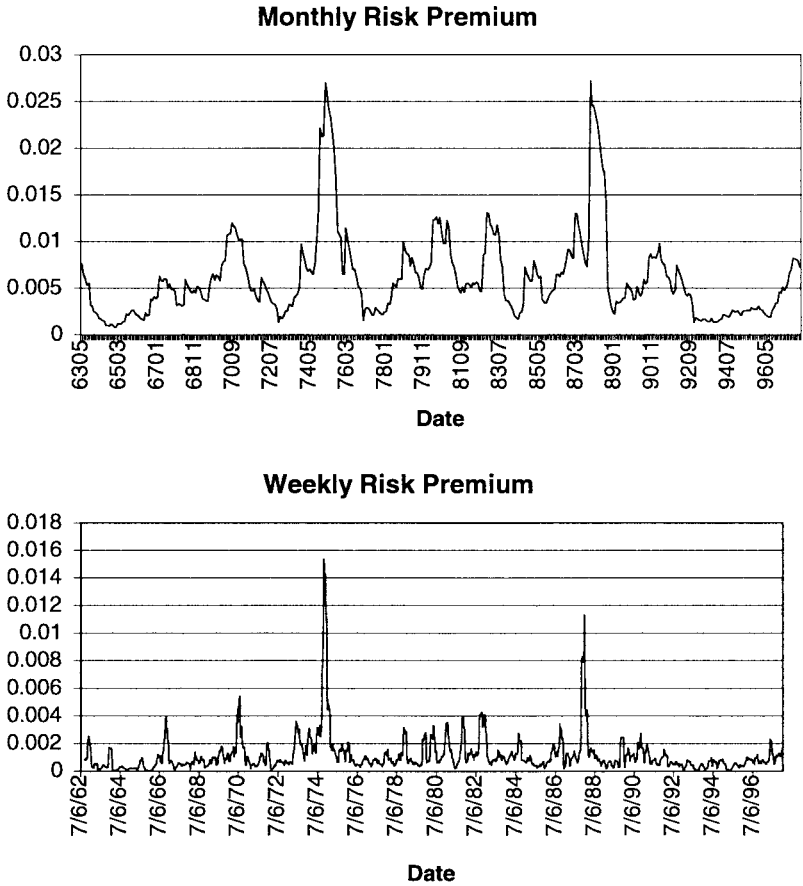


Figure 1
Estimated risk premium

This figure plots the estimated risk premium over the sample periods. The risk premium is the expected excess log return plus one-half the conditional variance. The upper and lower panels are results for the monthly and weekly data, respectively.

window of 20 and 10 periods for the monthly and weekly datasets, respectively. Certainly there are numerous ways to estimate conditional variance. Our qualitative conclusions regarding risk premia and the following decomposition of returns and asymmetry correlation do not change with a different variance estimation method.

From Equation (21), we see that the negative correlation between stock return and variance consists of two components. The first is due to the leverage effect, which is negative since the leverage correlation coefficient ρ_l is estimated to be negative. The second term is due to the volatility feedback effect, which is negative since λ_3 is positive. For the monthly data, $\rho_l = -0.8679$ with a standard error (henceforth s.e.) of 0.1402. Overall the

asymmetry correlation, $\text{corr}_t(r_{t+1}, \sigma_{r,t+1}^2)$, is estimated to be -0.9194 (s.e. 0.0960). Of the total correlation, -0.6868 (s.e. 0.1518) comes from the leverage effect and -0.2326 (s.e. 0.0625) comes from volatility feedback. Thus both the leverage effect and the volatility feedback effect are statistically significant. The leverage effect contributes more than twice as much to the negative correlation between return and return variance as the volatility feedback effect does. For the weekly data, $\rho_l = -0.5265$ (s.e. 0.1699). Overall the asymmetry correlation is -0.6074 (s.e. 0.1429) with -0.4919 (s.e. 0.1414) from the leverage effect and -0.1155 (s.e. 0.0713) from the volatility feedback effect.

Figure 2 shows the correlation between return and conditional variance as a function of the leverage correlation parameter ρ_l . The figure plots the asymmetry correlation as well as the two components against ρ_l while holding other parameters at the estimated level. It is clear from the figure that the leverage effect contribution to volatility asymmetry increases with ρ_l . The contribution from volatility feedback, however, is fairly stable. In fact, its contribution to asymmetric volatility declines slightly as the leverage effect becomes stronger.

The total return (r_{t+1}) in the volatility feedback model [Equation (17)] consists of three parts: the conditional mean is the sum of the risk-free rate and $\lambda_1 \sigma_{d,t}^2$, which is proportional to the risk premium; the impact of the dividend news ($\lambda_2 \epsilon_{d,t+1}$); and the volatility feedback effect ($-\lambda_3 \sigma_{d,t} v_{t+1}$). To help understand the economic significance of volatility feedback, I decompose historical returns into three parts and plot them in Figure 3 for the monthly data and Figure 4 for the weekly data. The scales of the individual panels in the figures are identical, except for expected returns, so that it is easy to see the relative magnitude of each part. The plots show clearly the relative importance of each part in generating the return series. We see that the expected return is of a smaller order of magnitude, and relatively stable compared to the other three plots. The news about dividends term seems to have the biggest impact on returns. The volatility feedback term is clearly economically significant, yet the magnitude of the feedback shock is usually less than half of the dividend news. However, the volatility feedback effect can be very large when innovations to volatility are large, such as in September 1974 and October 1987. The estimated model predicts that if return volatility increases unexpectedly by 10%, for example, from an annual rate of 20% to 22%, it will lead to a -2.81% return shock for the monthly data and -0.54% for the weekly data. If return volatility increases unexpectedly by 25% (e.g., from an annual rate of 20% to 25%), the implied drop is 7.52% for the monthly data and 1.45% for the weekly data. Hence, volatility feedback can be very important during volatile periods of the market. Its importance in determining the return dynamics under stable market conditions, however, seems to be secondary to dividend innovation itself.

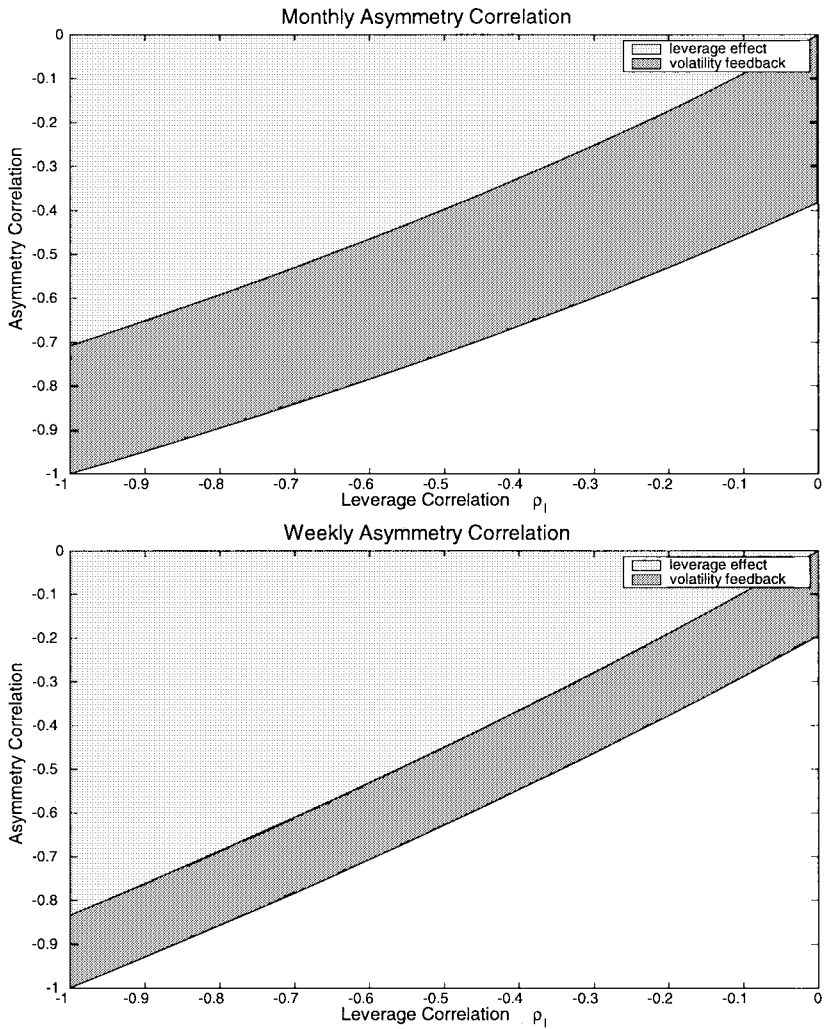


Figure 2

Asymmetry correlation and its components as a function of ρ_l

This figure shows the correlation between return and conditional variance as a function of the leverage correlation parameter ρ_l . The upper and lower panels are results for the monthly and weekly data, respectively. The total asymmetry correlation in Equation (21) is the sum of negative correlations caused by the leverage effect and the volatility feedback effect. The figure plots the asymmetry correlation as well as the two components against ρ_l while holding other parameters at the estimated level. The estimated ρ_l equals -0.8679 for the monthly data and -0.5265 for the weekly data.

Since volatilities tend to increase quickly but decline slowly, we generally see more negative return feedbacks than positive return feedbacks. This phenomenon is linked to the existence of negative skewness in stock returns. Even though our model has conditionally normally distributed innovations, the feedback mechanism can produce negative skewness for multiperiod

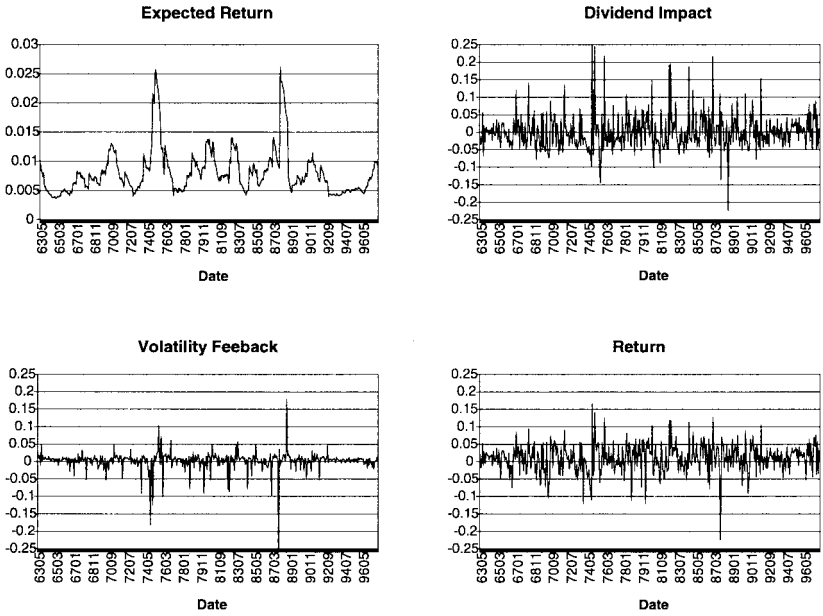


Figure 3
Monthly return components as implied by the volatility feedback model
This figure shows the historical monthly returns with estimates of the contributions of the three components as implied by our volatility feedback model [Equation (30)]: expected returns, return shocks from dividend news, and volatility feedback.

returns. Yet since the innovation to volatility is conditionally symmetric, the induced negative skewness is weak.

3. Conclusion

Volatility in equity markets is asymmetric: contemporaneous returns and conditional return volatility are negatively correlated. In this article I develop a volatility feedback model where dividend growth and dividend volatility are the two state variables. The model is estimated by simulated method of moments. I find that volatility feedback is significant both statistically and economically. I also find that the leverage effect and volatility feedback effect both play very important roles in generating asymmetric volatility. For the monthly and weekly CRSP value-weighted index, the leverage effect contributes more to the negative correlation between return and return volatility. I find dividend news and volatility feedback are both important in generating returns. Volatility feedback can be very large during volatile periods of the market. Its importance in determining the return dynamics under stable market conditions, however, seems to be secondary to dividend innovation itself.

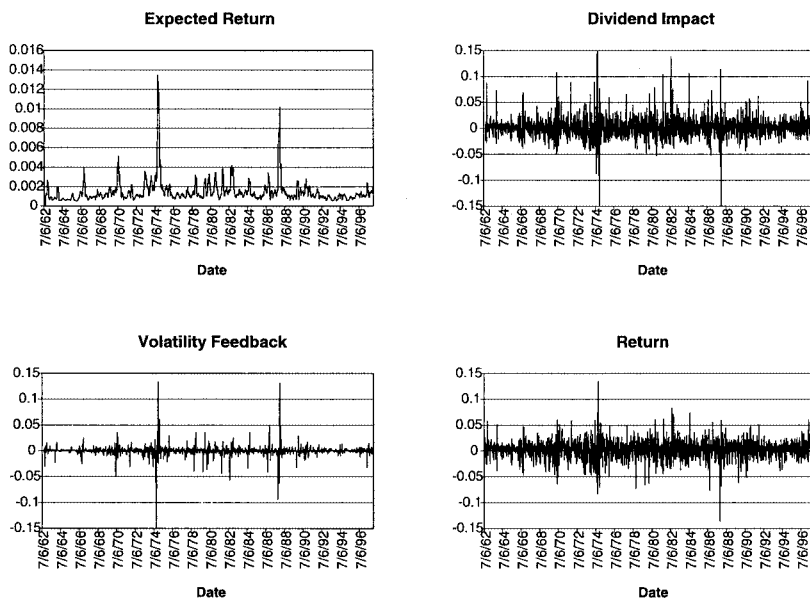


Figure 4
Weekly return components as implied by the volatility feedback model

This figure shows the historical weekly returns with estimates of the contributions of the three components as implied by our volatility feedback model [Equation (31)]: expected returns, return shocks from dividend news, and volatility feedback.

Appendix: Proof of Propositions

Proof of Proposition 1. We first postulate a solution to the log price-dividend ratio in terms of the state variables. We then verify this solution and solve for the parameters of the solution. The linear solution takes the following form,

$$p_t - d_t = c_0 + c_1 g_t + c_2 \sigma_{d,t}^2.$$

Substituting this solution to Equation (6), we get

$$1 = E_t \exp[A(\cdot)],$$

where

$$A(\cdot) = -r^f - \frac{1}{2} \sigma_{m,t}^2 + \epsilon_{m,t+1} + k + \rho(c_0 + c_1 g_{t+1} + c_2 \sigma_{d,t+1}^2) + g_{t+1} - (c_0 + c_1 g_t + c_2 \sigma_{d,t}^2). \quad (32)$$

Since $A(\cdot)$ is a normal random variable, we must have

$$E_t[A(\cdot)] + \frac{1}{2} \text{var}[A(\cdot)] = 0. \quad (33)$$

Substituting Equations (32) and (3) into Equation (33), we obtain

$$0 = \left[-r^f + k + (\rho - 1)c_0 + (\rho c_1 + 1)\alpha_0 + \rho c_2 \beta_0 \right] + \left[(\rho c_1 + 1)\alpha_1 - c_1 \right] g_t \\ + \left[(\rho \beta_1 - 1)c_2 + \frac{1}{2}(\rho c_1 + 1)^2 + \frac{1}{2}\rho^2 c_2^2 \eta_v^2 + (\rho c_1 + 1)\rho \eta_v \rho_t c_2 + (\rho c_1 + 1)\rho_m \right] \sigma_{d,t}^2$$

Note that for this equation to hold, the terms in the three brackets must be identically zero, since g_t and $\sigma_{d,t}^2$ are random variables. Solving the resulting three equations produces the solution to the log price:dividend ratio.

To prove the second part of the proposition we simply substitute the log price:dividend ratio solution [Equation (7) and Equation (3)] into the return equation [Equation (5)]. Q.E.D.

Proof of Proposition 2. We first note that $\lambda_3 = -\rho c_2$. Since ρ is the average ratio of the stock price to the sum of the stock price and the dividend, it is a positive number smaller than one. For λ_3 to be positive, c_2 has to be negative. Since ρ_t is likely to be a negative number due to the leverage effect, only the negative root in Equation (9) is feasible. Moreover, the second term in the equation must be larger than the first term, which leads to

$$\rho^2 \eta_v^2 [2\rho_m (1 - \rho \alpha_1) + 1] < 0.$$

This is clearly satisfied given $\rho_m < -\frac{1}{2(1-\rho\alpha_1)}$.

Since $\lambda_1 = -(1 - \rho \beta_1)c_2$, it is clearly positive. Finally, since $\lambda_2 = \frac{1}{1-\rho\alpha_1}$, it is positive. Q.E.D.

Proof of Proposition 3. We derive the news about volatility term [Equation (20)]. The news about dividends term [Equation (19)] can be derived in a similar fashion:

$$(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+j} = E_{t+1} \sum_{j=1}^{\infty} \rho^j r_{t+1+j} - E_t \sum_{j=1}^{\infty} \rho^j r_{t+1+j}.$$

Substituting in Equation (11) and taking expectations we get

$$(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+j} = \lambda_1 \sum_{j=1}^{\infty} \rho^j \left[E_{t+1}(\sigma_{d,t+j}^2) - E_t(\sigma_{d,t+j}^2) \right] = \lambda_1 \rho \sigma_{d,t} v_{d,t+1} \sum_{j=0}^{\infty} \rho^j \beta_1^j \\ = \frac{\lambda_1 \rho}{1 - \rho \beta_1} \sigma_{d,t} v_{d,t+1}.$$

Note that $\lambda_1 = -(1 - \rho \beta_1)c_2$, we finally obtain

$$(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+j} = -\rho c_2 \sigma_{d,t} v_{d,t+1} = \lambda_3 \sigma_{d,t} v_{d,t+1}. \quad \text{Q.E.D.}$$

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