An analysis of risk for defaultable bond portfolios

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Abstract

Purpose – The purpose of this article is to estimate value at risk (VaR) using quantile regression and provide a risk analysis for defaultable bond portfolios.

Design/methodology/approach – The method proposed is based on quantile regression pioneered by Koenker and Bassett. The quantile regression approach allows for a general treatment on the error distribution and is robust to distributions with heavy tails.

Findings – This article provides a risk analysis for defaultable bond portfolios using quantile regression method. In the proposed model we use information variables such as short-term interest rates and term spreads as covariates to improve the estimation accuracy. The study also finds that confidence intervals constructed around the estimated VaRs can be very wide under volatile market conditions, making the estimated VaRs less reliable when their accurate measurement is most needed.

Originality/value – Provides a risk analysis for defaultable bond using quantile regression approach.

Keywords Risk analysis, Bonds, Autogressive processes

Paper type Research paper

1. Introduction

A corporate institution faces many types of risks. These risks affect its financial well being on a regular basis. In the extreme case, as a result of its exposure to such risks, the institution may not be able to survive as a going concern. In addition to the business risks that are specific to its market environment, a firm faces market risk and credit risk. Market risk arises from adverse movements in prices of financial assets such as equity, and market rates such as interest rates and exchange rates. Credit risk is the risk of loss arising from the failure of a counter-party to make a promised payments. Market risk and credit risk are not only intrinsically related to each other, but also nonseparable (Jarrow and Turnbull, 2000). In this paper we provide a regression quantile method to estimate the value-at-risk (VaR), and apply the model to returns data on portfolios of treasury securities and rated, defaultable corporate bonds, thus capturing the dynamics of market risk and credit risk[1].

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VaR has become a popular tool in the measurement and management of market risks (see Beder, 1995; Duffie and Pan, 1997; Dowd, 1998; Saunders, 1999) for reviews of literature on VaRs). It is viewed as the best measurement for market risk (Group of Thirty, 1993). Specifically, VaR is the loss in market value that is exceeded with a certain probability over a given time horizon, such a probability is often set at 1 or 5 percent. In requiring all US publicly traded corporations to report their quantitative market risk exposures, the Securities and Exchange Commission (SEC) (1997) lists VaR as a disclosure method "expressing the potential loss in future earnings, fair values, or cash flows from market movements over a selected period of time and with a selected likelihood of occurrence." VaR disclosure communicates a single dollar amount for a public company's aggregate risk exposures, allowing for leverage, diversification of a variety of risk factors that affect the company's trading portfolios. Reporting VaR also forces companies to develop a systematic approach for risk measurement.

Given the importance of VaR in reporting market risk and its prominence in risk measurement and risk management, it is not surprising that its estimation has attracted much attention from researchers. One popular approach to estimate VaR assumes a conditionally normal stock return distribution. The estimation of VaR is equivalent to estimating conditional volatility of returns[2]. Another popular method is to compute the empirical quantile nonparametrically, for example, rolling historical quantiles or Monte Carlo simulations based on an estimated model[3].

However, these models are based on some restricted assumptions, such as normal, about the distributions of stock returns. There has been accumulated evidence that portfolio returns (or log returns) are usually not normally distributed. In particular, it is frequently found that market returns display structural shifts, negative skewness and excess kurtosis in the distribution of the time series. These market return characteristics suggest that more robust method in estimating VaR is needed. In this paper, we estimate VaR using a robust method based on quantile regressions. The quantile regression method is an extension of the empirical quantile methods. While classical linear regression methods based on minimizing sums of squared residuals enable one to estimate models for conditional mean functions, quantile regression methods offer a mechanism for estimating models for the conditional quantile functions, thus quantile regression is capable of providing a complete statistical analysis of the stochastic relationships among random variables (see Koenker and Bassett, 1978; Koenker, 1999).

In this paper, we estimate VaR via a quantile regression model that allows for ARCH effect. VaRs estimated by this quantile regression approach display certain nice properties: they track VaRs estimated from GARCH volatility models well during normal market conditions. However, during a market turmoil when market drops are followed by further drops or rebounds, GARCH volatility models tend to predict implausibly high VaRs. This is due to that GARCH models treat both large positive and large negative return shocks as indicators of higher volatility, while only large negative return shocks indicate higher value at risk. Therefore, volatility and VaRs are not synonymous. VaRs estimated by the ARCH quantile regression model, while predicting higher volatility in the ARCH component, assigns a much larger weight to a big negative return shock than to a big positive return shock. The resulting estimated VaRs are therefore closer to reality: a large drop in market return is indicative of a high probability in both a further market drop and a market rebound.

Other methods that allow for more general distributional assumptions include Engle and Manganelli (1999). Engle and Manganelli (1999) consider a different quantile regression based method. In particular, they consider an autoregression on the estimated VaRs. Comparing with these methods, the ARCH quantile regression model that we use in the current paper has the advantage of well-developed distributional theory that facilitates statistical inference and construction of confidence interval, as well as efficient computational advantages[4].

While most existing VaR estimation methods were developed for stock portfolios and foreign exchange positions, financial institutions such as pension funds, mutual funds, insurance companies and hedge funds generally hold large positions of treasury securities and defaultable bonds, especially in recent years[5]. It is conceivable that fixed income securities display different risk characteristics from that of stocks and foreign exchanges. For example, defaultable bond returns are closely linked to the term structure of interest rates and the dynamics of the default premiums. Therefore, understanding the unique features of bond VaRs not only recognizes the growing popularity of bond market in the current financial world, but also has important practical implication for managing risks of fixed income securities[6].

When applying our method to estimate bond portfolios, we add information in short-term interest rates and term spreads. Most existing methods of calculating VaR use univariate time series, ignoring information in related time series. However, information contained in other time series can be quite helpful and ignoring this information reduces the estimation efficiency, thus we propose a general model of calculating VaR that takes into accounts of useful covariates.

In addition to measuring bond VaRs using quantile regressions, we also provide a sense of accuracy to our individual VaR estimates. While current VaR estimation methods generally do not provide a systematic approach to examine how accurate the estimated VaRs are, a number of analysts and investors have expressed concerns to the SEC about the accuracy of the quantitative, probabilistic information contained in firms' VaR disclosures (Hodder *et al.*, 2001). Therefore, evaluating the accuracy of VaR estimates is important in VaR market risk reporting. In this study, we calculate confidence bands of the estimated VaRs under fairly general distribution assumptions. The results indicate that for the portfolios of treasury bonds, the estimated VaRs are generally small and the confidence intervals are narrow. However, large estimated VaRs and sometimes wide confidence intervals are associated with defaultable bond portfolios.

This paper makes the following contributions to research in VaR and risk management in general. First, we propose a quantile regression model using useful covariates in estimating VaR for defaultable bond portfolios. The model allows for conditional heteroskedasticity. This procedure is easy to implement and the estimation programs are available in standard statistical packages such as S-Plus, and can also be easily written in other programming languages. Given that one criticism of using VaR as a reporting alternative of companies' quantitative market risk exposure is its difficulty in estimation (see Hodder *et al.*, 2001), the approach we propose in this paper provides a convenient solution to estimating VaR. Second, we apply the proposed model to defaultable bond portfolios after considering information in related time series. In doing so, this paper illustrates the importance of using short-term interest rates and term spreads as covariates in defaultable bond VaR estimations. These covariates should be used in the ARCH component of the model. Lastly, we show that since a complete mean equation is hard to estimate with high statistical significance, keeping it as simple as an AR(1) model in returns is often sufficient. Therefore, covariates should not be used in the mean equation.

The remainder of the paper is organized as follows. Section 2 presents a quantile regression model to estimate VaR. It derives confidence bands and model specification tests. Section 3 provides data descriptions and conducts the estimation. Empirical results regarding estimated VaR, confidence bands and model performance are reported and discussed in section 4. Section 5 contains concluding remarks.

2. The model

2.1 Analyzing risk by VaR

For ease of exposition, we define value-at-risk as the percentage loss in market value over a given time horizon that is exceeded with probability τ. That is, for a time series of returns on an asset, $\{R_t\}_{t=1}^{n}$, find VaR_t such that:

$$\Pr\left(R_t < -VaR_t|\mathscr{I}_{t-1}\right) = \tau \tag{1}$$

where $\mathscr{I}_{t-1}$ denotes the information set at time $t-1$. From this definition, it is clear that finding a VaR essentially is the same as finding a 100τ percent conditional quantile.

A natural way of modeling a return process is to use some type of autoregressive specification. If we consider the following regression model for the defaultable bond returns process $\{R_t\}$:

$$R_t = \alpha_0 + \sum_{i=1}^{k} \alpha_i R_{t-1} + u_t \tag{2}$$

the 100τ percent value-at-risk of R_t is then determined by:

$$\alpha_0 + \sum_{i=1}^{k} \alpha_i R_{t-i} + Q_u\left(\tau|\mathscr{I}_{t-1}\right) \tag{3}$$

where $Q_u\left(\tau|\mathscr{I}_{t-1}\right)$ is the p-th conditional quantile of the residual process u_t. More generally, we may consider the following regression:

$$R_t = \alpha' x_t + u_t$$

where $x_t \in \mathscr{I}_{t-1}$ is the vector of regressors. Usually x_t include lag values of the dependent variable. In the case that $x_t = \left(1, R_{t-1}, \ldots, R_{t-k}\right)$, we get model (2). To calculate VaR_t, we need to model u_t and calculate $Q_u\left(\tau|\mathscr{I}_{t-1}\right)$.

An important property of financial time series is the presence of conditional heteroskedasticity. A natural way to capture this important characteristic in the returns process (equation (3)) is to let the variance of u_t depend on its lagged values. If we specify u_t in the following way:

$$u_t = \left(\gamma_0 + \gamma_1|u_{t-1}| + \ldots + \gamma_q|u_{t-q}|\right)\varepsilon_t$$

we obtain an ARCH type model for defaultable bond returns process.

2.2 Improving efficiency by adding covariates

In the context of evaluating asset prices in univariate time series, the convention is to ignore information in related time series. Sometimes, information contained in other time series can be quite helpful and ignoring this information may be costly. For this reason, we consider the following general model for u_t by including useful covariates in the conditional mean and conditional heteroskedasticity specification:

$$R_t = \alpha' x_t + u_t$$

where

$$x_t = \left(1, R_{t-1}, \cdots, R_{t-1}, z_{1t}, \cdots, z_{vt}\right)$$

and

$$u_t = \left(\gamma_0 + \gamma_1 |u_{t-1}| + \cdots + \gamma_q \left|u_{t-q}\right| + \gamma_{z,1} |z_{1t}| + \cdots + \gamma_{z,v} |z_{vt}|\right) \varepsilon_t \left(z_1, \cdots, z_v\right) \qquad (4)$$

were covariates and $\gamma_0 > 0$, $\left(\gamma_1, \cdots, \gamma_q, \gamma_{z,1} \cdots, \gamma_{z,v}\right)' \in \Re_{+}^{q+v}$. Here we assume that the innovations $\{\varepsilon_t\}$ have a general distribution $F(\cdot)$, including the normal distribution and other commonly used distributions in financial applications with heavy tails. This is a quite general setting that includes the popular ARCH model. Since an ARMA process can be asymptotically represented by AR processes, with an appropriate chosen number of lags. This model can also practically provide a good approximation for GARCH models, and avoids the technical and computational difficulties for GARCH models that have not yet been solved in the context of quantile regression. Our purpose is to estimate the above models and analyze these estimates.

3. Estimation procedure

3.1 The data

The data used in this paper are composed of monthly bond index returns obtained from Lehman Brothers. The indexes include a US Treasury index (all public obligations of the US Treasury), as well as corporate indexes for bonds rated Aaa, Aa, A, and Baa by Moody's investors service. These indices are further separated based on time to maturity. The intermediate-term indices include bonds with maturities of up to ten years and the long-term indices include those with maturities of ten years or longer. The return is the monthly total return as defined by Lehman Brothers, which takes into consideration any relevant accrued interests and coupon payments. The sample period begins in January 1973 and ends October 1998 for a total of 310 observations for each series. For the defaultable bonds, we use the first log differences of the six-month T-bill yields and the ten-year treasury note yields as covariates in the quantile regression.

Table I reports the summary statistics of the intermediate-term bond return data and the T-bill data. We see that the means of returns increase as we move from treasury notes to Aaa, Aa, A and Baa rated bonds, signifying increasing default premiums. The standard deviations also increase in general. It is interesting to note that all reported bond returns display positive skewness and moderate excess kurtosis. There are some evidence of positive serial AR(1) correlation and negative AR(2) and AR(3) correlation, indicating some persistence and mean reversion in monthly bond returns. Six-month T-bill yields display negative skewness and moderate excess kurtosis. The autocorrelation dynamics is similar to bond returns.

	T-note	Aaa	Aa	A	Baa	T-bill 1st Diff.
Mean	0.0072	0.0074	0.0075	0.0076	0.0082	−0.0009
Std. Dev.	0.0132	0.0158	0.0160	0.0159	0.0165	0.0616
Max	0.0857	0.1020	0.1018	0.0957	0.0937	0.1559
Min	−0.0446	−0.0571	−0.0576	−0.0562	−0.0582	−0.3981
Skewness	0.7056	0.9342	0.7886	0.7165	0.4111	−0.9660
Excess Kurtosis	4.8985	7.4123	6.5167	5.6442	4.7521	5.5810
AC(1)	0.1706	0.1654	0.1736	0.1891	0.2050	0.3507
AC(2)	−0.0730	−0.0734	−0.0510	−0.0381	0.0004	−0.0308
AC(3)	−0.0673	−0.0566	−0.0443	−0.0284	−0.0284	−0.0688
AC(4)	−0.0357	−0.0519	−0.0674	−0.0524	−0.0108	0.0085
AC(5)	0.0808	0.1303	0.1106	0.0990	0.0935	0.0495
AC(10)	0.0801	0.0578	0.0669	0.0573	0.0434	0.0543

Notes: This table shows the summary statistics for the monthly returns of five intermediate term bond indexes, and the log difference of the six-month T-bill yield; AC(k) denotes autocorrelation of order k; The sample period is from January 1973 to October 1998

Table I. Summary statistics of the intermediate term bond return data

Table II reports summary statistics of the long-term bond return data and the ten-year treasury note yield data. Overall the data displays similar characteristics to the intermediate-term data. The long-term return series are more volatile and display excess, but mild kurtosis. The autocorrelation structures of the long-term bonds and the ten-year T-note are similar to those of the intermediate-term bonds.

3.2 *The estimation method*

The idea of quantile regression provides a natural way of estimating value at risk. Quantile regression was introduced by Koenker and Bassett (1978) and has received a lot of attention in econometrics and business statistical research in the past two decades. To introduce quantile regression we consider a random variable Y that is characterized by its distribution function $F(y)$, the τ-th quantile of Y is defined by:

	T-bond	Aaa	Aa	A	Baa	10Y 1st Diff.
Mean	0.0082	0.0079	0.0079	0.0081	0.0087	−0.0011
Std. Dev.	0.0302	0.0287	0.0278	0.0274	0.0286	0.0373
Max	0.1433	0.1476	0.1456	0.1332	0.1427	0.1390
Min	−0.0798	−0.0862	−0.0886	−0.0895	−0.1029	−0.1235
Skewness	0.4624	0.5152	0.4339	0.3334	0.2168	−0.1548
Excess Kurtosis	1.6549	2.7971	2.8997	2.3456	3.0878	1.1379
AC(1)	0.1174	0.1424	0.1596	0.1769	0.1980	0.3673
AC(2)	−0.0352	−0.0536	−0.0509	−0.0690	−0.0370	−0.0733
AC(3)	−0.1034	−0.0681	−0.0524	−0.0531	−0.0392	−0.0211
AC(4)	−0.0042	−0.0542	−0.0489	−0.0232	0.0436	0.0218
AC(5)	0.0536	0.0826	0.0796	0.0915	0.1066	0.0526
AC(10)	0.0512	0.0244	0.0356	0.0500	0.0360	0.0466

Notes: This table shows the summary statistics for the monthly returns of five long term bond indexes, and the log difference of the ten year treasury note yield; AC(k) denotes autocorrelation of order k. The sample period is from January 1973 to October 1998

Table II. Summary statistics of the long-term bond return data

$$Q_Y(\tau) = \inf\{y|F(y) \geq \tau\}$$

Similarly, if we have a random sample $\{y_1, \cdots, y_n\}$ from the distribution F, the τ-th sample quantile can be defined as:

$$\hat{Q}_Y(\tau) = \inf\left\{y\middle|\hat{F}(y) \geq \tau\right\}$$

where $\hat{F}$ is the empirical distribution function of the random sample. Note that the above sample quantile may be found by solving the following minimization problem:

$$\min_{b\in\Re}\left[\sum_{t\in\{t:y_t\geq b\}}\tau\left|y_t - b\right| + \sum_{t\in\{t:y_t<b\}}(1-\tau)|y_t - b|\right] \tag{5}$$

Koenker and Bassett (1978) studied the analogue of the empirical quantile function for the linear models and generalized the concept of quantiles to the regression context.

If we consider a regression model:

$$y_t = b'x_t + u_t \tag{6}$$

where x_t is a k by 1 vector of regressors including an intercept term and lagged residuals, then, conditional on the regressor x_t, the τ-th quantile of y:

$$Q_Y(\tau|x_t) = \inf\{y|F(y|x_t) \geq \tau\}$$

is a linear function of x_t:

$$b_1 + F_u^{-1}(\tau) + b_2x_{2t} + \cdots + b_kx_{kt}$$

where $F_u(\cdot)$ is the cumulative distributional function of the residual. Koenker and Bassett (1978) show that the p-th conditional quantile of y can be estimated by an analogue of equation (5):

$$\hat{Q}_Y(\tau|x_t) = x'_t\hat{b}(\tau) \tag{7}$$

where

$$\hat{b}(\tau) = \arg\min_{b\in\Re^k}\left[\sum_{t\in\{t:y_t\geq x_tb\}}\tau|y_t - x'_tb| + \sum_{t\in\{t:y_t<x_tb\}}(1-\tau)|y_t - x'_tb|\right] \tag{8}$$

is called as the regression quantiles. As a special case, the least absolute error estimator is the regression median, i.e. the regression quantile for $\tau = 0.5$. The quantile regression theory can be extended to time series models with conditional heteroskedasticity. If we consider the following model with conditional heteroscedasticity:

$$y_t = (\gamma_0 + \gamma_1|y_{t-1}| + \cdots + \gamma_q|y_{t-q}|)\varepsilon_t$$

with $\gamma_0 > 0$, $(\gamma_1, \cdots, \gamma_q)' \in \Re^q_+$, then this is a time series with ARCH effect.

3.3 Estimating VaR for defaultable bond portfolios

We use quantile regression method to estimate the value at risk of bond portfolios. Given equation (4), if we denote the vector $(1, |u_{t-1}|, \cdots, |u_{t-q}|, |z_{1t}|, \cdots, |z_{vt}|)$ as Z_t and the corresponding coefficient vector as γ, then:

$$Q_u(\tau|\mathscr{I}_{t-1}) = \gamma(\tau)' Z_t$$

where

$$\gamma(\tau)' = (\gamma_0 Q_\varepsilon(\tau), \gamma_1 Q_\varepsilon(\tau), \cdots, \gamma_{q+v} Q_\varepsilon(\tau))$$

and $Q_\varepsilon(\tau) = F^{-1}(\tau)$ is the quantile function of ε. By definition, VaR_t, the conditional VaR at τ-percent level, is just the conditional quantile of R_t in the model of equation (3) given information to time $t-1$, i.e. $\mathscr{I}_{t-1}$. Thus:

$$-VaR_t(\tau) = \alpha_0 + \sum_{i=1}^{k} \alpha_i R_{t-i} + \gamma(\tau)' Z_t \tag{9}$$

In order to estimate the conditional VaR, we need to estimate $\gamma(\tau)$. In this paper, we use quantile regression method to estimate $\gamma(\tau)$ and thus $VaR_t(\tau)$. In particular, the parameters that determine the conditional heteroskedasticity, i.e. $\gamma(\tau)$, can be estimated by the following problem:

$$\hat{\gamma}(\tau) = \arg\min_{\gamma \in \Re^k} \left[\sum_{t \in \{t: u_t \geq Z'_t\gamma\}} \tau |u_t - Z'_t\gamma| + \sum_{t \in \{t: u_t < Z'_t\gamma\}} (1-\tau)|u_t - Z'_t\gamma| \right] \tag{10}$$

In practice, we can replace u_t and Z_t by their (say, OLS) estimators $\hat{u}_t = R_t - \hat{\alpha}_0 + \sum_{i=1}^{k} \hat{\alpha}_i R_{t-i} + \sum_{i=1}^{v} \hat{\alpha}_{k+i} z_{it}$. Under mild regularity conditions, it can be shown that the $\hat{\gamma}(\tau)$ estimated based but is still a (root-n) consistent estimator of $\gamma(\tau)$.

3.4 Computation and properties of the method

Quantile regression method has the important property that it is robust to distributional assumptions. This property is inherited from the robustness property of the ordinary sample quantiles. Quantile estimation is only influenced by the local behavior of the conditional distribution of the response near the specified quantile. As a result, the estimated coefficient vector $\hat{\gamma}(\tau)$ is not sensitive to outlier observations. Such a property is especially attractive in financial applications since many financial data such as portfolio returns (or log returns) are usually heavy-tailed and thus are not normally distributed.

The quantile regression model has a linear programming representation that makes the estimation easy. Notice that the optimization problem (10) may be reformulated as a linear program by introducing "slack" variables to represent the positive and negative parts of the vector of residuals (see Koenker and Bassett, 1978, for a more detailed discussion). Computation of the regression quantiles by standard linear programming techniques is very efficient. It is also straightforward to impose the non-negativity constraints on all elements of γ.

3.5 Model tests: choosing the lags

An issue that arises with the implementation of the quantile regression and related inference is the choice of lags. The distribution of the quantile regression coefficient estimates facilitates significance tests based on t-ratios or Wald statistics. Notice that the estimated coefficients of the lagged differences are asymptotically normally distributed, a general-to-specific modeling strategy that chooses between a model with q lags and a model with $r = q + k$ lags can be obtained based on a sequential test. Let $\hat{\gamma}_{q+1,r}(\tau)$ denote the vector of coefficients $\left(\hat{\gamma}_{q+2}(\tau), \cdots, \hat{\gamma}_{r+1}(\tau)\right)$. We also define Γ_k to be the lower-right $k \times k$ block of the covariance matrix. The Wald statistic for the null hypothesis that the coefficients on the last k lags are jointly equal to 0 is then given by:

$$\hat{J}_k(\tau) = \frac{N}{\tau(1-\tau)} \hat{\gamma}_{q+1,r}(\tau)' \hat{\Gamma}_k^{-1} \hat{\gamma}_{q+1,r}(\tau) \tag{11}$$

where $\hat{\Gamma}_k$ is a consistent estimator of Γ_k. Under the null hypothesis that the coefficients on the last k lags are zeros, $\hat{J}_{q,k}(\tau)$ converge to a chi-square distribution with k degrees of freedom. We now consider the following procedure for choosing q from a set of possible values $\{0, 1, 2, \cdots, q_{\max}\}$, where $q_{\max}$ is an upper bound selected *a priori*: starting with the most general model with $q_{\max} + k$ lags and test whether the coefficients of the last lags are significant at given level θ. If they are, then choose $q = q_{\max}$; otherwise, we consider regressions of order $q_{\max} + k - 1$ and perform the test again. We choose q to be $q^* + 1$ if, at given significance level, $\hat{J}_{q^*,k}(\tau)$ is the first in the sequence $\hat{J}_{i,k}(\tau)\left(i = q_{\max} - 1, \cdots 1\right)$, which is significantly different from zeros. If $\hat{J}_{i,k}(\tau)$ is not significantly different from zero for all $i = q_{\max} - 1, \cdots, 1$, we choose q to be 0. The above test reduces to a t test on the last lag if the test is performed with $k = 1$[7].

4. Empirical results

4.1 Estimated VaRs

Using the model (3) and (9) proposed in the previous section, we analyze the intermediate-term and long-term bond indexes. For each series, we estimate the model with various lags in the mean equation and the ARCH equation. We also use short-term interest rates as covariates to improve the model performance. In particular, we choose yield changes (with one period lag) in six-month T-bills and ten-year T-note as the covariates. We choose the lag length based on the sequential t-test and report the estimated results of the optimal model (that uses the optimal lag length). The estimated model parameters are reported in Tables III and IV, respectively.

We see from Table III that for the intermediate bond indexes the intercept term α_0 for all series are very stable and significant. The AR(1) coefficient α_1 is also positive and significant. The coefficient increases with the decline of bond ratings, indicating that more risky bonds tend to exhibit stronger first order autocorrelation. We experimented with different lags and found that more lags are not needed and should be avoided. Since the conditional standard deviations of the returns on bond portfolios are generally much smaller than those of stock returns, over-fitting the dynamics in the mean equation may result in excessive fluctuations in the estimated VaRs. This also explains why we only use the covariates in the ARCH equation instead of the mean equation.

	T-note	Aaa	Aa	A	Baa
α_0	0 0.0060	0.0061	0.0061	0.0062	0.0065
	(2.367)	(2.698)	(2.212)	(2.885)	(2.521)
α_1	1 0.1701	0.1655	0.1737	0.1892	0.2053
	(1.929)	(2.015)	(2.015)	(3.015)	(2.186)
$\gamma_{z,1}$	0.0443	0.0668	0.0804	0.0397	0.0338
	(1.519)	(2.218)	(1.999)	(2.117)	(1.562)
$\gamma_{z,2}$	−0.1305	−0.1738	−0.1329	−0.0752	−0.0855
	(−1.687)	(−2.521)	(−1.030)	(−1.725)	(−1.058)
γ_0	−0.0192	−0.0229	−0.0236	−0.0253	−0.0266
	(−16.48)	(−12.31)	(−12.36)	(−13.89)	(−16.29)
γ_1	−0.0191	−0.1762	−0.0090	0.0314	−0.0815
	(−0.1409)	(−1.027)	(−0.0434)	(0.1600)	(−0.4509)
γ_2	−0.2253	−0.0233	0.0677	0.1411	0.1791
	(−1.081)	(−0.2221)	(0.3284)	(0.8768)	(6.498)
γ_3	−0.1214	0.1214	0.1314	0.0734	0.1127
	(−1.175)	(1.934)	(1.131)	(0.7364)	(1.403)
γ_4	−0.1708	−0.0142	−0.0847	−0.0762	0.1109
	(−2.190)	(−0.1669)	(−0.8961)	(−0.7582)	(1.047)
γ_5		0.1619	0.2076	0.1722	0.1402
		(2.013)	(1.722)	(1.930)	(1.161)
γ_6		0.2178	0.1988	0.1202	0.1850
		(2.839)	(1.831)	(1.857)	(3.532)

Notes: This table reports the parameter estimates of the five intermediate term bond indexes; t-statistics are reported below the parameters in parentheses

Table III. Parameter estimates for the intermediate term bond indexes

In the ARCH equation, we find the coefficients on the covariates to be mostly significant. The coefficients for the six-month treasury bill yields ($\gamma_{z,1}$) are all positive, indicating that higher volatility is associated with higher short-term interest rates. However, the coefficients for the ten-year treasury note yields ($\gamma_{z,2}$) are negative. This indicates that increases in intermediate-term treasury yield reduce the conditional volatility of bond returns. Finally, Most preferred ARCH models have six lags. Typical of ARCH modeling, some of the coefficients are not significant. Table IV reports the parameter estimates for the long-term bond indexes. The results are similar to those of the intermediate term bond indexes.

In Figure 1, we plot the estimated 5 percent VaRs for the intermediate-term and long-term treasury indexes (T_I and T_L, respectively). The two series are strikingly different. We see that the VaRs for the intermediate-term bond index are fairly stable over time, hovering at a level slightly over 1 percent. They range mostly between 0.5 to 2 percent. For the long-term bond index VaRs average about 4 percent and ranges mostly between 2.5 to 6 percent. For both series, VaRs display the highest level of volatility during the summer of 1980. Clearly, longer maturity bond instruments are inherently more risky. This result also holds for all the rated defaultable bonds.

Figures 2 and 3 graph the estimated 5 percent VaRs for the rated intermediate-term and long-term bond indexes, respectively. To clearly show the results, we only plot the series for Aaa and Baa bonds. In order to make Figures 2 and 3 comparable, we draw these two to the same scale. It is striking that the long-term bonds have much larger VaRs and they display much higher variations. In contrast, for the same maturities, the

	T-bond	Aaa	Aa	A	Baa
α_0	0 0.0072	0.0067	0.0066	0.0067	0.0069
	(3.168)	(3.220)	(3.010)	(2.980)	(3.160)
α_1	1 0.1174	0.1425	0.1598	0.1772	0.1985
	(3.009)	(2.846)	(1.950)	(1.727)	(1.865)
$\gamma_{z,1}$	0.1085	0.2097	0.2005	0.1925	0.1309
	(1.543)	(4.329)	(4.219)	(3.099)	(2.107)
$\gamma_{z,2}$	−0.0647	−0.1113	−0.0787	−0.1832	−0.2059
	(−0.6317)	(−0.7058)	(−0.4597)	(−1.933)	(−1.632)
γ_0	−0.0463	−0.0437	−0.0455	−0.0462	−0.0485
	(−14.58)	(−18.15)	(−16.20)	(−17.83)	(−16.17)
γ_1	0.1196	0.1145	0.1515	0.0936	−0.1050
	(0.7175)	(0.7698)	(0.8626)	(0.7871)	(−0.7433)
γ_2	0.0653	0.2394	0.2224	0.0491	0.0906
	(0.2452)	(2.712)	(1.284)	(0.5214)	(1.182)
γ_3	−0.0358	0.0914	0.1278	0.1247	0.1599
	(−0.5760)	(0.9245)	(1.642)	(1.519)	(2.729)
γ_4	0.0744	0.0284	0.0415	0.0419	0.3411
	(1.004)	(0.6356)	(0.7415)	(0.5643)	(4.357)
γ_5	−0.0177	0.0449	−0.0136	0.1009	−0.0127
	(−0.1853)	(0.7284)	(−0.3027)	(1.605)	(−0.1682)
γ_6	0.1708	0.2771	0.3443	0.2665	0.3844
	(2.492)	(5.539)	(4.585)	(3.616)	(3.273)

Table IV.
Parameter estimates for the long term bond indexes

Notes: This table reports the parameter estimates of the five long term bond indexes; *t*-statistics are reported below the parameters in parentheses

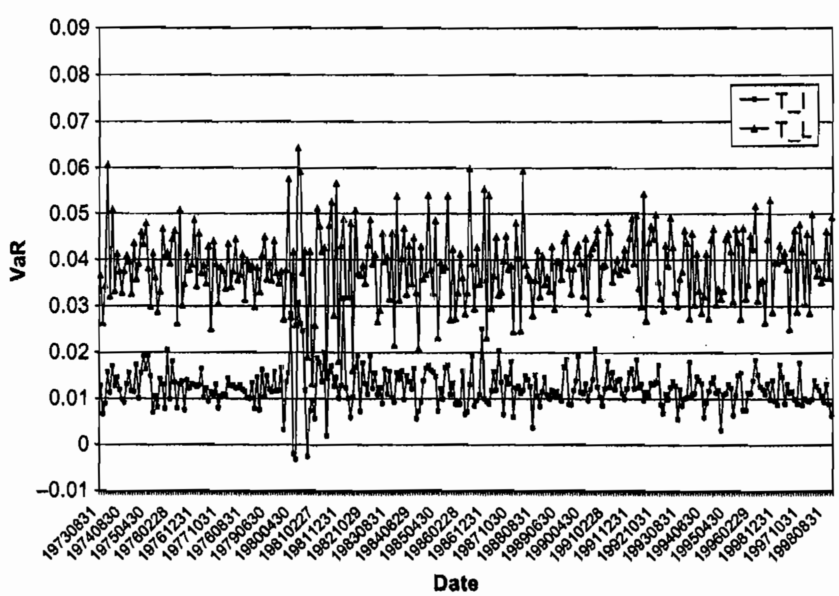
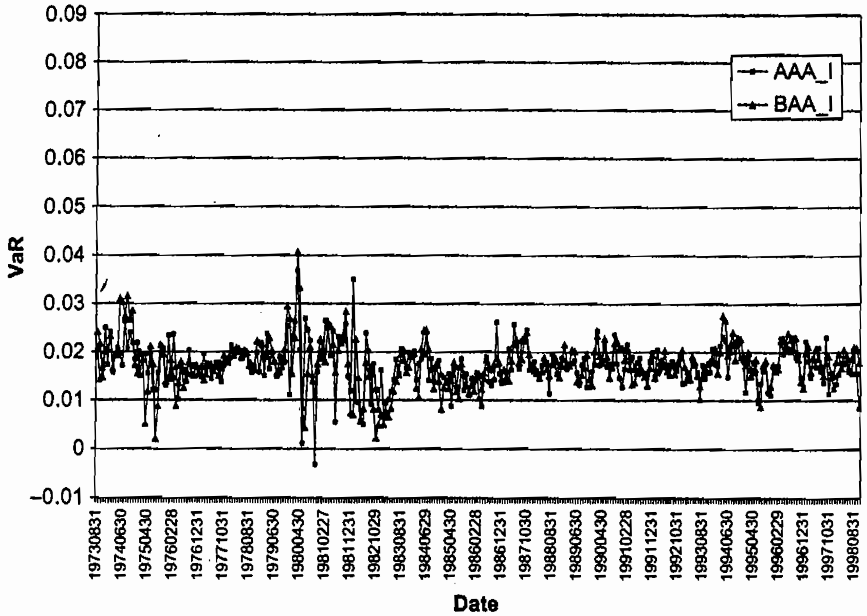


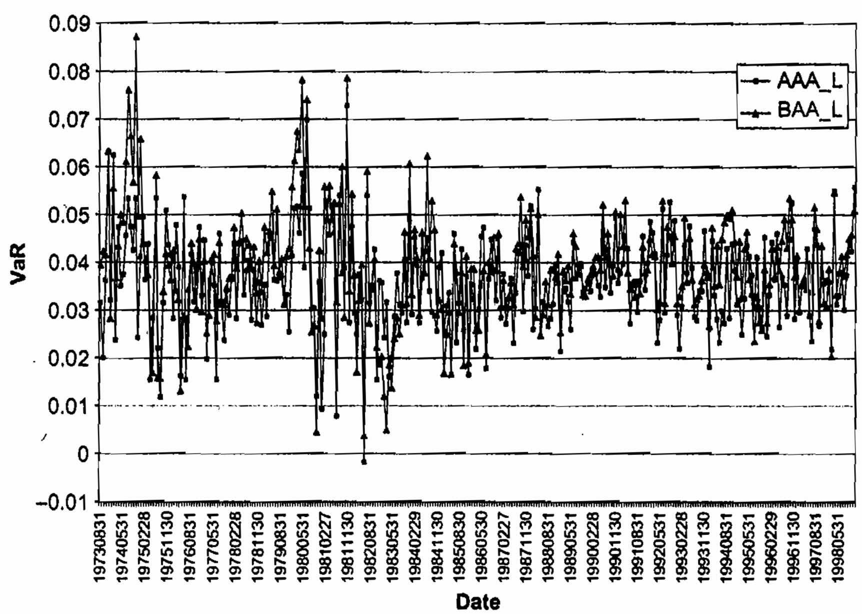
Figure 1.
5 percent VaR by regression quantile for treasury bonds

Note: This figure shows the 5% VaR by regression quantile for treasury bonds



Note: This figure shows the 5% VaR by regression quantile for intermediate term bonds

Figure 2.
5 percent VaR by regression quantile for intermediate term bonds



Note: This figure shows the 5% VaR by regression quantile for long term bonds

Figure 3.
5 percent VaR by regression quantile for long term bonds

Baa bonds only have marginally larger VaRs and the variability in estimated VaRs are about the same. This result shows that Treasury term structure dynamics dominate the dynamics of rated bond returns. Certainly, bonds with lower rating tend to have higher expected returns. This is clear from the parameter estimates in Tables III and IV. For example, the long-term expected return for intermediate Aaa rated bond index is 0.73 percent per month, or 8.77 percent per year. The intermediate Baa bond index, on the other hand, yields an expected return of 0.82 percent per month, or 9.82 percent per year. But the difference in expected return is too small to be visible from the plotted VaRs.

Our experience with estimating VaRs for rated bond portfolios suggest that information about the treasury term structure of interest rates is important. Whenever available they should be used as covariates in the regression relations.

4.2 The distribution of bond portfolio returns

Although VaR or volatility estimates are the most common measures of risk, a more complete description of the conditional probability distribution is very useful and is also frequently required. Currently the common approach used in the previous literature is based on the conditional Gaussian assumption. Given the accumulated empirical evidence that the distribution of many return time series are heavy-tailed, a more robust method in estimating the conditional distribution of financial returns is of particular interest. In this paper, besides estimating VaRs at 5 percent level, we also estimated the distribution of the return series using the robust quantile regression method.

Quantile regression method is attractive because not only it is robust against non-Gaussian errors in the way that least squares estimates are not, but also the models can be used to characterize the entire conditional distribution of a dependent variable. By considering the behavior of the regression at different quantiles, the quantile process conveys a more complete picture of the conditional distribution of the dependent variable than the single mean derived from a traditional approach. In our model, this provides an ideal way of estimating the conditional distribution of financial returns. In this section, we estimate the conditional quantile function (thus the conditional distribution function) for the last period and the average quantile distributions for each time series.

Figures 4 and 5 plot the estimated distribution functions of bond returns against those of normal distributions. The normal distribution function is plotted with the smooth line. The left two subplots are for the Treasury note and the right two are for the Baa-rated bond index. The top two are for the distributions averaged over the entire sample, i.e. sample means of returns, covariates and absolute residuals are used in fitting the estimated quantile models. The bottom two are for the distributions conditional on all information available on the last day of the sample.

Figure 4 presents the results for the intermediate term bonds. For the average distribution, we see that the treasury has a smaller standard deviation than the Baa bonds. Compared with the normal distribution, the treasury distribution is very close to being normal while the Baa index displays very strong fat-tailedness. The conditional distributions are quite different from the average distributions. The standard deviation of the treasury is very small. This reflects the fact that the Treasury yields change over a fairly wide range in the entire sample period but conditional on

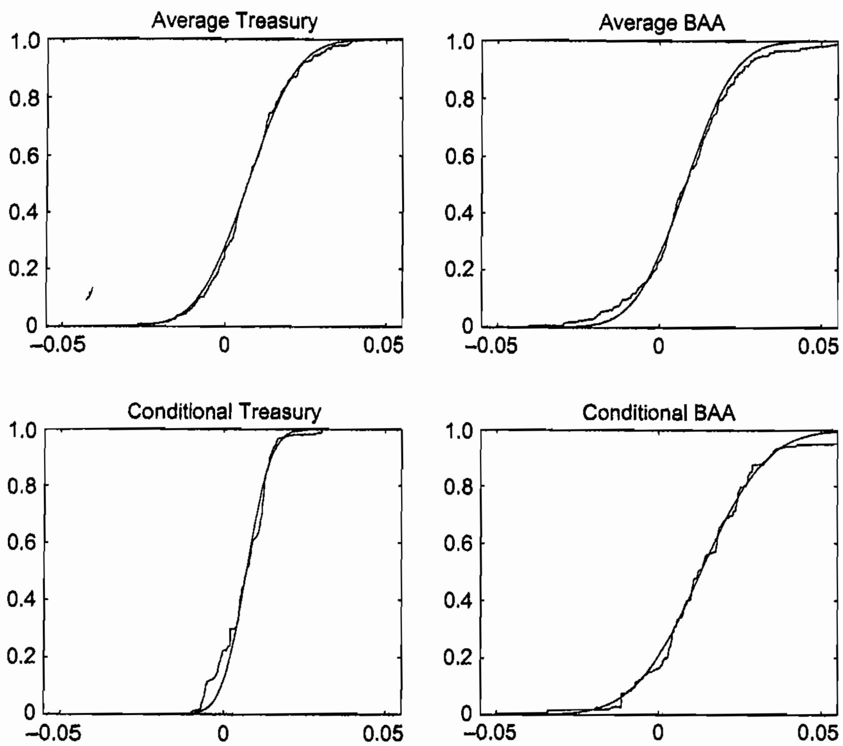


Figure 4. The results for the intermediate term bonds

information available at the end of the sample, large movement is very unlikely. The conditional distribution of the Baa index does not seem to have a similar pattern, indicating much higher conditional uncertainty of returns for the index. Fat-tailedness is very pronounced.

Figure 5 presents the results for the long-term bonds. Overall long-term bond indexes have very large variances, indicating higher risk for longer maturity fixed income instruments. Interestingly, long-term treasury does not have a smaller standard deviation than the Baa index. Comparing with the normal distribution, we see that the treasury distributions are very close to being normal while the Baa index displays very strong fat-tailedness in both the average distribution and conditional distribution.

4.3 VaR confidence bands

The above analysis provides point estimates of VaR at each period and specified τ. The distributional theory of the proposed model further facilitates the construction of confidence band for VaR estimates[8]. By a similar argument as Koenker and Zhao (1996), it can be shown that, under regularity conditions, the solution $\hat{\gamma}(\tau)$ of our optimization problem (10) is $\sqrt{N}$-consistent and asymptotic normal:

$$\sqrt{n}\left[\hat{\gamma}(\tau) - \gamma(\tau)\right] \Rightarrow N\left(0, \Omega(\tau)\right) \tag{12}$$

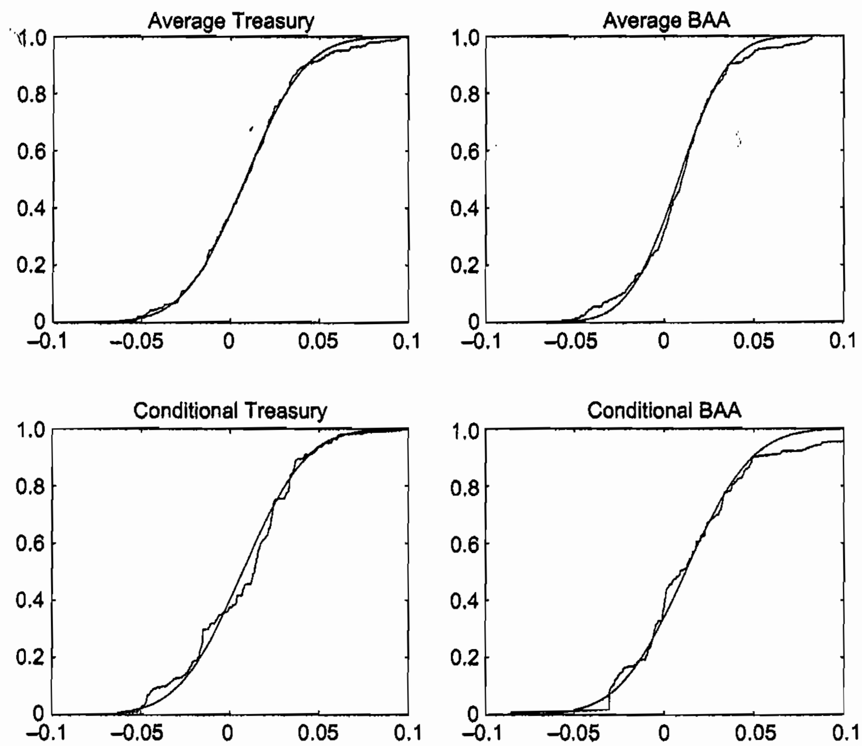


Figure 5.
The results for the long-term bonds

where

$$\Omega(\tau)=\frac{\tau(1-\tau)}{f^2\big(F^{-1}(\tau)\big)}H^{-1}DH^{-1}, H=\lim\frac{1}{n}\sum_{t=1}^{n}Z_tZ'_t/(Z'_t\gamma), D=\lim\frac{1}{n}\sum_{t=1}^{n}Z_tZ'_t$$

As a result, the limiting distribution of $\hat{\gamma}(\tau)'Z_t$ can be obtained based on equation (12), facilitating the construction of confidence intervals for the VaR estimates. Like the conventional approaches of confidence interval estimation, this approach requires that the covariance matrix $\tau(1-\tau)H^{-1}DH^{-1}/f^2\big(F^{-1}(\tau)\big)$ must be estimated. Notice that $n^{-1}\sum_{t=1}^{n}Z_tZ'_t$ is a natural candidate for the estimator of D, what we need is an estimator for $\sum(\tau)=f\big(F^{-1}(\tau)\big)H$. Fortunately there are quite a few methods in the existing literature on estimating $\sum(\tau)$. In particular, Hendricks and Koenker (1992) studied estimation of $\sum(\tau)$ based on sparsity estimation (see Siddiqui, 1960; Bofinger, 1975; Sheather and Maritz, 1983; Welsh, 1987, among others for related literature on estimating this quantity). In our empirical analysis, we adopt this approach[9] in constructing the confidence interval.

We estimate confidence bands of the estimated VaRs using the asymptotic distribution (12). To estimate the covariance matrix $\Omega(\tau)$, we need first estimate:

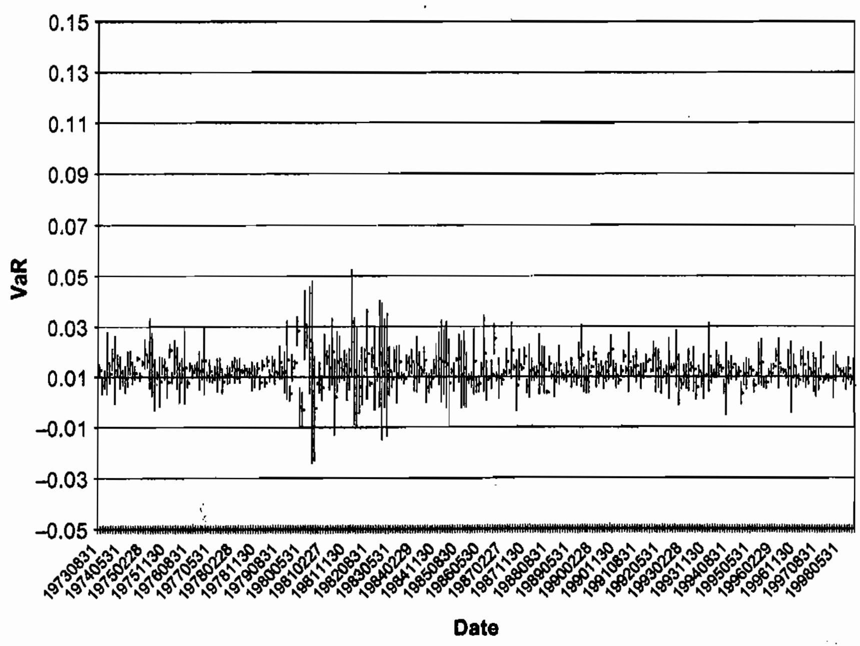
$$\sum(\tau) = \lim \frac{1}{n}\sum_{t=1}^{n} f(F^{-1}(\tau))Z_t Z'_t/(Z'_t\gamma)$$

Following Hendrick and Koenker (1992), we estimate $\sum(\tau)$ by:

$$\frac{1}{n}\sum_{t=1}^{n}\frac{2h_n}{Z'_t[\hat{\gamma}(\tau+h_n)-\hat{\gamma}(\tau-h_n)]} Z_t Z'_t/(Z'_t\hat{\gamma})$$

where $h_n = z_\alpha^{2/3}\left[1.5\phi^2(\Phi^{-1}(\tau))/(2(\Phi^{-1})(\tau))^2+1\right]^{1/3} n^{-1/3}$ where z_α satisfies $\Phi(z_\alpha) = 1 - \alpha/2$ for the construction of $1-\alpha$ confidence intervals and ϕ and Φ represents the standard normal density and distribution functions.

Figures 6 and 7 plot the estimated VaRs and their 95 percent confidence intervals for the intermediate-term Treasury index and the long-term Baa index. We see that for the treasury, the VaRs are fairly small and the confidence intervals are narrow, mostly smaller than 3 percent in total, except during a few occasions such as in the early 1980s. The long-term Baa index, by contrast, displays large VaRs and sometimes huge confidence intervals. For example, in the summer of 1980, the estimated VaR is about 7.5 percent, yet the confidence interval ranges from 0 to 15 percent. This result has important implications for risk management since the true value of VaR could be much larger than the estimated one. It is therefore prudent to assume a higher VaR in making hedging decisions with fixed income securities.



Notes: This figure shows the 5% VaR the intermediate term treasury index. A 95% confidence band is also indicated by vertical lines

Figure 6.
percent VaR with 95 percent confidence bands for intermediate term treasury index

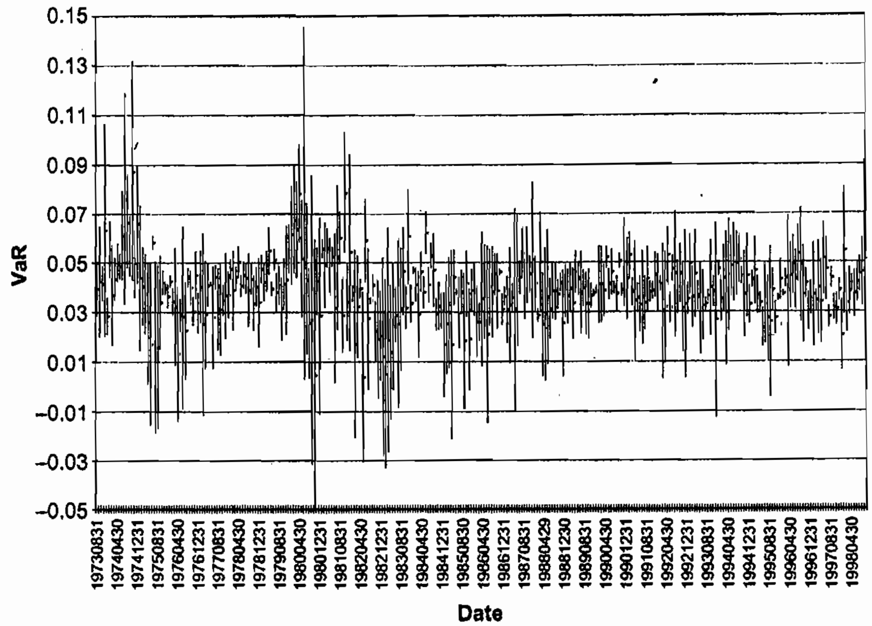


Figure 7.
5 percent VaR with 95 percent confidence bands for long term baa index

Notes: This figure shows the 5% VaR for the long term BAA index. A 95% confidence band is also indicated by vertical lines

5. Conclusions

The SEC has now listed VaR as one reporting alternative for quantifying companies' market risk exposures. Although the Securities and Exchange Commission (1998) stated that quantitative market risk disclosures provide "new and useful information" to the market, the quality of these disclosures, particularly the quality of VaR disclosures needs to be improved (Jorion, 2002). To improve the quality of VaR disclosure, a more robust estimation method is needed. In this paper we estimate value at risk using the quantile regression approach pioneered by Koenker and Bassett (1978). This method does not assume a particular conditional distribution for the returns. The model is applied to defaultable bond portfolios. We find that for defaultable bonds the use of information variables such as short term interest rates and term spreads as covariates improves the model performance significantly. We also find that confidence intervals constructed around the estimated VaRs can be very wide during volatile markets, making them less reliable when their accurate measurement is most needed.

Notes

1. Liquidity risks may also be priced in the holding period returns. This paper focuses on bonds rated Baa and higher to reduce the problem of liquidity premium due to infrequent trading.
2. Since this approach is linked to volatility modeling, there is a large and growing literature on this. The most popular and empirically robust method is the ARCH methodology proposed by Engle (1982). See Bollerslev *et al.* (1992) for an extensive review on this subject.

3. This approach includes the weighted moving average method by J.P. Morgan's Riskmetrics and the hybrid method by Boudoukh *et al.* (1998).
4. The estimation procedure can be easily implemented on a regular personal computer. The estimation programs are available in standard statistical packages, such as S-Plus. In addition, since GARCH models can be asymptotically represented by ARCH processes, an ARCH model with an appropriate chosen number of lags can practically provide a good approximation for GARCH effect, and retain substantial computational advantage.
5. For example, Calpers, the largest US pension fund, recently doubled the amount of investments in high-yield bonds. The popularity of corporate bonds is also revealed on the supply side. For instance, international bond issues (sovereign and corporate) reached a record high of $1,680 billion in 2001, which is 15 percent higher than 2000, due to aggressive Fed rate cuts and bear stock market. *Financial Times*, December 27, 2001.
6. An example of using VaR to manage bond portfolio risk is the long-term capital management saga. The firm enjoyed phenomenal return from 1994 to 1997, yet suffered huge losses in its fixed income derivative positions in 1998. The firm used VaR as a measure of risk and monitored its value continuously. The collapse of the firm serves as a wake-up call for improved risk measurement and management for fixed-income securities.
7. An alternative way for choosing lags is to test the assumption of i.i.d. in the estimated residuals based on the spectral density estimates of the residual process. If the models (3) and (4) are correctly specified, we should have $\Pr\{y_t < -VaR_t(\tau)\} = \tau$ at the true parameter. As a result, $\{e_t : e_t = I[y_t < -VaR_t(\tau)] - \tau\}$ should be i.i.d. In contrast, when the lags are incorrectly chosen, $\{e_t\}$ will be serially dependent. Therefore, to test the adequacy of lag choice, it suffices to check whether $\{e_t\}$ is i.i.d. There have been quite a few statistical procedures for testing the i.i.d. assumption, including Cowles and Jones (1937), Ljung and Box (1978), and Anderson (1993).
8. There are several approaches in estimating confidence intervals for regression quantiles: direct estimation of the asymptotic covariance matrix can be obtained based on an estimate of the reciprocal of the error density at the quantile of interest; inversion of rank tests by Gutenbrunner *et al.* (1993) provides an alternative approach of estimating confidence intervals for quantile regression without estimating the error density; several resampling/bootstrap methods have also been proposed for estimating confidence intervals for quantile-type estimators.
9. Another method was proposed by Powell (1986) based on kernel estimation.

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