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How Important Is Intertemporal Risk for Asset Allocation?*

We test a conditional asset pricing model that includes long-term interest rate risk as a priced factor for four asset classes—large stocks, small stocks, and long-term Treasury and corporate bonds. We find that the interest risk premium is the main component of the risk premiums for bond portfolios, while representing a small fraction of total risk premiums for equities. This suggests that stocks, especially small stocks, are hedges against variations in the investment opportunity set. We estimate that, at average market volatility levels, investors earn annual premiums between 3.6% during expansions and 5.8% during recessions for bearing intertemporal risk alone.

I. Introduction

Several authors have investigated whether the weak relation between equity market returns and market volatility is due to the omission of risk factors that link variations of the investment opportunity to changes in economic conditions. It has been well known since Merton (1971, 1973) that, when investment opportunities are time varying, dynamic hedging is necessary for forward-looking investors. The literature of active portfolio management is based almost exclusively on the traditional mean-variance analysis, and therefore the impact of dynamic hedging is not considered. Scruggs (1998) investigates the link between the equity market returns and long-term interest rates.

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He finds that, after taking into account long bond risk, equity returns are significantly positively related to their own variance and negatively related to their exposure to long bond risk. De Santis and Gerard (1999) hypothesize that the state variable driving changes in the investment opportunity set is unexpected changes in the inflation rate. They find that the price of inflation risk is statistically significant and time varying, and they estimate the inflation premium in stock returns to average -4.36% on an annual basis. They argue that the relevance of inflation risk stems not only from investors' concerns with real return volatility but also from the fact that inflation is a proxy for the variation of the investment opportunity set.

In this article, we propose a unified framework to investigate this issue and the optimal asset allocation strategies when investors do or do not take into account intertemporal risk. This issue is obviously important for asset pricing and intertemporal asset allocation decisions.[1] For example, the lack of a significant and positive relation between the first two moments of the returns on the market portfolio is puzzling because it is inconsistent with the prediction of one of the most widely used models in finance, the capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965), and Black (1972).[2] Yet, this evidence has been documented in a large number of studies and for may different asset classes and many national markets (Bollerslev, Engle, and Wooldridge 1988). For example, using U.S. data, Baillie and De Gennaro (1990) find that the relation between expected returns and own variance is weak, both at daily and monthly frequencies. Turner, Startz, and Nelson (1989), Nelson (1991), and Glosten, Jagannathan, and Runkle (1993) find that the relation becomes negative when returns are modeled with variations of a generalized autoregressive conditional heteroskedasticity (GARCH) model in which conditional variance is used to explain expected returns. In fact, even studies that document a positive relation, such as French, Schwert, and Stambaugh (1987) and Campbell and Hentschel (1992), find that their results are not robust to the use of different statistical methods.

We assume that the long-term interest rate is a proxy for the state variable that describes how the distribution of returns changes through time. Interestingly, in this model, investors may be willing to pay a premium for assets whose payoffs are negatively correlated with the changes in long-horizon interest rates. This is because these assets may provide a hedge against changes in the investment opportunity set. Our approach also provides a possible explanation for the weakness of the relation between expected returns and variance on the market portfolio discussed earlier. If the long bond rate is a priced risk factor, then tests of the CAPM are likely to produce biased estimates of the price of market risk and, thereby, of the market premium.

1. See Campbell (2000) and Campbell, Chan, and Viceira (2003) regarding long-term and strategic asset allocation, and Liu, Longstaff, and Pan (2003) regarding dynamic asset allocation analysis with event risk.

2. Backus and Gregory (1993), however, show that the theoretical relation between the market risk premium and market variance may not be positive.

In a recent paper, Chen (2003) develops a model with time-varying expected market returns and volatilities to reflect the change in the investment opportunity set in the economy. He finds that historical returns on the book-to-market effect and the momentum effect are too high to be explained as compensation for exposures to adverse changes in the investment opportunity set.[3] Brennan, Wang, and Xia (2004) develop and estimate a model of intertemporal risk with the real interest rate and the maximum Sharpe ratio as the two state variables. They find that the Fama-French three-factor model can be linked to their model.

In contrast with the studies cited above, we perform our estimation and test jointly on four large asset classes: portfolios of large stocks, small stocks, long maturity Treasury notes and bonds, and investment-grade corporate bonds. If the model we test correctly characterizes financial asset returns, it should hold for all asset portfolios. Hence, by considering several portfolios simultaneously, we improve the estimation of the price of risk and increase the power of our tests of the asset pricing model. Furthermore, there is mounting evidence that both prices of risk and risk exposure change over time (see, e.g., Harvey 1989; Ferson and Harvey 1993). Therefore, we model prices of risk, covariances, and correlations to be time varying.

We find that both prices of market risk and intertemporal risk are significant and that they vary with economic conditions. Not surprisingly, the exposure to market risk accounts for more than 90% of the premium of equity assets, while exposure to intertemporal risk is the dominant determinant of bonds returns. Intertemporal risk premiums account for more than 60% of total risk premiums for fixed-income assets. The risk premiums of the small firm equity portfolio is, however, unaffected by exposure to intertemporal risk. Overall, the evidence suggests that exposure to intertemporal risk has a pervasive effect on an asset's risk premium and should be explicitly accounted for in asset pricing tests.

The relatively insignificant intertemporal risk premium associated with small stocks has important implications. For example, investors with a long horizon care more about the risk of changing investment opportunity set and less about volatility. Our findings point to the merit of including small stocks in long-term strategic asset allocations for investors such as pension funds and insurance companies.

To further assess the economic importance of intertemporal risk, we investigate its impact on investors' portfolio holdings. Since our approach is fully parametric, we can use our model to construct the optimal period-by-period asset allocations of different classes of investors and decompose these holdings into their hedging and speculative (asset selection) components. First, we apply the framework of Glen and Jorion (1993) to analyze the portfolio holdings of classes of investors. We construct period by period the equity-

3. See also Brandt and Kang (2004) for a latent vector autoregressive (VAR) approach on intertemporal relationship between risk and return.

only optimal portfolio, the minimum variance intertemporal risk hedge for that equity-only portfolio, and the global optimal portfolio. We use the difference between the equity-only and global optimal portfolios as our proxy for the optimal intertemporal hedge portfolio. We find that the global hedge portfolio exhibits a significant negative correlation of −0.604 with the equity-only portfolio. Although earning only a small positive 1.1% annual excess return and having a volatility of similar magnitude as the equity-only portfolio, the global hedge significantly improves the risk-reward trade-off of the global optimal portfolio: it accounts for 22% of the global optimal portfolio risk premium, or approximately an additional 2.5% annual premium.

Second, we implement the multifactor efficient portfolio approach of Fama (1996) and use the orthogonal portfolio method of Roll (1980). We construct period by period the optimal portfolio orthogonal to market risk and the optimal portfolio orthogonal to intertemporal risk. The performance of the portfolio orthogonal to market risk provides direct insights into the rewards for market neutral strategies bearing only intertemporal risk. However, the difference between the global optimum portfolio and the portfolio orthogonal to intertemporal risk yields estimates of the incremental benefits of bearing intertemporal risk in addition to market risk. We find that, at average market volatility levels, bearing intertemporal risk only yields an annual premium of 3.6% during expansion, increasing to 5.8% during recessions. However, the incremental reward of bearing optimal amounts of intertemporal risk in addition to market risk varies between 1.1% per annum during expansions and 3% during recessions.

This article makes the following contributions to the literature. First, we provide a fully consistent empirical framework to test the two-factor ICAPM model. We subject the model to four diverse portfolios simultaneously, thus enhancing the power of the tests. Second, we model both the price of market risk and the price of risk associated with changes in the investment opportunity set, as well as allow all covariances and correlations to be time varying. This flexibility, and the presence of four diverse portfolios, enables us to study the relative importance of the two risk factors for these portfolios. Third, our general empirical framework makes it interesting to decompose the optimal portfolio holdings into speculative and hedging components. Since the market portfolio, small stocks, Treasury bills, and long bonds represent important segments of every investor's portfolio holdings, this exercise provides a unique perspective into the portfolio decisions of agents in changing economic environments, and into the economic impact of intertemporal risk.

The rest of this article is organized as follows. In Section II, we briefly review the ICAPM and discuss some of the testable implications that are relevant for our study. In Section III, we describe the empirical methodology. In Section IV, we describe the construction of the return series of the four portfolios and provide information on all data used in this study. In Section V, we discuss the tests of the asset pricing model. In Sections VI and VII, respectively, we investigate the intertemporal asset allocation and hedging implications of our results. Section VIII concludes.

II. Models and Testable Implications

In this section, we discuss an asset pricing model in which investors choose their optimal portfolios in the presence of a changing investment opportunity set. We assume that the investment opportunity set changes over time, as a function of a state variable x. Merton (1973) shows that, in this case, intertemporal risk—measured by the covariance of asset returns with the state variable—becomes a relevant pricing factor in addition to the traditional market risk.

Denote $E(R_{it})$ as the expected return on the asset i over the period $t-1$ to t; σ_{it} is the standard deviation of the returns. In addition, denote Σ_t as the covariance matrix of asset returns, with generic element $\sigma_{ij,t} = \sigma_{it}\sigma_{jt}\rho_{ij,t}$, and R_{ft} as the return on the nominally risk-free asset. To accommodate a stochastic investment opportunity set, as in Merton (1973), we assume that the first and second moments of the returns depend on one or more state variables. Here we assume that one state variable x is sufficient to describe the dynamics of the investment opportunity set.

Merton (1973) shows that, in equilibrium, when all investors are expected intertemporal utility maximizers, expected returns include compensation for market risk and an additional risk component, measured by the covariance between each asset return and the state variable x. Formally, the following set of pricing restrictions, expressed in terms of expected nominal return on asset i, obtains:

$$E_{t-1}(R_{it}) - R_{ft} = \alpha_t\sigma_{iM,t} + \lambda_t\sigma_{ix,t}, \tag{1}$$

where $\sigma_{iM,t} = \sigma_{it}\sigma_{Mt}\rho_{iM,t}$ is the covariance between R_{it} and the return on the market portfolio R_{Mt}, and $\sigma_{ix,t} = \sigma_{it}\sigma_{xt}\rho_{ix,t}$ is the covariance between R_{it} and the state variable x_t.[4] The quantity $\alpha_t = -J_{WW,t}W_t/J_{W,t}$ is a measure of aggregate relative risk aversion.[5] It is usually referred to as the price of market risk because it measures the sensitivity of the expected return to changes in market risk. For obvious reasons, the quantity $\lambda_t = -J_{Wx,t}/J_{W,t}$ can be interpreted as the price of intertemporal risk.[6] One feature that differentiates the price of market risk from the price of intertemporal risk is that, while the former must always be positive as long as investors are risk averse, the sign of the latter cannot be predetermined. More precisely, since utility is assumed to be in-

4. The market portfolio is defined, as usual, as the portfolio of all risky assets weighted by their relative market value. See Merton (1973) for a complete derivation.

5. We use $J(W, I, \tau)$ to denote the solution to the optimization problem. Obviously, the symbols $J_{W,t}$ and $J_{WW,t}$ denote the first and second derivatives of $J(W, I, \tau)$ with respect to W_t.

6. In this case, the solution to the investor's optimization problem is a function $J(W, I, x, \tau)$, and $J_{Wx} = \partial J_W/\partial x$.

creasing in wealth, J_W is strictly positive, and therefore the sign of the price of intertemporal risk depends on the sign of J_{Wx}. For example, assume that the state variable x is positively correlated with the return on asset i. If the marginal utility of wealth is increasing in x (i.e., $J_{Wx} > 0$), then λ is negative and so is the premium for intertemporal risk. This reflects the fact that investors are willing to accept a lower risk premium on asset i because the asset has a higher payoff when the marginal utility of wealth is higher. However, if the marginal utility of wealth is decreasing in x (i.e., $J_{Wx} < 0$), then λ is positive and so is the premium for intertemporal risk. In this case, investors require a higher risk premium on asset i because the asset has a higher payoff when the marginal utility of wealth is lower.

When the investment opportunity set is constant over time, the pricing restrictions simplify to the traditional Sharpe-Lintner-Mossin CAPM:

$$E(R_{it}) - R_{ft} = \alpha_t \sigma_{iM,t} \qquad i = 1, \ldots, n. \tag{2}$$

In this case, the model predicts that the premium of a risky asset is determined only by its market risk—measured by the covariance of the return on asset i with the return on the market portfolio.

In the absence of a general equilibrium model, it is not possible to identify the state variable x that describes the dynamics of the investment opportunity set. In this study, we use the return on the long Treasury bond portfolio as a proxy for x.[7] With this assumption, equation (1) can be rewritten as follows:

$$E(R_{it}) - R_{ft} = \alpha_t \sigma_{iM,t} + \lambda_t \sigma_{iTB,t}, \tag{3}$$

which states that the nominal premium on any asset is proportional to its exposure to market risk and to (long bond) interest rate risk.

As noted by Cochrane (1999), since Merton (1973) does not specify what exactly is the intertemporal risk factor, empirical researchers often attribute findings of "abnormal returns" to the factor. Our choice of the long Treasury bond return as a proxy for x is consistent with Merton (1977). Merton suggests that uncertainty about future rates of return may induce differential demands for long- and short-term bonds. He also listed potential candidates for the intertemporal risk factor, such as a short-term riskless asset, shifts in the wage-rental ratio, and changes in prices for basic groups of consumption goods (inflation). We were not able to find satisfactory data for the wage-rental ratio that matches our sample for rigorous econometric analysis. For the short-term riskless asset, inflation, and long-term Treasury bonds, we subject them to a horse race, using the Akaike Information Criteria (AIC) and the Bayesian Information Criteria (BIC), modified to make the models comparable. Long-term bond return is found to dominate short-term interest rate by both the

7. There is a long tradition of linking stock returns to the change in interest rates. See, e.g., Chen, Roll, and Ross (1986) and, more recently, Scruggs (1998) and Scruggs and Glabadanidis (2002).

AIC and BIC measures for all specifications of the model. For the inflation factor, the estimation failed to converge.[8] In sum, the specification analysis supports our choice of the long-term Treasury bond return as the proxy for the intertemporal risk.

III. Econometric Methods

To simplify our empirical analysis, we exploit the fact that the pricing restrictions of the model must be satisfied for any asset, including the market portfolio. Therefore, we can focus on any subset of the assets included in the investment opportunity set. Increasing the number of assets included in the test will increase the power of our tests while making estimation more difficult, and the more diverse the assets are, the more powerful the tests will be. To our knowledge, our four-asset empirical model is the most general yet studied, and the tests should be powerful.

If the long bond portfolio return can be used as a proxy of the state variable in the ICAPM with a time-varying investment opportunity set, as argued by Merton (1973) and Scruggs (1998), then the second model is nested into the first. The following equation can be used to test the restrictions of both models:

$$R_{it} - R_{ft} = \alpha_{t-1}\,\mathrm{Cov}_{t-1}(R_{it}, R_{Mt}) + \lambda_{t-1}\,\mathrm{Cov}_{t-1}(R_{it}, R_{TBt}) + \varepsilon_{it},\quad i = 1, \ldots, n. \tag{4}$$

The pricing equation (1) implies that, if the long Treasury bond return is perceived as a risk factor that accounts for intertemporal risk, then λ_{t-1} is different from zero. It can even become positive if the marginal utility of wealth is decreasing in the long bond rate. Later in this article, we discuss how to incorporate this information into the specification of α_{t-1} and λ_{t-1} when estimating and testing the model.

Equation (4) provides an interesting insight into standard tests of the CAPM. It is common practice to test the model by estimating the relation between expected return and the conditional covariance of each asset with the return on the market portfolio. Inspection of equation (4) reveals that this approach may be misleading. If intertemporal risk is priced and economically significant, any measure of the market premium obtained from the regression

$$R_{it} - R_{ft} = \alpha_{t-1}\,\mathrm{Cov}_{t-1}(R_{it}, R_{Mt}) + \eta_{it} \tag{5}$$

will be biased. For example, if the long rate premium is negative, equation (5) is likely to produce low, and possibly negative, estimates of the market premium. This could explain the weak relation between expected returns and volatility on the market documented in many recent studies.

Before we proceed to the empirical analysis, we need to complete the

8. These results are available from the authors upon request.

specification of the model. The right-hand side of equation (4) contains the conditional covariance of the asset returns with market portfolio returns and the conditional covariance between the asset returns and the long bond return as explanatory variables. For this reason, we need to include the long Treasury bond portfolio as well as a reasonable proxy of the market portfolio in the set of assets on which the estimation will be performed. Second, we need to specify a model that describes the dynamics of the conditional second moments.

For the conditional second moments, we assume that the disturbance vector $\varepsilon_t = [\varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{nt}]'$ is conditionally normally distributed,

$$\varepsilon_t | \Im_{t-1} \sim N(0, \Sigma_t),$$

and that the covariance matrix Σ_t follows an asymmetric GARCH (1,1) process,[9]

$$\Sigma_t = C'C + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'\Sigma_{t-1}B + D'\eta_{t-1}\eta'_{t-1}D, \quad (6)$$

where C is a $(n \times n)$ upper triangular matrix; A, B, and D are $(n \times n)$ matrices; η_{t-1} is the vector of negative shocks where $\eta_{i,t-1} = \varepsilon_{i,t-1}$ if $\varepsilon_{i,t-1} < 0$ and 0 otherwise.

Under the assumption of conditional normality, the log-likelihood function for the system of equations (4) and (6) for n assets can be written as follows:

$$\ln L(\Theta) = -\frac{Tn}{2}\ln 2\pi - \frac{1}{2}\sum_{t=1}^{T}\ln|\Sigma_t(\Theta)| - \frac{1}{2}\sum_{t=1}^{T}\varepsilon_t(\Theta)'\Sigma_t(\Theta)^{-1}\varepsilon_t(\Theta), \quad (7)$$

where Θ is the vector of unknown parameters in the model. Since the normality assumption is often violated in financial time series, we estimate the model and compute all our tests using the quasi-maximum likelihood (QML) approach proposed by Bollerslev and Wooldridge (1992). Under standard regularity conditions, the QML estimator is consistent and asymptotically normal and statistical inferences can be carried out by computing robust Lagrange Multiplier or Wald statistics. Optimization is performed using the Berndt, Hall, Hall, and Hausman (BHHH; Berndt et al. 1974) and the Broyden, Fletcher, Godfarb, and Shanno (BFGS) algorithms.

IV. Data

We perform our investigation simultaneously on four asset portfolios: a proxy for the equity market portfolio, a small-firm equity portfolio, long-term Treasury securities, and investment-grade long-term corporate bonds. We use monthly data for the period from November 1948 to December 2000, for a total of 626 observations. To measure the return on the market portfolio, we use end-of-month total returns on the NYSE-AMEX-NASDAQ total stock

9. See, e.g., Campbell and Hentschel (1992), Glosten et al. (1993), Kroner and Ng (1998), Bekaert and Wu (2000), and Wu (2001).

market index, computed by the Center for Research in Security Prices (CRSP) at the University of Chicago. The small stock portfolio includes all stocks traded on the three exchanges that belong to the smallest size quintile.[10] Both the market index and the small stock portfolio are market-value weighted, and the dividends accumulated during the month are reinvested at the closing price, at the end of each month. For the risk-free rate, we use the return on the U.S. T-bill closest to 30 days to maturity, as reported in the CRSP risk-free files.

The long-term Treasury securities portfolio returns are constructed from the CRSP U.S. Government Bills, Notes, and Bonds database. To construct the T-bond portfolio returns, we collected, at the end of each month, the realized return for all notes and bonds that traded at the beginning of the month, were still outstanding at the end of the month, and had a maturity of 5 or more years at the beginning of the month. We excluded all bonds with special tax status. We then weighted the realized return on each included bond by the ratio of bond's outstanding amount at the beginning of the month to the total amount outstanding of all included bonds. The long-term investment grade corporate bond return is extracted from Ibbotson and Associates (2001). For all portfolios, we use continuous compounding.

In addition, we use a number of instruments to model the dynamics of the various prices of risk. Specifically, we use the dividend price ratio on the CRSP market index in excess of the risk-free rate, the first difference in annualized yield to maturity on the 3-month T-bill, and the lagged default premium as measured by the difference between the yield to maturity on an AAA corporate bond and on the most recently issued 5-year Treasury bond or note. All Treasury securities yields are from the CRSP U.S. Government Bond Files.

Summary statistics for the portfolios return series, as well as the instruments, are reported in panel A of table 1. Over the entire sample, the average annual return on the CRSP stock market index is equal to 12.01%, whereas it is 12.05% for the small stock portfolio. The returns on the T-bond and corporate bond portfolios both averaged 5.89% per year, while the risk-free rate averaged 4.81% on an annual basis. However, some of these statistics are considerably different when computed over subsamples. Note that, while average return on small stocks is of similar magnitude as the average return on the market, the volatility of small stock returns is 50% greater than that of the market. In contrast, the Treasury and corporate bond portfolios exhibit a return volatility of similar magnitude.

As one would expect, the autocorrelation matrix in panel B of table 1 also shows that the risk-free rate and the excess dividend ratio exhibit significant positive autocorrelation. Similarly, the correlation table indicates that the risk-free rate and the excess dividend price ratio are highly correlated. For the

10. The CRSP uses only the stocks traded on the NYSE to determine size quintile cutoff values.

TABLE 1 Summary Statistics of the Returns and Information Variables

	R_{ft}	R_{Mt}	R_{St}	R_{TBt}	R_{CBt}	XDPR	ΔR_3	DefP
				A. Summary Statistics				
Mean	.405	1.001	1.004	.491	.491	−.096	.009	1.886
Median	.386	1.376	1.334	.315	.377	−.116	.017	1.800
Standard deviation	.236	4.184	6.007	1.951	2.221	.266	.497	.814
Minimum	.031	−25.463	−34.193	−7.381	−9.324	−1.037	−3.960	.110
Maximum	1.416	15.264	32.824	12.049	12.902	.711	2.509	4.696
Lag				**B. Autocorrelations**				
1	.965	.044	.188	.114	.165	.959	.126	.926
2	.939	−.038	−.006	−.023	−.016	.924	−.050	.871
3	.924	−.005	−.047	−.065	−.059	.896	−.036	.831
4	.904	.008	−.017	.051	.008	.873	−.067	.794
5	.890	.066	−.011	.040	.085	.859	.002	.763
6	.878	−.043	.018	.039	.059	.844	−.124	.729
12	.799	.039	.108	.016	.021	.778	−.094	.577
				C. Correlations				
	R_{ft}	R_{Mt}	R_{St}	R_{TBt}	R_{CBt}	$XDPR_{t-1}$	$\Delta R_{3,t-1}$	$DefP_{t-1}$
R_{ft}	1							
R_{Mt}	−.088	1						
R_{St}	−.090	.809	1					
R_{TBt}	.146	.202	.081	1				
R_{CBt}	.091	.311	.182	.870	1			
$XDPR_{t-1}$	−.899	.117	.134	−.131	−.069	1		
$\Delta R_{3,t-1}$	.061	−.158	−.140	−.073	−.126	−.076	1	
$DefP_{t-1}$	.295	.138	.149	.107	.177	−.258	−.271	1

NOTE.—The market (R_{Mt}) and the small stock portfolio (R_{St}) returns are measured as the returns on the CRSP value-weighted NYSE-AMEX-NASDAQ index and the smallest-size quintile portfolio. The risk-free rate (R_{ft}) is the return on the T-bill with maturity closest to 1 month, as reported in the CRSP risk-free files. The T-bond (R_{TBt}) portfolio returns are measured as the value-weighted returns on all T-bonds and notes with more than 5 years remaining to maturity traded at the beginning of the month and that remain outstanding at the end of the month. All T-bond and T-note data come from the CRSP Monthly Government Bond database. The corporate bond returns come from Ibboston and Associates. The information set includes the excess dividend price ratio (XDPR) on the CRSP value-weighted index, the first difference (ΔR_3) in the 3-month T-bill rate from CRSP, and the default premium (DefP), as measured by the lagged end of month yield difference between the AAA benchmark bond and the most recently issued 5-year T-note. All returns are continuously compounded and are in percent per month. The sample covers the period November 1948 through December 2000 (626 observations).

portfolio returns, the correlation between small stocks and the market is of similar magnitude as the correlation between the corporate and Treasury bond portfolios.

V. Empirical Evidence

A. Conditional CAPM and the Price of Market Risk

The theoretical model, which we discussed in Section II, states that the nominal premium on the market portfolio is proportional to market risk, measured by the conditional covariance of the portfolio returns with the return on the market index, as well as long bond risk, measured by the conditional covariance between the return on the asset and the bond portfolio return. For each source of risk, the model also identifies a shadow price, which is potentially time varying. A common approach, often used in tests of the conditional CAPM, is to assume that the price of market risk is a linear function of a number of instruments. However, this specification may be inappropriate because the theoretical model predicts that the price of market risk should be strictly positive.[11] As discussed in Merton (1980), this information should be taken into account in the specification of the empirical model if the aim is to obtain an unbiased estimate of the market premium.

To provide some empirical evidence on this issue, we start our analysis with a version of the conditional CAPM that has been widely used in the literature and that does not include long bond risk. The model postulates a linear relation between the conditional expected return on the asset and its conditional covariance with the market portfolio; therefore, it can be estimated and tested using the following equation:

$$R_{it} - R_{ft} = \tau_i + \alpha_{t-1} \mathrm{Cov}_{t-1}(R_{it}, R_{Mt}) + \eta_{Mt}, \tag{8}$$

where returns are measured in nominal terms, the disturbance η_{Mt} is conditionally normal and follows a standard GARCH(1, 1) process, and τ_i is an asset-specific constant.

We consider two alternative parametrizations for the price of market risk α_{t-1}. First, we assume a linear specification $\alpha_{t-1} = \kappa' z_{t-1}$, which does not account for the positivity restriction on the market premium. Then we reestimate the model assuming an exponential parameterization $\alpha_{t-1} = \exp(\gamma' z_{t-1})$, which imposes the restriction suggested by Merton (1980). In table 2, we report a number of diagnostic tests for the two specifications of the model.[12] The results in panel A support the hypothesis that the premium on the market portfolio is proportional to market volatility and that the price of market risk is time varying, no matter which specification is used for α_{t-1}. Note that the test of intercepts suggests that, for both specifications, the conditional CAPM is well specified for equity portfolios, while it does not perform as well for the Treasury and corporate bond portfolios. This suggests that the traditional conditional CAPM is not well specified to price all assets.

The diagnostic statistics in the table reveal that the model with a linear price of risk has a slightly better fit, as implied by a -0.014% average prediction error and a 6.55% pseudo-R^2 versus a -0.096% average prediction error and a 6.40% pseudo-R^2 for the model with exponential prices.[13] This may be due to the fact that the linear specification can accommodate negative values of the market premium, as shown by the summary statistics in panel C of table 2 and the graphs in figure 1. The linear version of the model generates a negative market premium in 124 out of 626 observations, roughly 20% of the months.

11. As mentioned earlier, the price of market risk is also a measure of the aggregate degree of risk aversion. Since the model is derived under the assumption that risk-averse investors maximize the expected utility of their future consumption stream, the aggregate level of risk aversion must be strictly positive.

12. We do not report individual parameter estimates for each model since they are not of particular interest for the issue that we want to address at this point.

13. The pseudo-R^2 is computed as ESS/TSS, where ESS is the explained sum of squares and TSS is the total sum of squares.

TABLE 2 Tests and Diagnostics for the Traditional CAPM

A. Specification Tests

Null Hypothesis	df	Linear χ^2	Linear *p*-Value	Exponential χ^2	Exponential *p*-Value
Are all the coefficients in the price of market risk equal to zero?	4	21.88	.000	52.75	.000
Is the price of market risk constant?	3	21.39	.000	13.65	.004
Are the equity portfolio intercepts jointly zero?	2	.46	.795	.35	.839
Are the bond portfolio intercepts jointly zero?	2	1.02	.600	.87	.647

B. Diagnostic Statistics

	$E(R_M - R_f)$ Average	Average Predicted Error	R^2	F	$Q_{12}(z)$
Linear	.596	−.014	6.55	42.79**	6.58
Exponential	.596	−.096	6.40	40.25**	7.54

C. Estimated Market Premia, $\hat{\alpha}_{t-1} = \widehat{\mathrm{Var}}_{t-1}(R_{Mt})$

	Average	SD	Minimum	Maximum	Negative Value	Observations
Linear	.566	.037	−3.599	7.205	124	626
Exponential	.759	.030	.065	6.632	0	626

NOTE.—Estimates are based on monthly, continuously compounded returns from November 1948 through December 2000 (626 observations). The risk premium on the market portfolio is measured by $\alpha_{t-1}\mathrm{Var}_{t-1}(R_{Mt})$. The shadow price of market risk is assumed to vary with a set of instruments z_{t-1}, which are known to the investor at the beginning of time t. The instruments include the lagged values of the market portfolio's dividend price ratio in excess of the risk-free rate (XDPR), the change in the 3-month T-Bill rate (ΔR_3), and the default premium (DefP). The estimated model is

$$R_{it} - R_{ft} = \tau_i + \alpha_{t-1}\mathrm{Cov}_{t-1}(R_{it}, R_{Mt}) + \eta_{it}, \quad i = 1, \ldots, 4,$$

where η_{it} is conditionally normal and follows an asymmetric GARCH(1,1) process. We present results for two versions of the model, which differ in the specification of the price of market risk α_{t-1}: the first one uses a linear specification, $\alpha_{t-1} = \gamma' z_{t-1}$; the second one uses an exponential specification, $\alpha_{t-1} = \exp(\gamma' z_{t-1})$. In panel B, the column labeled R^2 is the pseudo-R^2 computed as ESS/TSS, the column labeled F is the robust F-test (16, 610) of residual predictability using z_{t-1}, and the column labeled $Q_{12}(z)$ is the Ljung-Box test statistic of order 12 for the standardized residuals.

** Statistically significant at the 1% level.

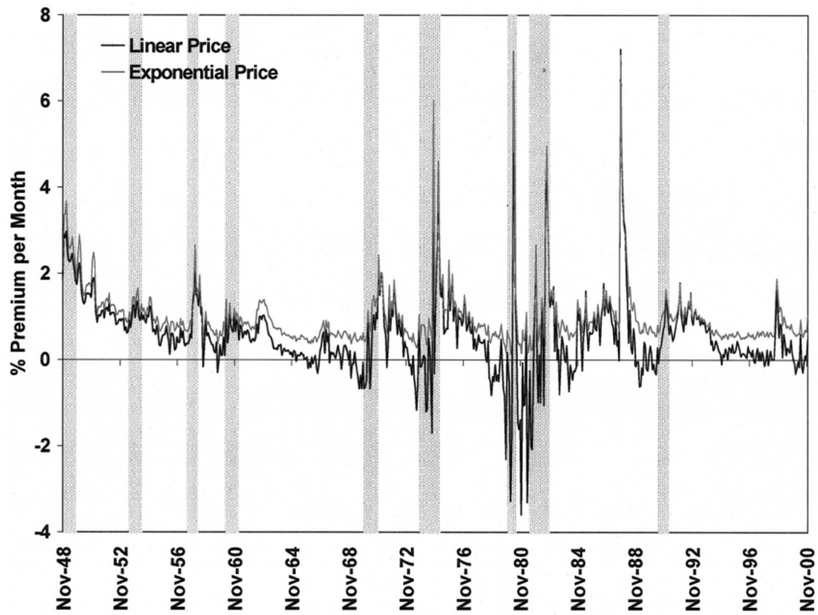

FIG. 1.—Estimated market premium for the conditional CAPM. The unrestricted premium is obtained assuming that the price of market risk is a linear function of the information variables in z_{t-1}. The restricted premium is obtained assuming that the price of market risk is an exponential function of z_{t-1}. Shaded areas highlight NBER recession periods.

It is interesting that figure 1 shows that most of the negative values are concentrated around the high interest rate and high inflation period during the 1970s and early 1980s.[14]

The average monthly premium estimated from the linear model is equal to 0.566% and is statistically significant over the whole sample, based on a Newey-West standard error of 0.037.[15] The exponential model yields an average monthly premium of 0.759%, which is also statistically significant (Newey-West standard error of 0.030). Note that the estimated premium from the linear model exhibits higher volatility than the premium estimated from the exponential model. Although both specifications yield sensible estimates for market risk premiums, neither model is very successful at explaining the cross section of returns. We find evidence of significant predictability of both

14. These results are consistent with the findings of Boudoukh, Richardson, and Smith (1993) for a similar subsample.

15. The standard errors are adjusted for autocorrelation and heteroskedasticity using the approach developed by Newey and West (1987). The purpose of this test is to determine whether the average of the estimated market premium is statistically significant conditional on the parameter estimates obtained from the model. However, the test does not account for estimation error.

models' residuals, using the information variables that were included as conditioning variables for the price of market risk. The *F*-tests of the regressions, reported in panel B of table 2, are respectively 42.79 and 40.25; both are highly significant at conventional confidence levels.

In summary, on the one hand, the conditional CAPM with a linear price of risk has a better statistical fit but contains a significant bias in the estimated market premium. On the other hand, a CAPM that imposes a nonnegativity restriction on the market premium generates predictable residuals, thus suggesting that other systematic factors are necessary to explain expected returns. This evidence motivates our attempt to determine the relevance of intertemporal risk.

B. Conditional CAPM with Intertemporal Risk

Our main objective is to determine whether it is possible to decompose the total risk premium into a market premium component and a long interest rate premium component. The evidence discussed in the previous section suggests that an appropriate specification for the dynamics of the price of market risk is important. For this reason, we impose the positivity constraint on α_{t-1} by assuming that the price of market risk is an exponential function of the instruments in z_{t-1}. For the price of bond risk λ_{t-1}, we consider a linear function of the instruments. Formally,

$$\alpha_{t-1} = \exp(\gamma' z_{t-1}) \text{ and } \lambda_{t-1} = \kappa' z_{t-1}.$$

Table 3 (in four parts [3A–3D]) reports the results of the estimation and tests. Tables 3A and 3B report parameter estimates, while table 3C reports specification tests and table 3D reports diagnostic tests for the residuals. The evidence supports a two-factor model in which both market risk and bond risk are priced. First, the robust Wald test for the hypothesis that the price of market risk is zero is equal to 31.85 with 4 degrees of freedom, which implies rejection at any standard level. Second, the null hypothesis that the price of intertemporal risk is equal to zero is also strongly rejected, given a Wald test of 13.06 with 4 degrees of freedom. The tests indicate that the prices of both market risk and intertemporal risk vary significantly with changes in economic conditions. Third, the likelihood ratio test of the intertemporal CAPM versus simple CAPM rejects the simple CAPM, since the $\chi 2$-statistic is 10.268. At 4 degrees of freedom, the associated *p*-value is 0.036. Although based on the BIC criterion, the simple CAPM is slightly favored, the intertemporal CAPM is favored according to the AIC criterion. The finding that intertemporal risk is priced and that it can explain the bias in the market premium estimated from the nominal CAPM is consistent with the recent work of Scruggs (1998).[16]

16. As in Scruggs (1998), we also estimate the model with constant prices for both market risk and intertemporal risk. In this case, however, our results differ from Scruggs's as neither our estimate of the price of market risk or the price of intertemporal risk is significantly different from zero, whether evaluated separately or jointly.

TABLE 3A Conditional CAPM with Stochastic Investment Opportunity Set Quasi-Maximum Likelihood Mean Equation Parameter Estimates

	Constant	$XDPR_{t-1}$	$\Delta R_{3,t-1}$	$DefP_{t-1}$
α_{t-1}	−3.396	1.601	−.377	.287
	(.984)	(.925)	(.263)	(.262)
λ_{t-1}	.175	−.013	−.010	−.065
	(.056)	(.057)	(.025)	(.024)
	Intercepts			
	Market	Small	T-Bond	Corporate Bond
τ_i	−.324	−.422	−.149	−.191
	(.461)	(.554)	(.078)	(.087)

NOTE.—Estimates are based on monthly, continuously compounded returns from November 1948 through December 2000 (626 observations). The total risk premium on each portfolio is decomposed into market premium, measured by $\alpha_{t-1}\text{Cov}_{t-1}(R_{it}, R_{Mt})$ and intertemporal premium, measured by $\lambda_{t-1}\text{Cov}_{t-1}(R_{it}, R_{TBt})$. The shadow prices of both sources of risk are assumed to vary with a set of instruments z_{t-1}, which are known to the investor at the beginning of time t. The instruments include the lagged values of the market portfolio's dividend price ratio in excess of the risk free rate (XDPR), the change in the 3-month T-bill rate (ΔR_3), and the default premium (DefP). The estimated model is

$$R_{it} - R_{ft} = \tau_i + \alpha_{t-1}\text{Cov}_{t-1}(R_{it}, R_{Mt}) + \lambda_{t-1}\text{Cov}_{t-1}(R_{it}, R_{TBt}) + \varepsilon_{it}, \quad i = 1, \ldots, 4$$

where $\alpha_{t-1} = \exp(\gamma' z_{t-1}), \lambda_{t-1} = \kappa' z_{t-1}$ and $\varepsilon_t | I_{t-1} \sim N(0, \Sigma_t)$. The conditional covariance matrix Σ_t follows an asymmetric GARCH process

$$\sum_t = C'C + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'\sum_{t-1}B + D'\eta_{t-1}\eta'_{t-1}D,$$

where C is a $(n \times n)$ upper triangular matrix; A, B, and C are $(n \times n)$ matrices; and η_{t-1} is the vector of negative shocks; $\eta_{i,t-1} = \varepsilon_{i,t-1}$ if $\varepsilon_{i,t-1} < 0$, and 0 otherwise. QML standard errors are reported in parentheses.

TABLE 3B Conditional CAPM with Stochastic Investment Opportunity Set Quasi-Maximum Likelihood Covariance Process Parameter Estimates

	C Matrix				A Matrix			
	R_{Mt}	R_{TBt}	R_{St}	R_{CBt}	R_{Mt}	R_{TBt}	R_{St}	R_{CBt}
R_{Mt}	1.034	. . .	. . .	. . .	.165	.017	. . .	. . .
R_{TBt}	−.176	.063	. . .	. . .	−.016	.232	. . .	. . .
R_{St}	1.244	.091	−.453	. . .	−.025	−.024	.211	. . .
R_{CBt}	−.048	.174	.018	.0001	.005	−.162	. . .	.292
	B Matrix				D Matrix			
	R_{Mt}	R_{TBt}	R_{St}	R_{CBt}	R_{Mt}	R_{TBt}	R_{St}	R_{CBt}
R_{Mt}	.941	.050	. . .	. . .	.178	−.235	. . .	. . .
R_{TBt}	.013	.933	. . .	. . .	−.010	.390	. . .	. . .
R_{St}	−.027	.063	.965	. . .	.266	−.298	−.109	. . .
R_{CBt}	.007	.082	. . .	.879	.001	−.139	. . .	.478

NOTE.—See note to table 3A.

TABLE 3C Conditional CAPM with Stochastic Investment Opportunity Set Specification Tests

Null Hypothesis	χ^2	df	p-Value
Are all the coefficients in the price of market risk equal to zero?			
Hypothesis: $\gamma_0 = \gamma_1 = \gamma_2 = \gamma_3 = 0$	31.852	4	.000
Is the price of market risk constant?			
Hypothesis: $\gamma_1 = \gamma_2 = \gamma_3 = 0$	16.301	3	.001
Is the price of intertemporal risk equal to zero?			
Hypothesis: $\kappa_0 = \kappa_1 = \kappa_2 = \kappa_3 = 0$	13.062	4	.011
Is the price of intertemporal risk constant?			
Hypothesis: $\kappa_1 = \kappa_2 = \kappa_3 = 0$	10.640	3	.013
Are the prices of market and intertemporal risk jointly constant?			
Hypothesis: $\gamma_1 = \gamma_2 = \gamma_3 = \kappa_1 = \kappa_2 = \kappa_3 = 0$	18.580	6	.005
Are the intercepts jointly different from zero?			
Hypothesis: $\tau_1 = \tau_2 = \tau_3 = \tau_4 = 0$	5.866	4	.221
Are the equity portfolios intercepts jointly different from zero?			
Hypothesis: $\tau_1 = \tau_3 = 0$	.602	2	.740
Are the bond portfolios intercepts jointly different from zero?			
Hypothesis: $\tau_2 = \tau_4 = 0$	5.675	2	.058

NOTE.—See note to table 3A.

Further support for the statistical relevance of intertemporal risk is contained in table 3D. In the two columns labeled R^2_m and R^2_{m+TB}, we report two different pseudo-R^2s for the CAPM equation: the first one (R^2_m) only accounts for market risk as a priced factor, while the second one (R^2_{m+TB}) includes both risks as explanatory variables for the total risk premium. It is interesting that the average R^2_{m+TB} is equal to 5.58%, whereas the average R^2_m is equal to 4.58%. For the equity assets, there is no difference in pseudo-R^2, whether intertemporal risk is included or not. For fixed income assets, however, considering intertemporal risk substantially improves the fit of the model. Note that, for the market portfolio, R^2_{m+TB} is larger than the pseudo-R^2's of the CAPM re-

TABLE 3D Conditional CAPM with Stochastic Investment Opportunity Set Residuals Diagnostics with Summary Statistics

	Average	Average Pre-dicted Error	RMSE	R^2_m	R^2_{m+TB}	$Q_{12}(z)$	$Q_{12}(z^2)$
Market	.596	−.084	4.127	9.16	9.13	7.81	4.89
Small	.599	−.094	5.926	5.76	5.72	30.50**	2.38
T-bond	.086	.016	1.928	1.02	3.81	15.82	9.56
Corporate bond	.086	.007	2.215	1.62	3.64	36.02**	5.97
Likelihood function						5,306.41	

NOTE.—The columns labeled R^2_m and R^2_{m+TB} are the pseudo-R^2's computed as ESS/TSS; the column labeled $Q_{12}(z)$ is the Ljung-Box test statistic of order 12 for the standardized residuals; the column labeled $Q_{12}(z^2)$ is the Ljung-Box test statistic of order 12 for the standardized residuals squared. See note to table 3A.

** Statistically significant at the 1% level.

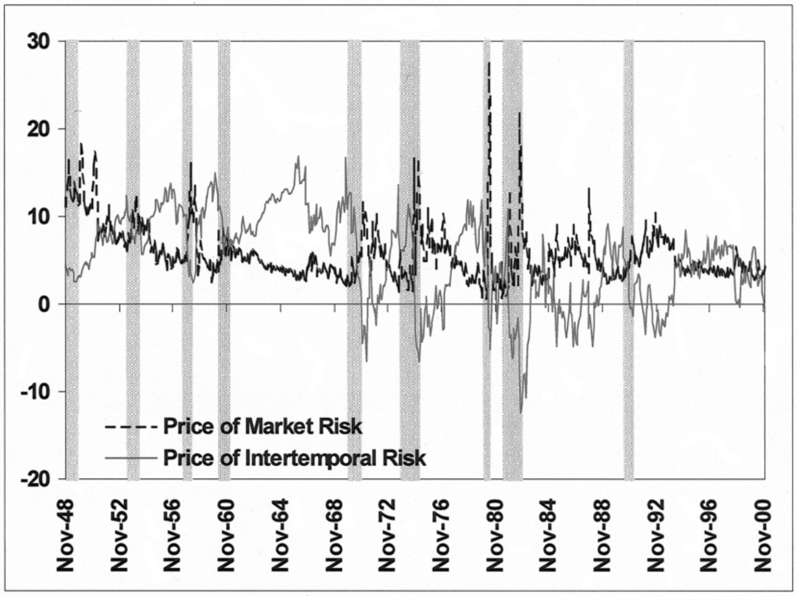

FIG. 2.—Estimated prices of market and intertemporal risk. The estimated series are obtained from the conditional CAPM with stochastic investment opportunity set. The price of market risk is an exponential function of the information variables in z_{t-1}. The price of intertemporal risk is a linear function of the variables in z_{t-1}. Shaded areas highlight NBER recession periods.

ported in table 2, even when the positivity restriction on the price of market risk is relaxed. Figure 2 plots the estimated prices of market risk and intertemporal risk. The estimated series are obtained from the conditional CAPM with stochastic investment opportunity set. We see that both series are volatile, yet the price of intertemporal risk seems to fluctuate much more dramatically, in particular during NBER recession periods in the 1970s and 1980s.

Of course, documenting the statistical significance of intertemporal risk as a pricing factor is not sufficient to conclude that the premium for bond risk is economically relevant. To address this issue, we perform a number of exercises. Define $\alpha_{t-1}\,\mathrm{Cov}_{t-1}(R_{it}, R_{Mt})$ as the premium for market risk and $\lambda_{t-1}\,\mathrm{Cov}_{t-1}(R_{it}, R_{TBt})$ as the premium for bond risk. First, we consider a graphical representation of the estimates of the two premia. As shown in figures 3–6, the two premia play quite different roles in the pricing of the stock portfolios and bond portfolios. The market risk seems to be significant for both portfolios, while the intertemporal risk seems most significant for the bond portfolio.

Second, we propose a more detailed statistical analysis of the fitted risk premia. In table 4, we report generalized method of moments (GMM) tests of the overall statistical significance of the market and intertemporal com-

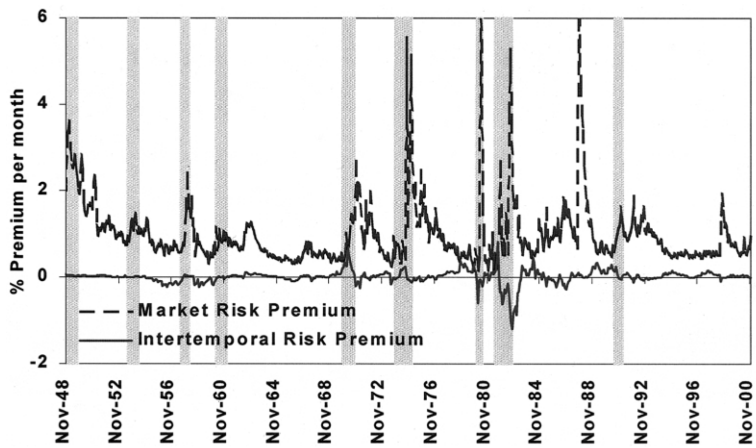

FIG. 3.—Equity market portfolio: estimated market and intertemporal premiums. Shaded areas highlight NBER recession periods.

ponents of the fitted premia for each asset and for the equity and fixed income portfolios jointly. The tests shows that the fitted market risk premiums are significantly different from zero for each individual asset and each group of assets. In contrast, the fitted intertemporal risk premiums are significant for the fixed income assets only and not for the equity portfolio when each asset is considered individually. However, the fitted intertemporal risk premiums are significant for equities when the two stock portfolios are considered jointly.

In tables 5 and 6, we provide summary statistics of the prices of market

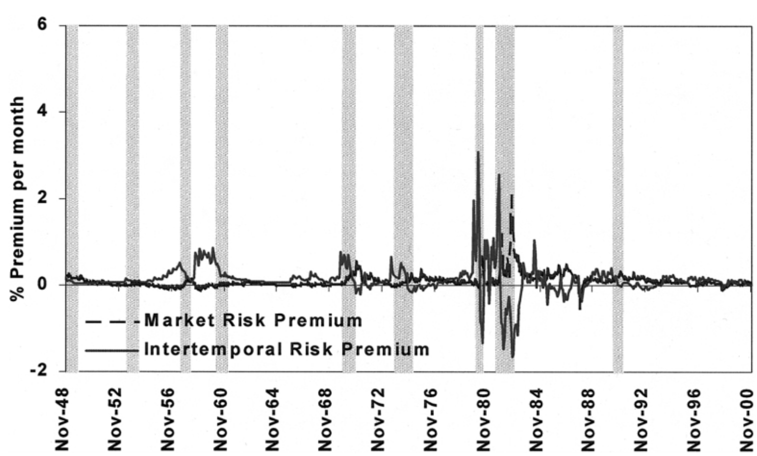

FIG. 4.—Long-term Treasury bond portfolio: estimated market and intertemporal premiums.

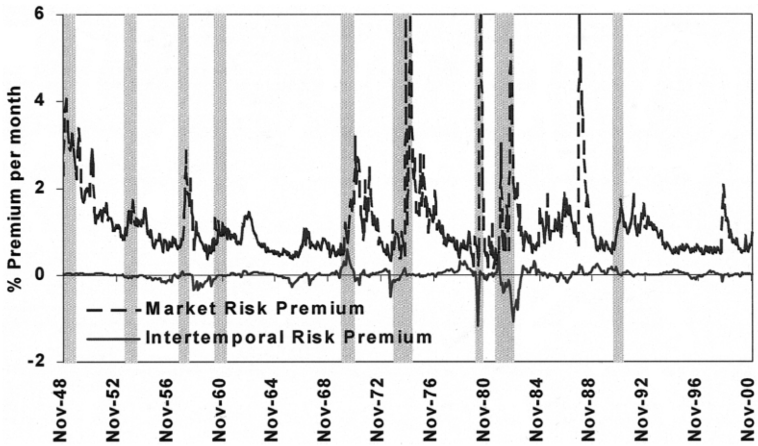

FIG. 5.—Small stocks portfolio: estimated market and intertemporal premiums.

and intertemporal risk as well as the estimated total premiums and their components for the overall sample as well as an analysis of how the prices and premiums change over time and across business expansions and recessions. Over the entire sample, the average prices of intertemporal risk and of market risk are both positive and of similar magnitude. However, the price of intertemporal risk exhibits twice as much volatility as the price of market risk. The average premium for intertemporal risk is equal to 0.1% and 0.13% per month for corporate bonds and Treasury bonds, respectively. Arguably, the annualized premiums of 1.3% and 1.5% are also economically relevant, especially considering that the average annualized market risk premiums for

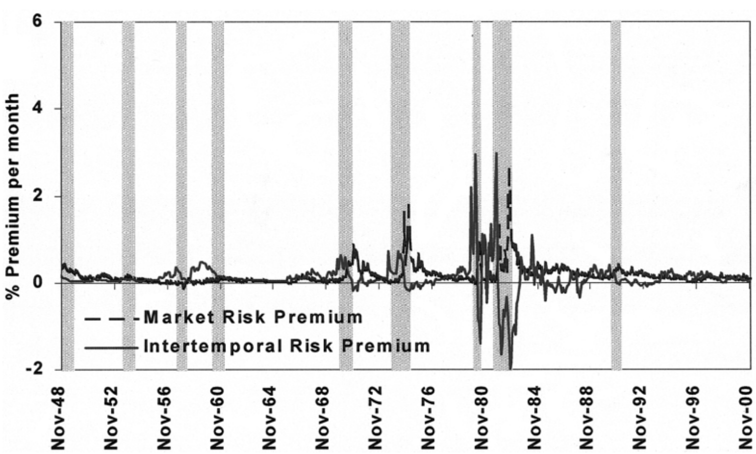

FIG. 6.—Corporate bond portfolio: estimated market and intertemporal premiums.

TABLE 4 **Statistical Tests on the Fitted Risk Premia**

	Individual Portfolio Tests ($\chi^2(1)$)			
	Market Risk	*p*-value	Intertemporal Risk	*p*-value
Market portfolio	122.30	.000	.76	.384
Long bonds	13.47	.000	13.65	.000
Small stocks	127.10	.000	.37	.546
Corporate bonds	27.51	.000	11.42	.001
	Joint Tests ($\chi^2(2)$)			
	Market Risk	*p*-value	Intertemporal Risk	*p*-value
Equity assets	127.11	.000	15.94	.000
Bond assets	35.95	.000	13.71	.000

NOTE.—This table reports GMM tests on the fitted risk premium series to check their difference from zero. Variances are computed using a Bartlett-kernel estimator where the bandwidth is selected according to Andrews (1991). The Newey and West (1987) method is used for serial correlation correction for the standard errors.

these assets were equal to 1.9% and 1.1%, respectively, as documented in panel B of table 5. Given that the estimated prices of risk are approximately equal and that the fixed income portfolios earn premiums of similar magnitudes for both market and intertemporal risk, we can infer that both bond portfolios have exposures of similar magnitudes to both market and intertemporal risk. For the market portfolio, the average intertemporal risk premium is 0.015% on a monthly basis. The corresponding market risk premium is 0.989% per month. These numbers are consistent with the commonly documented market premiums. In our case, the average total premium, obtained as the sum of the market premium and the bond premium (see panel C in table 5) is equal to 12% on an annual basis for the market portfolio and 13.2% for small stocks. For these two portfolios, exposure to market risk is several orders of magnitude larger than to intertemporal risk. The last three columns in tables 5 and 6 report joint tests of the hypothesis that the risk premiums are equal to zero. For the average premiums, these tests confirm the results of the tests reported in table 4.

Tables 5 and 6 also investigate whether the estimated prices of risk and fitted risk premia change over the business cycle (table 5), as well as from the first half of the sample period to the second half (table 6). The tests are performed using a robust dummy variable regression in which the constant represents the average premium or price during expansions or during the first half of the sample and the dummy variable coefficient is the estimate of the change in prices or premiums during NBER recessions or the second half of the sample. The results in panel A of table 5 indicate, surprisingly, that there is no significant change either in the price of intertemporal risk or in the fitted intertemporal risk premiums over the business cycle. Looking at individual assets, the intertemporal risk premiums average to zero during recessions for the market portfolio, while they become more negative for the small stock portfolio. However, the results in panel B of table 5 suggest that the price of market risk and the fitted market risk premiums increase significantly during

TABLE 5 **Decomposition of Total Risk Premiums and the Business Cycle**

	A. Intertemporal Risk Premium							
	$\widehat{\lambda_{t-1}}$	R_{Mt}	R_{TBt}	R_{St}	R_{CBt}	$\chi^2_2(eq)$	$\chi^2_2(bnds)$	$\chi^2_4(all)$
Average	5.17	.015	.126	−.009	.109	15.34	13.72	19.42
	(.83)	(.017)	(.052)	(.015)	(.032)	(.000)	(.001)	(.001)
Cst	5.41	.018	.124	−.005	.108	14.33	20.57	22.40
	(.89)	(.015)	(.029)	(.013)	(.024)	(.001)	(.000)	(.000)
D_{NBER}	−1.22	−.019	.013	−.022	.004	.32	.25	.69
	(1.34)	(.054)	(.089)	(.047)	(.097)	(.853)	(.884)	(.952)
	B. Market Risk Premium							
	$\hat{\alpha}_{t-1}$	R_{Mt}	R_{TBt}	R_{St}	R_{CBt}	$\chi^2_2(eq)$	$\chi^2_2(bnds)$	$\chi^2_4(all)$
Average	5.57	.989	.092	1.125	.161	153.5	40.56	159.4
	(.41)	(.081)	(.021)	(.091)	(.027)	(.000)	(.000)	(.000)
Cst	5.26	.891	.074	1.011	.131	156.7	70.56	156.7
	(.33)	(.075)	(.016)	(.082)	(.019)	(.000)	(.000)	(.000)
D_{NBER}	2.13	.586	.110	.678	.179	9.09	5.64	10.75
	(.79)	(.195)	(.063)	(.223)	(.077)	(.011)	(.060)	(.030)
	C. Total Risk Premium							
		R_{Mt}	R_{TBt}	R_{St}	R_{CBt}	$\chi^2_2(eq)$	$\chi^2_2(bnds)$	$\chi^2_4(all)$
Average		1.004	.218	1.115	.269	165.8	95.96	276.5
		(.079)	(.026)	(.091)	(.028)	(.000)	(.000)	(.000)
Cst		.909	.198	1.005	.239	156.0	143.8	327.4
		(.073)	(.023)	(.083)	(.020)	(.000)	(.000)	(.000)
D_{NBER}		.567	.122	.656	.184	9.28	7.11	16.23
		(.187)	(.055)	(.220)	(.071)	(.010)	(.029)	(.003)

NOTE.—The table reports summary statistics of the prices of intertemporal and market risk, as well as of the estimated risk premia and tests of whether the estimated prices and premiums differ between NBER recessions and expansions. The tests are performed by regressing the estimated prices and premiums on a constant *Cst* and a dummy variable D_{NBER} taking a value of 1 during NBER recessions. We compute the tests for the following three risk premiums:

$$\text{Intertemporal premium}: \quad \hat{\omega}_{it} = \hat{\lambda}_{t-1}\widehat{\text{Cov}}_{t-1}(R_{it}, R_{TBt}),$$

$$\text{Market premium}: \quad \hat{\phi}_{it} = \hat{\alpha}_{t-1}\widehat{\text{Cov}}_{t-1}(R_{it}, R_{Mt}),$$

$$\text{Total premium}: \quad \hat{\eta}_{it} = \hat{\alpha}_{t-1}\widehat{\text{Cov}}_{t-1}(R_{it}, R_{Mt}) + \hat{\lambda}_{t-1}\widehat{\text{Cov}}_{t-1}(R_{it}, R_{TBt}).$$

Tests of whether the estimated premiums are jointly significant for the equity assets, the fixed income assets, and all assets simultaneously are reported in the last three columns. All standard errors (in parentheses) are computed using a Bartlett-kernel estimator with bandwidth selected according to Andrews (1991) and Newey and West (1987) serial correlation correction.

recessions. This is true also for the fixed income portfolio, although the joint test that the fitted market premium for bond portfolios increases in recessions is only significant at the 6% level. Panel C indicates that the resulting increase in total premiums during recessions is significant across the board. In particular, the total premiums on all assets are approximately 60% higher during NBER recessions than during expansions.

The changes in the fitted prices and premiums from the first half of the sample to the second half are reported in table 6. One notices immediately the significant decrease in the prices of both intertemporal risk and market risk over the second half of the sample period. The average price of intertemporal risk decreases by 75%, while the price of market risk decreases by 20%. The decrease in the price of intertemporal risk induces a substantial

TABLE 6 Decomposition of Total Risk Premiums: Subsample Evidence

	A. Intertemporal Risk Premium							
	$\widehat{\lambda}_{t-1}$	R_{Mt}	R_{TBt}	R_{St}	R_{CBt}	$\chi_2^2(eq)$	$\chi_2^2(bnds)$	$\chi_4^2(all)$
Average	5.17	.015	.126	−.009	.109	15.34	13.72	19.42
	(.83)	(.017)	(.052)	(.015)	(.032)	(.000)	(.001)	(.001)
Cst	8.41	.006	.172	−.017	.126	9.60	32.33	33.31
	(.81)	(.017)	(.038)	(.018)	(.023)	(.008)	(.000)	(.000)
$D_{\text{Post}-1973}$	−6.12	.017	−.090	.015	−.033	.274	7.89	8.59
	(1.21)	(.034)	(.065)	(.030)	(.062)	(.853)	(.019)	(.072)
	B. Market Risk Premium							
	$\hat{\alpha}_{t-1}$	R_{Mt}	R_{TBt}	R_{St}	R_{CBt}	$\chi_2^2(eq)$	$\chi_2^2(bnds)$	$\chi_4^2(all)$
Average	5.57	.989	.092	1.125	.161	153.5	40.57	159.4
	(.41)	(.081)	(.021)	(.091)	(.027)	(.000)	(.000)	(.000)
Cst	6.21	.937	.035	1.101	.093	69.75	35.69	128.2
	(.63)	(.116)	(.017)	(.134)	(.025)	(.000)	(.000)	(.000)
$D_{\text{Post}-1973}$	−1.19	.100	.110	.045	.132	8.72	9.05	21.85
	(.67)	(.161)	(.037)	(.182)	(.048)	(.013)	(.011)	(.000)
	C. Total Risk Premium							
		R_{Mt}	R_{TBt}	R_{St}	R_{CBt}	$\chi_2^2(eq)$	$\chi_2^2(bnds)$	$\chi_4^2(all)$
Average		1.004	.218	1.115	.269	165.8	95.96	276.5
		(.079)	(.026)	(.091)	(.028)	(.000)	(.000)	(.000)
Cst		.943	.208	1.084	.218	66.61	62.39	107.67
		(.118)	(.034)	(.138)	(.028)	(.000)	(.000)	(.000)
$D_{\text{Post}-1973}$		.117	.021	.060	.098	11.95	8.05	30.86
		(.155)	(.051)	(.181)	(.051)	(.003)	(.018)	(.000)

NOTE.—The table reports summary statistics of the prices of intertemporal and market risk, as well as of the estimated risk premia and tests of whether the estimated prices and premiums differ between the first and second half of the sample period. The tests are performed by regressing the estimated prices and premiums on a constant *Cst* and a dummy variable $D_{\text{Post-1973}}$ taking a value of 1 after December 1973. We compute the tests for the following three risk premiums:

$$\text{Intertemporal premium}: \quad \hat{\omega}_{it} = \hat{\lambda}_{t-1}\widehat{\text{Cov}}_{t-1}(R_{it}, R_{TBt}),$$

$$\text{Market premium}: \quad \hat{\phi}_{it} = \hat{\alpha}_{t-1}\widehat{\text{Cov}}_{t-1}(R_{it}, R_{Mt}),$$

$$\text{Total premium}: \quad \hat{\eta}_{it} = \hat{\alpha}_{t-1}\widehat{\text{Cov}}_{t-1}(R_{it}, R_{Mt}) + \hat{\lambda}_{t-1}\widehat{\text{Cov}}_{t-1}(R_{it}, R_{TBt}).$$

Tests of whether the estimated premiums are jointly significant for the equity assets, the fixed income assets, and all assets simultaneously are reported in the last three columns. All standard errors (in parentheses) are computed using a Bartlett-kernel estimator with bandwidth selected according to Andrews (1991) and Newey and West (1987) serial correlation correction.

reduction in the fitted intertemporal risk premiums for the fixed income portfolio, which, although not significant for the individual portfolios, is highly significant jointly. The equity portfolios intertemporal risk premiums increase during the second half of the sample, although not significantly. Turning to the fitted market risk premiums in panel B of table 6, we see that the price of market risk decreased in the post-1973 period and that the fitted market risk premiums uniformly increased. Even though these increases are not always significant at the individual portfolio level, the tests reported in the last three columns of the table indicate that the market premium increases are jointly significant for the equity portfolios, the bond portfolios, and all assets simultaneously. For the total premiums, although the joint tests suggest a significant change from the pre- to the post-1973 period, the total premium change on individual assets is small and insignificant.

To summarize, we find that the prices of both market risk and intertemporal risk are significant and time varying. Further, we find that the fitted market risk premia and the intertemporal risk premia both are statistically and economically significant for equity and fixed income portfolios, although intertemporal risk premia represent a much larger fraction of total risk premia for bond portfolios than for equity portfolios. Moreover, we find that total fitted risk premiums tend to increase during NBER recessions and that increase is driven mainly by a significant increase of the price of market risk. Finally, we document a significant decrease in the price of both sources of risk in the post-1973 period. However, although the prices of risk decrease, estimated total premiums increase due to increased levels of risk. Our tests suggest that both market risk and intertemporal risk are important risk factors in the pricing of financial assets. Finally, we would like to note that, although we find evidence supporting the importance of an intertemporal hedge factor to price risky assets, our investigation in no way precludes the existence of additional priced factors.

VI. Intertemporal Asset Allocation and Hedging

To assess further the economic importance of intertemporal risk, we examine its effect on investors' portfolios. In this section, we investigate the gains that may arise from explicitly considering intertemporal risk in the asset allocation decision. In the next section, we explore the dynamics of the market risk and intertemporal risk premiums and their links with business cycles by investigating the properties of two specific portfolios.

A. Optimal Portfolio Weights

Our empirical analysis yields conditional expected returns and conditional covariances for the four asset classes. Consider an investor who faces market risk as well as intertemporal risk in her asset allocation decision. This investor can proceed in several ways. She could construct an optimal portfolio that includes equity assets only. She could decide to hedge her equity portfolio against intertemporal risk. Or she could invest in a portfolio that includes both equity and fixed income assets to optimally manage her exposure to market risk and intertemporal risk.

Assume that investors maximize the expected utility of future consumption. The investment opportunity set, available to all investors, includes the following securities:

- two risky equity assets, that is, a market index portfolio and a small stocks portfolio. Given that we know the composition of the market portfolio, this is equivalent to the two equity asset classes of large and small stocks;

- two risky bond assets, that is, the corporate bonds portfolio and the long-term T-bonds portfolio.

Each investor has access to a total of four ($N = 4$) risky securities and one risk-free asset (the 1-month Treasury bill). Let γ denote the investor's degree of risk aversion. Also, indicate with μ the ($N \times 1$) vector of expected returns in excess of the risk-free rate on the N risky assets, and with Σ the ($N \times N$) covariance matrix for the risky assets. Mean-variance optimization implies the following portfolio allocation:

$$\begin{bmatrix} \omega_N \\ \omega_{N+1} \end{bmatrix} = \frac{1}{\gamma}\begin{bmatrix} \Sigma^{-1}\mu \\ 1 - \iota'\Sigma^{-1}\mu \end{bmatrix} + \left(1 - \frac{1}{\gamma}\right)\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{9}$$

where ω_N is the ($N \times 1$) vector of optimal weights for the N risky assets, ω_{N+1} is the fraction of the portfolio invested in the risk-free asset, and ι is a vector of ones.

The optimal weights in equation (9) deserve further discussion. Consider an investor with a logarithmic utility function. This would be equivalent to the assumption that $\gamma = 1$. In this case, the portfolio weights in (9) simplify into

$$\begin{bmatrix} \omega_N \\ \omega_{N+1} \end{bmatrix} = \begin{bmatrix} \Sigma^{-1}\mu \\ 1 - \iota'\Sigma^{-1}\mu \end{bmatrix}, \tag{10}$$

where $\omega_N = \Sigma^{-1}\mu$ is the vector of optimal weights for the N risky assets. The portfolio $\Sigma^{-1}\mu$ in equation (10) is common among all investors and is usually referred to as the universal logarithmic portfolio. Investors with any degree of risk aversion γ would just scale their investment in the logarithmic portfolio by $1/\gamma$ by shifting funds to or from the risk-free asset.

Our discussion to this point implies that all investors hold a combination of two portfolios: the universal portfolio of risky assets and the risk-free asset. The allocation between the two depends on the degree of risk aversion of each investor. Specifically, investors exploit the correlation structure for the entire set of available assets and choose an allocation that maximizes the Sharpe ratio of their portfolio. This result is similar to the standard solution of a portfolio problem. However, when investors have access to long-term bond markets, the result has a number of additional implications, which can be derived after appropriately partitioning both μ and Σ:[17]

$$\mu = \begin{bmatrix} \mu_s \\ \mu_d \end{bmatrix} \qquad \Sigma = \begin{bmatrix} \Sigma_{ss} & \Sigma_{sd} \\ \Sigma_{ds} & \Sigma_{dd} \end{bmatrix},$$

where the letter s denotes the stock portfolios and the letter d indicates the bonds portfolios (long-term corporate bonds and long-term Treasury bonds). The bonds can be held both for speculative and/or hedging purposes.

Define $\Gamma = \Sigma_{dd}^{-1}\Sigma_{ds}$, the [$2 \times 2$] matrix of coefficients from the regression

17. A similar partitioning is used by Glen and Jorion (1993) and Jorion and Khoury (1995) to discuss the implications of optimal currency hedging on international portfolio performance.

of the stock returns on the bond returns. Also define $\Sigma_{s/d} = \Sigma_{ss} - \Gamma'\Sigma_{dd}\Gamma$, the (2×2) covariance matrix of the stock returns, conditional on the bonds. Hence, $\Sigma_{s/d}$ is the covariance matrix of fully hedged equity returns. Standard rules from the inversion of a partitioned matrix imply the following result:

$$\begin{bmatrix}\omega_s\\ \omega_d\end{bmatrix} = \Sigma^{-1}\mu = \begin{bmatrix}\Sigma_{s/d}^{-1}\mu_s - \Sigma_{s/d}^{-1}\Gamma'\mu_d\\ \Sigma_{dd}^{-1}\mu_d - \Gamma\omega_s\end{bmatrix} = \begin{bmatrix}\Sigma_{s/d}^{-1}(\mu_s - \Gamma'\mu_d)\\ \Sigma_{dd}^{-1}\mu_d - \Gamma\omega_s\end{bmatrix}, \tag{11}$$

where ω_s and ω_d are the vectors of optimal weights for, respectively, the equities and the bonds included in the universal portfolio. The first interesting feature of equation (11) is that the optimal choice of ω_s and ω_d should be made simultaneously to exploit the properties of both sets of asset classes. The equity positions are a function of the covariance of the fully hedged stock returns and mean equity returns adjusted for the cost of the hedge. The bond positions have two components. The expression $\Sigma_{dd}^{-1}\mu_d$ is the solution to a standard mean-variance problem for the optimal portfolio of bonds only and, therefore, can be interpreted as a purely speculative position in bonds. On the other hand, the expression $\Gamma\omega_s$ reflects the investment in bonds that minimizes the variance of the portfolio given the position in equities. In this sense, investors hold bonds for both speculative and hedging purposes.[18]

Two special cases are of interest. First, if the expected excess returns on bonds are zero, then the optimal portfolio weights simplify to

$$\begin{bmatrix}\omega_s\\ \omega_d\end{bmatrix} = \Sigma^{-1}\mu = \begin{bmatrix}\Sigma_{s/d}^{-1}\mu_s\\ -\Gamma\omega_s\end{bmatrix}. \tag{12}$$

In this case, the optimal strategy calls for selecting equity portfolio weights based not on the equity unhedged expected returns but on the covariance matrix of their fully hedged returns. However, the bond positions have only a hedging component. If the intertemporal risk is fully diversifiable, then $\Gamma = 0$ and $\Sigma_{s/d}^{-1} = \Sigma_{ss}^{-1}$ and the optimal portfolio includes only equity positions. It is also the solution to a standard mean-variance problem for the optimal portfolio of unhedged equity investments.

Our empirical exercise focuses on three dynamic strategies of particular interest: investing each month in the overall optimal portfolio, investing in an optimal portfolio of equities only, or investing in equities and hedging intertemporal risk. Our discussion of optimal portfolio choice will help identify the shortcomings of each strategy and the circumstances under which they may be optimal. Using the notation introduced above, the three strategies can be summarized as follows:[19]

18. The expression for ω_d is equivalent to the expression for the optimal hedge for a prespecified portfolio derived in Anderson and Danthine (1981); as they show, this would be valid for any choice of ω_s.

19. Implicitly, the weights in the table assume that the investor has a degree of relative risk aversion of 1. It is a simple matter to scale the portfolio weights of the risky assets by the inverse of the degree of relative risk aversion to get the optimal weights for any level of risk aversion.

	Portfolio Position	
	Equities	Long-Term Bonds
1. EO	$\omega_s = \Sigma_{ss}^{-1}\mu_s$	$\omega_d = 0$
2. EO + IH	$\omega_s = \Sigma_{ss}^{-1}\mu_s$	$\omega_d = -\Gamma\omega_s$
3. OPT	$\omega_s = \Sigma_{s/d}^{-1}\mu_s - \Sigma_{s/d}^{-1}\Gamma'_{\mu_d}$	$\omega_d = \Sigma_{dd}^{-1}\mu_d - \Gamma\omega_s$

1. *Optimal equity-only strategy* (EO): In this strategy, we take the point of view of a manager whose mandate prohibits her from taking direct positions in long-term bonds. She optimizes her portfolio holdings over the eligible equity positions only. This strategy would be optimal only if the intertemporal risk exposure of the equity investments is fully diversifiable and bonds have zero expected excess returns.

2. *Overlay intertemporal hedge strategy* (EO + IH): This strategy corresponds to the situation where the role of an equity portfolio manager is distinct from the role of a bond manager. First, the equity portfolio manager chooses her optimal equity portfolio weights in the same fashion as in strategy 1. Second, conditional on these equity portfolio weights, the plan sponsor optimally hedges her exposure to the intertemporal risk. This implicitly assumes that intertemporal risk commands a zero premium, and hence no speculative position in bond assets is allowed. However, the equity allocation is suboptimal since the equity positions are selected without taking into account the correlations between equity and bond assets.

3. *Overall optimal allocation* [OPT]: This strategy implements the unrestricted global optimum portfolio strategy. The portfolio weights of equity and bond assets are selected simultaneously, taking into account the covariances between all assets. In particular, the equity positions now reflect their covariance with the bond assets and the costs of the minimum variance hedges.

Note that the bond positions are always determined as a function of the positions in the equity portion of the portfolio. Hence, we can evaluate the benefits of overlay strategies for any given portfolio of equities as long as we have estimates of the excess returns on bonds and of the variance-covariance matrix of the equity and bond assets considered.

B. Portfolio Performance

We implement each strategy at the beginning of each month and record its performance and characteristic over the whole sample period. We use our estimated model to provide beginning-of-month forecasts of expected returns and volatility. Note that time variation in the price of market risk reflects time variation in the aggregate degree of relative risk aversion in the economy. All the strategies are scaled by one over the estimated relative risk aversion coefficient. Table 7 summarizes the performance and characteristics of the three portfolio strategies described in the preceding subsection. We also report the characteristic of the minimum variance hedge for the equity-only portfolio,

TABLE 7 Optimal Strategies Characterisitcs

	Overall Optimal Portfolio	Equity-Only Portfolio	Overall Hedge	Hedged Equity-Only Portfolio	Minimum Variance Hedge
	Fraction in Risky Fund				
Mean	.472	.930	−.458	.960	.030
SD	.443	.181	.486	.158	.108
	Realized Excess Returns				
Mean	.933	.842	.091	.569	−.273
SD	4.964	5.750	5.380	3.731	4.370
Minimum	−24.635	−23.72	−12.52	−25.20	−10.52
Maximum	32.31	14.94	34.67	11.93	7.55
Sharpe Ratio	.188	.146	.017	.152	−.062
	Correlations of Realized Excess Returns				
Optimal portfolio (OPT)	1.000				
Equity-only portfolio (EO)	.810	1.000			
Optimal hedge	.384	−.604	1.000		
EO + IH portfolio	.776	.650	.021	1.000	
Maximum variance hedge	−.000	−.761	.813	−.001	1.000
	Expected Excess Returns				
Mean	.978	.724	.254	.663	−.061
E[SD]	4.600	4.013	2.587	3.660	1.694
Minimum	.136	.015	−4.635	.000	−7.704
Maximum	7.269	7.713	3.144	5.776	1.030

NOTE.—This table reports the performance and characteristiscs of dynamic portfolio strategies. We consider three portfolio strategies: the overall optimal portfolio of all assets (OPT), the optimal portfolio restricted to equity only (EO), and the optimal equity-only portfolio hedged for intertemporal risk (EO+IH). Overall hedge reports the difference in portfolio characteristics between the overall optimum and the equity-only portfolio and can be thought of as a proxy for an overall optimum intertemporal hedge portfolio. Minimum variance hedge reports the mimimum variance intertemporal hedge for the equity-only portfolio. Expected returns and covariances are the fitted values from our general model. All returns are reported in percent per month.

as well as the difference between the equity-only and global optimal portfolios. We use the last one as our proxy for the optimal intertemporal hedge portfolio.

We first report the fraction of the overall portfolios invested in the four risky assets. For the equity-only optimal portfolio, on average 93% of the overall portfolio is invested in the two equity assets, the rest being invested in the riskless asset. The standard deviation of the fraction in the risky asset is 18%, which represents a rough measure of average monthly turnover. By comparison, the global optimal portfolio has 47% of assets in risky funds and a standard deviation of 44%. The minimum variance hedge of the equity-only portfolio has a substantially smaller impact on the fraction in risky assets. While the minimum variance hedge decreases both the returns and volatility of the equity only portfolio, leaving the portfolio realized Sharpe ratio basically unchanged, the global optimal hedge induces both higher returns and lower volatility in the global optimum portfolio than in the equity-only optimum portfolio. The realized monthly Sharpe ratio is 0.146 for the equity-only portfolio and 30% higher at 0.188 for the global optimum portfolio. To put these numbers in perspective, a portfolio of the risk-free asset and the optimum equity-only portfolio with a volatility equal to the realized volatility of the global optimum portfolio would have earned a 0.727% mean premium per month compared to a realized mean premium of 0.933% for the global optimum. Hence, for the global optimum, out of an annual average excess return of 11.20%, 2.47%, or a bit more than a fifth, can be directly traced back to the optimal consideration of intertemporal risk in the portfolio optimization process. The performance of the different strategies is also illustrated in figure 7, which plots the cumulative total returns of the three strategies from October 1948 to December 2000.

The correlations across the different portfolios are reported in the third panel of table 7. First, we see that the reason why combining the optimal hedge with the equity-only portfolio yields a superior risk-reward trade-off for the global optimum portfolio is the very large negative correlation of -0.604 between the two portfolios. Second, not surprisingly, the minimum variance hedge for the equity-only portfolio yields a position with even lower realized correlation of -0.761 with that portfolio. These realized correlations also confirm that our approach indeed identifies hedge portfolios. Finally, the fourth panel reports the average expected mean excess return and the average expected volatility for all the strategies and hedge portfolios. For all three, strategies expectations and realizations of mean returns and volatility are not

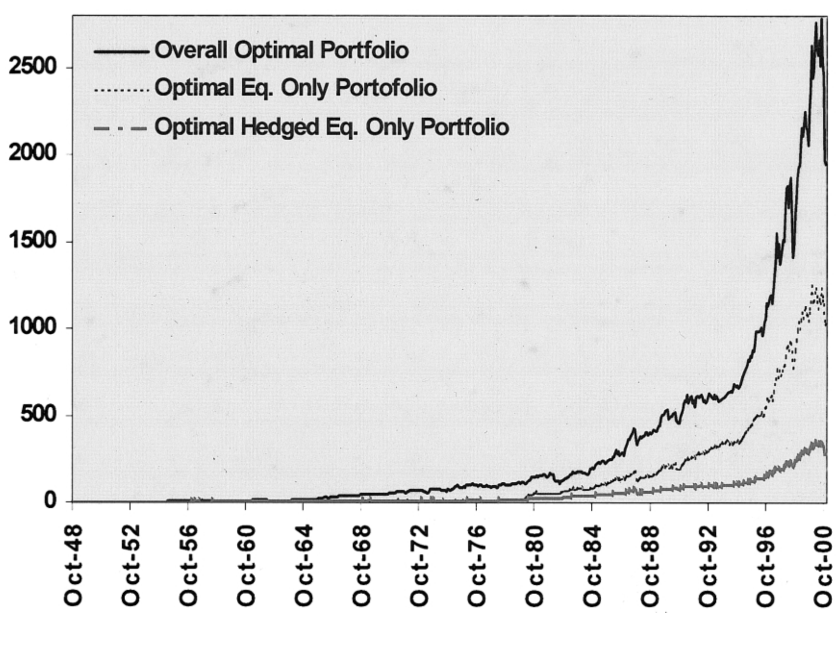

FIG. 7.—In sample performance of dynamic strategies. This figure plots the cumulative returns on three strategies: (*a*) overall optimal portfolio, (*b*) optimal equity-only portfolio, and (*c*) optimal equity-only portfolio hedged against intertemporal risk.

far apart. For both hedge portfolios, however, expectations and realizations depart more substantially.

In summary, we find that the global hedge portfolio exhibits a significant negative correlation of −0.604 with the equity-only portfolio. Although earning only a small positive 1.1% annual excess return and having a volatility of similar magnitude as the equity-only portfolio, the global hedge significantly improves the risk-reward trade-off of the global optimal portfolio: it accounts for 22% of the global optimal portfolio risk premium, or approximately an additional 2.5% annual premium.

VI. The Importance of Intertemporal Risk

In this section, we explore the dynamics of the market and intertemporal risk premiums and their links with business cycles by investigating the properties of two specific portfolios. These portfolios are constructed to yield the maximum Sharpe ratios among all portfolios with zero exposure to either intertemporal risk or market risk. Specifically, we construct efficient frontiers of portfolios that are orthogonal to intertemporal risk or market risk.

A. Portfolios Orthogonal to Intertemporal Risk

To hedge completely against the intertemporal risk, an investor is constrained to invest in portfolios orthogonal to the long-term Treasury bond portfolio,

$$\omega_Z' \Sigma \omega_b = 0, \tag{13}$$

where

$$\omega_b = [0 \;\; 0 \;\; 1 \;\; 0]'$$

is the vector of portfolio weights for the Treasury bond portfolio and ω_Z represents the weights for all the portfolios orthogonal to the Treasury portfolio. Roll (1980) shows that, for a portfolio that is off the mean-variance efficient frontier, the set of portfolios orthogonal to it is given by an area bounded by a quadratic function. Figure 8 illustrates the set of orthogonal portfolios. Point *b* represents the intertemporal risk portfolio proxied by the long-term Treasury bond portfolio. If we draw a line from the intertemporal risk portfolio through the global minimum variance portfolio, we find the expected return of the minimum variance portfolio z_0 orthogonal to the intertemporal risk. The hatched area represents the set of portfolios orthogonal to the intertemporal risk. We minimize the variance of portfolios subject to constraint (13) to find the risky asset weights for the orthogonal mean-variance frontier,

$$\begin{aligned} \omega_Z &= [\omega_b : \Sigma^{-1}\mu : \Sigma^{-1}\iota] H^{-1} [0 : \mu_z : 1]' \\ &\equiv M[0 : \mu_z : 1]', \end{aligned} \tag{14}$$

where μ_z is the mean excess return of the orthogonal portfolio and H is a

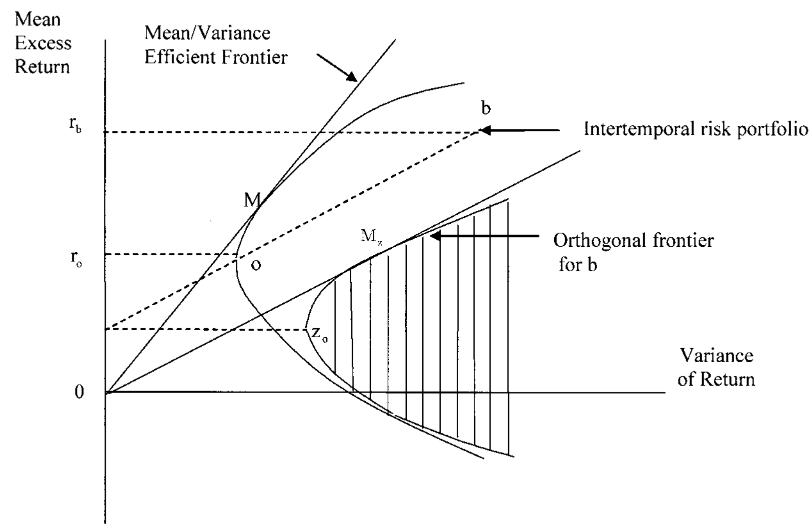

FIG. 8.—Constrained efficient frontier for portfolios orthogonal to the long-term Treasury portfolio. This figure shows the set of portfolios orthogonal to the long-term Treasury portfolio. The Sharpe ratios of the fully efficient frontier and the constrained efficient frontier are indicated by the two lines from the origin to the tangencies on the the frontiers.

(3 × 3) matrix of the following form:

$$H = [\Sigma\omega_b : \mu : \iota]'\Sigma^{-1}[\Sigma\omega_b : \mu : \iota]$$

$$= \begin{bmatrix} \sigma_b^2 & \mu_b & 1 \\ \mu_b & \mu'\Sigma^{-1}\mu & \mu'\Sigma^{-1}\iota \\ 1 & \mu'\Sigma^{-1}\iota & \iota'\Sigma^{-1}\iota \end{bmatrix}.$$

The efficient portfolio frontier for the orthogonal portfolios is then a straight line from the origin to the tangency on the hatched region. It is apparent that the constrained efficient frontier is dominated by the fully efficient frontier.

There are two ways that we can examine the relative importance of the intertemporal risk. First, we can study the time series absolute and relative differences in the Sharpe ratios. The Sharpe ratios for any constrained or unconstrained efficient frontier portfolio ω can be computed as

$$SR = \frac{\omega'\mu}{\sqrt{\omega'\Sigma\omega}}.$$

The Sharpe ratio for the overall efficient frontier is just the Sharpe ratio for the overall tangency portfolio. For the orthogonal frontier, the expected excess return of the tangency portfolio is

$$\mu_z^* = \frac{-M_3' \Sigma M_3}{M_2' \Sigma M_3},$$

where M_2 and M_3 are the second and third columns of M defined above in equation (14). By substituting μ_z^* into equation (14), we obtain the weights for the tangency portfolio and Sharpe ratios can be computed. We can test whether the Sharpe ratios for the fully efficient frontier and the orthogonal frontier are statistically different.

The second way to study the relative importance of the intertemporal risk is to analyze the difference in excess returns for a fixed level of portfolio risk and derive implications for the cost of hedging or the benefits of taking into account the intertemporal risk. We can quantify the cost or benefit in terms of premiums.

As a comparison, we also conduct the above exercises with respect to the market risk. The mathematics for the portfolio frontier orthogonal to the market risk is similar to that to the intertemporal risk. We simply use

$$\omega_b = [1 \;\; 0 \;\; 0 \;\; 0]'$$

instead of

$$\omega_b = [0 \;\; 0 \;\; 1 \;\; 0]'$$

in deriving the weights for the optimal orthogonal portfolios.

B. Empirical Evidence

Using the estimated conditional means and covariances of the four assets, we apply the constrained mean variance analysis derived above and compute the conditional Sharpe ratios of various portfolios. Figures 9 and 10 plot optimal Sharpe ratios over the entire sample for three types of strategies. The unhedged portfolio represents the tangency portfolio for the overall mean-variance frontier. Figure 9 graphs the Sharpe ratios for the unhedged portfolio with those from portfolios hedged against intertemporal risk, that is, the tangency portfolios from the frontier orthogonal to the intertemporal risk. The result is striking. We find that the unhedged Sharpe ratios coincide with the intertemporal risk-hedged Sharpe ratios during most periods in the sample, except from 1956–1961, 1969–1970, 1978–1982, and 1988–1990. These periods occur before or during recessionary periods.

In contrast, figure 10 graphs the Sharpe ratios for the unhedged portfolio with those from portfolios hedged against market risk, that is, the tangency portfolios from the frontier orthogonal to the market risk. The difference in Sharpe ratios for these two strategies is large and volatile throughout the entire sample. There is nothing special in terms of the difference in Sharpe ratio during the above-mentioned periods. In fact, the difference in Sharpe ratio is

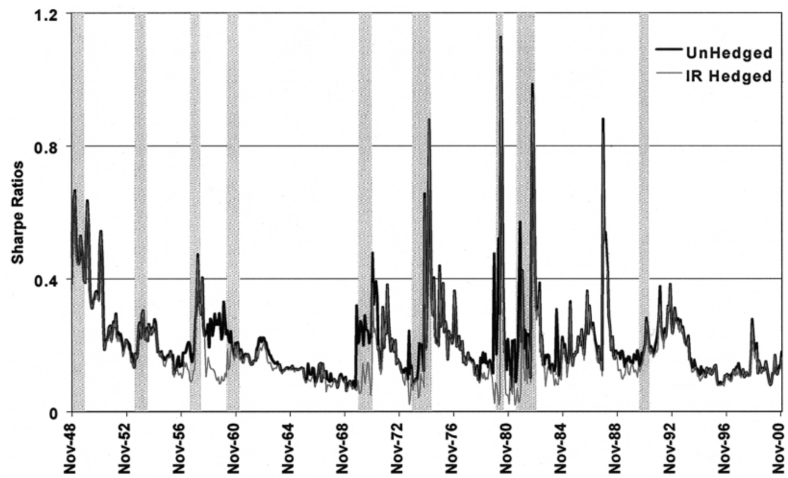

FIG. 9.—Expected Sharpe ratios on optimal unhedged and intertemporal-risk hedged portfolios. This figure plots the Sharpe ratios of an unconstrained efficient portfolio and the constrained efficient portfolios orthogonal to market risk. Shaded areas highlight NBER recession periods.

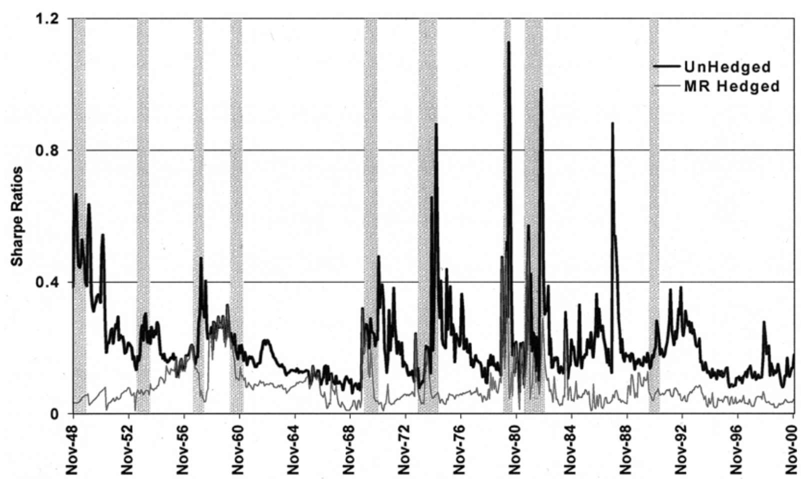

FIG. 10.—Expected Sharpe ratios on optimal unhedged and market-risk hedged portfolios. This figure plots the Sharpe ratios of an unconstrained efficient portfolio and the constrained efficient portfolios orthogonal to market risk. Shaded areas highlight NBER recession periods.

large and relatively stable outside those periods. For example, during the 1990s, the loss in Sharpe ratio by hedging against the market risk is consistently high and ranges between 0.05 and 0.31. These two graphs indicate that the cost of hedging against intertemporal risk is generally low, except during adverse market conditions. This may reflect the fact that the benefit of hedging against intertemporal risk is also low during normal market conditions. The cost of hedging against market risk, however, is shown to be very significant during the entire sample. The cost seems to be smaller and more volatile during the 1978–82 period, suggesting some benefit of hedging against market risk during severe market conditions.

One may argue that the conditional Sharpe ratios do not reflect the realized performance of the strategies. To answer this question, we also compute "realized Sharpe ratios" using realized excess returns under the three strategies. Figures 11 and 12 plot the difference in the expected (conditional) and realized Sharpe ratios between an unconstrained efficient portfolio and the constrained efficient portfolios orthogonal to either intertemporal or market risk. The realized differences in Sharpe ratio are certainly much more volatile, but overall they seem to vary around the expected differences.

Certainly, Sharpe ratios may not be a good measure of cost of hedging against a particular risk. Another measure is the difference in risk premium for a common benchmark risk level. We next compute expected risk premiums on unconstrained efficient portfolios (OPT) and on efficient portfolios either orthogonal to market (OPTHMR) or intertemporal risk (OPTHIR). The portfolios are constructed to have volatilities equal to the average market portfolio volatility during NBER recessions and to the average market volatility during NBER expansions. Table 8 reports summary statistics of these risk premiums. The last two columns of that table report the additional premium earned for bearing market risk or intertemporal risk when comparing the unconstrained optimal portfolio and the efficient portfolios orthogonal to market or intertemporal risk ($\text{MktRP} = \text{OPT} - \text{OPTHMR}$; $\text{IntRP} = \text{OPT} - \text{OPTHIR}$).

The average estimated risk premium for bearing intertemporal risk is 0.331% per month, which translates into a 4.0% annual risk premium. During NBER market expansion, the monthly intertemporal risk premium is 0.300% (3.6% annually), while during recession, the premium is 0.186% per month higher at 0.486% (5.8% annually). The mean difference between risk premiums for the unconstrained optimal portfolio and the efficient portfolios hedged against the intertemporal risk is 0.118% per month (1.4% annually). This implies that the incremental reward of bearing optimal amounts of intertemporal risk in addition to market risk is 1.4% annually. During expansion, this risk premium is 0.091% per month (1.1% annually), while during recession this premium increases by 0.160% to 0.251% per month (3.0% annually). Therefore, intertemporal risk is overall significant for asset allocation decisions, especially during down markets and business recessions.[20]

20. Ang and Bekaert (2004) show that, during a persistent bear market, investors tend to switch

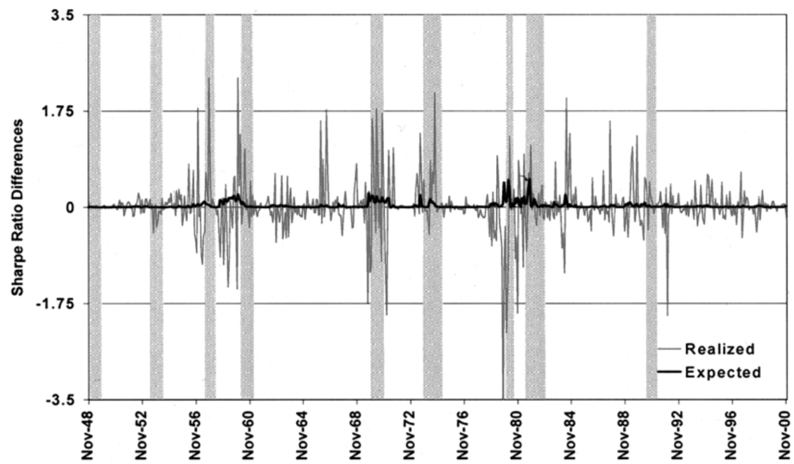

FIG. 11.–Difference in Sharpe ratios between the unconstrained opitmal portfolio and the constrained optimal portfolio orthogonal to intertemporal risk. Shaded areas highlight NBER recession periods.

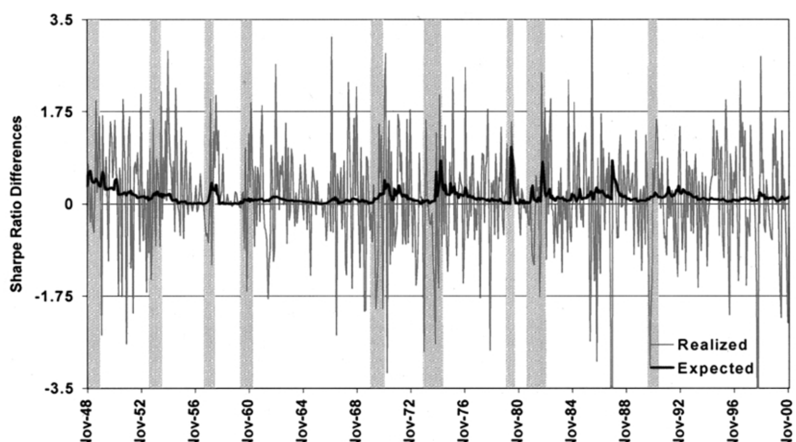

FIG. 12.—Difference in Sharpe ratios between the unconstrained opitmal portfolio and the constrained optimal portfolio orthogonal to market risk. Shaded areas highlight NBER recession periods.

TABLE 8 Decomposition of Optimal Portfolios Risk Premiums and the Business Cycle

	OPT	OPTHIR	OPTHMR	MktRP	IntRP
Average	.874	.756	.331	.543	.118
	(.062)	(.060)	(.034)	(.063)	(.063)
Expansion	.774	.683	.300	.474	.091
	(.049)	(.050)	(.031)	(.055)	(.021)
Δ Recession	.594	.434	.186	.408	.160
	(.152)	(.157)	(.069)	(.155)	(.071)
Due to ΔSR_t	.514	.363	.155	.359	.151
	(.153)	(.158)	(.070)	(.155)	(.071)
Due to $\Delta\sigma_M$	.080	.071	.031	.049	.009
	(.005)	(.005)	(.003)	(.005)	(.002)

NOTE.—The table reports summary statistics of expected risk premiums on unconstrained efficient portfolios (OPT) and on efficient portfolios either orthogonal to market (OPTHMR) or intertemporal risk (OPTHIR). The last two columns report the incremental premiums earned for bearing market risk or intertemporal risk when comparing the unconstrained optimal portfolio and the efficient portfolios orthogonal to market or intertemporal risk (MktRP = OPT − OPTHMR; IntRP = OPT − OPTHIR). The portfolios are constructed to have volatilities equal to the recession average market portfolio volatility during NBER recessions and to the expansion average market volatility during NBER expansions. We test whether the estimated premiums differ between NBER recessions and expansions and whether the change in premium is due to change in Sharpe ratios or in market volatility. Risk premiums are reported in percent per month. All standard errors (in parentheses) are computed using a Bartlett-kernel estimator with bandwidth selected according to Andrews (1991) and Newey and West (1987) serial correlation correction. The row "Δ Recession" indicates increases in risk premium during NBER recessions.

In comparison to intertemporal risk, the average estimated risk premium for bearing market risk is 0.756% per month, or 9.1% on an annual basis. During NBER market expansion, the monthly market risk premium is 0.683% (8.2% annually), while during recession, the premium is 0.434% per month higher at 1.112% (13.4% annually). The mean difference between risk premiums for the unconstrained optimal portfolio and the efficient portfolios hedged against the market risk is 0.543% per month (6.5% annually). This implies that the incremental reward of bearing optimal amounts of market risk in addition to intertemporal risk is 6.5% annually. During expansions, this risk premium averages 0.474% per month (5.7% annually), while during recessions it increases by 0.408% to 0.882% per month (10.1% annually). Not surprisingly, market risk is a more significant risk than intertemporal risk for asset allocation decisions.

Figures 13 and 14 plot the difference between the expected premium earned on an unconstrained efficient portfolio and on efficient portfolios fully hedged against intertemporal (fig. 13) or market risk (fig. 14). We see that, in terms of risk premiums, the cost of hedging the market risk is very high, averaging 0.543% per month, as compared to an average of 0.118% per month for hedging against intertemporal risk. Yet intertemporal risk can be important. For example, during the 1978–82 period, intertemporal risk premium reached a high of 2.8% per month (33.6% per year).

to all-cash investment. This may reflect the need for hedging during adverse market conditions. Our findings indicate that investors demand more intertemporal hedging and are willing to pay a higher premium for such hedging during recessions.

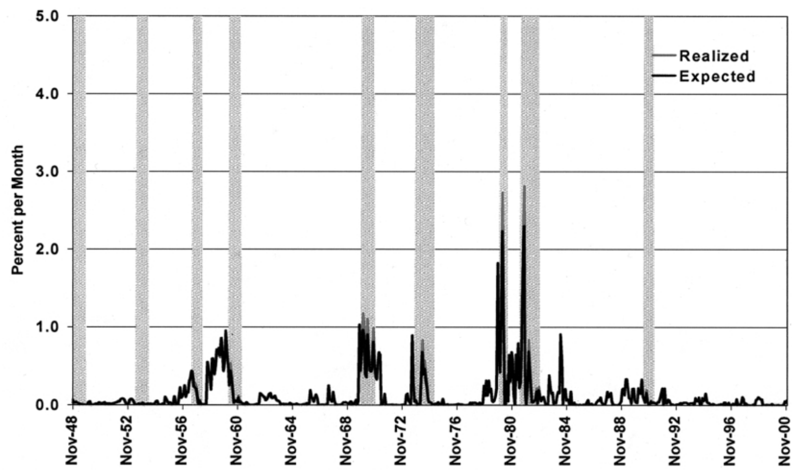

FIG. 13.—Difference in expected premiums between the unconstrained optimal portfolio and the optimal portfolios fully hedged against intertemporal risk. The portfolios are chosen to have a volatility equal to the recession average market portfolio volatility during NBER recesssions and to the expansion average market volatility during NBER expansions. Shaded areas highlight NBER recession periods.

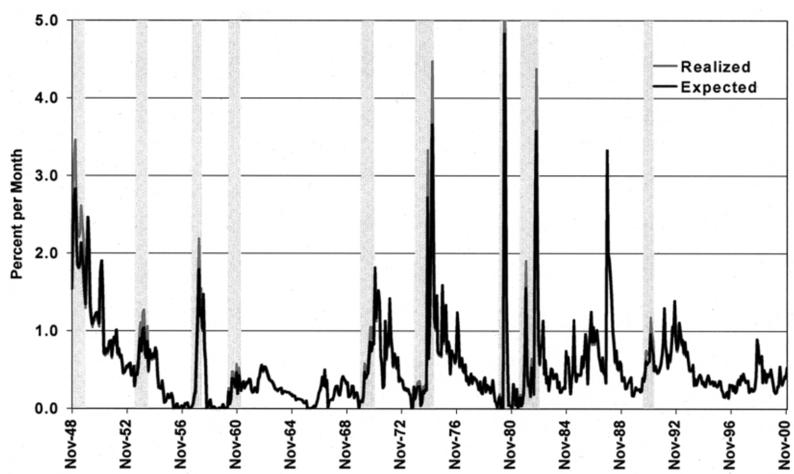

FIG. 14.—Difference in expected premiums between the unconstrained optimal portfolio and the opotimal portfolios fully hedged against market risk. The portfolios are chosen to have a volatility equal to the recession average market portfolio volatility during NBER recessions and to the expansion average market volatility during NBER expansions. Shaded areas highlight NBER recession periods.

VIII. Conclusions

In this article, we analyze the statistical and economic relevance of intertemporal risk in explaining the dynamics of the premium for holding stocks and bonds. We jointly estimate and test a conditional asset pricing model that includes long-term interest rate risk as a potentially priced factor for four broad classes of assets—large stocks, small stocks, long-term Treasury bonds, and corporate bonds. For the conditional CAPM, we find that market risk—measured by the conditional variance of the return on the market portfolio—is priced and that the reward-to-risk ratio for holding the market (i.e., the price of market risk) is time varying. We also find that the premium for long bond risk is the main component of the risk premiums of Treasury bond and corporate bond portfolios, while it represents a small fraction of total risk premiums for equities. Our results suggest that investors perceive stocks as hedges against variations in the investment opportunity set.

Since the four asset classes under study represent some of the most important for investors, we proceed to use our estimated premiums and covariances to compute the dynamic optimal asset allocations for investors with different risk preferences and trading strategies. We use two alternative approaches to decompose portfolio holdings into their hedging and speculative components. First, we construct period by period the equity-only optimal portfolio, the minimum variance intertemporal risk hedge for the equity-only portfolio, and the global optimal portfolio. To proxy for the optimal intertemporal hedge portfolio, we use the difference between the equity-only and global optimal portfolios. Although earning only a small premium of 1.1% per year and having a volatility of similar magnitude as the equity-only portfolio, the global hedge significantly improves the risk-reward trade-off of the global optimal portfolio due to its significant negative correlation of -0.604 with the equity-only portfolio. The intertemporal hedge accounts for 22% of the global optimal portfolio risk premium, or approximately 2.5% out of the 11.1% annual premium.

Second, we construct period by period both the optimal portfolio orthogonal to market risk and the optimal portfolio orthogonal to intertemporal risk. The performance of the portfolio orthogonal to market risk provides direct insights into the rewards for market-neutral strategies bearing only intertemporal risk. We find that, at average market volatility levels, bearing intertemporal risk only yields an annual premium of 3.6% during expansion, increasing to 5.8% during recessions. However, the incremental reward of bearing optimal amounts of intertemporal risk in addition to market risk varies between 1.1% per annum during expansions and 3% during recessions.

In sum, our evidence suggests that considering intertemporal risk is important both for asset pricing and for asset allocation decisions, especially to model changes in the risk-return trade-off between business recessions and expansions. Not only does intertemporal risk bear economically meaningful time-varying premiums but explicitly incorporating intertemporal risk considerations in portfolio decisions brings large benefits, especially during down markets.

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