Analyst Coverage and Two Volatility Puzzles in the Cross Section of Returns

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We examine a stylized version of Miller's (1977) hypothesis as the explanation of the puzzling findings of both Chordia, Subrahmanyam and Anshuman (2001) and Ang, Hodrick, Xing and Zhang (2006). Identifying stocks that are prone to disagreement by using low analyst coverage produces results that are strongly consistent with the model's predictions. The low returns to high return volatility stocks are corrections of optimistic mispricing that arises because information arrivals generate disagreement among traders. Disagreement also implies a negative relation between returns and shocks to trading volume. The abnormal returns to a trading strategy based on idiosyncratic return volatility are explained by the returns to a strategy based on turnover volatility, suggesting that the same economic forces underlie both relations as the model predicts.

Introduction

Ang, Hodrick, Xing and Zhang (2006, 2009) show that stocks with high idiosyncratic return volatility earn low average returns (hereafter referred to as the AHXZ result). Chordia, Subrahmanyam and Anshuman (2001) document a similar result with respect to the volatility of share turnover—high turnover volatility predicts low returns in the cross section. AHXZ's result has received a great deal of scrutiny because it seems obviously to contradict the notion that only systematic risk is priced, and also Merton's (1987) hypothesis that under diversification leads to a return premium for stocks with high idiosyncratic volatility. However, the Chordia et.al. (hereafter CSA) result is puzzling as well. Asset pricing models that incorporate a role for the trading environment predict return premiums for less liquid stocks [e.g., Brennan and Subrahmanyam (1996)]. Since trading interest in stocks with high turnover volatility is more difficult to predict, those stocks should earn return premiums.

In this paper, we present evidence that Miller's (1977) hypothesis explains both the AHXZ and CSA results. Miller argues that disagreement generates upward biased prices because short sales costs prevent pessimists from trading as aggressively as optimists.[1] As disagreement eventually is resolved, the bias dissipates and prices fall. This has been considered as an explanation for AHXZ's results before, using dispersion in analysts' forecasts to measure disagreement. In fact, AHXZ include this among the robustness tests in their study. They find that idiosyncratic volatility retains its significance in predicting cross sectional differences in returns after controlling for dispersion in analysts forecasts, which suggests that Miller's hypothesis does not fully explain the relation between returns and idiosyncratic volatility.

There are two reasons to reexamine this. First, dispersion among analysts' forecasts can only be computed if there are two or more analysts, which changes dramatically the cross section that can be included in the study. This is especially important when explaining AHXZ's and CSA's results, which relate to comparisons of extreme quintiles of the entire CRSP cross section. By omitting precisely the firms for which a paucity of

[1] See also Harrison and Kreps (1978), Chen, Hong and Stein (2002), Hong and Stein (2003) and Gopalan (2003).

analyst coverage leaves traders most prone to disagreement, the truncation of the sample could eliminate the differences that Miller's hypothesis explains, creating the appearance that Miller's hypothesis has weak explanatory power.

Second, controlling for dispersion is not likely to eliminate return volatility as a predictor of returns even if Miller's hypothesis is driving the AHXZ and CSA results. In order for Miller's hypothesis to affect returns, disagreement among traders must change through time—a constant bias in prices will not affect returns. Disagreement arises when traders' interpretations differ about information that arrives, and it recedes as time passes thereafter. In order for dispersion to eliminate return volatility, dispersion would have to reflect accurately time variation in information flows. Since analysts update forecasts with a lag, return volatility will still play a role by indicating whether information has arrived that leads to disagreement. As explained below, a simple equilibrium model indicates that an ex-ante measure of whether traders are prone to disagree can instead be combined with return volatility to test Miller's hypothesis. Using the entire sample to examine predictions from this model, our findings are remarkably consistent with Miller's hypothesis as an explanation for AHXZ and CSA's findings for returns horizons up to two years.

We model the arrival of information in a market where traders are strategic, short positions are costly, and traders can disagree about the interpretation of news. Consensus beliefs are correct despite disagreements, however. An important implication is that disagreement and volatility are complements in predicting returns—future returns *should* be related to volatility, but only among stocks for which traders are prone to disagreement. When news arrives, it changes consensus beliefs. This shock to beliefs is reflected in equilibrium prices, and it is the source of high return volatility. However, only the subset of news arrivals that change beliefs, *and also* generate disagreement, impart an upward bias into prices. Information arrivals that do not generate disagreement produce high return volatility, but they do not impart an upward bias into prices. Likewise, stocks for which traders are prone to disagreement, but that do not experience significant information arrivals, will also not have bias embedded in their prices. Consequently, only stocks for which traders are prone to disagreement *and* that experience significant information arrival are predicted by the model to earn low returns.

In addition, the anticipation of news and disagreement causes prices to rise even *prior* to the actual news arrival—traders bet on the appearance of a future bias in prices when they expect it. With the passage of time, the true value of the security is eventually revealed, disagreement dissipates, and the bias in prices disappears. This sequence of events implies a pattern of equilibrium returns that mirrors optimistic mispricing—i.e., a runup prior to a period of significant news arrival followed by low returns thereafter. Alternatively, if traders are not prone to disagreement, this pattern does not arise even though the arrival of news causes return volatility to be high.

We test this explanation of AHXZ's and CSA's results in several ways using low analyst coverage as a proxy for whether traders are prone to disagree. This approach has the advantage of allowing all firms to be included in the sample, which has a significant impact on the sample size—46% of the firm-month observations in the Table 4 regressions below would be lost under the requirement of two or more analysts.

Our first test examines whether the relation between returns and idiosyncratic return volatility (IVOL) is different depending on whether stocks have low coverage or not. We use three different measures of IVOL, ex-post return horizons of one month to two years, and raw and risk adjusted returns. Consistent with the model, most of the results indicate that the low returns following high IVOL rankings are attributable to low coverage stocks only. The exception is returns in the month immediately following the ranking, which are affected by liquidity related reversals. As expected, those returns are negative regardless of coverage.[2]

Disagreement generates trading in our model as it does in many other models [e.g., Harris and Raviv (1993)], so shocks to turnover occur when information arrives about which traders disagree. In our second test, we examine the relation between returns and the

[2]Stock returns are positively correlated with idiosyncratic volatility in the ranking month, so the beginning price from which the next month's return is computed is more likely at the ask than the bid. The ensuing reversal of price concessions to liquidity providers [see Kaul and Nimalendran (1990)] then leads to a negative relation between returns and idiosyncratic volatility in the month immediately following the ranking. This contributes to the AHXZ result in the month following ranking. Some have argued that this *explains* the AHXZ puzzle [see Fu (2009), Huang, Liu, Rhee and Zhang (2010), and Han and Lesmond (2010)]. However, our findings that the AHXZ result persists over one and even two-year return horizons, and that it is mirrored in stocks ranked by turnover volatility, indicate there is more to the AHXZ puzzle than liquidity reversals.

volatility of turnover. We find that the low returns to high turnover volatility documented by CSA are also attributable to stocks with low coverage. This finding is very strong and uniform across returns horizons, suggesting that Miller's hypothesis explains this puzzle as well.

Our third test considers whether return dynamics are consistent with optimistic mispricing of high volatility stocks—a runup followed by returns that are negatively related to the runup. This pattern is indeed significant, and it too is driven by stocks with low coverage. This finding is robust across the IVOL and turnover volatility measures, and it is incremental to the commonly observed return reversals at intermediate horizons [e.g., DeBondt and Thaler (1985)].

Our fourth test examines returns around earnings announcements. The concreteness of earnings as a measure of operating performance should *resolve* disagreement and reduce the bias in prices. We find that average earnings announcement returns are indeed significantly negative for high IVOL low coverage stocks, and insignificant for low IVOL stocks *and* for high IVOL stocks with high coverage. Similar results hold when the volatility of turnover is used instead of IVOL. The significant earnings related returns are attributable to stocks with low coverage just as in the other tests, suggesting again that the low returns are attributable to the resolution of disagreement.

The model predicts that if disagreement is large enough to create a bias in prices, returns vary with the cost of short sales. This is because the bias must compensate pessimists for the costs of short selling in equilibrium. Our fifth test examines whether the return pattern is strongest for stocks whose short sale costs are highest. We use institutional holdings as a proxy for cross sectional differences in short sale costs. This follows D'Avolio (2002) who finds a strong negative relation between institutional holdings and the direct costs of short selling in proprietary data on short sale costs. Indeed, we find that the negative relation between returns and volatility (both IVOL and turnover volatility) among low coverage stocks is stronger if institutional holdings are low.

All five tests support the hypothesis that mispricing associated with disagreement explains the low returns to stocks that sustain significant shocks to prices or turnover. In fact, the strength and uniformity of the *turnover* results suggest that turnover volatility coupled with low coverage is actually a better indicator of disagreement-induced mispricing than is IVOL and low coverage. Since the model predicts that the same economic forces drive both relations, we ask whether the time series of returns to IVOL ranked stocks is explained by the returns to stocks ranked by turnover volatility.

We construct long-short portfolios based on each, and we find that the significant risk adjusted returns to the IVOL portfolio disappear after controlling for its exposure to the returns to the turnover volatility portfolio. This supports the model's prediction, and it also has an important practical implication. Investment managers who wish to exploit these effects would earn greater returns by conditioning their strategies on low coverage and *turnover* volatility rather than idiosyncratic return volatility. Its economic magnitude is one of the largest among the anomalies documented in the literature—about 80bp per month risk adjusted over the first post-ranking year excluding January, even using generous quintile rather than decile cutoffs and excluding penny stocks.

Our paper makes several contributions. We begin by addressing directly whether the AHXZ result is weak or non-existent (cf. footnote 2). We show that after accounting for both the January effect and microstructure biases, the AHXZ result is robust and highly significant, and that it persists over return horizons up to *two years.* In addition, we show that the AHXZ result is not attributable to illiquid or small stocks, or to stocks with lottery-like payoffs. We then examine the predictions of a stylized dynamic model of Miller's hypothesis, and we find that the general returns patterns that characterize both the AHXZ and CSA results are consistent with this explanation, as are the returns associated with earnings announcements. These findings are robust to the definition of IVOL and to the choice of return horizon, and they provide a unified explanation for two separate puzzles.

The next section of the paper presents the model. Section 2 describes the sampling procedure and the volatility measures. Section 3 examines the robustness of the AHXZ result. Section 4 presents tests of the model's predictions. Section 5 concludes.

1. Model

We model a firm's transition from a period of no information flow to a period in which significant information arrives, then back again as uncertainty is resolved. This approach is consistent with the findings of Sonmez-Seryal (2008), which we have also confirmed. She examines *ex-post* changes in IVOL rankings to gauge how much of the AHXZ result is driven by stocks that change versus persist in their IVOL quintile rankings. She shows that returns are very high when stocks subsequently rise in IVOL quintile ranking, and returns are low when stocks fall in ranking. The relation between returns and IVOL is also positive for stocks that persist in their quintile ranking. She concludes that the AHXZ result therefore must be driven by the subset of high IVOL stocks that *fall* from a high ranking—i.e., the *transitions* are what matter. Our model explains why they should matter when short sales are costly and traders are prone to disagree about the interpretation of news.

This perspective is very different from existing explanations of the AHXZ and CSA results that view volatility as a stock characteristic and argue that investors prefer high volatility stocks to low volatility stocks [see Han and Kumar (2008) and Bali, Cakici and Whitelaw (2011)], or that volatility measures the sensitivity to a priced factor [Chen and Petkova (2012)]. We consider these explanations also, and the evidence strongly favors an explanation based on transitions and Miller's hypothesis.[3]

1.1 Traders' Beliefs and Strategy Choices

We assume that $2N$ traders participate in a market for a single security whose per-capita supply is $X > 0$. Traders have time additive mean-variance preferences over profit, with common risk aversion parameter α. Traders are strategic, and they account for their impact on prices when formulating trading strategies. For simplicity, we model a single

[3]We focus on the negative relation between returns and measures of realized volatility as in AHXZ and CSA. However, Fu (2009) and Spiegel and Wang (2005) show there is a positive relation between returns and expected volatility as predicted from time-series models of volatility that are estimated in-sample. Both relations are important in the cross section [see Banerjee (2012)], suggesting that returns may also contain a risk premium for the permanent component of volatility that is distinct from the effect related to realized volatility examined here. However, only the realized volatility relation leads to a profitable trading strategy. Guo, Kassa and Ferguson (2011) report that the return premium disappears when the parameters of the time series models are estimated on a rolling basis without looking ahead.

information arrival. We also do not distinguish between the idiosyncratic and systematic components of information.[4]

Trading occurs at dates 1 and 2, and the security pays off $\tilde{v}$ at date 3. Traders are identical at date 1. They all believe $\tilde{v}$ will be drawn at date 3 from a distribution whose expectation is v_o. With probability $1-q$, no information arrives at date 2, traders continue to hold this belief, and at date 3, $\tilde{v}$ is in fact drawn from a distribution having expectation v_o and variance σ_v^2.

However, with probability q, information arrives at date 2 that generates disagreement among the traders about the security's expected payoff. N traders adopt the optimistic belief that $\tilde{v} \sim (\overline{v}_H, \sigma_v^2)$, and the other N traders adopt the pessimistic belief that $\tilde{v} \sim (\overline{v}_L, \sigma_v^2)$ where $\overline{v}_H > \overline{v}_L$. One group's interpretation of the information will turn out to be correct, meaning that $\tilde{v}$ will be drawn at date 3 from one of these distributions. We assume the objective probability that $\tilde{v}$ is drawn from either distribution is 1/2. This means that, conditional on an information arrival at date 2, the expectation of the objective distribution from which $\tilde{v}$ will be drawn at date 3 is $\overline{\overline{v}} \equiv \frac{1}{2}\overline{v}_H + \frac{1}{2}\overline{v}_L$. Nevertheless, traders in both groups behave as though their subjective beliefs are correct. Figure 1 illustrates the sequence of possible events.[5]

We assume that traders are aware ex-ante of the level of disagreement that will exist if information arrives—i.e., $d \equiv \overline{v}_H - \overline{v}_L$ is common knowledge. Traders are aware of how different the beliefs adopted by the other group will be from their own. This assumption prevents us from having to model inferences from prices that traders would otherwise draw about the beliefs of others, and the strategic response of each trader to knowing that others attempt to forecast his beliefs from prices.

[4]This distinction is not important empirically either. The AHXZ result is the same whether we rank stocks by total return volatility or idiosyncratic return volatility. This is because the Fama-French (1993) model explains little of the variation in individual stock returns that generates cross sectional differences in return volatility. Nevertheless, the predictions below could be derived for idiosyncratic return volatility from a model that distinguishes between systematic and idiosyncratic news and in which disagreement arises over the idiosyncratic component.

[5]An *even* split in the population between optimists and pessimists is convenient for exposition, but not necessary. What is important is that the split has the same proportions as the probability of $\tilde{v}$ being drawn from the "H" and "L" distributions. This ensures that consensus beliefs are correct, and that any bias in prices at date 2 is the result of strategic choices and not because traders are misinformed on average.

At date 1, traders' common beliefs are consistent with Bayes rule. In particular, traders' date-1 expectation is consistent with the three possible conditional expectations they will adopt at date 2, and the probabilities of each:

$$v_o = q\mathrm{E}_1\left[\frac{1}{2}\overline{v}_H + \frac{1}{2}\overline{v}_L\right] + (1-q)v_o \qquad \Rightarrow \qquad v_o = \frac{1}{2}\mathrm{E}_1\left[\overline{v}_H\right] + \frac{1}{2}\mathrm{E}_1\left[\overline{v}_L\right].$$

This ensures that traders' prior expectation reflects what they know at date 1 about how the future will unfold, and a bias in prices does not arise merely because traders are misinformed.

A crucial assumption is that short sales are costly. This is comprised of the direct fees paid to a broker, the difficulty in locating shares to borrow, the opportunity cost associated with constraints on selling shares posted as collateral, and the extra effort involved in monitoring a short versus a long position [see Lamont (2004 and 2012)]. For simplicity, we assume there is a constant marginal cost c_s per share per period to maintaining a short position. The profit that trader j earns from holding x_{tj} shares between dates t and $t+1$ is therefore

$$\tilde{\pi}_{tj} \equiv (\tilde{p}_{t+1} - p_t)x_{tj} + c_s I_{tj} x_{tj}, \qquad \text{where} \qquad I_{tj} = \begin{cases} 1 & \text{if } x_{tj} < 0 \\ 0 & \text{otherwise,} \end{cases}$$

p_t is the market price per share at dates $t = 1$ and $t = 2$, and $p_3 \equiv v$.

At date 2, trader j selects a demand schedule, $x_{2j}(p_2)$, that maximizes his utility conditional on his date-2 beliefs about the distribution of $\tilde{v}$ and the date-2 demand schedules of the other traders:

$$J_{2j} = \max_{x_{2j}(\cdot)} \left\{\mathrm{E}_{2j}\left[\tilde{\pi}_{2j}\right] - \alpha \mathrm{Var}_{2j}\left[\tilde{\pi}_{2j}\right]\right\}.$$

The solution conditional on no information arrival at date 2 is denoted by $\hat{x}^*_{2j}(\cdot)$, and the solution conditional on an information arrival is denoted by $x^*_{2j}(\cdot)$—throughout the discussion, a "hat" means conditional on no information arrival at date 2.

Likewise, at date 1, trader j selects a demand schedule $x_{1j}(p_1)$, that maximizes his utility conditional on his date-1 beliefs about prices, the distribution of $\tilde{v}$, and the date-1 *and* date-2 demand schedules of the other traders:

$$J_{1j} = \max_{x_{1j}(\cdot)} \left\{\mathrm{E}_{1j}\left[\tilde{\pi}_{1j}\right] - \alpha \mathrm{Var}_{1j}\left[\tilde{\pi}_{1j}\right] + \mathrm{E}_{1j}\left[\tilde{J}_{2j}\right]\right\}.$$

We solve for the optimal schedules $\{x^*_{1j}(\cdot), \hat{x}^*_{2j}(\cdot), x^*_{2j}(\cdot) : j = 1, \ldots, 2N\}$ by backward induction. If a solution exists in which all traders' beliefs about the strategies of others are correct, such a solution constitutes a subgame perfect Nash equilibrium. Expressions for the equilibrium prices $\{p^*_1, \hat{p}^*_2, p^*_2\}$ are obtained from the market clearing conditions that equate per-capita demand and per-capita supply.

1.2 Equilibrium Strategies and Prices

Whenever traders have identical beliefs, they all hold long positions equal to their share of per-capita supply, and short sale costs have no impact on holdings or prices. In order for the cost of short sales to matter, the difference between optimistic and pessimistic beliefs must be large enough that the cost actually deters short positions the pessimists would otherwise enter. This occurs in a parameter region in which pessimists hold zero shares (but would short if it were costless) and in a region in which pessimists hold short positions (that are smaller in magnitude than they would be if shorting were costless). To simplify the exposition, we ignore the former region and consider levels of disagreement that are sufficient to generate non-zero short sales.

We show in the Appendix that there exists a $\underline{d} > 0$ such that if $d > \underline{d}$, there is a subgame perfect Nash equilibrium in symmetric linear strategies that has the following three features. First, in each subgame, the optimal strategies of traders whose beliefs are the same are identical linear functions of their expectation of the price change over the next period. Second, conditional on an information arrival at date 2, optimists hold long positions and pessimists hold short positions. Third, this equilibrium is unique in the class of symmetric linear equilibria. We now describe the dynamics of prices in this equilibrium to flesh out the connections between return volatility, disagreement and returns.

If no information arrives at date 2, traders have identical beliefs about the date-3 payoff—i.e., that $\tilde{v} \sim (v_o, \sigma^2_v)$. Their optimal demand schedules are of the form $\hat{x}^*_{2j}(\hat{p}_2) = \hat{\beta}(v_o - \hat{p}_2)$ for all j, and the (endogenous) market clearing price is

$$\hat{p}^*_2 = v_o - \frac{X}{\hat{\beta}}.$$

The second term is a discount to compensate traders with a positive return for bearing risk, and the first term is traders' consensus belief about the security's expected payoff.[6] Since v_o is the expected value of the distribution from which $\tilde{v}$ will be drawn if information does not arrive, the risk adjusted price is an unbiased estimate of the security's payoff. At equilibrium, all traders hold their per-capita share of supply: $\hat{x}^*_{2j}(\hat{p}^*_2) = X$ for all j.

If information does arrive at date 2, optimists' strategies are $x^*_{2H}(p_2) = \beta(\overline{v}_H - p_2)$, and pessimists' strategies are $x^*_{2L}(p_2) = \beta(\overline{v}_L - p_2 + c_s)$. It turns out that the β coefficients are the same for optimists and pessimists and equal to $\hat{\beta}$ in the case when information does not arrive. The positive c_s term in the pessimists' demand schedule reduces the aggressiveness with which they short because shorting is costly. The market clearing price is

$$p^*_2 = \overline{\overline{v}} + \frac{c_s}{2} - \frac{X}{\beta}.$$

The first term is traders' consensus belief about the security's expected payoff conditional on the information arrival, and the third term is a discount due to risk. Since $\tilde{v}$ is equally likely to be drawn from a distribution with expectation $\overline{v}_H$ or $\overline{v}_L$, the expectation of its *objective* distribution is $\overline{\overline{v}}$, so the consensus component of the price is unbiased as an estimate of the security's payoff. However, the risk adjusted price is biased upward because pessimists hold back when shorting is costly. This bias is reflected in the middle term, $c_s/2$. It rewards pessimists, who are short, with an expected drop in price to compensate for bearing the cost. The bias is analogous to the upward bias Miller (1977) argues will arise when beliefs are divergent and short sales are costly.[7]

The next observation follows from the fact that traders are not myopic—a bias is incorporated into prices *before* the information arrival because traders anticipate divergent beliefs in the future. When traders anticipate that future prices might be high because pessimists hold back, they bet now that prices will rise. This shifts current demand and

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[6] An explicit expression for $\hat{\beta}$ is obtained by combining Equations (A.14) and (A.23) in the Appendix.

[7] Such a bias does not exist by necessity in a carefully structured equilibrium model. For example, prices are not biased in Diamond and Verrecchia's (1987) analysis of costly short sales with divergent beliefs. Their model is based on Glosten and Milgrom (1985) wherein prices are set by the unbiased beliefs of a market maker who does not bear costs if his inventory is short. In our model, prices are determined by market clearing (i.e., liquidity is provided by the trading crowd), so beliefs move prices through the orders that traders submit. Even though beliefs are unbiased on average across traders, market clearing prices are biased upward because pessimists trade less aggressively on their beliefs than optimists.

prices upward, incorporating a bias into the *current* price. The implication is that an upward bias due to costly short sales is imparted into prices even before the arrival of the shock to beliefs that generates disagreement.[8]

At date 1, traders have identical beliefs and their demand schedules are of the form $x^*_{1j}(p_1) = \gamma(\mathrm{E}_1[p_2] - p_1)$ for all j.[9] The market clearing price is

$$p^*_1 = v_o + q\frac{c_s}{2} - \frac{X}{\beta} - \frac{X}{\gamma}.$$

The first three terms constitute traders' date-1 consensus expectation of the date-2 price. The $q\frac{c_s}{2}$ term is a bias in the date-1 price associated with traders' *anticipation* that information will arrive with probability q and generate a bias of $\frac{c_s}{2}$ in *next* period's price. The more strongly traders anticipate an information arrival at date 2, the more fully the date-1 price incorporates the future bias. The last term is a discount that compensates traders with a positive expected return for bearing risk between dates 1 and 2. At date 1, traders all hold their share of per-capita supply at equilibrium: $x^*_{1j}(p^*_1) = X$ for all j.[10]

This price sequence clarifies the connections between disagreement, short sale costs and return volatility in generating AHXZ's IVOL puzzle. First, return volatility is not a measure of disagreement because prices do not bounce between traders' divergent beliefs. Prices *aggregate* beliefs, so return volatility measures shocks to *consensus* (i.e., $\overline{\overline{v}} - v_o$). Second, *disagreement* generates an upward bias in prices whose magnitude depends on the cost of short sales. Third, the bias appears *prior* to information arrivals to the extent that information arrivals are anticipated. The bias reverses as the disagreement dissipates when $\tilde{v}$ is drawn.

[8] The result that a future bias affects the current price is quite robust to the model's specification. It appears also in Harrison and Kreps (1978) where traders are risk neutral and perfectly competitive, traders disagree for sure, and shorting is ruled out entirely. In our model, traders are risk averse and strategic, they disagree with probability q, and shorting is merely costly.

[9] An explicit expression for γ is obtained from substituting Equation (A.26) into Equation (A.21.1a) in the Appendix.

[10] The discount $\frac{X}{\gamma}$ is compensation for the risk that information will arrive, which also depends positively on q. Such compensation should be small for idiosyncratic news. For example, if we think of α as representing traders' aversion to the incremental risk associated with idiosyncratic news for this security in a diversified portfolio, then α will be small and indeed X/γ vanishes (and $q\frac{c_s}{2}$ does not) as $\alpha \rightarrow 0$ (by Equation (A.26) in the Appendix). Alternatively, if traders hold sufficiently concentrated portfolios as envisioned in Merton (1987), then the risk premium could be large enough to compete with the bias in affecting the price and *raw* returns.

If we define as date zero a time at which traders believe $q = 0$ then, by an argument similar to that used to derive p_1^*, the date-zero price will have the form $p_o^* = v_o +$ risk premium. Conditional on an information arrival at date 2, the sequence of equilibrium risk adjusted price *changes* is

$$R_{o,1} = q\frac{c_s}{2} \tag{1}$$

$$R_{1,2} = \left\{\overline{\overline{v}} - v_o\right\} + (1-q)\frac{c_s}{2} \tag{2}$$

$$R_{2,3} = \left\{\tilde{v} - \overline{\overline{v}}\right\} - \frac{c_s}{2}. \tag{3}$$

These equations illustrate how disagreement affects returns around significant information arrivals. A news arrival, $\left\{\overline{\overline{v}} - v_o\right\}$, contributes significant variation to prices, which leads to a high return volatility ranking. The expected values of the terms in curly brackets are equal to zero because the consensus is unbiased. Nevertheless, expected returns are positive both before, and coincident with, the news arrival. The expected subsequent return is negative as disagreement dissipates and the bias disappears. We refer to this pattern as "optimistic mispricing."

Equations (1) - (3) relate to the case in which information arrivals that shock consensus *also* generate disagreement. However, if traders are not prone to disagree (i.e., $d = 0$), there is an equilibrium in which traders have identical beliefs in each subgame, all traders hold their share of per-capita supply, and there is no bias in prices in either period whether information arrives or not. Equations (1) - (3) are then replaced by similar equations but *without* the terms involving c_s. If traders are not prone to disagree, pessimists do not (anticipate taking positions) short when information arrives, and the equilibrium returns need not compensate them for the costs of shorting.

Nevertheless, when information arrives, prices change because consensus shifts and $\left\{\overline{\overline{v}} - v_o\right\}$ still contributes significant variation to returns. Such a shock will lead to a high return volatility ranking that is *not* accompanied by a prior price runup and a subsequent price drop. Thus, a pattern of optimistic mispricing will *not* be associated with high return volatility if traders are not prone to disagreement. These ideas are summarized as follows.

<u>Proposition 1</u>. *If traders are prone to disagree, then a price path consistent with optimistic mispricing will be associated with information arrivals that generate high return*

volatility. The magnitude of the mispricing is greater, the greater is the cost of short sales. If traders are not prone to disagree, the price path will not be consistent with optimistic mispricing whether return volatility is high or not.

Disagreement in our model drives trading volume as well. If information does not arrive at date 2 (or if it does, but there is no disagreement), then all traders continue to hold their per-capita share of supply and there is no trading. However, if information arrives that generates disagreement, trading occurs between optimists and pessimists. A large value for a statistic that measures shocks to trading volume therefore indicates both the arrival of news *and* the presence of disagreement. Also conditioning on a proxy for whether traders are prone to disagree should produce an even clearer signal of the presence of disagreement by filtering out liquidity trading (outside our model but present in real markets) that is unrelated to differences in beliefs. If our model is correct, a pattern of optimistic mispricing should be at least as pronounced if a measure of turnover volatility is used in place of IVOL as a ranking variable in empirical tests.

These observations suggest several empirically refutable hypotheses. First, the negative returns following a high volatility ranking are reversals of biases that build prior to the ranking, so there should be an association between the size of the prior runup and the subsequent price drop. Second, and most important, information arrival *and* disagreement are both necessary to generate the AHXZ result in our model. Information arrivals that are not accompanied by disagreement will not generate a bias. Consequently, only the subset of high volatility stocks for which investors are prone to disagree should exhibit a pattern consistent with optimistic mispricing. Third, the pattern of mispricing should be at least as strong when a measure of shocks to turnover is used in place of IVOL, which measures shocks to prices.

In our empirical tests, we use low analyst coverage to identify stocks where traders are likely to disagree if significant news arrives.[11] When coverage is low, investors have

[11] This contrasts with measuring the presence of disagreement among analysts and is better suited to our particular setting for the reasons described in the introduction. See Chen, Hong and Stein (2002), Diether, Malloy and Scherbina (2002), Jones and Lamont (2002), Lamont (2004), Nagel (2005), Boehm, Danielsen and Sorescu (2006), Sadka and Scherbina (2007), and Yu (2011) for studies that use analyst disagreement effectively in other settings.

less a-priori guidance and perspectives available to help them interpret the value relevance of news. In contrast, if many analysts follow a firm, investors have a common and large set of professional opinions on which to anchor their beliefs and to help with interpreting the meaning of significant information when it arrives. Our empirical analysis is therefore a joint test of the model and low coverage as a proxy for whether traders are prone to disagree.

The model predicts greater mispricing when the costs of short sales are large. We use institutional holdings as a proxy for short-sale costs following D'Avolio (2002). He finds in regressions of the direct costs of short selling on various firm characteristics, that the most significant explanatory variable is institutional holdings—stocks with high short sale costs have low levels of institutional holdings.

2. Data and Methods

The data consist of monthly prices, returns and other characteristics of the NYSE, AMEX and Nasdaq companies covered by CRSP from 1963 through 2006. Price, return and volume data are obtained from CRSP. Financial information is obtained from Compustat. Data on analyst coverage are obtained from the Summary History data set compiled by the Institutional Brokerage Estimation System (I/B/E/S). Stocks are classified each month as having low coverage ($LCOV = 1$) if three or fewer analysts are listed as providing one-year earnings forecasts.[12] We refer to stocks outside the low coverage group as high coverage stocks. Although I/B/E/S coverage begins in 1976, we follow Diether, Malloy and Scherbina (2002) in limiting our sample period to begin in January 1983. Until 1983, the I/B/E/S coverage is sparse and unreliable. Data on institutional holdings are obtained from Thomson Financial, compiled from the Securities and Exchange Commission's Form 13-F, which must be filed by all U.S. institutions with over $100 million in assets.

Following AHXZ, we measure the idiosyncratic volatility of each stock as the standard deviation of residuals from a time series regression of stock returns on the Fama-French

[12] Partitioning between three and four divides the overall sample nearly in half. Low coverage stocks represent 45% of the sample of NYSE/AMEX stocks and 67% of the sample of Nasdaq stocks. The average number of analysts covering stocks in our sample is five.

(1993) factors:

$$R_{it} = \alpha_i + \beta_{i,MKT} MKT_t + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + \epsilon_{it}. \tag{4}$$

We construct three idiosyncratic volatility measures. The first is the original AHXZ idiosyncratic volatility measure ($IVOL20D$). It is estimated from regressions using one prior month of daily returns and factor data, including firm months with at least 20 observations. We also construct two other measures to identify firms by their volatility over longer periods of time. $IVOL200D$ is estimated using the prior 12 months of daily returns and factor data, requiring at least 200 non-missing observations in the past year. The third measure $IVOL60M$ uses the prior 60 months of monthly returns and factor data, requiring at least 24 months of non-missing observations [see also Fama and MacBeth (1973), Lehmann (1990), Malkiel and Xu (2006) and Spiegel and Wang (2005)]. Considering a variety of volatility measures provides a clearer picture of the robustness of the results than would be possible using a single measure alone.

Following CSA, we measure the volatility of trading volume as the standard deviation of share turnover ($TVOL$). Each month, turnover is calculated as trading volume divided by shares outstanding as reported by CRSP. $TVOL$ is calculated over 36 months ending in the *second-to-last* month prior to portfolio formation.[13] Nasdaq volume includes interdealer trades and NYSE/AMEX volume does not, so we divide volume by two in computing $TVOL$ for Nasdaq stocks. Trading volume data are not available prior to November 1982 for Nasdaq stocks.

We follow the Fama-MacBeth (1973) style regression approach taken in George and Hwang (2004) and Grinblatt and Moskowitz (2004) to measure and compare the returns to portfolios formed by different investment strategies. This approach has the advantage of using all the stocks in the sample. The regression coefficient estimates isolate the returns to portfolios exhibiting particular characteristics by hedging (zeroing out) the impact of other variables that are included as controls [see Fama (1976)].

We examine returns over future horizons of different lengths. This involves computing returns in a given month to portfolios that were formed in each of several past months.

[13]Note that since CSA skip a month between ranking and computing returns, the results reported for turnover volatility are not subject to the short term liquidity reversals discussed in footnote 2.

Consider the strategy of forming portfolios every month and holding the portfolios for the next T months. In a given month t, the return to pursuing this strategy is the equal-weighted average of the returns to T portfolios, each formed in one of the T past months $t-j$ (for $j=1$ to $j=T$). The contribution of the portfolio formed in month $t-j$ to the strategy's month-t return can be identified by the coefficient estimates of a cross sectional regression of month-t returns on portfolio selection criteria in month $t-j$.

The main cross sectional regression specification we work with is as follows:

$$\begin{aligned} R_{it} = b_{0jt} &+ b_{1jt} LVOL_{i,t-j} + b_{2jt} HVOL_{i,t-j} + b_{3jt} LCOV_{i,t-j} * LVOL_{i,t-j} \\ &+ b_{4jt} LCOV_{i,t-j} * HVOL_{i,t-j} + b_{5jt} BM_{i,t-1} + b_{6jt} Size_{i,t-1} + b_{7jt} R_{i,t-1} \\ &+ b_{8jt} 52WKHW_{i,t-j} + b_{9jt} 52WKHL_{i,t-j} + e_{ijt}, \end{aligned} \tag{5}$$

where R_{it} is the return to stock i in month t, $HVOL_{i,t-j}$ ($LVOL_{i,t-j}$) equals one if stock i is among the top (bottom) 20% of stocks in month $t-j$ when ranked by idiosyncratic volatility, and $LCOV_{i,t-j}$ takes the value of one if stock i has no more than three analysts covering it in month $t-j$. Equity market capitalization and book-to-market in month $t-1$ are used as control variables to capture the size and book-to-market effects in returns. We include the prior month's return as a control to capture predictability due to bid-ask bounce, though as we observe later, this does not fully capture short term reversals. These variables are included as deviations from cross sectional means to facilitate the interpretation of the intercept. We also include winner and loser dummies based on the 52-week high momentum measure in George and Hwang (2004), which they show dominates momentum measures based on past returns in predicting future returns.

In light of the control variables, the estimate of b_{0jt} is the return in month t to a "neutral" portfolio, formed in month $t-j$, that has neither low nor high idiosyncratic volatility (i.e. the portfolio that includes stocks in the middle three idiosyncratic volatility quintiles) and that hedges (zeros out) the effects of deviations from average prior month return, average size and average book-to-market, and also the effects of momentum in predicting returns. The sum of the estimates $b_{0jt} + b_{1jt}$ is the month-t return to the low volatility portfolio of high coverage stocks that was formed in month $t-j$ and that has hedged out all other effects. Similarly, the sum $b_{0jt} + b_{1jt} + b_{3jt}$ is the month-t return to a portfolio of low volatility stocks with low coverage that was formed in month $t - j$ that has hedged out all other effects. The individual coefficients are, therefore, excess returns that isolate specific characteristics. For example, b_{3jt} is the excess return in month t associated specifically with low coverage in a low volatility portfolio formed in month $t - j$. The remaining coefficients have similar interpretations.

For a given month t, the coefficients in Equation (5) are estimated in T separate cross sectional regressions—one regression for each $j = 1, ..., T$. The portfolio returns associated with various strategies and holding periods are calculated as averages of the appropriate coefficient estimates. For example, consider the strategy of investing in stocks with low volatility and low coverage. The excess return in month-t to such a strategy with a K-month holding period beginning p months after portfolio formation is $S_{3t} = \frac{1}{K} \sum_{j=p}^{p+K} b_{3jt}$. All the coefficients that comprise S_{3t} are estimated using the *same* cross section of month-t returns as the dependent variable, so the sums S_{3t} and S_{3s} are *non-overlapping* returns for $t \neq s$.

The numbers reported in the tables are time series means of these non-overlapping returns (e.g., $\overline{S}_1$ and $\overline{S}_3$), and the corresponding t-statistics are computed from the temporal distribution of these returns. We estimate regressions out to $T = 24$ and report results for horizons $\{p = 0, K = 1\}$, $\{p = 1, K = 11\}$ and $\{p = 12, K = 12\}$, and sometimes others.

2.1 Summary Statistics

Table 1 reports summary statistics for the variables used in our tests. The numbers reported are time series averages of the cross sectional mean, median, maximum, and minimum of each variable, and the correlations among the variables. $RET(-1, -12)$ is the past 12-month return, and $IVOL20D$, $IVOL200D$ and $IVOL60M$ are the idiosyncratic volatility measures defined earlier. The low coverage dummy $LCOV$ is a key variable in our tests. It has a mean of 0.55—on average, 55% of the sample stocks have three or less analysts covering them. Not surprisingly, the correlation between $LCOV$ and market capitalization is negative, and the correlations between $LCOV$ and measures of idiosyncratic volatility are moderate and positive—low coverage stocks tend to be smaller and have higher idiosyncratic volatility. The magnitude of the correlations is between 0.18

and 0.25. There is little correlation between $LCOV$ and $TVOL$ at -0.05—low coverage stocks tend only slightly to have low volatilities of share turnover.

3. Robustness of the Return-Volatility Relation

In this section, we confirm the findings of others that the AHXZ result is sensitive to sampling choices and microstructure biases. However, we show that after skipping the first month and accounting for the January effect, the AHXZ result is significant in returns up to two years after the ranking. We also consider whether the result is driven by small firms or illiquid stocks, and we find it is not (even though small and illiquid stocks do have high idiosyncratic volatility). The AHXZ result is also not explained by measures of whether stocks have lottery-like payoffs. Several high daily returns in one month do predict low returns in the following month, but not in later months. This suggests that such measures identify stocks that experience buying pressure, and the low return in the subsequent month is a reversal of the price concession to liquidity providers.

3.1 Idiosyncratic Return Volatility

Previous studies find the AHXZ result is nonexistent when portfolios are equally weighted [e.g., Bali and Cakici (2008) and Huang et al. (2010)]. We therefore begin with equally weighted portfolios by estimating Equation (5) with $HVOL$ and $LVOL$ dummies as the only independent variables:

$$R_{it} = b_{0jt} + b_{1jt}LVOL_{i,t-j} + b_{2jt}HVOL_{i,t-j} + e_{ijt}. \tag{6}$$

The results are reported in the top panel of Table 2 for $IVOL20D$. Columns 1 and 2 report average portfolio returns one month after portfolio formation $\{p = 0, K = 1\}$ with and without January, respectively. Columns 3 and 4 report the average monthly portfolio returns during the second month $\{p = 1, K = 1\}$, and columns 5 and 6 the third month $\{p = 2, K = 1\}$. Columns 7 and 8 report the first year after portfolio formation, excepting the first month $\{p = 1, K = 11\}$.

As discussed earlier, the numbers reported as the coefficients for $LVOL$ and $HVOL$ are time series means (and t statistics in parentheses) of non-overlapping returns such as $S_{1t} = \frac{1}{K}\sum_{j=p+1}^{p+K} b_{1jt}$ and $S_{2t} = \frac{1}{K}\sum_{j=p+1}^{p+K} b_{2jt}$. These are excess returns to equally weighted low and high volatility portfolios relative to a benchmark equally weighted portfolio of stocks in the middle three volatility quintiles.[14] In columns 2, 4, 6 and 8, January returns are excluded from the calculations.

We confirm the Bali and Cakici (2008) finding in Column 1. High idiosyncratic volatility stocks do not have low returns in the month following portfolio formation when equally weighted portfolios that include January returns are considered. Low IVOL stocks have significantly *lower* returns than the middle three quintile stocks, and the returns to high IVOL stocks are not significantly different from the returns to the middle quintile stocks.

The results are *opposite* when January returns are excluded, however. Column 2 shows that the equally weighted portfolio of top IVOL quintile stocks earns negative excess returns that are quite significant. The excess return is -0.87% ($t = -5.08$) in the first month after portfolio formation. This indicates why the original AHXZ result fails for equally weighted portfolios. High idiosyncratic volatility stocks are prime candidates for tax-loss selling [see Roll (1983), D'Mello, Ferris and Hwang (2003) and Grinblatt and Moskowitz (2004)]. They tend to have large positive January returns, which conceal the AHXZ result. This also explains why the AHXZ result is stronger in value weighted portfolios. Tax loss selling is more prevalent among small firms, which tend also to have high IVOL. Weighting by value minimizes the impact of the positive January returns to small firms on the returns to the high IVOL portfolio.

We examine separately the returns in the month immediately following the ranking and returns up to two years later. This follows the approach of Jegadeesh and Titman (1993) in studying the profits to momentum strategies. This is important because high IVOL rankings are contemporaneously correlated with high returns, so an ask-to-midquote reversal occurs in the first month after ranking that accentuates the AHXZ result [see Huang et.al. (2010) and Han and Lesmond (2011)]. Skipping the first month to avoid the liquidity reversal, we find that January returns conceal the AHXZ result at longer horizons as well.

[14]This approach has the advantage of indicating whether significant returns are attributable to either or both of the extreme rankings, which is not apparent from an analysis of returns to long-short portfolios.

When January returns are included (columns 3, 5 and 7), there is no significant return difference between either high or low IVOL stocks and stocks in the middle three quintiles. However, the results in columns 4, 6 and 8 show that the AHXZ result is robust and persistent in non-January months following portfolio formation. For example, the $\{p = 1, K = 11\}$ horizon in columns 7 and 8 shows that excluding January changes an insignificant premium to high volatility of 0.26% ($t = 1.35$) per month into a significant discount of -0.35% ($t = -2.18$) per month.[15]

These results are based on samples similar to those in AHXZ, Bali and Cakici (2008) and Huang et al. (2010), which include "penny stocks." The relative illiquidity of these stocks introduces noise and possibly bias into volatility rankings and measured returns even at longer horizons [see Amihud (2002)]. In Table 3, we repeat the analysis after excluding stocks with share prices smaller than $5 at the end of the month of portfolio formation. This has a noticeable effect. The results are even stronger and consistent with AHXZ when penny stocks are excluded.

The excess returns to the equally weighted portfolio of high volatility stocks are significant and negative in the first month and the first year after portfolio formation even when January is included. This is because the January effect is especially strong for penny stocks—their relative illiquidity means tax-loss selling pressure has a bigger impact on their prices. The exclusion of penny stocks leads to a uniformly significant pattern of negative returns to high IVOL stocks, and uniformly insignificant returns to low IVOL stocks (relative to stocks in the middle three quintiles). The average excess returns to high volatility portfolios are -1.24% ($t = -8.00$) in the first month and -0.40% per month ($t = -2.65$) in the next eleven months after portfolio formation even with January included.

When January returns are excluded, the excess returns become even more negative. The excess returns in the first month and first year are -1.39% ($t = -8.62$) and -0.70% ($t = -4.83$) per month, respectively.[16] Here again, the impact of short term liquidity

[15] Similar results hold under both the medium and long term idiosyncratic volatility measures, though they are not tabulated to save space.

[16] In all cases, both here and in Table 4, the coefficients on the high and low volatility dummies are also significantly different from each other. The untabulated results for medium and long term IVOL measures are similar to those reported, although somewhat smaller in magnitude. For example, the excess return to

reversals is apparent in accentuating the low return in the first month (-1.39% versus -0.70% per month thereafter), but liquidity reversals clearly do not explain away the AHXZ result.

3.2 Other Relations

The influence of January returns and penny stocks creates the appearance that the AHXZ result is non-existent. Their influence might also distort inferences concerning the *relative* importance of IVOL versus other variables that have been used to explain the AHXZ result.

3.2.a Lottery-Like Payoffs

A prominent example is the "MAX" variable of Bali, Cakici and Whitelaw (2011), which is designed to capture the degree to which investors view a stock as having lottery-like payoffs. It is defined as the average of the five highest daily returns during the prior month. Bali, et.al. document negative one-month returns for high MAX stocks and argue that this drives the AHXZ result because, after controlling for MAX, the return discount to high IVOL stocks becomes a premium.

Column 1 of the bottom panel in Table 2 confirms their finding. High MAX (low MAX) stocks earn a large and significant return discount (premium) in the first post-ranking month. After controlling for MAX, the relation between returns and IVOL is positive and significant. The relation is positive when January is excluded from the first month returns, though not significant.

However, in months two and three, and in the eleven months after the first month, the results are different. When January is included, both the high and low MAX dummies are insignificant, as are the high and low IVOL dummies. When January is excluded, the high IVOL dummies are *all* negative and significant despite having controlled for MAX. The coefficients of the MAX dummies are inconsistent in sign and significance across horizons. High MAX is significant and negative at month two, insignificant at month three and insignificant in the eleven months after month one. Low MAX is insignificant at month

the high $IVOL200D$ portfolios one month and one year after portfolio formation are -0.57% (t=−2.89) and -0.29% (t=−1.55) per month, respectively. The corresponding figures when excluding January are -0.81% (t=−3.99) and -0.67% (t=−3.70) per month.

two, significant and *negative* at month three and insignificant in the eleven months after month one.

The bottom panel of Table 3 reports the same analysis after excluding penny stocks. As in table 2, the first month returns are low for high MAX stocks whether January is included or not. However, in month two, neither of the MAX dummies is significant. In month three, and the eleven months following month one, the high MAX dummy is not significant while the low MAX dummy is significant and *negative*.

In contrast, the results for the IVOL dummies are quite consistent across horizons. Regardless of whether January is included or not, the coefficients of the high IVOL dummy are uniformly negative and strongly significant. In the eleven months following month one, with January included, the coefficient of the low IVOL dummy is insignificant and that of the high IVOL dummy is -0.41 ($t = -5.00$). Excluding January, the estimates are 0.18 ($t = 2.12$) for the low IVOL dummy and -0.63 ($t = -8.93$) for the high IVOL dummy. Interestingly, this evidence of a significant negative relation between returns and IVOL is *stronger* after having controlled for MAX than the evidence in the top panel of Table 3 where MAX is not included.

The relation between returns and IVOL is the more robust of the two relations. It is more persistent, and it survives after controlling for biases in returns due to tax-loss selling and the influence of penny stocks. If the MAX variable really does capture investors' willingness to pay premium prices for lottery-like stocks, investors' perception of which stocks possess this attribute is very fleeting—the price premium dissipates by the end of month one. The speed with which this disappears suggests that the MAX effect is a liquidity reversal. This interpretation would also explain the stronger IVOL results after controlling for MAX. If MAX accounts for liquidity effects that are not eliminated by skipping a month, then adding MAX as a control improves the specification so the true (negative) relation between returns and IVOL is estimated with greater precision.

3.2.b Risk Adjusted Returns

Table 4 reports the *risk adjusted* returns corresponding to all three measures of IVOL and over return horizons out two years after portfolio formation. Penny stocks are excluded from this and all later tables. The numbers reported here for $LVOL$ and $HVOL$ are intercepts (and t-statistics) from time series regressions in which the non-overlapping returns such as S_{1t}, S_{2t}, etc., are regressed on contemporaneous Fama-French (1993) factors.

Risk adjusting strengthens the AHXZ result in both high and low volatility portfolios. In sixteen of eighteen cases, the excess returns to high idiosyncratic volatility portfolios are significantly negative both with and without January. Low volatility portfolio returns are insignificant in ten cases and significantly positive in eight cases. The magnitude and significance of the relation for both high and low idiosyncratic volatility groups is again stronger when January is excluded. Outside January, the high volatility portfolios have risk adjusted excess returns that are *all* significantly negative, irrespective of the holding period and the volatility measure. The excess returns are strongest for $IVOL20D$. The weakest returns correspond to $IVOL60M$, which are still strong. The first month, first year and second year risk adjusted excess returns to the high $IVOL60M$ portfolios are -0.33% ($t = -3.07$), -0.50% ($t = -5.12$) and -0.47% ($t = -4.76$) per month, respectively.

3.2.c Size and Illiquidity

We examine the degree to which the AHXZ result is attributable to small versus large firms (the results are not tabulated to save space).[17] Regressions similar to those in Table 4 are estimated with the addition of a dummy variable, $SMALL$, defined as one for stocks whose market capitalization is below the cross sectional monthly median and zero otherwise. This dummy is included by itself, and also interacted with the $HVOL$ and $LVOL$ dummies. Two findings are noteworthy. First, the coefficients of the $HVOL$ dummy are significantly negative in all cases except two (they are significantly negative in *all* cases in risk adjusted returns). This indicates that the AHXZ result is strong among large firms. Second, the AHXZ result is actually weaker among small than large firms. There are many cases in which the *SMALL*HVOL* coefficient is significant, and in all those cases it is *positive*—i.e., the relation between returns and high idiosyncratic volatility is less negative among small firms than among large firms. The AHXZ result is therefore not attributable to small firms. Although high idiosyncratic volatility stocks tend to be

[17] Included as Table 4A in the Appendix.

small firms, those responsible for the negative *cross sectional relation* between returns and idiosyncratic volatility are not small.

We also examine whether the AHXZ result is driven by stocks that are relatively illiquid. Similar to $SMALL$, we define a dummy $ILLIQ$ to be unity if the Amihud (2002) liquidity measure is in the upper half of the cross section in a given month, and zero otherwise. The results, which are omitted to save space, are very similar to those reported for $SMALL$.[18] Controlling for $ILLIQ$ does not eliminate the significant negative coefficient on $HVOL$, and when the interaction between $HVOL$ and $ILLIQ$ is significant, it *weakens* the AHXZ result. In other words, the AHXZ result is stronger among liquid stocks than among illiquid stocks.

Summarizing, our results indicate that the influence of January returns is responsible for the seeming lack of robustness of the AHXZ result in equally weighted portfolios, particularly when the sample includes penny stocks. Once we exclude January returns and penny stocks, equally weighted portfolios of high idiosyncratic volatility stocks have low raw and risk adjusted returns in the first month, the first year, and even the second year after portfolio formation. Low idiosyncratic volatility portfolios have either insignificant or high returns. These findings hold for the short term idiosyncratic volatility measure used in AHXZ and also for medium and long term idiosyncratic volatility measures, and it is not attributable to small firms, illiquid stocks or stocks that have lottery-like payoffs.

4. Tests of the Model's Predictions

The results so far confirm the existence of a robust negative relation between returns and idiosyncratic return volatility. In this section we confirm that a stronger relation exists for the volatility of turnover and examine whether these and other relations conform to the predictions of the model.

4.1 Volatility of Share Turnover

Table 5 reports an analysis in which the $HVOL$ and $LVOL$ dummies are defined with respect to the volatility of share turnover (highest and lowest quintiles of $TVOL$)

[18] Included as Table 4B fin the Appendix.

rather than idiosyncratic return volatility. The results for raw returns in the top panel of the table confirm Chordia et. al.'s (2001) finding that there is a strong negative relation between turnover volatility and subsequent returns. The high turnover volatility portfolio has significant excess returns ranging from -0.63% to -0.44% per month. Only the year-two return with January included is insignificant at -0.23% per month. All but one of the excess returns to the low turnover volatility portfolios are insignificant in raw returns.

After adjusting for risk in the bottom panel, high (low) turnover volatility portfolios have uniformly significantly negative (positive) excess returns for all holding periods with and without January. For example, a zero investment strategy of taking a long position in the low volatility portfolio and a short position in the high volatility portfolio nets 0.70% (0.25% + 0.45%) per month in the second month (see footnote 12) after portfolio formation, and 0.69% (0.28% + 0.41%) per month in the following eleven months. Excluding January, the profit is 0.83% in the second month and 0.81% per month in the following eleven months. Similar monthly returns persist for holding periods extending out two years.

If exploitable, this is one of the most profitable investment strategies documented in the literature. Note that penny stocks have already been excluded, and we use 20% cutoffs rather than more extreme 10% cutoffs for ranking by $TVOL$. Because a month is skipped between ranking and computing returns, if there is a short term reversal in ranking by $TVOL$, it is not included here. These results are consistent with the model's prediction of a strong negative relation between returns and shocks to trading volume.

4.2 Analyst Coverage and the Return-Volatility Relations

Our next test examines whether the negative relations between returns and IVOL and returns and $TVOL$ are stronger among low coverage stocks than high coverage stocks. Indeed they are, and the low returns to high volatility low coverage stocks persist for years. Outside the low coverage subsample, the relation between returns and idiosyncratic return volatility (and turnover volatility) is mostly insignificant. Risk adjusted returns are reported in Table 6. The results for raw returns are similar and not tabulated to save space. These regressions include the control variables defined above in Equation (5), but the coefficient estimates for the control variables have been omitted from the tables to save space.

The first three panels report results for the three measures of idiosyncratic return volatility. In columns 3 - 6, which skip the first month, all the significant negative relations between returns and IVOL are attributable to low coverage stocks. When January is excluded, all the coefficients of interactions between low coverage and high IVOL are significantly negative, and the interactions between low coverage and low IVOL are significantly positive. When January is included, the results are weaker, but still significant in many cases.

In contrast, excess returns are either insignificant or, in two cases, significantly positive, for stocks with high IVOL and *high* analyst coverage in columns 3 - 6. For example, the results for the second year after portfolio formation using $IVOL200D$ show that outside January, high IVOL stocks with high coverage earn a positive excess return of 0.65% ($t = 2.19$) per month; but high IVOL stocks with low coverage earn a significant 1.12% less ($t = -3.88$), or -0.47% per month. The strong and significant negative relation between IVOL and future returns after the first month is therefore attributable to stocks that have low analyst coverage. Among high coverage stocks, there is an *insignificant or positive* relation between returns and idiosyncratic return volatility. The results in columns 3 - 6 are consistent with the model and the (joint) hypothesis that low coverage measures disagreement.

The results in columns 1 and 2 for returns in the month immediately following the ranking by IVOL are different. The return-volatility relation is negative for *both* high and low coverage stocks, and it is especially strong using the short horizon measure $IVOL20D$. For example, the difference between the coefficients of $HVOL$ and $LVOL$ with January included is -1.20%, which relates to high coverage stocks. For low coverage stocks, the difference is between $(HVOL + LCOV * HVOL)$ and $(LVOL + LCOV * LVOL)$, which is (-1.09 + 0.37) minus (0.11 - 0.04) or -0.79%. Comparing this to the results from columns 3 - 8 that skip the first month suggests there is a source of bias associated with a high IVOL ranking that is both unrelated to disagreement and that dissipates quickly. This is the liquidity reversal discussed in section 3.1. These reversals overstate the strength of the AHXZ result at the one-month horizon regardless of analyst coverage.

The results in the last panel of Table 6 are based on the volatility of share turnover. There is a significant negative coefficient on the interaction between low coverage and high volatility for all holding periods, and the low coverage low volatility coefficients are positive and, in all but one case, significant. As with IVOL, the negative relation between returns and $TVOL$ is attributable to low coverage stocks only. These results are quite strong and they too are consistent with the model's prediction.

4.2.a Robustness Tests

We conduct two robustness tests of the results in Table 6. The first considers whether $LCOV$ is merely a proxy for illiquidity by also including interactions between $ILLIQ$ and $HVOL$ and $LVOL$ in the regressions. The tables are omitted to save space.[19] Including these interactions does not affect the inference that the AHXZ and CSA results are attributable to low coverage. At both the one and two year horizons, the dummies for high and low volatility are insignificant, whereas the interactions between low coverage and the high and low volatility dummies remain significant and have signs consistent with Table 6. Moreover, the interactions between illiquidity and the high and low volatility dummies are *opposite* in sign to the AHXZ and CSA results. Reminiscent of the discussion in section 3.2.c, any additional explanatory power associated with liquidity per-se indicates that the AHXZ and CSA results are stronger for *liquid* than for illiquid stocks.[20]

The second robustness test includes the market average volatility factor (or AV) of Chen and Petkova (2012) alongside the Fama-French (1993) factors in computing risk adjusted returns.[21] Chen and Petkova find that the AV factor has a significant negative price—portfolios with high exposure to the factor have low returns. They argue that the AHXZ puzzle arises because high (low) IVOL stocks have high (low) exposure to this factor, and therefore low (high) returns in the cross section. Since AV is also a traded factor

[19]Included as Table 6A in the Appendix.

[20]The foregoing description relates to the one- and two-year horizons beyond the first month. As explained above, the first month is strongly affected by an ask-to-midquote reversal, which leads us to expect a high ranking by the Amihud (2002) measure would uniformly accentuate the negative return-volatility relation in the near month. Indeed it does for the $TVOL$ ranking. However, it modestly weakens the first month return following the IVOL ranking. This could be because we already screened out penny stocks, or because Amihud's measure is not uniformly a good proxy for month-end effective spreads.

[21]We are grateful to Zhanhui Chen for providing us with their factor data.

(i.e., returns to a mimicking portfolio), the intercept of a time series regression of portfolio returns on the Fama-French and AV factors is the average risk adjusted return. Results for $IVOL20D$ and $TVOL$ only are reported in Table 7 to save space. The estimates and inferences are nearly identical to those in Table 6. The significant excess returns to low coverage high volatility stocks are not explained by exposures of those stocks to the AV factor.

4.3 Analyst Coverage and the Mispricing Reflected in Past Returns

The model predicts a pattern of optimistic mispricing for high volatility stocks with low coverage. Returns prior to and including the ranking month should be greater than was justified by fundamentals, and the post-ranking correction of mispricing should be related to that increase in prices. This implies that the negative excess returns documented for high IVOL and high $TVOL$ stocks should be larger in magnitude if returns leading up to the ranking are high. This should hold after controlling for the general medium term continuations and long term reversals that exist in returns [see Jegadeesh and Titman (1993) and DeBondt and Thaler (1985)].

Table 8 reports regressions of returns on high and low volatility dummies, indicator variables for whether the prior three year return ranks in the top or bottom third of the cross section, and interactions between the high volatility dummy and the high and low prior three year return indicators. Each panel reports results for a different measure of idiosyncratic return volatility or the volatility of turnover. Within each panel, results are reported for the entire sample on the left, and separately for the high coverage subsample ($LCOV = 0$) on the right.

The results are quite consistent across volatility measures. Consider the full sample results on the left side of the table. The coefficients of the past return variables do pick up significant general continuation and reversal patterns in returns. Past loser stocks continue losing over the next year, and past winners reverse in year two following the ranking year irrespective of volatility rankings. The optimistic mispricing prediction is supported as well. Across volatility measures and return horizons, all but one of the coefficients on the interaction between $HVOL$ and the high past return dummy are highly significant and negative. Low returns to high IVOL stocks are strongest among stocks whose past returns are in the highest third of the cross section.

The magnitudes of the coefficients are striking. In all cases, the *incremental* reversal associated with high volatility is even larger than the general reversal associated with a high past return. For example, in Panel A using the short-run volatility measure including January, the general reversal in year two is -0.17% per month for past winners, but -0.46% per month (-0.17% plus -0.29%) for high volatility winners. This follows a first year excess return of an insignificant 0.00% for past winners and a significant -0.26% per month (0.00% plus -0.26%) for high volatility past winners.

The high coverage subsample results on the right side of the table are different. The relation is nearly non-existent even for past returns in the highest third of the cross section. All but one of the interactions between high past returns and high volatility are insignificant, and none are significant when January is excluded. Taken together, these results show there is a significant "extra" return reversal among high volatility stocks with the biggest price runups, and it is attributable to the stocks with low analyst coverage.[22]

4.4 Analyst Coverage and Earnings Announcement Returns

In this subsection, we examine whether earnings announcement returns corroborate the interpretation of the regression tests in the prior tables as consistent with the model. The regressions show that low returns are earned by high volatility stocks with low coverage. If this reflects the correction of an upward bias in prices associated with disagreement, then we expect earnings announcement returns for these stocks to be significantly *negative* on average, because realized earnings should *resolve* some of the disagreement. They should also be more negative than the announcement returns for stocks that appear not to be mispriced in the regression tests—i.e., all low volatility stocks, and high volatility stocks outside the low coverage group. This is indeed the pattern that appears below, both

[22]The results in Table 8 are similar to those of Stambaugh, Yu and Yuan (2012). They find that high IVOL accentuates both low and high ex-post returns to stocks identified as over and underpriced by a composite anomalies index, and that *over*pricing dominates the high-IVOL group. That is, their index of over and underpricing is used in place of past returns here, and the results are similar. They do not examine analyst coverage, disagreement, or the connection between the AHXZ and CSA puzzles as we do here, however.

when volatility is measured using idiosyncratic returns and turnover.

We follow the approach of Chopra, Lakonishok and Ritter (1992), La Porta (1996) and La Porta, Lakonishok, Shleifer and Vishny (1997). Each June, we sort stocks independently by volatility (of either idiosyncratic returns or turnover) and analyst coverage. As before, those with three or fewer analysts are defined as low coverage stocks, the rest as high coverage stocks. High, medium and low volatility groups consist of stocks in the top, middle three, and bottom volatility quintiles, respectively. For each stock, we record the cumulative announcement return over a 3-day window (-1, 0, +1) around the next four quarterly earnings announcements. We calculate "size adjusted" returns by subtracting the return of the firm with median book-to-market among stocks in the same size decile as the announcer. For each stock, the size adjusted "annual" return is the sum of the four quarterly size adjusted returns. The numbers reported in Table 9 are temporal averages of cross sectional means (one for each year) computed within each group. The p-values reported correspond to t tests conducted using the time series of yearly cross sectional means and differences in cross sectional means.

The announcement returns for high idiosyncratic volatility stocks with low analyst coverage range from -0.87% (p value 0.02) using $IVOL20D$ to -1.22% (p value 0.00) using $IVOL60M$. When investors receive information about these firms' fundamentals via earnings announcements, returns are negative *on average*. These are significant and in most cases significantly more negative than the insignificant return to high idiosyncratic volatility stocks with high analyst coverage.

Note also that the earnings announcement returns are not significantly different between low and high idiosyncratic volatility stocks with *high* coverage. This coincides with the earlier results in Table 6 where there is no significant difference between the returns of high and low idiosyncratic volatility stocks having high analyst coverage. Both sets of results suggest that a bias exists only in the pricing of stocks in the low coverage high IVOL group. These findings corroborate the interpretation of the evidence in the earlier tables as consistent with the model.

The results based on turnover volatility (reported at the bottom of Table 9) are much stronger. The average announcement return to high turnover volatility stocks with low analyst coverage is -2.27% (p value 0.00). When coverage is low, the announcement returns of the high turnover volatility stocks are much more negative than those of the low turnover volatility stocks—the difference is a striking -3.12% (p value 0.00). However, when coverage is high, the announcement returns of high turnover volatility stocks are not significantly different from zero and not different from low turnover volatility stocks. These results further support the hypothesis that both the AHXZ result and the turnover volatility puzzle of CSA are attributable to the optimistic mispricing of low coverage stocks predicted by the model.

4.5 Variation in Short Sale Costs

The model predicts that the magnitude of mispricing is larger the greater are short sale costs. In this section, we use low institutional holdings to identify stocks with the highest short sale costs [see D'Avolio (2002) and Boehm, Danielsen, Kumar and Sorescu (2009)]. We then examine whether the negative relation between returns and volatility, attributable to low coverage, is stronger for stocks with low institutional holdings.

We define a dummy variable $LINST$ to equal one for stocks with institutional holdings in the bottom 20% of the cross section in each month. This should capture the stocks with the highest borrowing fees as in D'Avolio (2002). The coefficient of the interaction between low coverage and high volatility should remain negative because shorting costs are higher than the costs of long positions even for stocks outside the bottom 20% of institutional holdings (i.e., borrowing fees, locating shares, monitoring positions, etc.). If the model is correct, the interaction between low coverage, low institutional holdings and volatility should also matter. Low institutional holdings should accentuate the AHXZ and CSA results among low coverage stocks. This is what we find.

The regression estimates are reported in Table 10.[23] The negative relation between returns and $IVOL$ for low coverage stocks is stronger among those with low institutional holdings. The difference between the coefficients of $LCOV*LVOL$ and $LCOV*HVOL$ is significant, and so is the difference between the coefficients of $LCOV*LINST*LVOL$ and $LCOV*LINST*HVOL$. The results are similar and stronger both in the magnitude of

[23]The first month after portfolio formation is omitted from this and later tables to save space.

the estimates and their significance for the volatility of turnover. For example, in the first year, excluding January, the discount to high versus low $TVOL$ stocks with low coverage is 0.52% per month (0.25 + 0.27). The discount is an *additional* 1.19% per month (0.17 + 1.02) for those with low institutional holdings as well.

Here again, the negative relation between returns and $TVOL$ is more pronounced than that of returns and IVOL. Consistent with the model, this negative relation is stronger for stocks with low institutional holdings (i.e., high shorting costs) for both volatility measures.

4.6 Do Both Puzzles Have the Same Source?

The similarities of the IVOL and $TVOL$ results so far suggest that a single economic explanation underlies both the AHXZ and CSA puzzles. To address this directly we now ask whether the returns to a long-short $TVOL$ strategy explain the risk adjusted returns to a long-short IVOL strategy. It turns out that the $TVOL$ strategy returns are highly significant in explaining the time series of IVOL strategy returns. Moreover, the risk adjusted return to the IVOL strategy is *insignificant* after controlling for the $TVOL$ strategy's return—the return to the long-short $TVOL$ strategy completely absorbs the abnormal return to the long-short IVOL strategy. We select the long-short $TVOL$ returns as the independent variable because it appears in the tests above to capture more strongly the effect of Miller's hypothesis.

We focus on $\{p = 1, k = 11\}$ strategies. Returns to portfolios that are long high IVOL and short low IVOL stocks are constructed as differences between the regression coefficients on the high and low IVOL dummies in the cross sectional regressions. Specifically, the returns in the "unconditional" test are differences between the coefficient estimates on the $HVOL$ and $LVOL$ dummies from regressions similar to Equation (5) except that the $LCOV$ interactions are excluded. The returns in the "conditional" (on $LCOV = 1$) test are differences between the coefficients on $LCOV * HVOL$ and $LCOV * LVOL$ from the Equation (5) regressions as reported in Table 6. These are excess returns to the long-short strategy conditional on low coverage. The returns to unconditional and conditional long-short $TVOL$ portfolios are constructed similarly.

In a given month t, the long-short IVOL strategy return is comprised of returns from eleven portfolios, each formed over the past eleven months. These non-overlapping monthly returns to the long-short IVOL strategy are regressed on the Fama-French (1993) factors and the (corresponding, non-overlapping) monthly returns to the long-short $TVOL$ strategy. In doing so, we leave intact the one-month skip in constructing $TVOL$ returns as in the earlier tests. This means the $TVOL$ rankings that define the strategy returns in the independent variable are a month old.

The results are reported in Table 11. The regressions for the unconditional (conditional) test appear in Panel A (Panel B). Both panels report estimates with and without January returns. When the $TVOL$ strategy returns are excluded, the intercepts are significant as expected from the results in Table 6—a strategy that is long high IVOL stocks and short low IVOL stocks earns a significantly negative risk adjusted return (intercepts in Columns 1 and 3 of Panel A). Moreover, the magnitude of the intercepts is larger when the strategy conditions on low coverage (Panel B), consistent with the hypothesis that low coverage stocks drive the IVOL puzzle.

Alternatively, when the TVOL strategy returns are included, the intercepts are not significantly different from zero in both panels. The abnormal returns to the long-short IVOL strategy are completely explained by their exposure to the long-short $TVOL$ strategy returns. The coefficients on the $TVOL$ strategy returns are highly significant, with t-statistics ranging from 12.5 to 20.5. These coefficient estimates are very similar across regressions, ranging between 0.62 and 0.68.

The consistency of these findings across the panels is striking given how different the returns properties of the strategies are. The unconditional long-short IVOL strategy in Panel A has positive exposure to market and size risk factors and negative exposure to the book-to-market factor. In contrast, the long-short IVOL strategy in Panel B has negative exposure to market and size factors and positive exposure to the book-to-market factor. In addition, the relatively low R-squares (about 20%) of the regressions in Panel B without the $TVOL$ returns indicates that the conditional strategy is well hedged against the risks captured by the Fama-French factors. In contrast, the R-squares of the corresponding regressions in Panel A are about 70%—the unconditional long-short strategy returns have a great deal in common with the Fama-French factors.

Despite these differences, the $TVOL$ strategy coefficients are remarkably similar across regressions, and including them uniformly eliminates the significant abnormal returns to the IVOL strategy. We cannot prove the two puzzles have a common economic source, but this evidence seems very unlikely if they have different sources.

5. Conclusion

We reexamine weaknesses others have found in the evidence of a negative relation between returns and idiosyncratic return volatility (IVOL) first documented by Ang, Hodrick, Xing and Zhang (2006) (AHXZ). We confirm the weaknesses exist, then show they are attributable to the January effect and penny stocks (< \$5 per share). Controlling for these effects, we find that the AHXZ result is robust to variations in the data frequency, the length of the time series used to construct idiosyncratic volatility, controls for firm size, illiquidity, and the degree to which security returns are lottery-like. Moreover, the significant lower returns to high IVOL portfolios last at least into the second year after portfolio formation.

We argue that the AHXZ result arises from mispricing that is consistent with Miller's (1977) hypothesis, which we capture in a stylized dynamic model of strategic trading with costly short sales. In our model, significant information arrivals generate disagreement among traders when analyst coverage is low, and costly short sales lead pessimistic beliefs to be underrepresented in prices. Strategic traders anticipate this, which biases prices upward *prior* to information arrivals. Since information arrivals cause return volatility, this leads to a price runup prior to a high volatility ranking and then a correction as disagreement dissipates, but *only* for low coverage stocks.

Our empirical results are consistent with this explanation of the AHXZ result. First, low average returns to high IVOL stocks occur almost exclusively among firms with low analyst coverage. Outside the low coverage group, the return premium to high IVOL is insignificant or positive. Second, returns to high IVOL stocks are lower, the greater are their returns in the prior three years. This relation also is attributable to low coverage stocks, suggesting the low returns are corrections of prior optimistic mispricing of low coverage stocks.

These conclusions are reinforced by an analysis of returns around earnings announcements. If low coverage high IVOL stocks are mispriced too high, their returns should be negative *on average* when earnings are announced because the concreteness of earnings news should reduce disagreement among traders. We find that earnings announcement returns are indeed significantly negative for stocks with high IVOL when coverage is low, and only in this case. This indicates that investors systematically revise their valuations downward with news on earnings, consistent with these stocks having been mispriced too high beforehand.

Our model also makes a prediction that is unrelated to return volatility—trading volume is driven by disagreement. This prediction is not unique to our model, but it provides another way to test it. Since disagreement coupled with costly short sales drives optimistic mispricing, shocks to trading volume should predict mispricing as well or better than shocks to returns—if we run our tests by substituting share turnover volatility for IVOL, the results should be similar or stronger. This is exactly what we find. We confirm the finding of Chordia, Subrahmanyam and Anshuman (2001) (CSA) that high turnover volatility predicts low returns, and we show this relation is attributable to stocks with low coverage. The results of all the tests described above are even *stronger* when turnover volatility is used in place of idiosyncratic return volatility.

We then consider whether the economic source of the AHXZ and CSA results is the same by testing whether the returns to a long-short IVOL strategy are explained by the returns to a long-short strategy based on the volatility of share turnover. We find that the significant risk adjusted returns to the IVOL strategy disappear when the returns to the turnover strategy are included alongside the Fama-French (1993) factors in a time series regression. We find no evidence that there is "something more" to the returns to the long-short IVOL strategy beyond that which is captured by the strategy based on the volatility of turnover.

Figure 1

Sequence of Events

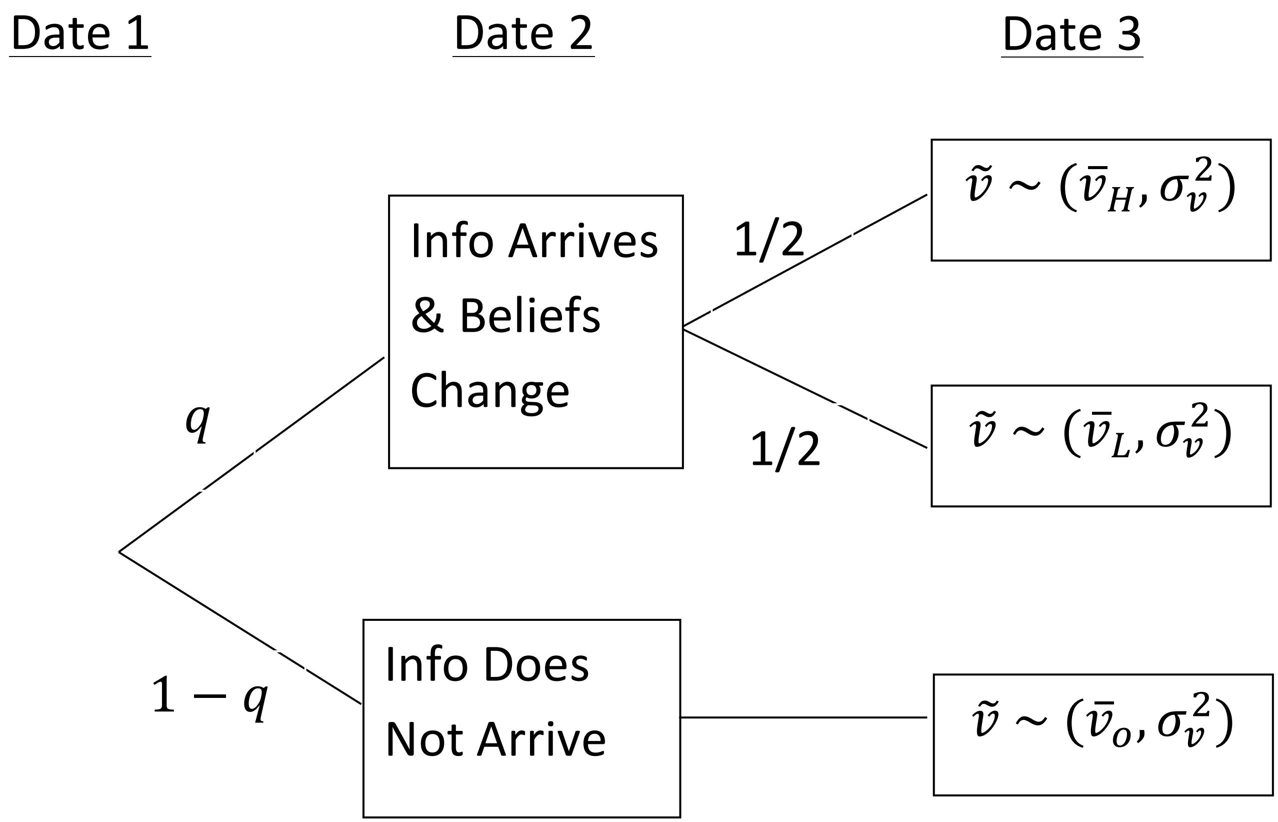

Table 1: Summary Statistics

Panel A reports time-series averages of equally-weighted monthly cross-sectional mean, median, maximum and minimum of each variables used in the paper. Panel B reports time-series averages of equally-weighted monthly cross-sectional correlations. Using monthly data from January 1963 to December 2006, we construct indicator variables for each of the measures described in the text. *Market CAP* is market equity capitalization, *Ret(-1,-12)* is the one year return prior to month *t*, *TVOL* is the standard deviation of turnover calculated over the past 36 months ending in month -1, *IVOL20D (IVOL200D)* is idiosyncratic volatility calculated from daily returns in the past month (year), *IVOL60M* is idiosyncratic volatility calculated from monthly returns over the past five years. *LCOV* is a dummy that takes the value 1 if the stock is covered by three or fewer analysts, and takes the value zero otherwise.

Panel A

	Mean	Median	Min	Max
Market Cap (Millions)	1460.53	1818.88	2.80	123198.14
Ret(-1,-12)	0.214	0.13	-0.72	8.05
Ret(-1,-36)	0.616	0.29	-0.84	25.45
TVOL	0.036	0.02	0.01	0.13
IVOL200D	0.024	0.02	0.01	0.11
IVOL20D	0.021	0.02	0.00	0.15
IVOL60M	0.095	0.09	0.03	0.49
LCOV	0.551	0.96	0.00	1.00

Panel B

	Market Cap	Ret(-1,-12)	Ret(-1,-36)	TVOL	IVOL200D	IVOL20D	IVOL60M	LCOV
Market Cap	1.000							
Ret(-1,-12)	-0.002	1.000						
Ret(-1,-36)	0.004	0.452	1.000					
TVOL	-0.093	0.087	0.194	1.000				
IVOL200D	-0.213	0.162	0.114	0.423	1.000			
IVOL20D	-0.161	0.076	0.070	0.275	0.639	1.000		
IVOL60M	-0.202	0.177	0.254	0.500	0.744	0.524	1.000	
LCOV	-0.210	0.070	0.006	-0.045	0.253	0.184	0.198	1.000

Table 2: Raw Returns of High and Low Idiosyncratic Portfolios (Including Penny Stocks)

Each month between January 1963 and December 2006, 24 (j=1,…,24) cross-sectional regressions of the following forms are estimated:

$$R_{it} = b_{ot} + b_{1jt} LVOL_{i,t-j} + b_{2jt} HVOL_{i,t-j} + e_{ijt} \quad and \quad R_{it} = b_{ot} + b_{1jt} LVOL_{i,t-j} + b_{2jt} HVOL_{i,t-j} + b_{3jt} LMAX5_{i,t-j} + b_{4jt} HMAX5_{i,t-j} + e_{ijt}$$

where R_{it} is the return to stock i in month t, $LVOL_{i,t-j}$ ($HVOL_{i,t-j}$) is the low (high) idiosyncratic volatility dummy that takes the value of 1 if the idiosyncratic volatility for stock i is ranked in the top (bottom) 20% in month $t-j$, and zero otherwise. $LMAX5_{i,t-j}$ ($HMAX5_{i,t-j}$) is the low (high) idiosyncratic volatility dummy that takes the value of 1 if the *MAX5* (the average of the five highest daily return in the month) for stock i is ranked in the top (bottom) 20% in month $t-j$, and zero otherwise. The coefficient estimates of a given independent variable are for j=1 for columns labeled (p=0,K=1), and averaged over j=2 to 12 for columns labeled (p=1,K=11), and j=13 to 24 for columns labeled (p=12,K=12). The numbers reported in the table are the time-series averages of these averages. They are in percent per month. The accompanying t-statistics are calculated from the time series. This sample includes penny stocks (price < \$5). *NOBS* is the average number of stocks used in the monthly cross-sectional regressions.

Raw Returns, IVOL20D (NOBS=5220)								
	Column 1 (p=0,K=1)	Column 2 (p=0,K=1) Jan. excluded	Column 3 (p=1,K=1)	Column 4 (p=1,K=1) Jan. excluded	Column 5 (p=2,K=1)	Column 6 (p=2,K=1) Jan. excluded	Column 7 (p=1,K=11)	Column 8 (p=1,K=11) Jan. excluded
Intercept	1.38	0.99	1.31	0.92	1.30	0.90	1.26	0.85
	(5.60)	(4.04)	(5.35)	(3.78)	(5.29)	(3.72)	(5.04)	(3.46)
LVOL	-0.27	-0.06	-0.11	0.10	-0.11	0.09	-0.09	0.13
	(-2.11)	(-0.49)	(-0.88)	(0.82)	(-0.91)	(0.74)	(-0.70)	(1.06)
HVOL	-0.21	-0.87	-0.08	-0.72	0.00	-0.62	0.26	-0.35
	(-1.01)	(-5.08)	(-0.39)	(-4.21)	(0.01)	(-3.71)	(1.35)	(-2.18)
Raw Returns, IVOL20D (NOBS=5220)								
Intercept	1.46	1.08	1.32	0.93	1.29	0.91	1.22	0.84
	(5.94)	(4.42)	(5.38)	(3.85)	(5.30)	(3.77)	(5.06)	(3.54)
LVOL	-0.49	-0.24	-0.19	0.05	-0.03	0.19	-0.06	0.16
	(-4.53)	(-2.33)	(-1.79)	(0.49)	(-0.27)	(1.97)	(-0.63)	(1.79)
HVOL	0.72	0.09	0.01	-0.54	-0.17	-0.68	0.17	-0.33
	(4.25)	(0.72)	(0.08)	(-4.30)	(-1.10)	(-5.44)	(1.25)	(-2.95)
LMAX5	0.16	0.09	0.10	0.06	-0.11	-0.16	-0.01	-0.05
	(2.06)	(1.16)	(1.35)	(0.76)	(-1.51)	(-2.04)	(-0.13)	(-0.72)
HMAX5	-1.29	-1.33	-0.14	-0.25	0.21	0.06	0.05	-0.08
	(-10.33)	(-10.11)	(-1.20)	(-2.17)	(1.93)	(0.54)	(0.56)	(-0.89)

Table 3: Raw Returns of High and Low Idiosyncratic Portfolios

Each month between January 1963 and December 2006, 24 (j=1,…,24) cross-sectional regressions of the following forms are estimated:

$$R_{it} = b_{ot} + b_{1jt} LVOL_{i,t-j} + b_{2jt} HVOL_{i,t-j} + e_{ijt} \quad and \quad R_{it} = b_{ot} + b_{1jt} LVOL_{i,t-j} + b_{2jt} HVOL_{i,t-j} + b_{3jt} LMAX5_{i,t-j} + b_{4jt} HMAX5_{i,t-j} + e_{ijt}$$

where R_{it} is the return to stock i in month t, $LVOL_{i,t-j}$ ($HVOL_{i,t-j}$) is the low (high) idiosyncratic volatility dummy that takes the value of 1 if the idiosyncratic volatility for stock i is ranked in the top (bottom) 20% in month $t-j$, and zero otherwise. $LMAX5_{i,t-j}$ ($HMAX5_{i,t-j}$) is the low (high) idiosyncratic volatility dummy that takes the value of 1 if the *MAX5* (the average of the five highest daily return in the month) for stock i is ranked in the top (bottom) 20% in month $t-j$, and zero otherwise. The coefficient estimates of a given independent variable are for j=1 for columns labeled (p=0,K=1), and averaged over j=2 to 12 for columns labeled (p=1,K=11), and j=13 to 24 for columns labeled (p=12,K=12). The numbers reported in the table are the time-series averages of these averages. They are in percent per month. The accompanying t-statistics are calculated from the time series. Penny stocks (price < \$5) are excluded. *NOBS* is the average number of stocks used in the monthly cross-sectional regressions.

	Column 1 (p=0,K=1)	Column 2 (p=0,K=1) Jan. excluded	Column 3 (p=1,K=1)	Column 4 (p=1,K=1) Jan. excluded	Column 5 (p=2,K=1)	Column 6 (p=2,K=1) Jan. excluded	Column 7 (p=1,K=11)	Column 8 (p=1,K=11) Jan. excluded
Raw Returns, IVOL20D (NOBS=3997)								
Intercept	1.33	1.04	1.26	0.95	1.24	0.94	1.20	0.86
	(5.58)	(4.29)	(5.28)	(3.97)	(5.22)	(3.91)	(4.89)	(3.54)
LVOL	-0.22	-0.08	-0.09	0.06	-0.09	0.05	-0.06	0.10
	(-1.82)	(-0.68)	(-0.75)	(0.49)	(-0.78)	(0.42)	(-0.49)	(0.88)
HVOL	-1.24	-1.39	-0.70	-0.87	-0.61	-0.84	-0.40	-0.70
	(-8.00)	(-8.62)	(-4.50)	(-5.44)	(-4.08)	(-5.58)	(-2.65)	(-4.83)
Raw Returns, IVOL20D (NOBS=3997)								
Intercept	1.38	1.08	1.26	0.96	1.25	0.95	1.16	0.86
	(5.79)	(4.50)	(5.35)	(4.04)	(5.28)	(3.99)	(4.98)	(3.68)
LVOL	-0.37	-0.21	-0.06	0.10	0.03	0.18	0.02	0.18
	(-3.92)	(-2.18)	(-0.64)	(1.12)	(0.34)	(1.95)	(0.22)	(2.12)
HVOL	-0.68	-0.86	-0.70	-0.84	-0.73	-0.89	-0.41	-0.63
	(-7.03)	(-8.99)	(-7.35)	(-8.70)	(-7.58)	(-9.57)	(-5.00)	(-8.93)
LMAX5	0.13	0.09	-0.06	-0.08	-0.18	-0.19	-0.10	-0.12
	(2.04)	(1.30)	(-0.92)	(-1.17)	(-2.83)	(-2.90)	(-1.94)	(-2.16)
HMAX5	-0.77	-0.74	-0.01	-0.06	0.15	0.05	-0.01	-0.10
	(-5.40)	(-4.85)	(-0.07)	(-0.45)	(1.21)	(0.40)	(-0.10)	(-0.88)

Table 4: Risk Adjusted Returns of High and Low Idiosyncratic Volatility Portfolios

Each month between January 1963 and December 2006, 24 (j=1,…,24) cross-sectional regressions of the following form are estimated:

$$R_{it} = b_{ot} + b_{1jt} LVOL_{i,t-j} + b_{2jt} HVOL_{i,t-j} + e_{ijt}$$

where R_{it} is the return to stock i in month t, $LVOL_{i,t-j}$ ($HVOL_{i,t-j}$) is the low (high) idiosyncratic volatility dummy that takes the value of 1 if the idiosyncratic volatility for stock i is ranked in the top (bottom) 20% in month t-j, and zero otherwise. The coefficient estimates of a given independent variable are for j=1 for columns labeled (p=0,K=1), and averaged over j=2 to 12 for columns labeled (p=1,K=11), and j=13 to 24 for columns labeled (p=12,K=12). To obtain risk-adjusted returns, we further run times-series regressions of these averages (one for each average) on the contemporaneous Fama-French factor realizations to hedge out the factor exposure. The numbers reported for risk-adjusted returns are intercepts from these time-series regressions. They are in percent per month and their t-statistics are in parentheses. Penny stocks (price < $5) are excluded. *NOBS* is the average number of stocks used in the monthly cross-sectional regressions.

	Column 1 (p=0,K=1)	Column 2 (p=0,K=1) Jan. excluded	Column 3 (p=1,K=11)	Column 4 (p=1,K=11) Jan. excluded	Column 5 (p=12,K=12)	Column 6 (p=12,K=12) Jan. excluded
Risk Adjusted Returns, IVOL20D (NOBS=3997)						
Intercept	0.08	0.04	-0.06	-0.13	-0.07	-0.15
	(1.92)	(0.87)	(-1.33)	(-2.74)	(-1.24)	(-3.08)
LVOL	-0.01	0.02	0.14	0.20	0.10	0.17
	(-0.28)	(0.33)	(2.72)	(3.81)	(1.76)	(3.12)
HVOL	-1.22	-1.31	-0.46	-0.66	-0.14	-0.38
	(-10.95)	(-11.57)	(-4.47)	(-6.90)	(-1.30)	(-4.15)
Risk Adjusted Returns, IVOL200D (NOBS=3664)						
Intercept	-0.01	-0.04	-0.05	-0.11	-0.04	-0.12
	(-0.13)	(-0.91)	(-1.04)	(-2.15)	(-0.81)	(-2.39)
LVOL	0.12	0.16	0.13	0.20	0.10	0.18
	(2.04)	(2.82)	(2.22)	(3.31)	(1.60)	(2.92)
HVOL	-0.59	-0.75	-0.32	-0.60	-0.22	-0.54
	(-4.14)	(-5.32)	(-2.31)	(-4.70)	(-1.53)	(-4.44)
Risk Adjusted Returns, IVOL60M (NOBS=2519)						
Intercept	0.02	0.02	0.01	0.00	0.00	-0.03
	(0.47)	(0.33)	(0.23)	(-0.06)	(0.05)	(-0.54)
LVOL	0.06	0.08	0.05	0.08	0.03	0.07
	(1.08)	(1.43)	(0.95)	(1.41)	(0.59)	(1.22)
HVOL	-0.21	-0.33	-0.31	-0.50	-0.25	-0.47
	(-1.93)	(-3.07)	(-2.93)	(-5.12)	(-2.37)	(-4.76)

Table 5: Raw and Risk Adjusted Returns of High and Low Turnover Volatility Portfolios

Each month between January 1963 and December 2006, 24 (j=1,…,24) cross-sectional regressions of the following form are estimated:

$$R_{it} = b_{ot} + b_{1jt}\ LVOL_{i,t\text{-}j\text{-}1} + b_{2jt}\ HVOL_{i,t\text{-}j} + e_{ijt}$$

where R_{it} is the return to stock i in month t, $LVOL_{i,t\text{-}j\text{-}1}$ ($HVOL_{i,t\text{-}j\text{-}1}$) is the low (high) turnover volatility dummy that takes the value of 1 if the volatility of share turnover for stock i is ranked in the top (bottom) 20% in month t-j-1, and zero otherwise. Turnover volatility is measured as the standard deviation of the share turnover using data for the past 36 months, ending in month t-2. The coefficient estimates of a given independent variable are for j=1 for columns labeled (p=0,K=1), and averaged over j=2 to 12 for columns labeled (p=1,K=11), and j=13 to 24 for columns labeled (p=12,K=12). To obtain risk-adjusted returns, we further run times-series regressions of these averages (one for each average) on the contemporaneous Fama-French factor realizations to hedge out the factor exposure. The numbers reported for risk-adjusted returns are intercepts from these time-series regressions. They are in percent per month and their t-statistics are in parentheses. Penny stocks (price < $5) are excluded. *NOBS* is the average number of stocks used in the monthly cross-sectional regressions.

	Column 1 (p=0,K=1)	Column 2 (p=0,K=1) Jan. excluded	Column 3 (p=1,K=11)	Column 4 (p=1,K=11) Jan. excluded	Column 5 (p=12,K=12)	Column 6 (p=12,K=12) Jan. excluded
Raw Return, TVOL (NOBS=3084)						
Intercept	1.24	1.00	1.17	0.88	1.18	0.83
	(4.79)	(3.80)	(4.37)	(3.27)	(4.41)	(3.14)
LVOL	0.10	0.19	0.14	0.26	0.04	0.21
	(0.80)	(1.48)	(1.10)	(2.08)	(0.33)	(1.73)
HVOL	-0.46	-0.58	-0.44	-0.63	-0.23	-0.48
	(-2.99)	(-3.58)	(-2.83)	(-4.22)	(-1.61)	(-3.59)
Risk Adjusted Returns, TVOL (NOBS=3084)						
Intercept	-0.01	-0.03	-0.07	-0.04	-0.05	-0.01
	(-0.23)	(-0.63)	(-1.21)	(-0.74)	(-0.84)	(-0.23)
LVOL	0.25	0.29	0.28	0.31	0.19	0.25
	(4.01)	(4.60)	(4.30)	(4.85)	(2.90)	(4.01)
HVOL	-0.45	-0.54	-0.41	-0.50	-0.27	-0.45
	(-5.09)	(-5.98)	(-4.14)	(-5.72)	(-2.74)	(-5.09)

Table 6: Analyst Coverage and Risk-Adjusted Returns of High and Low Volatility Portfolios

Each month between January 1983 and December 2006, 24 (j=1,…,24) cross-sectional regressions of the following form are estimated:

$$R_{it} = b_{0jt} + b_{1jt}LVOL_{i,t-j} + b_{2jt}HVOL_{i,t-j} + b_{3jt}LCOV_{i,t-j} * LVOL_{i,t-j} + b_{4jt}LCOV_{i,t-j} * HVOL_{i,t-j}$$
$$+ b_{5jt}BM_{i,t-1} + b_{6jt}Size_{i,t-1} + b_{7jt}\mathrm{Re}t_{i,t-1} + b_{8jt}52WKHW_{i,t-j} + b_{9jt}52WKHL_{i,t-j} + e_{ijt}$$

where R_{it} is the return to stock i in month t, $LVOL_{i,t-j}$ ($HVOL_{i,t-j}$) is the low (high) idiosyncratic volatility dummy that takes the value of 1 if the idiosyncratic volatility for stock i is ranked in the top (bottom) 20% in month t-j, and zero otherwise. $LCOV_{i,t-j}$ is a dummy that takes the value of 1 if the number of analyst coverage for stock i is three or less in month t-j, and is zero otherwise. The control variables are defined in the text, and their coefficients are omitted to save space. The coefficient estimates of a given independent variable are for j=1 for columns labeled (p=0,K=1), and averaged over j=2 to 12 for columns labeled (p=1,K=11), and j=13 to 24 for columns labeled (p=12,K=12). To obtain risk-adjusted returns, we further run times-series regressions of these averages (one for each average) on the contemporaneous Fama-French factor realizations to hedge out the factor exposure. The numbers reported for risk-adjusted returns are intercepts from these time-series regressions. They are in percent per month and their t-statistics are in parentheses. Penny stocks (price < $5) are excluded. *NOBS* is the average number of stocks used in the monthly cross-sectional regressions.

	Column 1 (p=0,K=1)	Column 2 (p=0,K=1) Jan. excluded	Column 3 (p=1,K=11)	Column 4 (p=1,K=11) Jan. excluded	Column 5 (p=12,K=12)	Column 6 (p=12,K=12) Jan. excluded
Risk Adjusted Return, IVOL20D (NOBS=3549)						
Intercept	-0.11	-0.16	-0.06	-0.13	-0.02	-0.11
	(-1.43)	(-2.15)	(-0.75)	(-1.75)	(-0.28)	(-1.59)
LVOL	0.11	0.12	0.04	0.04	-0.03	-0.05
	(1.67)	(1.76)	(0.61)	(0.65)	(-0.48)	(-0.82)
HVOL	-1.09	-0.92	0.01	-0.02	0.22	0.16
	(-5.88)	(-4.88)	(0.06)	(-0.17)	(1.91)	(1.36)
LCOV*LVOL	-0.04	-0.01	0.09	0.14	0.17	0.25
	(-0.71)	(-0.09)	(1.69)	(2.67)	(2.58)	(4.07)
LCOV*HVOL	0.37	0.18	-0.21	-0.31	-0.25	-0.33
	(1.81)	(0.88)	(-1.60)	(-2.30)	(-1.84)	(-2.33)
Risk Adjusted Return , IVOL200D (NOBS=3525)						
Intercept	-0.15	-0.21	-0.05	-0.12	-0.02	-0.10
	(-1.95)	(-2.63)	(-0.70)	(-1.63)	(-0.27)	(-1.46)
LVOL	0.16	0.19	0.03	0.03	0.00	-0.03
	(2.02)	(2.46)	(0.39)	(0.34)	(0.01)	(-0.46)
HVOL	0.02	0.14	0.34	0.25	0.66	0.65
	(0.04)	(0.34)	(1.23)	(0.89)	(2.30)	(2.19)
LCOV*LVOL	-0.02	0.00	0.10	0.14	0.16	0.24
	(-0.29)	(-0.07)	(1.64)	(2.45)	(2.42)	(3.72)
LCOV*HVOL	-0.13	-0.35	-0.47	-0.58	-0.91	-1.12
	(-0.33)	(-0.85)	(-1.79)	(-2.17)	(-3.25)	(-3.88)

Table 6 (cont.)

	Column 1 (p=0,K=1)	Column 2 (p=0,K=1) Jan. excluded	Column 3 (p=1,K=11)	Column 4 (p=1,K=11) Jan. excluded	Column 5 (p=12,K=12)	Column 6 (p=12,K=12) Jan. excluded
Risk Adjusted Return IVOL60M (NOBS=2596)						
Intercept	-0.09	-0.11	0.02	-0.01	0.05	0.02
	(-1.05)	(-1.37)	(0.30)	(-0.10)	(0.72)	(0.21)
LVOL	0.16	0.19	0.04	0.06	-0.02	-0.04
	(1.98)	(2.31)	(0.58)	(0.70)	(-0.25)	(-0.44)
HVOL	0.21	0.26	-0.08	-0.06	0.11	0.08
	(0.89)	(1.09)	(-0.39)	(-0.31)	(0.53)	(0.37)
LCOV*LVOL	0.09	0.11	0.12	0.15	0.16	0.21
	(1.48)	(1.68)	(2.06)	(2.39)	(2.41)	(3.20)
LCOV*HVOL	-0.26	-0.42	-0.30	-0.47	-0.50	-0.64
	(-1.01)	(-1.62)	(-1.53)	(-2.33)	(-2.42)	(-3.01)
Risk Adjusted Return TVOL (NOBS=3089)						
Intercept	-0.11	-0.15	-0.04	-0.10	0.02	-0.05
	(-1.29)	(-1.81)	(-0.46)	(-1.28)	(0.29)	(-0.68)
LVOL	0.12	0.16	0.04	0.06	0.08	0.08
	(1.35)	(1.87)	(0.47)	(0.76)	(1.04)	(0.96)
HVOL	-0.11	-0.13	-0.09	-0.12	-0.04	-0.13
	(-1.02)	(-1.23)	(-0.86)	(-1.21)	(-0.33)	(-1.07)
LCOV*LVOL	0.15	0.16	0.29	0.34	0.18	0.28
	(1.85)	(2.05)	(3.87)	(4.77)	(1.97)	(3.29)
LCOV*HVOL	-0.32	-0.40	-0.39	-0.51	-0.37	-0.44
	(-2.26)	(-2.73)	(-3.09)	(-4.10)	(-3.16)	(-3.65)

Table 7: Fama-French and Chen-Petkova Factor Risk Adjusted Returns

Each month between January 1983 and December 2006, 24 (j=1,…,24) cross-sectional regressions of the following form are estimated:

$$R_{it} = b_{0jt} + b_{1jt}LVOL_{i,t-j} + b_{2jt}HVOL_{i,t-j} + b_{3jt}LCOV_{i,t-j} * LVOL_{i,t-j} + b_{4jt}LCOV_{i,t-j} * HVOL_{i,t-j}$$
$$+ b_{5jt}BM_{i,t-1} + b_{6jt}Size_{i,t-1} + b_{7jt}\mathrm{Ret}_{i,t-1} + b_{8jt}52WKHW_{i,t-j} + b_{9jt}52WKHL_{i,t-j} + e_{ijt}$$

where R_{it} is the return to stock i in month t, $LVOL_{i,t-j}$ ($HVOL_{i,t-j}$) is the low (high) idiosyncratic volatility dummy that takes the value of 1 if the idiosyncratic volatility for stock i is ranked in the top (bottom) 20% in month $t-j$, and zero otherwise. $LCOV_{i,t-j}$ is a dummy that takes the value of 1 if the number of analyst coverage for stock i is three or less in month $t-j$, and is zero otherwise. The control variables are defined in the text, and their coefficients are omitted to save space. The coefficient estimates of a given independent variable are for j=1 for columns labeled (p=0,K=1), and averaged over j=2 to 12 for columns labeled (p=1,K=11), and j=13 to 24 for columns labeled (p=12,K=12). To obtain risk-adjusted returns, we further run times-series regressions of these averages (one for each average) on the contemporaneous Fama-French factors and Chen-Petkova AV factor realizations to hedge out the factor exposure. The numbers reported for risk-adjusted returns are intercepts from these time-series regressions. They are in percent per month and their t-statistics are in parentheses. Penny stocks (price < $5) are excluded. *NOBS* is the average number of stocks used in the monthly cross-sectional regressions.

Risk Adjusted Return, IVOL20D (NOBS=3549)						
	Column 1 (p=0,K=1)	Column 2 (p=0,K=1) Jan. excluded	Column 5 (p=1,K=11)	Column 6 (p=1,K=11) Jan. excluded	Column 7 (p=12,K=12)	Column 8 (p=12,K=12) Jan. excluded
Intercept	-0.16	-0.22	-0.10	-0.18	-0.04	-0.14
	(-2.15)	(-3.06)	(-1.29)	(-2.46)	(-0.59)	(-2.01)
LVOL	0.09	0.10	0.01	0.02	-0.04	-0.06
	(1.31)	(1.47)	(0.21)	(0.29)	(-0.60)	(-0.93)
HVOL	-1.00	-0.84	0.11	0.08	0.32	0.26
	(-5.39)	(-4.41)	(1.03)	(0.73)	(2.84)	(2.19)
LCOV*LVOL	-0.05	-0.01	0.09	0.14	0.14	0.22
	(-0.78)	(-0.16)	(1.68)	(2.73)	(2.09)	(3.60)
LCOV*HVOL	0.42	0.23	-0.20	-0.30	-0.28	-0.37
	-2.02	(1.09)	(-1.50)	(-2.22)	(-2.07)	(-2.59)
Risk Adjusted Return, TVOL (NOBS=3087)						
Intercept	-0.16	-0.22	-0.08	-0.15	0.00	-0.08
	(-2.03)	(-2.70)	(-0.93)	(-1.89)	(0.01)	(-1.06)
LVOL	0.10	0.15	0.01	0.04	0.06	0.05
	(1.14)	(1.73)	(0.13)	(0.48)	(0.74)	(0.67)
HVOL	-0.07	-0.10	-0.03	-0.07	0.03	-0.06
	(-0.67)	(-0.92)	(-0.27)	(-0.70)	(0.25)	(-0.55)
LCOV*LVOL	0.13	0.15	0.27	0.32	0.15	0.25
	(1.67)	(1.83)	(3.57)	(4.48)	(1.60)	(2.93)
LCOV*HVOL	-0.25	-0.34	-0.39	-0.52	-0.38	-0.46
	(-1.74)	(-2.27)	(-3.01)	(-4.11)	(-3.20)	(-3.73)

Table 8: Past Returns and High and Low Volatility Portfolios

Each month between January 1963 (1983 for the high coverage sample) and December 2006, 24 (j=1,…,24) cross-sectional regressions of the following form are estimated:

$$R_{it} = b_{0jt} + b_{1jt} Low\ 3Y\ Ret_{i,t-j} + b_{2jt} High\ 3Y\ Ret_{i,t-j} + b_{3jt} Low\ 3Y\ Ret_{i,t-j} * HVOL_{i,t-j} + b_{4jt} High\ 3Y\ Ret_{i,t-j} * HVOL_{i,t-j} + b_{5jt} LVOL_{i,t-j} + b_{6jt} HVOL_{i,t-j}$$
$$+ b_{7jt} BM_{i,t-1} + b_{8jt} Size_{i,t-1} + b_{9jt} Ret_{i,t-1} + b_{10jt} 52WKHW_{i,t-j} + b_{11jt} 52WKHL_{i,t-j} + e_{ijt}$$

where R_{it} is the return to stock i in month t, $LVOL_{i,t-j}$ ($HVOL_{i,t-j}$) is the low (high) idiosyncratic volatility dummy that takes the value of 1 if the idiosyncratic volatility for stock i is ranked in the top (bottom) 20% in month $t-j$, and zero otherwise. $High\ 3Y\ Ret_{i,t-j}$ ($Low\ 3Y\ Ret_{i,t-j}$) is a dummy that takes the value of 1 if the past three year return for stock i is ranked in the top (bottom) 30% in month $t-j$, and is zero otherwise. The control variables are defined in the text, and their coefficients are omitted to save space. The coefficient estimates of a given independent variable are averaged over j=2 to 12 for columns labeled (p=1,K=11), and j=13 to 24 for columns labeled (p=12,K=12). To obtain risk-adjusted returns, we further run times-series regressions of these averages (one for each average) on the contemporaneous Fama-French factor realizations to hedge out the factor exposure. The numbers reported for risk-adjusted returns are intercepts from these time-series regressions. They are in percent per month and their t-statistics are in parentheses. Penny stocks (price < \$5) are excluded. *NOBS* is the average number of stocks used in the monthly cross-sectional regressions.

Panel A: 20-Day Idiosyncratic Return Volatility Measure

	Risk Adjusted Returns IVOL20D All Firms (NOBS=2330)				Risk Adjusted Returns IVOL20D High Coverage Sample (NOBS=1174)			
	Column 1 (p=1,K=11)	Column 2 (p=1,K=11) Jan. excluded	Column 3 (p=12,K=12)	Column 4 (p=12,K=12) Jan. excluded	Column 1 (p=1,K=11)	Column 2 (p=1,K=11) Jan. excluded	Column 3 (p=12,K=12)	Column 4 (p=12,K=12) Jan. excluded
Intercept	0.05	0.04	0.08	0.05	-0.02	0.02	0.02	0.06
	(1.04)	(0.69)	(1.60)	(1.12)	(-0.18)	(0.20)	(0.25)	(0.70)
Low 3 yr Ret	-0.09	-0.21	-0.04	-0.12	-0.07	-0.24	-0.17	-0.30
	(-1.26)	(-2.97)	(-0.61)	(-1.81)	(-0.48)	(-1.75)	(-1.18)	(-2.12)
High 3 yr Ret	0.00	0.02	-0.17	-0.17	-0.03	-0.01	-0.10	-0.13
	(0.05)	(0.43)	(-3.44)	(-3.32)	(-0.40)	(-0.13)	(-1.27)	(-1.54)
Low 3yr Ret*HVOL	0.03	0.04	-0.06	-0.07	0.32	0.23	-0.19	-0.11
	(0.29)	(0.42)	(-0.58)	(-0.59)	(1.42)	(0.99)	(-0.69)	(-0.40)
High 3yr Ret*HVOL	-0.26	-0.23	-0.29	-0.34	-0.02	0.09	-0.25	-0.22
	(-2.97)	(-2.63)	(-3.24)	(-3.68)	(-0.11)	(0.47)	(-1.13)	(-0.96)
LVOL	-0.01	0.01	-0.02	0.00	0.07	0.11	0.03	0.08
	(-0.18)	(0.28)	(-0.50)	(0.04)	(1.24)	(1.86)	(0.57)	(1.40)
HVOL	-0.15	-0.24	0.08	0.02	-0.03	-0.08	0.23	0.13
	(-1.97)	(-3.04)	(1.10)	(0.29)	(-0.26)	(-0.68)	(1.55)	(0.87)

Panel B: Volatility of Share Turnover

	Risk Adjusted Returns TVOL All Firms (NOBS=2030)				Risk Adjusted Returns TVOL High Coverage Sample (NOBS=1068)			
	Column 1 (p=1,K=11)	Column 2 (p=1,K=11) Jan. excluded	Column 3 (p=12,K=12)	Column 4 (p=12,K=12) Jan. excluded	Column 1 (p=1,K=11)	Column 2 (p=1,K=11) Jan. excluded	Column 3 (p=12,K=12)	Column 4 (p=12,K=12) Jan. excluded
Intercept	0.02	0.01	0.07	0.05	0.02	0.08	0.01	0.07
	(0.39)	(0.18)	(1.28)	(0.99)	(0.24)	(0.81)	(0.13)	(0.76)
Low 3 yr Ret	-0.08	-0.22	-0.02	-0.10	0.04	-0.08	-0.11	-0.17
	(-0.92)	(-2.62)	(-0.23)	(-1.26)	(0.26)	(-0.45)	(-0.65)	(-1.01)
High 3 yr Ret	0.08	0.11	-0.13	-0.13	0.01	0.03	-0.10	-0.12
	(1.55)	(2.11)	(-2.81)	(-2.71)	(0.13)	(0.37)	(-1.36)	(-1.45)
Low 3yr Ret*HVOL	0.05	0.09	0.00	0.02	0.04	-0.05	0.11	0.01
	(0.49)	(0.80)	(-0.04)	(0.15)	(0.15)	(-0.21)	(0.47)	(0.04)
High 3yr Ret*HVOL	-0.34	-0.28	-0.16	-0.14	-0.27	-0.16	-0.08	-0.11
	(-4.49)	(-3.60)	(-2.21)	(-1.89)	(-2.08)	(-1.25)	(-0.66)	(-0.91)
LVOL	0.13	0.15	0.08	0.11	0.04	0.07	0.10	0.15
	(2.41)	(2.90)	(1.49)	(2.12)	(0.53)	(0.84)	(1.24)	(1.73)
HVOL	-0.13	-0.23	-0.12	-0.23	0.05	-0.08	0.06	-0.04
	(-1.67)	(-3.06)	(-1.55)	(-3.04)	(0.45)	(-0.66)	(0.54)	(-0.35)

Table 9: Earnings Announcement Returns for Portfolios Sorted on Volatility and Analyst Coverage

Every June from 1983 to 2006, we sort firms independently into two groups by analyst coverage (three or less analysts is low coverage, greater than three is high coverage) and three groups by idiosyncratic volatility (top 20%, middle 60% and bottom 20%), and form portfolios based on these groupings. For each firm, we then compute the average abnormal return over the four quarterly announcement returns following portfolio formation and annualize this number by multiplying by four. Following La Porta et al (1997), we benchmark each earnings announcement return by the firm with median book-to-market in the same size decile as the announcer. The numbers in the table are the equally weighted average annualized earning announcement abnormal (net of benchmark) returns in percent. The column labeled H-L is the difference between the returns to high and low leverage groups, and p-values relate to a test of the null hypothesis that the difference between the mean abnormal returns of high and low leverage groups is zero. Penny stocks (price < $5) are excluded.

Cumulative Abnormal Returns

	IVOL20D				
Coverage	L	M	H	H-L	p-value
L	0.34	-0.05	-0.87	-1.21	0.00
p-value	0.12	0.67	0.02		
H	-0.14	0.51	-0.87	-0.73	0.18
p-value	0.43	0.00	0.10		

	IVOL200D				
Coverage	L	M	H	H-L	p-value
L	0.38	0.03	-1.03	-1.41	0.00
p-value	0.09	0.84	0.00		
H	0.04	0.47	-0.60	0.19	0.38
p-value	0.87	0.01	0.39		

	IVOL60M				
Coverage	L	M	H	H-L	p-value
L	0.05	0.51	-1.22	-1.27	0.00
p-value	0.85	0.00	0.00		
H	-0.26	0.49	-0.16	0.72	0.81
p-value	0.21	0.00	0.66		

	STURN				
Coverage	L	M	H	H-L	p-value
L	0.85	0.13	-2.27	-3.12	0.00
p-value	0.00	0.29	0.00		
H	-0.05	0.45	-0.47	-0.42	0.26
p-value	0.85	0.00	0.13		

Number of Stocks

	IVOL20D		
Coverage	L	M	H
L	264	855	380
H	273	811	159

	IVOL200D		
Coverage	L	M	H
L	259	855	385
H	296	799	147

	IVOL60M		
Coverage	L	M	H
L	189	649	265
H	228	593	143

	STURN		
Coverage	L	M	H
L	333	737	213
H	139	765	283

Table 10: Low Institutional Holdings, Low Coverage and Volatility

Each month between January 1983 and December 2006, 24 (j=1,…,24) cross-sectional regressions of the following form are estimated:

$$R_{it} = b_{0jt} + b_{1jt} LVOL_{i,t-j} + b_{2jt} HVOL_{i,t-j} + b_{3jt} LCOV_{i,t-j} * LVOL_{i,t-j} + b_{4jt} LCOV_{i,t-j} * HVOL_{i,t-j} + b_{5jt} LCOV_{i,t-j} * LINST_{i,t-j} * LVOL_{i,t-j} + b_{6jt} LCOV_{i,t-j} * LINST_{i,t-j} * HVOL_{i,t-j}$$

$$+ b_{7jt} BM_{i,t-1} + b_{8jt} Size_{i,t-1} + b_{9jt} \mathrm{Re}t_{i,t-1} + b_{10jt} 52WKHW_{i,t-j} + b_{11jt} 52WKHL_{i,t-j} + e_{ijt}$$

where R_{it} is the return to stock i in month t, $LVOL_{i,t-j}$ ($HVOL_{i,t-j}$) is the low (high) idiosyncratic volatility dummy that takes the value of 1 if the idiosyncratic volatility for stock i is ranked in the top (bottom) 20% in month t-j, and zero otherwise. $LCOV_{i,t-j}$ is a dummy that takes the value of 1 if the number of analyst coverage for stock i is three or less in month t-j, and is zero otherwise. $LINST_{i,t-j}$ is a dummy that takes the value of 1 if the institutional holdings for stock i is ranked in the bottom 20% in month t-j, and is zero otherwise The control variables are defined in the text, and their coefficients are omitted to save space. The coefficient estimates of a given independent variable are for j=1 for columns labeled (p=0,K=1), and averaged over j=2 to 12 for columns labeled (p=1,K=11), and j=13 to 24 for columns labeled (p=12,K=12). To obtain risk-adjusted returns, we further run times-series regressions of these averages (one for each average) on the contemporaneous Fama-French factors realizations to hedge out the factor exposure. The numbers reported for risk-adjusted returns are intercepts from these time-series regressions. They are in percent per month and their t-statistics are in parentheses. Penny stocks (price < $5) are excluded. *NOBS* is the average number of stocks used in the monthly cross-sectional regressions.

	Risk Adjusted Return, IVOL20D(NOBS=3549)				Risk Adjusted Return, TVOL(NOBS=3087)			
	Column 1 (p=1,K=11)	Column 2 (p=1,K=11) Jan. excluded	Column 3 (p=12,K=12)	Column 4 (p=12,K=12) Jan. excluded	Column 5 (p=1,K=11)	Column 6 (p=1,K=11) Jan. excluded	Column 7 (p=12,K=12)	Column 8 (p=12,K=12) Jan. excluded
Intercept	-0.06	-0.13	-0.02	-0.11	-0.04	-0.10	0.02	-0.05
	(-0.75)	(-1.76)	(-0.28)	(-1.59)	(-0.45)	(-1.26)	(0.30)	(-0.67)
LVOL	0.04	0.04	-0.03	-0.05	0.05	0.07	0.08	0.08
	(0.65)	(0.71)	(-0.47)	(-0.79)	(0.56)	(0.88)	(1.10)	(1.04)
HVOL	0.01	-0.02	0.22	0.17	-0.08	-0.12	-0.04	-0.13
	-0.07	(-0.15)	(1.93)	(1.38)	(-0.85)	(-1.21)	(-0.36)	(-1.10)
LCOV*LVOL	0.11	0.17	0.14	0.24	0.19	0.25	0.12	0.21
	(2.41)	(3.94)	(2.35)	(4.30)	(2.59)	(3.39)	(1.32)	(2.48)
LCOV*HVOL	-0.10	-0.15	-0.13	-0.15	-0.21	-0.27	-0.27	-0.29
	(-0.75)	(-1.13)	(-1.01)	(-1.10)	(-1.95)	(-2.35)	(-2.34)	(-2.38)
LCOV*LINST*LVOL	-0.07	-0.10	0.04	0.00	0.18	0.17	0.14	0.14
	(-0.90)	(-1.26)	(0.52)	(0.04)	(2.52)	(2.26)	(1.75)	(1.72)
LCOV*LINST*HVOL	-0.26	-0.34	-0.26	-0.39	-0.77	-1.02	-0.53	-0.77
	(-2.17)	(-2.83)	(-2.32)	(-3.54)	(-3.66)	(-4.96)	(-2.97)	(-4.55)

Table 11: TVOL Returns Explain IVOL Returns

Each month between January 1983 and December 2006, (j=1,…,12) cross-sectional regressions of the following form are estimated:

$$R_{it} = b_{0jt} + b_{1jt}LVOL_{i,t-j} + b_{2jt}HVOL_{i,t-j} + b_{3jt}BM_{i,t-1} + b_{4jt}Size_{i,t-1} + b_{5jt}\,\mathrm{Ret}_{i,t-1} + b_{6jt}52WKHW_{i,t-j} + b_{7jt}52WKHL_{i,t-j} + e_{ijt}$$

For Panel A and the following cross-sectional regressions of the following form are estimated:

$$R_{it} = b_{0jt} + b_{1jt}LVOL_{i,t-j} + b_{2jt}HVOL_{i,t-j} + b_{3jt}LCOV_{i,t-j} * LVOL_{i,t-j} + b_{4jt}LCOV_{i,t-j} * HVOL_{i,t-j} + b_{5jt}BM_{i,t-1} + b_{6jt}Size_{i,t-1} + b_{7jt}\,\mathrm{Ret}_{i,t-1} + b_{8jt}52WKHW_{i,t-j} + b_{9jt}52WKHL_{i,t-j} + e_{ijt}$$

each month between January 1963 and December 2006, 24 (j=1,…,12) for Panel B, where R_{it} is the return to stock i in month t, for Panel A and $LVOL_{i,t-j}$ ($HVOL_{i,t-j}$) is the low (high) volatility dummy that takes the value of 1 if the idiosyncratic volatility for stock i is ranked in the top (bottom) 20% in month t-j, and zero otherwise. $LCOV_{i,t-j}$ is a dummy that takes the value of 1 if the number of analyst coverage for stock i is three or less in month t-j, and is zero otherwise. The coefficient estimates b_{2jt} - b_{1jt} (Panel A) and b_{4jt} - b_{3jt} (Panel B) are averaged over j=2 to 12. This procedure is performed each month first with IVOLD20 as the volatility measure, and is repeated with TVOL as the volatility. The resultant time series from IVOLD20 is regressed on that from TVOL (denoted as LS-STRUN in panel A and LS-STRUN-LCOV in panel B) along with Fama-French three factors (Market, SMB, HML.) The intercepts reported are in percentage per month and t-statistics are in parentheses. Penny stocks (price < $5) are excluded.

Panel A: Unconditional

	Column 1 (p=1,K=11)	Column 2 (p=1,K=11)	Column 3 (p=1,K=11) Jan excluded	Column 4 (p=1,K=11) Jan excluded
Intercept	-0.32	-0.07	-0.43	-0.12
	(-3.48)	(-1.08)	(-4.72)	(-1.67)
Market	0.34	0.06	0.34	0.03
	(15.08)	(2.81)	(14.75)	(1.48)
SMB	0.49	0.22	0.45	0.17
	(16.86)	(8.80)	(14.78)	(6.49)
HML	-0.29	-0.05	-0.30	-0.08
	(-8.57)	(-1.77)	(-8.38)	(-2.91)
LS-TVOL		0.62		0.65
		(20.49)		(19.13)
R-square	0.70	0.85	0.72	0.85

Panel B: Conditional on Low Coverage

	Column 1 (p=1,K=11)	Column 2 (p=1,K=11)	Column 3 (p=1,K=11) Jan excluded	Column 4 (p=1,K=11) Jan excluded
Intercept	-0.34	0.11	-0.50	0.08
	(-2.18)	(0.93)	(-3.30)	(0.64)
Market	-0.07	-0.13	-0.08	-0.12
	(-1.79)	(-4.26)	(-1.99)	(-4.02)
SMB	-0.20	-0.28	-0.22	-0.25
	(-4.23)	(-7.47)	(-4.47)	(-6.71)
HML	0.16	0.04	0.17	0.06
	(2.66)	(0.90)	(2.83)	(1.18)
LS-TVOL-LCOV		0.67		0.68
		(13.84)		(12.53)
R-square	0.18	0.53	0.23	0.53

APPENDIX

Date 2

Whether or not information arrives at date 2, trader j solves

$$J_{2j} = \max_{x_{2j}(\cdot)} \mathrm{E}_{2j}\left[(\tilde{v} - p_2)x_{2j}(p_2) + c_s I_{2j} x_{2j}(p_2) - \psi_2 x_{2j}(p_2)^2\right]. \tag{A.1}$$

The ψ_2 parameter captures the utility cost associated with risk aversion and equals the product of the risk aversion coefficient and the variance of profit for trader j between dates 2 and 3. Since the variance of profit in each subtree is endogenous, we first solve the model for unspecified ψ parameters, then we close the model at the end by solving for the equilibrium ψ parameters in terms of the underlying model parameters.

Pointwise optimization of (A.1) yields a family of first-order conditions

$$(\mathrm{E}_{2j}[\tilde{v}] - p_2) - \frac{\partial p_2}{\partial x_{2j}} x_{2j} + c_s I_{2j} - 2\psi_2 x_{2j} = 0$$

that the trader's optimal choice must satisfy at each p_2. It will be apparent later that the second order condition for a maximum, $\frac{\partial p_2}{\partial x_{2j}} + 2\psi_2 > 0$, is satisfied in equilibrium. Rearranging the FOC yields an expression for trader j's optimal demand schedule at date 2:

$$x^*_{2j}(p_2) = \frac{\mathrm{E}_{2j}[\tilde{v}] - p_2 + c_s I_{2j}}{\frac{\partial p_2}{\partial x_{2j}} + 2\psi_2}. \tag{A.2}$$

<u>Date 2 with Information Arrival</u>

We now establish the existence of a symmetric Nash equilibrium conditional on an information arrival at date 2. (In what follows, time subscripts are dropped where this creates no ambiguity.) Suppose a given pessimist j conjectures the other traders follow symmetric linear strategies, where the strategies can differ by "type" (pessimist vs. optimist). Specifically, trader j conjectures:

$$x_k = \begin{cases} \beta_L(\overline{v}_L - p - c_s) & \text{for all } k = L \text{ and } k \neq j \\ \beta_H(\overline{v}_H - p) & \text{for all } k = H. \end{cases} \tag{A.3}$$

Under this conjecture, pessimist j perceives the market clearing condition to be

$$x_j + (N-1)x_L + N x_H = 2NX.$$

Substituting from (A.3) and solving for p,

$$p = \frac{(N-1)\beta_L(\overline{v}_L + c_s) + N\beta_H\overline{v}_H}{(N-1)\beta_L + N\beta_H} + \frac{x_j + 2NX}{(N-1)\beta_L + N\beta_H}.$$

Therefore, trader j perceives

$$\frac{\partial p}{\partial x_j} = \frac{1}{(N-1)\beta_L + N\beta_H} \tag{A.4.1}$$

if he is a pessimist (i.e., $j = L$). Similar reasoning implies

$$\frac{\partial p}{\partial x_j} = \frac{1}{N\beta_L + (N-1)\beta_H} \tag{A.4.2}$$

if $j = H$. Combining (A.4.1) and (A.4.2) with (A.2) implies that if trader j conjectures the others follow the strategies in (A.3), then trader j's optimal strategy is

$$x_{2j}^* = \begin{cases} \frac{\overline{v}_L - p + c_s I_L}{\frac{1}{(N-1)\beta_L + N\beta_H} + 2\psi} & \text{if } j = L \\ \frac{\overline{v}_H - p + c_s I_H}{\frac{1}{N\beta_L + (N-1)\beta_H} + 2\psi} & \text{if } j = H. \end{cases} \tag{A.5}$$

This is the same form as the conjectured strategies in Eq. (A.3) provided that $I_L = 1$ and $I_H = 0$. Thus, if trader j conjectures that others follow the strategies in (A.3), it is optimal for trader j to follow the same strategy if the following conditions are satisfied:

$$\frac{1}{\beta_L} = \frac{1}{(N-1)\beta_L + N\beta_H} + 2\psi \quad \text{and} \quad \frac{1}{\beta_H} = \frac{1}{N\beta_L + (N-1)\beta_H} + 2\psi \tag{A.6.1}$$

$$\beta_L > 0 \quad \text{and} \quad \beta_H > 0 \tag{A.6.2}$$

$$\beta_L(\overline{v}_L - p^* + c_s) < 0 \quad \text{and} \quad \beta_H(\overline{v}_H - p^*) > 0. \tag{A.6.3}$$

Eq. (A.6.1) says that pessimists share a common strategy coefficient, and optimist share a common strategy coefficient. Eq. (A.6.2) ensures the second-order condition is satisfied for both trader types. Eq. (A.6.3) says that pessimists hold short positions ($I_L = 1$) and optimists hold long positions ($I_H = 0$) at the price, p^*, that clears the market. Therefore, a symmetric equilibrium exists with optimists taking long positions and pessimists short positions if (A.6.1) - (A.6.3) are satisfied.

First, we find a solution to the pair of equations in (A.6.1) that satisfies (A.6.2). Rewrite the equations in (A.6.1) as

$$\begin{aligned} N\beta_L + N\beta_H &= 2\beta_L + 2\psi\beta_L\left[(N-1)\beta_L + N\beta_H\right] \\ N\beta_L + N\beta_H &= 2\beta_H + 2\psi\beta_H\left[(N-1)\beta_H + N\beta_L\right]. \end{aligned} \tag{A.7}$$

Equating these

$$2\beta_H \{1 + \psi [(N-1)\beta_H + N\beta_L]\} = 2\beta_L \{1 + \psi [(N-1)\beta_L + N\beta_H]\}$$
$$\beta_H - \beta_L = \psi \{\beta_L [(N-1)\beta_L + N\beta_H] - \beta_H [(N-1)\beta_H + N\beta_L]\}$$
$$\beta_H - \beta_L = \psi(N-1)(\beta_L + \beta_H)(\beta_L - \beta_H).$$

Either $\beta_H - \beta_L = 0$ or $\beta_H + \beta_L = \frac{-1}{\psi(N-1)}$. The latter possibility is not consistent with (A.6.2). Consider then the possibility that $\beta_H = \beta_L = \beta$. Using either of the equations in (A.7) to solve for β:

$$2N\beta = 2\beta + 2\psi\beta[(2N-1)\beta]$$
$$\beta = \frac{N-1}{\psi(2N-1)} > 0 \tag{A.8}$$

Thus, (A.6.1) has a unique solution that satisfies (A.6.2).

To verify (A.6.3), we need an expression for the market-clearing price. In the proposed equilibrium, the market-clearing condition and price are

$$N\beta(\overline{v}_L - p^* + c_s) + N\beta(\overline{v}_H - p^*) = 2NX$$
$$p^* = \frac{1}{2}(\overline{v}_L + \overline{v}_H) + \frac{c_s}{2} - \frac{X}{\beta}. \tag{A.9}$$

The equilibrium price equals the consensus valuation at date 2 plus an upward bias equal to one half the short-sale cost, minus a risk premium. Using (A.9), the condition in (A.6.3) that pessimists have short positions in equilibrium can be written as

$$\beta \left[\overline{v}_L - \frac{1}{2}(\overline{v}_L + \overline{v}_H + c_s) + \frac{X}{\beta} + c_s\right] < 0,$$

or equivalently as

$$\overline{v}_H - \overline{v}_L > c_s + 2\frac{X}{\beta}. \tag{A.10}$$

The difference between optimists' and pessimists' beliefs about value must be greater than the short sale cost plus two times the price discount due to risk in order for pessimists to hold short positions in equilibrium. The condition in (A.6.3) that optimists hold long positions in equilibrium can be written as

$$\beta \left[\overline{v}_H - \frac{1}{2}(\overline{v}_L + \overline{v}_H + c_s) + \frac{X}{\beta}\right] > 0,$$

or equivalently as

$$\overline{v}_H - \overline{v}_L > c_s - 2\frac{X}{\beta} \tag{A.11}$$

which is implied by (A.10).

Therefore, if an information arrival at date 2 generates divergence in beliefs that is large enough to satisfy (A.10), then there is a symmetric Nash equilibrium in linear strategies at date 2 where optimists hold long positions and pessimists hold short positions. In that equilibrium, the market clearing price has the form:

$$\text{price} = \text{consensus beliefs} + \text{bias} - \text{risk premium}.$$

To solve for strategies at date 1 below, we need expressions for equilibrium (i.e., optimized) expected utility at date 2 for each trader type. Using Eq. (A.1), expected optimist utility is

$$\begin{aligned} J_{2H}(p_2^*) &\equiv J_{2H}|_{p_2^*} \\ &= (\overline{v}_H - p_2^*)x_{2H}^*(p_2^*) - \psi_2 x_{2H}^*(p_2^*)^2 \\ &= \beta(1-\psi\beta)\left(\overline{v}_H - p_2^*\right)^2, \end{aligned} \tag{A.12.1}$$

and expected pessimist utility is

$$\begin{aligned} J_{2L}(p_2^*) &\equiv J_{2L}|_{p_2^*} \\ &= (\overline{v}_L - p_2^* + c_s)\beta(\overline{v}_L - p_2^* + c_s) - \psi_2\beta^2(\overline{v}_L - p_2^* + c_s)^2 \\ &= \beta(1-\psi\beta)(\overline{v}_L - p_2^* + c_s)^2, \end{aligned} \tag{A.12.2}$$

where β is defined in (A.8) and p_2^* is defined in (A.9). The key observation is that neither $J_{2H}(p_2^*)$ nor $J_{2L}(p_2^*)$ depend on choices or prices at date 1, so $J_{2H}(p_2^*)$ and $J_{2L}(p_2^*)$ are irrelevant to the optimization of date-1 holdings.

Date 2 without Information Arrival

If no information arrives at date 2, all traders continue to believe $E[\tilde{v}] = v_o$. If trader j conjectures the other $2N-1$ traders follow a strategy of the form

$$\hat{x}_k = \hat{\beta}(v_o - \hat{p}) \qquad \text{for all } k \neq j,$$

then his perception of market clearing is

$$\hat{x}_j + (2N-1)\hat{\beta}(v_o - \hat{p}) = 2NX$$
$$\hat{p} = v_o + \frac{1}{(2N-1)\hat{\beta}}\left(\hat{x}_j - 2NX\right)$$

and therefore

$$\frac{\partial \hat{p}}{\partial \hat{x}_j} = \frac{1}{(2N-1)\hat{\beta}}.$$

By (A.2), trader j's optimal strategy is then

$$\hat{x}_j^* = \frac{v_o - \hat{p} + c_s I_j}{\frac{1}{(2N-1)\hat{\beta}} + 2\psi},$$

which is the same form as the conjecture above provided that $I_j = 0$ (i.e., trader j holds a long position). A symmetric equilibrium with all traders holding long positions will exist if the following conditions are satisfied:

$$\frac{1}{\hat{\beta}} = \frac{1}{(2N-1)\hat{\beta}} + 2\psi \tag{A.13.1}$$
$$\hat{\beta} > 0 \tag{A.13.2}$$
$$\hat{\beta}(v_o - \hat{p}^*) > 0. \tag{A.13.3}$$

The interpretations of these equations are analogous to those of (A.6.1) - (A.6.3). Solving (A.13.1) implies

$$\hat{\beta} = \frac{N-1}{\psi(2N-1)} > 0 \tag{A.14}$$

which satisfies (A.13.2). The market-clearing price derives from the market clearing condition

$$2N\hat{x}_j^*(\hat{p}^*) = 2NX$$
$$\hat{p}^* = v_o - \frac{X}{\beta}, \tag{A.15}$$

so (A.13.3) is satisfied because

$$\hat{\beta}(v_o - \hat{p}^*) = \hat{\beta}\left(v_o - v_o + \frac{X}{\hat{\beta}}\right) = X > 0.$$

Thus, if information does not arrive, there is a symmetric Nash equilibrium in linear strategies at date 2 where all traders hold (identical) long positions. In this equilibrium, trader j's expected utility is

$$\begin{aligned} J_{2o}(\hat{p}_2^*) &\equiv J_{2o}|_{\hat{p}_2^*} \\ &= (v_o - \hat{p}_2^*)\, \hat{x}_2^*(\hat{p}_2^*) - \psi \hat{x}_2^*(\hat{p}_2^*)^2 \\ &= \hat{\beta}\,(v_o - \hat{p}_2^*)^2 - \psi\hat{\beta}^2\,(v_o - \hat{p}_2^*)^2 \\ &= \hat{\beta}\left(1 - \psi\hat{\beta}\right)(v_o - \hat{p}_2^*)^2\,, \end{aligned}$$

where $\hat{\beta}$ and $\hat{p}_2^*$ are given in (A.14) and (A.15). The key observation is that $J_{2o}(\hat{p}_2^*)$ does not depend on choices or prices at date 1, so $J_{2o}(\hat{p}_2^*)$ is irrelevant to the optimization of date-1 holdings.

Date 1

At date 1, trader j seeks to maximize expected long-run utility, given a probability q that information will arrive next period and generate divergent beliefs, and a probability $1 - q$ that information will not arrive:

$$\begin{aligned} \max_{x_{1j}(\cdot)} \quad & q\ \mathrm{E}_1\left[(p_2^* - p_1)x_{1j}(p_1) + \tilde{J}_{2j}(p_2^*)\right] + (1-q)\ \mathrm{E}_1\left[(\hat{p}_2^* - p_1)x_{1j}(p_1) + \tilde{J}_{2o}(\hat{p}_2^*)\right] \\ & + c_s I_{1j} x_{1j}(p_1) - \psi_1 x_{1j}(p_1)^2. \end{aligned}$$

To simplify notation, the term involving the short-position indicator, I_{1j}, is suppressed. It eventually drops out (just as in the analysis of date 2 when no information arrives) because traders are identical at date 1.[1] Trader j's problem is therefore

$$\begin{aligned} \max_{x_{1j}(\cdot)} \quad & \left(q\mathrm{E}_1\left[p_2^*\right] + (1-q)\mathrm{E}_1\left[\hat{p}_2^*\right] - p_1\right) x_{j1}(p_1) - \psi_1 x_{1j}(p_1)^2 \\ & + q\mathrm{E}_1\left[\tilde{J}_{2j}(p_2^*)\right] + (1-q)\mathrm{E}_1\left[\tilde{J}_{2o}(\hat{p}_2^*)\right]. \end{aligned} \tag{A.16}$$

Now, $\mathrm{E}_1\left[\tilde{J}_{2o}(\hat{p}_2^*)\right] = J_{2o}(\hat{p}_2^*)$ because no new information arrives between dates 1 and 2 in the "hat" case. For the case where information does arrive, there is equal liklihood that trader j will adopt optimistic and pessimistic beliefs, so

$$\mathrm{E}_1\left[\tilde{J}_{2j}(p_2^*)\right] = \frac{1}{2}\mathrm{E}_1\left[\tilde{J}_{2H}(p_2^*)\right] + \frac{1}{2}\mathrm{E}_1\left[\tilde{J}_{2L}(p_2^*)\right].$$

[1] When traders are identical and there is a symmetric equilibrium, all traders hold their share of per-capita supply of the security, which results in long positions because the security is in positive net supply.

We established earlier that $J_{2o}(\hat{p}_2^*)$, $J_{2H}(p_2^*)$ and $J_{2L}(p_2^*)$ are all independent of date-1 holdings and prices, so these terms in (A.16) are irrelevant to the date-1 optimization of (A.16). Consequently, the family of first-order conditions that characterize $x_{1j}(\cdot)$ is

$$q\mathrm{E}_1\left[p_2^*\right] + (1-q)\mathrm{E}_1\left[\hat{p}_2^*\right] - p_1 - \frac{\partial p_1}{\partial x_{1j}} x_{1j} - 2\psi_1 x_{1j} = 0$$

for each p_1, noting that both p_2^* and $\hat{p}_2^*$ are independent of date-1 choices. It will be apparent later that the second order condition for a maximum, $\frac{\partial p_1}{\partial x_{1j}} + 2\psi_1 > 0$, is satisfied in equilibrium. Rearranging the FOC yields an expression for trader j's optimal demand schedule at date 1:

$$x_{1j}^*(p_1) = \frac{q\mathrm{E}_1\left[p_2^*\right] + (1-q)\mathrm{E}_1\left[\hat{p}_2^*\right] - p_1}{\frac{\partial p_1}{\partial x_{1j}} + 2\psi_1}. \tag{A.17}$$

Suppose trader j conjectures that other traders follow the strategy

$$x_{1k} = \gamma\left(v_o + q\frac{c_s}{2} - \frac{X}{\beta} - p_1\right) \qquad \text{for all } k \neq j. \tag{A.18}$$

Then trader j's perception of market clearing is that

$$x_{1j} + (2N-1)\gamma\left(v_o + q\frac{c_s}{2} - \frac{X}{\beta} - p_1\right) = 2NX$$
$$p_1 = v_o + q\frac{c_s}{2} - \frac{X}{\beta} + \frac{x_{1j} - 2NX}{(2N-1)\gamma}.$$

and so

$$\frac{\partial p_1}{\partial x_{1j}} = \frac{1}{(2N-1)\gamma}. \tag{A.19}$$

Substituting (A.19) into (A.17) and using the fact that j has a rational prior about how his beliefs will change if information arrives (i.e., $v_o = \mathrm{E}_1\left[\frac{1}{2}\overline{v}_L + \frac{1}{2}\overline{v}_H\right]$) implies that trader j's demand schedule is

$$x_{1j}^*(p_1) = \frac{q\left(v_o + \frac{c_s}{2} - \frac{X}{\beta}\right) + (1-q)\left(v_o - \frac{X}{\hat{\beta}}\right) - p_1}{\frac{1}{(2N-1)\gamma} + 2\psi_1} = \frac{v_o + q\frac{c_s}{2} - \frac{X}{\beta} - p_1}{\frac{1}{(2N-1)\gamma} + 2\psi_1}. \tag{A.20}$$

where we have used the fact that $\hat{\beta} = \beta$ in the two date-2 equilibria above. Eq. (A.20) is the same form as the conjecture in (A.18). A symmetric equilibrium with all traders holding long positions will therefore exist if the following conditions are satisfied:

$$\frac{1}{\gamma} = \frac{1}{(2N-1)\gamma} + 2\psi_1 \tag{A.21.1}$$

$$\gamma > 0 \tag{A.21.2}$$

$$\gamma\left(v_o + q\frac{c_s}{2} - \frac{X}{\beta} - p_1^*\right) > 0. \tag{A.21.3}$$

The interpretation of these is the same as (A.13.1) - (A.13.3). Solving (A.21.1) implies

$$\gamma = \frac{N-1}{\psi_1(2N-1)} > 0 \tag{A.21.1a}$$

which satisfies (A.21.2). The market-clearing price satisfies

$$2Nx_{1j}^*(p_1^*) = 2NX$$

$$p_1^* = v_o + q\frac{c_s}{2} - \frac{X}{\beta} - \frac{X}{\gamma}, \tag{A.22}$$

so (A.21.3) is satisfied because

$$\gamma\left(v_o + q\frac{c_s}{2} - \frac{X}{\beta} - p_1^*\right) = X > 0.$$

Therefore, there is a symmetric Nash equilibrium in linear strategies at date 1 where all traders hold identical long positions. In that equilibrium, the market clearing price (A.22) has the form:

$$\text{price} = \text{consensus beliefs} + (q \times \text{date-2 bias}) - \text{risk premium}.$$

The bias here is exactly the bias in the date-2 price conditional on an information arrival, scaled by the probability of an information arrival.

Solving for ψ_1, ψ_2 and $\underline{d}$

Since traders maximize mean-variance preferences, the quadratic cost modeled above using ψ parameters arises from risk aversion. The common risk aversion parameter is α, so the equivalence between the quadratic costs and the variance component of mean-variance preferences is as follows:

$$\psi_t\left(x_{tj}^*\right)^2 = \alpha\mathrm{Var}_{tj}\left[\tilde{\pi}_{jt}\right] = \alpha\mathrm{Var}_{tj}\left[\tilde{p}_{t+1} - p_t\right]\left(x_{tj}^*\right)^2$$

$$\psi_t = \alpha\mathrm{Var}_{tj}\left[\tilde{p}_{t+1} - p_t\right]$$

so, for each sub-tree of the game,

$$\psi_t = \begin{cases} \alpha \text{Var}\left[\tilde{v} - p_2^*\right] & \text{if info arrives at } t = 2 \\ \alpha \text{Var}\left[\tilde{v} - \hat{p}_2^*\right] & \text{if info does not arrive at } t = 2 \\ \alpha \text{Var}\left[\tilde{p}_2 - p_1^*\right] & \text{if } t = 1. \end{cases}$$

where $\tilde{p}_2$ equals p_2^* with probability q and $\hat{p}_2^*$ with probability $1 - q$. We derive explicit expressions for the equilibrium values of the ψ parameters next.

Case 1: If information arrives at $t = 2$ then

$$p_2^* = \overline{\overline{v}} + \frac{c_s}{2} - \frac{X}{\beta}$$
$$\tilde{v} - p_2* = \tilde{v} - \overline{\overline{v}} - \frac{c_s}{2} - \frac{X}{\beta}.$$

At $t = 2$, agent j knows $\overline{\overline{v}}$ because he knows his own beliefs $\overline{v}_j$ and those of the other group. Therefore,

$$\text{Var}_{2j}\left[\tilde{v} - p_2^*\right] = \text{Var}_2\left[\tilde{v}\right] = \sigma_v^2.$$

Case 2: If information does not arrive at $t = 2$ then

$$\hat{p}_2^* = v_o - \frac{X}{\beta}$$
$$\tilde{v} - \hat{p}_2^* = \tilde{v} - v_o + \frac{X}{\beta}$$

therefore

$$\text{Var}_{2j}\left[\tilde{v} - \hat{p}_2^*\right] = \text{Var}_2\left[\tilde{v}\right] = \sigma_v^2.$$

Cases 1 and 2 together imply that ψ_2 is the same in both date-2 subtrees:

$$\psi_2 = \alpha\sigma_v^2. \tag{A.23}$$

Case 3: Recall from above that the definition of $\tilde{p}_2$ is

$$\tilde{p}_2 = \begin{cases} p_2^* = \overline{\overline{v}} + \frac{c_s}{2} - \frac{X}{\beta} & \text{with probability } q \\ \hat{p}_2^* = v_o - \frac{X}{\beta} & \text{with probability } 1 - q. \end{cases}$$

Subtracting Eq. (A.22) from these expressions we have

$$\tilde{p}_2 - p_1^* = \begin{cases} \overline{\overline{v}} - v_o + (1 - q)\frac{c_s}{2} + \frac{X}{\gamma} & \text{with probability } q \\ -q\frac{c_s}{2} + \frac{X}{\gamma} & \text{with probability } 1 - q. \end{cases} \tag{A.24}$$

The form of this is

$$\tilde{Y} = \begin{cases} \tilde{Z} & \text{with probability } q \\ K & \text{with probability } 1-q, \end{cases}$$

where K is a constant, and $\tilde{Z}$ is a random variable conditional on the top state occurring. The variance of a random variable of this form is

$$\text{Var}\left[\tilde{Y}\right] = q\text{Var}\left[\tilde{Z}\right] + q(1-q)\left(E\left[\tilde{Z}\right] - K\right)^2. \tag{A.25}$$

Substituting from (A.24) into (A.25) yields

$$\text{Var}_1\left[\tilde{p}_2 - p_1^*\right] = q\text{Var}_1\left[\overline{\overline{v}}\right] + q(1-q)\left(\frac{c_s}{2}\right)^2,$$

so

$$\psi_1 = \alpha q\left\{\text{Var}_1\left[\overline{\overline{v}}\right] + (1-q)\frac{c_s^2}{4}\right\}. \tag{A.26}$$

The differences between the expressions (A.23) and (A.26) arise because the uncertainty resolved between dates 1 and 2 relates to whether or not information arrives that shifts the mean of the distribution of $\tilde{v}$ and by how much, whereas the uncertainty resolved between dates 2 and 3 is the realization of $\tilde{v}$.

Finally, combining (A.10) and (A.23) yields an expression, in terms of exogenous variables, for the extent of divergence in beliefs required to support an equilibrium where pessimists hold short positions and optimists long positions:

$$d = \overline{v}_H - \overline{v}_L > c_s + 2\alpha\sigma_v^2 X\left(\frac{2N-1}{N-1}\right) \equiv \underline{d}. \tag{A.27}$$

Table 4A
Small Firms and Raw Returns of High and Low Idiosyncratic Volatility Portfolios

Each month between January 1963 and December 2006, 24 (j=1,…,24) cross-sectional regressions of the following form are estimated:

$$R_{it} = b_{0jt} + b_{1jt}SMALL_{i,t-j} + b_{2jt}LV4L_{i,t-j} + b_{3jt}HVOL_{i,t-j} + b_{4jt}SMALL_{i,t-j} * LVOL_{i,t-j} + b_{5jt}SMALL_{i,t-j} * HVOL_{i,t-j} + e_{ijt}$$

where R_{it} is the return to stock i in month t, $LVOL_{i,t-j}$ ($HVOL_{i,t-j}$) is the low (high) idiosyncratic volatility dummy that takes the value of 1 if the idiosyncratic volatility for stock i is ranked in the top (bottom) 20% in month $t-j$, and zero otherwise. $SMALL_{i,t-j}$ is a dummy that takes the value of 1 if firm i's market capitalization is below the median of the sample in month $t-j$, and is zero otherwise. The coefficient estimates of a given independent variable are for j=1 for columns labeled (p=0,K=1), and averaged over j=2 to 12 for columns labeled (p=1,K=11), and j=13 to 24 for columns labeled (p=12,K=12). The numbers reported in the table are the time-series averages of these averages. They are in percent per month. The accompanying t-statistics are calculated from the time series. Penny stocks (price < $5) are excluded. *NOBS* is the average number of stocks used in the monthly cross-sectional regressions.

	Column 1 (p=0,K=1)	Column 2 (p=0,K=1) Jan. excluded	Column 3 (p=1,K=11)	Column 4 (p=1,K=11) Jan. excluded	Column 5 (p=12,K=12)	Column 6 (p=12,K=12) Jan. excluded
Raw Returns, IVOL20D (NOBS=3990)						
Intercept	1.22	1.00	1.08	0.86	1.02	0.75
	(5.03)	(4.05)	(4.53)	(3.54)	(4.13)	(3.03)
SMALL	0.31	0.09	0.22	-0.02	0.18	-0.05
	(3.19)	(0.93)	(2.32)	(-0.18)	(1.91)	(-0.58)
LVOL	-0.12	-0.04	-0.01	0.09	-0.05	0.07
	(-0.98)	(-0.28)	(-0.12)	(0.73)	(-0.43)	(0.61)
HVOL	-1.34	-1.41	-0.58	-0.82	-0.23	-0.55
	(-7.20)	(-7.16)	(-3.42)	(-4.86)	(-1.43)	(-3.64)
SMALL*LVOL	-0.27	-0.16	-0.09	0.00	-0.01	0.09
	(-3.96)	(-2.31)	(-1.62)	(0.07)	(-0.20)	(1.55)
SMALL*HVOL	0.00	-0.04	0.20	0.24	0.19	0.26
	(-0.03)	(-0.31)	(2.26)	(2.53)	(2.30)	(3.05)
Raw Returns, IVOL200D (NOBS=3534)						
Intercept	1.17	0.95	1.13	0.89	1.05	0.77
	(4.62)	(3.67)	(4.50)	(3.50)	(4.05)	(2.98)
SMALL	0.25	0.03	0.22	0.01	0.20	-0.01
	(2.49)	(0.36)	(2.30)	(0.11)	(2.13)	(-0.14)
LVOL	-0.06	0.06	-0.07	0.06	-0.06	0.08
	(-0.40)	(0.38)	(-0.53)	(0.43)	(-0.48)	(0.60)
HVOL	-0.82	-1.00	-0.57	-0.86	-0.45	-0.83
	(-3.29)	(-3.78)	(-2.62)	(-3.95)	(-2.12)	(-4.08)
SMALL*LVOL	-0.16	-0.05	-0.05	0.04	-0.07	0.02
	(-2.00)	(-0.68)	(-0.76)	(0.51)	(-0.94)	(0.28)
SMALL*HVOL	0.24	0.25	0.33	0.34	0.26	0.30
	(1.46)	(1.49)	(2.73)	(2.67)	(2.13)	(2.34)

Table 4A (Continued)

Raw Returns, IVOL60M (NOBS=2420)						
Intercept	1.26	1.06	1.23	1.03	1.14	0.91
	(5.53)	(4.58)	(5.47)	(4.52)	(5.01)	(3.98)
SMALL	0.24	0.01	0.24	0.01	0.20	-0.02
	(2.57)	(0.12)	(2.60)	(0.16)	(2.26)	(-0.21)
LVOL	-0.13	-0.05	-0.14	-0.05	-0.14	-0.03
	(-1.11)	(-0.42)	(-1.26)	(-0.46)	(-1.23)	(-0.27)
HVOL	-0.28	-0.42	-0.43	-0.62	-0.35	-0.63
	(-1.43)	(-2.05)	(-2.41)	(-3.40)	(-2.07)	(-3.79)
SMALL*LVOL	-0.03	0.04	-0.07	0.00	-0.05	0.03
	(-0.38)	(0.53)	(-1.01)	(-0.06)	(-0.71)	(0.37)
SMALL*HVOL	-0.02	-0.02	0.15	0.11	0.14	0.17
	(-0.20)	(-0.20)	(1.50)	(1.05)	(1.28)	(1.45)

Table 4B

	Column 1 (p=0,K=1)	Column 2 (p=0,K=1) Jan. excluded	Column 3 (p=1,K=11)	Column 4 (p=1,K=11) Jan. excluded	Column 5 (p=12,K=12)	Column 6 (p=12,K=12) Jan. excluded
Raw Returns, IVOL20D (NOBS=3990)						
Intercept	1.31	1.17	1.05	0.91	0.92	0.71
	(4.20)	(3.57)	(3.47)	(2.89)	(2.95)	(2.20)
ILLIQ	0.02	-0.08	0.21	0.10	0.15	0.07
	(0.18)	(-0.56)	(1.65)	(0.75)	(1.30)	(0.60)
LVOL	-0.08	0.00	0.13	0.22	0.08	0.20
	(-0.41)	(-0.02)	(0.74)	(1.20)	(0.47)	(1.17)
HVOL	-1.59	-1.67	-0.94	-1.21	-0.35	-0.75
	(-5.70)	(-5.69)	(-3.74)	(-4.85)	(-1.47)	(-3.45)
ILLIQ *LVOL	-0.11	-0.06	-0.12	-0.09	-0.04	-0.04
	(-1.02)	(-0.52)	(-1.32)	(-0.96)	(-0.50)	(-0.40)
ILLIQ *HVOL	0.25	0.24	0.48	0.53	0.25	0.37
	(1.32)	(1.26)	(3.56)	(3.73)	(1.88)	(3.03)
Raw Returns, IVOL200D (NOBS=3534)						
Intercept	1.25	1.12	1.08	0.94	0.93	0.72
	(3.96)	(3.38)	(3.47)	(2.91)	(2.92)	(2.18)
ILLIQ	0.01	-0.09	0.22	0.11	0.18	0.11
	(0.11)	(-0.62)	(1.65)	(0.83)	(1.39)	(0.83)
LVOL	0.05	0.13	0.14	0.24	0.10	0.25
	(0.23)	(0.62)	(0.75)	(1.20)	(0.54)	(1.29)
HVOL	-0.92	-1.11	-0.91	-1.26	-0.57	-1.08
	(-2.55)	(-2.95)	(-3.00)	(-4.21)	(-1.82)	(-3.75)
ILLIQ *LVOL	0.00	0.06	-0.12	-0.10	-0.08	-0.08
	(0.04)	(0.50)	(-1.23)	(-0.99)	(-0.81)	(-0.81)
ILLIQ *HVOL	0.29	0.30	0.63	0.66	0.33	0.48
	(1.22)	(1.21)	(3.82)	(3.79)	(1.78)	(2.67)

Table 4B (Continued)

Raw Returns, IVOL60M (NOBS=2420)						
Intercept	1.34	1.26	1.22	1.14	1.04	0.90
	(4.71)	(4.21)	(4.35)	(3.89)	(3.61)	(2.99)
ILLIQ	0.03	-0.09	0.23	0.10	0.18	0.09
	(0.23)	(-0.76)	(1.95)	(0.86)	(1.55)	(0.73)
LVOL	-0.05	0.00	0.01	0.07	0.01	0.10
	(-0.28)	(-0.01)	(0.09)	(0.40)	(0.05)	(0.59)
HVOL	-0.38	-0.58	-0.71	-0.95	-0.50	-0.84
	(-1.29)	(-1.87)	(-2.64)	(-3.57)	(-1.99)	(-3.42)
ILLIQ *LVOL	0.11	0.21	-0.10	-0.02	-0.05	0.01
	(1.14)	(2.27)	(-1.13)	(-0.21)	(-0.56)	(0.08)
ILLIQ *HVOL	0.00	0.06	0.33	0.34	0.26	0.32
	(-0.02)	(0.33)	(2.40)	(2.39)	(1.59)	(1.88)

Table 6A

Risk Adjusted Return, IVOL20D (NOBS=3549)						
	Column 1 (p=0,K=1)	Column 2 (p=0,K=1) Jan. excluded	Column 5 (p=1,K=11)	Column 6 (p=1,K=11) Jan. excluded	Column 7 (p=12,K=12)	Column 8 (p=12,K=12) Jan. excluded
Intercept	-0.10	-0.16	-0.06	-0.13	-0.02	-0.10
	(-1.42)	(-2.14)	(-0.75)	(-1.76)	(-0.24)	(-1.53)
LVOL	0.13	0.14	0.02	0.02	-0.04	-0.07
	(1.91)	(1.97)	(0.36)	(0.35)	(-0.69)	(-1.15)
HVOL	-1.12	-0.94	-0.14	-0.16	0.10	0.01
	(-5.73)	(-4.77)	(-1.18)	(-1.36)	(0.78)	(0.08)
LCOV*LVOL	0.03	0.06	0.07	0.11	0.13	0.20
	(0.42)	(0.95)	(1.31)	(2.01)	(2.24)	(3.31)
LCOV*HVOL	0.32	0.21	-0.53	-0.62	-0.54	-0.68
	(1.40)	(0.88)	(-3.92)	(-4.53)	(-4.05)	(-5.11)
ILLIQ*LVOL	-0.22	-0.20	0.15	0.17	0.15	0.22
	(-2.70)	(-2.40)	(2.36)	(2.82)	(2.11)	(3.35)
ILLIQ*HVOL	-0.02	-0.12	0.63	0.61	0.59	0.71
	(-0.11)	(-0.55)	(4.96)	(5.04)	(4.11)	(5.92)
Risk Adjusted Return, TVOL(NOBS=3087)						
Intercept	-0.10	-0.15	-0.04	-0.10	0.02	-0.05
	(-1.28)	(-1.79)	(-0.47)	(-1.28)	-0.27	(-0.70)
LVOL	0.13	0.19	0.01	0.03	0.05	0.04
	(1.56)	(2.23)	(0.08)	(0.41)	(0.66)	(0.53)
HVOL	-0.06	-0.08	-0.09	-0.12	-0.06	-0.15
	(-0.55)	(-0.71)	(-0.85)	(-1.19)	(-0.49)	(-1.23)
LCOV*LVOL	0.08	0.11	0.16	0.22	0.10	0.18
	(0.98)	(1.42)	(2.30)	(3.18)	(1.26)	(2.32)
LCOV*HVOL	-0.07	-0.13	-0.39	-0.49	-0.45	-0.53
	(-0.49)	(-0.83)	(-2.96)	(-3.85)	(-4.08)	(-4.61)
ILLIQ*LVOL	0.07	0.02	0.21	0.20	0.15	0.18
	(0.89)	(0.22)	(3.16)	(2.98)	(2.04)	(2.55)
ILLIQ*HVOL	-0.93	-1.00	0.01	-0.04	0.33	0.33
	(-5.69)	(-5.89)	(0.11)	(-0.33)	(2.32)	(2.50)

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