Unresolved Issues in Modeling Credit Risky Assets

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Abstract

There are many unresolved issues in the modeling and calibration of credit risky instruments that directly affect pricing and risk management: from the modeling of the determinants of recovery rates, credit spreads, contagion, default dependence, to the testing of models. In this paper we describe the current state of understanding regarding these different areas and identify some of the unresolved issues.

To quantify the risk of a portfolio of credit risky instruments for regulatory purposes, it is necessary to describe the probability distribution of possible portfolio values at some specified horizon. This involves two distinct steps. First, at the horizon it is necessary to determine the value of the portfolio in a given state of nature, implying that we must have a valuation model that can price the different types of credit sensitive instruments in the portfolio. The second step involves describing the probability distribution of portfolio values under the natural measure.

For pricing credit sensitive instruments, two different approaches can be used: either structural based models or reduced form models. In either case two natural questions arise. First, can the chosen model describe the existing term structure of spreads and second, if the model is used for dynamic hedging, can it accurate describe the evolution of spreads[1]. Regulators also raise the question about the testing of a credit pricing model, given the limited default data. All models must be calibrated. For loans and bond related instruments, bond spreads – if available - are the natural choice, while credit default swap prices are often used for credit derivatives[2]. In either case information about the recovery in the event of default is usually a necessary input. The criterion used for calibration depends on whether the model is simply required to match a chosen set of prices or whether the time series properties of the model are important.

Apart from the issue of pricing individual instruments, it is necessary to describe the probability distribution of a portfolio as it evolves over time. In the original CreditMetrics framework, it is assumed that the credit quality of an obligor could have one of n possible credit ratings including the state of default. The term structure of credit spreads was assumed to be deterministic. At the horizon it is necessary to enumerate every possible combination of credit qualities. The Merton (1974) model is employed and a multivariate normal distribution for asset returns assumed. It is found that small changes in correlation could have a major impact on the measured risk of the portfolio and its economic capital. In applying this framework it is necessary to specify a ratings transition matrix over the horizon that reflects the current state of the economy. The modeling of default dependence in this framework is achieved by combining the structural model of Merton (1974) with a rating transition matrix. The original CreditMetrics method of modeling default dependence with a multi-variate normal probability distribution was

[1] In some markets dynamic hedging is rarely used, so the ability of the model to describe the evolution of spreads is of little relevance. It is only necessary to generate a term structure of spreads.
[2] The use of bond spreads for loan pricing is explored in Turnbull (2003).

generalized by Li (2000), who introduced a normal copula function[3]. Since then the use of copula functions has become the standard way of modeling default dependence, in risk management and for pricing structural products.

In the New Basel Capital Accord, the foundation internal ratings based (IRB) approach allows banks to develop and use their own internal ratings. The loss given default (LGD) and exposure at default (EAD) are fixed and based on values set by the regulators. Under the advanced IRB approach, banks can use their own loss given default and exposure at default values, subject to regulatory approval. The flexibility to determine these three parameters so that they reflect the nature of a bank's portfolio, provides a bank with the incentives to move to the advanced IRB approach. However a bank's specification of these parameters is subject to supervisory review. This raise the question of what constitutes a reasonable approach to estimating expected loss. The issue of model testing is of central importance to financial regulators and to financial institutions employing the advanced internal ratings based methodology. Consequently it is important to recognize what we actually know about some of the issues in credit and what remain unresolved issues.

The purpose of this paper is to examine some of the major unresolved areas in credit pricing and risk management. This paper is not intended to be a comprehensive review with its focus confined to recent academic findings. In Section 2 we examine our current knowledge about modeling two of the important components of expected loss: the loss given default and the probability of default. To model the loss given default requires that we describe the determinants of the recovery rate. We start with reviewing the modeling of recovery rates and then describe recent advances in academic default prediction models, which can be extended to multi-period horizons.

In Section 3 we discuss current empirical knowledge of the behavior of credit spreads. We examine the ability of structural models to examine the cross sectional and time series variation in credit spreads and the ability of reduced form models to examine the time series variation in spreads.

[3] A copula function links together the individual marginal distributions to form the multi-variate distribution. Copula functions will be described in more detail later.

In Section 4 we consider some of the problems that can arise in the pricing of single name credit default swap instruments. We first consider how calibration can affect pricing and hedging. We start by considering a digital credit default swap. The great advantage of a digital is that the payoff if a credit event occurs is independent of the recovery rate on the reference entity. Digital credit swaps are only a small part of the credit swap market and consequently we use regular credit default swaps to calibrate our model for pricing. Unfortunately this form of calibration causes problems, especially if we hedge our position. Contrary to the theoretic pricing model, the calibrated price is dependent on the recovery rate. Second, we consider the specification of the credit event that we are modeling. For regular credit default swaps, the protection buyer owns a cheapest to deliver option. If a credit event occurs, the protection buyer has some choice over the asset to deliver, assuming physical delivery. The set of deliverable obligations must satisfy certain restrictions with respect to maturity[4] and be pari passu, usually with respect to senior unsecured debt. This is especially important if the credit event is a restructuring. We have a number of different restructuring clauses: Mod-R and Mod-Mod-R versus No-R. How do we estimate the difference in pricing? The third issue we address is that of hedging. How do we hedge a simple credit default swap? This issue raises the fundamental question of the methodological approaches to pricing that assume market completeness. The fourth issue is that of the presence of counterparty risk and how it affects the pricing of single name credit default swaps due to default dependence. It automatically arises if we consider a portfolio of credit default swaps. How do we model this type of dependency?

In Section 5 we address some of the issues that arise in the pricing of multi-name products and in the modeling of the distribution of portfolio values. Modeling default dependence is central to modeling the probability distribution of a portfolio of credit risky assets, pricing multi-name products and pricing counterparty risk. Despite its importance, we have little empirical information about default dependence. This is the first issue that we discuss. Second, is how default of an obligor or news about changes in its credit worthiness affects the credit worthiness of other obligors. The third issue is that of actually modeling default dependence. We tackle this in two ways. First we discuss the use of structural models and copula functions, and second we discuss the use of the reduced form methodology. The final issue that we address is the testing of models.

A summary is given in Section 6.

[4] The contract specifies a maximum maturity, usually 30 years.

2 Modeling Expected Loss

Both the loss given default[5] and the probability of default are key inputs when estimating the expected loss at a specified horizon. We cannot directly test a model of expected loss, due to unobservable nature. However, we can test the models used to generate estimates of the loss given default and the probability of default. We start by examining our knowledge about the determinants of recovery rates and then we review recent academic models estimating the probability of default.

2.1 Recovery Rates

In the event of default claim holders receive some fraction of their claim. This fraction is referred to as the recovery rate[6]. Recovery rate information is a critical input into estimating the expected loss and a critical input for pricing the majority of credit sensitive instruments. Most of our knowledge about the determinants of recovery rates comes from actual recovery data and can immediately be used for calculating expected loss. This is not the case for pricing. At present there is little information about how to model recovery rates under the appropriate pricing measure.

There are many issues that need to be addressed in the modeling of recovery rates. First is the definition of default. The BIS[7] reference definition of default covers a wide range of corporate events:

- it is determined that an obligor is unlikely to pay its debt obligations in full;
- distressed restructuring involving the forgiveness or postponement of principal, interest or fees;
- missed payments of credit obligations;
- obligor has filed for bankruptcy or similar protection from creditors.

The heterogeneity in the list of events used to define default implies that, everything else equal, the recovery will depend on the type of event that triggered default. This range of events is

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[5] The loss given default is calculated as one minus the recovery rate.
[6] Recovery rates for bonds and loan are usually expressed as a fraction of the par value.
[7] See Section III.F, in Basel Committee on Banking Supervision (2001).

similar to the range used to define a credit event in the credit default swap markets and it is this diversity of events and the resulting disparity in recovery that lead to the introduction of Modified Restructuring, Modified - Modified Restructuring, and No Restructuring default swap contracts.

Second is the measurement of recovery. The recovery rate is often taken as the price of a bond within a one month period of default. The event of default may affect a bond's liquidity and thus its price, implying that any measure of recovery is affected by the bond's liquidity. The issue of liquidity is usually ignored in any discussion of recovery. Guha (2002) looks at bond prices just after default. Prices of bonds of the same seniority converge to almost the same price, independent of maturity. He argues that the amount recovered is best modeled as a fraction of the face value[8]. An alternative measure is the recovery at emergence of default. Given that restructuring of claims often occurs during bankruptcy and the absolute priority rule is frequently violated, it is far from clear how to measure recovery. The duration of the bankruptcy process is highly uncertain. All of these considerations greatly complicate the process of estimating the present value of the payments to claim holders. In practice various ad hoc discount rates are used in the estimation, implying that such estimates probably have a larger standard deviation than those generated by the first method[9].

Altman *et al* (2005) and Schuermann (2004) lists some stylized facts about recoveries:

1. dependent on seniority and collateral;
2. vary over the credit cycle;
3. vary across industries;
4. distribution is often bi-modal.

Moody's (2003) – see Exhibits 22 and 36 - documents a monotonic relation between average (and also median) recovery rates[10] with priority in the capital structure over the twenty year period 1982-2002. Gupton, *et al* (2000) and Acharya *et al* (2003) find that syndicated loan recoveries are substantially greater than recoveries on senior unsecured debt. That recovery rates vary with the credit cycle is to be expected. Frye (2000) and Schuermann (2004, Exhibit 4) document that the distribution of recovery rates is shifted towards the left during a recession, compared to the

[8] An alternative approach of inferring recovery rates from traded bond prices is described in Bakshi *et al* (2001). They find empirical support for recovery as a fraction of the discounted par value.
[9] See Table 1, Acharya *et al* (2003).
[10] Recovery rate is the bond price within one month of default. If the coupon on the debt is large relative to the prevailing term structure, it is possible for the recovery rate to be greater than unity.

distribution during an expansion. Altman *et al* (2005), Frye (2003) and Moody's (2003, Exhibits 25 and 26) find a negative relation between aggregate default rates and recovery rates. Average recovery rates vary across industries. In Moody's (2003, Exhibit 23) for the period 1982-2002, public utilities had the highest recovery rates, followed by financial institutions, with telecommunications being last.

There is often wide cross sectional variation in recoveries, even after keeping seniority, security and industry fixed. Altman *et al* (2005) consider aggregate recovery rates. They find that the logarithm of the weighted average aggregate default rate, changes in the weighted average aggregate default rate and the total amount of high yield bonds outstanding for a particular year[11] are important explanatory variables. Macro economic variables were generally not statistically significant[12].

Acharya *et al* (2003) regress recovery rate, measured by the bond price within one month of default, against a number of variables classified to one of four groups: Contract Specific Characteristics, Firm Specific Characteristics, Industry Specific Characteristics and Macro Economic Conditions[13]. The following variables are found to be statistically significant at the one or five percent level-

Contract Specific Characteristics: (1) logarithm of issue size[14], and (2) seniority;

Firm Specific Characteristics: (3) profit margin (defined as EBITDA/Sales one year prior to default) and (4) debt concentration[15] (defined as the Herfindahl index measured using the amount of the debt issues of the defaulted company);

Industry Specific Characteristics: (5) industry distress[16] (defined as a dummy variable that equals one if the industry is in distress[17], zero otherwise);

[11] This is interpreted as a supply variable. Substituting the bond default amount produced similar results.

[12] It is interesting to observe that the constant term is highly significant, suggesting that the explanatory variables can not explain the level of the aggregate recovery rate.

[13] The variables consider for the different catalogues are: Contract specific characteristics: seniority, maturity, current asset; firm specific characteristics: profit margin, tangibility, logarithm of total assets, Q-ratio, firm return, firm volatility, debt concentration; industry specific characteristics: industry distress, median industry Q-ratio, median industry concentration, industry liquidity, median industry leverage; and macro economic conditions: Fama-French factors, S&P 500 stock return, annual gross domestic product, product growth rate, aggregate weighted average default rate, and total face value amount of defaulted debt.

[14] It is argued that large issues may earn higher recoveries than small issues, as large stakeholders in the bankruptcy proceedings may be able to exert greater bargaining power.

[15] Firms with a greater number of issues and more dispersed creditor base, that is lower debt concentration, may experience greater coordination problems and in turn lower recovery rates.

[16] If an industry is distress, recovery rates are expected to reduced as a consequence.

Macro Economic Conditions: in the presence of industry specific characteristics macro economic variables were insignificant.

If a firm has been struggling for a long time to avoid default, its assets may be run down and hence the recovery rate will be lower than for the case where demise occurred quickly. None of these papers address whether the history of a firm's condition prior to default influences the recovery rate. The modeling of the recovery rate to depend upon firm specific and macro economic variables is useful for risk management purposes. However for such models to be useful for pricing, it is necessary to change the probability measure to that used for pricing. Apart from the work of Bakshi *et al* (2001), there is little guidance as to the appropriate modeling of recovery under the pricing measure.

Unresolved Issues

We still do not know the following.

1. Whether the history of a firm's condition prior to default influences the recovery rate.
2. We have no information about the out of sample performance of models. If models are going to be used for regulatory capital reasons or for pricing, we need to test their accuracy.
3. There is little guidance as to the modeling of recovery rates under the appropriate pricing probability measure, given that recovery rates depend on systematic factors.

2.2 New Developments in Default Prediction Models

Hillegeist et al (2004) employ a discrete duration model and examine the relative performance of a Merton (1974) based model compared to Altman's (1968) Z-Score and Ohlson (1980) O-Score models. They find that the Merton based model outperforms the accounting based models. Interestingly they find the performance of the Merton model to be relatively insensitive to the precise specification of the critical liability value used in the distance to default variable. They find that the distance to default variable does not entirely explain the variation in

[17] An industry is in distress if the median stock return for the industry of the defaulting firm in the year of the default is less than or equal to -30%. A second definition of industry distress combined the first definition with the median industry sales growth. If the median growth was negative over a one or two year period prior to default then the industry was classified as in being in distress. The two definitions produced similar results.

the failure probabilities across firms. Bharath and Shumway (2004), Campbell et al (2004), Chava and Jarrow (2003) reach a similar conclusion.

Janosi, Jarrow and Yildirim (2002) specify an intensity function depending on the spot default free interest rate and the cumulative unanticipated rate of return on the market index. They estimate the stochastic processes describing these processes. The probability of default over a finite horizon will depend on the parameters of theses variables.

Chava and Jarrow (2003) use a number of standard accounting variables, and a number of market variables: relative size, excess return on equity and the stock' volatility. The relative size is defined as the logarithm of the value of firm's equity divided by the total market value of equity. The excess return is defined as the monthly return of firm's equity minus the value weighted market index return. The firm's equity volatility is computed as the standard deviation over the last sixty days of observable daily market prices. Chava and Jarrow also include industry dummy variables. They find that these dummy variables are statistically significant. Their model provides a small improvement in the out of sample performance compared to Shumway's (2001) model.

Duffie and Wang (2003) employ a reduced form methodology. They use five covariates: distance to default, personal income growth, sector performance, firm level earnings and firm size. Sector performance, measured by sector average across firms of the ratio of earnings to assets, was found to statistically insignificant when the first two covariates were used and hence dropped from the analysis. They did not test the relative performance of their model against other models. Similar to Janosi et al (2002), they estimate the underlying stochastic processes for these state variables.

The models of Janosi *et al* (2002), and Duffie and Wang (2003) can all be applied to estimating multi-period default probabilities[18]. This will provide an alternative methodology to estimating state dependent probability transition matrices.

Unresolved Issues

All prediction models utilize accounting and macro variables as basic inputs. While a firm's reported accounting variables must satisfy accounting standards, a large number of

[18] The Janosi et al (2002) model is used by Kamakura to generate multi-period estimates of default probabilities. Moody's KMV model also generates a term structure of default probabilities.

unidentified assumptions have been employed in the production of these numbers. Additionally there are many factors such as managerial ability, firm culture and ability to response to changes in market and production conditions that are imperfectly measured, if measured at all, by the types of covariates employed.

1. There is no empirical work that attempts to model contagion arising from information effects if investors are imperfectly informed about some common factors that affect the true probability of default.

2.3 Expected Loss

A number of recent studies directly address the dependence between recovery and default rates by assuming that both depend on a single common factor. Frye (2000) describes the determinants of the probability of default using the single factor model suggested by Gordy (2000) and Finger (1999). The rate of return on the asset value for the j th firm, denoted by $A_j(t)$, is assumed to depend in a linear way on a common factor

$$A_j(t) = p\,X(t) + (1-p^2)^{1/2}\,e_j(t)$$

where X(t) is a common factor; $e_j(t)$ is an idiosyncratic factor; and p the correlation between the asset return and the common factor. It is assumed that X(t) and $e_j(t)$ are independent normally distributed random variables. Default occurs if the asset return is below some critical value. The rate of recovery is described by

$$R_j(t) = \mu_j + \sigma\,q\,X(t) + \sigma\,(1-q^2)^{1/2}\,Z_j(t)$$

where $Z_j(t)$ is a idiosyncratic factor. It is assumed that X(t) and $Z_j(t)$ are independent normally distributed random variables, which implies that the recovery rate is also normally distributed. The standard deviation of the recovery rate is σ and q is a measure of the sensitivity of the recovery rate to the macro factor X(t). The term μ_j depends on the seniority of the debt. Using a two stage estimation procedure, Frye (2000) finds evidence of negative correlation between default and recovery rates. In a similar one factor framework, Schönbucher (2003, pp 147-150) assumes that recovery rates are described by a logit normal transformation[19], while Pykhtin

[19] This implies that recovery rates lie within the range zero to one. For the normal distribution the recovery rate lies between minus infinity to plus infinity and for the lognormal distribution between zero and plus infinity.

(2003) assumes a lognormal distribution. He derives closed form expressions for the expected loss and the economic capital.

These three specifications are tested in Dullmann and Trapp (2004), assuming homogeneity among obligors[20]. They find that the normal distribution and the logit normal transformation provide a better fit than the log-normal distribution. They also find empirical evidence of a systematic factor and negative correlation between recovery rates and default rates.

Unresolved Issues

The Dullmann and Trapp (2004) results do not favor the logit normal transformation over the normal distribution. Given that recovery rates are defined over a finite interval, usually between zero and one, their finding suggests model misspecification.

1. We know from the work of Acharya *et al* (2003) and Altman *et al* (2005) that recovery rates depend on more than a single systematic factor. We need to model the probability of default and recovery rates in a multi-factor setting.

3 What Do We Know About Credit Spreads?

In many applications of the reduced form methodology the intensity function is calibrated to match the term structure of the credit spread. This assumes that the whole spread is generated by credit risk and consequently raises the question about what do we know about credit spreads. Many structural models form the basis for default prediction and for modeling default dependence. One of the ways to test these models is to examine their ability to describe the properties of credit spreads.

In this section we briefly describe some of the known empirical properties of credit spreads. We then discuss the ability of structural and reduced form models to describe the different properties of credit spreads.

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[20] This is a strong and unrealistic assumption, which may affect the empirical results. It is employed due to the mathematical simplifications it produces and justified on the basis that it represents a "good" approximation for large internationally well diversified banks.

3.1 Empirical Evidence

Elton *et al* (2001) argue that three factors affect corporate bond spreads: expected loss, a tax premium and a risk premium. The tax premium exists because corporate bonds are taxed at the state level, whereas interest income on government bonds is not. The risk premium exists because credit spreads depend on systematic factors. They use a simple model that assumes risk neutrality, stationary default probabilities and constant recovery rates to estimate the spread due to default. They find that these estimated spreads are substantially lower than observed spreads. Expected loss accounts for only a small part of observed credit spread, while State taxes explain a substantial portion of the difference. They regress the residual against a three factor Fama-French model and then argue that these factors go a long way in explaining the differences.

Collin-Dufresne, Goldstein and Martin (2001) (CGM) argue that a number of variables should affect the change in credit spreads:

- changes in the spot rate (negative relationship) [21]
- changes in the slope of the Treasury yield curve (negative relationship)[22]
- changes in the leverage (positive relationship)
- changes in volatility of the assets of the firm (positive relationship)
- changes in the probability or magnitude of a downward jump in the value of the firm (positive relationship)
- changes in the business climate (negative relationship)[23]

CGM find:

- Negative correlation between the spot interest rate and credit spreads;
- Convexity and slope were not significant and often had the wrong sign;
- Volatility was significant
- Return of the S&P 500 index was economically and statistically significant;
- Changes in the S&P smirk were significant.

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[21] The usual argument is that an increase in the spot rate increases the drift term and lowers the probability of default (under the pricing measure) and hence lowers the spread. Longstaff and Schwartz (1995) and Duffee (1998) find a negative relationship. The argument used to justify a negative correlation is a partial result, as an increase in the spot rate may decrease the value of the firm and hence increase the probability of default.

[22] Litterman and Scheinkman (1991) find the spot rate and the slope of the yield curve important explanatory variables. An increase in the slope increases the expected spot rate and hence a decrease in the credit spread.

[23] Monthly S&P 500 returns are used. They did not use sector returns.

Financial and economic variables explain only 25 percent of observed credit spread changes. A principal component analysis on the residuals shows that they are mostly driven by some unspecified single factor. An expanded regression, which included (a) some liquidity measures, (b) some non-linear spot interest rate terms, and (c) some lagged variables could only explain about 34 percent of the spread changes. A principal components analysis still indicated that residuals were highly correlated.

To check that their finding of a systematic factor is not spurious, CGM use both the Longstaff-Schwartz (1995) and the Collin-Dufresne and Goldstein (2001) models to generate a data set of credit spreads. They undertake a regression of changes in spreads against changes in leverage and spot interest rates, and find, contrary to their empirical findings, that changes in leverage is a major explanatory factor, with a R-squared of about 90%. CGM conclude that it is difficult to reconcile their empirical findings with existing structural models. While models may be able to fit existing term structure of spreads, they can not explain the time series variation.

The view that credit risk only explains a small part of credit spreads is challenged by Longstaff *et al* (2003). Using credit default swap and corporate bond data to jointly estimate the expected loss under the pricing measure, they find that the proportion of the instantaneous spread[24] explained by credit risk is 62 % for AAA/AA-rated bonds, 63% for A-rated bonds, 79% for BBB-rated bonds and 89% for BB-rated bonds. They also show that the proportion of the spread not explained by credit risk appears to be related to different measures of liquidity. There is only weak support for the hypothesis that the non-credit risk component of spreads is due to taxation.

Macro economic factors also affect credit spreads. Jaffee (1975) found that different macro-economic variables could explain a substantial amount of the cyclical variation in corporate bond yields. Altman (1983), looking at aggregate business failure in the U. S. economy found four macro economic variables had explanatory power[25]. Wilson (1997) used macro-economic variables to explain aggregate corporate default rates across different countries. Yang (2003) shows that macro-economic variables such as inflation and real activity can explain a significant amount of the variation in corporate bond prices.

[24] The spread is measured relative to the Treasury curve. The percentages increase when the Refcorp and swap curves are used as the risk less curve.

[25] The variables were real economic growth, stock market performance, money supply growth and business formation.

Summary

To summarize the empirical evidence we find:

1. negative correlation between credit spreads and the short-term default free interest rate;
2. credit risk explains only part of the credit spread;
3. credit spreads dependent on systematic factors;
4. credit spreads dependent of firm specific factors;
5. credit spreads dependent on liquidity and possibly taxation.

All of these factors should be considered when modeling the evolution of credit spreads.

Unresolved Issues

There is still a lot of uncertainty as to the major determinants of credit spread.

3.2 Structural Models

In the application of structural models, two different approaches have been used for calibration and the approaches reach quite different conclusions. In the first approach, the models are calibrated to the term structure of default probabilities under the natural probability measure, while the second approach uses firm value, leverage, payout ratio and estimation of exogenous parameters, such as the default interest rate process.

For the first approach, we review the work of Huang and Huang (2003) who calibrate a wide array of structural models, including jump-diffusion models, and generate a term structure of credit spreads for different credit classes[26]. The models are calibrated to match (1) the average probability of default under the natural probability measure over different horizons; (2) the average loss as a fraction of the face value of debt; (3) the average leverage ratio; and (4) the equity premium. They find that these models for investment grade firms can explain less than 30 percent of the average credit spread. For high yield the models can explain between 60 to 80 percent of the average credit spread.

A similar conclusion is reached by Turnbull (2003) using a reduced form model that is calibrated to match the term structure of default probabilities under the natural measure over a

[26] They use following models: Longstaff and Schwartz (1995), Leland and Toft (1996), Anderson, Sundaresan and Tychon (1996), Mella-Barral and Perraudin (1997), Collin-Dufresne and Goldstein (2001), and a jump diffusion model.

five year horizon. For each obligor the term structure of default probabilities is generated using KMV's Credit Monitor. The probability measure was changed to the "risk-neutral" measure[27] and a term structure of credit spreads generated, assuming some average recovery rate. It was found that credit spreads were one third to one half of those observed in the market.

In the second approach, Eom, Helwege and Huang (2004) test five different structural models. They use firm specific parameters such as firm value, leverage, payout ratio etc using historical corporate data. They find that the Merton (1994) model generates spreads that are too small. The Geske (1977) compound option model also generates spreads that are too small, but is an improvement over Merton. Leland and Toft (1996) model over estimates spreads, even for short maturity bonds. The Longstaff and Schwartz (1995) model also generates spreads that are too high on average. The model generates excessive spreads for risky bonds and under estimates the spreads for low risk bonds. The Collin-Dufresne and Goldstein (2001) model over estimates spreads on average. Eom *et al* conclude that structural models do not systematically under predict credit spreads, but accuracy is a problem. They make no attempt to infer the term structure of default probabilities under the natural measure or to examine the time series properties of their models.

In any application of a structural model that utilizes accounting information, it is implicitly assumed that such information is accurate. An interesting extension of the structural modeling approach is given in the recent work Duffie and Lando (2001), who show that with incomplete accounting information, it becomes necessary to model the event of default as an inaccessible stopping time, along the lines of the reduced form models introduced by Jarrow and Turnbull (1992, 1995). Yu (2002a) provides some empirical evidence relating accounting transparency and credit spreads.

Summary

In summary, the structural models are far from accurate.

Unresolved Issues

1. How does one calibrate a structural model?
2. Why are structural models so inaccurate?
3. Modeling of information asymmetry and the implications for instrument dynamics.

[27] The default premium was assumed to be unity.

3.3 Reduced Form Models

In the modeling of credit spreads Duffee (1999), assumes that the risk neutral intensity function depends on a firm specific factor and two default free interest rate factors. All of these variables are described by square root processes. It is found that credit spreads are weakly negatively correlated with the short-term interest rate. The firm specific factor and its price of risk are significant. It is also found that the magnitude of these factors varies over credit ratings. The firm specific factors are found to be correlated across firms, implying a possible misspecification of the intensity function. There is no estimation of the default risk premium.

Janosi *et al* (2000), assume that the intensity function depends on the short-term default free interest rate and the cumulative unanticipated return on the equity index[28]. They also include a liquidity term modeled as a convenience yield. Similar to Duffee, they find a negative relation between spreads and the short-term interest rate. They find that the coefficient on the equity market return is statistically insignificant, contradicting the finding in CGM. The different models tested by Janosi *et al* have relatively low R-squared values, implying that can only explain a small part of the variation in credit spreads.

Driessen (2005) models the intensity function as dependent on two short term default free interest rate factors, two latent factors common to all firms and a firm specific factor. These factors are described by square root processes. These factors are found to be statistically significant. The prices of risk for the two common factors are found to be statistically significant, while the price of risk for the firm specific factor is close to zero for the median firm. Lower rated firms were found to be more sensitive to the common factors than the higher rated firms. The default risk premium, μ, is defined as the ratio of the intensity function under the pricing measure Q to the intensity function under the natural measure P. It is assumed to be constant and identical across firms.[29] The default risk premium is estimated by equating estimates of the historical probability of default to the probability of default estimated from the model. The estimated default risk premium can not be estimated with high statistical precision, even after

[28] See Jarrow and Turnbull (2000) for details.

[29] It is assumed that $\lambda^{Q}_{j,t} = \mu\, \lambda^{P}_{j,t}$, for all t and j.

adjusting for differential taxation and liquidity. Using S&P data for the period 1981-2000 the estimate is 2.91 with standard error 1.11 and using Moody's data for the period 1970-2000, the estimate is 2.15 with standard error 1.03[30].

Unresolved Issues

1. How to relate the state variables of the intensity function to firm specific and macro-economic variables.
2. The out of sample performance of models needs to be examined.
3. In the current formulations, the intensity function is assumed to be a linear function of state variables. Even if the state variables are assumed to follow square roots processes, given the empirical estimation, there is no guarantee that the intensity function is positive. We need a convenient formulation, where the intensity function is always positive.

4 Issues in Pricing Single Name Credit Instruments

There are many issues that affect the pricing of single name products that must be addressed before it is meaningful to discuss multi-name products. In this section we consider some of the issues that arise in the pricing of credit default swaps, perhaps the simplest form of credit derivative. The issues that we discuss are however generic to all single name credit risky derivative instruments. For any model there is always imprecision in parameter values. Often simple point estimates are used as inputs for a model to price an instrument, such as a credit default swap, implying that the estimated price is an implicit function of the point estimates. For many credit derivatives the recovery rate is an input parameter. The usual assumption is to treat the recovery rate as a constant, though it is well known that recovery rates are stochastic and vary over the credit cycle.

The first issue we consider arises because of input parameter imprecision and how this can affect calibration. The second issue we consider is the application of the standard model to pricing credit default swaps, when there is more that one type of credit event. This raises the issue of clearly identifying the stochastic processes for the different types of credit events and recognizing that cash and default swap markets may depend on different factors. Note that the

[30] See Table 6.

issue of defining what constitutes a credit event arose when we discussed recovery rates. The third issue we discuss is the question of hedging. We argue that the standard static hedge is only an approximation. Arbitrage free models assume that markets are dynamically complete and hence hedging and pricing are intrinsically related. Again we argue that due to different definitions about what constitutes a credit event, uncertain recovery rates and a lack of hedging instruments, markets are also dynamically incomplete. The last issue that we address is counterparty risk. The credit default swap market is an inter-bank market and consequently counterparty risk is always present. This affects the pricing and hedging in the market.

4.1 Calibration and Pricing Credit Digital Swaps: Theory and Practice.

The protection buyer promises to make payments, S_D, to the protection seller at dates T_1, …, T_n, provided there is no credit event. The present value of these payments is given by[31]

$$S_D\, E_t[PV_{PB}(t)] \equiv S_D \sum_{j=1}^{n} E_t[B(t, T_j) \perp (\Gamma_{RE} > T_j) | F_t]$$

where Γ_{RE} is the time of default by the reference entity; $B(t, T_j)$ is the discount factor; F_t is the information set at time t; and $t < T_1$. If there is a credit event the protection buyer must pay accrued interest. We will ignore this complication[32]. If there is a credit event, the protection seller makes a payment of L to the protection buyer. Without loss of generality, we set $L = 1$. The present value of protection leg is given by

$$E_t[PV_{PS}(t)] \equiv E[B(t, \Gamma_{RE}) \perp (T_n \geq \Gamma_{RE}) | F_t]$$

where $T = T_n$. The price, S_D, is set such that present value of the premium leg equals the present value of the protection leg:

$$S_D\, E_t[PV_{PB}(t)] = E_t[PV_{PS}(t)]$$

Note that the price does not depend on the recovery rate for the reference entity. How do we actually price this instrument?

The credit digital swap market is a small market and consequently the usual way is to infer the intensity function from the prices of regular default swaps. The present value of premium payments is given by $S_{CDS}\, E_t[PV_{PB}(t)]$ and the present value of the protection leg is given by $E_t[(1-\delta) PV_{PS}(t)]$, where δ is the recovery rate on the reference entity. This is

[31] An introduction to the reduced form methodology is given in Duffie and Singleton (2003) and Schönbucher (2003a).
[32] See O'Kane and Turnbull (2003) for a detailed discussion.

determined by the cheapest to deliver bond. We will ignore the optionality associated with the cheapest-to-deliver option. The price is determined by again equating the present value of the premium payments and the present value of the protection payment:

$$S_{CDS}\, E_t[\, PV_{PB}(t)\,] = E_t[\,(1-\delta)\, PV_{PS}(t)\,]$$

Using the above expression, we can relate the premium of a regular swap to the premium on the digital swap:

$$\frac{S_{CDS}}{E_t(1-\delta)} + \frac{\mathrm{cov}_t[\delta, PV_{PS}(t)]}{E_t(1-\delta)\, E_t[PV_{PB}(t)]} = S_D$$

Frye (2000), Altman *et al* (2003) and Acharya *et al* (2003) find a negative correlation between the recovery rate and the frequency of default. This is under the natural probability measure. Assuming a negative correlation holds under the pricing measure, implies that

$$S_{CDS} \geq S_D\, E_t(1-\delta)$$

Taking the value of S_{CDS} as given, then the spread for the digital swap is inversely related to our estimate of the expected loss. There is normally a lot of uncertainty about the expected loss and small changes in the expected loss can produce large changes in the estimated digital premium. This problem is compounded given the differences in the definitions of what constitutes a credit event in the cash and swap markets. For instruments where we need the recovery rate and do not want to employ the assumption of the rate being a known constant, we have very little information about recovery rates under the measure pricing measure Q[33]. The method of calibration can infer false dependence.

4.2 Defining the Credit Event: Restructuring[34]

The issue of defining a credit event affects default prediction models, the measurement of recovery rates, and the pricing and hedging of credit risky bonds and credit derivatives. In the credit default swap market, the last two years has seen a major attempt to increase the clarity of the definition of a credit event and the resulting recovery rate. In this section we briefly describe the definition of a credit event and the implications for pricing.

[33] See Bakshi *et al* (2001) who infer estimates of the recovery rate under the measure Q.
[34] This material draws on the discussion given in O'Kane, Pedersen and Turnbull (2003).

In the current standard default swap contract linked to a corporate (non-sovereign) reference credit, there are three credit events that can trigger the payment of protection: bankruptcy, failure to pay, and restructuring. Both bankruptcy and failure to pay are called *hard* credit events because following either, *pari passu* bonds and loans of the issuer should trade at (or very close to) the same price. In what follows we also use the term *default* to refer to a hard credit event. Restructuring is different, and is known as a *soft* credit event. It is different in the sense that a restructuring does not cause the debt of a company to become immediately due and payable so that the assets of a company will generally continue to trade, with long-dated assets trading below short-dated assets.

Two years ago the mechanism for settling a default swap following all of these three credit events was the same. If a credit event occurred, protection buyers would settle by delivering the face value amount of deliverable obligations to the protection seller in return for the face value amount paid in cash. The market standard was to specify a basket of delivery obligations consisting of bonds or loans that are at least *pari passu* with the reference obligation. There was usually a limit on the maximum maturity of 30 years. Protection buyers can choose from this basket of deliverables. They are effectively long a cheapest-to-deliver option.
In order to define what restructuring means in the context of a credit default swap, we must refer to the 2003 ISDA definitions. These state that a debt obligation is considered restructured if there is:

1) an interest rate reduction,
2) a reduction in principal or premium,
3) a postponement or deferral (maturity extension),
4) a change in the priority ranking of payments, or
5) a change in currency or composition of payment of principal or interest.

In addition, more than three unaffiliated holders must hold the restructured obligation and at least two-thirds of the holders must have consented to the restructuring. It is also important to note that a restructuring is viewed as a credit event, if there is deterioration in the creditworthiness or financial condition of the reference entity.

Following the release of the ISDA 2003 definitions, there are now four types of restructuring clause: Old Restructuring (Old-R)[35], Modified Restructuring (Mod-R)[36], Modified-

[35] This was used widely in the US until 2001. In this case the maximum maturity deliverable is 30 years.

Modified Restructuring (Mod-Mod-R)[37] and No Restructuring (No-R)[38]. Note that Mod-Mod-R allows a greater range of deliverables than Mod-R. The standard for US credits is Mod-R and the only other contract traded in the US market is No-R. In Europe the standard contract is now Mod-Mod-R.

For each of the four types of contracts, the occurrence of a credit event will affect the recovery rate. In pricing these contracts it is necessary to model the probability of an occurrence of a credit event and the resulting recovery rate – see O'Kane *et al* (2003 a). One of the major challenges is the calibration of such models. A number of questions must be addressed, such as what is the probability of a restructuring and what impact does a restructuring have for the pricing of bonds and hence the cheapest to deliver bond? The current prices of bonds for a particular obligor should reflect the possibility of (a) default, (b) restructuring event and (c) default after restructuring. It is impossible to infer all these parameters from market prices.

4.3 Hedging a Credit Default Swap

Pricing theory assumes the existence and uniqueness of an equivalent martingale measure. This can be achieved either in an equilibrium context or using arbitrage arguments. If we rely on arbitrage, then we must be able to hedge a credit default swap. This raises the question of how one hedges a credit default swap. Traditionally a static hedge argument has been used to price a default swap. However, the hedge is only an approximation. If static hedging fails, this leaves dynamic hedging. We argue that dynamic hedging also fails.

The premium payments in a default swap contract are defined in terms of a default swap spread, S, which is paid periodically on the protected notional until maturity or a credit event. It is possible to show that the default swap spread can, to a first approximation, be replicated by a par

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[36] This clause is now standard in the US market. It limits the maturity of deliverable obligations to the maximum of the remaining maturity of the swap contract and the minimum of 30 months and the longest remaining maturity of a restructured obligation. It only applies when the protection buyer has triggered the credit event.

[37] The new European standard limits the maturity of deliverable obligations to the maximum of the remaining maturity of the CDS contract and 60 months for restructured obligations and 30 months for non-restructured obligations. It also allows the delivery of conditionally transferable obligations rather than only fully transferable obligations. This should widen the range of loans that can be delivered. It also only applies when the protection buyer has triggered the credit event.

[38] This is the removal of restructuring as a credit event leaving just the hard (default) credit events. Some insurance companies and commercial banks favor this type of contract and it remains to be seen if it will become liquid.

floater bond spread (the spread to LIBOR at which the reference entity can issue a floating rate note of the same maturity at a price of par) or the asset swap spread of an asset of the same maturity provided it trades close to par.

Consider a strategy in which an investor buys a par floater issued by the reference entity with maturity T. The investor can hedge the credit risk of the par floater by purchasing protection to the same maturity date. Suppose this par floater (or asset swap on a par asset) pays a coupon of LIBOR plus F. Default of the par floater triggers the default swap, as both contracts are written on the same reference entity. With this portfolio the investor is effectively holding a default free investment, ignoring counterparty risk. The purchase of the asset for par may be funded on balance sheet or on repo – in which case we make the assumption that the repo rate can be locked in to the bond's maturity. The resulting funding cost of the asset is LIBOR plus B, assumed to be paid on the same dates as the default swap spread S. It is shown in O'Kane and Turnbull (2003) that default swap spread, S, is given by

$$S = F - B.$$

However the hedge is not exact, as it relies on several assumptions that do not hold in practice and hence the above result is only an approximation, resulting in small but observable differences.

If a static hedge is limited, then what about dynamic hedging? To hedge a credit swap, we form a replicating portfolio. Suppose that we are long protection, the value of the credit swap is the difference between the value of the protection minus the cost of the protection. To replicate this portfolio, we must replicate the changes in value without default and the change in value when default occurs. This requires two instruments written on the reference entity and default free instrument. Being long protection is equivalent to being short the reference entity. It is not easy to short corporate names and the recovery rates may differ, unless we short the cheapest to deliver bond. We could use credit swaps of different maturities to form the replicate portfolio. However, the secondary credit swap market is not a liquid market. The inability to be able to replicate a credit swap raises the question about pricing credit derivatives and the minimum price one should charge for such instruments.

4.4 Counterparty Risk

Counterparty risk arises when there is the risk that the writer of a contract might default and thus be unable to perform. This affects the pricing of the contract, as now it is necessary to consider the default risk of the counterparty and the default dependence between the reference entity and the counterparty. The effect on pricing was first considered by Jarrow and Turnbull (1992, 1995), and for the case of independence between the reference entity and the counterparty a relatively simple expression can be derived.

A credit default swap is a bilateral contract, where the magnitude of the exposure to the counterpart is quite different for the protection buyer and seller[39]. There are two facets to counterparty risk: default risk and mark-to-market risk. If the protection buyer defaults, the protection seller suffers (a) at most one payment of the swap premium and (b) the cost of replacing the protection buyer – a mark-to-market risk. If the protection seller defaults, there are two cases to consider. First, if the reference entity defaults first and protection seller defaults before the settlement of the contract, this leaves the protection buyer with no credit protection. Second, if the protection seller defaults first, it is necessary to replace the protection seller. This can be particularly costly if the credit quality of the reference entity has deteriorated.

To determine the effects of counterparty risk it is necessary to specify the default dependence between the reference entity and the protection buyer and seller. This raises the question of modeling default dependence, which we discuss in the next section. To compute the mark-to-market risk it is necessary to price a replacement contract at the time of default by the counterparty. Given that default by the counterparty can occur at any time over the life of the contract, this raises a computational problem, especially for structured products[40].

Summary

Imprecision about parameter values can have unforeseen consequences when calibrating instruments, even those that do not explicitly depend on the particular parameter. Different definitions about what constitutes a credit event affect the recovery rate and pricing. While it is possible to model this diversity, the challenge is to calibrate such a model. Markets are not complete and eventually the issue of pricing in incomplete markets and data availability must be addressed.

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[39] For interest rate and foreign exchange swaps, see Duffie and Huang, (1996) and Jarrow and Turnbull (1997).

[40] See the recent work of Mashal and Naldi (2005) on the pricing of contracts in the presence of counterparty risk and Turnbull (2004) for the effects of counterparty risk on P&L.

Unresolved Issues

1 How to incorporate event definition diversity into a model that can be used in practice.

2 How to incorporate pricing in an incomplete market given the data availability.

3 How to incorporate a standard approach to address the effects counterparty risk on pricing.

5. Pricing Multi-Name Products

The complication in the pricing of multi-name products arises from modeling how the credit spreads of the different obligors evolve over time and from the effects that the default of an obligor has on the remaining obligors. An obligor in a particular sector may experience some form of difficulty that affects the market's assessment of its credit worthiness. Other obligors in the same sector may also see their credit spreads widening simply because they are in the same sector: "guilt by association".

In the academic literature we have a number of different approaches: the structural approach, the reduced form methodology and the use of copulae. Practitioners tend to use a combination of these approaches. We review these different approaches and raise the question about testing of models.

We first start with a discussion about what we actually know about default dependence. We briefly discuss the structural approach, the copula methodology and reduce form approaches. The last part of this section addresses the issues of testing models and the effects of data limitations.

5.1 What Do We Know Theoretically About Default Correlation?

Zhou (2001) extends the typical barrier framework[41] to consider two obligors. By assuming either that the critical value is a constant or exponential, a closed form expression is derived for the joint probability of default. Based on this simple model, Zhou derives a number of conclusions[42].

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[41] In the typical barrier model, default is said to occur the first time the value of the firm falls some critical valued called the barrier. In order to derive closed form solutions, it is assumed that the barrier is either a constant or exponential – see Schönbucher (2003a, chapter 9) for details.

[42] These conclusions are based on the case that the expected rate of return on the asset equals the growth rate of the critical value.

1 The default correlation and asset correlation are the same sign. This is an intuitive result. If two assets are positively correlated, and one firm defaults, then the value of the other firm's assets has probably declined and the probability of default increased.

2 Default correlations are generally small over short horizons and then they first increase and then decline. Over short horizons, the defaults are rare and are primarily idiosyncratic. This explains why the default correlations are generally small over short horizons. Given the assumption that the asset's expected rate of return equals the growth rate of the barrier, the probability of default converges to one as the time horizon increases. Consequently the default correlation eventually converges to zero.

3 Firms of "high" credit quality tend to have a low default correlation over a typical horizon. For high credit quality firms the initial probability of default is low and hence the default correlation is low. Default by one firm signals that the value of another firm may have declined. However, over short intervals the probability of default is still very small.

4 The time of peak default correlation depends on the credit quality of the underlying firms. The high credit quality firms take a longer time to peak. The intuition is similar to point (1).

5 Default correlation is dynamic, as the credit quality of firms is changing over time.

The last point has important practical implication: default correlations can change even though the underlying processes for asset values are stationary. This highlights the need for care when attempting to develop internally consistent models for default dependence. By using a simple model, and assuming that the distributions for the two firms is bi-variate lognormal, Zhou derives a number of interesting conclusions that do not depend on modeling the credit cycle of the economy.

What Do We Know Empirically About Default Correlation?

Servigny and Renault (2002) (S&R) analyze all defaults and credit rating changes recorded by Standard and Poors' for a 21year period from 1981 to 2001, using U. S. annual data.

For each year, they estimate probabilities of default and rating transitions. They also estimate these probabilities for different industry sectors. For investment grade firms, the average default correlation is 0.1 percent per year and for non-investment grade firms 1.7 percent[43]. There is wide dispersion across different industries.

Using a one factor model first described by Merton (1974) and extended in CreditMetrics, the critical asset values are calibrated to match the estimated probabilities. Under the assumption of multivariate normality, S&R derive an expression for the joint probability of default for two obligors. This bivariate distribution depends on the correlation of asset returns. Given that they have an estimate of the joint probability, they can infer the implied asset correlation[44] and calculate the default correlation. This approach allows them to address the question of the relation between equity default correlation and asset default correlation. They use 60 months of equity returns to estimate the equity correlation. They regress the default correlation from equity as the independent variable against the asset default correlation[45]. They find the intercept is "small" (0.0027) and the slope coefficient 0.7794. This leads S&R to conclude that (a) equity default correlation is greater than the asset default correlation and (b) equity default correlation is a poor proxy for the asset default correlation, given a low R-squared value of 0.2981.

To test the bi-variate normal distributional assumption, they also use a t-copula. Keeping the same marginal distributions, they estimate the degrees of freedom. They find that in 88 % percent of their cases the joint probability of default had an implied degree of freedom greater that 50, implying that the differences between using either a normal or t–copula were minimal.

S&R also examined the effect of time horizon on the estimated default correlation. They found that in general the estimated default correlation increased in magnitude as the horizon increased from one to three to five years. The one exception was the empirical estimate of the default correlation between the automotive and financial sectors. The finding that default

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[43] This contradicts the work of Das, Freed, Geng and Kapadis (2002) and Zhou (2001).

[44] The estimated probabilities of joint default will be estimated with large standard errors. This will affect the estimation of the implied correlation. The effect of estimation error could be large when the joint default probabilities are "small". Unfortunately S&R give no error statistics such as standard deviations throughout their paper and consequently it is impossible to judge the significance of their results.

[45] Using the equity correlation of returns, they estimate the joint probability of default. This allows them to calculate the default correlation using equity returns. We refer to this estimate as the equity default correlation.

correlation increased in magnitude is consistent with the theoretical work of Zhou (2001). In Zhou (2001) default correlation could increase even with constant asset correlation. S&R also examine whether asset correlation increases with horizon. Given estimated joint probabilities of default over different horizons, they calculate inferred asset correlation and argue that asset correlations increase with horizon. They also found that the implied asset correlation changes over the business cycle.

Das *et al* (2002) use U. S. data supplied by Moody's, covering the period January 1987 to October 2000. For each firm, Moody's RiskCalc model is used to generate monthly estimates of the probability of default over a one year horizon. A default intensity for each obligor, $\lambda(t)$, is calculated, given an estimate of the probability of default[46].

It is shown that

(a) default intensities of individual firms within a given rating class vary substantially over time and the variation depends on the initial credit class and industry sector;

(b) there appears to be a systematic component to default intensities;

(c) correlations between default intensities vary through the business cycle;

(d) firms of the highest credit quality have the highest default correlations[47].

A simple regime model is used to model this dependence. A more detailed model is described in Das and Geng (2002).

However results for this type of study must be treated with caution. The "raw" data for the study is generated by a black box, a proprietary model developed by Moody's. Hence the findings of the study reflect properties of the model used by Moody's and not necessarily the raw data. Each "raw" observation is an estimate of the true value. Servigny and Renault (2002) do not provide any details about the standard errors of the estimates and the properties of the standard errors, so it is unclear if the point estimates are statistically significant. Issuing a "user beware" caution about both studies, we can summarize the two studies.

Summary

Based on these two studies, four conclusions can be stated:

1 Default and asset correlations are positively correlated.

[46] The probability of default over the next period, given survival up to that point is given by the expression $1 - \exp(-\lambda(t))$.

[47] This contradicts the finding of the case considered by Zhou (2001).

2 Default correlations vary cross-sectionally, even after controlling for industry and initial credit rating.
3 Default correlations are time varying, even if we have stationary asset distributions.
4 Default correlations increase with time horizon.

5.2 The Effect of a Default on Other Obligors

It is generally assumed that the effect of a default by an obligor will have a negative effect on the remaining obligors. The economy may be in such poor condition that the failure of one firm sends out a clear signal about the dire conditions within the industry. Presumably investors will revise downwards their expectations about the credit worthiness of the remaining firms. However investors may be only too well aware of the poor economic conditions within the sector and the default by a firm conveys little or no new information about the remaining obligors.

The state of the economy and the competitive structure of different sectors affect both the probabilities of default and the effects of a default on the credit worthiness of the remaining obligors. It is well known that defaults tend to increase as the economy goes into recession, and different sectors will have different responses to the state of the economy. In a competitive industry, the default of any one obligor may have short run positive effects for other firms in the industry before firms adjust their production or new firms enter the industry. In other sectors with different industrial structures, the effects of a default may be beneficial, as there will be less competition and remaining firms can pick up the slack. Consequently if we want to model default dependence, we should address the issue of how the credit worthiness of an obligor depends on the state of the economy and the industrial structural nature of the different sectors.

In Davis and Lo (1999, 2001), it is assumed if default by an obligor occurs, the default intensities of the other obligors are increased by a factor called the “risk enhancement factor,” that is assumed to be greater than one. This is a “guilt by association” type of model. It is also assumed that after an exponentially distributed time, the default intensities return to their original level. Davis and Lo call their model “macroeconomic shock”. This is a misnomer given their assumption that the probability of ‘direct’ default and the probability of a jump given a default are constants independent of the state of the economy. Perhaps a more appropriate name is “macro shock”. While their model is interesting to practitioners, Davis and Lo do not discuss how to implement their model using market data. In a similar vein Jarrow and Yu (2000) consider "large" and "small" firms. If a "small" firm defaults it has no effect on the credit worthiness of the remaining firms, while if a "large" firm defaults the credit worthiness of the remaining firms is assumed to deteriorate. It is difficult to generalize this approach to multiple sectors. There is no discussion about implementation.

Gagliardini and Gourieroux (2003) and Schönbucher (2003b) have introduced the idea of unobservable heterogeneity or frailty into the modeling of information driven contagion[48]. Frailty represents the effects of unobservable variables that affect the hazard function. The frailty variable is represented as a random variable that multiplies the hazard function. In this approach, default contagion arises from informational effects given investors are imperfectly informed about parameters that affect the hazard function. Beliefs about the frailty parameter are updated when a credit event occurs and consequently default intensities of remaining obligors jump. This affects the level of default dependence among obligors. This approach provides a simple way to model how a sector is affected by a credit event and how credit spreads will recover given no events. However, to-date there is no published empirical papers in the area of credit using this form of specification.

Unresolved Issues

At present we have no model that (a) recognizes the industrial structure of a particular sector and (b) can be calibrated and applied to real data.

5.3 The Structural Approach

The structural approach forms the basis for the current regulatory approach to modeling credit risk. Based on the Merton (1974) model, for two obligors we have the well known result that the probability of the two obligors defaulting over the horizon T is given by

$$\Pr(\Gamma_1 \le T \text{ and } \Gamma_2 \le T) = N_2[N^{-1}(p_1), N^{-1}(p_2), \rho] \qquad (1)$$

where ρ is the correlation between the rates of return on the firm/asset values; p_j, j = 1, 2 denotes the probability of obligor j defaulting over the horizon; N(.) is the normal distribution function; and N_2(., ., .) is the bi-variate normal distribution function. Note that the Merton model does not consider any forms of market frictions, such as corporate/personal taxation, so there is no difference between firm value and asset value.

[48] Frailty is common in the statistical analysis of biological and health data – see Hougaard (2000, chapter 7). In economics it is referred to as unobservable heterogeneity – see Kiefer (1988).

The above expression, generalized to an arbitrary number of assets, underlies the methodology used in CreditMetrics. The correlation matrix is estimated using a factor model of the form

$$R_j = \sum_{k=1}^{F} \beta_{jk} I_k + \varepsilon_j$$

where R_j is the rate of return on equity for the j th name, I_k is the rate of return on some traded index, ε_j is the idiosyncratic risk, and F is the number of factors. Various forms of this factor model are employed to accommodate global portfolios. In KMV the correlation matrix is for asset returns while for CreditMetrics it is for equity returns[49].

In the Merton (1974) model default can only occur when the zero coupon debt matures. An alternative approach to circumvent this limitation, is to assume that if the value of the firm's assets falls below some critical value, $K_j(t)$, default occurs. The time to default for firm j is defined as

$$\Gamma_j = \inf\{t;\, V_j(t) \leq K_j(t) | V_j(0) > K_j(0)\}$$

For this approach to be operational, it is necessary to specify the critical asset value $K_j(t)$. Usually it has been assumed that $K_j(t)$ is deterministic[50]. The standard assumptions are: (a) $K_j(t)$ is a constant[51]; (b) $K_j(t)$ is exponential[52]; or (c) $K_j(t)$, is a deterministic function calibrated to match some market observables, such as the term structure of credit spreads or prices of credit default swaps. However extending this approach to more than two assets – see Zhou (2001)- is complicated and close form solutions do not seem to exist.

One of the objections to the Merton/CreditMetrics approach is the assumption that asset values are lognormally distributed. In practice equity returns are often used to estimate the correlation matrix. In this case it is assumed that equity returns are lognormally distributed, an assumption that is inconsistent with the Merton model. It is well known that equity return distributions are fat tailed. This has lead to the use of different types of copula functions to model the multivariate distribution of stopping times.

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49 In Merton type diffusion models equity and asset correlations are locally equal.
50 When the credit quality of the obligor is declining and/or the economy is going into recession, the availability of funding usually declines, implying that the critical value will in general be state dependent.
51 Leland and Toft (1996) provide a theoretical justification for such an assumption.
52 It is assumed that $K_j(t) = K_j \exp(\lambda_j t)$, where λ_j is a constant.

Copula Approach

This approach takes the marginal distributions as given and "stitches" them together with a copula function in order to specify the joint distribution. This "stitching" is achieved using Sklar's theorem, which describes the conditions for the existence of a copula function.

Examples of Copulae

Two frequently employed copula functions are the normal copula and t-copula.

<u>The Bi-variate Normal Copula</u>

The normal-copula function is defined by

$$C(u_1, u_2) = N_2[N^{-1}(u_1),\ N^{-1}(u_2), \gamma]$$

where γ is the correlation coefficient. The joint distribution of default times can be written as

$$\Pr[\Gamma_1 \le t_1 \text{ and } \Gamma_2 \le t_2] = N_2[N^{-1}[F_1(t_1)], N^{-1}[F_2(t_2)], \gamma] \qquad (2)$$

Comparing expressions (1) and (2), we that

$$F_j(t_j) = p_j \text{ and } \gamma = \rho.$$

This allows us to identify the correlation parameter.

<u>The t-Copula</u>

The t-copula function is defined by

$$C(u_1, ..., u_n) = t_{v,\Sigma}[t_v^{-1}(u_1), ...,\ t_v^{-1}(u_n)]$$

where $t_{v,\Sigma}$ is a multi-variate standard t distribution with v degrees of freedom, Σ is the correlation matrix, and t_v is a univariate standard t – distribution. The joint distribution of default times can be written as

$$\Pr[\Gamma_1 \le t_1 \text{ and } ...., \Gamma_n \le t_n] = t_{v,\Sigma}[t_v^{-1}[F_1(t_1)], ..., t_v^{-1}[F_n(t_n)]]$$

To apply this model we need to specify the degree of freedom and we need to estimate the correlation matrix for either approach. How do we estimate these parameters?

The Standard Application of the Copula Approach[53]

[53] See Mashal and Naldi (2001, 2003) and Schönbucher (2003a) for details.

In practice the structural and reduced form approaches are combined with the copula methodology. To specify the distribution of stopping times the Merton model is used, implying that it is necessary to specify the correlation matrix of asset returns. Asset returns cannot be observed and consequently equity returns are used as a proxy. It is sometimes argued that due to different degrees of leverage, it is inappropriate to use equity returns to model default dependence, and this problem is particularly severe for low quality obligors. The Merton model refutes this type of argument[54].

Practitioners combine both the structural and reduced form approaches. Using a normal copula, which is the industry norm, the basic assumptions are:

1. Univariate default times are calibrated to market data[55].
2. Univariate equity returns follow a normal-distribution.
3. Dependence is modeled as a normal-copula.
4. Dependence structure among default times is identical to the dependence structure among equity returns.

Mashal *et al* (2003) investigate two questions: (1) the use of equity compared to asset returns; and (2) the assumption of using a normal copula. Using equity and asset returns generated by the KMV model, they argue that as there is little difference in their results, it is immaterial whether one uses equity or asset returns. This is an encouraging result, though it is not surprising. Asset returns cannot be directly observed. KMV start with equity returns to generate a time series of asset returns using the Merton (1974) model. Consequently their asset generating process implicitly assumes that asset and equity returns are instantaneously perfectly correlated[56]. Marshal *et al* compare a normal-copula to a t-copula and provide empirical evidence that the t-copula provides a more appropriate description of equity returns. This has a significant impact on the pricing of multi-name credit instruments.

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[54] In the Merton model the conclusion that asset and equity correlations are locally equal does not depend on leverage. See Schönbucher (2003a), chapter 10.

[55] A simple reduced form model is calibrated to credit default swap data.

[56] The counter result by Servigny and Renault (2002) must also be treated with caution. Previous work published by Standard and Poors' provides estimates of joint probabilities and due to the small number of observations, the standard errors have been large. The inversion procedure employed by Servigny and Renault is sensitive to measure errors: they are measuring tail probabilities and small differences may have a significant impact on the estimate of the implied correlation.

There are a number of limitations to the current application of the copula methodology to modeling default dependence. The current approach employs the following hypothesis:

Hypothesis H1

The description of the dependence among equity returns can be used to describe dependence among stopping times, given the copula function.

First, we have no evidence justifying this type of hypothesis. It might be true that a particular copula provides a more accurate description of equity returns than a normal copula. However there is no direct way to test the second part of the hypothesis that this copula is appropriate for describing the structure of stopping times. The current approach of specifying for each obligor some marginal distribution for the stopping time, specifying some form of copula to model the joint distribution of stopping times and then using asset or equity returns to estimate required parameters for the copula, is problematic. One possible route would be to modify the Merton (1974) framework. Instead of assuming a lognormal distribution to describe the value of the firm, use a more general stochastic process that allows for fat tails and allows jumps in the value of the firm. Introduce some form of barrier model, so that default can occur at any point. Hence the marginal distribution of the stopping time is implicitly defined. The specification of the copula function is dictated by the specification of the joint dependence of asset values. Alternatively one could specify some general process describing the stopping time for an obligor and some arbitrary copula function to describe the joint distribution of stopping times along the lines described by Schönbucher and Schubert (2001). The challenge that this approach faces is empirical justifying the underlying assumptions and calibration.

Second, for the most part the choice of copula is arbitrary and seems to be dictated by ease of use. For practitioners ease of use and calibration are important issues. However for modeling default dependence even “nice” copulae become “complicated and messy” once obligors start to default – see Schönbucher and Schubert (2001). Fan (2003) argues that for modeling equity return or foreign exchange dependence a mixed copula should be used. Given hypothesis H1, this implies that a mixed copula should be used for modeling default dependence and this will impact the pricing of multi-name credit instruments.

Third, in the standard application the marginal distribution for the stopping time is assumed to be exponential. In general this is inconsistent with the underlying structural model used to justify the use of equity correlations.

Fourth, once an obligor defaults, the effect on the remaining obligors is dictated by the initial and arbitrary choice of the copula – see Schönbucher and Schubert (2001)[57]. This approach is independent of the state of the economy and independent of the industrial structure of the different sectors of the economy – see Jarrow andYu (2001) and Yu (2003).

Unresolved Issues

1. To be of practical use any methodology describing the joint distribution of stopping times must be calibrated to extant data. This places a severe restriction on the types of models that can be used. We require an internally consistent model that can be calibrated.

5.4 The Reduced Form Approach

Reduced form models provide a natural way to describe the evolution of credit spreads. If the intensity function is modeled as a Cox process, then this will in general imply that defaults are correlated. Typically, a set of state variables is specified and for each obligor a time series analysis of credit spreads is employed to estimate the coefficients. For many obligors, time series and cross-sectional analysis can be combined. Given that for each obligor the default process depends on the same set of state variables then defaults are correlated – see Jarrow and Deventer (2005). Some academics and practitioners have argued that reduced form models are unable to generate "acceptable" levels of default correlations. This view is challenged by Yu (2002b).

Yu (2002b) first considers the models developed by Duffee (1999) and Driessen (2005). The intensity function is assumed to be identical under the natural and pricing measures. This is a strong and questionable assumption, as Driessen presents empirical evidence that the default

[57] For the bivariate case, Schönbucher (2003, p358/9) shows that for the normal and t-copula functions, given default by one obligor the relative jump size in the intensity function of the remaining obligor has a singularity at the origin.

premium is greater than one. Yu adjusts for liquidity and taxation and estimates the adjustment coefficients by calibrating to Moody's one year transition matrix[58].

The Driessen model generates default correlations that are substantially larger than those produced by the Duffee model, which is not surprising given the two additional economy wide factors. It is shown that the adjustment for liquidity and taxation has a major impact on the estimates of the default correlation inferred from credit spreads. For example, for a horizon of five years the unadjusted default correlation for A rated firms is 0.93 percent and the adjusted value is 11.78 percent. Default correlations increase monotonically with maturity. For maturities greater than one year, default correlations decrease as credit quality declines.

Yu also develops a number of different models that address how different industrial structures affect default correlations. The current applications of the copula methodology do not have the ability to address this type of issue. Again the issue of calibration is not addressed.

Limitations

Consider the problem facing a practitioner who is trying to price a tranche of a CDO involving over a hundred names. To apply the reduced form methodology you need to estimate the intensity function for each obligor. It is an unfortunate reality that for many obligors we simply do not have enough bond/spread data with which to undertake the necessary econometric analysis. Until such data exist, or we have sufficient data from other markets, such as the credit default swap market, then the reduced form methodology can only be applied to a small part of the fixed income market. Using data from other markets also generates additional problems, if these markets are not liquid, as we will have different liquidity premiums in the different markets.[59]

5.5 Testing a Model

[58] The adjusted intensity function is assumed to be of the form $\lambda_t^{Adj} = \lambda_t - \frac{a}{t+b}$, where a and b are coefficients that must be estimated. It is not clear why Yu did not estimate the adjustment factor directly from spreads, given its deterministic nature.

[59] See Jarrow and Turnbull (2000) and Longstaff *et al* (2003).

In testing an option pricing model, often some form of replicating portfolio is formed and the properties of the residual error, the difference between the price of the option and the replicating portfolio are examined. This approach assumes that we have an active secondary market for trading the option. For both single and multi-name credit derivatives this is not the case and this prevents direct testing of pricing models. For investment grade obligors default is a rare event, so it is not possible to directly test a model of default dependence. Even for high yield obligors we have limited data, which restricts testing of models. How do we test a model of default dependence given data limitations? One of the main complaints from regulators is the inability to test models that are used to generate risk measures for credit portfolios.

For testing copula models describing default dependence, the usual approach is to test the ability of the model to describe the joint distribution of equity returns. It is then assumed, given hypothesis H1, that the model is capable of describing the distribution of stopping times. Clearly this is not a test of the model. We do have bond data and this allows us to test models describing the evolution of credit spreads for different obligors. In employing the reduced form methodology, individual intensity functions are calibrated using a time series of credit spreads. The combining of time series and cross-sectional data and developing hypotheses about the joint co-movements of spreads remains to be done. However, this approach does provide a way for testing models. The copula methodology has not addressed the issue of modeling the evolution of credit spreads and there has been no real testing of hypothesis H1.

Summary

We have no reliable empirical knowledge about default correlations. Given that default is a rare event, it is unlikely that reliable statistical estimates can be generated. There does not seem to be any models that can used to model contagion in credit markets, though the frailty approach offers hope. There is no general economic framework that links the probability of default of single firms together with a particular copula, apart from the original CreditMetrics framework. The reduced form methodology must be adopted to avoid its reliance on credit spreads.

Unresolved Issues

1. We need models to (a) describe contagion in credit markets, (b) can be calibrated and (c) can be empirically tested.

2. We need to extend the reduced form methodology so that it can be more widely employed.

6 Summary

We have explored a number of the important areas that directly affect risk management and the pricing of credit risky instruments. There are many unresolved issues. The works of Acharya *et al* (2003) and Altman *et al* (2005), which identify some of factors that affect recovery rates, are a major step forward. However, the work needs to be generalized to incorporate past performance of a firm and out of sample testing is required. Similarly the work on the modeling of the probability of default to a multi-period setting needs to be generalized to incorporate contagion arising from information effects if investors are imperfectly informed about common factors that affect the true probability of default. Testing these two areas of research – recovery rate and default probabilities - will always be inherently difficult, given that scarcity of data. Combining these two streams of work will provide an important contribution to the area of risk management, as it will facilitate the modeling of expected loss through the credit cycle, conditional on the current state of the economy.

We need to incorporate these empirical findings into pricing models for individual instruments and for modeling default dependence. At present default dependence is modeled using ad hoc copula functions with ad hoc parameter identification. More work is needed to relate the use of copulae to the specifics of firms and to the state of the economy[60]. The reduced form approach provides a natural means to model default dependence. More work is required to justify the transition from the natural probability measure to the pricing measure and identifying the default risk premium. Once this is done, then by calibrating the model to actual default data and by changing to the pricing probability measure, this approach provides a natural way to model default dependence for pricing multi-name products and addressing the pricing effects of counterparty risk. Both structural and reduced form approaches must address the issue that spreads may also be affected by other factors, apart from the risk of default.

[60] The original CreditMetrics approach provides in a limited context, a justification for the normal copula.

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