On Pricing Credit Default Swaps With Observable Covariates^{*}

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Abstract

Observable covariates are useful for predicting default, but several findings question their value for explaining credit spreads. We introduce a discrete time no-arbitrage model with observable covariates, which allows for a closed form solution for the value of credit default swaps (CDS). The default intensity is a quadratic function of the covariates, specified such that it is always positive. The model yields economically sensible results in terms of fit and the economic impact of the covariates. Macroeconomic and firm-specific information can explain most of the variation in CDS spreads over time and across firms, even with a parsimonious specification. These findings resolve the existing disconnect in the literature regarding the value of observable covariates for credit risk pricing and default prediction. Our results also suggest that although CDS spreads are highly auto-correlated, analyzing spread levels may be preferable to analyzing differences for daily CDS data.

JEL Classification: G12

Keywords: credit default swap; no-arbitrage; observable covariates; volatility; leverage; distanceto-default.

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1 Introduction

Many studies have shown that observable covariates are very useful in predicting default. Shumway (2001) demonstrates that firm specific variables such as the excess stock return, stock return volatility, the ratio of net income to total assets, and the ratio of total liabilities to total assets can explain the probability of default. Duffie, Saita and Wang (2007) use distance to default, the firm's stock return, the three month Treasury bill yield, and the one-year trailing S&P 500 index return as explanatory variables to estimate the probability of default. Given these studies on default prediction under the natural probability measure, it is reasonable to ask whether observable covariates are also key determinants of the prices of credit risky securities. The main contribution of this paper is to answer this question using data on corporate credit default swaps (CDS), derivatives contracts contingent on a firm's default.

When pricing corporate bonds and CDSs, it is necessary to estimate the loss distribution under the pricing probability measure. There are several approaches to identifying the observable determinants of credit spreads for corporate bonds and CDSs. One approach uses structural models of default, following Merton (1974).¹ In these models, the observable covariates are determined by the underlying theory. For example, in the simplest structural models suggested by Merton (1974) and Black and Cox (1976), credit spreads are determined by interest rates, firm asset volatility, and firm leverage. However, several authors have come to the conclusion that structural models do a poor job of explaining credit spreads for corporate bonds and CDSs.² These findings cast doubt on the value of observable covariates for explaining credit risk.

There is a large literature that attempts to explain credit spreads, or credit spread changes, by regressing on observable covariates. Overall, this literature questions the explanatory power of observable covariates for credit spreads; see for instance Collin-Dufresne, Goldstein, and Martin (2001). The evidence from linear regressions, together with that from structural models, suggests a disconnect between the literature on default prediction, where observable covariates are highly successful, and the literature explaining credit spreads, where observable covariates are much less useful.

An alternative to the use of structural models for pricing corporate bonds and CDSs is the reduced-form approach, introduced by Jarrow and Turnbull (1992, 1995). Presumably in part because of the shortcomings of models with observable covariates, the reduced form approach is

¹See also Black and Cox (1976), Collin-Dufresne and Goldstein (2001), Cremers, Driessen, and Maenhout (2008), Geske (1977), Kim, Ramaswamy and Sundaresan (1993), Leland (1994), Leland and Toft (1996), and Longstaff and Schwartz (1995).

²See for example Eom, Helwege, and Huang (2004) and Huang and Zhou (2008).

usually implemented using latent factors.³ Latent factor models usually provides a good in-sample fit. However, while the estimated latent factors can be compared to observables, they do not provide much intuition with respect to the economy-wide and firm-specific determinants of credit risk.

We follow Lando (1998) and assume that the default intensity in a reduced-form model is a Cox process depending on macroeconomic and observable firm specific covariates. We introduce a new discrete time no-arbitrage model with observable covariates, where the dynamics for the stopping time are described by a quadratic function of these covariates, and we derive a recursive closed form solution for the pricing of CDSs. The advantage of our no-arbitrage model is that estimated spreads are positive by construction without restricting model coefficients, and that estimated spreads are internally consistent across maturities.

The model is estimated using daily data for eighty-three firms for the period January 2002 to March 2008. We use one, three, and five year maturity CDS spreads in estimation.⁴ We show that observable covariates are adequate at explaining the variation in credit spreads over time and across firms. Our preferred model is a very parsimonious specification, with four covariates: two covariates extracted from the riskless term structure, the firm's distance-to-default computed using option-implied volatility, and the VIX. These covariates are suggested by the structural model of Merton (1974), and the estimated signs on the covariates in the resulting specification are therefore easily interpretable from a theoretical perspective. This model provides a good fit, and the impact of the VIX and the firm's distance-to-default on credit spreads have the expected sign for more than 93% of estimated firms. We also investigate richer models with more extensive sets of covariates. While richer models may achieve somewhat better fit, the improvements on the more parsimonious specification are modest.

While our focus is on the performance of the no-arbitrage model, we also investigate linear regressions, because our results seem to contradict the findings in extant literature regarding the limited explanatory power of observable covariates for credit spreads. We argue that the evidence from both approaches is in fact consistent, both in terms of fit and economic impact of observable covariates. However, the interpretation of results depends on whether one analyzes credit spreads and observable covariates in levels or differences.

This raises the question which specification is preferable, levels or differences? We provide a detailed analysis of the economic impact of covariates on credit spreads using regressions, and

 $^{^{3}}$ For latent-factor reduced-form studies of corporate bonds, see for example Duffie and Singleton (1999), Duffee (1999), Driessen (2005), and Feldhutter and Lando (2007). For studies of credit default swaps, see Houweling and Vorst (2005) and Chen, Cheng, Fabozzi, and Liu (2008). Longstaff, Mithal, and Neis (2003) link the implied probabilities of default in corporate bond and credit default swap markets, and find pricing inconsistencies between both markets.

⁴Bakshi, Madan, and Zhang (2006) provide a related analysis of observable determinants of corporate bond spreads within a no-arbitrage model.

the role of statistical assumptions in measuring this impact. We argue that while the analysis of spread differences facilitates the interpretation of certain measures of fit, the resulting estimation is potentially statistically more inefficient than the analysis of spread levels, and this may complicate inference; see Harvey (1980) and Maeshiro and Vali (1988). This problem is particularly acute for daily differences of CDS spreads, because the resulting signal-to-noise ratio is very low. There is additional motivation for the analysis of credit spread levels. First, while the time series of credit default swap spreads and observable covariates are highly auto-correlated, they are mean-reverting and not characterized by long term stochastic trends, as opposed to many other financial and economic variables.⁵ Second, the covariates we analyze are explicitly suggested by theory, and it is therefore less likely that we would be uncovering spurious relationships using levels regressions.

We expand on this argument by demonstrating that while differencing a time series of daily spreads yields very different results compared to a levels regression, differencing lower frequency data confirms the results from levels regressions. We conclude that the choice of levels versus differences in the credit risk literature should depend on the frequency of the data. For the monthly bond data analyzed in Collin-Dufresne, Goldstein, and Martin (2001), differencing the data may be the best choice, whereas for daily CDS data, analyzing levels may be preferable.

In summary, this paper makes four contributions. First, it describes a new discrete time, closed form model with observable covariates for pricing credit risky assets. The intensity function is a quadratic function of the covariates, specified such that it is strictly positive without restrictions of the signs of the coefficients. Second, the no-arbitrage model yields sensible results in terms of fit and the economic impact of covariates on spreads. The methodological advantages of the no-arbitrage model and the consistency across maturities imposed in estimation do not come at the cost of empirical fit and statistical significance. Third, we analyze the underlying statistical assumptions appropriate for different data frequencies. We argue that for the analysis of daily CDS spreads, analyzing spread levels may be preferable to analyzing spread differences. Fourth, the empirical results provide strong evidence that observable covariates are useful in explaining credit spreads, which is consistent with the evidence regarding default prediction, therefore resolving an important disconnect in the existing literature.

The paper proceeds as follows. In Section 2, we introduce the new discrete time no-arbitrage model for CDSs. Both the term structure of interest rates and the process for the stopping time are described by quadratic functions of observable covariates. Section 3 presents a case study of the firm The Gap, Inc., to provide more intuition for the model's features. The data are described in Section 4. In Section 5, we present empirical results for eighty-three firms using a parsimonious specification with four observable covariates, and compare the estimates to those obtained using

⁵See Cremers, Driessen, Maenhout, and Weinbaum (2008) for a similar argument.

linear regression. We also examine the implications of alternative statistical assumptions. In Section 6 we consider various robustness tests. Section 7 concludes.

2 Model Description

In this section we describe the pricing models for the default free term structure and for CDSs. We work in discrete time and assume factors are described by compound autoregressive processes. See Gourieroux and Jasiak (2006) for an overview of these processes. We use a quadratic model. See Bekaert, Cho and Moreno (2006) and Ang and Piazzesi (2003) for applications of discrete time Gaussian frameworks, and Gourieroux, Monfort and Polimenis (2006) for an application of the discrete equivalent of the CIR model.

2.1 Default Free Bonds

The spot interest rate over a given period is assumed to be a quadratic function of the form

$$r_t = \left(\delta_0 + \sum_{k=1}^n \delta_k X_{k,t}^r\right)^2,$$

where $\{X_{k,t}^r\}$ are factors.⁶ It is assumed that these can be described by the following AR(1) dynamics:

$$X_t^r = \mu_r + \rho_r X_{t-1}^r + e_t, \tag{2.1}$$

where X_t^r denotes a (n, 1) vector, $e_t \sim N(0, \Sigma_r)$, μ_r is a (n, 1) vector and ρ_r and Σ_r are (n, n) matrices. The price of a default-free zero coupon bond is given by

$$B(t, t+h) = E[\exp(-r_t - \dots - r_{t+h-1})|r_t].$$

It is shown in Appendix A that this can be written in the form

$$B(t, t+h) = \exp(A_h + B'_h X^r_t + X^{r'}_t C_h X^r_t), \qquad (2.2)$$

where the explicit definitions of the coefficients A_h , B_h and C_h are derived recursively.

⁶See for example Longstaff (1989), Ahn, Dittmar, and Gallant (2002), Constantinides (1992), Brandt and Chapman (2002), Ang, Boivin and Dong (2008), and Leippold and Wu (2002) for quadratic term structure models. See Gourieroux and Monfort (2007) for an application of a discrete time quadratic factor model to mortality intensity modeling.

2.2 Credit Default Swap Valuation

We follow the discrete time modeling of default described in Gourieroux, Monfort, and Polimenis (2006). A stopping time has an intensity process $\lambda(t)$. Given no default up to time t, the probability of no default over the next interval is $exp(-\lambda(t))$. A default time for an obligor generates a default process N(t) that is zero before default and one after default. The probability for an obligor surviving until at least interval h is given by

$$P_t[\tau > t+h] = E_t\left[\exp\left(-\sum_{j=0}^{h-1} \lambda_{t+j}\right)\right],\tag{2.3}$$

where τ denotes the time of default.

Default can arise from events that affect a particular sector or the whole economy or are unique to the obligor. For example, in the current credit crisis the fall in house prices has been one of the major drivers of default by home owners. The fall in house prices occurred across many different states, eventually triggering default by mortgage originators and financial institutions. However, a particular institution's leverage and portfolio composition affected its chances of survival.⁷ We assume that default for an obligor depends on a set of measurable covariates - see Lando (1994, 1998). The intensity is assumed to depend on the same factors that affect the default free term structure $X_{k,t}^r$ and on other macro and obligor specific factors denoted by $\{X_{k,t}^d\}$, and is also assumed to be a quadratic function of these covariates

$$\lambda_t = \left(\alpha_0 + \sum_{k=1}^n \alpha_k X_{k,t}^r + \sum_{k=1}^m \alpha_{k,X^d} X_{k,t}^d\right)^2.$$
 (2.4)

The advantage of a quadratic specification is that the intensity function will be strictly positive. This is not the case for a linear specification if the state variables are assumed to be Gaussian. If they are assumed to follow CIR processes, then it is necessary to restrict the parameters of the process and the coefficients restricted to be positive. Let

$$X_{t+j} \equiv \begin{bmatrix} X_{t+j}^r \\ X_{t+j}^d \end{bmatrix}$$

denote a (q, 1) vector, where q = n + m, where n is the number of term structure factors and m the

 $^{^7\}mathrm{See}$ Crouhy, Jarrow, and Turnbull (2008) for a description of the many different factors that contributed to the crisis.

number of additional covariates. It is shown in Appendix B that

$$r_{t+j} + \lambda_{t+j} = \gamma_0 + \gamma_1' X_{t+j} + X_{t+j}' \Omega X_{t+j}.$$
(2.5)

It is assumed that

$$X_t = \mu + \rho X_{t-1} + e_t, \tag{2.6}$$

where $e_t \sim N(0, \Sigma)$, μ is a (q, 1) vector and ρ and Σ are (q, q) matrices.

For a CDS, we first consider the payments by the protection buyer. When entering into a contract, the protection buyer may possibly make an initial payment, U, and a series of quarterly payments. Let S denote the CDS spread. The protection buyer promises to make payments $S\Delta$ each quarter, conditional on no default by the reference obligor, where Δ is the time between payment dates. If a credit event occurs, the protection buyer receives a payment from the protection seller and the contract terminates. The present value of the payments by the protection buyer is

$$PB_{t} = U + E_{t} \left[S\Delta \sum_{j=1}^{h} 1_{(\tau > t+j)} A(t+j) \right].$$
(2.7)

where A(t+j) is the riskless discount rate $\exp(-r_t - \dots - r_{t+j-1})$. In Appendix B we show that

$$E_t[1_{(\tau > t+j)}A(t+j)] = \exp(F_j + G'_j X_t + X'_t H_j X_t),$$
(2.8)

where the coefficients F_j , G_j and H_j are derived recursively.

The protection seller will make a payment of (1 - R), where R is the recovery rate, if a default event occurs. We assume that if a default event occurs during the interval (t+j-1, t+j], payment by the protection seller is made at the end of the interval. The present value of the promised payment by the protection seller is

$$PS_t = E_t \left[(1-R) \sum_{j=1}^h 1_{(t+j-1 < \tau \le t+j)} A(t+j) \right].$$

We assume that the recovery rate is known. We can relax this assumption, though with insufficient data, it is difficult to estimate the additional parameters. We can write the above expression in the form

$$PS_t = (1 - R) \left(E_t \left[\sum_{j=1}^h (1_{(\tau > t+j-1)} A(t+j) \right] - E_t \left[\sum_{j=1}^h 1_{(\tau > t+j)} A(t+j) \right] \right).$$
(2.9)

The second term of the right side of the above expression is given by expression (2.8). To evaluate

the first term, consider

$$E_t \left[1_{(\tau > t+j-1)} A(t+j) \right].$$
(2.10)

It is shown in Appendix B that this also can be written in the form

$$E_t \left[1_{(\tau > t+j-1)} A(t+j) \right] = \exp \left(M_j + N'_j X_t + X'_t P_j X_t \right),$$
(2.11)

where the coefficients M_j , N_j and P_j are derived recursively.

The spread of the CDS is set such that

$$PB_t = PS_t. (2.12)$$

In what follows the price of default protection refers to the spread $S^{.8}$

3 A Case Study: The Gap, Inc.

To illustrate the main findings and implications of our study, we begin with a case study of a single firm: The Gap, Inc., a company whose fortunes have varied significantly. Table 1, Panel A, reports descriptive statistics. The average CDS spread over the 2002 to 2008 period is 156 basis points for one-year protection, 176 basis points for three-year protection, and 197 basis points for five-year protection. Figure 1 plots the market CDS spread for the five-year tenor between 2002 and 2008, together with model spreads, the company's stock price, the firm's option-implied volatility, leverage, and distance-to-default. In the earlier part of our sample, the price of five-year default protection fluctuated around 600 basis points, and Standard & Poors assigned the company a BB+ credit rating on their long term debt. At the time, distance-to-default was at its lowest and leverage at its highest during our sample period.

During 2002, the company, faced with stiffening competition, repeatedly reported losses. A failed change in marketing strategy led to the demise of CEO Michael Drexler. As of mid 2002, the company changed strategy, a change in leadership took place, and the firm's outlook started to improve. As a result, the stock rallied and the price of five-year protection was steadily lowered, and eventually bottomed out at approximately 100 basis points. Implied volatility decreased steadily from approximately 60% to about 20%. The firm was upgraded to investment grade in early 2005 and maintained this rating until late 2006, when it was downgraded back to BB+. However, this downgrade did not cause the CDS spread to increase to its earlier highs. Interestingly, option-

⁸Recently, changes in the CDS market make the up-front fee U the pricing parameter. However, this does not affect our sample.

implied volatility and leverage also remained at lower levels, and the distance to default did not significantly decrease until the end of the sample.

Importantly, a visual inspection of Figure 1 suggests strong univariate relationships between the candidate covariates and the CDS spread. While it is of course important to confirm these impressions using other approaches that are both more formal and multivariate in nature, all suggested relationships are consistent with available theory: higher firm-implied volatility and higher leverage are associated with higher spreads, and higher distance-to-default is associated with lower spreads. Higher stock prices are associated with lower spreads, which can also be obtained as an implication of structural models, or alternatively can be understood because of the robust negative correlation between stock returns and firm volatility. Because of space constraints, it is of course not possible to include figures for all 83 firms used in the empirical analysis, but similarly strong associations between spreads and candidate covariates are apparent from visual inspection of the data for almost all firms.

The two top panels of Figure 1 illustrate the performance of a parsimonious linear regression and the no-arbitrage model. In both cases, we limit the covariates to two term structure factors, the VIX, and distance-to-default, a metric that combines leverage and operating risk. The data are discussed in more detail in Section 4.1. To save space, we only provide figures for the five-year tenor. Table 1, Panel B provides measures of fit for the one-year, three-year, and five-year tenors. The R-square of the linear regression is high for all three tenors, around 85%, but because the CDS spreads and some of the covariates are highly autocorrelated, the interpretation of this R-square is subject to problems, as discussed by Granger and Newbold (1974).

We estimate the no-arbitrage model using all three tenors jointly to impose consistency in pricing, and the fitting exercise is therefore more demanding. Despite this, the RMSEs for the no-arbitrage model are about 35% lower on average compared to the regression, at about 53 basis points averaged across the tenors. Figure 1 indicates that both the no-arbitrage and regression models provide an adequate fit. Note in this respect that, following the argument of Granger and Newbold (1974), while the high R-square for the regression may indicate a spurious relationship, the relationship may of course also be genuine. Importantly, visual inspection of the spreads and the covariates in Figure 1 clearly indicates that the time series under study are not characterized by stochastic trends, which is the case Granger and Newbold (1974) had in mind because of the properties of time series such as aggregate consumption or aggregate GDP. Instead, the CDS spread and the firm implied volatility in Figure 1 confirm our intuition that they should be stationary time series. We will discuss these observations in more detail below.

Figure 1 also clearly indicates that the no-arbitrage model performs well in pricing the CDS. The good fit obtains in spite of the discipline imposed by the no-arbitrage approach, which imposes consistency in pricing across maturities and avoids negative spreads. Note that after the turnaround in the company's fortunes, and the reduction in its CDS spread to lower levels as of 2004, we observe several episodes of negative predicted spreads for the linear regressions. The most notable excursion into negative territory occurs in mid 2005, when the five-year spread approaches minus 100 basis points. The pattern is even more dramatic for the one-year and three-year tenors. The one-year model spread for the linear regression approaches minus 200 basis points. We also observe negative predicted spreads for one-year tenors in late 2006, and across tenors in 2007.

Negative spreads, in particular of such magnitudes, constitute arbitrage opportunities, and render the model useless for practical purposes during such episodes. The no-arbitrage credit risk model presented in this paper rules out such scenarios by design, without restricting model coefficients. Figure 1 also indicates that this model is able to match the low levels and low spread volatility from 2004 onwards quite easily, whereas the fitted spreads from linear regressions are too volatile and negative.

We conclude that in the case of The Gap, Inc., the greater economic consistency of the noarbitrage model does not come at the cost of increased fitting errors, despite the fact that the fitting exercise is more demanding.

In Section 5 we report results for all 83 firms in our sample. First we discuss our data and estimation methodology.

4 Data and Estimation Method

4.1 Data

Our sample period is from January 1, 2002 to March 7, 2008. Reliable data on the CDX index are available starting in October 2004. CDS and CDX spreads are obtained from Markit. We collect data for all single name CDS contracts that were part of the DJ.CDX.NA.IG (CDX henceforth) index at any time between October 2004 and March 2008. To have sufficiently long time series, we require that the obligors have CDS data that starts in 2002 and is available until March 2008, and that data on observable covariates, as discussed below, are available during this period. With these requirements, we obtain a sample of 83 firms. We obtain CDS spreads for 1, 3, and 5 year maturities for all firms. The top panel of Figure 2 presents the average spread across all 83 firms for the 5-year tenor. The variation over the sample for the 1-year and 3-year tenors is similar, though we do not include these figures to save space.

To estimate the risk-free term structure model, we use daily Libor rates with 6 month maturity and interest swap rates with maturities of 1, 2, 3, 4, 5, 7, and 10 years. The Libor and interest swap rates are obtained from Bloomberg.

In addition to the CDS and Libor data, we require the following firm-specific and economy-wide data: firm-specific option implied volatility, total liabilities, the market value of equity, liquidity, the implied volatility of the index, and the one-year trailing S&P500 return. We obtain the data on 30-day at-the-money put option implied volatility from Optionmetrics. We obtain data on equity prices, the number of shares outstanding, and the daily stock return for each firm, from CRSP. For total liabilities, we use Compustat variable LTQ. Since balance sheet information is reported at a quarterly frequency, we transform it into daily data through linear interpolation. We define leverage as the ratio of total liabilities and the sum of the market value of equity and total liabilities. We use the number of contributors for the 5-year maturity spread as our measure of CDS liquidity. We obtain the S&P500 return from CRSP, and the daily VIX from Optionmetrics. The bottom two panels of Figure 2 depict the one-year trailing S&P500 return and the VIX. Over the sample period, the S&P500 return is 4.5% on average with a standard deviation of 14.6%, and the VIX is 18.21% on average with a standard deviation of 6.83%.

Panel A of Table 2 provides summary statistics for all 83 firms in the data set.⁹ The table includes the averages and standard deviations of spreads for the 5-year tenor, as well as for leverage, option implied volatility, distance-to-default, and the CDS liquidity measure. The table also includes the firms' average rating over the sample period.¹⁰ Most firms in the sample are rated A or BBB, which is not surprising given the composition of the CDX index. Despite the relative homogeneity of the firms in the sample in terms of ratings, the table indicates substantial differences in the average spread levels, as well as in the descriptive statistics for the observable covariates. The median 5-year CDS spread is just over 60 basis points, with a standard deviation of 34 basis points. Distanceto-default ranges between 2.3 and 14.6, with a median of 9. The smallest distance-to-default is for Visteon which has the largest average spread in our sample, while the largest distance-to-default is for Home Depot, which has a fairly low average spread. Firms with average spread higher than 100 basis points have an average distance to default of 7.2, while firms with average spread lower than 100 basis points have an average distance to default of 9.6, suggesting the expected negative relationship between distance to default and spreads. The liquidity variable reported in the table averages 11 with a minimum time series average of just less than 7, and a maximum of 14. The firms in our sample have an average leverage that ranges from a minimum of 17% to a maximum of 87%, and average option-implied volatility ranges from a minimum of 19% to a maximum of 56%.

⁹Panel B of Table 2 is discussed in Section 5.

¹⁰At each point in time, we assign a numerical code to the firm's rating. Subsequently we average over time and map the average back to a rating category.

4.2 Estimation Method

All estimates of the no-arbitrage CDS model are obtained using nonlinear least squares. The pricing model with observable covariates described in Section 2 provides a closed form solution, up to a recursion. Estimation is relatively straightforward compared to a model with latent covariates, because we proceed under the pricing probability measure, and the residuals can simply be obtained using the pricing formulas as a function of the observable data, which allows straightforward construction of the sum of squares.

In our implementation, we use stochastic term structure factors as covariates, and their estimation is somewhat more complex, because the latent state variables have to be filtered from the data. The Kalman filter offers a convenient framework for the estimation of these models. For our application, the transition function is Gaussian, but the measurement function is highly nonlinear. In most term structure and credit risk applications the nonlinearity in the measurement equation is addressed by the use of the extended Kalman filter, which approximates the nonlinearity using a Taylor expansion.¹¹ We instead use the unscented Kalman filter, which directly allows for non-linearities. We use a particular implementation of the unscented Kalman filter, the squareroot unscented Kalman filter proposed by Van der Merwe and Wan (2001), which we found to be numerically stable and computationally feasible.¹²

5 Empirical Results

In this section, we estimate the no-arbitrage model for all 83 firms in our sample using a parsimonious specification, with four covariates: two covariates extracted from the riskless term structure, the firm's distance-to-default computed using option-implied volatility, and the VIX. These covariates are suggested by a simple structural model, as in Merton (1974), and the estimated signs on the covariates are therefore easily interpretable from a theoretical perspective. We report on other specifications in Section 6.1 below.

It can be seen from equation (2.6) that the dynamic of each covariate contains three parameters, the drift μ , the persistence ρ , and the standard deviation σ . The estimation proceeds in two steps. In the first step we estimate the dynamics of the macro factors. This estimation only has to be performed once. This first step, in turn, consists of two parts. First, we estimate the six parameters describing the dynamics of the two term structure factors from the term structure of interest rate

¹¹See Chen and Scott (1995), Duan and Simonato (1999), and Duffee (1999) for applications of the extended Kalman filter to term structure models.

¹²See Chen, Cheng, Fabozzi, and Liu (2008) for an application of the unscented Kalman filter to credit risk models with latent factors. See Carr and Wu (2007) and Bakshi, Carr, and Wu (2008) for applications to equity options.

swaps. To estimate the three parameters characterizing the dynamics of the VIX, we use the noarbitrage model to determine the spread on the CDX index.¹³ For the index, the no-arbitrage model has three covariates: the two term structure factors and the VIX.

In the second step, we estimate for each firm the parameters characterizing the dynamics of the distance-to-default process, as well as the loadings on all four factors.

5.1 The Risk-Free Term Structure and Macro Factors

Table 3 reports on the estimation of the macro covariates. Panels A and B examine the risk-free term structure and Panel C the VIX. Panel A reports on the dynamics of the risk free term structure factors. The risk free term structure is captured using the two latent factors. Panel B reports the pricing errors as well as the measurement error standard deviation. The pricing errors for the swap curve are quite small, ranging from about nine basis points for the 6-month Libor to five basis points for the 5-year swap rate. These results seem to be in line with other studies.¹⁴ Estimated parameters are reasonable. Both factors are very persistent.

Panel C reports on the dynamics of the third macro factor, the CBOE VIX implied volatility index. These dynamics are estimated using the CDX spreads. The purpose of the estimation exercise that uses the CDX index is to uncover the risk-neutral dynamics of the VIX. The process is very persistent. The implied unconditional average of the VIX is 38.3%. Note that these are risk-neutral estimates, and that the estimate for the VIX will therefore exceed physical market volatility. The fit is very good, and we do not report the pricing errors to save space.

5.2 Firm-by-Firm Results

We now turn to a discussion of our findings for all 83 firms, using the parsimonious covariate specification with four observable covariates. The top panel of Figure 2 provides a summary of the time series behavior of the average CDS spread across firms for the five-year tenor. The average spread is approximately 100 basis points in 2002, and reaches a peak of just under 200 basis points

¹³Because the CDX index is not available over the entire sample period, we approximate it by averaging the spreads of the 83 firms in our sample. For the 2004-2008 time period, when the CDX is available, the correlation of the resulting time series with the CDX is over 99%.

¹⁴Jagannathan, Kaplin, and Sun (2003) estimate a multi-factor CIR model using the term structure of Libor-swap rates. Using a two-factor CIR model, they find mean absolute pricing errors between 2.2 and 7.5 basis points for swap rates with maturities between 3 and 7 years, while the pricing error for the three-month maturity Libor rate is 31 basis points. Li and Zhao (2006) estimate a quadratic term structure model using zero coupon Libor rates. They report RMSEs ranging from 0.5 to 7 basis points. Duffie and Singleton (1997) also estimate the term structure model using Libor-swap rates and find similarly low pricing errors. Chen, Cheng, Fabozzi, and Liu (2008) report RMSEs ranging from 13 to 23 basis points.

later that year. In the middle of the sample, from 2004 to early 2007, spreads are stable and low, although the term structure is steeper, with spreads ranging from 10 to 20 basis points at the short end, and reaching 50 to 60 basis points for the five-year tenor. Later in 2007, as the credit crisis starts developing, spread volatility increases and spreads again reach levels observed in 2002.

Figure 2 also reports on the VIX, the option-implied index volatility, as well as the one-year trailing return on the S&P 500, which we use in Section 6.1. Our sample begins with a period of high volatility and negative returns. As of early 2003, volatility begins to stabilize and the stock market rallies. The rest of our sample consists of a long period of fairly stable index returns and volatility, but in mid to late 2007, volatility increases and returns decrease in the run-up to the financial crisis.

Importantly, Figure 2 confirms the observation discussed in Section 3 that there seems to be a strong association between candidate covariates and CDS spreads. The positive correlation between the VIX in the middle panel and the average spread in the top panel is quite apparent. Similarly, Figure 2 suggests a negative correlation between the S&P return and the average spread.

Panel B of Table 2 reports estimation results on a firm-by-firm basis for the no-arbitrage model described in Section 2. In order to conserve space, we only report on the model's firm-by-firm fit for the five-year tenor. The qualitative conclusions for the one- and three-year tenors are similar and we will present some summary statistics later in Table 6. We use the estimates for the macro factors from Panels A-C of Table 3 and the estimated dynamics of the only firm-specific covariate, the distance-to-default process. Panel D of Table 3 reports on this process, which is found to be very persistent for nearly all firms. We also report goodness of fit measures for the linear regression

$$S_t = \gamma + \beta_r X_t^r + \beta_d X_t^d + \varepsilon_t, \tag{5.1}$$

where S_t is the CDS spread at time t, X_t^r is the vector of term-structure factors at time t, which for this specification consists of the two term-structure factors, and X_t^d is the vector of other factors at time t, which for this specification consists of distance-to-default and the VIX.

Table 2, Panel B indicates that for the five-year tenor, the level regressions yield on average high R-squares at 63.6%, ranging from 20% to 92% for individual firms. Once again, following Granger and Newbold (1974), these high R-squares may indicate a spurious relationship. However, the RMSEs confirm that the fit of the regressions is adequate. The no-arbitrage model outperforms the linear regression on average, yielding a 10% lower mean RMSE of 27.1 basis points. For the one-year and three-year tenors, the improvement in fit offered by the no-arbitrage model is even larger.

In summary, the no-arbitrage model provides a good fit, and performs well compared to the

regression approach. This is notable, because the model's enhanced economic consistency biases it towards a poorer fit. Note however that this improvement in fit is not the focus of our study; instead, the regressions simply serve as a benchmark to demonstrate that the no-arbitrage model performs well in terms of fit. What is more important is that two important methodological advantages are imposed by the no-arbitrage setup. First, the no-arbitrage approach rules out negative spreads. Second, for the regression approach, there is no obvious way to impose consistency in the pricing across tenors, and regressions are implemented one tenor at a time, which also provides arbitrage opportunities. The methodological advantages of the no-arbitrage approach come at no cost in fit.

The no-arbitrage model therefore provides a useful framework to investigate the impact of observable covariates on CDS spreads. However, before we can further explore estimates of this impact, we need to discuss in more detail the implications of statistical assumptions that are critically important in the credit risk literature.

5.3 Statistical Assumptions and Model Fit

5.3.1 Levels and Difference Regressions for Analyzing Credit Spreads

The specification of the no-arbitrage model in Section 2 defines the intensity - see (2.4) - in terms of the levels of the covariates. It therefore seems sensible to compare its empirical performance with the levels regression (5.1), see Table 2, Panel B. However, the statistical specification of linear regression models of credit spreads, which dates back to at least as far as Fisher (1959), has been the subject of some debate. Collin-Dufresne, Goldstein, and Martin (2001), using monthly data on corporate bonds and regressions using differences rather than levels, argue that covariates suggested by economic theory have limited explanatory power. Other credit risk studies use levels regressions, and some authors report results using both specifications.¹⁵

The choice between levels and difference regressions for credit spread analysis is a complex one, and no consensus has emerged in the literature. From a statistical perspective, differencing is preferred if the dependent variable and/or regressors are characterized by stochastic trends and integrated, because regression analysis using integrated or nearly integrated variables may yield spurious regression results, in that R-squares and t-statistics may be misleading. A first important remark in this respect is that stochastic trends may not be the most obvious representation of the variables used in (5.1). While bond spreads, CDS spreads, and covariates such as volatility are typically highly auto-correlated, economic intuition suggests that they are stationary, in the

¹⁵Recent work on bond spreads relying on levels regressions includes Campbell and Taksler (2003) and Cremers, Driessen, Maenhout, and Weinbaum (2008). Avramov, Jostova, and Philipov (2007) report on difference regressions for bond spreads. Blanco, Brennan, and Marsh (2005) use difference regressions on CDS spreads, while Zhang, Zhou, and Zhu (2009) and Ericsson, Jacobs, and Oviedo (2009) report on both levels and difference regressions for CDSs.

sense that they are not inherently characterized by a positive drift like stock prices or aggregate consumption. This intuition is confirmed by the time series of aggregate spreads in Figure 2. The time series of spreads for The Gap, Inc. in Figure 1, and the graphs for other firms in the sample (not reported) confirm this conclusion. Spreads are not trending up or down through time in our sample period.

One may argue that differencing ought to be preferred regardless of statistical considerations, because if a theory holds in levels, it should also hold in differences. However, this implicitly assumes that differencing is costless; this reasoning is incorrect. There is an important potential cost to differencing, because the difference regression may be less statistically efficient than the levels regression, and this can affect the estimated magnitudes of the impact of the covariates. On this issue, see for instance Harvey (1980), Zellner (1979), Gospodinov (2009), and especially Maeshiro and Vali (1988). Moreover, in many realistic scenarios, measurement error may further lower the signal to noise ratio in a difference regression compared to a levels regression.

5.3.2 Interpreting Measures of Fit

In summary, there are good arguments in favor of both approaches, and the choice is not obvious. We consider the implications of this choice for the specification of no-arbitrage models of credit risk with observable covariates, such as the one outlined in Section 2. Consider the difference regression

$$\Delta S_t = \omega + \beta_r \Delta X_t^r + \beta_d \Delta X_t^d + \xi_t.$$
(5.2)

Panel A of Table 4 presents the average R-squares for the level and difference regressions (5.1) and (5.2) for all three maturities. The R-squares for levels are in the 63 to 69 percent range, while for differences the R-squares are dramatically lower, at around 1 to 2 percent. This evidence is consistent with existing studies, in the sense that R-squares from levels regressions are consistently higher than those from difference regressions.¹⁶

Granger and Newbold's (1974) warning regarding the R-squares of the levels regression is well known. However, the low R-squares from the difference specification must also be interpreted with caution: they may simply indicate that much of the spread is explained by lagged spreads. Most importantly, a comparison between the R-squares of the levels and difference regressions is uninformative, even if the R-squares are free of problems in both cases.¹⁷ More generally, as argued

¹⁶Our R-squares for difference regressions may seem rather low. Zhang, Zhou, and Zhu (2009) also report R-squares between 1 and 5%. Collin-Dufresne, Goldstein, and Martin (2001) report higher R-squares for bond spread differences, and Blanco, Brennan, and Marsh (2005) and Ericsson, Jacobs, and Oviedo (2009) report higher R-squares using CDS spread differences. We comment on differences with the existing literature in more detail in Section 6.4.

¹⁷It is well known that it may be problematic to compare models using R-squares. See for instance chapter 3 in

by Harvey (1980), comparing the fit of levels and difference regressions is difficult. To appreciate the inherent difficulties, consider instead the R-square from the regression of the spread S_t on the fitted value $\hat{S}_t = S_{t-1} + \hat{\omega} + \hat{\beta}_r \Delta X_t^r + \hat{\beta}_d \Delta X_t^d$, where $\hat{\omega}$, $\hat{\beta}_r$ and $\hat{\beta}_d$ indicate the estimates from the difference regression (5.2). The R-squares for this regression are upward of 98% across tenors, suggesting very good fit. Alternatively, one can compare RMSEs for difference and level specifications. Panel B of Table 4 presents average RMSEs. For the level regressions, the average RMSE is between 29 to 31 basis points. For the difference regressions, the RMSEs are between 4 to 6 basis points, again suggesting that the difference regressions perform well.

Measures of fit must thus be interpreted carefully, dependent on the statistical specification, and are hard to compare across specifications. From a finance perspective, the economic impact of covariates on spreads is even more important than model fit. As mentioned above, there is a tradeoff between the advantages and disadvantages of levels and difference regression in this respect, with levels regressions offering advantages in assessing economic impact, as estimates are consistent, but subject to disadvantages when assessing model fit. For a level specification, the coefficients are consistently and efficiently estimated. The main potential disadvantage of a level specification is in the interpretation of the measures of fit, notably R-square, in the presence of autocorrelation. Difference regressions also yield consistent estimates, but as explained in Maeshiro and Vali (1988), the estimates are inefficient. This important point is often ignored, implicitly assuming that the problem is minor, but Maeshiro and Vali (1988) use a Monte Carlo experiment to demonstrate that the loss in statistical efficiency can be very large. It therefore seems prudent to estimate both levels and difference specifications, and to carefully analyze differences in the estimated impact of covariates on spreads.

5.3.3 Cochrane-Orcutt Regressions

An alternative approach to deal with highly auto-correlated variables is a Cochrane-Orcutt regression, which effectively imposes an AR(1) structure on the error term.¹⁸ Consider the levels regression in (5.1), and subtract the lagged spread pre-multiplied by ρ . This gives us

$$S_t - \rho S_{t-1} = \gamma (1 - \rho) + \beta_r (X_t^r - \rho X_{t-1}^r) + \beta_d (X_t^d - \rho X_{t-1}^d) + (\varepsilon_t - \rho \varepsilon_{t-1}).$$
(5.3)

The difference regression can be thought of as the special case of $\rho = 1$, and the levels regression obtains in case $\rho = 0$. Table 4 indicates that the results for (5.3) are very similar to the results for

Davidson and MacKinnon (2004).

¹⁸We have been unable to find any extant studies on the determinants of bond or CDS spreads that use the Cochrane-Orcutt methodology.

the difference regression (5.2), reflecting the very high auto-correlation in the daily CDS spreads.

The no-arbitrage results in Table 2 are obtained using nonlinear least squares, where the error term is defined to be white noise. To incorporate the statistical assumptions underlying (5.2) and (5.3) into a no-arbitrage framework, we also implement nonlinear least squares assuming that the error term is given by an AR(1) process, similarly to the specification (5.3). The results from this approach are labeled as NA-AR(1). Table 4, Panel B, indicates that the RMSEs for the NA-AR(1) model are very similar to the RMSEs for the difference and Cochrane-Orcutt regressions, indicating that the no-arbitrage approach is very flexible, and can accommodate alternative assumptions regarding the error distribution and the autocorrelation in spreads.

5.4 Observable Covariates and Credit Spreads

We now turn to a detailed study of the quantitative impact of covariates on CDS spreads, using different statistical assumptions. Note that the loadings α in equation (2.4) are not directly interpretable because the default intensity is quadratic in the state variables. We therefore focus on the numerical derivatives or "deltas" of the credit spreads with respect to changes in the covariates, which we refer to as sensitivities. These sensitivities also make it easier to compare the results of the no-arbitrage specification and the regression approach, because in the no-arbitrage specification (2.4) it is the default intensity that is specified as a function of the covariates, whereas for the regression (5.1) it is the credit spread.

Figure 3 depicts the cross-sectional distribution of the sensitivities of default swap spreads with respect to the VIX and the firm specific distance-to-default factor. Table 5 reports cross-sectional means and standard deviations for sensitivities. Panel A of Table 6 reports on the statistical significance of each of the covariates.

We expect the VIX to have a positive effect on the spread, and distance-to-default to have a negative impact. These predictions hold not only in the Merton (1974) model, but also in all more recent models. It is less clear what to expect from the term structure factors.¹⁹ The evidence for the term structure factors is indeed quite mixed, as can be seen from the standard deviations in Panels A and B of Table 5.

Results for the market wide volatility VIX and distance-to-default confirm the theory. For the no-arbitrage model, Figure 3 indicates that the sensitivities to the VIX factor have the expected positive sign for 96% of all firms. Panel C of Table 5 indicates that on average across all firms

¹⁹Empirically, the link between the level of the risk-free term structure and credit spreads tends to be negative (see e.g. Duffee (1998) and Collin-Dufresne, Goldstein, and Martin (2001)). This empirical finding is often motivated by referring to the Merton (1974) model, but this is based on a comparative static where asset value is taken to be exogenous.

in the no-arbitrage model, if the VIX increases by 1%, the credit spread increases by 1.49 basis points. When using linear regressions, the estimated effect is larger on average, at 3.03 basis points, and Figure 3 indicates that we obtain a positive sign for 93% of firms.²⁰ For distance-to-default, the sensitivities have the a priori expected negative sign in 98% of the cases for both the linear regression and the no-arbitrage model. For the no-arbitrage model, credit spreads decrease by 8.50 basis points on average when distance-to-default increases by one unit. Results obtained using linear regression are on average very similar. Panel A of Table 6 indicates that these results are statistically significant for most firms.

Figure 3 clearly indicate the *economic* importance of statistical assumptions. In the models with auto-correlated errors, we again obtain the a priori expected positive sign for the VIX and negative sign for the distance-to-default in the majority of cases, but less often than in the case of the levels regression and the no-arbitrage model with white noise errors. Moreover, the magnitude of the estimated coefficients for the Cochrane-Orcutt and no-arbitrage-AR(1) cases is very different from that of the levels and no-arbitrage estimates. Panel C of Table 5 indicates that on average, a 1% increase in the VIX increases the credit spread by 0.15 basis points for the Cochrane-Orcutt regressions, compared to 3.03 basis points for the case of the levels regressions. For distance-to-default, an increase of one standard deviation leads to a decrease of -1.94 basis points when using Cochrane-Orcutt regressions, versus -8.16 basis points for the levels regressions. However, Table 6 indicates that this comparison has to be interpreted cautiously: in contrast to the results for the no-arbitrage model and levels regressions, estimates in models with auto-correlated errors are less often statistically significant. Note that the t-statistics in Table 6 are corrected for serial correlation in the residuals.

How similar are these estimates to existing results? The literature does not yet contain a wealth of evidence on the impact of macroeconomic and firm-specific variables on CDS spreads, even using simple regressions. For the NA and levels models, our estimates of the impact of the VIX obtained are roughly similar to the findings of Cao, Yu, and Zhong (2010), who report that a 1% increase in firm implied volatility increases CDS spreads by 2 to 3 basis points, and Ericsson, Jacobs, and Oviedo (2009), who report that a 1% increase in volatility raises the CDS premium on average by 0.8 to 1.5 basis points. Zhang, Zhou, and Zhu (2009) report a larger impact of firm volatility on CDS spreads.

In summary, the no-arbitrage approach is attractive: it avoids the pitfalls of the regression approach, while yielding economically plausible results, and it can accommodate a variety of statistical assumptions used in the linear regression literature. We also document that in linear regressions,

 $^{^{20}}$ To address serial correlation in the residuals, standard errors are corrected using a Newey-West correction with seven lags.

the statistical assumptions have a very strong impact on the economic magnitude of the loadings, which to the best of our knowledge has not yet been discussed in the literature.

6 Robustness

We now discuss the robustness of the empirical results in Section 5. We first discuss alternative covariate specifications and quadratic covariates. Subsequently we comment on the estimation of the risk-free term structure, discuss the importance of data frequency, and discuss overall implications.

6.1 Alternative Covariates Specifications

The analysis of alternative covariate specifications is of interest for several reasons. First, it is important to verify that the good performance of the no-arbitrage model in Section 5 extends to other specifications of the covariates. Second, the question arises by how much the fit can be improved by including additional covariates. Third, it is of interest to measure the economic impact of alternative covariates on spreads.

Because estimating the no-arbitrage model for many permutations of covariates is computationally costly, we proceeded as follows. We conducted an extensive specification analysis of regression models to find the covariates that best explain the CDS spreads, using the firm-specific and economy-wide variables described in Section 4.1. Analysis of a few random firms showed that the relative ranking of models is similar for the no-arbitrage models and linear regressions. Based on the results of this specification search, we subsequently estimated the no-arbitrage model for all 83 firms for a small number of alternative specifications of the covariates, and compared the results to the parsimonious specification in Section 5.

Here we summarize our main conclusions regarding the specification of no-arbitrage and regression models. First, the overall evidence is encouraging for the no-arbitrage setup, and for the relevance of observable covariates in explaining credit spreads more in general. Using more elaborate models, most of the coefficients are estimated with economically plausible sensitivities, as in the case of the model in Section 5. Second, while it is possible to improve on the fit of the parsimonious model, the improvement in fit is modest, and the parsimonious model more often yields the theoretically expected sign. This is interesting from a theoretical perspective, as the volatility, interest rate, and distance-to-default factors in the parsimonious model are directly suggested by structural models.

We now briefly discuss the results of a second covariate specification to illustrate these points. Three of the covariates are the same as in the parsimonious covariate specification studied in Section 5 : the same two stochastic term structure factors and the VIX. The other covariates are the oneyear trailing return on the S&P500, the 30-day option implied volatility, firm leverage, and the liquidity measure. The additional covariates included in this specification are intuitively appealing, and have been extensively analyzed in the existing literature. On average, across firms, each of these variables, except for the liquidity variable, contributes significantly to explanatory power.²¹ The RMSE is approximately 25 basis points on average across the three tenors, which is lower than the RMSE for the parsimonious specification in Table 4; however it is clear that the improvement in fit from three extra covariates compared to the results in Section 5 is modest.

Figure 4 reports how often the observable covariates are estimated with the a priori expected sign. For the S&P500, we obtain the expected negative impact on credit spreads in 92% of the cases for the levels regression, and in 89% of the cases for the no-arbitrage model. The VIX yields the expected positive sensitivity in the majority of cases, but less often so than for the parsimonious covariate specification in Figure 3. Firm-implied volatility is estimated with the expected positive sign in 98% of the cases for the levels regression, and in 93% of the cases for the no-arbitrage model.

Estimated sensitivities are therefore intuitively plausible. They are also consistent with existing results. The firm-implied volatility yields estimates in a similar range found by Cao, Yu, and Zhong (2010) and Ericsson, Jacobs, and Oviedo (2009). Further, Ericsson, Jacobs, and Oviedo (2009) find that a 1% increase in the leverage ratio increases the CDS premium by 5-7 basis points, whereas Cao, Yu, and Zhong (2010) report increases of approximately 3 basis points. Our estimate obtained using the levels regressions is on average 2.7 basis points, but the no-arbitrage estimates are smaller. An increase of 1% in the S&P500 leads to a decrease in the spread of at most half a basis point.

6.2 Quadratic Regression Specifications

It could be argued that the comparison of our no-arbitrage model with regression specifications such as (5.1) is flawed, because the regression model is linear in the covariates, whereas the noarbitrage model is quadratic. It is important to note in this respect that our objective is not to run a horse race between no-arbitrage and regression models, which are fundamentally different tools with different uses. The fit of the regression models is merely provided as a benchmark to verify that the fit of the no-arbitrage models is adequate. Our emphasis is on the methodological advantages of our approach, and we provide the fit of regression models to indicate that these methodological

 $^{^{21}}$ The liquidity variable performed poorly in general. Note that this is perhaps due to sample selection, because our sample exclusively consists of investment grade firms. Also, Bongaerts, de Jong and Driessen (2009) demonstrate that the impact of liquidity on CDS spreads is harder to sign than in the case of corporate bonds. Finally, a more detailed analysis of alternative liquidity measures, as in Tang and Yan (2007) for example, might yield more conclusive results.

advantages do not come at the cost of empirical fit.

Nevertheless, it is useful to compare the no-arbitrage model with a regression that includes quadratic terms of the covariates. We investigate this for the parsimonious model in Section 5. As expected, including quadratic terms improves the performance of the regression model, as the analysis is in-sample. On average, the fit for the regression model is slightly better than that of the no-arbitrage model when including quadratic regressors. However, fitted spreads are again negative on many occasions. It is therefore possible that the inclusion of quadratic terms in a regression model may harm the model in an out-of-sample exercise, but such an exercise is beyond the scope of this paper.

Alternatively, negative spreads can be avoided in a regression context by specifying

$$S_t = (\varrho + \phi_r X_t^r + \phi_d X_t^d)^2 + \varepsilon_t, \tag{6.1}$$

which can be estimated by Nonlinear Least Squares. We estimated this specification for all 83 firms, and the resulting fit is very similar to that of our no-arbitrage model, but of course this specification still has the disadvantage that the no-arbitrage restrictions are not imposed across tenors.

In summary, we confirm that the no-arbitrage model allows us to impose no-arbitrage and consistency across tenors, and this methodological advantage comes at no cost in terms of empirical fit.

6.3 Observable Term Structure Factors

We have reported on results for two specifications of the covariates: a parsimonious one in Section 5.2, and a richer one in Section 6.1. Both specifications use two stochastic term structure factors, as reported in Table 3. It is preferable to use stochastic term structure factors in the context of no-arbitrage models, but it is of interest to verify whether our results are robust to using deterministic term structure factors, especially because several existing studies have used the level and slope of the term structure in linear regressions.

We therefore re-estimated the levels regressions using two term structure factors: the level of the term structure, represented by the ten-year yield, and the slope, represented by the difference of the ten-year and three-month yields. For the no-arbitrage models the intensity function includes the same two factors. The cash flow are discounted using the extant term structure. The resulting estimates are very similar to the results for the stochastic term structure; they are therefore not reported but available from the authors on request. The fit of the resulting model is very similar to the fit reported in Table 4, and the histograms for the sensitivities of the VIX and distance-todefault are similar to those in Figure 3. The signs of the estimated sensitivities of the spreads with respect to the level and slope of the term structure vary considerably, consistent with the evidence for the stochastic term structure factors in Table 5. These findings are not necessarily surprising, as the correlation of the level with the first stochastic term structure factor is 90%, and the correlation of the slope with the second stochastic term structure factor is -93%.

6.4 Difference Frequency

In Section 5.3, we report on the impact of statistical assumptions on model fit and estimated covariate sensitivities. We now study this question in more detail by investigating regressions at different data frequencies.

Table 4 indicates that for our sample, the R-squares of the difference regressions and Cochrane-Orcutt regressions are very small, around 2%. This may seem somewhat surprising in light of the findings in the existing literature. While the difference regressions in Zhang, Zhou and Zhu (2009) also yield low R-squares in the 1 to 5 percent range, Ericsson, Jacobs and Oviedo (2009) and Blanco, Brennan, and Marsh (2005) report higher R-squares in the 22 to 26 percent range using CDS data. Moreover, Collin-Dufresne, Goldstein, and Martin (2001) and Avramov, Jostova, and Philipov (2007) obtain much higher R-squares, in the 19 to 42 percent range, using bond spread differences.

We argue that these differences are due to data frequency. We investigated the effects of data frequency by repeating our analysis for the parsimonious covariate specification, using weekly, monthly, and yearly data. For the weekly and monthly data, we simply use the data described in Section 4.1, and sample weekly or monthly. Sampling yearly yields a sample that is too small for meaningful analysis; we therefore use daily data and take one-year differences.²²

Our first objective is to investigate how data frequency impacts on measures of fit. Panels A and B in Table 7 report measures of fit for the weekly data, Panels C and D report on the monthly data, and Panels E and F on the yearly data. Table 7 shows that for the levels regressions, the R-squares and the RMSEs are very similar to those in Table 4. However, the R-squares for the weekly difference regressions are much higher than those obtained for daily difference regressions in Table 4, and the R-squares for the monthly and yearly difference regressions are even higher. The reason is that if credit spreads are highly auto-correlated, autocorrelation decreases with lower data frequency. Consequently, while today's spreads are very reliable predictors for tomorrow's spreads, they are less useful predictors of next week's or next month's spreads. Therefore, daily spread

²²Existing studies on credit spreads use weekly or monthly, but not yearly data. We use yearly data merely to illustrate the effects of data frequency for differenced data. Taking one-year differences yields overlapping data. Results of the Cochrane-Orcutt and no-arbitrage model with autocorrelated errors are not insightful with overlapping data, and therefore we do not report on all cases for the yearly data.

differences yield largely noise, but this is less the case when taking weekly or monthly differences; as a result, in the difference regressions more is left unexplained when the sampling frequency is lower, thereby providing a better opportunity for observable covariates to explain the data. This intuition is confirmed by the patterns in RMSEs and by the Cochrane-Orcutt regressions.

These observations explain the differences between our results using daily differences in Table 4 and some of the existing literature. The results for bonds in Collin-Dufresne, Goldstein, and Martin (2001) and Avramov, Jostova, and Philipov (2007) are obtained using monthly data. Ericsson, Jacobs and Oviedo (2009) and Blanco, Brennan, and Marsh (2005) use daily CDS data, but report gaps in the data, effectively yielding lower frequency data.

From an economic perspective, the most important question is if data frequency affects the estimated sensitivities of spreads with respect to covariates. Figure 5 presents results for monthly data. The histograms and estimated magnitudes for the levels regressions are very similar to those obtained using daily data in Figure 3. For the difference and Cochrane-Orcutt regressions, the percentage of a priori expected signs for the weekly, monthly, and yearly data is not too different from the daily case in Figure 3, but the estimated magnitudes of the coefficients are very different. We do not report weekly and yearly results because of space constraints, but the lower the data frequency, the more similar the estimation results from the difference and Cochrane-Orcutt regressions are to those of the levels regressions. For the yearly differences (not reported), the distribution of estimated magnitudes is very similar to that of the levels regressions.

Table 6 reports the percentage of statistically significant loadings on the covariates for different frequencies.²³ We report on the five-year tenor; results for the other maturities are very similar.²⁴ Lower data frequency leads to more statistically significant results for the difference and Cochrane-Orcutt regressions. Statistical significance for yearly differences is similar to that obtained for levels after correcting for serial correlation.

6.5 Implications

The robustness analyses confirm the usefulness of the no-arbitrage model. We also uncover additional evidence regarding regression specifications. While this is not the main focus of our paper, it is important to put these findings in perspective, because the regressions serve as a benchmark.

²³To address serial correlation in the residuals, standard errors are corrected using a Newey-West correction. We use four lags for weekly data, three for monthly data, and seven for daily data and yearly differences. We tried including additional lags for the yearly differences because of the overlapping nature of the data, but this did not affect the results.

²⁴Care must be exercised in interpreting the results, as the samples differ in size, which may affect the t-statistics. The daily and yearly samples are similar in size, but sample sizes for weekly and monthly regressions are much smaller.

These findings therefore indirectly indicate what we can learn from the no-arbitrage models with respect to the economic impact of observable covariates.

Our original no-arbitrage model in Section 2 is specified in levels. While we show that it is possible to estimate no-arbitrage models that incorporate assumptions similar to the ones underlying difference regressions, our interpretation of the evidence is that for daily CDS data, the model specified in levels is most appropriate to learn about observable covariates and credit spreads.

We believe that estimated covariate sensitivities from daily difference regressions are hard to interpret, because these regressions are statistically inefficient. The potential problem with the levels regressions is that measures of fit such as R-squares may be biased, and that t-statistics may be unreliable. However, t-statistics can be corrected for serial correlation, and RMSEs as well as a visual inspection of model fit indicate that observable covariates are very helpful in fitting credit spreads. Moreover, while credit default swap spreads are highly auto-correlated, they may not be natural candidates for differencing. Economic intuition suggests that these spreads are not characterized by a stochastic trend, and plotting the spreads suggests that they are mean-reverting.

This interpretation is confirmed by analyzing different data frequencies. For differences based on lower frequency data, the performance of observable covariates improves, and results are more reliable. Note also that as data frequency decreases, the estimates of covariate sensitivities from difference regressions get closer to those of the levels regressions, which are more efficient.

Both levels and difference specifications have advantages and disadvantages, and the trade-off needs to be carefully evaluated. Our results suggest that in the credit risk literature, the choice of levels versus differences should perhaps depend on the data frequency. For the monthly bond data analyzed in Collin-Dufresne, Goldstein, and Martin (2001), differencing the data may be the best alternative, but for higher data frequencies levels regressions may be preferable. This is critically important for the analysis of CDS data, because the short available sample periods make it unavoidable to use daily data.

7 Concluding Remarks

We make four contributions. First, we introduce a no-arbitrage model with observable covariates, which allows for a closed form solution for the value of CDSs. We specify the default intensity as a quadratic function of the covariates, such that the intensity function is always positive. Our approach enables us to study the effects of observable covariates, while maintaining the discipline of a no-arbitrage model, and imposing pricing consistency across maturities.

Our second contribution is empirical. We demonstrate that macroeconomic and firm-specific information can explain most of the variation in CDSs over time and across firms. A parsimonious model with four covariates suggested by theory performs very well, and richer models with variables commonly used in the literature do not add much explanatory power. The model provides sensible results from an economic perspective: the impact of covariates such as volatility and distance-todefault on CDSs is entirely consistent with economic intuition, as well as with the logic of structural credit risk models such as Merton (1974). Moreover, we find that requiring our no-arbitrage model to simultaneously fit CDS prices for different maturities does not come at the cost of empirical fit.

Third, when analyzing the determinants of credit risk, the economic interpretation of estimates depends on the statistical assumptions, such as whether the dependent variable should be expressed in levels or changes. We find that this choice involves clear trade-offs, and that it should partly depend on the available data frequency. We argue that the analysis of credit spread levels may be more valuable than previously thought, especially for daily CDS data. The intuition for this is that although time series of credit default swap spreads and the observable covariates are highly auto-correlated, they are mean-reverting and not characterized by long term trends.

Our fourth contribution is our overall conclusion: observable covariates are very useful to explain credit spreads. This resolves an important disconnect in the existing literature, making valuation results consistent with the evidence regarding default prediction.

In future research, it might prove interesting to investigate the modeling choice between levels and differences in more detail. Here we merely comment on this issue from the perspective of the specification of the no-arbitrage model, and an in-depth discussion is outside of the scope of this paper. However, the existing literature that investigates the trade-off involved in estimating levels versus differences is largely motivated by the analysis of economic variables that are presumably characterized by (stochastic) trends, such as aggregate consumption or GDP. Existing Monte Carlo evidence may therefore not be informative for highly autocorrelated but ostensibly mean-reverting variables such as credit spreads and volatilities, as well as many other financial variables such as interest rates. Careful evaluation of this trade-off, using Monte-Carlo experiments with parameters relevant for credit risk applications, would seem to be a topic worthy of further study.

Appendix A: Risk-Free Bond Pricing

If ϵ is a (n, 1) vector described by a multi-variate normal distribution $\epsilon \sim N(0, \Sigma)$, Σ being nonsingular, then the Laplace transform of a quadratic form

$$Q = \epsilon' A \epsilon + a' \epsilon + d \tag{7.1}$$

is given by 25

$$E[\exp(tQ)] = \exp(-\frac{1}{2}\ln(\det(I - 2t \ \Sigma \ A)) + td + \frac{1}{2}ta'(\Sigma^{-1} - 2tA)^{-1}at)$$
(7.2)

We want to price a default free zero coupon bond

$$E_t[\exp(-r_t - \dots - r_{t+h-1})] \equiv L_{t,h}$$

Let $r_{t+j} = (\delta_0 + \delta' X_{t+j}^r)^2$ where X_{t+j}^r and δ are (n, 1) vectors, j = 0, 1, ..., h - 1. We can re-write this in the form

$$r_{t+j} = (\delta_0 + \delta' X_{t+j}^r)' (\delta_0 + \delta' X_{t+j}^r)$$

$$= \delta_0^2 + 2\delta_0 \delta' X_{t+j}^r + X_{t+j}^{r'} \delta \delta' X_{t+j}^r$$
(7.3)

Assume that

$$X_t^r = \mu_r + \rho_r X_{t-1}^r + e_t \tag{7.4}$$

where $e_t \sim N(0, \Sigma_r)$, μ_r is a (n, 1) vector and ρ_r and Σ_r are (n, n) matrices.

First, consider

$$L_{t+h-1,1} \equiv E_{t+h-1}[\exp(-r_{t+h-1})] = \exp(-r_{t+h-1})$$

Substituting expression (7.3) we have

$$L_{t+h-1,1} = \exp(A_1 + B'_1 X^r_{t+h-1} + X^{r'}_{t+h-1} C_1 X^r_{t+h-1})$$

where

$$A_{1} = -\delta_{0}^{2} \qquad \text{scalar}$$
$$B_{1} = -2\delta_{0}\delta \quad \text{a} (n, 1) \text{ vector}$$
$$C_{1} = -\delta\delta' \quad \text{a} (n, n) \text{ matrix}$$

To determine $L_{t,h}$ we use iterative expectations. We first consider $L_{t+h-2,2}$

$$L_{t+h-2,2} = \exp(-\delta_0^2 - 2\delta_0 \delta' X_{t+h-2}^r - X_{t+h-2}^{r'} \delta \delta' X_{t+h-2}^r) E_{t+h-2}[L_{t+h-1,1}]$$

and then use expression (7.2) and simplify. This process is repeated to give, after much simplification

$$L_{t,h} = \exp(A_h + B'_h X_t + X'_t C_h X_t)$$

²⁵The proof is given in Mathai, A. M. and S. B. Provost (1992, p. 40).

where for k = 2, ..., h

$$A_{k} = \frac{-\delta_{0}^{2} + (A_{k-1} + B_{k-1}' \mu_{r} + \mu_{r}' C_{k-1} \mu_{r})}{+\frac{1}{2} (B_{k-1} + 2C_{k-1} \mu_{r})' (\Sigma_{r}^{-1} - 2C_{k-1})^{-1} (B_{k-1} + 2C_{k-1} \mu_{r}) - \frac{1}{2} \ln[\det(I - 2\Sigma_{r} C_{k-1})]}$$

$$B'_{k} = -2\delta_{0}\delta' + (B_{k-1} + 2C_{k-1}\mu_{r})'\rho_{r} + 2(B_{k-1} + 2C_{k-1}\mu_{r})'(\Sigma_{r}^{-1} - 2C_{k-1})^{-1}C_{k-1}\rho_{r}$$

and

$$C_{k} = -\delta\delta' + \rho_{r}'C_{k-1}[I + 2(\Sigma_{r}^{-1} - 2C_{k-1})^{-1}C_{k-1}]\rho_{r}$$

Appendix B: Default Intensity Modeling

The intensity function is also a quadratic function of the form

$$\lambda_{t+j} = (\alpha_0 + \alpha' X_{t+j}^r + \alpha'_d X_{t+j}^d)' (\alpha_0 + \alpha' X_{t+j}^r + \alpha'_d X_{t+j}^d) = \alpha_0^2 + 2\alpha_0 \alpha' X_{t+j}^r + 2\alpha_0 \alpha'_d X_{t+j}^d + 2X_{t+j}^{r\prime} \alpha \alpha'_d X_{t+j}^d + X_{t+j}^{r\prime} \alpha \alpha' X_{t+j}^r + X_{t+j}^{d\prime} \alpha_d \alpha'_d X_{t+j}^d$$

where X_{t+j}^d and α_d are (m, 1) vectors, j = 0, 1, ..., h - 1. The sum of the interest rate plus the intensity is given by

$$r_{t+j} + \lambda_{t+j} = (\delta_0 + \delta' X_{t+j}^r)' (\delta_0 + \delta' X_{t+j}^r) + (\alpha_0 + \alpha' X_{t+j}^r + \alpha'_d X_{t+j}^d)' (\alpha_0 + \alpha' X_{t+j}^r + \alpha'_d X_{t+j}^d)$$

which can be written in the form

$$r_{t+j} + \lambda_{t+j} = \delta_0^2 + \alpha_0^2 + 2(\delta_0 \delta' + \alpha_0 \alpha') X_{t+j}^r + 2\alpha_0 \alpha'_d X_{t+j}^d$$

$$+ 2X_{t+j}^{r'} \alpha \alpha'_d X_{t+j}^d + X_{t+j}^{r'} (\delta \delta' + \alpha \alpha') X_{t+j}^r + X_{t+j}^{d'} \alpha_d \alpha'_d X_{t+j}^d$$
(7.5)

Define

$$\gamma_0 \equiv \delta_0^2 + \alpha_0^2$$

a scalar. Let q = n + m and define

$$\gamma_1 \equiv \left[\begin{array}{c} 2(\delta_0 \delta + \alpha_0 \alpha) \\ 2\alpha_0 \alpha_d \end{array} \right]$$

a (q, 1) vector. Define

$$\Omega \equiv \begin{bmatrix} \delta \delta' + \alpha \alpha' & \alpha \alpha'_d \\ \\ \alpha_d \alpha' & \alpha_d \alpha'_d \end{bmatrix}$$

a (q,q) matrix and

$$X_{t+j} \equiv \begin{bmatrix} X_{t+j}^r \\ X_{t+j}^d \end{bmatrix}$$

a (q, 1) vector, then

$$r_{t+j} + \lambda_{t+j} = \gamma_0 + \gamma'_1 X_{t+j} + X'_{t+j} \Omega X_{t+j}$$
(7.6)

It is assumed that

$$X_t = \mu + \rho X_{t-1} + e_t \tag{7.7}$$

where $e_t \sim N(0, \Sigma)$, μ is a (q, 1) vector and ρ and Σ are (q, q) matrices.

The derivation of expression (2.8) requires evaluating

$$E_t[\exp(-\sum_{j=0}^{h-1} r_{t+j} + \lambda_{t+j})]$$
(7.8)

This expression is isomorphic to expression (7.3), so the derivation follows that given in Appendix

A. Expressions (7.6) and (7.7) are similar to expressions (7.3) and (7.4) in Appendix A.

First, consider

$$L_{t+h-1,1} \equiv E_{t+h-1}[\exp(-r_{t+h-1} - \lambda_{t+h-1})] = \exp(-r_{t+h-1} - \lambda_{t+h-1})$$

Substituting expression (7.6) we have

$$L_{t+h-1,1} = \exp(F_1 + G'_1 X_{t+j-1} + X'_{t+h-1} H_1 X_{t+j-1})$$

where

$$F_{1} = -\gamma_{0} \qquad \text{scalar}$$

$$G_{1} = -\gamma_{1} \quad \text{a } (q, 1) \text{ vector}$$

$$H_{1} = -\Omega \quad \text{a } (q, q) \text{ matrix}$$

Repeating the logic used in Appendix A, gives

$$L_{t,h} = \exp(F_h + G'_h X_t + X'_t H_h X_t)$$

where for k = 2, ..., h

$$F_{k} = \frac{-\gamma_{0} + (F_{k-1} + G'_{k-1}\mu + \mu'H_{k-1}\mu) + \frac{1}{2}(G_{k-1} + 2H_{k-1}\mu)'(\Sigma^{-1} - 2H_{k-1})^{-1}(G_{k-1} + 2H_{k-1}\mu)}{-\frac{1}{2}\ln[\det(I - 2\Sigma H_{k-1})]}$$

$$G'_{k} = -\gamma'_{1} + (G_{k-1} + 2H_{k-1}\mu)'\rho + 2(G_{k-1} + 2H_{k-1}\mu)'(\Sigma^{-1} - 2H_{k-1})^{-1}H_{k-1}\rho$$

and

$$H_k = -\Omega + \rho' H_{k-1} [I + 2(\Sigma^{-1} - 2H_{k-1})^{-1} H_{k-1}]\rho$$

The Derivation of Expression (2.10)

First, consider

$$L_{t+h-1,1} = \exp(-r_{t+h-1})$$

Substituting expression (7.3) we have

$$L_{t+h-1,1} = \exp[-(\delta_0^2 + 2\delta_0 \delta' X_{t+h-1}^r + X_{t+h-1}^{r'} \delta \delta' X_{t+h-1}^r)]$$

$$\equiv \exp(M_1 + N_1' X_{t+h-1}^r + X_{t+h-1}^{r'} P_1 X_{t+h-1}^r)$$

where

$$M_{1} = -\delta_{0}^{2} \qquad \text{scalar}$$

$$N_{1} = -2\delta_{0}\delta \quad \text{a} (n, 1) \text{ vector}$$

$$P_{1} = -\delta\delta' \quad \text{a} (n, n) \text{ matrix}$$

Next, consider

$$L_{t+h-2,2} = \exp(-r_{t+h-2} - \lambda_{t+h-2})E_{t+h-2}[L_{t+h-1,1}]$$

Using expressions (7.2) and (7.5) we have

$$L_{t+h-2,2} = \exp(M_2 + N_2' X_{t+h-2} + X_{t+h-2}' P_2 X_{t+h-2})$$
(7.9)

where

$$M_{2} \equiv -(\delta_{0}^{2} + \alpha_{0}^{2}) + (M_{1} + N_{1}'\mu_{r} + \mu_{r}'P_{1}\mu_{r}) + \frac{1}{2}(N_{1} + 2P_{1}\mu_{r})'(\Sigma_{r}^{-1} - 2P_{1})^{-1}(N_{1} + 2P_{1}\mu_{r}) - \frac{1}{2}\ln[\det(I - 2\Sigma_{r}P_{1})]$$

$$N_{2}' \equiv \begin{bmatrix} (N_{1} + 2P_{1}\mu_{r})'\rho_{r} + 2(N_{1} + 2P_{1}\mu_{r})'(\Sigma_{r}^{-1} - 2P_{1})^{-1}P_{1}\rho_{r} - 2(\delta_{0}\delta' + \alpha_{0}\alpha') \\ -2\alpha_{0}\alpha'_{d} \end{bmatrix}$$

$$P_2 \equiv \begin{bmatrix} \rho_r P_1 \rho_r + 2\rho_r P_1 (\Sigma_r^{-1} - 2P_1)^{-1} P_1 \rho_r - (\delta \delta' + \alpha \alpha') & -\alpha \alpha'_d \\ -\alpha_d \alpha & -\alpha_d \alpha'_d \end{bmatrix}$$

From this point, the analysis is similar to the derivation of expression (2.8).

References

- Ahn, D., R. Dittmar, and A.R. Gallant, 2002, "Quadratic Term Structure Models: Theory and Evidence," *Review of Financial Studies*, 15, 243-288.
- [2] Ang, A., J. Boivin, and S. Dong, 2008, "Monetary Policy Shifts and the Term Structure," Working Paper, Columbia University.
- [3] Ang, A., and M. Piazzesi, 2003, "A No-Arbitrage Vector-Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables," *Journal of Monetary Economics*, 50, 745-787.
- [4] Avramov, D., G. Jostova, and A. Philopov, 2007, "Understanding Changes in Corporate Credit Spreads," *Financial Analysts Journal*, 62, 90-105.
- [5] Bakshi, G., P. Carr, and L. Wu, 2008, "Stochastic Risk Premiums, Stochastic Skewness in Currency Options, and Stochastic Discount Factors in International Economies," *Journal of Financial Economics*, 87, 132–156.
- [6] Bakshi, G., D. Madan, and F. Zhang, 2006, "Investigating the Role of Systematic and Firm-Specific Factors in Default Risk: Lessons from Empirically Evaluating Credit Risk Models," *Journal of Business*, 4, 1955-1988.
- [7] Bekaert, G., S. Cho, and A. Moreno, 2006, "New-Keynesian Macroeconomics and the Term Structure," Working Paper, Columbia University.
- [8] Black, F., and J. Cox, 1976, "Valuing Corporate Securities: Some Effects of Bond Indenture Provisions," *Journal of Finance*, 31, 351-367.
- [9] Blanco, R., S. Brennan, and I. Marsh, 2005, "An Empirical Analysis of the Dynamic Relation Between Investment Grade Bonds and Credit Default Swaps," *Journal of Finance*, 60, 2255-2281.
- [10] Bongaerts, D., F. de Jong, and J. Driessen, 2009, "Derivative Pricing with Liquidity Risk: Theory and Evidence from the Credit Default Swap Market," *Journal of Finance*, forthcoming.
- [11] Brandt, M., and D. A. Chapman, 2002, "Comparing Multifactor Models of the Term Structure," Working Paper, Duke University.
- [12] Campbell, J., and G. Taksler, 2003, "Equity Volatility and Corporate Bond Yields," *Journal of Finance*, 58, 2321-2349.

- [13] Cao, C., F. Yu, and Z. Zhong, 2010, "The Informational Content of Option Implied Volatility for Credit Default Swap Valuation," *Journal of Financial Markets*, 13, 321-343.
- [14] Carr, P., and L. Wu, 2007, "Stochastic Skew in Currency Options," Journal of Financial Economics, 86, 213-247.
- [15] Chen, R-R., and L. Scott, 1995, "Interest Rate Options in Multifactor Cox-Ingersoll-Ross Models of the Term Structure," *Journal of Derivatives*, 3, 53-72.
- [16] Chen, R-R., X. Cheng, F. J. Fabozzi, and B. Liu, 2008, "An Explicit Multi-Factor Credit Default Swap Pricing Model with Correlated Factors," *Journal of Financial and Quantitative Analysis*, 43, 123-160.
- [17] Collin-Dufresne, P., and R. S. Goldstein, 2001, "Do Credit Spreads Reflect Stationary Leverage Ratios?," *Journal of Finance*, 56, 1929-1958.
- [18] Collin-Dufresne, P., R. S. Goldstein, and S. J. Martin, 2001, "The Determinants of Credit Spreads," *Journal of Finance*, 56, 2177-2207.
- [19] Constantinides, G., 1992, "A Theory of the Nominal Term Structure of Interest Rates," *Review of Financial Studies*, 5, 531-552.
- [20] Cremers, M., J. Driessen, and P. Maenhout, 2008, "Explaining the Level of Credit Spreads: Option Implied Jump Risk Premia in a Firm Value Model," *Review of Financial Studies*, 21, 2209-2242.
- [21] Cremers, M., J. Driessen, P. Maenhout, and D. Weinbaum, 2008, "Individual Stock-Option Prices and Credit Spreads," *Journal of Banking and Finance*, 32, 2706-2715.
- [22] Crouhy, M. G., R. A. Jarrow, and S. M. Turnbull, 2008, "Insights and Analysis of Current Events: The Subprime Credit Crisis of 2007," *Journal of Derivatives*, 16, 81-110.
- [23] Davidson, R., and J. G. MacKinnon, 2004, "Econometric Theory and Methods," Oxford University Press, New York.
- [24] Driessen, J., 2005, "Is Default Event Risk Priced in Corporate Bonds?," Review of Financial Studies, 12, 2203-2241.
- [25] Duan, J., and J. Simonato, 1999, "Estimating and Testing Exponential-Affine Term Structure Models by Kalman Filter," *Review of Quantitative Finance and Accounting*, 13, 111-135.

- [26] Duffee, G., 1999, "Estimating the Price of Default Risk," Review of Financial Studies, 12, 197-226.
- [27] Duffie, D., and K. J. Singleton, 1997, "An Econometric Model of the Term Structure of Interest-Rate Swap Yields," *Journal of Finance*, 52, 1287-1321.
- [28] Duffie, D., and K. J. Singleton, 1999, "Modeling Term Structures of Defaultable Bonds," *Review of Financial Studies*, 12, 687-720.
- [29] Duffie, D., L. Saita, and K. Wang, 2007, "Multi-Period Corporate Default Prediction with Stochastic Covariates," *Journal of Financial Economics*, 83, 635-665.
- [30] Eom, Y. H., J. Helwege, and J. Huang, 2004, "Structural Models of Corporate Bond Pricing: An Empirical Analysis," *Review of Financial Studies*, 17, 499-544.
- [31] Ericsson, J., K. Jacobs, and R. Oviedo, 2009, "The Determinants of Credit Default Swap Premiums," *Journal of Financial and Quantitative Analysis*, 44, 109-132.
- [32] Feldhutter, P., and D. Lando, 2007, "Decomposing Swap Spreads," Journal of Financial Economics, 88, 375-405.
- [33] Fisher, L., 1959, "Determinants of Risk Premiums on Corporate Bonds," Journal of Political Economy, 67, 217-237.
- [34] Geske, R., 1977, "The Valuation of Corporate Liabilities as Compound Options," Journal of Financial and Quantitative Analysis, 12, 541-552.
- [35] Gospodinov, N., 2009, "A New Look at the Forward Premium Puzzle," Journal of Financial Econometrics, 7, 312-338.
- [36] Gourieroux, C., and J. Jasiak, 2006, "Autoregressive Processes," Journal of Forecasting, 25, 129-152.
- [37] Gourieroux, C., A. Monfort, and V. Polimenis, 2006, "Affine Models for Credit Risk Analysis," *Journal of Financial Econometrics*, 4, 494-530.
- [38] Gourieroux, C., and A. Monfort, 2007, "Quadratic Stochastic Intensity and Prospective Mortality Tables," Working Paper, University of Toronto.
- [39] Granger, C., and P. Newbold, 1974, "Spurious Regressions in Econometrics," Journal of Econometrics, 2, 111-120.

- [40] Harvey, A., 1980, "On Comparing Regression Models in Levels and First Differences," International Economic Review, 21, 707-720.
- [41] Houweling, P., and T. Vorst, 2005, "Pricing Default Swaps: Empirical Evidence," Journal of International Money and Finance, 24, 1200-1225.
- [42] Huang, J., and H. Zhou, 2008, "Specification Analysis of Structural Credit Risk Models," Working Paper, Federal Reserve Board.
- [43] Jagannathan, R., A. Kaplin, and S. Sun, 2003, "An Evaluation of Multi-Factor CIR Models using LIBOR, Swap Rates, and Cap and Swaption Prices," *Journal of Econometrics*, 116, 113-146
- [44] Jarrow, R. A., and S. M. Turnbull, 1992, "Drawing the Analogy," Risk, 5, 63-70.
- [45] Jarrow, R. A., and S. M. Turnbull, 1995, "The Pricing and Hedging of Options on Financial Securities Subject to Credit Risk," *Journal of Finance*, 50, 53-85.
- [46] Kim, J., K. Ramaswamy, and S. Sundaresan, 1993, "Does Default Risk in Coupons Affect the Valuation of Corporate Bonds," *Financial Management*, 22, 117-131.
- [47] Lando, D., 1994, "Three Essays on Contingent Claims Pricing," Ph. D. Thesis, Cornell University.
- [48] Lando, D., 1998, "On Cox Processes and Credit Risky Securities," Review of Derivatives Research, 2, 99-120
- [49] Leippold, M., and L. Wu, 2002, "Asset Pricing under the Quadratic Class," Journal of Financial and Quantitative Analysis, 37, 271-295.
- [50] Leland, H. E., 1994, "Corporate Debt Values, Bond Covenants, and Optimal Capital Structure," Journal of Finance, 49, 1213-1252.
- [51] Leland, H. E., and K. Toft, 1996, "Optimal Capital Structure, Endogeneous Bankruptcy, and the Term Structure of Credit Spreads," *Journal of Finance*, 51, 987-1019.
- [52] Li, H., and F. Zhao, 2006, "Unspanned Stochastic Volatility: Evidence from Hedging Interest Rate Derivatives," *Journal of Finance*, 61, 341-378.
- [53] Longstaff, F., 1989, "A Nonlinear General Equilibrium Model of the Term Structure of Interest Rates," *Journal of Financial Economics*, 23, 195-224.

- [54] Longstaff, F., S. Mithal, and E. Neis, 2005, "Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Credit Default Swap Market," *Journal of Finance*, 60, 2213-2253.
- [55] Longstaff, F., and E. S. Schwartz, 1995, "A Simple Approach to Valuing Risky Fixed and Floating Rate Debt," *Journal of Finance*, 50, 789-819.
- [56] Maeshiro, A., and S. Vali, 1988, "Pitfalls in the Estimation of a Differenced Model," Journal of Business and Economic Statistics, 6, 511-515.
- [57] Mathai, A. M., and S. B. Provost, 1992, "Quadratic Forms in Random Variables," Marcel Dekker, Inc., New York.
- [58] Merton, R. C., 1974, "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," Journal of Finance, 29, 449-470.
- [59] Shumway, T., 2001, "Forecasting Bankruptcy More Accurately: A Simple Hazard Model," Journal of Business, 74, 101–124.
- [60] Tang, D., and H. Yan, 2007, "Liquidity and Credit Default Swaps," Working Paper, University of South Carolina.
- [61] Van der Merwe, R., and E. A. Wan, 2001, "The Square-Root Unscented Kalman Filter for State and Parameter Estimation," Working Paper, Oregon Health and Science University.
- [62] Zellner, A., 1979, "Causality and Econometrics," Carnegie-Rochester Conferences on Public Policy, 9-54.
- [63] Zhang, B., H. Zhou, and H. Zhu, 2009, "Explaining Credit Default Swap Spreads with Equity Volatility and Jump Risks of Individual Firms," *Review of Financial Studies*, 22, 5099-5131.


Figure 1: The Gap, Inc.: Model Spreads, Market Spreads, and Firm-Specific Covariates.

Notes to Figure: The top panels show the time series of linear regression and no-arbitrage model spreads together with the market spread for the contract with five-year maturity. The middle panels show the time series of the stock price and the 30-day at-the-money firm implied volatility. The bottom panels show the time series of leverage, defined as the ratio of total liabilities to the sum of total liabilities and market value of equity, and distance-to-default.



Figure 2: Average Market Spreads, VIX and SP500 Returns.

Notes to Figure: We show the time series of the average market spread for the contract with five-year maturity, the VIX, and the one-year trailing S&P 500 return.



Figure 3: Sensitivity of Market Spread to VIX and Distance-To-Default. Various Models.

Notes to Figure: We show the sensitivities of the market spread to the VIX and distance-to-default for various models. The sensitivities are based on the contract with five-year maturity. Level indicates the linear regression model where the dependent variable is the level of the spreads, NA indicates the no-arbitrage model, CO indicates the Cochrane-Orcutt model, and NA-AR(1) indicates the no-arbitrage model with AR(1) errors. The reported numbers indicate the change in spreads (in basis points) for a 1% change in VIX, and the change in spreads (in basis points) for a one unit change in distance-to-default. The covariate specification includes two stochastic term structure factors, the VIX, and distance-to-default.



Figure 4: Alternative Covariate Specification: Factor Sensitivities.

Notes to Figure: We show the sensitivities of the market spread to different factors. The sensitivities are based on the contract with five-year maturity. The y-axis indicates the model for which the sensitivity is computed. SP500 stands for the S&P 500 return, LEV stands for firm leverage, FIV stands for the firm's implied option volatility, and LIQ indicates the liquidity measure. For SP500, VIX, LEV and FIV, the reported numbers indicate the change in spread (in basis points) for a 1% change in the corresponding covariate. For LIQ, the numbers indicate the change in spread (in basis points) for a one unit change in LIQ. The covariate specification includes two stochastic term structure factors, the one-year trailing return on the S&P500, the VIX, firm leverage, option-implied volatility, and liquidity.



Figure 5: Sensitivity of Market Spread to VIX and Distance-to-Default. Various Models, Monthly Frequency.

Notes to Figure: We show the sensitivities of the market spread to the VIX and distance-to-default (DTD) for various models using monthly data. The sensitivities are based on the contract with fiveyear maturity. The y-axis denotes the model for which the sensitivity is computed. Level indicates the linear regression model where the dependent variable is the level of the spreads, NA indicates the no-arbitrage model, CO indicates the Cochrane-Orcutt model, and NA-AR(1) indicates the no-arbitrage model with AR(1) errors. The reported numbers indicate the change in spreads (in basis points) for a 1% change in VIX, and the change in spreads (in basis points) for a one unit change in distance-to-default.

Panel A: D	escriptive S	tatistics		
	Mean	Std.		
1 Yr (bps)	156.3	227.5		
3 Yr (bps)	176.5	215.1		
5 Yr (bps)	196.7	197.2		
DTD	10.9	5.2		
LEV	22.7%	6.1%		
FIV	40.8%	15.1%		
LIQ	10.8	5.9		

Table 1: The Gap, Inc.: Summary Statistics

Panel B: R-squares (%) and RMSEs										
	Linear Regression No Arbitrage									
Maturity	R-2	RMSE(bps)	RMSE(bps)							
1 Yr	85.4	88.4	56.9							
3 Yr	85.3	83.9	51.5							
5 Yr	84.6	78.6	52.0							

Notes: Panel A reports averages and standard deviations for the market spreads for 1, 3, and 5 year maturities, distance to default (DTD), leverage (LEV), 30 day option implied volatility (FIV), and the number of quote contributors (LIQ) for The Gap, Inc. Panel B reports the R-square (R-2) from the linear regression, and the RMSEs in basis points from the linear regression and no-arbitrage models. The data are for the period January 1, 2002 to March 7, 2008.

						Pan	al A							Panel	R
				Fi	rm-Spe	cific Des		Statisti	cs				Мо	odel Fit	
	Rating	Mkt Sp	r. (5 Yr)	D		LEV		FIV		LI	Q			r Regr.	NA
		Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	1	R2 %	RMSE	RMSE
Sun Microsystems	BB	116.1	64.3	7.8	3.8	29.8	8.3	54.1	19.7	10.8	4.5	1	64.3	35.2	32.9
, Honeywell Intl	А	33.3	22.4	10.6	4.1	38.5	4.6	30.6	11.5	10.9	4.3	1	73.8	11.4	11.2
Fortune Brands	BBB	41.4	27.6	11.7	2.4	35.7	7.1	22.3	5.8	7.7	4.9	1	40.2	21.9	22.3
du Pont	А	22.6	9.7	11.0	3.4	36.1	2.8	24.3	7.5	11.2	5.1	1	62.0	6.1	6.6
Eastman Kodak	В	174.4	77.4	6.0	1.9	57.5	6.0	34.7	6.7	13.3	5.8		54.5	50.4	64.6
Goodrich Corp	BBB	85.3	66.5	7.3	3.2	53.1	9.0	32.5	10.0	12.4	5.4	1	79.5	30.7	28.0
Ingersoll Rand	А	41.8	20.4	9.1	2.9	38.6	8.3	30.1	7.7	10.2	4.0		59.3	12.3	12.1
Altria Gp	BBB	110.4	73.8	11.5	4.2	33.3	8.3	26.1	8.0	12.1	5.6	1	73.3	40.7	44.1
RadioShack	BB	102.6	55.6	7.1	2.5	26.5	6.1	41.7	12.6	11.1	6.1		69.9	31.2	38.3
Wyeth	А	43.7	33.1	11.5	4.6	25.5	3.7	27.5	9.2	11.9	5.9		52.3	23.9	24.2
Kroger	BBB	66.0	25.2	7.7	2.8	51.7	5.4	28.9	7.2	12.5	6.8		64.1	15.7	13.2
Gen Mills	BBB	43.3	19.9	12.1	3.7	40.5	4.5	19.3	5.5	10.3	4.9		83.8	8.1	8.5
J C Penney	BBB	247.8	217.6	7.0	3.3	53.0	14.7	39.6	13.6	8.0	4.6		82.9	67.9	54.7
Caterpillar	А	31.2	15.7	6.7	1.7	54.5	5.5	29.4	6.8	10.5	4.3		73.5	8.0	8.3
Deere	А	37.1	20.8	6.2	1.4	60.0	5.8	30.5	7.2	10.3	4.1		68.1	11.3	10.5
Dow Chemical	А	54.1	38.2	9.0	3.0	44.4	4.2	29.1	8.9	13.2	6.2		89.7	13.0	12.1
Lockheed Martin	BBB	42.7	22.2	13.7	5.0	43.5	7.1	26.1	9.2	10.3	5.3		79.5	8.4	11.4
Cardinal Health	BBB	43.5	22.8	11.6	4.4	32.1	5.8	28.0	8.9	11.2	7.1		44.7	18.3	18.7
Intl Paper	BBB	70.1	24.2	7.3	1.6	55.7	3.5	27.3	7.5	13.7	6.5		60.8	14.5	16.2
Motorola	BBB	122.1	114.6	7.8	3.9	34.6	7.0	42.6	15.9	12.8	4.5	1	80.4	50.1	41.3
Sara Lee	BBB	37.6	15.7	10.2	2.7	42.0	3.6	21.8	4.9	9.5	5.8	1	49.5	11.6	12.8
Halliburton	BBB	124.0	160.9	8.2	3.5	35.0	14.5	41.1	15.8	12.2	5.6	1	85.7	60.5	42.7
Rohm & Haas	А	35.8	14.0	9.8	2.9	40.3	6.0	27.8	7.5	10.2	5.0		38.9	9.7	9.9
Cl Channel Comms	BB	182.7	152.2	8.4	3.7	36.8	3.2	33.4	17.2	13.4	6.6		62.3	99.5	120.3
Amern Elec Power	BBB	88.5	104.5	8.2	4.0	68.8	5.5	24.0	12.1	12.0	4.4	1	63.6	64.8	50.7
Constellation Engy	BBB	72.5	72.8	9.4	4.1	61.2	6.2	26.0	9.2	11.6	5.9	1	58.8	48.5	45.7
Alcoa	А	35.1	17.7	7.4	2.1	41.1	4.8	33.8	8.5	11.4	5.9		36.2	15.0	15.8
Northrop Grumman	BBB	52.9	37.0	12.7	5.5	47.9	7.4	23.0	8.3	10.2	5.5		78.0	16.0	13.0
Raytheon	BBB	73.0	54.0	11.9	5.7	47.2	10.2	27.1	12.1	11.7	5.0	1	92.1	13.5	11.9
Campbell Soup	А	29.7	12.1	12.9	4.5	32.5	4.1	22.4	6.9	9.9	5.0		58.2	6.8	6.5
Whirlpool	BBB	58.6	25.5	6.8	1.9	58.4	3.6	33.5	7.4	11.7	6.7		19.7	17.1	18.2
Walt Disney	А	48.2	33.2	9.0	3.5	34.0	5.2	29.7	10.6	13.4	6.1		81.7	14.8	13.2
Loews	А	56.7	37.2	10.8	4.7	80.8	6.9	25.1	7.4	10.8	5.8		84.7	14.8	14.2
Hewlett Packard	А	45.0	34.6	10.0	4.5	33.4	5.4	37.1	13.7	11.3	5.0		83.7	13.8	12.2
Baxter Intl	А	37.1	19.3	12.6	5.9	26.1	8.2	27.1	9.0	10.7	5.0		75.9	9.3	9.2
Arrow Electrs	BBB	161.4	125.9	6.4	3.3	56.5	8.6	38.8	10.6	12.7	6.2	1	78.7	56.3	43.2
Omnicom Gp	BBB	64.1	62.8	11.9	4.7	41.7	4.4	28.6	14.1	10.9	4.2		74.2	32.3	23.8
Sherwin Williams	А	45.3	24.1	11.8	3.7	31.8	3.7	29.3	9.5	9.8	5.5		65.6	13.0	13.0
Wells Fargo	AA	25.5	15.1	10.2	3.2	79.3	1.8	21.9	8.6	10.3	4.7	1	60.0	9.9	8.7
Weyerhaeuser	BBB	70.4	31.2	7.3	2.2	56.2	6.1	29.5	8.9	13.1	6.3		54.5	22.2	23.1
Computer Sciences	А	60.2	32.1	7.6	2.9	43.6	4.8	36.6	12.5	12.7	6.2		29.7	27.2	28.4
McDonalds	А	26.7	11.3	11.2	4.4	26.3	5.7	26.4	7.5	10.9	5.5		64.4	7.1	7.4
Supervalu	В	146.2	79.8	7.3	3.1	59.1	10.0	29.6	8.3	11.5	6.3		68.3	40.8	47.0
Target	А	29.3	18.9	9.0	3.3	33.0	4.1	31.7	10.3	9.3	4.3		70.3	10.9	11.0
Liz Claiborne	BBB	72.4	55.3	12.0	3.7	25.0	5.7	28.8	10.0	7.1	3.8		53.2	36.5	25.1
Burl Nthn Santa Fe	BBB	37.7	16.2	9.2	2.6	54.1	8.6	26.3	5.1	10.7	5.2		64.4	9.5	9.7
Centex	BBB	114.7	97.4	3.9	2.2	67.0	4.1	43.2	13.0	14.2	7.2		57.5	68.2	79.9
Wal Mart Stores	AA	17.7	7.7	12.9	2.4	24.0	6.3	24.1	8.1	10.5	4.4		61.6	4.8	5.1
ConAgra Foods	BBB	45.6	16.8	11.9	3.5	42.7	5.1	21.0	5.2	11.3	5.6		56.5	9.7	10.5
Southwest Airls	BBB	61.5	32.9	8.9	2.4	33.0	7.8	34.7	9.5	12.9	6.3		69.8	15.5	15.9
The Gap	BB	196.7	197.2	10.9	5.2	22.7	6.1	40.8	15.1	10.8	5.9		84.6	78.6	52.0
Amern Express	А	39.2	29.8	7.3	3.3	69.3	5.1	28.1	11.8	10.6	4.8		53.4	21.2	18.8
Chubb	А	35.3	24.9	11.2	5.8	67.0	4.5	24.9	8.5	10.2	4.2		74.7	18.1	20.1

Table 2: Firm-by-Firm Descriptive Statistics and Model Fit.

					1	Table 2 C	ontinu	ed				_			
						Pan	el A							Panel I	В
			Firm-Specific Descriptive Statistics									Mo	odel Fit (5 Yr)	
	Rating	Mkt Sp	r. (5 Yr)	DT	D	LEV	(%)	FIV	(%)	LI	Q		Linea	r Regr.	NA
		Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.		R2 %	RMSE	RMSE
Newell Rubbermaid	BBB	51.8	17.9	8.8	2.6	40.7	3.7	26.6	6.8	11.5	6.0		41.6	12.6	12.8
CSX	BBB	55.6	24.5	7.2	2.6	62.2	8.7	30.8	7.8	11.4	5.8		61.5	14.6	15.2
Ltd Brands	BBB	79.7	55.2	9.5	2.8	24.6	7.0	36.5	11.8	11.7	6.3		47.1	39.2	35.2
Norfolk Sthn	BBB	44.2	21.3	6.7	2.7	55.6	8.5	32.1	8.4	10.2	5.5		65.6	10.5	10.4
Ctrywde Home Lns	А	98.5	152.4	5.7	3.9	87.7	3.8	40.3	20.8	11.9	6.4		54.9	110.5	85.1
Dominion Res	BBB	57.6	32.7	9.7	3.6	59.7	3.4	20.9	8.4	11.6	5.6		66.1	20.1	20.3
Verizon Comms	BBB	67.8	63.1	8.4	4.3	50.1	3.6	26.0	10.7	6.8	4.0		62.8	41.0	32.1
Temple Inland	BBB	103.8	45.1	5.0	1.7	82.6	5.2	30.4	8.1	10.8	6.8		71.0	24.9	26.1
Home Depot	А	26.7	26.9	14.6	5.3	17.4	7.9	30.1	10.8	9.3	5.0		52.2	19.6	15.1
Amern Intl Gp	AA	33.0	26.2	7.1	3.1	77.7	7.4	25.9	10.2	12.6	5.7		55.5	18.6	17.6
Anadarko Petr	BBB	46.9	21.4	7.7	2.8	47.5	9.7	33.8	8.0	10.6	5.3		46.7	12.6	12.3
Carnival	А	66.5	54.2	9.4	3.0	27.8	4.1	32.2	11.1	11.3	4.7		71.6	24.5	23.2
MBIA Ins	AAA	66.0	82.5	5.6	2.4	72.7	9.3	37.3	26.6	11.8	6.2		57.5	54.9	32.4
Safeway	BBB	62.2	18.3	6.5	2.8	46.4	8.9	31.2	6.9	13.4	7.0		51.1	13.1	13.2
Autozone	BBB	77.5	40.2	9.5	2.4	33.2	4.8	29.8	8.0	11.4	7.1		34.7	19.7	22.7
Jones Apparel	BBB	105.7	66.2	7.5	2.2	30.9	5.7	33.2	10.2	11.1	6.2		66.8	38.3	48.2
Time Warner	BBB	102.5	116.2	8.7	4.6	41.9	9.1	34.3	17.9	10.7	6.6		68.2	69.6	48.4
Boston Scientific	BBB	71.8	67.3	8.2	2.8	20.8	12.6	39.6	10.3	10.8	6.3		40.8	46.8	46.8
Tyson Foods	BB	104.1	42.8	5.8	2.5	63.1	8.7	36.1	10.8	13.5	6.3		44.2	30.1	30.4
ACE	А	67.0	46.8	10.2	4.1	77.4	2.9	31.8	11.5	12.1	6.3		75.1	20.4	18.6
Transocean	BBB	57.9	34.1	7.3	3.2	26.9	11.9	41.2	9.4	9.1	4.3		73.5	15.5	15.6
Allstate	А	33.1	20.9	13.8	4.3	78.9	1.8	22.1	7.7	9.9	5.0		76.7	9.6	10.9
Eastman Chem	BBB	67.1	29.4	8.4	3.1	54.2	7.7	26.7	6.3	12.7	6.9		65.0	16.4	15.5
McKesson	BBB	68.0	41.7	11.7	4.5	51.5	4.2	29.5	8.2	10.6	5.8		75.2	15.2	15.1
Electr Data Sys	BB	135.1	90.2	7.7	4.7	43.7	11.7	36.8	15.7	12.4	5.3		66.0	53.3	50.0
Marriott Intl	BBB	65.5	41.3	14.0	3.3	30.7	4.8	30.5	10.5	11.1	5.4		49.5	17.3	16.3
Sempra Engy	BBB	68.2	56.6	9.4	3.6	67.8	6.4	25.5	9.2	11.3	5.2		73.8	28.1	21.3
Devon Engy	BBB	62.8	48.8	7.2	2.4	46.2	11.3	34.2	7.8	11.3	5.7		66.2	23.2	19.2
Visteon	CCC	440.1	304.4	2.3	1.6	87.4	4.5	56.6	13.9	11.4	5.2		54.7	203.0	178.6
Aetna	А	75.8	72.6	10.4	3.2	71.8	11.7	34.1	10.0	10.2	4.7		72.5	30.7	24.3

Notes: we report averages and standard deviations for the market spreads for 1, 3, and 5 year maturities, distance to default (DTD), leverage (LEV), 30 day option implied volatility (FIV), and number of contributors (LIQ) for the eighty-three firms in the sample. For each firm, we also report the R-square (R-2) from the linear regression, and the RMSEs in basis points from the linear regression and no-arbitrage models. The data are for the period January 1, 2002 to March 7, 2008. The covariates include two stochastic term structure factors, the VIX, and distance-to-default. NA stands for the no-arbitrage model.

Panel A: Risk-Free Term Structure Factor Loadings and Dynamics										
Factor 1 Factor 2										
δ0	0.03859									
δ1	0.18497									
δ2		0.23072								
ρ	0.99941	0.99781								
σ x 100	0.00712	0.00705								
μ/(1-ρ)	-0.06031	-0.04994								

Table 3: Risk-Free Term Structure and Factor Estimates

Panel B: Risk Free Term Structure Model RMSE (bps) and Measurement Error Standard Deviation (bps)										
	6 Months	1 Year	2 Year	3 Year	4 Year	5 Year	7 Year	10 Year		
RMSE	8.99	5.50	8.10	6.60	5.13	4.74	4.57	8.67		
ME-Std	1.25	1.15	2.30	4.41	1.60	2.50	2.90	1.12		

Panel C: VIX Dynamics										
ρ	ρ σ x 100 μ/(1-ρ)									
0.9995	0.45214	0.383								

A	Panel D: DTD Dynamics										
	ρ μ/(1-ρ) σ x 100										
Mean	0.998	9.010	27.814								
2.5%	0.984	-1.814	12.832								
25%	0.999	3.586	17.973								
50%	0.999	6.853	22.252								
75%	0.999	13.341	30.711								
97.5%	0.999	31.321	68.871								
Std Dev	0.004	9.658	14.663								

Notes: Panel A reports parameter estimates for the risk-free term structure factors. The two latent risk-free term structure factors are estimated using the Unscented Kalman Filter. The factor dynamics and short rate loadings for the risk-free term structure are estimated using the 6 month Libor rate, and 1, 2, 3, 4, 5, 7 and 10 year maturity swap rates. Panel B reports RMSEs and measurement error standard deviations for the risk free term structure. ME-Std in Panel B indicates the Measurement Error Standard Deviation and RMSE indicates the Root Mean Squared Error. Panel C reports the factor dynamics for the VIX, which are estimated by fitting a three factor (two term structure factors and the VIX) model to the term structure of CDX Index spreads. Panel D reports the estimated parameter distribution for the distance-to-default dynamic for the no-arbitrage model.

Panel A: Average R-squares (%)											
	1 Yr 3 Yr 5 Yr										
Level	68.9	66.9	63.6								
Diff	1.0	1.7	2.6								
CO	1.4	1.8	2.7								

Table 4: Average RMSEs and R-Squares.

Panel B: Average RMSEs (bps)											
	1 Yr 3 Yr 5 Yr										
Level	31.2	29.8	29.3								
Diff	6.3	4.4	3.9								
CO	6.4	4.4	3.9								
NA	26.7	25.4	27.1								
NA-AR(1)	6.3	4.4	3.9								

Notes: We report the average R-squares for the regression models. In addition, the table includes the average RMSEs for the linear regression based models as well as the no-arbitrage models. Level indicates the regression model where the dependent variable is the level of credit spreads, Diff indicates the regression model where the dependent variable is the change in credit spreads, CO indicates the Cochrane-Orcutt regression model, NA indicates the no-arbitrage model without autocorrelated errors, and NA-AR(1) indicates the no-arbitrage model with autocorrelated errors. The covariate specification includes two stochastic term structure factors, the VIX, and distance-to-default.

	Panel A: Term-Structure Factor 1											
One-Year Maturity Three-Year Maturity Five-Year Maturity										rity		
	Level	NA	CO	NA-AR(1)	Level	NA	CO	NA-AR(1)	Level	NA	CO	NA-AR(1)
Mean	3.55	3.61	-4.42	-0.79	3.94	3.53	-3.11	-0.53	5.76	3.15	-2.70	-0.37
Std Dev	23.32	8.99	7.90	6.40	18.50	8.92	7.01	6.03	17.36	8.39	5.26	5.77

	Panel B: Term-Structure Factor 2											
	One-Year Maturity			Т	Three-Year Maturity			Five-Year Maturity				
	Level	NA	CO	NA-AR(1)	Level	NA	CO	NA-AR(1)	Level	NA	CO	NA-AR(1)
Mean	-0.72	2.81	-3.67	0.71	1.39	2.26	-2.35	0.90	4.47	1.70	-1.47	0.92
Std Dev	20.21	15.32	7.90	7.60	17.84	11.96	9.11	6.62	18.19	9.31	7.34	6.28

	Panel C: VIX											
	(One-Yea	ır Matu	Maturity Three-Year Maturity Five-Year Matur				rity				
	Level	NA	CO	NA-AR(1)	Level	NA	CO	NA-AR(1)	Level	NA	CO	NA-AR(1)
Mean	3.03	1.49	0.15	0.08	3.01	1.56	0.15	0.10	2.85	1.53	0.19	0.11
Std Dev	3.55	1.76	0.40	0.35	3.59	1.78	0.37	0.34	3.43	1.74	0.38	0.34

	Panel D: Distance to Default											
	One-Year Maturity Three-Year Maturity Fiv				Five-Yea	ar Matu	rity					
	Level	NA	CO	NA-AR(1)	Level	NA	CO	NA-AR(1)	Level	NA	CO	NA-AR(1)
Mean	-8.16	-8.50	-1.94	-1.77	-7.60	-8.05	-1.73	-1.82	-7.09	-7.40	-1.78	-1.73
Std Dev	15.33	17.00	5.98	6.05	12.11	15.63	5.10	5.56	10.55	13.53	4.71	5.19

Notes: We report the cross-sectional average and standard deviation of the sensitivities for each factor from different models. The reported sensitivities indicate the change in spreads (in basis points) for a 1% change in the covariates in Panel A through C, and the change in spreads (in basis points) for one unit change in distance-to-default in Panel D. The covariate specification includes two stochastic term structure factors, the VIX, and distance-to-default.

Panel A: Daily						
	Constant	T-1	T-2	VIX	DTD	
Level	71.1%	67.5%	65.1%	89.2%	91.6%	
Diff	1.2%	14.5%	8.4%	36.1%	38.6%	
CO	8.4%	21.7%	15.7%	38.6%	41.0%	

Table 6: Percentage Firms with Significant Loadings on Covariates Different Data Frequencies

Panel B: Weekly						
Level	71.1%	48.2%	54.2%	84.3%	81.9%	
Diff	1.2%	4.8%	4.8%	19.3%	33.7%	
CO	47.0%	10.8%	14.5%	25.3%	41.0%	

		Panel C:	Monthly		
Level	48.2%	30.1%	37.3%	75.9%	57.8%
Diff	3.6%	34.9%	36.1%	44.6%	31.3%
CO	30.1%	33.7%	39.8%	59.0%	36.1%

		Panel D	: Yearly		
Level	65.1%	69.9%	73.5%	90.4%	89.2%
Diff	69.9%	81.9%	69.9%	91.6%	72.3%

Notes: We report the percentage firms with significant factor loadings at the 5% level using daily, weekly, monthly, and yearly time series data. Level indicates the linear regression model with the level of credit spreads as the dependent variable, Diff indicates the significance for the linear regresson model with the spread difference as the dependent variable, CO indicates the Cochrane-Orcutt model, NA indicates the no-arbitrage model, and NA-AR(1) indicates the no-arbitrage model with AR(1) errors. All results are for the 5-year contract. All standard errors are adjusted for autocorrelation using a Newey-West correction.

Panel A: Average R-squares, Weekly (%)							
1 Yr 3 Yr 5 Yr							
Level	69.2	67.2	63.9				
Diff	5.1	7.2	8.3				
СО	8.4	9.3	9.6				

Panel B: Average RMSEs, Weekly (bps)							
	1 Yr	3 Yr	5 Yr				
Level	31.6	29.7	29.2				
Diff	13.6	10.5	10.1				
CO	13.3	10.3	9.9				
NA	27.0	25.5	28.1				
NA-AR(1)	13.9	11.2	10.9				

Panel C: Average R-squares, Monthly (%)							
1 Yr 3 Yr 5 Yr							
Level	71.6	69.1	65.6				
Diff	22.2	23.9	23.3				
СО	34.1	34.0	31.7				

Panel D: A	verage RM	SEs, Month	ly (bps)
	1 Yr	3 Yr	5 Yr
Level	29.5	28.4	28.1
Diff	20.6	18.7	18.7
CO	19.5	17.7	17.9
NA	26.1	24.9	27.8
NA-AR(1)	19.5	18.9	19.6

Panel E: Average R-squares, Yearly (%)			Panel F:	Panel F: Average RMSEs, Yearly (bps)			
	1 Yr	3 Yr	5 Yr		1 Yr	3 Yr	5 Y
Level	69.4	67.7	64.1	Level	29.0	27.8	27.
Diff	55.0	55.8	54.4	Diff	35.6	34.4	34.4

Notes: We report the average R-squares for the regression models using weekly and monthly data as well as for yearly differences. In addition, the table includes the average RMSEs for the linear regression based models as well as the no-arbitrage models. Level indicates the regression model where the dependent variable is the level of credit spreads, Diff indicates the regression model where the dependent variable is the change in credit spreads, CO indicates the Cochrane-Orcutt regression model, Yearly Diff. indicates the regression model where the dependent variable is the yearly change in credit spreads, NA indicates the no-arbitrage model without autocorrelated errors, and NA-AR(1) indicates the no-arbitrage model with autocorrelated errors. The covariate specification includes two stochastic term structure factors, the VIX, and distance-to-default.