
METALLGESELLSCHAFT: A PRUDENT HEDGER RUINED, OR a WILDCATTER ON NYMEX?

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INTRODUCTION

The travails of the firm Metallgesellschaft (MG) have received much attention in both academic circles and the financial press. The battle lines on the issue are clearly drawn. On one side, critics of MG [including Mello and Parsons, (1995)] claim that the firm's energy market trading was rashly speculative, and as a result of adverse movements in oil prices, the firm suffered real mark-to-market losses of as much as one billion dollars. On the other side, defenders of the firm—notably Culp and Miller (1994)—claim that the firm employed a prudent and potentially very lucrative strategy of hedging long-term energy delivery obligations with short-term futures and swaps. In this view, MG's bankers mistook a mere liquidity problem resulting from margin calls on futures positions for a full-blown insolvency crisis, unwisely unwound the firm's hedge position, and prematurely terminated some of its long-term delivery contracts.

Which view is correct ultimately depends upon the dynamics of energy prices, and how these dynamics affect optimal hedge ratios. MG implemented a barrel-for-barrel hedge. That is, it bought one barrel of short-term energy futures or swaps for each barrel of oil it was committed to deliver, regardless of whether it was obligated to deliver in 6 months or 10 years. There are strong reasons to believe *a priori* that this hedging strategy forced the firm to bear more risk than necessary. However, Culp and Miller defend the one-for-one hedge, claiming that MG employed an innovative synthetic storage (or carrying charge hedging) strategy that

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increased firm value while protecting MG against spot price increases over the 10-year life of the program. They recognize that this strategy forced the firm to bear basis risk, but claim that this basis risk was small relative to the risk inherent in the firm's fixed price contracts.

A definitive resolution of this debate cannot be achieved through a *a priori* argument; the data must be the ultimate arbiter. This article undertakes a thorough analysis of the dynamics of crude oil futures prices to determine the riskiness of the barrel-for-barrel strategy relative to alternative strategies available to the firm. This analysis of variance minimizing hedge ratios is more thorough and employs more sophisticated econometric techniques than previous studies by Mello and Parsons (1995) and Edwards and Canter (1995). It thus allows a more complete critique of the prudence of the barrel-for-barrel strategy.

The empirical results are starkly revealing. Given the behavior of crude oil prices, the variance-minimizing hedge ratio during 1993 was far less than 1. Indeed, for delivery obligations with maturities as short as 15 months, the variance-minimizing hedge ratio was around 0.5, implying that MG's barrel-for-barrel hedge actually increased the firm's exposure to oil price risk. Even under very conservative assumptions the data imply that MG's exposure to energy price risk was greater with a barrel-for-barrel futures and swap hedge than it would have been if the firm had not hedged its long-term delivery commitments at all! Consideration of options embedded in the firm's cash market contracts does not alter the fundamental result. Moreover, the prospect of earning gains every time it rolled over its futures positions did not justify taking this additional risk. Thus, it is impossible to view the firm's strategy as a prudent exercise in risk management.

The empirical results imply that the combined futures-long-term contract position exposed the company to severe losses in the event of a steepening of the term structure of energy prices. This indeed occurred in 1993. Simulation estimates of the profitability of the barrel-for-barrel strategy during this period imply that the firm lost approximately \$800 million on a mark-to-market basis. These estimates, which correspond closely to accounting estimates of MG's losses, contradict the claim that the firm's losses were a mirage caused by misleading accounting standards that failed to reflect mark-to-market gains on its deferred delivery contracts.

II. MG'S ENERGY MARKET ACTIVITIES

The details of MG's energy market activities have been the subject of much coverage, so a short overview will suffice here. In 1991, an MG

subsidiary—MG Refining and Marketing—entered the business of supplying American heating oil and gasoline retailers. To do so, it offered these retailers unprecedented 5- and 10-year fixed-price contracts. These contracts were of two types. The firm-fixed contracts specified delivery schedules. The firm-flexible contracts allowed buyers to choose the delivery schedule with certain restrictions. Under the firm-flexible contracts, buyers were allowed to defer or accelerate purchases, but were required to buy all quantities deferred by the end of the contract. Thus, these contracts permitted buyers to choose the timing of deliveries, but not their quantity. These contracts also allowed the buyers to terminate at will. At termination of the firm-fixed contracts, the buyers received a payment of one-half of the difference between the prevailing spot price of West Texas Intermediate light crude oil and the fixed price in the contract, multiplied by the quantity remaining under the contract. Under the firm-flexible contracts, the buyers received the full difference between the 2-month futures price and the contract price.

By September 1993, MG had entered contracts obligating it to deliver 102 million barrels of refined products under firm-fixed contracts. The tenor of 94% of these contracts was 10 years; the remainder had a 5-year tenor. MG was obligated to deliver 47.5 million barrels of products under 10-year firm-flexible contracts, and 10.5 million under 5-year firm-flexible deals. Approximately one-third of these obligations were entered into during September 1993. In addition, MG entered into an arrangement to purchase refined products from Castle Energy Corp., a small U.S. refiner. MG agreed to supply the refinery with most of the 100,000 barrels per day (b/d) of crude oil it required, and agreed to purchase the refinery's daily output of 40,000 b/d of gasoline and 35,000 b/d of heating oil and other distillates.

To protect itself against increases in energy prices, MG purchased crude oil and gasoline futures contracts, and entered into OTC energy swaps. Rather than matching the expiration dates of the futures contracts with the dates of its delivery obligations to Castle and its customers, MG bought primarily near-month (i.e., next to expire) crude oil and gasoline contracts. In the terminology of the futures trade, this is referred to as a stacked hedge, because all hedging positions are stacked on a single contract month rather than spread over several contract months. MG's OTC swaps were also of relatively short maturity. The expirations of these contracts were predominately less than or equal to three months. MG purchased one 1000 barrel futures contract or its swap equivalent for each 1000 barrels of the firm's short position regardless of the expiration date of the short position. That is, the firm hedged barrel-for-barrel, and thus

by mid-to-late 1993 had bought 160,000,000 barrels of futures and swaps to cover its 160,000,000 barrel cash market position.

III. THE DETERMINATION OF HEDGE RATIOS TO CREATE A SYNTHETIC FORWARD POSITION

A. Introduction

The riskiness of a barrel-for-barrel hedging strategy depends crucially upon the dynamics of energy prices. To see why, consider a firm that desires to minimize the variance of the payoff on a deferred forward delivery obligation (the short position) in crude oil. This focus on variance minimization is not intended to imply that only variance-minimizing hedges were appropriate for MG. Instead, variance minimization serves as a benchmark against which to measure actual trading strategies; by comparing actual hedge ratios to variance minimizing ratios, it is possible to quantify (a) the speculative component of a trading strategy, and (b) the risk of the actual trading strategy.

The fixed price in the forward obligation to be hedged is f . MG was short a bundle of many forward positions, but because the analysis is identical for each forward contract in the bundle, for simplicity the analysis focuses upon hedging a single element of the bundle; repeating the following analysis for each element produces the appropriate hedge ratio for MG's entire swap position.

Assume that the firm is constrained to employ a single hedging instrument—the next-to-expire crude oil futures contract (the nearby contract).¹ The deferred obligation expires at time T . The firm can adjust the number of nearby contracts it buys at M equally spaced times between time t_0 (the present) and T . That is, the firm can hedge dynamically. Each interval is $\Delta t = (T - t_0)/M$ in length. Because the nearby crude contracts expire monthly, each interval is less than or equal to 1 month in duration. As M grows arbitrarily large, the firm effectively employs a continuously adjusted dynamic hedging strategy. Through this strategy, the firm attempts to replicate the payoffs to a forward position, thereby creating a synthetic forward contract.

¹In general, it is not optimal to rely upon only a single hedging instrument. If there are multiple sources of risk in oil prices (e.g., the term structure shifts up and down *and* twists) then a firm can enjoy better hedging effectiveness if it uses several hedging instruments. The approaches described below can be used to determine multiple-instrument hedge ratios. The very fact that MG employed only the nearby contract strongly suggests that they were not interested in hedging alone, but were also speculating on movements in the term structure.

The change in the price of a unit of the deferred over a time interval ending at t equals $\Delta F_{t,T} \equiv F_{t,T} - F_{t-\Delta t,T}$, where $F_{t,T}$ is the forward price at t for delivery at T . Because the payoff to the deferred delivery obligation occurs at time T , and because MG is short, the change in the present value of the deferred obligation is $v_{t,T} \equiv -e^{-r(T-t)} \Delta F_{t,T}$.² The change in the price of the nearby contract over the same time interval equals $\Delta S_t \equiv S_t - S_{t-\Delta t}$. Because futures contracts are marked to market, the hedger realizes this gain or loss when it occurs. At $t - \Delta t$ the firm buys $\beta_{t,T}$ units of the nearby contract to hedge each unit of its deferred obligation over the interval, $[t - \Delta t, t]$. By T , the firm's realized profit/loss equals

$$\Pi_T = \sum_{i=1}^M e^{r(T-t_0-i\Delta t)} [\beta_{t_0+i\Delta t,T} \Delta S_{t_0+i\Delta t} - \Delta v_{t_0+i\Delta t,T}] - F_{t_0,T}$$

The firm's objective at t_0 is to minimize $E_{t_0} (\Pi_T - E_{t_0} \Pi_T)^2$.³

Determination of the variance-minimizing hedge strategy for each t requires the solution of an extremely complex dynamic programming problem that allows the hedge ratio at t to depend upon expected hedge ratios for $t' > t$ [Chan (1992); Duffie and Jackson (1991), and Lien and Luo (1994)]. If mean price changes are nonzero, even in relatively simple cases involving time-varying spot and forward price dynamics, solution of this dynamic programming problem is not practical even for M on the order of 2 or 3. It is therefore necessary to approximate the optimal dynamic variance minimizing hedging solution by a sequence of myopic hedge ratios which minimize the variance of the one-period hedge gain/loss; that is, the $\beta_{t,T}$ that minimizes $E_{t-\Delta t} [\beta_{t,T} \Delta S_t - \Delta v_{t,T} - E_{t-\Delta t} \Pi_t]^2$ for $t = t_0 + i\Delta t$, $i = 1, \dots, M$, where $\Pi_t = \beta_{t,T} \Delta S_t - \Delta v_{t,T}$. This is the approach taken in other studies of hedging with time-varying parameters, such as Kroner and Sultan (1991).

²For simplicity, the analysis assumes that interest rates are nonstochastic, and the term structure of interest rates is flat. This expression holds because the value of the forward contract equals $e^{-r(T-t)}(f - F_{t,T})$. If interest rates are stochastic, futures prices and forward prices may differ due to the effect of marking to market on the timing of cash flows. Cox, Ingersoll, and Ross (1981) demonstrate that this effect is important only to the extent that changes in interest rates and futures prices are correlated. Because the correlations between crude oil returns and percentage changes in interest rates are extremely small, this consideration is ignored hereafter. Specifically, over the July, 1987–June, 1994 period, the correlation between the percentage change in the 3-month T-bill rate and the percentage change in the spot oil price is 0.01; the correlations between the percentage change in the percentage changes in the 6- and 12-month T-bill rates and the percentage changes in futures prices with 6 and 12 months to expiration, respectively, are less than 0.005.

³For simplicity, it is assumed that the firm's hedging horizon corresponds to the maturity of the delivery obligation. This is not necessary. It is possible to choose a hedging horizon that is less than this maturity. Under the martingale assumption employed below, however, the firm optimally employs a myopic hedge ratio that is independent of the hedging horizon.

In the present case, the use of a series of single-period variance-minimizing hedges to approximate dynamically optimized hedges likely involves little cost in terms of accuracy. The Appendix shows that myopic hedge ratios are equal to those produced as the solution to the dynamic programming problem if S_t and $F_{t,T}$ are martingales. [See also Duffie and Jackson, (1991)], Section V.B shows that one cannot reject the hypothesis that past price changes do not explain current price changes for either nearby and deferred futures, which justifies the use of myopic hedge ratios even in a dynamic hedging problem.

It is well known that the optimal $\beta_{t,T}$ for one-period-ahead hedging is given by

$$\beta_{t,T} = - \frac{\text{cov}(\Delta S_t, \Delta v_{t,T})}{\text{var}(\Delta S_t)} = \frac{e^{-r(T-t)} \text{cov}(\Delta F_{t,T}, \Delta S_t)}{\text{var}(\Delta S_t)}. \quad (1)$$

This can be rewritten as:

$$\beta_{t,T} = e^{-r(T-t)} \frac{\sigma(\Delta F_{t,T})}{\sigma(\Delta S_t)} \text{corr}(\Delta F_{t,T}, \Delta S_t), \quad (2)$$

where $\sigma(\Delta F_{t,T})$ is the standard deviation in interval in the change in the price of the deferred obligation i $\sigma(\Delta S_t)$ is the standard deviation of the change in the nearby price, and $\text{corr}(\Delta F_{t,T}, \Delta S_t)$ is the correlation between the change in the nearby price and the change in the deferred price.⁴

These correlations and variances may change over time for a variety of reasons. First, it is plausible *a priori* that oil prices are stationary [Dixit and Pindyck (1994)]. Stationarity causes the volatility of the deferred to rise as time passes. Second, the theory of storage implies that the variances and correlations should depend upon the spread between spot and deferred prices (net of interest and storage costs). When supplies are short, the market is in backwardation. An increase in the severity of backwardation causes an increase in both spot and deferred volatilities, a decrease in the ratio of deferred volatility to spot volatility, and a decline in the correlation between spot and deferred prices [Ng and Pirrong (1994)].

⁴The discount factor multiplying the correlation/standard deviation term reflects the fact that cash flows on the forward contracts are not received until the delivery date. Adjusting for this deferral of cash flows by reducing the hedge ratio by the discount factor is called *tailoring the hedge*. This consideration is relevant only to the extent that (a) the hedge position is large enough to permit a match between the size of the tailed hedge and an integer number of futures contracts, and (b) the time to delivery is long enough to make the effect of discounting appreciable. Both cases are certainly relevant in the MG case. Therefore, MG could have and should have tailed its hedges to reflect cash-flow timing mismatches between forwards and futures.

Because backwardation is a random variable, this implies that hedge ratios should change randomly as well. Third, shocks to the oil market (due to OPEC policy changes, for example) can cause changes in the relevant variances and correlations, and, thus, in hedge ratios. Given these three factors, variance-minimizing hedging requires a methodology for quantifying how the relevant correlation and variances change. There are a variety of means to address this problem. This next section describes a GARCH-based methodology because it can take each of these factors into account.⁵

B. Backwardation-Adjusted GARCH

Backwardation-adjusted GARCH (BAG) is a two-stage technique that adjusts variances and covariances to reflect the three factors noted in the prior section. See Ng-Pirrong (1994, 1996) for a detailed presentation of this technique. In the first stage, to model the mean return of the nearby and the deferred, one regresses the change in the nearby (deferred) price against lagged changes in nearby and deferred prices and the lagged level of backwardation. This latter variable is defined as

$$z_{t-1} = \{\ln[F_{t-1,T} - w(T - t + 1)] - \ln S_{t-1}\} / (T - t + 1) - r.$$

In words, it is the percentage difference between the actual futures price and the full-carry price calculated from the spot price, the cost of storage, w , and the interest rate.⁶ The residual from the spot equation is ε_t , and the residual from the futures equation is η_t . In the second stage, one uses the residuals from the mean equations to estimate jointly a modified GARCH model of the conditional variances and covariances of the nearby and deferred return. In addition to the traditional GARCH terms, this model includes the squared lagged backwardation as an explanatory variable. This allows variances and covariances to depend upon the degree of backwardation in the market. This model is estimated with the use of quasimaximum likelihood. Formally, the equations for the conditional variance of the deferred return, $h_{F,t}$, and the conditional variance of the nearby return, $h_{S,t}$ are

⁵Earlier drafts of this article included hedge-ratio estimates based on alternative methodologies, including backwardation adjusted regression, and factor models. These methods are cruder in crucial aspects than the GARCH, so these results are not reported here. The relevant results are available on request.

⁶One cannot observe the actual value of w . It is estimated in the following way. Arbitrage precludes $z_t > 0$; that is, prices cannot be above full carry. Therefore, the value of the smallest w is found, such that, $z_t < 0$ for all maturities and all days. This value is used as the estimate of w .

$$h_{S,t} = \omega_S + \delta_1 h_{S,t-1} + \delta_2 \varepsilon_{t-1}^2 + \delta_3 z_{t-1}^2, \quad (3)$$

$$h_{F,t} = \omega_F + \phi_1 h_{F,t-1} + \phi_2 \eta_{t-1}^2 + \sigma_3 z_{t-1}^2. \quad (4)$$

The inclusion of the z_{t-1}^2 terms allows the degree of backwardation to affect volatility. The conditional spot-forward covariance is

$$\sigma_{S,F,t} = \rho \sqrt{h_{S,t} h_{F,t}} + \theta z_{t-1}^2 \quad (5)$$

A GARCH model that does not include a backwardation term can also be employed to determine hedge ratios:

$$h_{S,t} = \omega_S + \delta_1 h_{S,t-1} + \delta_2 \varepsilon_{t-1}^2, \quad (3')$$

$$h_{F,t} = \omega_F + \phi_1 h_{F,t-1} + \phi_2 \eta_{t-1}^2, \quad (4')$$

$$\sigma_{S,F,t} = \omega_{S,F} + \mu_1 \sigma_{S,F,t-1} + \mu_2 \varepsilon_{t-1} \eta_{t-1}. \quad (5')$$

This model does not allow the variance-covariance matrix of spot and futures returns to depend upon backwardation, but does allow the covariance between spot and futures residual returns at t to depend upon the lagged covariance, and the product of the lagged residuals.

In each model, hedge ratios are given by

$$\beta_{i,T} = e^{-r(T-t)} \frac{F_{i,T}}{S_t} \frac{\sigma_{S,F,t}}{h_{S,t}}.$$

C. Summary and Implications

The BAG model allows the estimation of time-varying variance-minimizing hedge ratios that reflect how fundamental supply-and-demand conditions affect the dynamics of energy prices. On *a priori* grounds there are strong reasons to believe that hedge ratios should be far less than 1, especially for distant-deferred obligations.

Culp and Miller (CM) (1995a) object to the variance-minimizing framework for a variety of reasons. First, they claim that estimates of variance-minimizing hedge ratios are imperfect because data are “subject to considerable error.” This is true, but estimates of hedge ratios that are conditional upon data, and consistent with an understanding of the fundamental dynamics of commodity prices, are better than naive estimates of hedge ratios that are conditional upon no data at all and inconsistent with theoretical understanding.

Second, CM argue that a variance-minimizing hedge does not necessarily maximize firm value. This is correct. Variance-minimizing strat-

egies are not the only legitimate hedges. Instead, the variance-minimizing hedge should be used as a benchmark to evaluate the relative importance of the hedging and speculative components present in most derivative trading strategies. Firms trade higher variance for higher expected returns. Anderson and Danthine (1981) demonstrate that in addition to variance, the optimal hedge ratio also depends upon a firm's estimate of the drift in the futures price. Similarly, Working (1962) notes that most hedgers do not strive to minimize risk, but also take positions on expected movements in the basis due to their possession of private information. That is, most hedges involve a speculative component when firms underhedge or overhedge (relative to the variance-minimizing hedge ratio) to exploit perceived differences between futures prices and their expectations of future spot prices or future basis movements. Perhaps MG's managers possessed information that led them to expect a rise in spot oil prices/widening of the basis, and this led them to choose a barrel-for-barrel hedge. Such a justification for their strategy is completely different than risk avoidance, however. Deviations between the barrel-for-barrel ratio and the variance-minimizing ratio therefore measure the importance of the speculative component of MG's strategy. Because it will be shown that these deviations are large, it may be concluded that MG's strategy was largely speculative.⁷

IV. VARIANCE-MINIMIZING HEDGING IN THE CRUDE OIL MARKET

A. Introduction

This section analyzes data from the crude oil futures market for the March 20, 1989–June 20, 1994 period to determine whether MG overhedge. Oil futures began trading in 1983; the analysis is based on data starting in 1989 because there are gaps in the trading of the 13–15-month maturity contracts prior to March 1989. Moreover, since the Gulf War period (August 2, 1990–February 28, 1991) is plausibly structurally different from the preceding and succeeding periods, the model is also estimated using post–Gulf War data only. This sample spans the period, 3/1/91–6/20/94.

⁷Edwards and Canter (1995) suggest that a hedge ratio of less than 1 was appropriate on variance-minimization grounds, but claim that MG had a defensible rationale for its barrel-for-barrel strategy. In brief, they attribute MG's strategy to the firm's beliefs that oil prices would rise over the life of the hedge. This is essentially a speculative rationale like that advanced in theory by Anderson and Danthine.

MG's cash market commitments extended 10 years into the future. As a result, it would be desirable to analyze the relationships between nearby futures prices and the prices of crude oil for all delivery periods between 2 months and 10 years into the future. Unfortunately, there are no continuous time series of reliable data on forward or futures prices of maturities longer than 15 months. Even this somewhat limited analysis provides valuable information. As will be seen, the data exhibit a monotonically decreasing relationship between the variance-minimizing hedge ratio and the maturity of the forward obligation being hedged. This implies that the 14- or 15-month hedge ratio is a conservative estimate of the 2-year or 10-year hedge ratio. The results, based on an analysis of the 15-month and earlier hedge ratios are, therefore, conservative.

B. Exploratory Data Analysis

Recall that single-period (myopic) hedge ratios are appropriate for a dynamic hedge when nearby and deferred futures prices are martingales. The data provide strong evidence that oil futures prices are martingales. Regressions of the spot price change versus 10 lagged spot price changes, 10 lagged 15-month futures price changes, the difference between the nearby and 15-month deferred futures prices, and a constant have very low R^2 's, and one cannot reject the null that all coefficients in this regression equal 0. The p value in this test equals 0.41. Similarly, in regressions of the 15-month deferred futures price change versus 10 lagged nearby futures price changes, 10 lagged 15-month futures price changes, the nearby 15-month price difference, and a constant, one cannot reject the null that all coefficients are jointly 0; the p value equals 0.64. Comparable results are obtained for different deferred month futures price changes. Moreover, the bicornelation test developed by Hsieh (1989) also fails to reject the hypothesis that expected price changes at t , conditional on all earlier price changes, equal 0 for nearby and deferred futures prices. No individual test statistic is significant for the first 15 lags, and the Q statistic testing the hypothesis that the first 15 bicornelations are jointly zero equals 14.30 for the nearby futures price change. The p value on this test equals 0.5. Thus, one cannot reject the hypothesis that the expected price change of the next expiring oil futures contract (conditional on past price changes) equals zero. Similar results are obtained for longer maturities. This implies that single-period hedge ratios are appropriate.

A preliminary analysis of the data also strongly suggests that a one-for-one hedge is not variance minimizing. Tables I and II present futures

TABLE I

Daily Crude Oil Futures Return Variances 3/21/89–6/20/94

<i>Maturity</i>	<i>Variance</i>
1	6.09E-4
2	4.59E-4
3	3.72E-4
4	3.25E-4
5	2.95E-4
6	2.73E-4
7	2.54E-4
8	2.40E-4
9	2.27E-4
10	2.19E-4
11	2.13E-4
12	2.07E-4
13	2.03E-4
14	1.96E-4
15	1.92E-4

TABLE II

Daily Crude Oil Futures Return Variances 3/1/91–6/20/94

<i>Maturity</i>	<i>Variance</i>
1	2.22E-4
2	1.95E-4
3	1.67E-4
4	1.47E-4
5	1.32E-4
6	1.21E-4
7	1.10E-4
8	1.03E-4
9	9.88E-5
10	9.51E-5
11	9.23E-5
12	8.92E-5
13	8.68E-5
14	8.50E-5
15	8.30E-5

return variances in the complete and post–Gulf War samples, respectively. Variances decrease monotonically with time to expiration, consistent with oil price stationarity. This would tend to induce a variance-minimizing hedger to choose a hedge ratio of less than 1. Tables III and IV present correlations between the 1 month oil futures return and returns on contracts with maturities greater than 1 month. The correlations

TABLE III

Correlations between Nearby Crude Oil Futures Return and Deferred Crude Oil Futures Returns 3/21/89–6/20/94

<i>Maturity</i>	<i>Correlation</i>
2	0.947
3	0.934
4	0.925
5	0.913
6	0.899
7	0.885
8	0.870
9	0.855
10	0.841
11	0.826
12	0.794
13	0.796
14	0.759
15	0.749

are monotonically decreasing with expiration, and are far below 1 for maturities of 10 months or more. Combined with the effect of tailing, these preliminary results strongly suggest that a barrel-for-barrel hedge is far larger than necessary to minimize variance.

C. BAG Hedge Ratios

The BAG analysis provides very strong evidence that variance-minimizing crude oil hedge ratios are substantially less than 1. To carry out this analysis, the model described in Section III.B is first estimated with the use of returns on 1 and 10–15-month crude oil futures contracts for the 3/20/89–6/20/94 period. The resulting parameter estimates from the 3/20/89–6/20/94 sample demonstrate that the variances of spot and deferred returns and the covariance between these returns depend upon z_{t-1}^2 in a statistically significant way. Table V reports these estimates for the 11- and 15-month maturities. (Results for other maturities are similar, so are omitted.) Moreover, as theory predicts, the spread has a more pronounced effect on spot returns than forward returns. Furthermore, as expected, the covariance between nearby and deferred returns falls as z_{t-1}^2 increases. Post-Gulf War results (not reported) are somewhat different. In this case, the coefficients on z_{t-1}^2 are of the right sign, but are not significant in either the variance equations or the covariance equation.

TABLE IV

Correlations between Nearby Crude Oil Futures Return and Deferred Crude Oil Futures Returns 3/1/91–6/20/94

<i>Maturity</i>	<i>Correlation</i>
2	0.989
3	0.978
4	0.967
5	0.954
6	0.941
7	0.922
8	0.909
9	0.896
10	0.885
11	0.875
12	0.864
13	0.854
14	0.845
15	0.836

To calculate hedge ratios with the BAG model, parameter values are updated by reestimating the model on a weekly basis. Thus, for hedge ratios for the 7-day period commencing 9/1/92, parameters estimated over the 3/21/89–8/31/92 period are used. For hedge ratios for the 7-day period commencing 9/8/92, parameters estimated on a 3/21/89–9/7/92 sample are used, and so on. This ensures that hedge ratio estimates are based on information available to MG when it was making its decisions. Using these parameters, the fitted value of the spot return variance and the spot-deferred return covariances are calculated to determine variance-minimizing hedge ratios.

Figure 1 illustrates the variance-minimizing hedge ratios for the 13–15-month maturities over the late 1992–early 1994 period, during which MG's hedging strategy was in place. For the September 1992–June 1993 period, variance-minimizing hedge ratios were typically less than 0.5 for these longer times to expiration. For the June–December, 1993 period, hedge ratios ranged between 0.5 and 0.6. Thus, the barrel-for-barrel hedge was not variance increasing for these maturities, but was still considerably overhedged. Figure 2 illustrates the variance-minimizing hedge ratios estimated from the post–Gulf War subsample. Although the hedge ratios are somewhat higher than those depicted in Figure 1, they are still consistently smaller than 1. In sum these results provide strong evidence that the barrel-for-barrel strategy did not substantially reduce MG's risk.

TABLE V

Backwardation Adjusted Garch Estimates (T Statistics in Parentheses) 3/21/89–6/20/94

	Maturity	
	11 Months	15 Months
ω_s	1.0E-6 (0.3460)	1.0e-6 (0.659)
δ_1	0.9168 (83.80)	0.9176 (76.81)
δ_2	0.0724 (6.99)	0.0726 (6.29)
δ_3	9.4E-5 (2.45)	5.6E-5 (2.01)
ω_F	1.0E-6 (0.217)	1.0e-6 (0.364)
ϕ_1	0.9125 (73.94)	0.9155 (68.11)
ϕ_2	0.0750 (6.78)	0.0745 (3.99)
ϕ_3	5.0E-5 (2.79)	3.2E-5 (2.55)
ρ	0.8343 (53.66)	0.801 (41.23)
θ	-2.8E-5 (-1.53)	-1.16E-4 (-2.48)
Θ	0.1760 (12.09)	0.1944 (9.26)
Log L	8329	8186

These inflated hedge ratios increased the variance of MG's position. To estimate the effects of overhedging on this variance, the fitted variances for the spot and deferred futures returns are used to calculate the variance of the returns on MG's positions in the 13–15-month maturities. Formally, this variance is equal to

$$h_t = (1 - \beta_{i,T})^2 S_t^2 h_{S,t} + (1 - R_{F,t}^2) F_{i,T}^2 e^{-2r(T-t)} h_{F,t},$$

where $R_{F,t}^2 = \sigma_{S,F,t}^2 / h_{S,t} h_{F,t}$ is the squared correlation between the spot and futures returns. Figure 3 depicts

$$h_t / [(1 - R_{F,t}^2) e^{-2r(T-t)} F_{i,T}^2 h_{F,t}] = 1 + (1 - \beta_{i,T})^2 S_t^2 h_{S,t} / [(1 - R^2) e^{-2r(T-t)} F_{i,T}^2 h_{F,t}]$$

for the September 1992–December 1993 period, where the hedge ratios and squared correlations are calculated based on the estimates from the

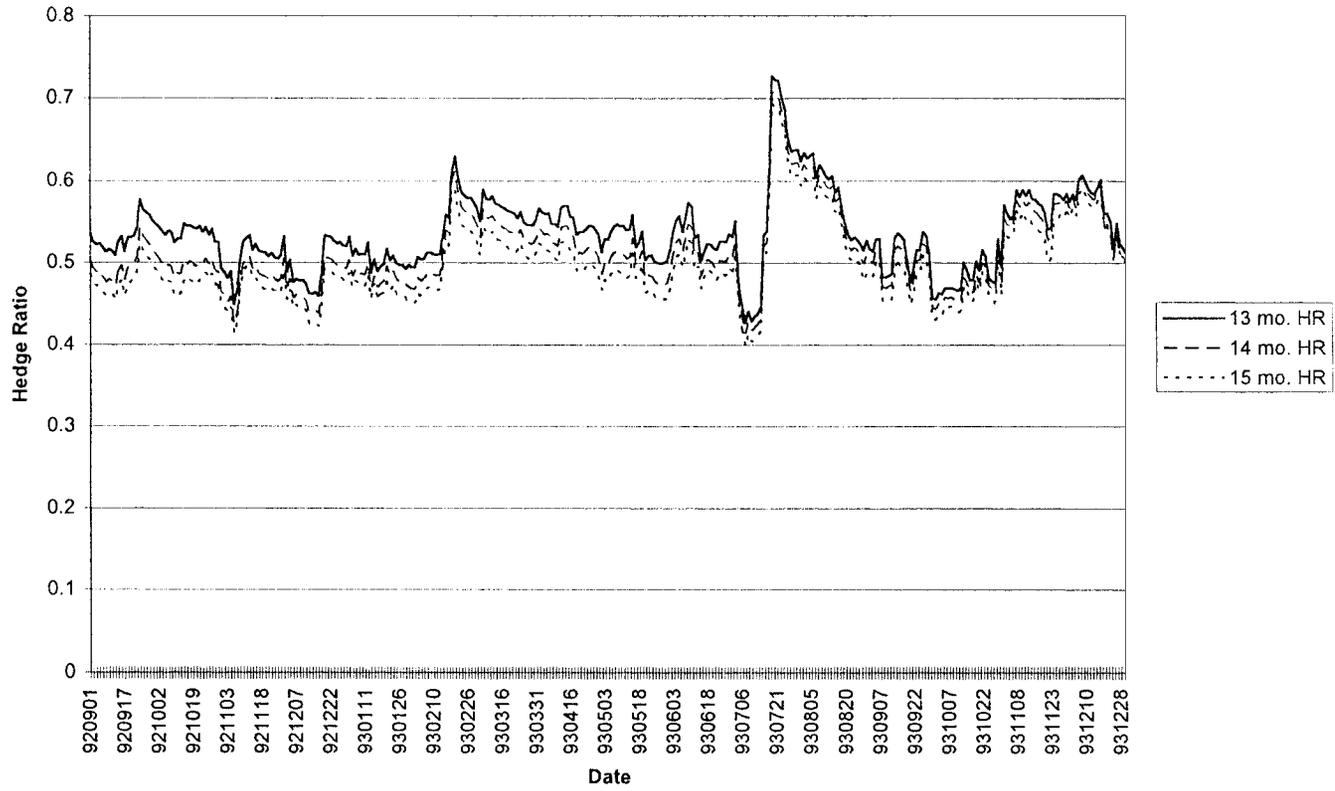


FIGURE 1
13–15 month BAG CL hedge ratios, 9/92–1/94.

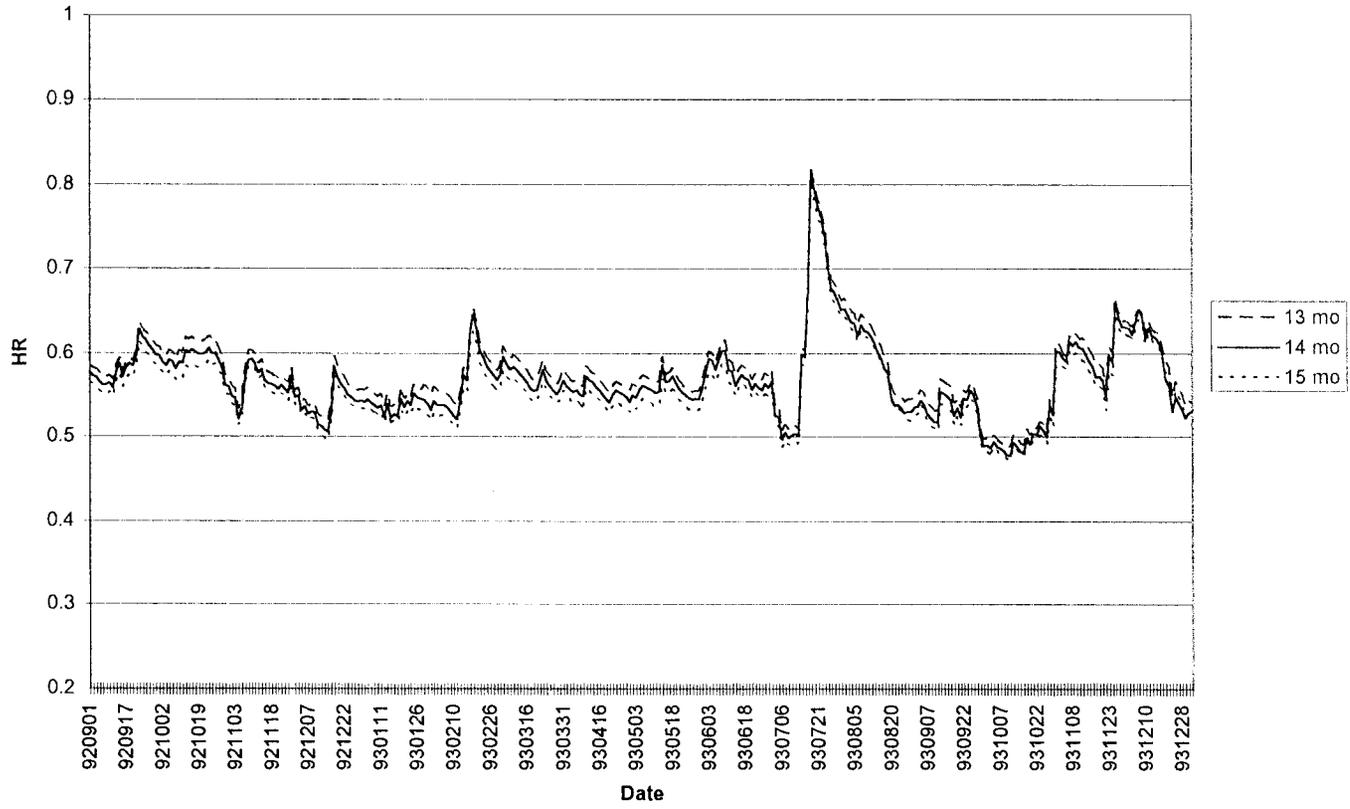


FIGURE 2
13–15 month BAG CL hedge ratios. (Gulf War data excluded from estimation)

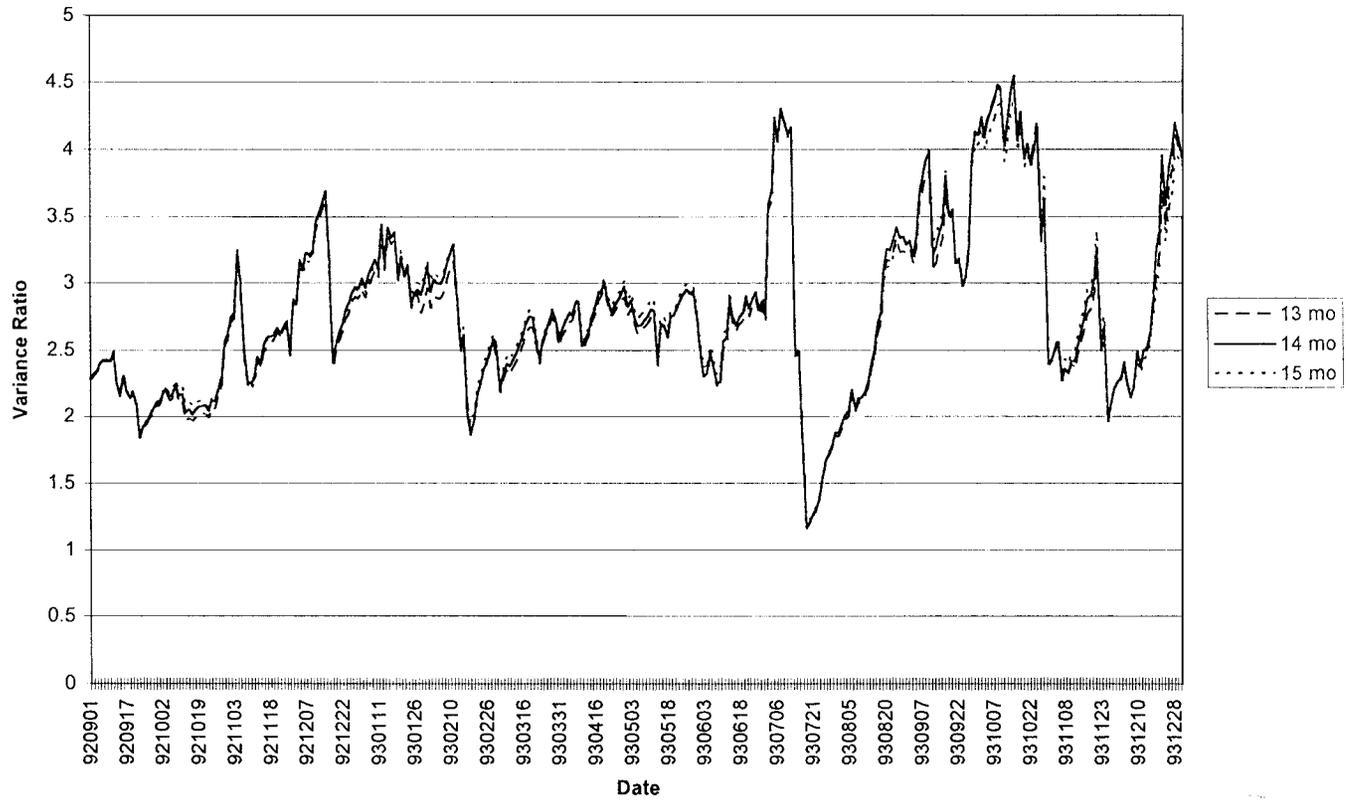


FIGURE 3
Ratio of hedged variance to minimized variance. (Gulf War period excluded from estimation)

samples that exclude the Gulf War. That is, this figure depicts the ratio between the variance of the barrel-for-barrel hedge and that of the variance-minimizing hedge for the 13–15-month maturities. The ratio ranges from between 1.19 and 4.50. Thus, even for these relatively short maturities (recall that MG was hedging obligations dated out to 120 months) MG bore between 19 and 350% more risk than was necessary.

The variance of MG's position was not only larger than that of the variance-minimized position—at times it was larger than the variance of the unhedged forward contracts. Figure 4 depicts the ratio of the variance of the hedged, barrel-for-barrel position to the variance of the unhedged forward contracts, for the 13-, 14-, and 15-month maturities. Even with the use of the more conservative post-Gulf War sample estimates, the variance of the MG position, was typically between .6 and .8 times the variance of the unhedged position for the 13–15-month maturities. At times—especially during the fall of 1993—the variance of the barrel-for-barrel position was larger than the variance of the unhedged forward contracts for 13–15-month delivery dates.

Although these results from the 13–15-month maturities clearly indicate that a barrel-for-barrel hedge was far riskier than alternatives available to the firm, they do not provide a complete picture of just how risky the strategy was. Theory strongly suggests that the 15-month variance-minimizing hedge ratios should be higher than hedge ratios for more distant delivery obligations. Indeed, hedge ratios should decrease monotonically with maturity because of tailing, the stationarity of oil prices, and the declining substitutability between spot oil and more distant deferred oil.

The latter two effects cannot be estimated because of the limitations of the data. *Taking only the tailing effect into account*, however, it is possible to show that even when the hedged-to-unhedged variance ratio was less than 1 for the 15-month contracts, MG's entire hedged position was riskier than its unhedged contracts. If we make the conservative assumptions that (a) MG discounted all cash flows at a 6% rate, and (b) the covariance between spot oil returns and the forward price returns for maturities beyond 15 months equaled the spot 15-month covariance, it is possible to estimate the variance ratio for the aggregate position. Specifically, under conservative assumptions, it is possible to calculate a *downward-biased* measure of the ratio between the variance of MG's entire position and the variance of its forward contract portfolio alone.

It is necessary to know the variance-covariance matrix of all 120 forward price changes to calculate this ratio. This is computationally impossible in the BAG model, but by assuming that all forward price

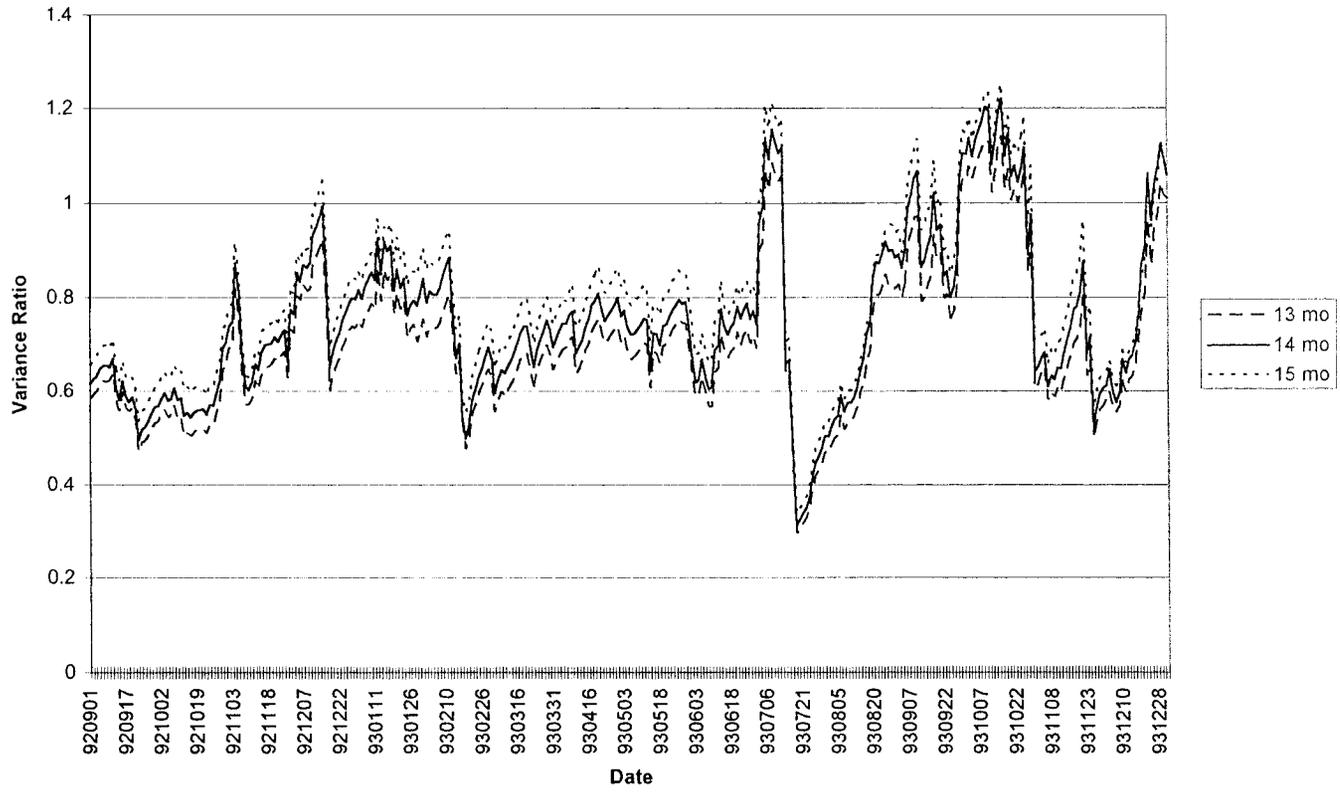


FIGURE 4
Ratio of hedged variance to unhedged variance. (Gulf War period excluded from estimation)

changes are perfectly correlated, one can construct an *upward-biased* measure of the variance of the unhedged position:

$$V_U = \sum_{i=1}^{120} \text{var}(e^{-ri\delta} \Delta F_{t,t+i\delta}) + 2 \sum_{i=1}^{120} \sum_{j \neq 1}^{120} [\text{var}(e^{-rj\delta} \Delta F_{t,t+j\delta}) \text{var}(e^{-ri\delta} \Delta F_{t,t+i\delta})]^{0.5},$$

where δ is 1 month (i.e., 1/12th year). This estimate is biased upward because correlations between different forward prices are in fact less than 1. Also, assume that $\text{var}(\Delta F_{t,t+i\delta}) = \text{var}(\Delta F_{t,t+15\delta})$ for $j > 15$. This contributes additional upward bias because stationarity causes variances to decline as j increases.

Under the same conservative assumptions, a *downward-biased* measure of the difference between the variance of MG's total position (including long futures and short forwards) and V_U equals

$$V_H = (120)^2 \text{var}(\Delta S_t) - (2)(120) \sum_{i=1}^{120} e^{-ri\delta} \text{cov}(\Delta S_t, \Delta F_{t,t+i\delta}),$$

where for $i \geq 15$, $\text{cov}(\Delta S_t, \Delta F_{t,t+i\delta}) = \text{var}(\Delta S_t) \beta_{t,t+15\delta}$. This creates a downward-biased measure of the variance difference because it assumes that covariances between spot price changes and forward price changes for more than 15 months to delivery do not decline with time to expiration as theory suggests.

A downward-biased estimate of the ratio of the hedged position variance to the unhedged variance equals $1 + (V_H/V_U)$. *Despite the downward bias of this measure (which may be extreme), the ratio exceeds 1 for all but 8 days in 1993. Indeed, at times this ratio is in excess of 2.5; on October 12, 1993, the variance of the hedged position was at least 160% larger than the variance of the unhedged position. On average, during 1993 the variance of the hedged position was at least 60% greater than the variance of the unhedged position.* Ironically, the variance ratio rose precipitously around the same time as MG dramatically increased its position in September, 1993. In effect, the firm was increasing the size of an increasingly risky position. Thus, the BAG hedge ratio estimates provide extremely strong evidence that the MG strategy increased, rather than reduced, the firm's risk.

In sum, the data provide no support for a barrel-for-barrel hedging strategy as a prudent means to synthesize a distant-deferred forward position. The most favorable hedge ratio estimates (from the post-Gulf War

BAG model) imply that the barrel-for-barrel strategy was at least 2–4 times riskier than the variance-minimized position. Moreover, extremely conservative estimates imply that the barrel-for-barrel strategy substantially increased the riskiness of MG's position for virtually all of 1993. A severely downwards-biased estimate implies that MG's hedged position was almost always riskier-and sometimes substantially so-than its position in the delivery contracts alone. *Thus, all of the evidence strongly demonstrates that rather than serving to protect the firm against oil price movements, MG's futures trades actually increased the risk for the firm.*⁸

It should also be recognized that in addition to forcing MG to bear more variance than necessary, the barrel-for-barrel strategy also resulted in substantial kurtosis. The GARCH models all demonstrate that the distribution of oil returns is very fat tailed. The point estimates of Θ in these models fall around 0.2. Because this parameter estimates the inverse of the number of degrees of freedom of the joint distribution of spot and futures returns, this implies that crude oil returns follow a t distribution with only 5 degrees of freedom. Thus, the excessive spot crude oil futures position (excessive relative to the variance-minimizing position) also imposed substantially more kurtosis on the firm than was necessary. Risk-averse parties with consistent preferences dislike both variance and kurtosis (Ingersoll, 1987). Therefore, the barrel-for-barrel hedge was even more costly for the firm than the excess variance alone would imply.

It is important to emphasize that the riskiness of the strategy is not primarily attributable to stacking all positions on the nearby contract. The maximum variance reduction at any t equals 1 minus the squared correlation between spot and forward returns at that t . Setting the squared correlation at t equal to $\sigma_{S,F,t}^2/h_{S,t}h_{F,t}$ (using the daily projected values from the BAG and GARCH models) demonstrates that a stacked hedge with a variance-minimizing hedge ratio would have reduced variance for 13–15-month forward positions by between 70 and 80% throughout 1993. Although including deferred futures contracts and longer-term swaps in the hedge could have reduced risk further, it is certainly possible that the additional transactions costs attributable to the lower liquidity of these contracts would have outweighed the benefits of the additional risk re-

⁸It should also be noted that the hedge ratios estimated herein for maturities less than 15 months are almost certainly conservative estimates of the hedge ratios for maturities extending from 16 months to 10 years. First, holding variances and covariances constant, tailing the hedge causes hedge ratios to fall with time to maturity. Second, in the 10–15-month maturity range, both the correlation between the spot and deferred futures and the ratio between the deferred futures variance and the spot variance decline as maturity increases. This is consistent with the stationarity of oil prices and the fact that more distant contracts are progressively poorer substitutes for spot oil. If this trend continues as maturities are extended beyond 15 months, this would also induce a fall in hedge ratios.

duction.⁹ Thus, it was not the stacking *per se* that presented problems. Instead, it was the overhedging of the stack that grossly inflated the risk of MG's position.

The main effect of MG's futures strategy was to transform the nature of the risk it faced. Without futures, MG was vulnerable to a rise in the level of oil prices. With a futures position that was larger than the variance-minimizing position stacked on the nearby contract, MG was vulnerable to a steepening of the term structure of crude oil prices. The firm's position hedged against some risks (a parallel shift in the term structure), but raised its exposure to others (a steepening of this structure). Thus, the strategy embedded both speculative and hedging components: it speculated on the basis between nearby and deferred oil prices, while hedging against spot oil price changes.

Even a cursory visual analysis of the basis between nearby and deferred prices illustrates the potential dangers of this strategy. Figure 5 plots the difference (i.e., basis) between the spot and 15-month crude futures prices for the March 1989–June 1994 period. Note that the basis is quite volatile. Moreover, with the Gulf War period excluded (the huge spike in the basis resulting from the war goes off the graph, which makes it impossible to evaluate basis variability as in more normal periods), it is clear that substantial basis risk was inherent in a barrel-for-barrel strategy. The graph shows that the spot price fell relative to the 15-month futures price in mid-1989, late 1989 to mid-1990, late 1991, and late 1992 by amounts approximately equal to or larger than the amount by which the basis fell during the period MG's strategy was in place. Thus, the behavior of the spot–15-month basis illustrates the pitfalls inherent in a barrel-for-barrel strategy.

Culp and Miller (1995a, 1995b, 1995c, 1995d) characterize MG's strategy in virtually identical terms, but they apparently fail to appreciate just how risky this basis speculation was.¹⁰ Although CM recognize the

⁹Mello and Parsons (1995) suggest matching the maturity dates of the hedge instrument to the dates of the delivery obligations. It should be noted that the R^2 for the entire position is not linearly related to the R^2 s of the individual contracts, because hedging errors for different maturities are correlated. This correlation is almost certainly positive for all combinations of maturities. Moreover, to calculate the percentage variance reduction, it is also necessary to know the variance of the unhedged position in all 120 forward contracts. Determination of the correlation structure of hedging errors and the variance of the unhedged position requires knowledge of the entire 121×121 variance–covariance matrix of spot and forward price changes. This is computationally impossible in the GARCH model. It is possible to calculate an *upward-biased* measure of percentage variance reduction by making same assumptions about the covariance structure as when calculating the variance ratio in Section III. C. During 1993, this measure ranged between 0.65 and 0.98, with a mean of 0.87.

¹⁰To provide some perspective on the size of the speculative component, the hedge ratio estimates imply that roughly 55% of the 160 million bbl futures and swap position was speculative. The resulting 88 million bbl figure is 88 times the speculative position limit for crude oil.

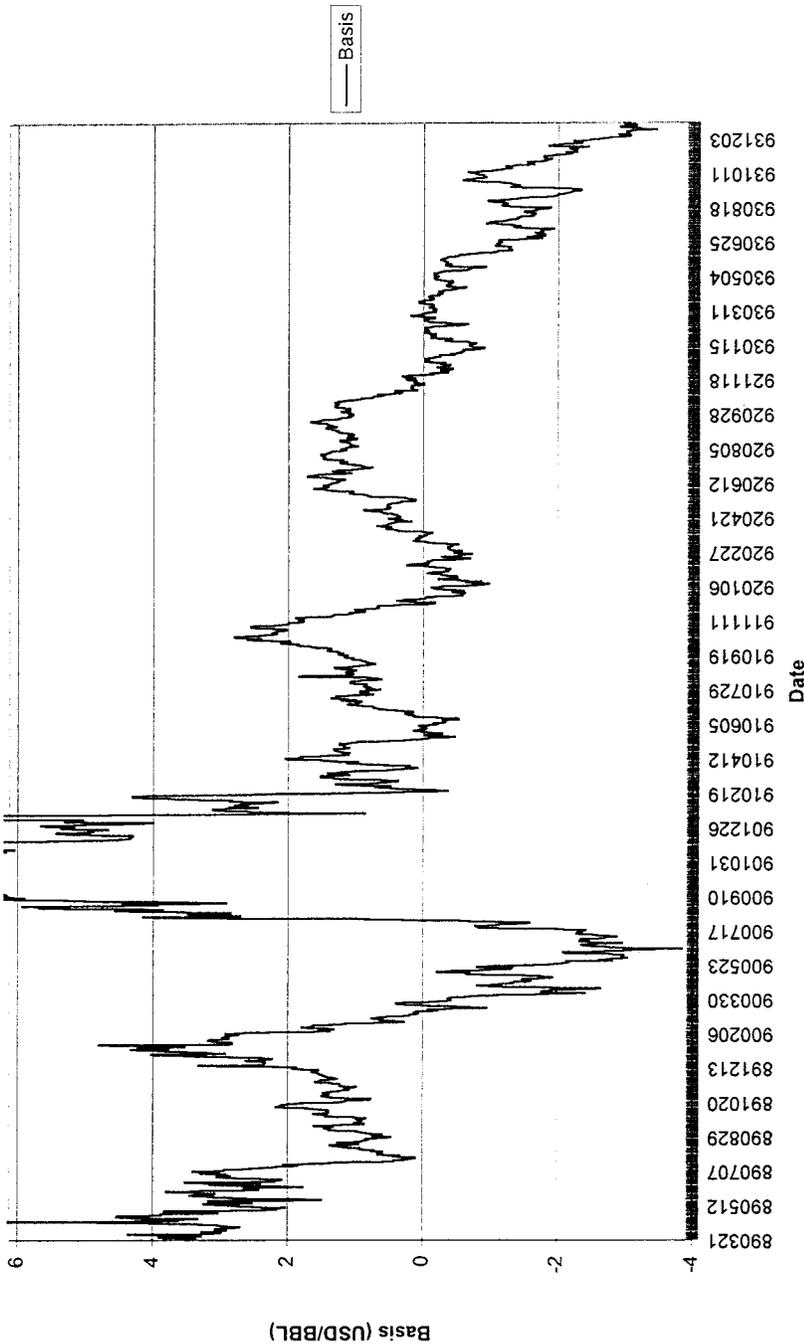


FIGURE 5
Spot-15-month CL basis.

possibility for term structure shifts (which they refer to as “covariance risk”) they do not quantify the risks these shifts actually create for a firm with a barrel-for-barrel stacked hedge. They claim (1995d) that the basis risk inherent in MG’s strategy was so small that it exposed MGRM to no real threat of bankruptcy whereas naked spot price exposure may well have. They justify this assertion by noting that the correlation between spot and nearby futures prices is high. This is not the relevant correlation, however. Instead, the correlations between the nearby futures price and deferred forward prices determine basis risk. All of the empirical results contained herein demonstrate conclusively that this correlation is small enough to make basis risk considerable. *Indeed, the evidence implies that this basis risk was substantially greater than the risk of the short forward contracts alone!*

Overhedging also exacerbated the pressures on MG’s liquidity. Whereas its cash market contracts did not impose substantial demands on MG’s cash flows, its futures contracts were marked to market daily. As a result, the firm needed cash to finance margin calls as the nearby futures price fell in late 1993. It was the inability to finance these margin flows that forced the firm to seek assistance from its bankers. The liquidity strains resulting from overhedging could have impaired the efficient operation of the firm. In the presence of information asymmetries, a fall in liquidity can force a firm to forgo positive NPV projects (Froot, Scharfstein, and Stein, 1994). Therefore, overhedging was undesirable not only because it exacerbated MG’s solvency risk; the liquidity risk inherent in overhedging made it even less desirable for the firm to use a barrel-for-barrel strategy. Put differently, whereas the objective function in (11) and the hedge ratios explicitly consider only solvency, expanding the analysis to include liquidity considerations strengthens the conclusion that barrel-for-barrel hedging was inappropriate.

The barrel-for-barrel strategy was undesirable even if one accepts CM’s claim that Deutsche Bank and other MG creditors mistook a liquidity crisis for a solvency crisis, and thus intervened unwisely by forcing the firm to scale back its oil market activities. Barrel-for-barrel hedging increased the likelihood of such a mistaken intervention because it increased MG’s liquidity needs. Thus, regardless of whether one examines liquidity or solvency considerations, barrel-for-barrel hedging was ill-advised.

The results presented in this section provide compelling evidence that MG’s strategy was highly speculative. It is the most reliable evidence pertaining to this question presented to date. Mello and Parsons (1995) calculate hedge ratios based on the model estimates of Gibson and

Schwartz (1991). Their evidence is somewhat suspect because (as admitted by Gibson and Schwartz) the estimates imply an implausibly high risk premium in oil futures prices. It is also somewhat dated, as the Gibson–Schwartz sample period ends 4 years prior to the beginning of MG's involvement in the oil market. Edwards and Canter (1995) use a simple regression analysis to calculate hedge ratios. This methodology does not take into account the stochastic nature of volatility and covariances in the energy market. Moreover, it does not take into account how backwardation affects variance-minimizing hedge ratios. Thus, the results presented here are based on a more flexible and complete analysis of oil price dynamics than utilized in previous studies of MG.

Unless one is willing to argue that MG's managers possessed appreciable information advantages regarding future basis movements, it is difficult to conclude that the barrel-for-barrel strategy was prudent. An analysis of the *ex post* performance of the hedge casts considerable doubt upon the prescience of MG's managers. The next section addresses this issue in detail.

V. THE MAGNITUDE OF LOSSES ATTRIBUTABLE TO BARREL-FOR-BARREL HEDGING

To determine the gains/losses attributable to barrel-for-barrel hedging, the payoffs to this strategy are estimated with the use of some assumptions about the size of the cash and futures positions, and the behavior of the term structure for maturities greater than 15 months. The parameters necessary to calculate gains/losses, namely, the size and maturity of the cash position, are set equal to the public descriptions of MG's activities. However, because the exact details of MG's strategy are not known, the simulation results are merely illustrative. It is important to remember that the exact details of MG's cash market, futures market, or swap positions at each relevant date are not known. Moreover, these simulations do not take into account the option features of the MG cash market contracts. Given these caveats, the simulation results do suggest that a barrel-for-barrel strategy on a 10-year, 160-mm bbl position could have led to economic losses of upwards of \$800,000,000 over the January 2, 1993–January 3, 1994 period.

Simulated profits/losses are calculated as follows. It is assumed that as of 1/2/93 MG was obligated to deliver $107 \text{ mm bbl}/120 = 893,333 = Q_1^*$ bbl of crude oil each month from 1/2/93 to 9/1/2002. To reflect the increase in MG's contractual obligations in September, 1993, it is as-

sumed that, as of 9/15/93 MG, was obligated to deliver $(160 \text{ mm} - 7.14 \text{ mm bbl})/120 = 1,273,777 = Q_2^*$ bbl of crude oil in each of the next 120 months; the reduction of 7.14 mm bbl reflects deliveries from February to September, 1993. It is assumed that MG held one barrel in the nearby futures contract for each barrel in cash contract delivery commitments. That is, as of 1/2/93, it is assumed that MG was long 107 mm bbl of the February contract, and on 9/15/93 MG they were long 152,853,333 mm bbl of the October contract. On the first business day of each month, this position is reduced by Q_i^* barrels to reflect the expiration of a delivery commitment, where $i = 1$ before 9/15/93, and $i = 2$ afterwards.

On each business day, the gain/loss on the nearby futures position is determined by multiplying the change in the nearby price by the size of the nearby futures position. Moreover, on the first business day of each month, the gains on the expiring delivery commitment are calculated as follows. The per-barrel gain/loss on the first business day is set equal to -1 times the difference between the price of the nearby contract and the price of that month's contract as of 1/2/93. For example, on February 1, 1993, the per barrel gain/loss is set equal to -1 times the difference between the March 1993 futures price on that date and the March, 1993 futures price as of 1/2/93. This difference is then multiplied by Q_i^* .

For each business day in the estimation period, the gains/losses on the long nearby futures position and any expiring delivery commitment are added to determine a daily gain/loss. In addition, interest on the cumulative gain/loss carried over from the previous business day is calculated with the use of the 3-month T-bill rate. Given that MG's financing cost was larger than the T-bill rate, this is a conservative assumption. For each business day t in the sample period, the gain/loss on the nearby position, the gain/loss on any expiring delivery commitment, and the net interest expense at t are then added to the cumulative gain/loss carried over from $t - 1$ to determine the cumulative gain/loss at t .

This process is repeated daily until 1/3/94. On that day, the unexpired delivery commitments are valued as follows. It is assumed that as of 1/2/93 the forward prices for delivery commitments for all months from March 1994 and beyond are equal to the April 1994 futures price on 1/2/93. That is, as of 1/2/93, the term structure of crude oil prices beyond 15 months is assumed flat. Call this price, $F_{1/2/93,15}$. Similarly, as of 9/15/93 the term structure beyond 15 months is flat, with a price, $F_{9/15/93,15}$. For each month, the difference is calculated between the price of the futures contract expiring closest to (but after) March 1994 and $0.67F_{1/2/93,15} + 0.33F_{9/15/93,15}$. The averaging reflects that MG entered into one-third of its contracts in September, 1993. This difference is then

discounted back to 1/3/94 by the appropriate Treasury rate and multiplied by Q_2^* . For example, for the July 1, 1994 delivery commitment, the difference is calculated between the August 1994 futures price and $0.67F_{1/2/93,15} + 0.33F_{9/15/93,15}$, this difference is discounted back to 1/3/94 with the use of the 6-month T-bill rate, and this discounted price difference is multiplied by Q_2^* .

To value as of 1/3/94 the forward commitments dated after May 1995, it is assumed that the forward prices for these delivery dates equal the June 1995 futures price. That is, it is assumed that the term structure of crude oil prices for delivery more than 15 months hence is flat as of 1/3/94. Call this price, $F_{1/3/94,15}$. The change in the forward price on these delivery commitments over the 1/2/93–1/3/94 period is set equal to $0.67F_{1/2/93,15} + 0.33F_{9/15/93,15} - F_{1/3/94,15}$. For each delivery date this difference is discounted back to 1/3/94 by the appropriate Treasury rate and multiplied by Q_2^* . For instance, the price difference for the 1/2/96 delivery commitment is discounted with the use of the yield on the 2-year T-note.

The marked-to-market values of these forward commitments on 1/3/94 are added to the cumulative gain/loss on the nearby futures and the gains/losses on the 2/1/93–12/1/93 delivery commitments to calculate the cumulative gain/loss on the entire MG position over the 1/2/93–1/3/94 period. Because all outstanding forward commitments are marked to market, the resulting total is an estimate of the economic gain/loss of a barrel-for-barrel strategy.

This methodology implies that the losses on a barrel-for-barrel strategy over the 1/2/93–1/3/94 period were equal to approximately \$800,000,000. This is due to a loss of \$1,090,000,000 on crude oil futures and expired delivery commitments and a gain of about \$290,000,000 on unexpired delivery commitments. If MG had used variance-minimizing hedge ratios rather than a barrel-for-barrel futures position throughout the period, their losses would have fallen by almost 78%, to only \$181,000,000. To derive this estimate, variance-minimizing hedge ratios are based on the post-War BAG estimates. Ratios for maturities in excess of 15 months are set equal to the 15-month hedge ratio multiplied by the relevant tailing factor. For example, on each day the 24-month hedge ratio is set equal to the 15-month hedge ratio multiplied by the discount factor relevant between month 15 and month 24. Because this almost certainly leads to upward-biased estimates of variance-minimizing hedge ratios for delivery commitments 16 months and more into the future, the loss estimate of \$181,000,000 is biased upwards as well.



FIGURE 6
MG's 1993 losses.

Figure 6 illustrates how these losses grew over the course of 1993. Prices actually moved in MG's favor in the first five months of 1993, and the firm profited accordingly. In June and in subsequent months, however, the term structure steepened appreciably; MG's ballooning losses during this period reflect these price movements.

It is interesting to note that these losses are comparable to those presented in an audit by the German accounting firms C&L Treuarbeit Deutsche Revision and Wollert-Elmendorff Industrie Treuhand. Based on an analysis of MGRM's accounting data, these firms report a gross loss on the futures and forward positions of \$1.277 billion, and a gain of \$245 million on the unexpired delivery commitments, for a net loss of \$1.06 billion. (The residual is attributable to losses on firm-flexible contracts not considered in this study.)

It should also be noted that if liquidity shortages imposed costs upon the firm, or made mistaken intervention by creditors more likely (as posited by CM) the marked-to-market losses do not reveal the full scope of MG's problems. The roughly \$1.1 billion loss on the futures position (net of gains on expired delivery commitments) represents the immediate demand on MG's cash flows. If the firm had used a variance-minimizing hedge ratio (that is, if it minimized solvency risk), its cash outflows under the hedging program would have equaled only \$.471 billion—about 57% less than the loss actually realized. Thus, if liquidity strains reduce firm value, the \$800 million marked-to-market loss understates the costs of barrel-for-barrel hedging because the firm also incurred costs attributable to the extra liquidity drains attributable to overhedging.

The large losses quantified here underscore the speculative nature of MG's strategy, and cast doubt upon the prescience of its managers. Although even the most well informed market participants lose money at times, the magnitude of the losses realized in 1993 strongly suggests that MG's management did not possess so acute an ability to forecast basis movements to justify the immense risks inherent in their strategy.

VI. THE EFFECTS OF EMBEDDED OPTIONS ON HEDGE RATIOS

The preceding analysis calculates hedge ratios under the assumption that MG's forward contracts did not embed any options. Recall, however, that the firm-fixed supply contracts did permit the buyers to terminate their contracts and, upon said termination, receive one-half of the difference between the prevailing spot price (measured by the nearby futures price) and the fixed price established in the contract times the volume remaining

under the contract. Formally, if a customer were to exercise this option at time t , he or she would receive $0.5Q(S_t - f)$, where f is the fixed price under the contract, Q is the volume remaining under the contract, and (as before) S_t is the spot price. Upon exercise, the customer would terminate his right to receive refined products at the fixed price for the remaining life of the contract.

CM claim that this feature of the MG contracts made it even more desirable to use nearby futures contracts to hedge the energy price risk inherent in the supply contracts. They state that "MGRM could liquidate an equivalent amount of futures positions to cover the required cash outlay. Because both the hedge and the early exercise option relied upon the front-month futures contract, the price in MGRM's hedge was the same as that governing early termination options. MGRM therefore faced no covariance risk . . . from the risk of early exercise."

In reality, the effect of the embedded option is much more complicated; it could have either exacerbated or mitigated the overhedging problem. In essence, it is necessary to account for the fact that oil price changes also affect the value of the forward contracts that customers would forfeit when exercising the option. When this factor is taken into account, the values of the early-out option and the position necessary to hedge it both depend upon the entire term structure of oil prices.

From MG's perspective, the firm-fixed contracts were equivalent to a portfolio consisting of a receive fixed-pay floating energy swap and a short position in a call option on this swap. There are N delivery months remaining on the swap, and MG must deliver Q/N units of petroleum products on each delivery date. Call Z_t the value of the swap to MG at time t . That is,

$$Z_t = \sum_{i=1}^N e^{-ri\delta} \frac{Q}{N} (f - F_{t,t+i\delta}).$$

As before, $F_{t,t+i\delta}$ gives the forward price for delivery at $t + i\delta$ as of t , and δ equals 1 month (i.e., 1/12 of a year). The value of the swap to MG's customers (i.e., its counterparties) is $-Z_t$.

If the customer exercises the option embedded in the MG contracts, he or she receives

$$0.5(S_t - f)Q - (-Z_t) = 0.5(S_t - f)Q + Z_t \equiv A_t.$$

The $-(-Z_t)$ term enters the expression because, upon exercising the option, the customer gives up his swap position, which is worth $-Z_t$ to him.

Define $q(A_t, T, t)$, the value of the option to terminate the contract and receive a payment of $0.5(S_t - f)Q$ as a function of A_t , the ending date on the contract, and the current date. This option is a call on the portfolio A_t , with a strike price of 0. Then, the value of MG's position at time t is

$$\Pi = Z_t - q(A_t, T, t). \quad (6)$$

To determine how many nearby futures contracts to purchase to hedge this obligation, first recognize that

$$\begin{aligned} \frac{\partial \Pi}{\partial F_{t,t+i\delta}} &= \frac{\partial Z_t}{\partial F_{t,t+i\delta}} \left[1 - \frac{\partial q(A_t, T, t)}{\partial A_t} \right] \\ &= -e^{-ri\delta} \frac{Q}{N} \left[1 - \frac{\partial q(A_t, T, t)}{\partial A_t} \right] \equiv \Delta_F(t, t + i\delta) > -\frac{Q}{N}. \end{aligned}$$

The inequality follows because the option increases in value as A_t increases (i.e., $\partial q/\partial A_t > 0$). Moreover,

$$\frac{\partial \Pi}{\partial S_t} = -\frac{\partial q(A_t, T, t)}{\partial A_t} \frac{\partial A_t}{\partial S_t} = -0.5Q \frac{\partial q(A_t, T, t)}{\partial A_t} \equiv \Delta_S < 0.$$

If only the nearby contract is used to hedge, the total number of nearby contracts to buy to hedge the entire swap and embedded option is

$$\begin{aligned} H_T &\equiv -\sum_{i=1}^N [e^{ri\delta} \Delta_F(t, t + i\delta) \beta_{t,t+i\delta}] - \Delta_S \\ &= \sum_{i=1}^N \frac{Q}{N} \left[\beta_{t,t+i\delta} \left(1 - \frac{\partial q}{\partial A_t} \right) + 0.5 \frac{\partial q}{\partial A_t} \right]. \end{aligned}$$

This may be either larger or smaller than the total number of nearby contracts required to hedge the swap alone, depending on whether the average hedge ratio (absent the option) is less than or greater than 0.5. Recall that a position is variance increasing if it is more than twice as large as the total variance-minimizing hedge. The barrel-for-barrel strategy is thus risk increasing in the absence of the embedded option if

$$\sum_{i=1}^N \frac{Q}{N} \beta_{t,t+i\delta} < 0.5Q$$

If this expression holds, rewriting H_T implies

$$\begin{aligned}
 H_T &= \left(1 - \frac{\partial q}{\partial A_t}\right) \sum_{i=1}^N \frac{Q}{N} \beta_{t,t+i\delta} + \frac{\partial q}{\partial A_t} 0.5Q < \left(1 - \frac{\partial q}{\partial A_t}\right) 0.5Q \\
 &+ \frac{\partial q}{\partial A_t} 0.5Q = 0.5Q.
 \end{aligned}$$

Therefore, if the barrel-for-barrel position increases variance in the absence of the embedded option, it increases the variance if the contract includes the option as well. The option feature mitigates the overhedging somewhat, but not enough to turn the barrel-for-barrel strategy into a true hedge.

In sum, the analysis of this section demonstrates that the options embedded explicitly in MG's firm-fixed contracts cannot reverse, and may strengthen, the conclusions drawn in the previous sections. Because the empirical results presented earlier demonstrate that buying nearby contracts barrel-for-barrel resulted in substantial overhedging in the absence of these options, the option analysis strengthens the conclusion that MG's strategy almost certainly increased the variance of the firm's payoffs.

The options embedded in the firm-flexible contracts are more difficult to analyze than those in the firm-fixed deals. The 15-month hedge ratios estimated earlier are likely to provide an upper bound on the hedge ratios for delivery commitments 2 years and beyond even in the presence of this option, however. The drop in oil prices (combined with the mean reversion in oil prices) during the life of the program gave buyers a strong incentive to defer, rather than accelerate, deliveries. Because more distant deliveries require smaller hedge ratios, this bias towards deferral suggests that the no-option hedge ratios overestimate the with-option hedge ratios for firm-flexible contracts as well.

VII. ROLLOVER PROFITS AND THE PRUDENCE OF THE BARREL-FOR-BARREL HEDGE

It has been argued that MG's policy allowed the firm to profit from the backwardation typical in energy markets by rolling over its futures at a profit. That is, when the market is in backwardation, at the expiration date of each contract the firm could expect to sell the expiring future at a price that exceeded that at which it purchased the next-to-expire contract. Arthur Benson, the main architect of MG's strategy, apparently relied upon such reasoning (Benson Affidavit, 1994). Edwards and Canter (1995, p. 224) state that "it does not seem unreasonable for MGRM to have expected that over a long period of time (such as ten years) its

hedging strategy would have produced a net rollover gain.” Edwards and Canter, however, recognize that there were appreciable risks in such a strategy.

An analysis of this argument reveals that the expected gain from rolling over nearby futures for K periods in a market that is in backwardation is equal to the current difference between the spot price of oil and the K -period forward price. If each successive nearby futures price is expected to exceed the next expiring futures price for K consecutive months, the sum of these differences equals the difference between the current spot price and the K -month futures price. As a result, in a driftless futures market, the expected cost of oil incurred in a rollover strategy ending in month K equals the current forward price of oil for delivery in month K . This strategy is riskier than the variance-minimized replication of the K -month forward contract, however, so it is dominated by that strategy.

These points are readily grasped by expressing the firm’s cost of acquiring oil to satisfy its contractual obligation (denoted by C) to deliver oil in K months as follows:

$$C = S_K - \sum_{i=1}^K [F_{i,i} - F_{i-1,i}] = S_K + F_{0,1} - F_{K,K} + \sum_{i=1}^{K-1} [F_{i,i+1} - F_{i,i}].$$

Here $F_{i,j}$ is the forward price of oil in month i for delivery in month j , and S_K is the spot price of oil in month K . For simplicity, this expression assumes that the interest rate equals 0, which simplifies the notation but has no effect on the results. In this expression, the total cost equals the spot price of oil in month K — S_K —minus the total realized rollover gains on futures contracts. The summation term is the total rollover gain, where in month i the rollover gain is defined as the deferred price minus the expiring price, $F_{i,i+1} - F_{i,i}$. With driftless futures prices, this expression implies that $E_0(C) = E_0(S_K) = F_{0,K}$. Also note that the convergence of spot and futures implies $S_K = F_{K,K}$. Therefore,

$$E_0(C) = E_0 \left\{ F_{0,1} - \sum_{i=1}^{K-1} [F_{i,i+1} - F_{i,i}] \right\} = F_{0,K}.$$

Because $F_{0,1}$ equals the price of acquiring oil 1 month after the initiation of the strategy, this expression states that the 1 month forward price net of the rollover gains expected over K month equals the K -period forward

price of oil. In essence, the expected rollover gains reduce the expected cost of acquiring oil for delivery in K months below the current spot price of oil. But in a market in backwardation, the K -month forward price is also below the current spot price by the same amount. That is, the expected total rollover gain equals the amount of backwardation over K months. The barrel-for-barrel rollover strategy is riskier than the variance-minimizing replication of the K -period forward price, however. Thus, there were less-risky ways for MG to exploit the backwardation in the market than a barrel-for-barrel rollover every month.

VIII. SUMMARY AND CONCLUSIONS

A thorough analysis of the behavior of oil prices demonstrates clearly that MGRM's strategy of purchasing one barrel of spot crude oil to hedge the sale of crude oil months into the future was almost certainly risk increasing, rather than risk reducing. The reasons for this are clear. First, the stationarity of oil prices implies that volatilities decline systematically with time to expiration. Second, the correlation between spot and deferred prices is imperfect, and this correlation also declines systematically as time to expiration of the deferred increases. A variance-minimizing hedger should reduce hedge ratios far below 1 in response to these factors: MG did not.

Empirical estimates provide extremely compelling evidence that, due to this overhedging, MG's position of long futures and short forwards was substantially riskier than its short position in forward contracts alone. Therefore, this strategy subjected the firm to the risk of real economic loss, not just accounting loss. The firm was vulnerable to a steepening of the oil price term structure, an event that occurred soon after it implemented its strategy. A simulation of the economic losses a firm employing such a strategy would have incurred produces figures that are comparable to the magnitude of the losses publicly recognized by MG. Thus, the data provide compelling evidence that MG's strategy imposed substantial risk upon the firm *ex ante*, and that the *ex post* losses were substantial.

This is not to say that all firms should employ variance-minimizing hedges when trading derivatives: informed speculation is a common part of any risk-management strategy. The relevant question is whether MG possessed the information advantage required to justify its immense speculative position. There is substantial reason to doubt that any firm, let alone a relative newcomer to the energy markets like MG, has a large enough informational advantage to justify the immense risks of what was arguably the largest time spread ever undertaken in commodity markets.

The losses incurred in the last half of 1993 certainly cast significant doubt upon the firm's ability to predict the movements of oil prices. Given the huge losses incurred in late 1993, a Bayesian estimating the probability distribution of MG's information advantage would almost certainly place little weight on the possibility that the firm was well informed, and great weight on the possibility that it did not possess superior information, regardless of the charitability of his priors concerning the prescience of MG's managers.

APPENDIX

If the futures prices are martingales, then $E_{t_0} [\Pi_T - E_{t_0} \Pi_T]^2 = E_{t_0} \Pi_T^2$. Consider the determination of the first hedge ratio ($\beta_{t_0+\Delta t, T}$) in a dynamic hedge that accounts for possible dependencies between hedge ratios at any time t and hedge ratios at subsequent times $t' > t$. The relevant first-order condition is

$$\begin{aligned} \frac{dE_{t_0} \Pi_T^2}{d\beta_{t_0+\Delta t}} = 0 &= 2E_{t_0} \{ \beta_{t_0+\Delta t} \Delta S_{t_0+\Delta t}^2 - \Delta S_{t_0+\Delta t} \Delta v_{t_0+\Delta t, T} \\ &+ \Delta S_{t_0+\Delta t} \sum_{i=2}^M [\beta_{t_0+i\Delta t, T} \Delta S_{t_0+i\Delta t} - \Delta v_{t_0+i\Delta t, T}] \}. \end{aligned}$$

The first two terms in this expression are present in a single-period variance-minimizing hedge ratio. The product of the first period spot price change and the sum of gains and losses in subsequent periods reflects the possible intertemporal dependencies among hedge ratios. Consider a representative term:

$$E_{t_0} [\Delta S_{t_0+\Delta t} \beta_{t_0+i\Delta t, T} \Delta S_{t_0+i\Delta t}].$$

The hedge ratio at $t_0 + i\Delta t$ may depend upon previous realizations of ΔS_t and $\Delta v_{t, T}$. However, by the law of iterated expectations:

$$\begin{aligned} E_{t_0} [\Delta S_{t_0+\Delta t} \beta_{t_0+i\Delta t, T} \Delta S_{t_0+i\Delta t}] \\ = E_{t_0} \{ \Delta S_{t_0+\Delta t} \beta_{t_0+i\Delta t, T} [E_{t_0+(i-1)\Delta t} (\Delta S_{t_0+i\Delta t})] \}, \end{aligned}$$

where the inner expectation is conditional on all price changes up to $t_0 + (i-1)\Delta t$. Because S_t is a martingale by assumption, this inner expectation is 0, the entire expression equals 0. Therefore, this expression disappears from the first-order condition, as do all other terms included in the summation. Consequently, the hedge ratio produced as the solution to the dynamic programming problem collapses to the single-period hedge

ratio. It is possible to demonstrate that this result obtains for $t > t_0$ as well.

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