

**Energy Derivatives**  
**Professor Craig Pirrong**  
**Homework 3**

1. The December Natural Gas futures price is \$4.75/MMBtu. The annualized volatility ( $\sigma$ ) for the natural gas is .7. The continuously compounded, annualized risk free interest rate is .035.
  - a. Construct a binomial tree of possible futures values in 4 weeks assuming one week time intervals (i.e.,  $\Delta t = 1/52$ ). (Hint: what is  $u$ ? What is  $d$ ?)
  - b. What is the value of  $p$  for this futures contract?
  - c. Determine the value of a European call option on December NG that expires in 4 weeks that has a strike price of \$4.75. What is the value of an American call option on this contract? What is the delta of the American call? How many December futures do I need to hold to hedge a long position in the call? Is my futures position for hedging long or short? How many contracts do I need to replicate a *short* position in the call? Is my futures position for replicating long or short?
  - d. Determine the value of a European put option on the December NG that expires in 4 weeks that is struck at \$4.75. What is the delta of the put? How many contracts do I need to hold to hedge a long position in the put? Is my hedging position long or short? How many contracts do I need to hold to replicate a long position in the put? Is my replicating position long or short?
  - e. Determine the value of an American put option on December NG futures that expires in 4 weeks that is struck at \$4.75.
  - f. Are the American values noticeably different from the Europeans? If so, why? If not, why not?
2. Answer 6a-6f under assuming the December NG volatility is only .1 and the interest rate is .25. Briefly discuss why the option values are so different under these different volatility and interest rate assumptions. Comment particularly on the effect of these changes on the differential between American and European option prices.