The Price of Power

Craig Pirrong
Martin Jermakyan

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- The deregulation of the electricity industry has resulted in the development of a market for electricity.

- Electricity derivatives, including forwards, "vanilla" options and various exotic options have been introduced and are currently traded both OTC and on exchange.

- Pricing electricity derivatives faces acute difficulties.

Challenges: The Uniqueness of Power

- Power is non-storable. Moreover, production capacity for power is limited. These factors conspire to create unique price dynamics including mean reversion with huge price "spikes" especially during peak demand periods.

- These features of power prices are not readily captured by standard "reduced form" models that are the basis for most derivatives pricing models.

- Price diffusion-type models clearly inappropriate.

- Traditional price jump models also clearly inadequate. Models with only positive jumps and mean reversion cannot capture the fact that jumps are very short-lived. Need model that has: (1) up jumps precede down jumps; (2) the time between up jumps and down jumps is very short; (3) high correlation between the magnitude of up jump and subsequent down jump; (4) seasonal jump intensity; (5) jumps correlated with loads. Even if you can write down such a model, it is very hard to calibrate it and use it to price power contingent claims.

- More advanced jump-based models (e.g., Geman-Roncoroni) still have problematic features, and are not readily fit to the data.

- Traditional approaches (e.g., lognormal price diffusion) implicitly assume storability.

- All of these are incomplete market models, and hence must change measure to price contingent claims. What is the right measure? Complicated calibration problem given limited data.

A Better Way

- An equilibrium, fundamentals-driven approach is the best way to address these issues. In this approach, the spot price of power at any instant is a function of two state variables, a demand variable and a cost variable. The demand variable is load or temperature. The cost variable is the price of fuel.

- A two factor approach is implemented first because of tractibility. We are aware that higher-dimension approaches may prove fruitful. For example, loads in multiple regions or modelling at the generating unit level are conceptually superior. "Curse of dimensionality" requires prudent choice of state variables. Improvements in computational power and experience will facilitate implementation of richer models.

The Intuition Behind this Approach

- The basic idea is that the non-linearities in prices result because a linear demand process is filtered through a non-linear cost function. Price rises rapidly as demand nears capacity.

- This approach exploits the transparency of fundamentals in the power market. You can't use this approach in financial markets because you don't know or can't measure the fundamentals.

Demand and Fuel Processes

- The load and fuel variables follow diffusion processes. In contrast to price, load is a well-behaved process.

- Load is a "controlled process." The system operator can intervene to control load to ensure that it does not exceed system generating and transmission capacity.

- Violations of system constraints can impose extreme costs. These include failure of the entire grid with blackouts as a result.

- Results of Harrison and Taksar (1983) imply that if system controller can "push" on load hard enough, load will exhibit reflection at system capacity. That is, if system capacity is X, the load process has a reflecting barrier at X.

The Reflecting Barrier Characterization

- The conditions assumed by H-T are not literally applicable to the problem of controlling power systems. Nonetheless, the reflecting barrier approach is a tractible method for incorporating the physical constraints in power systems into the analysis.

- Increasing the realism of the characterization of load dynamics requires solution of a complex stochastic control problem that takes the real constraints of the power system into account. These constraints include ramping constraints, transmission constraints, and so on.

- The basic framework for solving such control problems (Pontryagin approach) is well developed. It is our goal to pursue implementation of this approach in power.

- Thus, for now the reflecting barrier characterization should be viewed as a preliminary attempt to incorporate physical constraints into the analysis of power pricing problems.

Load and Fuel SDEs

- The load reverts to a time varying mean. Mean load peaks during summer months, with a smaller peak in the winter. The load process is:

  $$dq_t = \alpha_q(q_t, t)q_t dt + \sigma_q q_t du_t - dL_t^u$$

  where

  $$\alpha_q(q_t, t) = \mu(t) + k[\theta_q(t) - \ln q_t]$$

  du_t is a standard Brownian motion, and dL_t^u is the "local time" of the process. $\mu(t)$ is the deterministic drift in the load at t. Note that $dL_t^u > 0$ iff $q_t = X$, and $dL_t^u = 0$ if $q_t < X$.

- The fuel futures price follows a diffusion process:

  $$\frac{df_t}{f_t} = \alpha_f dt + \sigma_f dz_t$$

The Market Price of Risk

- Since load (or temperature) is not traded, valuation formula must have a market price of risk. In fact, any power derivatives pricing formula must have a market price of risk because of the non-storability of power.

- Thus, if the "true" probability measure is $\mathcal{P}$, we need to find a new measure $\mathcal{Q}$ under which deflated prices of contingent claims are martingales.

- The true probability measure $\mathcal{P}$ and the new measure $\mathcal{Q}$ must share sets of measure zero. That is, if an event cannot occur under $\mathcal{P}$ it cannot occur under $\mathcal{Q}$.

- Since $q_t > X$ is impossible under $\mathcal{P}$, it must also be impossible under $\mathcal{Q}$. Thus, q_t must reflect at X under $\mathcal{Q}$.

- Under $\mathcal{Q}$, q_t solves the SDE:

  $$dq_t = [\alpha_q(q_t, t)dt - \sigma_q \lambda(q_t, t)]q_t + \sigma_q q_t du_t - dL_t^u$$

- In this expression, $\lambda(q_t, t)$ is the market price of load risk.

- Under $\mathcal{Q}$, f_t solves:

  $$\frac{df_t}{f_t} = \sigma_f dz_t$$

The Fundamental Valuation Equation

- Define the discount factor $Y_t = \exp(-\int_0^t r_s ds)$ where r_s is the (assumed deterministic) interest rate at time s.

- Under $\mathcal{Q}$, the evolution of a deflated power price contingent claim C is:

$$Y_t C_t = Y_0 C_0 + \int_0^t C_s dY_s + \int_0^t Y_s dC_s$$

- Using Ito's lemma, this can be rewritten as:

$$
\begin{aligned}
Y_t C_t &= C_0 + \int_0^t Y_s(\mathcal{A}C + \frac{\partial C}{\partial s} - r_s C_s)ds \\
&\quad + \int_0^t [\frac{\partial C}{\partial q} du_s + \frac{\partial C}{\partial f} dz_s] - \int_0^t Y_s \frac{\partial C}{\partial q} dL_s^u
\end{aligned}
$$

where $\mathcal{A}$ is an operator such that:

$$
\begin{aligned}
\mathcal{A}C &= \frac{\partial C}{\partial q_t}[\alpha_q(q_t, t) - \sigma_q \lambda(q_t, t)] q_t \\
&\quad + .5 \frac{\partial^2 C}{\partial q_t^2} \sigma_q^2 q_t^2 + .5 \frac{\partial^2 C}{\partial f_t^2} \sigma_f^2 f_t^2 \\
&\quad + \frac{\partial^2 C}{\partial q_t \partial f_t} \sigma_f \sigma_q \rho_{qf} q_t f_t .
\end{aligned}
$$

- For the deflated price of the power contingent claim to be a martingale, it must be the case that:

$$E[\int_0^t Y_s(\mathcal{A}C + \frac{\partial C}{\partial s} - r_s C_s)ds] = 0$$

and

$$E[\int_0^t Y_s \frac{\partial C}{\partial q} dL_s^u] = 0$$

for all t.

- Since (1) $Y_t > 0$, and (2) $dL_t^u > 0$ only when $q_t = X$, with a constant interest rate r, we can rewrite these conditions as:

$$\mathcal{A}C + \frac{\partial C}{\partial t} - rC = 0 \qquad (1)$$

and

$$\frac{\partial C}{\partial q} = 0 \text{ when } q_t = X \qquad (2)$$

- Expressions (1) and (2) are necessary and sufficient to ensure that C is a martingale under $\mathcal{Q}$.

- Expression (1) can be rewritten as the fundamental valuation PDE:

$$
\begin{aligned}
rC &= \frac{\partial C}{\partial t} + \frac{\partial C}{\partial q_t}[\alpha_q(q_t,t) - \sigma_q \lambda(q_t,t)]q_t \\
&+ .5\frac{\partial^2 C}{\partial q_t^2}\sigma_q^2 q_t^2 + .5\frac{\partial^2 C}{\partial f_t^2}\sigma_f^2 f_t^2 \\
&+ \frac{\partial^2 C}{\partial q_t \partial f_t}\sigma_f \sigma_q \rho_{qf} q_t f_t \qquad (3)
\end{aligned}
$$

- For a forward contract, the relevant PDE is:

$$
\begin{aligned}
-\frac{\partial F_{t,T}}{\partial t} &= \frac{\partial F_{t,T}}{\partial q_t}[\alpha_q(q_t,t) - \sigma_q \lambda(q_t,t)]q_t \\
&\quad + .5\frac{\partial^2 F_{t,T}}{\partial q_t^2} q_t^2 \sigma_q^2 + .5\frac{\partial^2 F_{t,T}}{\partial f_t^2}\sigma_f^2 f_t^2 \\
&\quad + \frac{\partial^2 F_{t,T}}{\partial q_t \partial f_t} q_t f_t \sigma_f \sigma_q \rho_{qf} \qquad (4)
\end{aligned}
$$

where $F_{t,T}$ is the price at t for delivery of one unit of power at $T > t$.

- It is sometimes convenient to change the time variable to time to expiration $\tau = T - t$. In this case, the PDE can be rewritten as:

$$
\begin{aligned}
\frac{\partial F(\tau)}{\partial \tau} &= \frac{\partial F(\tau)}{\partial q_t}[\alpha_q(q_t,t) - \sigma_q \lambda(q_t,t)]q_t \\
&\quad + .5\frac{\partial^2 F(\tau)}{\partial q_t^2} q_t^2 \sigma_q^2 + .5\frac{\partial^2 F(\tau)}{\partial f_t^2}\sigma_f^2 f_t^2 \\
&\quad + \frac{\partial^2 F(\tau)}{\partial q_t \partial f_t} q_t f_t \sigma_f \sigma_q \rho_{qf} \qquad (5)
\end{aligned}
$$

Tying the Forward Contract to Market Conditions

- The PDE must be solved subject to certain boundary conditions.

- Equation (2) is a boundary condition of the Neumann type; this is a consequence of the controlled nature of the load process.

- Many ways to "skin the cat." In earlier versions of this paper, we used non-parametric econometric techniques. In this latest version, we use PJM bid data to construct a "bid stack."

- There are several complications that have to be addressed to implement this approach, but it does allow us to capture the economics of generation (including the impact of market power) as revealed by market participant's actions.

Estimating the Load Process

- It is also necessary to estimate a drift function for load in order to implement this approach.

- This proceeds in two steps: (1) estimating the mean load function $\theta_q(t)$ and deterministic drift function $\mu(t)$, and (2) estimating the speed of mean reversion k.

- We estimate the load function non-parametrically. This is very flexible. As the attached figure (from PJM) shows, it captures the salient features (seasonal and intra-day) variations in average load.

- To determine k, we estimate the following regression using ordinary least squares:

$$\frac{\Delta q_t}{q_t} - \mu(t) = \widehat{k}[\theta_q(t) - \ln q_t] + \epsilon_t$$

- We use the standard error from the regression as our estimate of σ_q and the $\widehat{k}$ coefficient from the regression for k (the speed of mean reversion).

- For the PJM data studied, k is quite large indicating that load shocks dissipate very quickly; the half-life of a load shock is approximately 5 hours. This is important as it implies that current load shocks shouldn't affect forward prices much. Some empirical evidence suggests that market overreacts to load shocks.

Estimating the Market Price of Risk

- The fundamental valuation equation is a conventional (parabolic) PDE that can be solved using fairly conventional means *if one knows the market price of risk function.* This is not observed. Can one assume it is equal to zero? If not, how does one estimate it?

- An analysis of data from the Pennsylvania-New Jersey-Maryland (PJM) market illustrates that an assumption of a zero market price of risk is not plausible. The average difference between the one day forward price and the realized spot price one day later is $.92 per MWh. The median difference is $1.36.

- The forward price of power (for day ahead) exceeds the realized spot price far more than would be expected by chance.

The Importance of the Market Price of Risk

- The importance of the market price of risk ("MPR") cannot be overstated. As will become apparent momentarily, it is a major component of power prices.

- Moreover, comparisons of implied market prices of risk across products and across markets can provide valuable trading signals.

- Therefore, any valuation and risk management methodology must take MPR into account.

- It is not a trivial problem to estimate the MPR.

Inverse Techniques and the MPR

- How does one therefore determine the market price of risk? We utilize inverse problem techniques to do so. We find a function $\lambda(q_t, t)$ that matches the forward prices generated by the model to the forward prices quoted in the market.

- An arbitrary number of functions can fit the limited number of forward prices quoted in the market. Therefore, this problem is "ill-posed." We use regularization techniques to address this difficulty.

Some Illustrative Results

- Solution of the inverse problem given forward prices and loads for PJM for June 2005.

- The results of this solution provide striking results. The results imply that the risk premium is substantially positive for delivery at the peak load dates in July and August.

- This premium observed throughout 1998-2006 period.

- These results are sensible. Spot power price for PJM highly right skewed (especially in the summer). Therefore, short seller of power faces possibility of extreme loss due to a spike in demand. Market price of risk/risk premium compensates them for this risk.

- Negative MPR for shoulder months consistent with economic analysis of hedging pressure in power markets (Bessembinder-Lemmon, 2000).

- Other evidence suggests that market price of risk is important. Power prices for distant delivery dates exhibit substantial variation even though stationarity of main price driver–load–suggests that forward prices for delivery dates months into the future should vary little. This could be due to substantial variation in the risk premium as hedging interest varies.

Extensions of the Approach

- It is possible to incorporate outages into the analysis through the boundary conditions using unit-specific information on outage frequencies and durations; simulate the bid stack, and exploit (1) the lack of correlation between outages, load, and fuel price, and (2) the stationarity of outages. At present, assume outage risk is unpriced.

- The model also allows integrated risk measurement and management.

- The solution to the PDEs generates load and fuel "Greeks" (Δ, Γ, and Θ) for each instrument in a portfolio.

- Since many market participants face quantity as well as price risk, the explicit incorporation of load into the model facilitates integrated risk measurement and management, as well as the pricing of load sensitive claims.

- Given load and fuel correlation structures across markets, can construct an integrated, multi-market risk measurement.

- This model can be extended to price options on power. These include highly path dependent options such as “swings” which are frequently embedded in power supply contracts.

- Pirrong (2006a and 2006b) values power options using splitting methods for solving PDEs.

- This model can also be used to price and hedge volume sensitive claims. This is important for utilities because their revenues depend on both price and load. It is also useful in valuing generation assets. This is another big issue because of industry restructuring.

- Finally, the model can be adapted to price and hedge power and weather derivatives in a single, unified framework. This involves using some weather variable (e.g., temperature) as the main state variable.

Pricing Power Options in a P-J Framework

Craig Pirrong
University of Houston
Houston, TX 77204
713-743-4466
cpirrong@uh.edu

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Introduction

- The PDE for solving options is the same as for forwards.

- The problem is that the dimensionality reduction is not feasible for futures.

- Need to grasp the nettle and implement a 2D numerical PDE solver.

Doing the Splits

- ADI is not suitable for this problem.

- I've looked at integration-based approaches, but they are clunky.

- Splitting is the way to go.

$$\frac{rC}{3} = \frac{\partial C}{\partial t} + \frac{\partial C}{\partial q_t}[\alpha_q(q_t, t) - \sigma_q \lambda(q_t, t)] + .5\sigma_q^2 \frac{\partial^2 C}{\partial q_t^2} \quad (1)$$

The second split handles the cross derivative term:

$$\frac{rC}{3} = \frac{\partial C}{\partial t} + .5\sigma_f \sigma_q \rho_{qf} f_{t,T} \frac{\partial^2 C}{\partial q_t \partial f_{t,T}} \quad (2)$$

The third PDE split, which handles the purely f-related terms, is:

$$\frac{rC}{3} = \frac{\partial C}{\partial t} + .5\sigma_f^2 f_{t,T}^2 \frac{\partial^2 C}{\partial f_{t,T}^2} \quad (3)$$

Results

- Load mean reversion rules!

- Short maturity daily strikes convex in load; long dated daily strikes, monthly strikes are not.

- This reflects strong mean reversion in load.

- Longer dated daily strikes, monthly strikes like options on fuel.

- Little time decay due to load dynamics until very close to maturity–another reflection of mean reversion.

- Implied vols increase dramatically as one nears expiration–mean reversion strikes again.