# Commodity forward curves: models and data

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# Commodity Forward Curves

- A forward curve is a locus of points relating the forward price to the associated delivery date
- Forward curves can take many shapes
- Upward sloping
- Downward sloping
- Humped

# Elementary Theory: Cash and Carry

- The simple theory of forward curves is that for storable commodities, forward prices should exceed nearby prices plus the cost of carrying inventory (interest, warehousing charges, insurance) to the expiration date
- Holds for virtually all financial forwards
- Seldom holds for commodity forwards: why not?

# Commodity Forward Curves

- Full carry (“carrying charge market”)
- Contango: forward prices are increasing with time to expiry, but do not necessarily cover carrying charges
- Backwardation (“inverted market”): prices fall with time to expiration

# Why the Diversity of Forward Curve Shapes?

- We need an explanation of the diversity of forward curve shapes
- We can draw on the basic insight of our earlier analysis: calendar spreads are price signals that guide the *intertemporal* allocation of commodities
- Intertemporal allocation economics will depend on the type of commodity
- Therefore need to have a basic taxonomy of commodity types

# A Commodity Taxonomy

- The Most Basic Divide
  - Storables
  - Non-storables (or effectively non-storables)
- Costs of storage in fact lie along a continuum, and it is perhaps better to think of things that are very costly to store vs. those that aren't quite so costly

# Non-Storables

- Electricity
  - Very costly to store, although hydro introduces an element of storability into some markets
- Freight
- Weather (duh!)
- Livestock

# A Taxonomy of Storables

- Continuously Produced
  - Exhaustible (e.g., oil)
  - Non-exhaustible (e.g., CU, AL)
- Periodically Produced
  - Grains and oilseeds
  - Tree crops

# Storables Forward Curves

- Again, the forward curve is providing price signals on how to allocate resources over time
- In a well functioning market, forward prices will reflect—and guide—the optimal intertemporal allocation
- So, to understand the forward curve in a well-functioning storables market, you need to understand the economics of intertemporal resource allocation

# Cash-and-Carry Arbitrage

- The simple theory of forward curves is based on the principle of arbitrage.
- "Cash-and-carry" arbitrage is a riskless strategy: buy the underlying, finance the purchase with borrowing sell futures/forwards.

# Cash-and-Carry Arbitrage Cash Flows

| TRANSACTION | Date $t$ | Date $T$ |
|---|---|---|
| Buy spot | $-S_t$ | $S_T$ |
| Borrow | $S_t + s$ | $-e^{r(T-t)}[S_t + s]$ |
| Sell Futures | 0 | $F_{t,T} - S_T$ |
| Pay storage | -$s$ | 0 |
| Net Cash Flows | 0 | $F_{t,T} - e^{r(T-t)}[S_t + s]$ |

# No Arbitrage Pricing Relation

- All of the cash flows at *T* are known as of *t* so this is riskless.
- Costless to initiate the position, so the cash flow a *T* must be 0 to prevent arbitrage.
- This implies:

$$F_{t,T} = e^{r(T-t)}[S_t + s]$$

# Implications

- This theory implies that the futures price is increasing with time to maturity (when interest rates are positive—a sufficient but not necessary condition)
- This is sometimes called "full carry" pricing: futures price covers the cost of carrying inventory.
- Obviously does not explain a lot of forward curves, so isn't the whole story: what is?

# Intertemporal Allocation Basics

- Want to carry goods from where they are abundant to where they are scarce but . . .
- With intertemporal allocation, can only move in one direction: from the present to the future
- Can't move in the other direction: from the future to the present (though that would be nice)
- This constraint is essential in understanding forward curves

# Scenario 1

- Supplies of the commodity are currently abundant because demand is low
- Substantial excess capacity
- Already have a lot of inventory
- Demand is expected to rebound, so commodity is abundant today relative to what we expect in the future
- Under these circumstances, optimal to store commodity for future use

# Forward Curves in Scenario 1

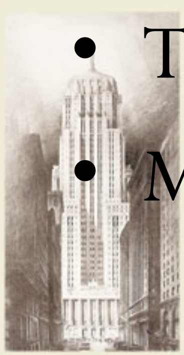
- Forward curve should adjust to provide an incentive to store
- Futures prices at or near full carry provides this incentive

# Examples

- Financial Crisis: Immediate and large transient decline in demand
- Supply increase and demand decline in 2014-2015 oil market
- COVID crisis

# Scenario 2

- Current supplies very tight
- Inventories low, and operating at nearly full capacity
- Commodity scarce today relative to what is expected in the future
- Would like to bring stuff from the future to today, but can't
- The best we can do is NOT to store
- May want to consume all inventories



# Forward Curves in Scenario 2

- Want to punish storage
- If spot price is above forward price net of carrying costs, anticipate losing money by storing: certainly lose money if you hedge inventory
- Therefore, backwardation in time of scarcity gives the right price signals

# Examples

- Polar vortex: Immediate short term demand shock
- Mine closure (e.g., Escondido Mine in Chile): Immediate short term supply shock
- Major refinery outage: Immediate short term supply shock

# Future Shock

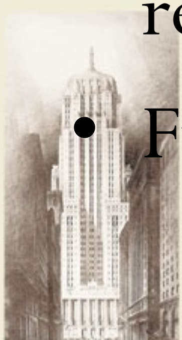
- Most shocks are to current supply and demand, but there can be shocks to anticipated future S&D
- These lead to different inventory, price and forward curve movements
- These are often considered to be anomalous

# Shock to Expected Future Supply

- Future supply expected to decline (e.g., OPEC considering extending output cuts beyond April, 2017)
- Desirable to store more today to mitigate future supply loss: inventories rise
- Spot price rises to reduce current consumption necessary to increase storage
- Futures rises relative to spot to encourage more storage

# Shock to Expected Future Demand

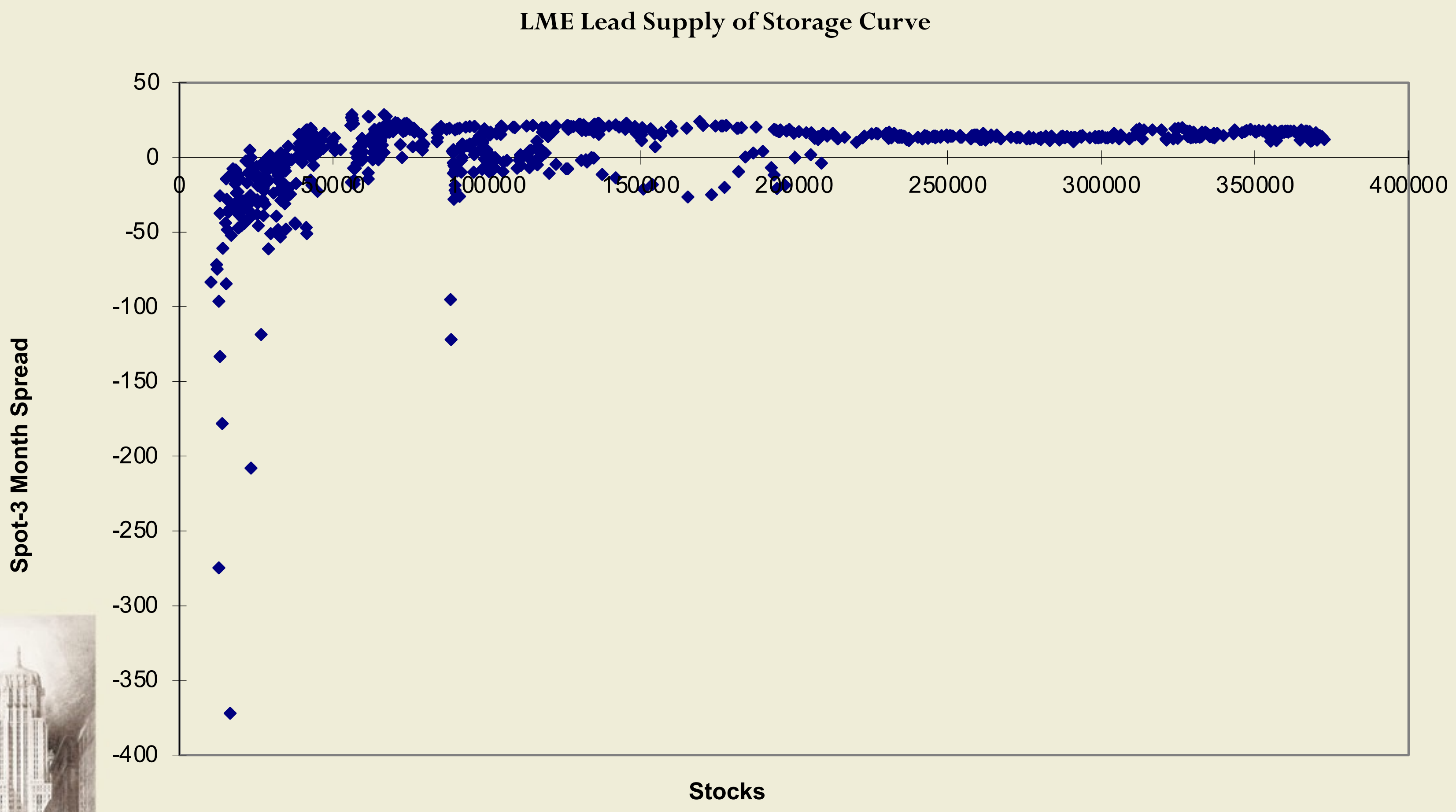
- Increase in expected future demand (e.g., Energy Policy Act of 2005 which mandated increased biofuel consumption in 2006 and beyond)
- Store more today to meet anticipated future demand increase
- Supply fixed in short run, so spot price must rise to reduce consumption to allow more storage
- Futures rise relative to spot to reward storage



# Implication

- Markets should be in full carry when inventory is very high
- Markets should be in backwardation when inventory is low
- “Supply of Storage Curve”

# Supply of Storage Curve (Lead)




# A Refinement

- Market may be in full carry for very short-dated deliveries, but below full carry for more distant dates
- EG, LME metals: one day prices may be at carry, cash-3 month spread at less than full carry
- This occurs when there are positive inventories, but there is some possibility of a stockout in the period prior to the expiry of the forward in question
- Departure from full carry/amount of backwardation depends on likelihood of stockout over period prior to expiry: higher the likelihood, bigger departure from full carry
- In a competitive market, backwardation between spot and 1 day forward occurs only during a stockout

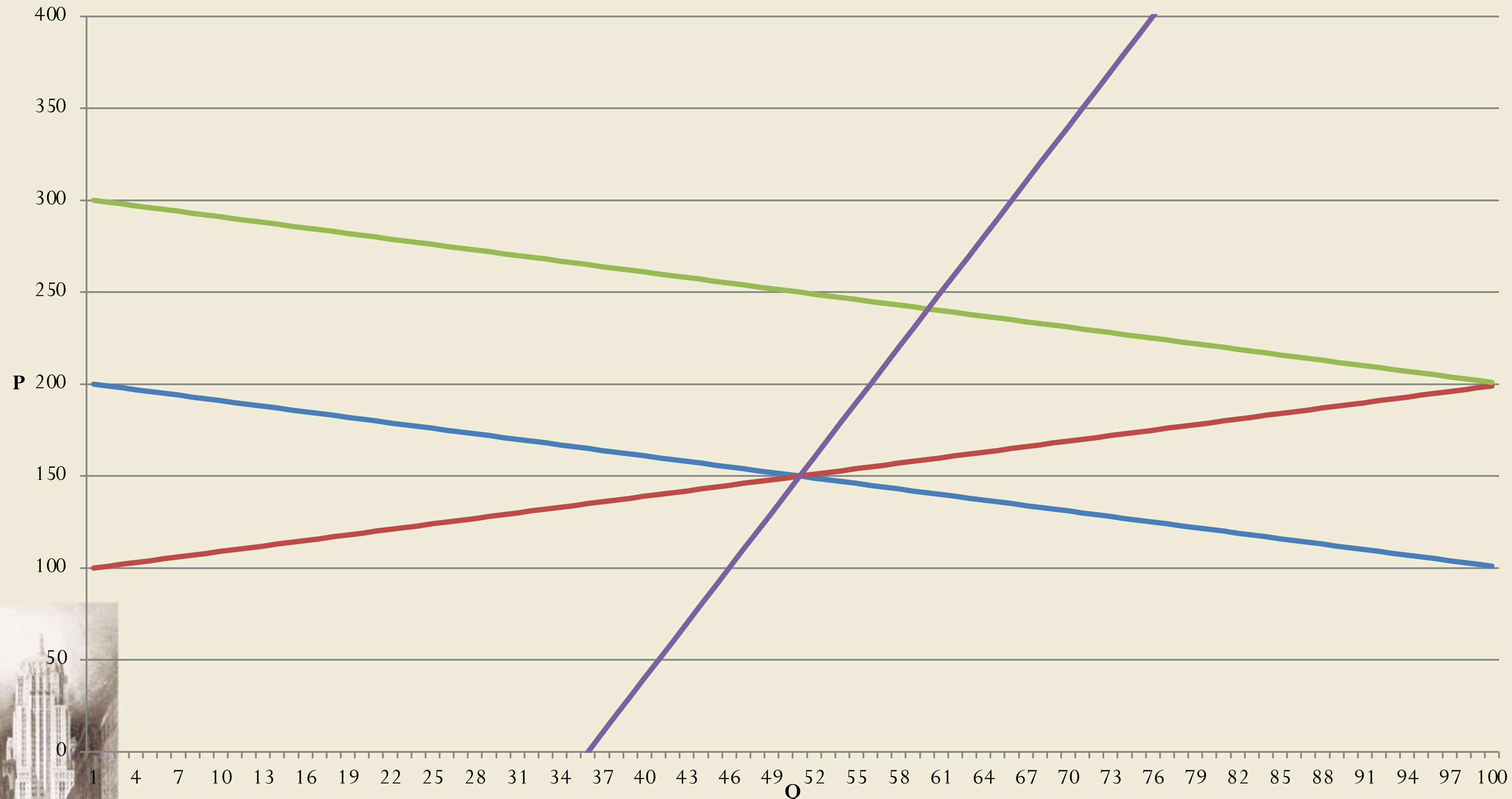
# Additional Implications for Continuously Produced Commodities

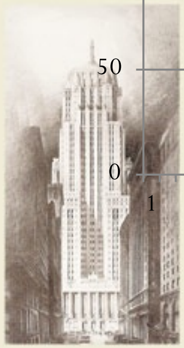
- The foregoing analysis implies that the shape of the forward curve depends on scarcity
- Price volatility and correlations should also depend on scarcity
- Therefore, volatility and correlation should be related to the shape of the forward curve

# Implication: Volatilities

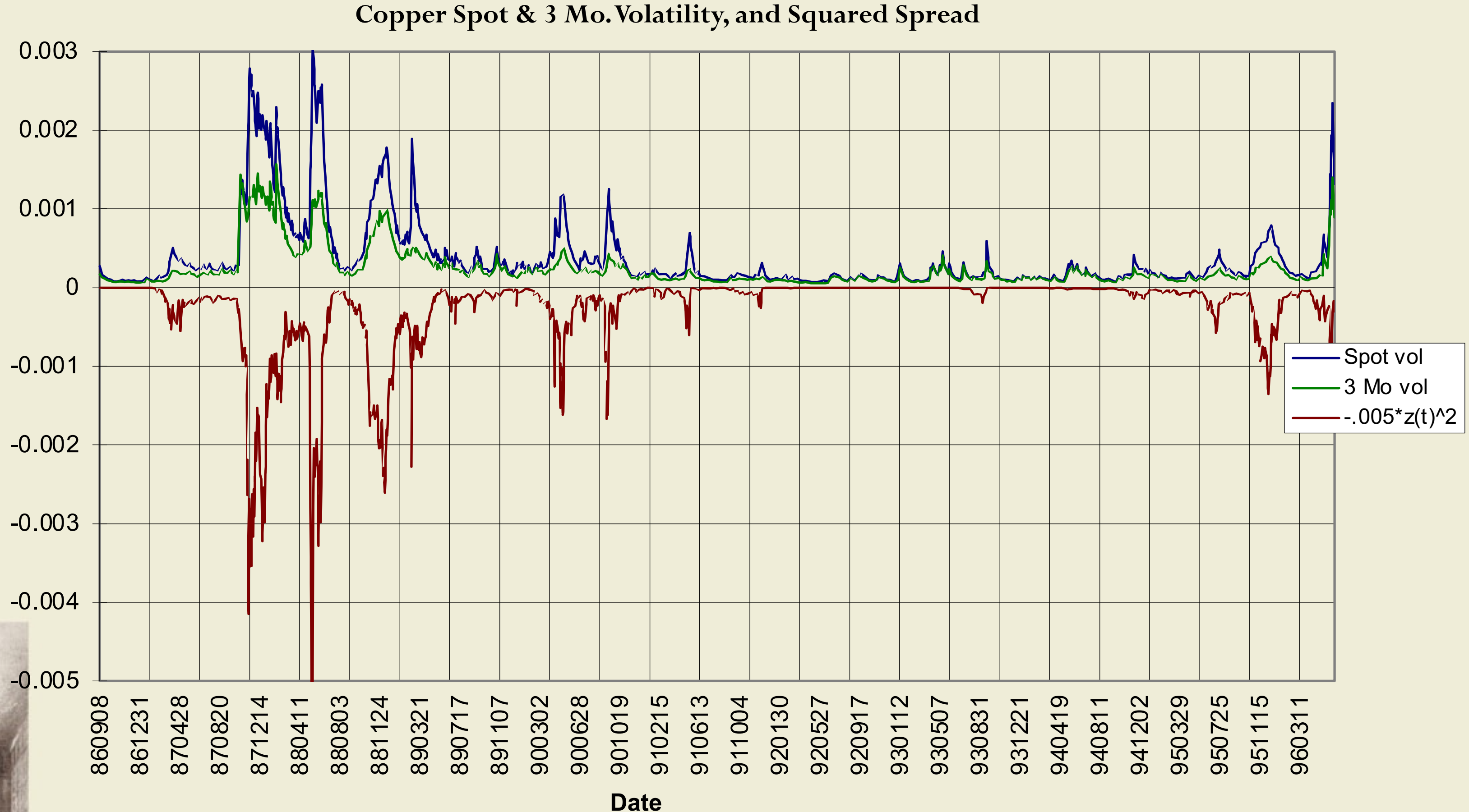
- Volatilities should be higher when the market is in backwardation than in contango
- Spot and forward prices should be equally volatile when the market is at full carry because storage connects the prices
- Spot prices should be more volatile than forward prices when the market is in backwardation because there is more flexibility to adjust quantity over a longer period of time, and this ability reduces the burden placed on price to respond to demand shocks
- Some empirical evidence supports this, but cannot explain all of the dynamics of volatility

# Scarcity, Supply Elasticity & Volatility



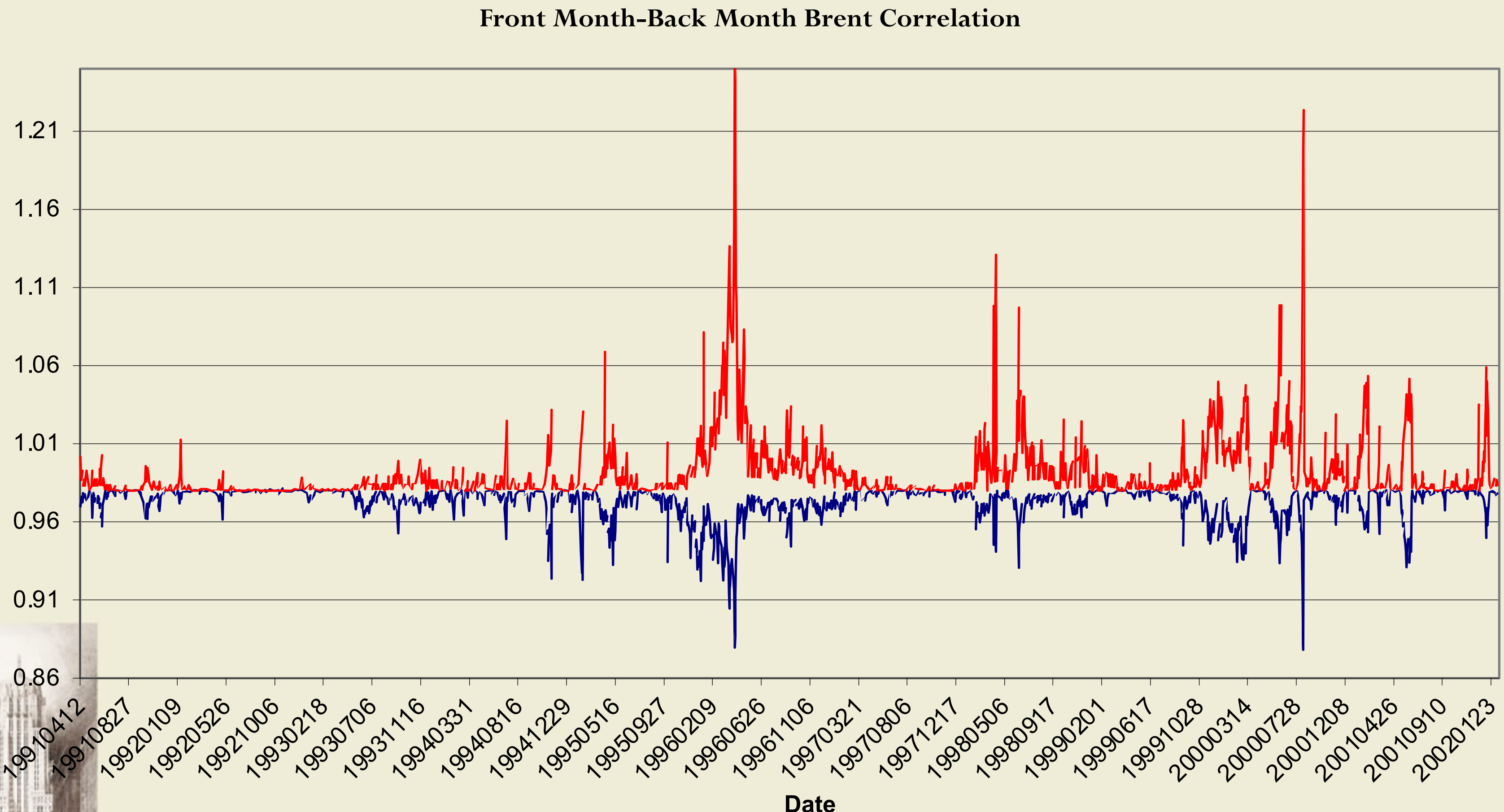
# Variances/Volatilities




# Implication: Correlations

- Correlations between spot and forward prices should be near 1.00 when market is at full carry, because storage connects these prices
- Since backwardation signals the possibility of a stockout that would break the direct link between spot and forward prices, correlations along the curve should be lower in backwardated markets
- Data support this

# Correlations (Brent Crude)




# Implication: Mean Reversion

- "Mean reversion" means that prices tend to rise (fall) when they are below (above) their long run mean values
- Prices tend to mean revert in this model for two reasons
  - Shocks are mean reverting
  - Equilibrium effects: capacity tends to enter when prices are high (thereby driving down prices) and exit when prices are low

# Empirical Analysis: Continuously Produced Commodities

- These theories difficult to test because the interemporal linkage greatly complicates modeling
- I've done some empirical analysis to see how well the model works
- Verdict: mixed

# Basic Idea

- 1 day decision horizon: can choose amount of inventory to hold every day
- Two demand shocks, one persistent (integrated or very close), one more transitory (half-life in days)
- Maximizing agents choose carry-out conditional on demand shocks and carry-in
- 1 day forward prices solves PDE:

$$0 = F_t + \mu_z z_t F_z + \mu_y y_t F_y + .5\sigma_z^2 F_{zz} + .5\sigma_y^2 F_{yy} + \rho_{yz}\sigma_z\sigma_y F_{yz}$$

# Solution Methodology

- Make an initial guess for spot price function at each point in a grid of $z$, $y$, and carry-in $x$. Use this as a boundary condition for the PDE, and solve the PDE
- At each point in the grid, choose carry-out so that spot price equals discounted forward price: if this carry-out is negative (an impossibility) set it to zero. Market can be in daily backwardation in this case
- Wash, rinse, repeat until spot price function converges

# Some Empirical Work

- Pirrong (2010) examines copper market data using a dynamic programming model calibrated to copper market data, and an extended Kalman filter
- Two demand shocks—one has very long persistence, another a short half life
- Can explain some of the dynamics of forward curves and inventory
- Does an OK job of explaining spot price dynamics, medium tenor forward prices, storage; poorer job explaining longer tenor prices, volatilities

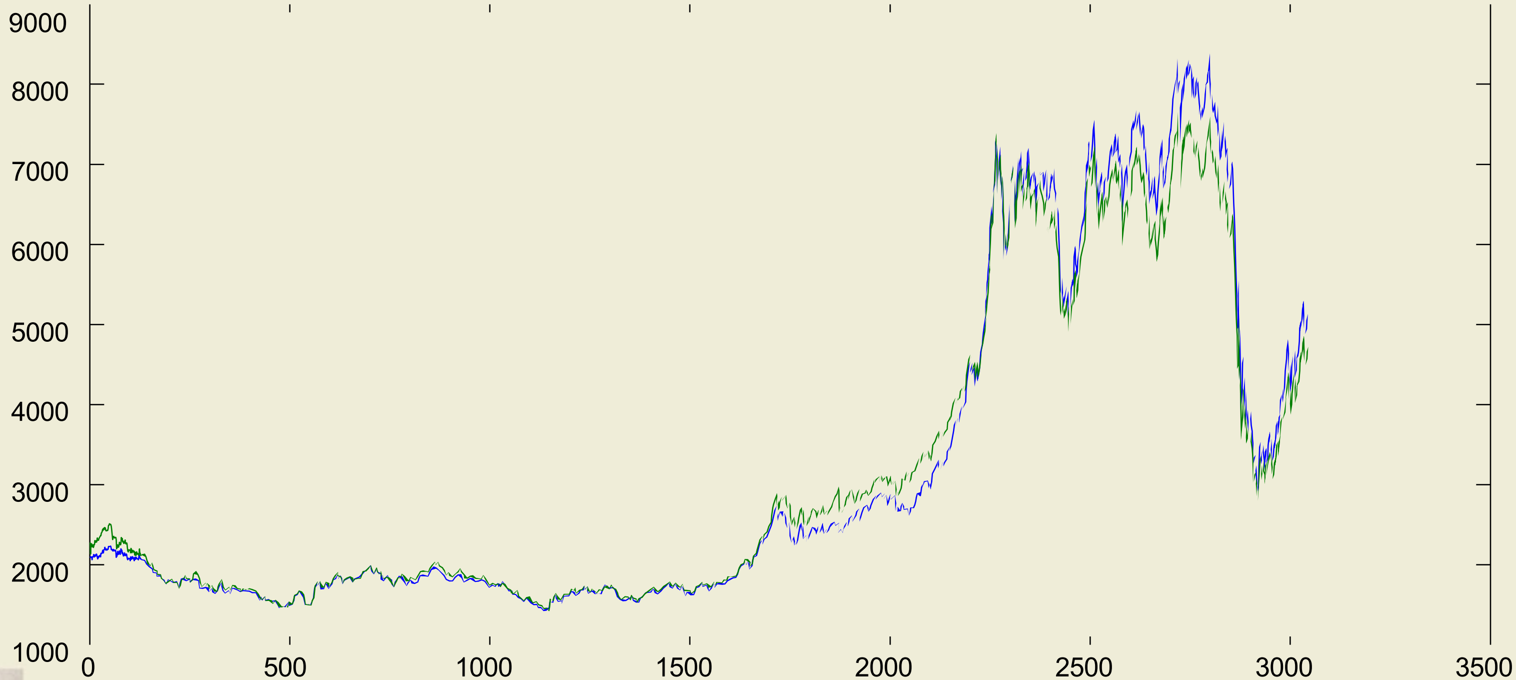
Spot Price

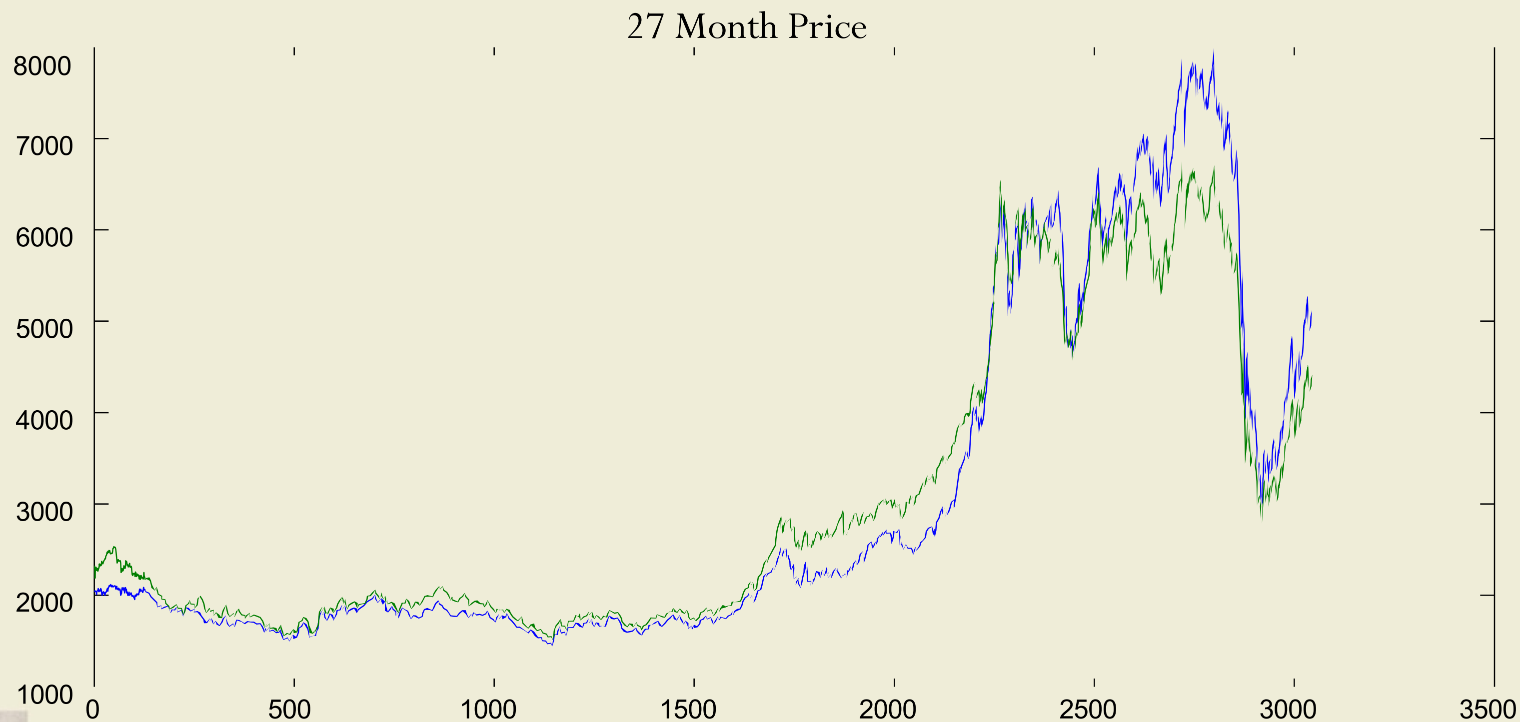
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
500
1000
1500
2000
2500
3000
3500



Three Month Price
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
500
1000
1500
2000
2500
3000
3500

## Fifteen Month Price

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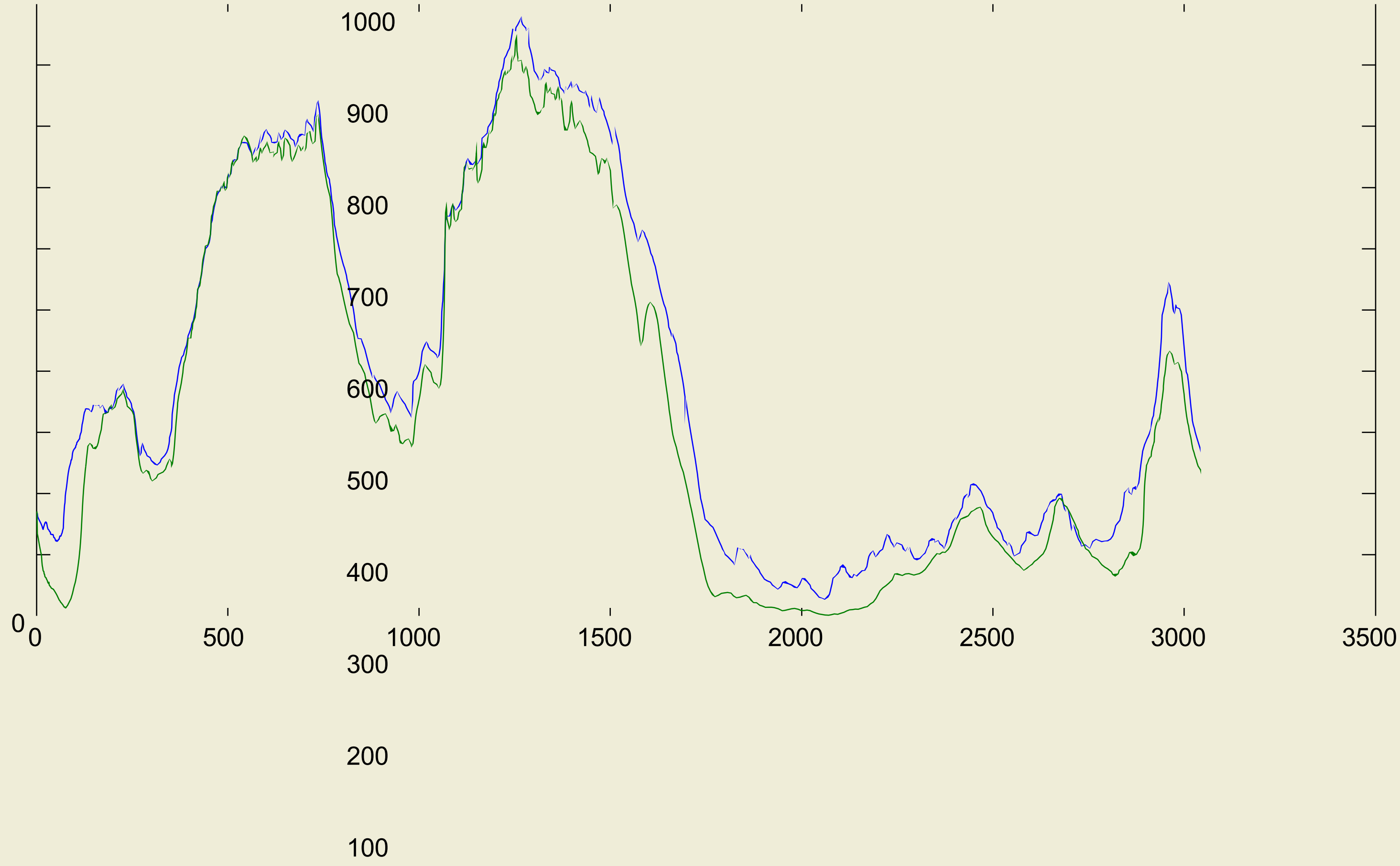
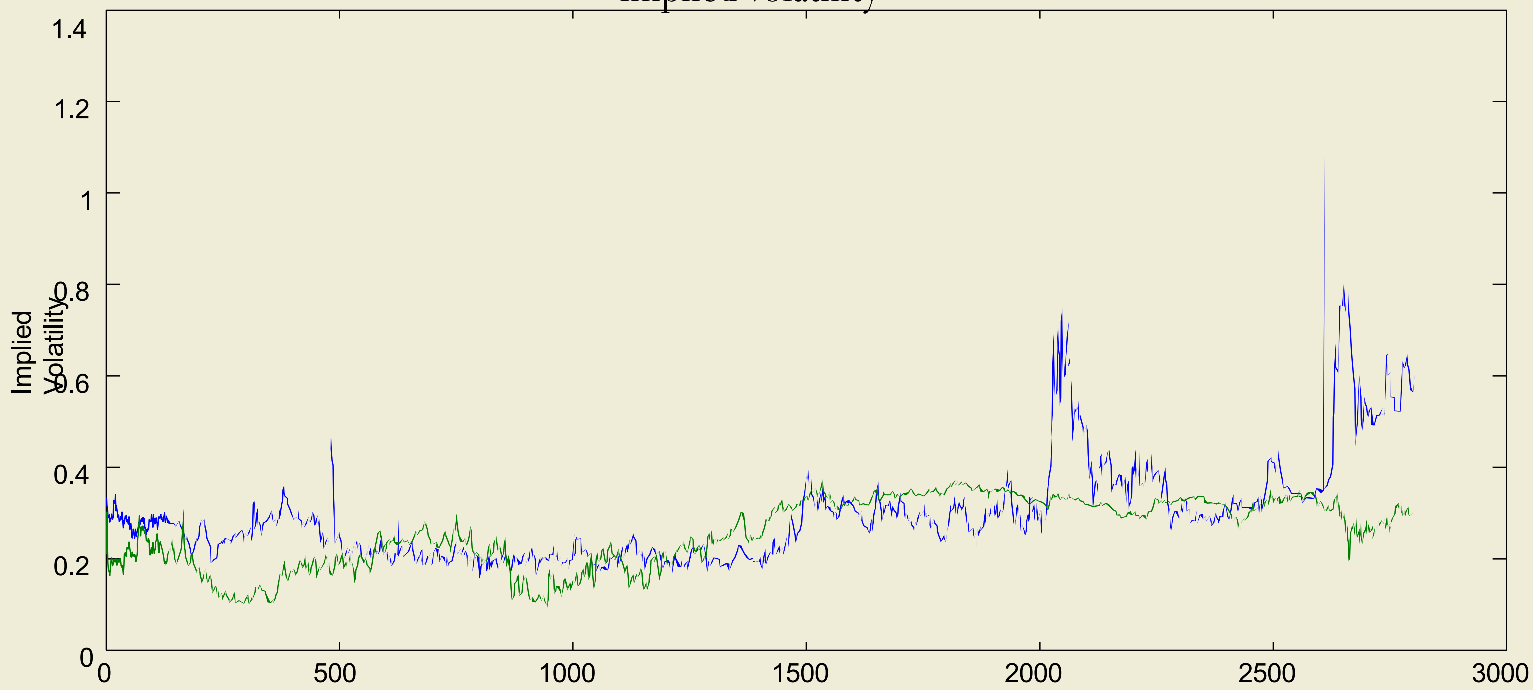
# Inventories

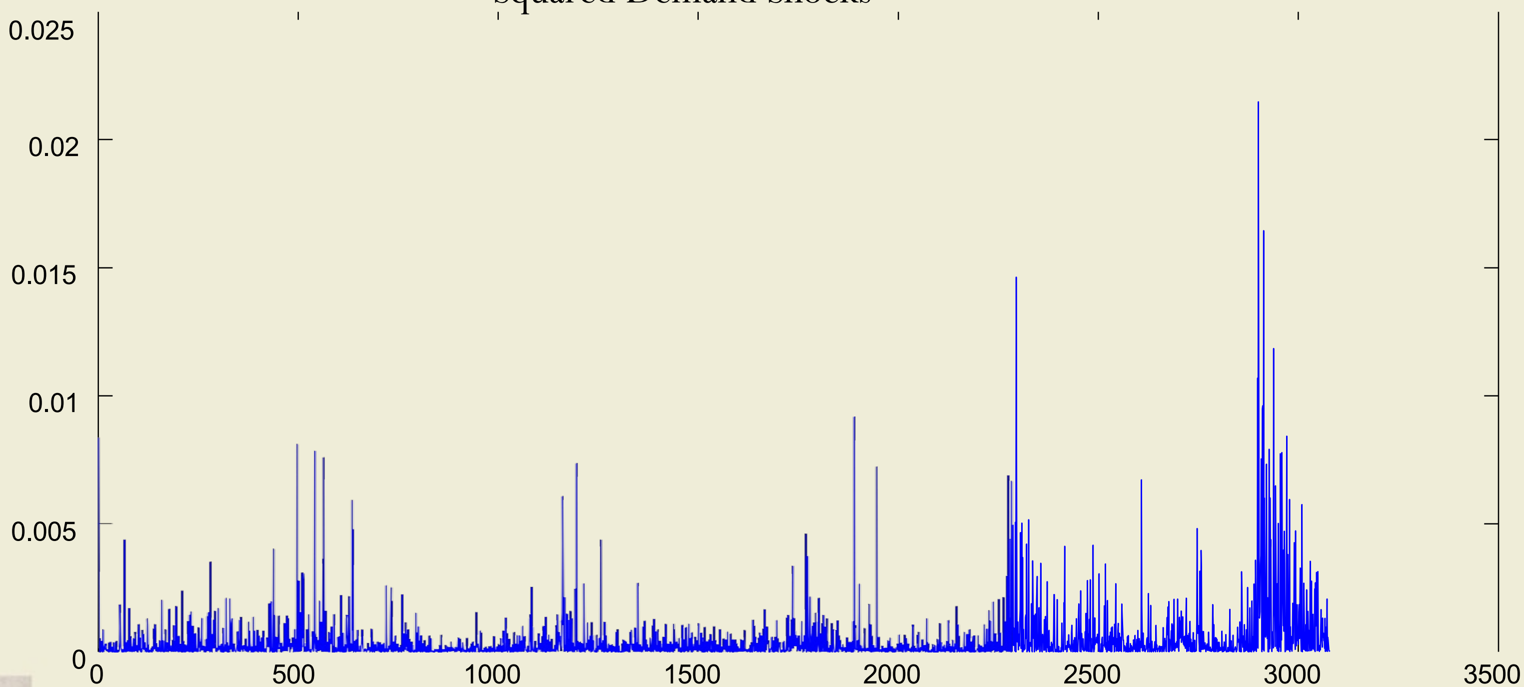



## Long Run Demand Shock


# Implied Volatility




## Squared Demand Shocks



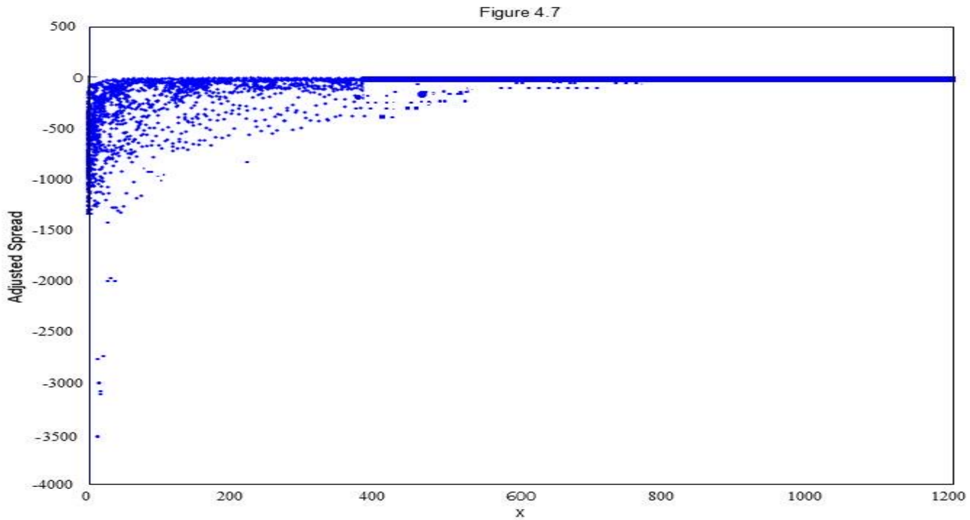



Figure 4.7
Adjusted Spread
500
0
-500
-1000
-1500
-2000
-2500
-3000
-3500
-4000
0
200
400
600
800
1000
1200
x

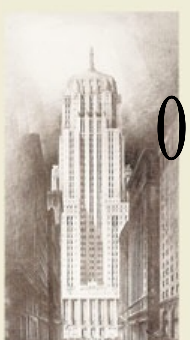
# Extension: Stochastic Fundamental Volatility

- Weakest aspect of this model is inability to explain volatility behavior: although fundamental factors generate time varying volatility, fluctuations in volatility aren't big enough
- Introduce stochastic fundamental volatility
- EG, 1 demand shock $z$, the variance of which is stochastic, such as:

$$dV_t = \kappa(\theta - V_t)dt + \sigma_v V_t^{.5} dB_t$$

# Same Basic Routine

- Forward price solves a PDE. Make an initial guess for spot price as a function of variance, demand shock, and carry-in. Solve PDE. Choose carry-out to equate spot price and expected forward price.
- Only difference is that forward price PDE is now:



$$0 = F_t - \mu z_t F_z + \kappa(\theta - V_t) F_V + .5 V_t F_{zz} + .5 \sigma_v^2 V_t \, F_{VV} + \rho_{zV} \sigma_v \sqrt{V_t} F_{zv}$$

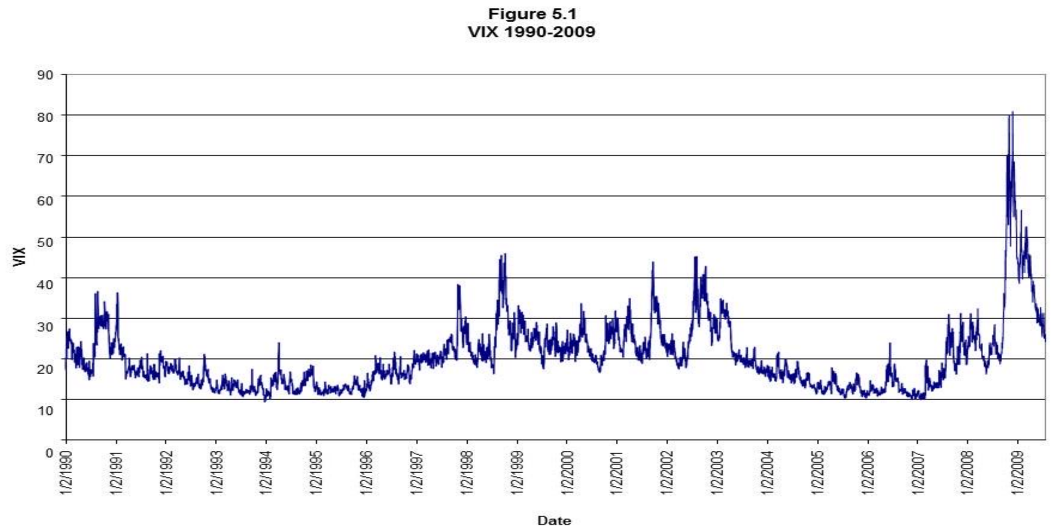
# Benefits of This Model

- This model can generate much more realistic dynamics for implied volatilities
- Plausible that economic fundamentals do exhibit stochastic volatility
- Look at the time-varying volatility of the stock market (which reflects macroeconomic volatility)
- Commodity specific factors, e.g., hurricane risk in the US Gulf, military conflict or political risk in oil producing regions

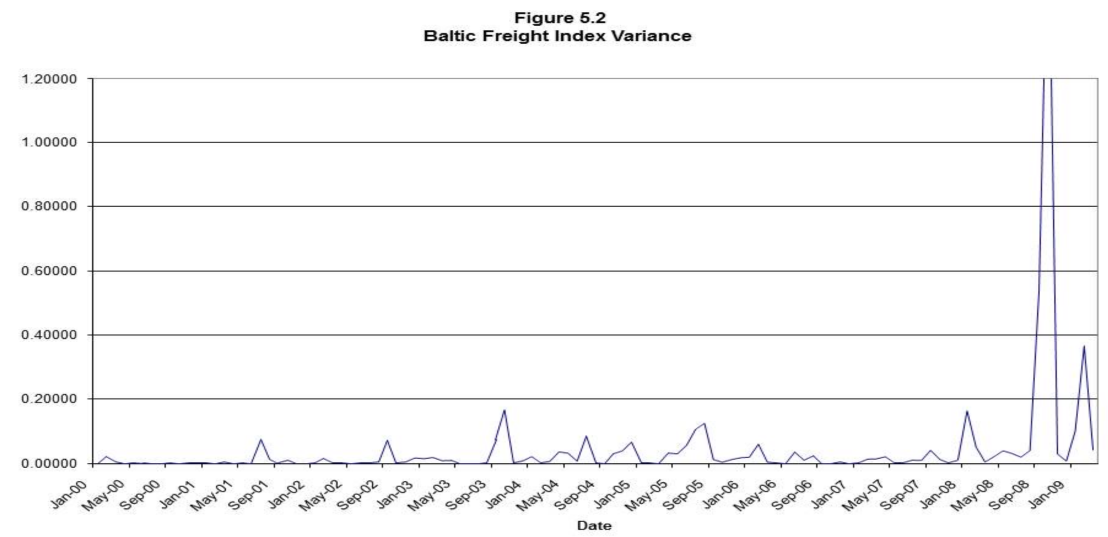
# Preliminary Findings

- This model can provide much more realistic dynamics of commodity price volatility
- It can also explain some otherwise anomalous things, such as simultaneous rises in inventories and prices (which some argue are an indicator of speculative distortion)
- Greater uncertainty makes it more advantageous to hold inventory as a precaution against future shocks
- The only way to accumulate inventory is to encourage production and discourage consumption: higher prices do this
- Therefore, a positive volatility shock should be associated with higher prices *and* inventories

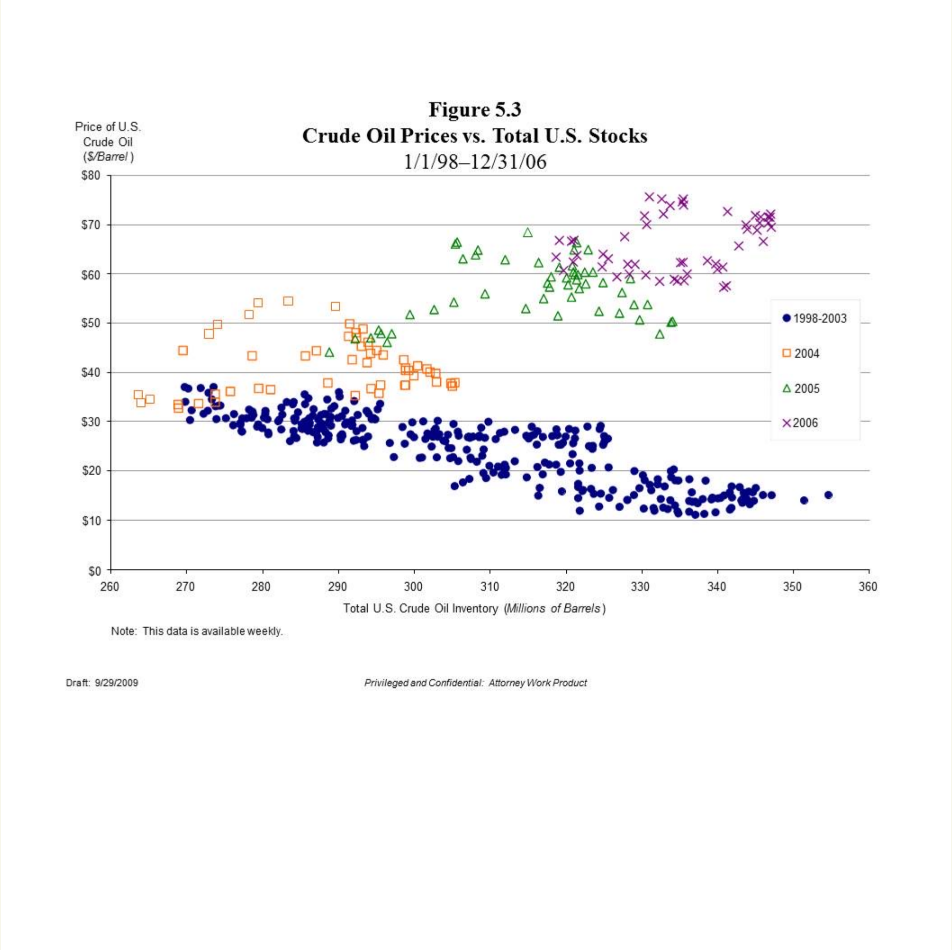
# VIX 1990-2009



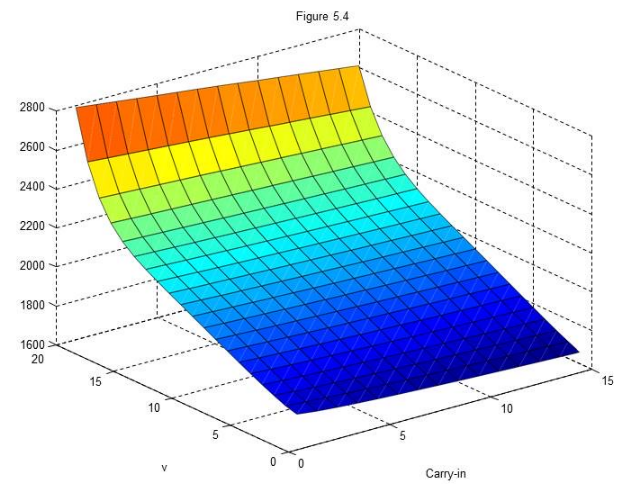

# BFI Variance



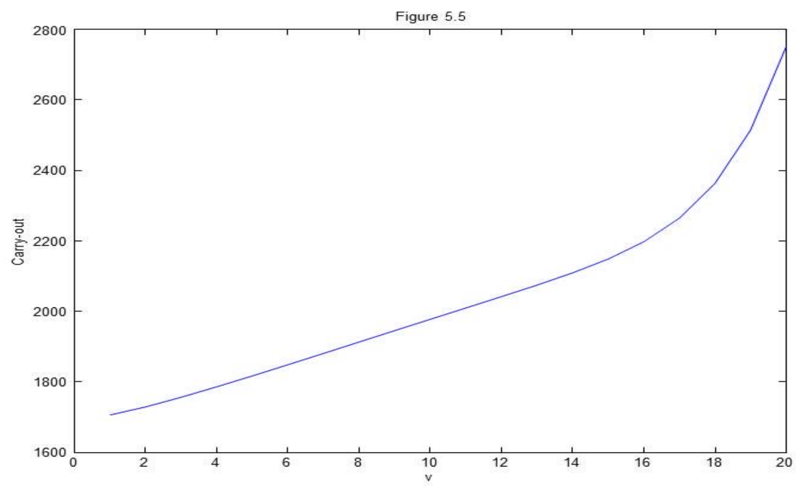

# Crude Prices vs. Total US Stocks



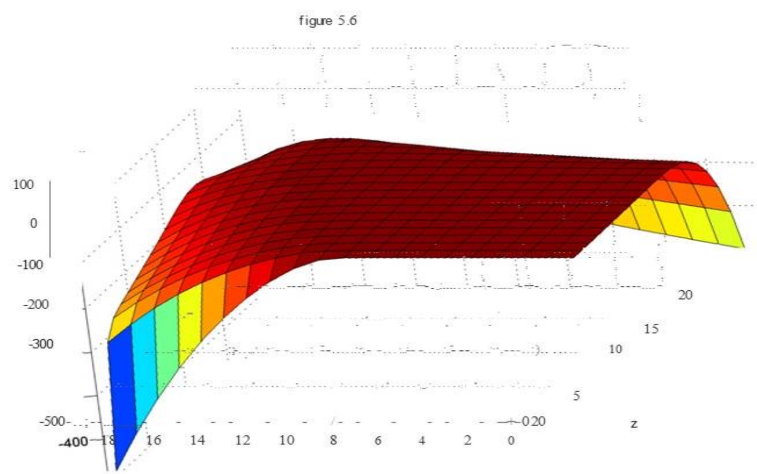

# Prices vs. Variance & Carry-In



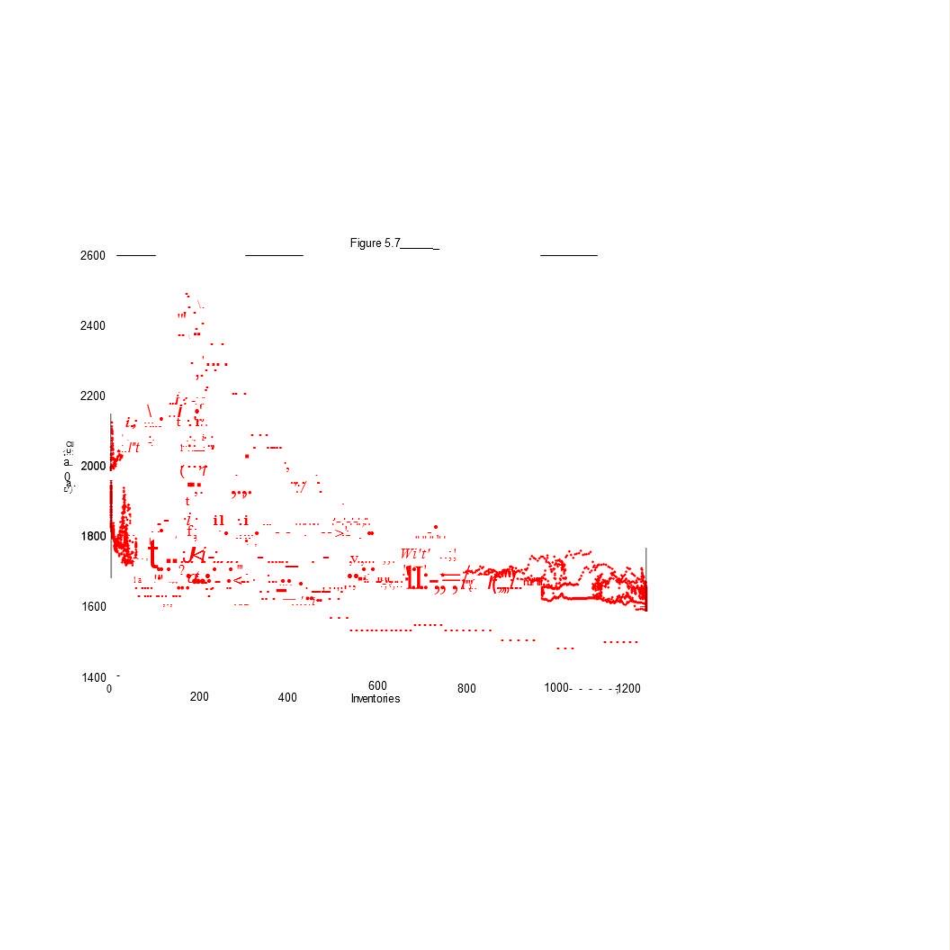
# Carry-Out vs. Variance



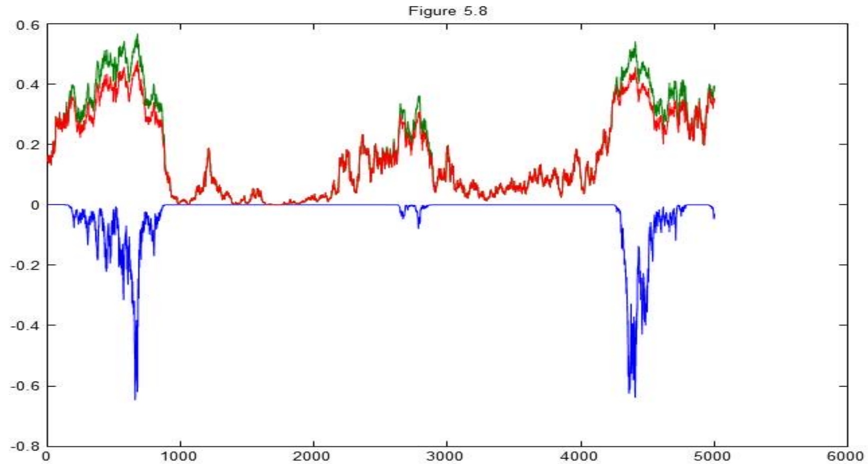

# Spreads vs. Variance & Carry-In



# Inventory-Spread Scatter




# Volatilities & Backwardation




# Seasonal Commodities

- The previous analysis focuses on continuously produced commodities—they are easier to handle and understand
- Seasonal commodities (such as grains) are more challenging

# Forward Curves for Seasonals

- The basic economic insight about intertemporal allocation suggests that prices should be at full carry for delivery dates prior to the next harvest, but “new crop” forward prices are likely to be less than “old crop” prices
- Big slug of new supply due to harvest: should want to consume current inventory before that new supply is available
- Only if expected harvest is expected to be really small should it be optimal to carry inventory to the new crop year; only then should there be a carry between old crop and new crop prices

# Empirical Evidence

- Foregoing provides a pretty good rough characterization of seasonals forward curves
- But there are some anomalies: old crop-old crop prices sometimes fall below full carry
- Aggregation phenomenon that reflects spatial dispersion of stocks and costs of transportation?

# An Anomaly

- The basic story suggests that old crop and new crop prices should not be that highly correlated, and that old crop prices should not respond much to news about the coming harvest
- New crop prices driven by anticipated crop size, but old crop price should not be because little or no storage between old and new crop
- But: (a) old crop and new crop prices are highly correlated, and (b) old crop prices respond almost as much to government crop forecasts as new crop prices