

MIDTERM 1

You will be given a sheet of formulas. You can have a calculator, but the calculator cannot have a programmable memory or formula feature. Those of you who do have a programmable feature will have to prove to me that the calculator memory is empty.

Please read all the questions before you start on the exam. Think before you answer. Also, keep in mind that the more you write, the more likely it is that you will make a mistake. Show all work

Unless specified otherwise, assume well functioning (normal) capital markets with no taxes. You must show all work. Anything you want me to read must be written on the exam papers.

The exam will last for 120 minutes.

1. (30 points) You have a portfolio consisting of one share in each of three securities. You are uncertain about how the economy will do over the year, but expect that at the end of the year, each security will have the following payoffs, conditional on either a weak or a strong economy:

	Weak Economy	Strong Economy
A	0	100
B	100	0
C	100	100

You don't care about what happens after the first year since you (and the rest of the world) expect that life, as we know it will end in 1 year. In addition, you believe that the probability of a weak economy equals the probability of a strong economy, at 1/2 each.

- If the risk free rate of interest is 10%, what should security C sell for?
- If the required return to security A is 20%, what should it be selling for?
- What is the value of Security B in a normal market?
- What is the required return to Security B?
- Why is the required return of Security B less than 10%, the return on risk free securities?
- You are considering buying a fourth security, which you estimate, will return 100 in a weak economy and 200 in a Strong Economy. What is the maximum you would be willing to pay for that security?

$$a. P_C = \frac{E[CF]}{(1+r)} = \frac{100}{1.1} = \boxed{90.91}$$

$$b. P_A = \frac{E[CF]}{(1+r)} = \frac{\frac{1}{2}(0) + \frac{1}{2}(100)}{1.2} = \frac{50}{1.2} = \boxed{41.67}$$

c.  $P_A + P_B = P_C$  because of law of one price

$$41.67 + P_B = 90.91$$

$$P_B = 90.91 - 41.67 = \boxed{49.24}$$

d.) The required return is the solution to  $r$  in

$$P = \frac{E(CF)}{1+r},$$

$$\text{so } r = \frac{E(CF)}{P} - 1$$

$$= \frac{50}{49.24} - 1 = 1.54\% \quad \checkmark$$

e) This security pays off in weak economies. So it is like insurance, paying off when other securities are doing poorly.

f) The payoff: 100 200

is the same payoff as holding 1 unit of B + 2 units of A

The cost therefore must be

$$P_B + 2P_A = 49.24 + 2(41.67) = 132.58$$

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Alternative Portfolio is

1 unit of C, 1 unit of A

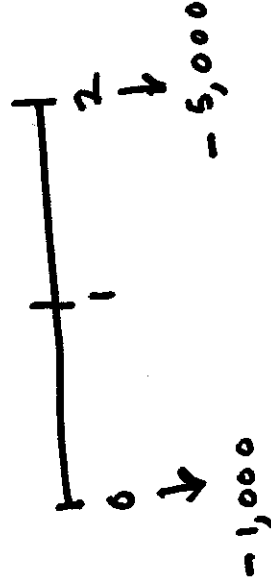
$$P_C + P_A = 90.91 + 41.67 = 132.58$$

2. (30 points) You are considering an investment project that is expected to generate \$4,000 per year forever, where the first payment will be made 1 year from today. The project will require an initial investment of \$1,000 immediately, and an additional investment of \$5,000 two years from today. The Project has a cost of capital (required return) of 10%

- What is the present value of the \$4,000 perpetuity?
- What is the present value of the required investments?
- What is the Net Present Value of the project?
- You decide to make the investment but after two years (that is in 2010), you discover that the additional investment required is no longer \$5,000 but is \$9,000 and that the future cash flows are only \$1,000 not \$4,000 (assume that you have already received the \$4,000 cash flow in the second year). You have the option of abandoning the project or making the second investment of \$9,000. What do you do?
- How would your answer to question d above change if the \$1,000 cash flow lasted only fifteen years?

a.  $PVP = \frac{E[CF]}{r} = \frac{4,000}{.10} = \boxed{40,000}$

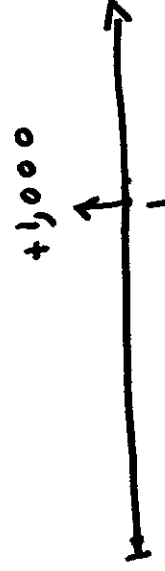
b.



$PV = -[1,000 + \frac{5,000}{(1.1)^2}] = 1,000 + 4132.23 = \boxed{5132.23}$

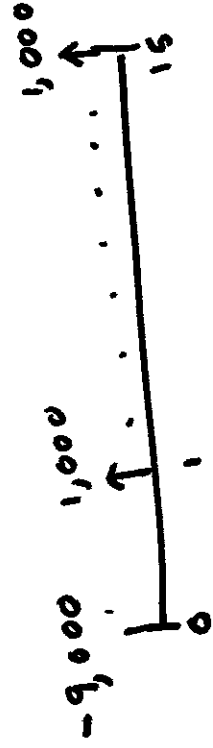
c.  $NPV = PV(\text{BENEFITS}) - PV(\text{COSTS}) = 40,000 - 5,132.23 = \boxed{34,867.77}$

d.



$NPV = \frac{1,000}{.10} - 9,000 = 10,000 - 9,000 = \boxed{1,000}$

Do not abandon but continue



e.  $NPV = -9,000 + PV_A [1,000, 10\%, 15 \text{ years}]$

$$= -9,000 + 7,606.08 = -1,393.92$$

$$PV_A = 7 \quad N = 15$$

$$i = 10$$

$$PMT = 1,000$$

$$FV = 0$$

$$CPT \quad PV = 7 \quad -7,606.08$$

ABANDON PROJECT  $NPV < 0$

3. (20 points) XYZ Corp. has been making widgets from mushrooms for the past three years. It is considering expanding its production facilities. The expansion will be financed with long-term debt requiring interest payments of \$ 2 million per year. Although the expansion will have a long term impact on the firm's cash flows, management has projected the following data resulting from the new facilities for the first two years. All dollar numbers are in million, and to be considered incremental, resulting from the expansion project alone. Find the incremental **Gross Profits**, **EBIT**, **Operating Income (unlevered income)**, and **Free Cash Flow** from the expansion project for these two years.

	Year 1	Year 2
Revenue (Sales)	250	320
Cost of Goods Sold (Operating expenses other than depreciation)	80	80
Depreciation	50	72
Increase in working capital	10	16
Capital Expenditure	60	80
Expected Marginal corporate tax rate	35%	30%

Gross Profits 170 240

EBIT 120 168

Operating Income (Unlevered Income) 78 117.60

Free Cash Flow 58 93.60

	Year 1	Year 2
Gross Profits	170	240
EBIT	120	168
Operating Income	78	117.60

$$\text{Gross Profits} = \text{Revenue} - \text{COGS} =$$

$$\text{EBIT} = \text{Gross Profits} - \text{Depreciation} =$$

$$\text{OPERATING INCOME} = \text{EBIT}(1-t)$$

$$\text{Free Cash Flow} =$$

$$\text{Operating Income} + \text{Dep.} - \Delta \text{WC}$$

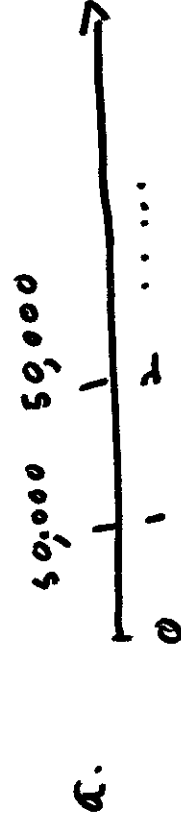
- Cap Ex

$$= 78 + 50 - 10 - 60 = 58$$

$$= 117.60 + 72 - 16 - 80 = 93.60$$

4. (20 points) You are about to retire and your financial advisor suggests you purchase an annuity (a fixed annual payment for your entire life) of \$50,000 per year, with the first payment to be made 1 year from today. The cost of the annuity (today) is \$600,000, and the relevant interest rate (long term government bonds yield) is 6%.

- a. How long do you have to live to make the annuity worthwhile (i.e. to get more in *value* than what you paid in)?
- b. Suppose you believe that you will live for another 25 years. If you save \$5,000 per year by purchasing long term government bonds, how much will your heirs inherit from that savings? (Ignore taxes)
- c. How much per year will you have to save in order to have \$1,000,000 for your heirs in 25 years?



$$\begin{aligned}
 & \downarrow \\
 & 600,000 \\
 & N = ?? \\
 & I/YR = 6 \\
 & PV = -600,000 \\
 & PMT = 50,000 \\
 & FV = 0
 \end{aligned}$$

$$CPT N \Rightarrow 21.846$$

MUST LIVE ~~FOR~~ MORE THAN

21 years,  
309 Days

or

22 ~~days~~ years

b.

$$274,322.56$$

$$N = 25$$

$$I = 6$$

$$PV = 0$$

$$PMT = 5,000$$

$$CPT FV = 274,322.56$$

c.

$$FV = 1,000,000$$

$$CPT PMT = 18,226.72 \text{ per year}$$