

## Homework 2

1.

The following 20 observations are drawn from a censored normal distribution:

16.0905, 1.283, 6.2015, .00000, .00000, 3.4132, 5.8801, 8.6012, 0.7971, 0.0828, .00000,  
.80260, 0.0000, 4.3211, .00000, 8.6801, .00000, .00000, 1.1021, .00000, 1.9654, 4.1762

The applicable model is

$$y_i = \mu + \varepsilon_i$$
$$y_i = y_i \text{ if } \mu + \varepsilon_i > 0,$$
$$= 0 \text{ otherwise.}$$
$$\varepsilon_i \sim N[0, \sigma^2].$$

All exercises in this section are based on the preceding.

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The OLS estimator of  $\mu$  in the context of this Tobit model is simply the sample mean.

Compute the mean of all 20 observations. Would you expect this estimator to over- or underestimate  $\mu$ ? If we consider only the nonzero observations, the truncated regression model applies. The sample mean of the nonlimit observations is the least squares estimator in this context. Compute it, then comment on whether this should be an overestimate or an underestimate of the true mean.

2. We now consider the Tobit model that applies to the full data set.

(a) Formulate the log-likelihood for this very simple tobit model.

(b) Reformulate the log-likelihood in terms of  $\theta = 1/\sigma$  and  $\gamma = \mu/\sigma$ . Then, derive the necessary conditions for maximizing the log-likelihood with respect to  $\theta$  and  $\gamma$ .

(c) Discuss how you would obtain the values of  $\theta$  and  $\gamma$  to solve the problem in part (b).

(d) Obtain the maximum likelihood estimators of  $\mu$  and  $\sigma$ .

3. Using only the nonlimit observations, repeat the Exercise 2 in the context of the truncated regression model. Estimate  $\mu$  and  $\sigma$  by using the method of moments estimator outlined in Example 20.4. Compare your results to those in the previous problems.

4. LAD Estimator

Show that in the usual conditional regression model, the LAD estimator is asymptotically normally distributed. Derive its variance.

5. The attached data set (adrsam-2.xls) was used to estimate the returns of arbitrage strategies in 147 Asian companies that have also ADRs traded in NY. Every time, the Asian prices (in USD) were smaller than the ADR prices by 2 standard deviations, an arbitrageur buys the Asian stock and then sells the ADR. When prices converge, the

arbitrager closes the position. The duration (in days) of the arbitrage strategies was also estimated. The data contains individual firm and liquidity characteristics (MKcap=Market capitalization, SR=short ratio, avgvol=average spread volatility, propzeret=proportion of zero returns in the domestic Asian market, RollD and RollF=Roll estimator of transactions costs in the domestic Asian market and ADR market, respectively. have average and median returns (avret and medret, respectively);

We are interested in the relation between arbitrage returns and liquidity. As conditioning variables use RollD, RollF, SR, and propzeret. Include Mkcap and avgvol as control variables. We suspect the data has several outliers.

- a) Do an OLS regression. Using the standardized residuals, check for outliers, (use 3 as the cut-off point).
- b) Use kernel density estimation (KDE), using a Gaussian kernel, to estimate the distribution of the standardized residuals. Is it normal?
- c) If you find outliers, take them out and re-run the OLS regression.
- d) Write a program to estimate a model for median returns using robust regression. Use the Huber weights and bisquare weights.
- e) Are the results in b) and c) similar?
- f) Back to d). Do the results change if you use median returns, instead of average returns?
- g) You believe that the best liquidity indicator for emerging markets is the proportion of zero returns. Run a non parametric regression (using Epanechnikov kernel) to estimate the relation between average arbitrage returns and liquidity. (You do not have to write a program. R, SAS, etc, is OK.)
- g) Run a quantile regression on the same data set, but use duration as the dependent variable. Estimate the regression for the .25, .50, and .75 quantiles. Plot the estimates for the different quantiles, along with C.I. (You do not have to write a program. R, SAS, etc, is OK.) Does liquidity have any effect on the duration of arbitrage strategies?