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Econometrics 1

# Homework 5

**1 .** Apply the Method of Steepest Descent to the function

$$f(x_1; x_2) = 4 x_1^2 - 4 x_1 x_2 + 2 x_2^2$$

with initial guess $\mathbf{x_0} = (2; 3)$. Compute the first three iterations.

**2**. Using the Newton-Raphson method, find the roots of the following function:

$$y = f(x) = x^4 - 2*x^2 - x - 5,$$

1. Find the roots using the Newton-Raphson method
2. Do initial values matter? Try $x_0=0$, $x_0=1$ and $x_0=3$.
3. Use a step-size, λ, in the updating step. Does this improve convergence?

**3.** Propose a Gauss-Newton algorithm to estimate the following non-linear model

$$q_t = \mu + \alpha\, q_{t-1} + \beta\, q_{t-1}[1 - \exp\{-\lambda(q_{t-d} - \mu)^2\}] + \varepsilon_t$$

where μ, α, β, λ are the unknown parameters, $q_t$ is our series of interest –say, abnormal returns relative to the market- and d is a delay factor. This model is called the ESTAR(1,d) model.

**4.** Go to Ken French's website to download the Average Value Weighted Returns for the 6 portfolios formed on size and book-to-market (2 x 3). You are going to use monthly returns. Also, download the Fama-French Factors –i.e., returns on excess market portfolio, SMB, HML- and the risk-free rate. Use a Gauss-Newton algorithm to estimate the following non-linear CAPM model:

$$R_{i,t} - R_{f,t} = \alpha + \beta_i (R_{m,t} - R_f) + \delta_i [|HML_{,t}|^{\lambda} - 1]/\lambda + \varepsilon_t \quad 0 \leq \lambda \leq 1 \qquad (*)$$

where α, β, δ and λ are the unknown parameters. You are using a Box-Cox transformation.

(i) Estimate α, β, δ and λ for the six portfolios. Calculate standard errors –use delta method when needed.

(ii) Test $H_0$: $\lambda=1$ against $H_1$: $\lambda\neq1$. Does the CAPM hold?

Note: If you want to experiment a bit more, estimate the following model instead of (*):

$$R_{i,t} - R_{f,t} = \alpha + \beta_i (R_{m,t} - R_{f,t}) + \delta_i [|HML|^{\lambda} - 1]/\lambda + \gamma_1 SMB_t + \gamma_2 HML_t + \varepsilon_t, \quad 0 \leq \lambda \leq 1$$