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Econometrics 1

Homework 4

When characterizing investment strategies, the most common statistic is the Sharpe Ratio (SR). The Sharpe Ratio (SR) is one of the most commonly cited statistics in financial analysis.

Let R_t be the one-period simple return of a portfolio or fund between dates $t-1$ and t. We denote by μ and σ^2 its mean and variance:

$$\mu \equiv \mathrm{E}(R_t)$$

and

$$\sigma^2 \equiv \mathrm{Var}(R_t)$$

The Sharpe ratio (SR) is defined as the ratio of the excess expected return to the standard deviation of return:

$$\mathrm{SR} = \frac{\mu - R_f}{\sigma}$$

where the excess expected return is usually computed relative to the risk-free rate, R_f.

Given a sample of historical returns ($R_1, R_2, \ldots, R_T$), the estimators for these moments are the sample mean and variance. Then, the estimator of the Sharpe ratio ($\hat{\mathrm{S}}\mathrm{R}$) follows immediately:

$$\hat{S}R = \frac{\hat{\mu} - R_f}{\hat{\sigma}}$$

Questions

We are interested in testing H_0: SR for S&P500 = 0.

1. Assume that $\{R_t\}$ is *iid* normal. Derive the asymptotic distribution of $\hat{\mathrm{S}}\mathrm{R}$. (Hint: First derive the joint distribution for the sample mean and variance. Then, use delta method. You should get a variance for SR, Var(SR), equal to Var(SR) = 1 + ½ SR^2.)

For the next questions, please download Data Set 2 (S&P500 index and 3-mo US T-bill rate, monthly since 1934:Jan).

2. Using the asymptotic distribution, test H_0.
3. Obtain a probability distribution for SR, using a bootstrap. For this you sample with replacement from the returns series, and compute each time the SR. Use B=999 and T=1076. Draw a histogram.
4. Using the bootstrap, calculate the *p-value* for the observed SR.

<u>Note</u>: Turn in the computer code.