Regime-Switching Stochastic Volatility and Short-term Interest Rates,
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Appendix A: The Gibbs Algorithm for Estimating the RSV Model

In the RSV model (2), we need to estimate the parameter vector $\theta = \{\beta, \gamma, \sigma_\eta, \phi_1, p_{01}, p_{10}\}$ along with the two latent variables $H_t = \{h_1,...,h_t\}$ and $S_t = \{s_1,....,s_t\}$. Thus, the parameter set consists of $\omega = \{H_t, S_t, \theta\}$ for all t. We use Bayes theorem to decompose the joint posterior density as follows.

$$f(H_n, S_n, q) \propto f(Y_n|H_n) f(H_n|S_n, q) f(S_n|q) f(q)$$

We next draw the marginals $f(H_t| Y_t, S_t, \theta)$, $f(S_t|Y_t, H_t, \theta)$, and $f(\theta|Y_t, H_t S_t)$, using the Gibbs sampling algorithm described below:

Step 1:

Specify initial values $\theta^{(0)} = \{\beta_1^{(0)}, \gamma^{(0)}, \sigma_\eta^{(0)}, \phi^{(0)}, p_{01}^{(0)}, p_{10}^{(0)}\}$. Set i =1.

Step 2:

Draw the underlying volatility using the multi-move simulation sampler described in De Jong and Shephard (1995), based on parameter values from step 1. The multi-move simulation sampler is used to draw the underlying volatility vector for all the data points as a single block (see De Jong and Shephard (1995) for details). Consider the RSV model (3), reproduced below:

$$\begin{aligned}
&\Delta r_t - (\hat{a}_0 + \hat{a}_1 r_{t-1}) \equiv RES_t \\
&RES_t = \sqrt{h_t r_{t-1}^{2a}}\, e_t, \qquad a = 0.5 \\
&\left(\ln(h_t) - m_{s_t}\right) = f_1\left(\ln(h_{t-1}) - m_{s_{t-1}}\right) + \sqrt{s_h^2}\, h_{t-1} \\
&m_{s_t} = b + g s_t \qquad g > 0 \quad s_t = \{1,2\}
\end{aligned} \tag{3}$$

The conditional mean equation can be written as,

$$\ln(RES_t^2) = \ln(h_t) + \ln(r_{t-1}) + \ln(e_t^2) \tag{A -1}$$

The term $\ln(\varepsilon_t^2)$ can be approximated by a mixture of seven normal variates (Chib, Shephard, and Kim (1998)).

$$\ln(e_t^2) = z_t$$

$$f(z_t) = \sum_{i=1}^{7} f_N\left(z_i \middle| m_i - 1.2704, v_i^2\right) \quad i = \{1,2,....7\} \qquad \text{(A - 2)}$$

Now, (A-1) can be written as

$$\ln(RES_t^2) = \ln(h_t) + \ln(r_{t-1}) + \left[z_t \middle| k_t = i\right] \qquad \text{(A - 3)}$$

where k_t is one of the seven underlying densities that generates z_t. Once the underlying densities k_t, for all t, are known, (A-3) becomes a deterministic linear equation and, along with the RSV model (3), can be represented in a linear state space model. Next, apply the De Jong and Shephard (1995) simulation smoother to extract the underlying log volatility from the observed data.

In order to estimate α as a free parameter, rewrite (A-1) as

$$\ln(RES_t^2) = \ln(h_t) + 2a\ln(r_{t-1}) + \ln(e_t^2) \qquad \text{(A - 1)}'$$

Then estimate α, approximating $\ln(\varepsilon_t^2)$ by a lognormal distribution. Once α is known, follow (A-3) and extract the latent volatility.

Step 3:

Based on the output from steps 1 and 2, the underlying k_t in (A-3) is sampled from a normal distribution as follows -see Chib, Shephard and Kim (1998):

$$f\left[z_{t=i} \middle| \ln(y_t^2), \ln(h_t)\right] \propto q_i f_N\left(z_i \middle| \ln(h_t) + m_i - 1.2704, v_i^2\right) \quad i \le k \qquad \text{(A - 4)}$$

For every observation t, we draw the normal density from each of the seven normal distributions $\{k_t = 1,2,..,7\}$. Then, we select a "k" based on draws from uniform distribution.

Step 4:

Based on the output from steps 1, 2 and 3, we draw the underlying Markov-state following Carter and Kohn (1994). We use the smoother for the above state-space model (3), to derive the vector of underlying state variable S_t, t = 1,2,...,n.

Step 5:

Cycle through the conditionals of parameter vector $\theta = \{\beta, \gamma, \sigma_\eta, \phi_1, p_{01}, p_{10}\}$ for the volatility equation using Chib (1993), based on the output from steps 1-4. Assuming that f (θ) can be decomposed as:

$$f(q|Y_n,H_n,S_n) \propto f(b|Y_n,H_n,S_n,q_{-b})\, f(g|Y_n,H_n,S_n,q_{-g})\, f(s^2|Y_n,H_n,S_n,q_{-s^2})$$
$$f(f|Y_n,H_n,S_n,q_{-f})\, f(p_{01},p_{10}|Y_n,H_n,S_n,q_{-p_{ij}}) \qquad \text{(A-5)}$$

where θ_{-j} refers to the θ parameters excluding the jth parameter. The respective conditional distributions (normal for β, γ and ϕ, inverse gamma for σ^2 and beta for p_{ij}) are described in Chib (1993). The parameter γ is drawn using an inverse CDF with the restriction that it is positive. The prior means and standard deviations are specified in Tables 3 and 5.

Step 6: Go to step 2.

Estimation of SSV model (2) has the same steps as in RSV model (3), except that we do not have to draw the latent states and transition probabilities. For the Gibbs estimation, we leave out the first 4000 draws (i.e., burn–in iterations are 4000) and sample from the next 6000 draws. We choose every fifth observation to minimize, and if possible eliminate, any possible correlation in the draws. Our effective number of draws therefore drops to 1200 (i.e., effective test iterations are 1200). We construct 95% confidence intervals for the parameters, based on 1200 draws. We construct the *standard errors* for the parameters using the batch-means method -see Chib (1993). We estimate the density functions for the parameters using a Gaussian kernel estimator (Silverman (1986)). The kernel estimator with kernel *K* is defined as:

$$\hat{f}(x) = \frac{1}{nh} \sum_{i=1}^{n} K\left(\frac{x - X_i}{h}\right)$$

where *h* is the band width, X_i {1,...…,*n*} are the observations from the given sample and *n* is the sample size. When *K* is a normal density, we get the Gaussian kernel.

Appendix B: A Monte Carlo Experiment of the RSV, using the Gibbs Algorithm

We perform a Monte Carlo experiment of the RSV model (2), without level effects, i.e., we consider the following version of model 2:

$$RES_t = \sqrt{h_t}\, e_t$$
$$\left(\ln(h_t) - m_{s_t}\right) = f_1\left(\ln(h_{t-1}) - m_{s_{t-1}}\right) + \sqrt{s_h^2}\, h_{t-1}$$
$$m_{s_t} = b + g s_t \qquad g > 0 \quad s_t = \{1,2\}$$

We first assume values for the parameter set $\theta = \{\beta, \gamma, \sigma_\eta, \phi_1, p_{01}, p_{10}\}$-we designate them as "true" values; the true parameter values used in the simulation are listed in the table below. Using the true transition probabilities p_{01} and p_{10}, we generate a state vector (with values 0 or 1) of size 1000. Using the state vector and the true parameters β, γ, σ_η, and ϕ_1, we generate stochastic volatility, i.e h_t. Then, we generate the residual vector RES_t based on the RSV model described above. Then, taking RES_t as given, we estimate the parameter set θ using the MCMC algorithm as explained in Appendix A. We set the number of burn–in iterations equal to 4000 and the number of effective test iterations equal to 1200. We construct the 95% confidence intervals for the parameters based on 1200 draws. We estimate the standard errors for the parameters using the batch-means method -see Chib (1993). The results are reported in Table B.1.

Table B.1
Results from a Monte Carlo experiment

Parameter	True values	Prior Values		Posterior Values		
		Mean	Std. Deviation	Mean (Std. error)	Std. deviation	95% Confidence Interval
β	0.7	0	50	0.759 (0.003)	0.112	(0.500- 0.986)
γ	1.5	1	50	1.477 (0.018)	0.191	(1.051 -1.819)
ϕ	0.4	0	1	0.391 (0.005)	0.085	(0.202- 0.540)
σ^2	0.6	-	-	0.733 (0.005)	0.109	(0.538 - 0.976)
p_{01}	0.01	0.2	0.16	0.011 (0.000)	0.007	(0.003-0.035)
p_{10}	0.04	0.2	0.16	0.074 (0.003)	0.036	(0.026-0.166)

*Prior distribution of σ^2 (inverse gamma) is improper. Sample size is T: 1000.

We find that the posterior means of parameters are quite close to the true values. The standard errors are small, indicating a high precision of the posterior means. For the variance and the transition probability p_{10}, the posterior means are slightly higher than true values. However, they clearly lie within the 95% confidence bounds.

Figure B.1 shows the latent volatility and states. The top panel consists of simulated residuals RES_t obtained using the true parameter vector θ. The second panel presents both the true and latent volatility, the latter obtained using the simulation smoother. The latent smoother volatility appears to be smoother compared to the true volatility. This is not surprising because the smoother volatilities are smoother by construction compared to the filter volatilities. To investigate how close the simulated and true h_t are to each other, we calculate the Wilcon test statistic and we find that it is 0.79, i.e., insignificant at 5% level. The third panel presents the true states –i.e., either 0 or 1- and smoother probabilities of being in the high volatility state. We see that the smoother probabilities track the latent volatility quite well. For example, for the second and third high volatility regimes, we find that the smoother probabilities begin to drop as the latent volatility begins to drop.

Figure B.1. Simulated Monte-Carlo Residuals and Corresponding Latent Volatility and States