CHAPTER XV

EUROCURRENCY FUTURES AND OPTIONS

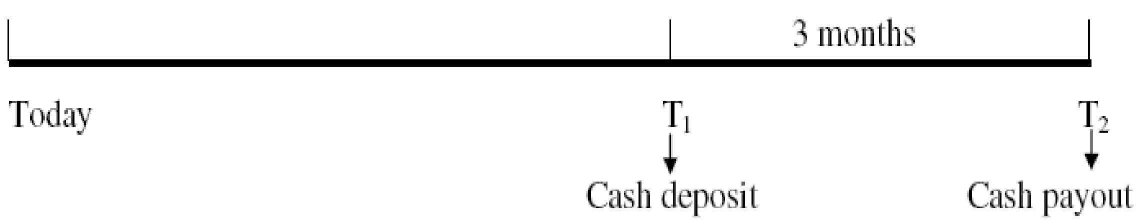
In Chapter XII, we were introduced to the Euromarkets. In that chapter, we briefly discussed the Eurocurrency market, which is a market for short-term deposits, or Eurodeposits. The most popular instrument in this market is the *certificate of deposit* (CD), which is a negotiable and often bearer instrument. We also mentioned that for long-term CDs (up to ten years), there is the possibility of selecting a CD with floating-rate coupons. For CDs with floating-rate coupons, the life of the CD is divided into subperiods of usually six months. The interest earned over such period is fixed at the beginning of the period, the *reset date*. This interest rate is based on the prevailing market interest rate at the time.

In this chapter, we will describe the derivative instruments that have Eurocurrency instruments as the underlying asset. Then, we will focus on how to use those derivative instruments, especially Eurocurrency futures and options, to hedge interest rate risk.

I. Eurocurrency Futures

Eurocurrency futures are very simple futures contracts. To see this simplicity, consider a domestic futures contract on a time deposit (TD), where the expiration day of the futures (when the cash deposit is made), T_1, precedes the maturity date T_2 of the TD (when settlement occurs), by typically three months. The timing of the cash flows are shown in Exhibit XV.1. This futures TD contract locks you in a 3-mo. interest rate at time T_1.

Exhibit XV.1
Timing of a futures Time Deposit (TD)



Example XV.1: In June you agree to buy in mid-September a TD that expires in mid-December. The value of the TD is 100, and the price you agree to pay is 96. The return you will realize on the TD during the last three months of its life is (100-96)/96 = 4.167% (or 16.67% on yearly basis). ¶

Eurocurrency futures work in the same way as the above mentioned time deposit futures. A Eurocurrency futures contract calls for the delivery of a 3-mo Eurocurrency time deposit at a given interest rate (LIBOR). Eurocurrency time deposits are the underlying asset in Eurocurrency futures. Actually, you can consider the interest rates (LIBOR) on the future Eurocurrency deposit as the underlying asset. The Eurocurrency futures contract should reflect the market expectation for the future value of LIBOR for a 3-mo deposit. Like with any other futures a trader can go long a Eurocurrency futures (an agreement to make a future

3-mo deposit) or short Eurocurrency futures (an agreement to take a future 3-mo. loan). A trader can go long a Eurocurrency futures -assuring a yield for a future 3-mo deposit- or go short a Eurocurrency futures -assuring a borrowing rate for a future 3-mo loan.

Eurocurrency futures contracts are traded at exchanges around the world. As we mentioned above, the reset rate is based on the prevailing market interest rate. This market rate is usually the LIBOR or the Interbank Offer Rate in the currency's domestic financial center. In the late 1990s, Eurocurrency futures contracts included:

i. Eurosterling (at LIFFE, the London International Financial Futures Exchange).
ii. Euroyen (at TIFFE, the Tokyo International Financial Futures Exchange, LIFFE, and the Singapore International Monetary Exchange, SIMEX).
iii. Euroswiss Franc (at LIFFE).
iv. Eurodollar (at the CME in Chicago, LIFFE, SIMEX, and TIFFE).
v. Euribor (at LIFFE, MATIF in Paris, MEFF, the Mercado Español de Futuros Financieros, in Barcelona, and Belfox in Brussels).
vi. Euro (LIBOR) (at LIFFE).
vii. HIBOR (at the Hong Kong Futures Exchange).
viii. JIBOR (at SAFEX, the South African Futures Exchange in Johannesburg).
ix. SIBOR (at SIMEX).

We will focus on the three-month Eurodollar futures at the Chicago Mercantile Exchange in Chicago. The mechanics of the other Eurocurrency futures contracts are very similar to the mechanics of the Eurodollar futures contract. The Eurocurrency futures market provides some of the key reference rates for interbank trading in Eurocurrencies. Eurocurrency futures contracts are like any other futures contracts (see Chapter VI). If the price goes up, the long side wins and the short side loses.

The Eurodollar futures price is based on three-month LIBOR, the offered interest rate for three-month Eurodollar deposits (for forward delivery) with a face value of USD 1,000,000. The interest rate on a (forward) three- month deposit is quoted at an annual rate.

The Eurodollar futures price is defined to be:

100 - (the interest rate of a 3-mo Eurodollar deposit for forward delivery).

Example XV.2: If the interest rate on the forward 3-mo. deposit is 6.43%, the Eurocurrency futures price is 93.57. ¶

♦ **Interest rates and Eurodollar futures: Inverse Relation**
If interest rates go up, the Eurodollar futures price goes down, so the short side of the futures contract makes money. ♦

Since the face value of the contract is USD 1,000,000, one basis point (.01%) has a value of USD 100 for a 360-day deposit. For a three-month deposit, the value of one basis point is thus logically USD 25.

Example XV.3: If the futures price drops 2 bps, from 89.64 to 89.62, then the short rate side gains:

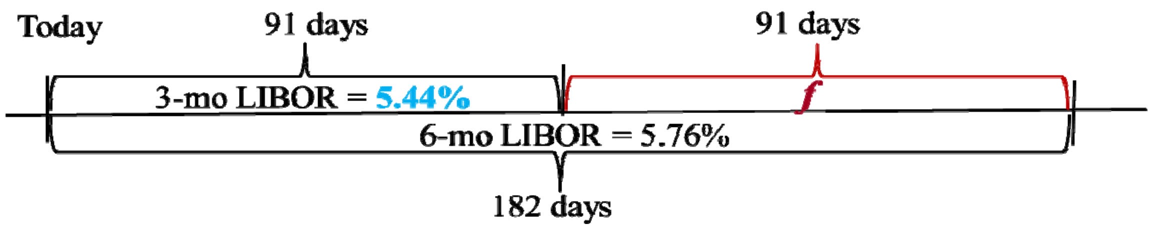
2 x USD 25 = USD 50. ¶

Eurocurrency futures reflect market expectations of forward 3-month rates. An *implied forward rate* (***f***) indicates approximately where short-term rates may be expected to be sometime in the future.

Example XV.4: 3-month LIBOR spot rate = **5.44%** (91 day period)

6-month LIBOR spot rate = **5.76%** (182 day period)

3-month forward rate = ***f***



$(1 + .0576 * 182/360) = (1 + .0544 * 91/360) * (1 + f * 91/360)$

$\Rightarrow \quad f = [(1 + 0.0576 * 182/360)/(1 + 0.0544 * 91/360) – 1] * (360/91)$

$f = 0.059975$ (**6.00%**). ¶

At all exchanges Eurodollar futures are traded for delivery in March, June, September, December, plus four serial months and spot month. Delivery at the CME is hypothetically made on the third Wednesday of the month. The last day of trading is two days prior to the third Wednesday. Three-month Eurodollar contracts are traded with delivery dates as far as ten years. However, they are only liquid out to about four years. In Eurocurrencies, delivery is only "in cash." No Eurocurrency deposit is actually delivered on the CME Eurocurrency futures contract.

The CME forces the Eurodollar futures price to converge to the spot price by the way the exchange determines the settlement price on the final day of trading. The CME keeps a list of at least twenty London banks that deal in Eurodollar deposits. On the last day of trading, at 3:30 PM London time and at a random time interval in the last ninety minutes of trading, they take a sample of twelve of these banks and obtain their quotations for three-month LIBOR. The arithmetic average of these two values is the final settlement price on the final day of trading.

Example XV.5: The Wall Street Journal on October 24, 1994 quotes the following Eurodollar contracts:

EURODOLLAR (CME) - $ 1 million; pts of 100%

	Yield	Open

	Open	High	Low	Settle	Chg	Settle	Chg	Interest
Dec	94.00	94.03	93.97	94.00	....	6.00	....	447,913
Mr95	93.56	93.60	93.53	93.57	+ .01	6.43	- .01	402,624
June	93.11	93.15	93.07	93.12	+ .01	6.88	- .01	302,119
Sept	92.77	92.80	92.73	92.77	....	7.23	....	243,103
Dec	92.45	92.49	92.42	92.46	....	7.54	....	176,045
Mr96	92.38	92.41	92.34	92.38	....	7.62	....	152,827
June	92.26	92.28	92.24	92.25	- .01	7.75	+ .01	124,984
Sept	92.16	92.18	92.12	92.15	- .01	7.85	+ .01	117,214
Dec	92.04	92.06	92.01	92.03	- .01	7.97	+ .01	95,555
Mr97	92.05	92.06	92.01	92.04	- .01	7.96	+ .01	83,127
June	91.98	92.01	91.95	91.97	- .01	8.03	+ .01	69.593
Sept	91.92	91.94	91.89	91.91	- .01	8.09	+ .01	55,103
Dec	91.81	91.82	91.77	91.79	- .01	8.21	+ .01	53,103
Mr98	91.83	91.84	91.79	91.81	- .01	8.19	+ .01	43,738
June	91.77	91.78	91.73	91.75	- .01	8.25	+ .01	37,785
Sept	91.71	91.73	91.67	91.69	- .01	8.31	+ .01	26,751
Dec	91.60	91.61	91.56	91.58	- .01	8.42	+ .01	24,137
Mr99	91.62	91.63	91.58	91.60	- .01	8.40	+ .01	21,890
June	91.56	91.57	91.52	91.54	- .01	8.46	+ .01	15,989
Sept	91.50	91.51	91.46	91.48	- .01	8.52	+ .01	10,263
Dec	91.40	91.41	91.36	91.37	- .01	8.63	+ .01	7,354
Mr00	91.44	91.44	91.40	91.41	- .01	8.59	+ .01	7,536
June	91.39	91.40	91.35	91.36	- .01	8.64	+ .01	4,971
Sept	91.34	91.35	91.30	91.31	- .01	8.69	+ .01	7,691
Dec	91.23	91.24	91.19	91.21	....	8.79	....	6,897
Mr01	91.28	91.28	91.24	91.26	....	8.74	....	7,312
June	91.24	91.24	91.20	91.22	....	8.78	....	5,582
Sept	91.21	91.20	91.17	91.19	....	8.81	....	5,040
Dec	....	....	....	91.09	....	8.91	....	3,845
Mr02	91.13	91.13	91.13	91.14	....	8.86	....	2,689
June	....	....	....	91.10	....	8.90	....	2,704
Sept	91.06	91.06	91.06	91.08	....	8.92	....	2,016
Dec	90.97	90.97	90.97	91.00	....	9.00	....	1,341
Mr03	91.01	91.03	91.01	91.04	....	8.96	....	1,589
June	90.97	90.97	90.97	91.00	....	9.00	....	1,367
Sept	90.85	90.98	90.95	90.98	....	9.02	....	1,354
Dec	90.85	90.88	90.85	90.89	- .01	9.11	+ .01	1,227

Est vol 437,328; vol Thur 615,913; open int 2,576,727. +17,451.

Let's focus on the Mar 95 contract. The "open" price of 93.56 was the market-determined price at which a contract traded (someone going long, someone going short) when the market opened at the beginning of the day.

The final settlement price for the day was 93.57, which was one basis points higher ("change" = +.01) than the previous day's settlement price. That is, between the close of each of the two days, the long position holders had their account balances increased by (credited with) USD 25 (1 x USD 25). The "yield" is an unnecessary column, since the 6.44% interest rate on three-month Eurodollar deposit can be obtained as

100 - 93.56 = 6.44. ¶

♦ The LIFFE "Euro-euro" Contract: Euribor and Euro (LIBOR)

Several European Futures Exchanges offer a Eurocurrency futures contract based on the Euro. The contract specifications are similar to the Eurodollar specifications. The LIFFE offers a Eurocurrency futures based on the Euribor (the European Bankers Federation's Interbank Offered Rate). The LIFFE also offers a Eurocurrency futures (LIBOR). The contract based on Euribor is the most liquid contract at the LIFFE. The LIFFE Euribor futures price is based on three-month Euribor, the offered interest rate for three-month Euro deposits (for forward delivery) with a face value of EUR 1,000,000. The interest rate on a (forward) three- month deposit is quoted at an annual rate. The minimum price movement is .005 and it has a value of EUR 12.50. Delivery months are March, June, September, December and two serial months. The contracts are settled in cash based on the Exchange Delivery Settlement Price. The settlement price is set to 100.00 minus the Euribor. The MATIF, the MEF, and the Belfox also offer a Eurocurrency futures contract based on Euribor. ♦

1.A Some Terminology

Several factors must always be kept in mind when hedging with Eurodollar or other interest-rate futures.

Amount: A Eurodollar futures contract involves a face amount of USD 1 million. To hedge USD 7 million borrowing for three months, we would need seven futures contracts.

Duration: Duration measures the time at which cash flows take place. For money market instruments, all cash flows generally take place at the maturity of the instrument. A six-month deposit has approximately twice the duration of a three-month deposit. Therefore, the value of one basis point for six months is approximately USD 50.

One could hedge a USD 1 million six-month deposit beginning in March, with two March Eurodollar futures (*stack hedge*). One could also hedge it with one March Eurodollar future and one June Eurodollar future (*strip hedge*).

Slope: Eurodollar contracts may be used to hedge other interest rate assets and liabilities, such as T-bills, commercial paper, banker's acceptances, etc. The rates on these instruments may not be expected to change one-for-one with Eurodollar interest rates.

If we define f as the interest rate in a Eurodollar futures contract, then

$$\text{slope} = \Delta \text{ underlying interest rate} / \Delta f.$$

If the rate of change of T-bill rates with respect to Eurodollar rates is .9 (slope = .9), then we would only need nine Eurodollar futures contracts to hedge USD 10 million of three-month T-bill.

Let F_A and D_A be the face amount and duration of the underlying asset to be hedged. Then the number n of Eurodollar futures needed to hedge is

$$n = (F_A/1{,}000{,}000) \text{ x } (D_A/90) \text{ x slope}.$$

Example XV.6: To hedge USD 10 million of 270-day commercial paper with a slope of .935 would require approximately twenty-eight contracts. ¶

Margin: Eurodollar futures require a deposit of initial margin. In September, this was typically USD 800 per contract, while maintenance margin was USD 600.

1.B The Eurostrip Yield Curve and the CME Swap

Successive Eurocurrency futures give rise to a strip yield curve. The March future involves a three-month rate of interest that begins in March and ends in June. The June future involves a three-month rate of interest that begins in June and ends in September and so on. There are some holes in the curve so defined: between the third Wednesday of March and the third Wednesday of June there will be 91 or 98 days (multiples of 7). But the Eurocurrency futures only cover 90 of these days. For example, one Eurodollar future does not quite hedge USD 1 million for the complete three months. The approximation, however, is close. This strip yield curve is called *Eurostrip*.

If we compound the interest rates for four successive Eurocurrency contracts, we define a one-year rate composed from the four three-month rates. Obviously, the one-year rate so constructed cannot differ much from the implied forward rate for the same period, for example, as calculated from the yield curve.

Example XV.7: Replicating a zero-coupon bond with a Eurodollar strip.
Suppose Mr. Peterson, a money manager, expects to receive USD 2,000,000 on the date that the December 95 Eurodollar futures expires. He wants to invest the money in the Eurodollar market. He is concerned that interest rates may fall during the next three months. Mr. Peterson uses the long Eurodollar strip to hedge against the interest rate uncertainty.

Today, October 24, 1994, the December, March, June and September futures are trading at 94.00, 93.57, 93.12 and 92.77, respectively -see Example XV.5. The December-March interest rate con be fixed today on the forthcoming USD 2,000,000 by purchasing two December Eurodollar futures. The rate is fixed at 6% annually, or 1.5% for the December-March period.

If in December 1994 the 3-mo Eurodollar rate falls to 4%, then a 1% rate of return would earn over the December-March period on the USD 2,000,000 spot investment, that is, USD 20,000. But to this spot return we should add the return on the December Eurodollar contract, which is

200 bps x USD 25 = USD 5,000 per contract.

Then, the effective return is USD 20,000 + USD 10,000 = USD 30,000 (or 1.5%).

Therefore, in March 1995, the spot three-month investment will have accrued to USD 2,030,000. The 6.43% annual rate, or 1.6075% quarterly rate, for the March-June interval can be locked in today on that amount by purchasing 2 x 1.015 March Eurodollar contracts.

The total return for this second 3-mo. period amounts to USD 32,632.25 (USD 203,000,000 x .016075). The total amount accrued by the end of the March-June period contract is:

USD 2,000,000 x 1.015 x 1.016075 = USD 2,062,632.30.

Extending the above reasoning, to lock in a reinvestment rate today for the June-September interval on the resulting USD 2,062,632.30, Mr. Peterson needs to buy 2 x 1.05 x 1.01675 June contracts today. This purchase locks in a 1.72% (6.88/4) quarterly rate on that balance. And to fix the final September-December's reinvestment rate at 1.8325% on the September balance of USD 2,000,000 x 1.015 x 1.016075 x 1.0172, Mr. Peterson needs to buy 2 x 1.015 x 1.016075 x 1.0172 September futures today.

Mr. Peterson has fixed a one-year investment today at the rate of

(1.015 x 1.016075 x 1.0172 x 1.018325) - 1 = .0682787 (or 6.82787%).

We should note that since all the proceeds are reinvested, Mr. Peterson receives the proceeds in a year at the end of the September contract. Thus, buying the one-year Eurostrip is equivalent to buying a one-year zero-coupon bond. ¶

A CME swap involves a trade whereby one party receives one-year fixed interest and makes floating payments of the three-month LIBOR. The payments dates in a CME swap correspond to Eurodollar futures expiration dates.

Example XV.8: On August 15, one might do a September-September swap. The floating-rate payer would make payments on the third Wednesday in December, as well as on the third Wednesday of the following March, June, and September. The fixed-rate payer would make a single payment on the last of these dates.¶

♦ **CME swaps and the Eurostrip: Arbitrage works**
Because the floating payment dates correspond to CME settlement dates, arbitrage ensures that the one-year fixed rate of interest in the CME swap does not diverge materially from the one-year rate constructed from the Eurostrip. ♦

1.B.1 Pricing Short-Dated Swaps

The swap coupons for short-dated fixed-for-floating interest rate swaps are routinely priced off the Eurostrip. The key to pricing the swap coupon is to equate the present values of the fixed-rate side and the floating-rate side of the swap. Eurocurrency futures contracts provide a way to do that.

The estimation of the fair mid-rate is complicated by the fact that (a) the convention is to quote swap coupons for generic swaps on a semiannual bond basis, and (b) the floating side, if pegged to LIBOR, is usually quoted on a money market basis (for consistency, we will assume that the swap coupon is on quoted a bond basis).

For purposes of notation, if the swap is to have a tenor of m months (m/12 years) and is to be priced off three-month Eurodollar futures, then pricing will require n sequential futures series, where n=m/3 or equivalently, m=3n.

The procedure by which the dealer obtains an unbiased mid-rate for pricing the swap coupon involves three steps:

i. Calculate the implied effective annual LIBOR for the full duration (full-tenor) of the swap from the Eurostrip.

$$r_{0,3n} = \pi_{t=1}^{n} [1 + r_{3(t-1),3t}\ A(t)/360]^{\tau} - 1, \quad \tau = 360/\Sigma_t A(t)$$

A(t) is the number of days covered by the Eurodollar futures contract that starts at time t.

ii. Convert the full-tenor LIBOR, which is quoted on money market basis, to its fixed-rate equivalent $FRE_{0,3n}$, which is stated as an effective annual rate (annual bond basis).

$$FRE_{0,3n} = r_{0,3n} \text{ x } (365/360).$$

iii. Restate the fixed-rate equivalent on the same payment frequency as the floating side of the swap. The result is the swap coupon SC. This adjustment is given by

$$SC = [(1 + FRE_{0,3n})^{1/k} - 1] \text{ x } k, \qquad k\text{=frequency of payments.}$$

Example XV.9: On October 24, 1994, Housemann Bank, a swap dealer, wants to price a one-year fixed-for-floating interest rate swap against 3-month LIBOR, that starts on December 94. The fixed rate will be paid quarterly and, therefore, is quoted quarterly bond basis. Housemann Bank wants to find the fixed rate that has the same present value (in an expected value sense) as four successive 3-mo. LIBOR payments.

In Table XV.A, we summarize the Eurostrip for the first five contracts from Example XV.5.

TABLE XV.A

Eurodollar Futures, Settlement Prices (October 24, 1994)

	Price	Implied 3-mo. LIBOR	Notation	Number of Days Covered
Dec 94	94.00	6.00	0 x 3	90
Mar 95	93.57	6.43	3 x 6	92
Jun 95	93.12	6.88	6 x 9	92
Sep 95	92.77	7.23	9 x 12	91
Dec 95	92.46	7.56	12 x 15	91

First, Housemann Bank calculates the implied LIBOR rate corresponding to the full-tenor of the swap using (i). Since the swap is for twelve months, n=4. The implied one-year LIBOR is:

$$f_{0,12} = [(1+.06x(90/360)) \text{ x } (1+.0643x(92/360)) \text{ x } (1+.0688x(92/360)) \text{ x } (1+.0723x(91/360))]^{360/365} - 1$$
$$= .06760814 \text{ (money market basis).}$$

Second, Housemann Banks converts this money market rate to its effective equivalent stated on an annual bond basis.

$$FRE_{0,12} = .06760814 \text{ x } (365/360) = .068547144.$$

Third, since the coupon payments are quarterly, k=4, Housemann Bank must restate this effective annual rate on an equivalent quarterly bond basis.

$SC = [(1.068547144)^{1/4} - 1] \times 4 = .0668524$ (quarterly bond basis)

Therefore the swap coupon mid-rate is 6.68524%. ¶

Example XV.10: Go back to Example XV.9. Suppose now, that Housemann Bank, instead of pricing a one-year swap with quarterly fixed-rate payments against 3-month LIBOR, wants to price a one-year swap with semiannual fixed-rate payments against 6-month LIBOR. The swap coupon mid-rate is calculated to be:

$SC = [(1.068547144)^{1/2} - 1] \times 2 = .06741108$ (semiannual bond basis). ¶

The simple procedure described above allows a dealer to quote swaps having tenors out to the limit of the liquidity of Eurodollar futures on any payment frequency desired and to fully hedge those swaps in the Eurostrip. The latter is accomplished by purchasing the components of the Eurostrip to hedge a dealer-pays-fixed-rate swap or selling the components of the Eurostrip to hedge a dealer-pays-floating-rate swap.

1.C Gap Risk Management

Now, consider the use of Eurocurrency futures to hedge *gap risk*. Gap risk arises in Eurobanking when assets and liabilities have different maturities. Such gaps tend to be managed with futures, forward rate agreements (FRA), and interest rate swaps because these use less capital than do time deposits.

Example XV.11: It is March 20. A Swiss bank is quoted a rate of 4% on three-month Euribor deposits in the interbank market. If a EUR 2,000,000 deposit is borrowed today, the value date will be March 24. Thus, the deposit will mature on June 24 (91 days). Today, the bank can lend a six-month Euribor deposit at 4 1/4%, with a value date on March 24 and maturity date on September 24 (183 days).

The Swiss bank receives a three-month deposit, but makes a six-month loan. Therefore, the Swiss bank is exposed to gap risk. The Swiss bank is uncertain over the interbank deposit interest rate on August 24: the implied interest rate for June 24, f, might be quite different than the actual rate on June 24.

In these cases, the Swiss bank follows a very simple rule: the bank will take the three-month deposit only if it is possible to hedge the risk and make a profit. The bank decides to manage this gap risk using June Euribor futures.

LIFFE lists a very active Euro (Euribor) contract. The characteristics of this contract are the same as the CME's Eurodollar contract. In particular, the face value is EUR 1,000,000. On March 20, June Euribor futures are trading at 96.13. Taken together, these prices imply the bank can lend a six-month deposit, funding this loan by a sequence of two three-month deposits, and be assured of a profit for the six months.

The situation of the bank is straightforward. If the bank lends for six months, it will receive interest at 4 1/4%. But for the first three months it will only pay interest on its borrowed three-month deposit at 4%. The bank should calculate the implied rate to see if a profit can be locked in on the three-month deposit.

The Swiss bank calculates the implied forward rate, *f*, as the rate the bank can afford to pay on the second three-month deposit and still break even:

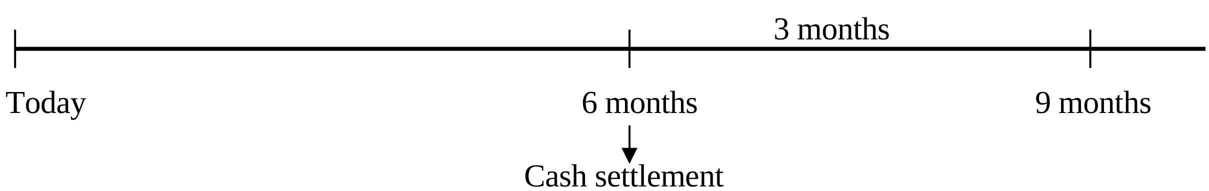
$$[1 + .0425\text{x}(183/360)] = [1 + .04\text{x}(91/360)]\text{x}[1 + f \text{ x } (92/360)] \quad \Rightarrow \quad f = 4.452\%.$$

If the Swiss bank can be certain that it will pay a rate less than 4.452% for the second three-month period, the bank will make a profit. June Euribor is trading at 3.87%. Then, by going *short* two June contract at 96.13 (3.87%) the Swiss bank locks in a profit. ¶

II. Forward Rate Agreements (FRA)

A forward rate agreement (FRA) is cash-settled forward contracts on interest rates traded among major international banks active in the Eurodollar market. You can think of an FRA as the OTC equivalent of a Eurodollar futures contract. Banks use FRAs to fix interest costs on anticipated future deposits or interest revenues on variable-rate loans. A bank that sells an FRA agrees to pay the buyer the increased interest cost on some notional principal amount if interest rates rise above the agreed ("forward") rate on the contrat maturity or settlement rate. On the other hand, the buyer agrees to pay the seller the increased interest cost if interest rates fall below the forward rate.

Example XV.12: An FRA on a three-month interest rate for a three-month period beginning six months from the present and terminating nine months from the present. This agreement is called a "six against nine" (6x9) FRA because it fixes the interest rate for a deposit to be placed after three months and maturing nine months from today. The contract would be settled in cash in six month – i.e., at the beginning of the FRA period.



Let f be the forward rate (expressed as a decimal); i, the settlement rate; N, the nominal contract amount; ym, the number of days in the FRA period; and yb, the year basis (360 or 365). Thus, if i > *f*, the seller pays the buyer:

$$\frac{N \text{ x } (i-f) \text{ x } (ym/yb)}{1 + i \text{ x } (ym/yb)}$$

If i < f, the buyer pays the seller the absolute value of the above amount. Since cash settlement is made at the beginning of the FRA period, the denominator discounts the payment back to that point. In general, the settlement rate is LIBOR.

♦ **FRA: Just Another Forward Contract**
An FRA is a cash-settled interbank forward contract on the interest rate. The cash flows at the beginning ot the FRA period might confuse you. You can consider these cash flows, however, as being exchanged when the forward contract expires. ♦

Example XV.13: A bank buys a "three against six" (or "3X6") FRA for GBP 4,000,000 for a three-month period, beginning three months from now and ending six months from now, at an interest rate of 5.5%. There are actually ninety-two days in the FRA period.

Three months from now, at the beginning of the FRA period, LIBOR is 6%. The bank receives *cash* at the beginning of the FRA-period from the selling party in the amount of

$$\text{GBP } 4{,}000{,}000 \times \frac{(.06 - .055) \times (92/360)}{1 + .06\times(92/360)} = \text{GBP } 5{,}033.92.$$

The bank's net borrowing cost on GBP 4,000,000 at the end of the FRA period is therefore:

	GBP 4,000,000 x .06 x (92/360)	= GBP 61,333.33
minus		
	GBP 5,033.92 x [1 + .06 x (92/360)]	= <u>GBP -5,111.11</u>
	Net borrowing cost	= GBP 56,222.22

The net borrowing cost is equivalent to borrowing GBP 4,000,000 at 5.5%:
GBP 4,000,000 x .055 x (92/360) = GBP 56,222.22.

Note that if instead of an FRA, the bank uses Eurosterling futures. Then, in six months the bank receives GBP 25 x 50 x 4 = GBP 5,000. (Recall that the value of a basis point change is GPB 25.) ¶

FRAs are tailor-made contracts. They are usually traded for dates between one of the standard Eurodeposit maturities, such as three, six, nine and twelve months, but nonstandard dates are also available. FRAs are off-balance sheet items and have smaller capital requirements than interbank loans. Therefore, FRAs are regularly used to close maturity gaps in an Eurobank's balance sheet. An FRA is an interbank-traded equivalent of the implied forward rate.

Example XV.14: On July 20, Leblanc Bank wants CHF 50 million of three-month deposit but is only offered CHF 50 million of six-month deposit by a customer at the bank's bid rate. At the current market, the other rates are these:

	Cash			FRA	
	bid	asked		bid	asked
3 months	5.2185	5.2500	3X6	5.22	5.29
6 months	5.2500	5.3125			

Should the manager of Leblanc Bank take the deposit?

The manager of the bank converts the six-month deposit to a three-month deposit by selling a three against six FRA. That is, Leblanc Bank sells off (lends) the last three months of the deposit in the FRA market. Then, the manager calculates the profit on this transaction.

There are 184 days from settlement of the deposit on July 22 to maturity on January 22. The three-month FRA period is 92 days from October 22 to January 22. The interest paid at the end of six months to the depositor is:

CHF 50,000,000 x .05250 x (184/360) = CHF 1,341,666,67.

Interest earned on lending for three months in the interbank market and then for another three months at the FRA rate is:

CHF 50,000,000 x [(1 + .052185x(92/360)) x (1 + .0522x(92/360)) - 1] = CHF 1,342,703.56.

There is a net profit of CHF 1,036.89 at the end of six months. This profit is possible because the bank was offered a six-month deposit at the bank's *bid* rate of 5.25%.

You should note that Leblanc Bank is not able to do arbitrage at the above rates. Usually, Leblanc Bank would have to borrow for six months in the interbank market at 5.3125%. Thus, the interest paid on the deposit would be CHF 1,357,638.90, and the bank would have a loss. ¶

III. Hedging with Eurocurrencies and FRAs

Contrary to an FRA, Eurocurrency future's cash flow is based on exactly ninety days and is not discounted to the beginning of the period. Thus, the interest rate on Eurocurrency futures cannot be identified with the implied forward rate used to calculate break-even loan costs. The FRA rate, by contrast, is the same as the implied forward rate. For hedging purposes, therefore, we must calculate a hedge ratio for Eurocurrency futures against FRA's. (For simplicity, we will not consider the interest opportunity cost associated with the daily cash flows on the futures contract).

The cash flow (ΔV) to the buyer of a currency FRA is given by the equation (from above):

$$\Delta V = \frac{N \text{ x } (i - f) \text{ x } (ym/yb)}{1 + i \text{ x } (ym/yb)} \qquad \text{(XV.1)}$$

If we let f=1 -.01Z_t also denote the forward interest rate at which a Eurocurrency future was opened (e.g., 0.09 for a futures price of 91.00), and let i=1 - .01Z_{t+1} also denote the forward interest rate at which the Eurocurrency futures price was closed out, then

$$i - f = (1 - .01Z_{t+1}) - (1 - .01Z_t) = .01(Z_t - Z_{t+1}) = -.01\ \Delta Z.$$

Therefore, the corresponding cash flow on a Eurodollar futures contract (where N = USD 1,000,000, ym = 90) is

$$\Delta V = -\ 1{,}000{,}000(.01\Delta Z)(90/360) = -\ 2500\ \Delta Z. \qquad \text{(XV.2)}$$

(Recall, there is no discount to the beginning of the period for a Eurodollar future's cash flow.)

Therefore, the number of Eurodollar futures contracts b needed to hedge an FRA or the implied forward rate is the ratio of (XV.1) to (XV.2):

$$
\begin{aligned}
b &= [N \times (-.01 \times \Delta Z)(ym/yb)/ (1 + i \times (ym/yb)]/[-2500\ \Delta Z] \\
&= N/[250{,}000(i + yb/ym)] \\
&= N/[250{,}000(1 + yb/ym - .01Z_{t+1})]. \qquad \text{(XV.3)}
\end{aligned}
$$

The hedge ratio depends on the current Eurodollar futures price Z_{t+1}. Note that if the forward interest rate were zero Z_{t+1}=100 and if the days in the FRA period were 90 (ym=90), and if the dollar amount of the FRA were N = USD 1,000,000, then b=1.

The number of Eurodollar futures contracts needed to hedge an FRA goes down as the discount rate S increases (as the current futures price Z_{t+1} decreases).

Example XV.15: There are ninety-one days in an FRA period beginning at the expiration of a Eurodollar contract. The current price of the futures is 92.00. The year basis, yb, is 360 days. We want to calculate the amount of the FRA hedges a short position in 100 Eurodollar contracts?

The Eurodollar hedge ratio, b, is prespecified as b = -100. We only need to calculate N.

N = -100 x 250,000 x [1 + 360/91 -.01(92.00)] = USD -100,901,099.

Against the short position of 100 Eurodollar futures contracts, the trader should *sell* an FRA in the amount of USD 100,901,099. To verify this answer, suppose that interest rates fall by 1 basis point (.0001). Then, since rates go from 8.00 to 7.99, the cash flow to the seller of the FRA is:

$$\Delta V = \frac{\text{USD } 100{,}901{,}099 \times (.0001) \times (91/360)}{1 + .0799 \times (91/360)} = \text{USD } 2{,}500.$$

The cash flow on the short Eurodollar futures position is

ΔV = USD 100 x 2,500 x (92.00 - 92.01) = USD -2,500.

For these value changes to remain equal and offsetting, however, the amount of the FRA would have to be continually adjusted because b changes with the current futures price. ¶

Example XV.16: Go back to Example XV.15. Suppose b is left unchanged in the original position, when Z changes to 93.00 (ΔZ equals to 100 basis points). Now, the cash flow to the FRA seller is:

$$\Delta V = \frac{\text{USD } 100{,}901{,}099 \times (.01) \times (91/360)}{1 + .07 \times (91/360)} = \text{USD } 250{,}621.$$

The cash flow on the short Eurodollar futures position is

ΔV = USD 100 x 2,500 x (92.00 - 93.00) = USD -250,000.

The total position has a net change of USD 621. ¶

Example XV.17: Go back, again, to Example XV.15. Now suppose interest rates rise by 1% from 8.00 to 9.00, the FRA seller has a cash flow of

$$\Delta V = \frac{\text{USD } 100{,}901{,}099 \text{ x } (-.01) \text{ x } (91/360)}{1 + .07 \text{ x } (91/360)} = \text{USD } -249{,}382.$$

The cash flow on the short Eurodollar futures position is

ΔV = USD 100 x 2,500 x (93.00 - 92.00) = USD -250,000.

The total position has a net change of USD 618. ¶

Thus, the position of being short Eurodollar futures and selling FRAs, at the fixed ratio b, makes money for interest rate moves in either direction. Such a position can be characterized in terms of options as a *delta-neutral, long-gamma position*.

Example XV.18: In Example XV.11, we compared the implied forward rate to the Euribor futures price, and determined that the Swiss bank could hedge its gap risk in the futures market at a profit. Suppose the Swiss bank wants to hedge EUR 20,000,000. We want to determine the number of futures contracts the bank needs to go short. (Recall Z_{t+1} = 96.13.)

The futures price is Z_{t+1} = 96.13. Thus, the number of contracts to sell is given by equation (IX.3) as b = 20,000,000 / [250,000 x (1 + 360/91 -.01(96.13)] ≈ 20. ¶

As noted before, the hedge ratio b given by (XV.3) does not take into consideration the variation margin flows on the Eurocurrency contract. Since variation margin must be paid in cash, there is in most circumstances an interest cost associated with the need to borrow negative variation margin flows, or an interest profit to be earned on positive variation margin flows.

To adjust the futures position for this interest rate effect (a process called *tailing*), we discount the cash flows on the FRA from now to the beginning of the FRA period. That is, we present value the FRA cash flows to make them comparable to the Eurocurrency futures cash flows.

Let i be the relevant discount rate from now to the beginning of the FRA period, which occurs in T days. (In Example XV.11, i = 4% and T =92 days). Then, from (XV.3), the adjusted futures hedge ratio, b*, is

$$b^* = b/[1 + i \text{ x } (T/360)].$$

As $T \to 0$, $b^* \to b$.

Example XV.19: In Example XV.18, i=4%, T=92. Therefore, at the beginning of the FRA period,

$b^* = 19.80 \approx 20$. That is, in this case, the initial position should be maintained at 20 contracts as $T \to 0$. ¶

The calculations associated with the FRA/Eurocurrency hedge ratio become more complicated once we allow for the general case that the FRA periods and Eurocurrency periods do not correspond. Because the Eurocurrency futures market is more liquid than the FRA market, an FRA period that overlaps two or more Eurocurrency futures periods would result in the FRA rate being calculated by an interpolation of the rates implied in the futures contracts. In addition, the hedge for the FRA would be split up among the Eurocurrency futures used in the interpolation.

IV. Interest Rate, Eurocurrency Futures Options and Other Derivatives

Options on Eurocurrency futures are traded at many exchanges including the CME, LIFFE and SIMEX. A Eurocurrency futures *call* gives the buyer the right to go long a Eurocurrency futures contract, while a Eurocurrency futures *put* gives the buyer the right to go short.

More specifically, a CME Eurodollar put is a contract between a buyer and a writer whereby the put buyer pays a price (the put premium) to the writer in order to acquire the right, but not the obligation, to go short one CME Eurodollar futures contract at the opening price given by the put's strike price.

If the put buyer exercises the right to go short, the put writer must go long the Eurodollar contract. Once option trades have cleared, the CME Clearing House becomes the writer to every option buyer, and the buyer to every other option writer.

The options are American and expire on the last trade date for the futures contract.

Strike prices are in intervals of .25 in terms of the CME index. Thus, one may buy a put on June Eurodollar futures with a strike of 9375. If exercised, it gives the right to go short one CME Eurodollar futures contract at an opening price of 93.75.

Recall that Eurodollar futures contracts involve a face amount of USD 1 million and are traded for delivery in March, June, September and December. For monthly delivery, the CME trades a futures option on LIBOR.

Example XV.20: On Tuesday, November 1, 1994, the *Wall Street Journal* published the following quotes for Eurodollar futures options.

EURODOLLAR (CME)
$ million; pts. of 100%

Strike	Calls-Settle			Puts-Settle		
Price	Dec	Mar	Jun	Dec	Mar	June
9350	0.56	0.29	0.18	0.01	0.18	0.53
9375	0.33	0.16	0.10	0.03	0.30	0.69
9400	0.14	0.07	0.05	0.09	0.45	0.89
9425	0.03	0.03	0.02	0.23	0.66	1.11

9450	0.00	0.01	0.10	0.45	0.89	1.36
9475	0.00	0.00	0.00	0.70	1.14	1.61

Est. vol. 56,820;
Fri vol. 80,063 calls; 72,272 puts
Op. Int. Fri 939,426 calls; 1,016,455 puts

Eurodollar futures option premium quotations are in percentage points, with each basis point representing USD 25.

Example XV.21: Consider the June 95 put, with a strike price of 93.75, in Example XV.20. A price of .69 would represent USD 25 x 69 = USD 1,725. ¶

Example XV.22: A trader goes short a June 1995 Eurodollar future at a price Z=93.75, simultaneously buying a June 1995 call with a strike price of 93.50. The premium on the call is C = .18 (see Example XV.20). The spot interest rate is 6%.

We want to calculate the payoffs on the last day of trading, thirty days from today, for each of the following possible prices for the expiring future: 93.00, 94.00, 95.00, and 96.00.

The call premium paid today is USD 25x18=USD 450. Adding the 6% carrying charge for thirty days gives USD 450x[1 + .06x(30/360)] = USD 452.25 at option expiration. If the future expires at 93.00, the payoff will be 93.75-93.00=.75 or USD 1,875 (.75x2,500). The call will expire worthless, so the net payoff is USD 1,875 - USD 452.25 = USD 1422.75.

Futures Price	Future Payoff	Call Payoff	Option Cost	Carrying Cost	Total	Total (USD)
93.00	0.75	0.00	.18	.0009	.5691	1422.75
94.00	-0.25	0.50	.18	.0009	.0691	172.75
95.00	-1.25	1.50	.18	.0009	.0691	172.75
96.00	-2.25	2.50	.18	.0009	.0691	172.75

By buying the call, the trader has limited his/her possible loss on the future to 6.91 basis points or USD (172.75). This amount can be approximated (ignoring carrying cost, which, in general, are small in magnitude) as Z - X - C = 93.75 - 93.50 - .18 = .07. ¶

4.A Valuation of futures options

To price Eurocurrency futures options, we will use the Black-Scholes formula, derived in Appendix V. The key lies with the forward interest rate f embodied in the futures price. The value of a European (exercise is only allowed at expiration) call on the forward interest rate f is given by:

$$c_t = B_t(T)[f\,N(d1) - X\,N(d2)], \qquad \text{(XV.4)}$$

$$d1 = \frac{\ln(f/X) + .5\,\sigma^2\,T}{\sigma\,T^{.5}} \qquad \text{and} \qquad d2 = \frac{\ln(f/X) - .5\,\sigma^2\,T}{\sigma\,T^{.5}},$$

where $B_t(T)$ is the price of futures contract with expiration date T, N(.) is the cumulative normal distribution, and σ^2 is the variance of B_t.

The European put price is obtained from the put-call conversion equation:

$$p = c + B\,(X-f). \qquad \text{(XV.5)}$$

The latter equation shows the European put and call will have equal values when the forward interest (or FRA) rate is equal to the strike price.

Example XV.23: Table XV.B gives some sample values for European options on interest rates. Assume σ=.20, T=90/365, discount rate 7% (B=98.28), the option premium is paid today, and the cash value of the option payoff is paid at option expiration.

Table XV.B
Value of European Options on Forward Interest Rates

		Call			Put		
	f:	6.0	7.0	8.0	6.0	7.0	8.0
X:	6.0	.234	.999	1.966	.234	.016	.001
	7.0	.016	.272	1.013	.999	.272	.030

Calculations for the call and put option with X=6 and *f*=7

A. Call.
Substituting into (XV.4):
$d1 = [\ln(7/6) + .5 \text{ x } (.20^2) \text{ x } .2466]/[.20 \text{ x } .2466^{.5}] = 1.6018$
$d2 = [\ln(7/6) - .5 \text{ x } (.20^2) \text{ x } .2466]/[.20 \text{ x } .2466^{.5}] = 1.5025$

Now, look for the cumulative normal distribution at z=1.6018 The number you get is .4454. But, since d1 (z) is positive, in order to get the whole area under the curve up to z=1.6018, you need to add .50 to .4454.

N(d1=.9635) = .9454
N(d2=.8890) = .9335

c = .9828 x [7.0 x .9454 - 6 x .9335] = .99931

B. Put.
Substituting into (XV.5):
p = .99931 + .9828 (6 -7.0) = .0172. ¶

The following example shows how these values should be interpreted.

Example XV.24: Consider the option in Table XV.B with strike X = 6.0 and forward interest rate f = 7.0. It has value of c = .99923. Let's analyze the meaning of this number.

Since X and f are in percent, the price c is also stated in percent. In order to translate this price to a dollar amount, we have to know the option size and the duration in days of the forward interest period.

Suppose the option is based on three-month LIBOR and involves a nominal amount of USD 10 million. Also suppose there are ninety-two days in the three-month period. Then the dollar cost of the option is:

.99923 x (1/100) x (92/360) x USD 10,000,000 = USD 25,535.88.

The values in Table XV.A also assume that the option premium is paid today, and that the cash in the option payoff is received at expiration, which is the beginning of the forward interest or FRA period. If either of these assumptions is false, there will have to be a further interest adjustment.

For example, suppose the cash in the option payoff will not be received until the **end** of the forward interest period (92 days). Then the table value (for X=6.0, f=7.0) must be discounted by the forward interest rate f=7.0 for 92 days:

.99923/[1 + .07 x(92/360)] = .98167.

This corresponds to an option premium of USD 25,087.12. ¶

We now proceed to price options on Eurodollar futures. Since the price of a Eurodollar future Z may be written as

$Z = 100 - f \ \Rightarrow \ f = 100 - Z.$

Note that $f \geq X \ \Rightarrow \ Z \leq 100 - X.$

Thus a call on f, which pays off when $f > X$, is equivalent to a put on Z, which pays off when $Z < 100 - X$.

Example XV.25: Let X=6. A call on the interest rate f has a positive exercise value when $f > 6$. But this is equivalent to the condition the Eurodollar futures price Z < 100 - 6 = 94. So the value of an interest rate call with strike price 6 is equal to the value of a Eurodollar futures put with strike price 94.

Note: The value of a call on the forward interest rate f with strike price X is equal to the value of a put on the Eurodollar future $Z = 100 - f$ with strike price 100 - X. ¶

At the CME, Eurodollar options are American in style, so to price Eurodollar options we need to use the American counterpart to equations (IX.4) and (IX.5), which allow for early exercise of the option.

Example XV.26: The Eurodollar future is at 94. To obtain the value of a future call with strike price of 93, we calculate f=100 - 94 = 6 and X=100 - 93 = 7. Thus the value of the Eurodollar futures call is the value of an American put on the forward rate f = 6 with a strike price of X = 7.

Table XV.C is the same as Table XV.B, the Eurodollar futures prices and strikes, however, represent their interest rate equivalent, and the options are American.

Table XV.C

		Value of Eurodollar Futures Options					
			Call			Put	
	Z:	92	93	94	92	93	94
X:	93.00	1.020	.277	.017	.001	.277	1.009
	94.00	1.977	1.005	.238	.033	.018	.238

Thus, the call value of .999, corresponding to X=6 and *f*=7 in Table XV.B, should be compared to the put value 1.005, corresponding to X=94 and Z = 93 in Table XV.C. The two numbers differ; for example, .999 is the European value while 1.005 is the corresponding American value. ¶

4.B Caps, Floors, and Collars

Many financial contracts in the international financial markets are based on periodic, but variable or "floating," interest payments, such as six-month LIBOR. Some parties to these contracts are at risk (as borrowers) if these series of interest payments rises dramatically. A solution to reduce this upside interest rate risk is to place a ceiling (or *cap*) on interest rate levels. Other parties are at risk (as lenders) if the level of interest payments falls. A solution for this downside interest rate risk is to place a *floor* under interest rates, so they do not fall too low.

Caps and floor evolved from interest rate guarantees that fixed a maximum or minimum level of interest on floating-rate loans. The advent of trading in over-the-counter interest rat caps and floor dates back to 1985, when banks began to strip such guarantees from floating-rate notes to sell to the market. The leveraged buyout boom of the 1980s spurred the evolution of the market for interest rate caps. Firms engaged in leveraged buyouts typically took on large quantities of short-term debt, which made them vulnerable to interest rates increases. As a result, lenders began requiring such borrowers to buy interest-rate caps to reduce the risk of financial distress.

The mechanics of these contracts are very simple. For example, suppose a LIBOR borrower buys an interest rate cap on six-month LIBOR. The buyer of the cap pays the seller a premium in return for the right to receive the difference in the interest cost on some notional principal amount any time LIBOR rises above the stipulated cap. Payments amounts are determined by the value of LIBOR on a series of interest rate reset dates. The market interest rate on the first six-month interval is already known, and is typically excluded from the cap. Cap buyers typically schedule interest rate reset and payment intervals to coincide with interest payments on outstanding floating-rate debt.

Example XV.27: A one-year floor.
On October 24, 1999 a Euroswiss Franc (LIBOR) borrower buys a one-year interest rate floor of 6.85%, with three-month Euroswiss payments on May 20, August 20, November 20, and February 20. The nominal amount specified in the floor is CHF 5,000,000. A new three-month interval will begin on February 20 and extend to the following May 20. The interest rate for this period will be fixed on February 20, 1995 but interest will be paid on the following May 20. Three-month Euroswiss is fixed at 6.50% on February 20. On May 20 (89 days later) the floor writer will pay the floor buyer:

CHF 5,000,000 x (6.68 - 6.5)/100 x (89/360) = CHF 2,225. ¶

♦ **A cap is a portfolio of calls**

The cap is a series of European call options on the interest rate (i), where the strike price is the cap rate (i_c). The amount of the payment is determined by:

Notional amount x Max(0, i - i_c) x days to reset date/360.

Similarly, a floor is a series of European put options on the interest rate, where put strike price is the floor rate (i_f). The amount of the payment is determined by:

Notional amount x Max(0, i_f - i) x days to reset date/360.

The first option begins at the beginning of the period and expires on the first interest reset date. ♦

Example XV.28: In Example XV.27, the first option begins on November 20, 1995 and expires on February 20, 1996 (a total of 91 days). The underlying variable in the option is the 3-mo implied forward (or FRA) rate from February 20 to the following May 20. The option expires on February 20 because the rate is set (reset) or determined on that date. But the cash value of the option will not be received for another 89 days (on the following May 20):

Rate Fixed
↓
November 20 February 20 May 20
91 Days 89 Days
Option Begins Option matures Cash Received

A collar is a combination of calls and puts. The buyer of an interest rate collar purchases an interest rate cap, while selling a floor indexed to the same interest rate. Borrowers with floating-rate loans buy collars to limit effective borrowing costs between some maximum, determined by the cap, and a minimum, determined by the floor. Buying a collar limits a borrower's ability to benefit from a significant drop in market interest rates. Collars, however, have the advantage of having a significant lower premium. If the strike prices are equal (i_c=i_f) , one is left with a series of regular forward or FRAs with the strike price as the forward rate.

Example XV.29: A two-year collar.
On April 13, 2000 two parties negotiate a two-year collar to cap six-month Euroyen (LIBOR) at 4.5%, while placing a floor at 3.75%. Euroyen LIBOR is currently at 3.20%. The reset dates are October 15 and April 15. The collar is based on a notional amount of JPY 200,000,000. On October

15, 2000, Euroyen LIBOR is 3.65%. Therefore, the collar writer will receive from the collar buyer on April 15, 2001 (183 days later):

JPY 200,000,000 x (3.75 - 3.65)/100 x (183/360) = JPY 101,667.

On April 15, 2001, Euroyen LIBOR is 4.63%. Then, the collar writer will pay the collar buyer on October 15, 2001 (182 days later):

JPY 200,000,000 x (4.63 - 4.50)/100 x (182/360) = JPY 131,444. ¶

4.B.1 Valuation of a Cap

A cap is a series of European options. The value of the cap is equal to the sum of the value of all the options imbedded in the cap.

Example XV.29: Valuation of a cap as a series of options.
Consider a nine-month interest rate cap of 6% on three-month Euribor. The cap amount is EUR 10 million. The cap trades on March 25 for effect on March 27. The Euribor reset rates are determined on each June 25 and September 25, and take effect two days later. There are 92 days from March 25 to June 25. At the time the cap is purchased, offered rates on time deposits are:

Period	Offered Rate
3-month	6.00
6-month	6.30
9-month	6.65
12-month	6.75
18-month	6.90

There are two options in the cap. The first three months' rate of interest is already determined at 6%. Option #1 is thus written on the second six-month period, and is an option on the implied forward or FRA rate between the three-month rate of 6% and the one-year rate of 6.30%. Option #2 is written on the third three-month period, and is an option on the implied forward or FRA rate between the 6-mo. rate of 6.30% and the 9-mo. rate of 6.65%.

Calculating the implied forward rate from the formula for Option #1:

$$[1 + .063x(183/360)] = [1 + .06 \text{ x } (92/360)] \text{ x } [1 + f \text{ x } (91/360)] \Rightarrow f = .065036.$$

The option expires in three months, but does not settle until the end of the second three-month period, which is six months from today. So the discount rate on the option is 6.30%. This gives a discount factor of

$$[1 + .063 \text{ x } (183/360)] = 1.0320.$$

The other forward (FRA) rates and discount factors may be calculated in a similar way.

Option #	Implied Forward Rate	Discount Factor
1	6.5036	1.0320
2	7.1206	1.0502

The next step is to impute volatilities to each time period. Based on recent activity in the market for caps, these are assumed to maturity be 10 percent.

We now have the information needed to price each option:

Option #	t=Tx365	*f*	X	σ	B (adjusted)	Call Value	EUR Amount
1	92	6.5036	6	.10	1/1.0320	.4949	EUR 12,510.5
2	183	7.1206	6	.10	1/1.0502	1.0681	EUR 26,998.7

Option # 1 has a value of .4949. There are 183 - 92 =91 days in the interest period, this corresponds to a dollar amount of (.4949/100) x (91/360) x EUR 10,000,000 = EUR 12,510.5.

The total value of the cap is equal to the value of Option #1 plus the value of Option #2. That is, the value of the cap is EUR 39,509.2. ¶

♦ **Valuation of a Floor**
Recall that a floor is a series of European put options on the interest rate, where the put strike price is the interest floor. Therefore, the value of a floor contract is equal to the value of the puts imbedded in the contract. ♦

4.B.1.i Cap Packaging

Banks often hedge their option writing by borrowing funds at a variable rate with an interest cap. In addition, cap packaging might help the bank to increase its profit margin without taking additional risks.

Example XV.31: Matsua Bank faces the following alternative operations:
a. Lend money to company A at TIBOR + 7/8%.
b. Borrow money from investors at TIBOR + 3/8% with a cap at 3%.
c. Sell a cap option at 3% to company B for 1/2% per year.

An alternative for Matsua Bank is to lend to company A at (TIBOR + 7/8) and borrow from investors at (TIBOR + 1/8) without any cap. In effect the margin is equal to 3/4%.

Let's analyze the operation. Matsua Bank's net income is given by:
(TIBOR + 7/8) - min(TIBOR + 3/8,3) + ½ -max(0,TIBOR-3).

If TIBOR remains below 3%, Matsua Bank's net income per year is:
(TIBOR + 7/8) - (TIBOR + 3/8) + ½ = 1%

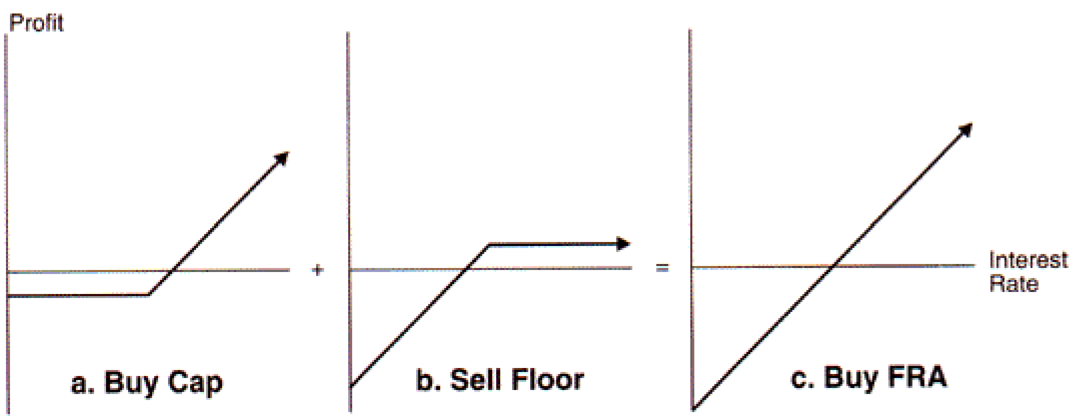
If TIBOR increases beyond 10%, Matsua Bank's net income per year is:
(TIBOR + 7/8) - (3) + 1/2 - (TIBOR - 3)= (7/8 + 1/2) = 1.375% ¶

4.C LIBOR Options and FRAs

At inception, an FRA has zero value to both parties. Therefore, it has no credit risk. Potential credit risk is bilateral: a party to an FRA is exposed to credit risk when the value of the agreement becomes positive to him or her. The value of an FRA changes as the forward rate changes. Banks can use interest rate options to value FRAs and estimate their associated credit risks.

Since caps and floor can be decomposed into a sequence of European call and put options, buying a cap and selling a floor with the same strike price and interest rate reset and payment dates effectively creates a sequence of FRAs, all with the same forward rate. Exhibit XV.3 shows a one-period collar (long the cap, short the floor). The payoff of the collar replicates the payoff of an FRA with a forward interest rate equal to the cap/floor rate. This result is a consequence of put-call parity.

Exhibit XV.3
Put-call Parity



▶ **FRAs and LIBOR Options: Cash Flow Timing** The equivalence is in terms of *value*. But the cash flow on an FRA is received at the beginning of the FRA period, whereas the cash flow for the options is received at the end of the FRA period. ◀

To summarize:

Long a LIBOR call + Short a LIBOR put = FRA bought.

Similarly,

Short a LIBOR call + Long a LIBOR put = FRA sold.

Example XV.32: Go back to Example XV.23. You want to buy an FRA with a strike price of 6, when f is equal to 7.0. From Table XV.B, we obtain the value of a call and a put with X=6.0 and f=7.0. Therefore, the value of an FRA is .983 (=.999-.016). ¶

Interesting reading: Chapter 9 and Chapter 10 of **International Financial Markets**, by J. Orlin Grabbe. (This chapter is partially based on those Chapters.)

<u>Exercises</u>:

1. Go back to Example XV.7. Calculate, on October 24, 1994, the price of the synthetic one-year zero-coupon bond (replicated with the one-year Eurostrip).

2. Ms. Mueller, a money manager, expects to receive CHF 4,000,000 on the date that the December 97 Euro Swiss franc futures expires. LIFFE has an active Euro-CHF futures market. LIFFE's Euro-CHF contract works like the CME's Eurodollar futures contract. The size of each contract is CHF 1 million. Ms. Mueller wants to invest the money in the Euro-CHF market. She is concerned about the potential short-term volatility of CHF interest rates.

Today, January 2, 1997, the March, June, September and December futures are trading at 96.88, 96.82, 96.66 and 96.45, respectively.

(a) Construct a long Euro-CHF strip.
(b) Outline a hedge for Ms. Mueller.
(c) How much should Ms. Mueller pay for a zero-coupon CHF bond?

3. Go back to Example XV.9. Now, on October 24, 1994, Housemann Bank wants to price a two-year fixed-for-floating interest rate swap against 6-month LIBOR, which starts on December 94. The fixed rate will be paid semiannually and, therefore, is quoted semiannually bond basis. Find the swap coupon mid-rate.

4. On September 9. Okinawa Bank, a Japanese bank, is quoted a rate of .78 % on three-month Euroyen deposits in the interbank market. The amount to be deposited at this rate is JPY 300,000,000. The bank can lend a nine-month Euroyen deposit at .91%, with a value date on September 13 and maturity date on June 13. Okinawa Bank decides to manage this gap risk using December and March Euroyen futures.

TIFFE lists a Euroyen contracts. The characteristics of this contract are the same as the CME's Eurodollar contract, but with a face value is JPY 100,000,000. On September, December Euroyen futures are trading at 99.24 and March Euroyen futures are trading at 99.15.

Should Okinawa Bank take the JPY deposit?

5. Consider a Hong Kong bank facing the following operations:
a. Lend HKD 10,000,000 to company Dideraux at the HIBOR + ½.
b. Could borrow HKD 10,000,000 at HIBOR + ¼ with a cap of 6% or borrow at HIBOR + ½.
c. Sell a cap option at 7% to company Belabu for ¾ a year.
Should the Hong Kong bank engage in cap packaging?

6. Reconsider Example XV.30, but now you buy three-year interest rate cap of 9% on an annual Euribor. (Hint, now you have two options in the cap.)

7. Consider a contract that floors the interest on a GBP 1,000,000 deposit at 7% for three months starting in one year. LIFFE has an active Eurosterling contract with a size of GBP 500,000. Suppose that the LIFFE's Eurosterling futures interest rate for a 3-month period starting one year is 7.15%; the current one-year interest rate is 6.7%; and the volatility of the 90-day forward rate is 20% (σ=.25). Value the floor.

8. Many swap dealers are also cap dealers. Consider the following situation. AAA Co. has a comparative advantage in the fixed-rate market, but wants rate-capped floating-rate debt. Structure a solution for AAA.

9. Bertoni Bank has a large client that is looking for a three-month loan, starting in three months, for EUR 100 million. The Bertoni's client wants to fix the loan's interest rate now. Bertoni Bank will make a loan commitment only if it can lock in the cost of funds through the FRA market or Eurodollar futures market.

Bertoni Bank seeks on December 1, 1999 to fund a Euro 100 million three-month loan at Euribor plus 50 basis points, which is 6 9/16 plus 1/2 of 1 percent. You work for Bertoni Bank. Using the data provided below, you have to advise the board of directors on the feasibility of the EUR 100 million loan. Estimate the cost and benefit of interest rate hedging using FRA and Euribor futures.

	Euribor -3 mo.		FRA -3 mo.
1MO	6 1/4 - 6 3/8	1X4	6.45 - 6.51
2MO	6 3/8 - 6 1/2	2X5	6.51 - 6.57
3MO	6 3/8 - 6 1/2	3X6	6.53 - 6.59
4MO	6 13/32 - 6 9/16	4X7	6.54 - 6.60
5MO	6 7/16 - 6 9/16	5X8	6.52 - 6.58
6MO	6 7/16 - 6 9/16	6X9	6.58 - 6.64

10. The Case of Banco del Suquia.
Banco del Suquia is a small, conservative bank, located in Córdoba, Argentina. Banco del Suquia has total assets of USD 78.3 million and variable costs are around 2% of loans. Córdoba, the second largest city in Buenos Aires, is the capital of the Province of Córdoba, and it is located 600 miles northwest of Buenos Aires, the Argentinean capital and financial center.

In July 1992, two longtime and reliable clients of Banco del Suquia, Tantor SA and La Sierrita SA, have approached Banco del Suquia management in order to secure two loans. Since the Argentinean economy is experiencing a boom in consumer demand, both companies are currently modernizing and expanding their production facilities. Tantor SA wants a USD 1 million fixed-rate loan for one year and La Sierrita SA wants a two-year floating-rate loan for USD 1 million.

Tantor's fixed-rate loan would have an interest rate of 17%, and La Sierrita's floating-rate loan would have a spread of 8 points over LIBOR.

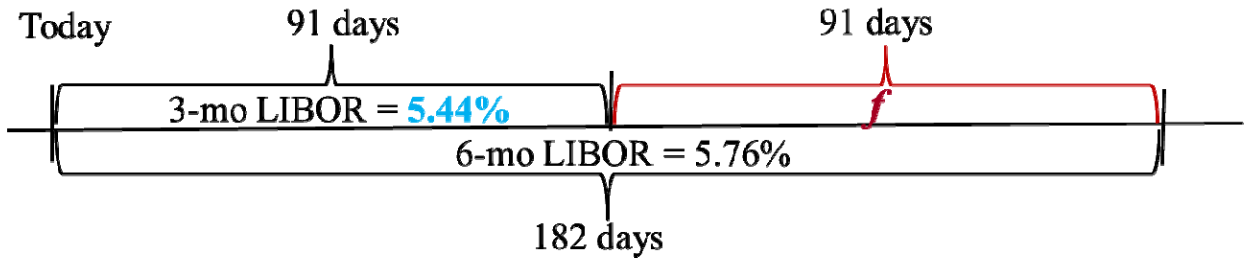
Given the magnitude of the loans, management decides to consider issuing bonds for a total amount of USD 2 million. In Argentina, similar bonds have a semiannual coupon rate of 11.5%.

Since Banco del Suquia has a very conservative management, it will proceed with the bond issue only if interest rate risk is minimized. Therefore, Banco del Suquia is considering a FRA for the fixed-rate loan and a floor for the floating-rate loan. Casullo Financial Services, CFS, an Argentinean Investment Bank, quotes one year against two year FRAs at 14.25-15.125. Using its international connections, CFS will buy a floor for Banco del Suquia, if needed.

The annual offered rates are:

6 mo.	5.4%
12 mo.	5.5%
18 mo.	5.9%
24 mo.	6.9%

Assume volatility is equal to .15. Calculate the value of the floor. Would you recommend management to issue the bonds? Consider the effect of different interest rate scenarios in your decision.


Today
91 days
91 days
3-mo LIBOR = 5.44%
f
6-mo LIBOR = 5.76%
182 days