CASE III: SOLUTIONS

1) Annualized Data (1993:2019):

Market	USD return_i	SD_i	$B_{i\,world}$	RVOL
Brazil	0.1635867	0.371214	**1.528336**	**0.091324**
India	0.1109195	0.282615	**0.947257**	**0.085486**
Switzerland	0.0957667	0.156978	**0.842868**	**0.08513**
US	0.0889213	0.142635	**0.93311**	**0.069561**
South Korea	0.1166363	0.351077	**1.379531**	**0.067141**
South Africa	0.0966861	0.260408	**1.158285**	**0.062742**
Mexico	0.0900206	0.274592	**1.231778**	**0.053587**
France	0.0706108	0.194335	**1.177949**	**0.039558**
Singapore	0.067527	0.241256	**1.164969**	**0.037352**
China	0.0470356	0.325592	**1.150143**	**0.020017**
UK	0.0425986	0.153744	**0.940358**	**0.019764**
World	0.07046	0.143626	-	

Risk-free Rate	**0.0**
SD_World	**0.1436**

Equally weighted Return	0.08267
Equally weighted Beta	1.09262
RVOL-equally weighted	0.05369

2) Optimal portfolio:

	USD r_t	SD	B_{world}	RVOL	$\sigma^2_{\varepsilon i}$	$(r_i-r_f)(\beta_i/\sigma^2_{\varepsilon i})$	$\sigma^2_m\beta_i^2/\sigma^2_\varepsilon$	C_i
Brazil	0.16359	0.37121	1.52834	0.09132	0.0896	2.3803	0.5377	**0.03193**
India	0.11092	0.28262	0.94726	0.08549	0.0614	1.3416	0.3017	**0.04174**
Switzerland	0.09577	0.15698	0.84287	0.08513	0.0100	6.0558	1.4674	**0.06100**
US	0.08892	0.14264	0.93311	0.06956	0.0024	25.4074	7.5346	**0.06695**
South Korea	0.11664	0.35108	1.37953	0.06714	0.0840	1.5212	0.4674	**0.06696**
South Africa	0.09669	0.26041	1.15829	0.06274	0.0401	2.0972	0.6895	**0.06671**
Mexico	0.09002	0.27459	1.23178	0.05359	0.0441	1.8436	0.7097	**0.06598**
France	0.07061	0.19434	1.17795	0.03956	0.0091	6.0035	3.1307	**0.06076**
Singapore	0.06753	0.24126	1.16497	0.03735	0.0302	1.6781	0.9268	**0.05946**
China	0.04704	0.32559	1.15014	0.02002	0.0787	0.3364	0.3466	**0.05867**
UK	0.04260	0.15374	0.94036	0.01976	0.0054	3.2389	3.3805	**0.05225**

Included: Brazil (12.40%), India (8.56%), Switzerland (45.92%), USA (32.92%), & S. Korea (0.21%).

Optimal Portfolio
Beta   0.9676
RVOL   0.0819

3) Constrained (50% US) Optimal portfolio:

	Z_i	ω_i	ω_i
Brazil	0.4197	**0.1240**	0.0924
India	0.2898	**0.0856**	0.0638
Switzerland	1.5542	**0.4592**	0.3422
US	1.1143	**0.3292**	**0.5**
South Korea	0.0070	**0.0021**	0.0015
South Africa	0.0000	0.0000	0.0000
Mexico	0.0000	0.0000	0.0000
France	0.0000	0.0000	0.0000
Singapore	0.0000	0.0000	0.0000
China	0.0000	0.0000	0.0000
U.K.	0.0000	0.0000	0.0000

Constrained Portfolio
Beta   0.9588
RVOL   0.0788

No, performance of portfolio is down. Lower RVOL (and lower raw returns, too.) The well-known risk-return trade-off appears: Caps can reduce the risk –in this case the beta- of the portfolio; but they can reduce returns. It can limit exposure and/or improve diversification.

4) 1993-2019 performance of portfolios:

The optimal portfolio is, obviously, the best –it was chosen to specifically do well!. But, it looks too undiversified. As it is usually pointed out in the mutual fund industry, past returns do not necessarily help to predict future performance.

5) 2020-2021 (Using the Betas computed during this period. If you used the old Betas, OK)

Out-of-sample	USD retruns	SD	B_{world}	RVOL
Brazil	-0.041487	0.452467	1.70446	-0.0257
India	0.2955463	0.301783	1.12304	0.26104
Switzerland	0.1422436	0.14356	0.56654	0.24687
US	0.2645527	0.20718	0.95441	0.27469
South Korea	0.2562338	0.245836	0.87622	0.28971
South Africa	0.0620658	0.310347	1.16513	0.05122
Mexico	0.1557071	0.319469	1.23041	0.12461
France	0.1408318	0.271096	1.14506	0.12091
Singapore	-0.003076	0.249298	1.01515	-0.0054
China	0.0817959	0.199171	0.31261	0.25403
UK	0.0037133	0.236316	1.02976	0.00129

R_f	0.002385
SD_World (σm)	**0.1480**
C*	**0.0675**

Optimal Portfolio

Beta	0.8836
RVOL	0.1932

Constrained
Portfolio

Beta	0.9016
RVOL	0.2152

Equal Portfolio

Beta	1.0093
RVOL	0.1318

It is clear that using the optimal portfolio weights from 1993-2019 data did not produce the best return for the 2-year period 2020-2021. The constrained portfolio provided the best return. The foreign investment cap proved to be beneficial in this instance, since it minimizes the negative impact of Brazil and magnifies the impact of the good USA performance.