# **FX RISK & HEDGING**

FX Futures, Options & FX Exposure

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# Last Class Model of FX Rate Determination not very successful. Parity Condition: PPP, IFE, EH Rejected by data, some long-run support. Structural Model: BOP (Balance of Trade) & Monetarist Approach Rejected by data, some event studies support. Random Walk Difficult to beat in forecasting performance. Foresting S<sub>t</sub> with different approaches: Fundamental Models: Based on macroeconomic data Example with USD/EUR exchange rate. TA Models: Based on past prices (S<sub>t-1</sub>, S<sub>t-2</sub>, S<sub>t-3</sub>, ...)

#### • This Class

Main take away: Forecasting is difficult, especially in the short-run.
 ⇒ Managing FX Risk becomes very important.

• Hedging FX Risk with Market-based Tools:

- Market-tools:
  - Futures/Forward
  - Money Market (IRPT strategy)
  - Options

#### - FX Exposure:

- Transaction Exposure (TE)
- Translation Exposure
- Economic Exposure (EE)

# **FX Risk**

**Example**: Spec's, the Texas liquor store chain, imports wine from Europe. Spec's has to pay **EUR 5,000,000** on May 2. Today, February 10, the exchange rate is  $S_t = 1.10$  USD/EUR.

Situation: Payment due on May 2: EUR 5M.  $S_{t=Feb \ 10} = 1.10 \ \text{USD/EUR}.$ 

Problem:	S <sub>t</sub> is difficult to forecast	$\Rightarrow$ Uncertainty.
	Uncertainty	$\Rightarrow$ Risk.
	<u>Example</u> : on May 2, $S_{t=N}$	$_{May 2}$ > or < 1.10 USD/EUR
Λ + <b>S</b>	Spag's total payment way	ld bo

At  $S_{t=Feb \ 10}$ , Spec's total payment would be: EUR 5M \* 1.10 USD/EUR = USD 5.50M.





<u>Definition</u>: A forward contract is an agreement written today, between two parties (one party is usually a bank), to exchange a *given amount* of currencies at a given *future date* at a *pre-specified exchange rate*,  $F_{t,T}$ .

Given amount: Size		
Future Date: Maturity = $Delivery Date = T$		
Forward markets: Tailor-made contracts (& illiquid)		
	Location: None.	
	Reputation/collateral guarantees the contract.	
Futures markets.	Standardized contracts (& liquid)	
i utures manets.	Location: Organized exchanges	
Clearinghouse guarantees the contract.		

	Futures	Forward
Size	Standardized	Negotiated
Delivery Date (T)	Standardized	Negotiated
Counter-party	Clearinghouse	Bank
Collateral	Margin account	Negotiated
Market	Auction market	Dealer market
Costs	Brokerage and exchange fees	Bid-ask spread
Secondary market	Very liquid	Highly illiquid
Regulation	Government	Self-regulated
Location	Central exchange floor	Worldwide

#### FX Futures/Forwards: Basic Terminology

Two parties: - A *buyer*, with the *long* FC position; - A *seller*, with the *short* FC position.

*Short*: Agreement to **Sell**.

*Long*: Agreement to **Buy**.

Contract Size: Number of units of FC in each contract.

*CME Expiration dates*: Mar, June, Sep, and Dec + Two nearby months (on the third Wednesday of expiration month)

*Margin account*. Funds deposited with a broker to cover possible losses involved in a futures/forward contract.

Initial Margin: Initial level of margin account.

Maintenance Margin: Lower bound allowed for margin account.

Settlement: FX futures can be cash-settled or physically delivered.

#### • Margin Account

A margin account is like a checking account you have with your broker, but it is *marked to market*. At the end of the day, if your contracts make (lose) money, money is added to (subtracted from) your account

**Example**: March GBP/USD CME futures (contract size = **GBP 62,500**) Today, a traders starts a **long 2 March GBP** contract position (= GBP 125,000).

Tomorrow, the March GBP futures increases by **USD 0.01**, then, USD 1,250 (=**USD .01** \* 125,000) are added to the trader's margin account.

If in 2 days, the March GBP futures decreases by **USD 0.02**, then, USD 2,500 (= **-USD .02** \* 125,000) are subtracted from the trader's margin account. ¶

If margin account goes below maintenance level, a *margin call* is issued: ⇒ you have to add funds to restore the account to the initial level. Example: GBP/USD CME futures (contract size = GBP 62,500) Initial margin: USD 2,800 Maintenance margin: USD 2,100 If losses do not exceed USD 700 ⇒ no margin call. If losses accumulate to USD 850 ⇒ margin call: add USD 850 to account. ¶

• Settlement of FX futures

Prior to expiration, traders can:

- Close out open positions, taking their contract to expiration
- **Extend** open positions without holding the trade to expiration.

When traders take their contract to expiration, settlement occurs.

# Details for FX futures:

The **last trading day** is, in general, the 2nd B-day prior to the 3rd Wednesday of the contract month (usually, last trading day = **Monday**).

Expiring contract stops trading  $\Rightarrow$  it needs a **settlement price**.

# Computation of Settlement price:

- From last 30" of trading of the contract, CME gets a value weighted average price (VWAP).
- **Calendar spread** (= Price of expiring contract Price of next contract)
- Settlement price = VWAP + Calendar spread.

• Settlement of FX Futures

Example: GBP/USD CME futures

On the Monday preceding expiration, the expiring Dec contract is trading at **1.5530**, and the next deferred contract, March, is trading at **1.5610**.

The *calendar spread* is **-0.0080**, or 80 ticks (= **1.5530** - **1.5610**).

GBP futures stop trading at 9:16 AM CT on that Monday. In the final 30" of trading, CME Clearing determines the VWAP for Dec: 1.5620.

- Settlement Price (of expiring contract): 1.5620 - 0.0080 = 1.5540.

- Short side deposits GBP 62,500/contract in approved agent bank.

- Long position deposits **USD 97,125** (= **1.5540** \* **62,500**)/contract in approved delivery bank.

- Cash vs FC are exchanged over bank wire. All completed by **10:00 AM CT** on the **3rd Wednesday** of Dec (2 B-days after last trading day). ¶

# Settlement

CME FX futures can be cash-settled (BRL, RUB) or physically delivered (GBP, EUR, JPY). We went over a detailed GBP futures example.

<u>Note</u>: For cash-settled FX futures, process is simpler. Any profit/loss is calculated as the difference between the final **settlement price** (1.5540) and previous day's mark-to-market.

Like any other futures contract, an FX trader with an open position may decide to offset or roll forward a position to avoid expiration and delivery. This is the usual situation for FX speculators, who have no desire to take delivery.

# **Using FX Futures/Forwards**

• Iris Oil Inc. will transfer **CAD 300 million** to its USD account in 90 days. To avoid FX risk, Iris Oil decides to *short* a USD/CAD Forward contract.

<u>Data</u>:  $S_t = .8451 \text{ USD/CAD}$  $F_{t,90\text{-}day} = .8493 \text{ USD/CAD}$ 

In 90-days, Iris Oil will receive with certainty:

(CAD 300M) \* .8493 USD/CAD = USD 254,790,000.

<u>Note</u>: The exchange rate at in 90 days  $(S_{t+90})$  is, now, irrelevant.



# **Hedging with FX Futures Contracts**

• FX Hedger

FX Hedger reduces the exposure of an *underlying position* to currency risk using (at least) another position (*hedging position*).

Basic Idea of a Hedger

A change in value of an underlying position is compensated with the change in value of a hedging position.

Goal: Make the overall position insensitive to changes in FX rates.

Hedger has an overall portfolio (OP) composed of (at least) 2 positions:

(1) Underlying position (UP)

(2) Hedging position (HP) with negative correlation with UPValue of OP = Value of UP + Value of HP.

 $\Rightarrow$  Perfect hedge: The Value of the OP is insensitive to FX changes.

• Types of FX hedgers using futures:

i. *Long hedger*: UP: *short* in the foreign currency. HP: *long* in currency futures.

ii. *Short hedger*: UP: *long* in the foreign currency.HP: *short* in currency futures.

Note: Hedging with futures is very simple: Take an opposite position!

Q: What is the proper size of the Hedging position?
1. Dasie (Naive) Approach. Equal heuge
Modern Approach: Optimal hedge
• The Basic Approach: Equal hedge
Equal hedge:
Size of $UP = Size$ of HP.
Example: Long Hedge and Short Hedge
(A) <i>Long</i> hedge.
A U.S. investor has to pay NOK 2.5M (Norwegian kroners) in 90 days
$\Rightarrow$ UP: Short NOK 2.5M.
HP: <i>Long</i> 90 days futures for NOK 2.5M.
(B) <b>Short</b> hedge
A US investor has CBD 1M invested in British silts
IN U.S. INVESTOR HAS ODT IN INVESTED IN DITUSIT BILS.
$\Rightarrow$ UP: Long <b>GBP</b> 1 <b>M</b> .
HP: <i>Short</i> futures for <b>GBP 1M</b> .

Define:

 $V_t$ : value of the portfolio of foreign assets measured in GBP at time t.  $V_t^*$ : value of the portfolio of foreign assets measured in USD at time t.

**Example (continuation)**: Calculating the *short hedge*r's profits. It's September 12 (t=0). The investor in (b), with a long GBP 1M position, is uncertain about  $S_{t=Dec}$ . Decides to hedge using Dec futures.

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Situation: UP = GBP 1M in British bonds.

Data:

F_{Sep 12,Dec} = 1.55 USD/GBP

Futures contract size: GBP 62,500.

S_{Sep 12} = 1.60 USD/GBP.

Number of contracts = ?

HP: Investor shorts (sells Dec futures)

GBP 1M / (62,500 GBP/contract) = 16 contracts.
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<b>Example</b> (continuation): Calculating the <i>short hedge</i> r's profits.				
• On October 29, prices (S <sub>t</sub> & $F_{t,T=Dec}$ ) have changed. Now we have:				
	<u>Sep 12</u>	Oct 29	Change	
$V_t$ (GBP)	1,000,000	1,000,000	0	
$V_t^*$ (USD)	1,600,000	1,500,000	-100,000	
S <sub>t</sub>	1.60	1.50	0.10	
$F_{t,T=Dec}$	1.55	1.45	0.10	
USD change in UP ("long GBP bond position"): $V_t^* - V_0^* = V_t S_t - V_0 S_0 = V_0 * (S_t - S_0)$ USD 1.5M - USD 1.6M = -USD 0.1M. ( $V_t = V_0 = GBP \ 1M$ )				
USD change in HP ("short GBP futures position"): $-V_0 * (F_{t,T} - F_{0,T}) = \text{Realized gain}$ (-GBP 1M) * USD/GBP (1.45 - 1.55) = USD 0.1M.				
USD Change in OP = USD Change of UP + USD Change of HP = $0$ $\Rightarrow$ This is a <i>perfect</i> hedge! ¶				

Note: In this example, we had a perfect hedge. We were lucky!  $V_{Sep \ 12} = V_{Oct \ 29} = \text{GBP 1M}$   $(\mathbf{F}_{Sep \ 12, Dec} - \mathbf{S}_{Sep \ 12}) = (\mathbf{F}_{Oct \ 29, Dec} - \mathbf{S}_{Oct \ 29}) = \text{USD .05}$ An equal position hedge is not a perfect hedge if: (1)  $V_t$  changes.  $(V_t \neq V_0)$ (2) The basis  $(F_{t,T} - S_t)$  changes.

• Changes in V<sub>t</sub> If  $V_t$  changes, the value of OP will also change. **Example**: Reconsider previous example.  $F_{Oct 29,Dec} = 1.45 \text{ USD/GBP.}$ On October 29:  $S_{Oct 29} = 1.50 \text{ USD/GBP.}$  $V_{oct 29}$  increases 2% because of interest payment. Size of UP (long): GBP 1.02M (2% higher) Size of HP (short): GBP 1M. USD change in UP (long GBP bond) is  $V_{Oct 29}^{*}$  (long) = **GBP 1.02M** \* 1.50 USD/GBP = USD 1,530,000  $V_{Sep \ 12}^{*}$  (long) = **GBP 1.0M** \* **1.60 USD/GBP** = <u>USD 1,600,000</u> USD Change in  $V_t^*$  (long) = USD -70,000

USD Change in HP (short GBP futur	ees) = USD 100,000.
Net change on the overall portfolio	= USD -70,000 + USD 100,000 = <b>USD 30,000</b> . ¶
$\Rightarrow$ Not a perfect hedge: <u>Only the prin</u>	ncipal (GPB 1M) was hedged!
• To get a perfect hedge we need to he	edge principal plus interest.

Basis Change Definition: Basis = Futures price – Spot Price = F<sub>t,T</sub> – S<sub>t</sub>. Basis risk arises if F<sub>t,T</sub> – S<sub>t</sub> deviates from a constant basis per period.
If there is no basis risk ⇒ completely hedge the underlying position (including changes in V<sub>v</sub>).
If the basis changes ⇒ "equal" hedge is not perfect.
In general: if (F<sub>t,T</sub> – S<sub>t</sub>) ↑ (or "*weakens*"), the short hedger loses. if (F<sub>t,T</sub> – S<sub>t</sub>) ↓ (or "*strengthens*"), the short hedger wins.
Example (continuation): Now, on October 29, the market data is: F<sub>Sep 12,Dec</sub> = 1.55 USD/GBP & S<sub>Sep 12</sub> = 1.60 USD/GBP ⇒ Basis = -.05 F<sub>Oct 29,Dec</sub> = 1.50 USD/GBP & S<sub>Oct 29</sub> = 1.50 USD/GBP ⇒ Basis = 0 ⇒ Basis ↑ (it changes from -.05 to 0): Basis weakens.

**Example** (continuation):  $\begin{array}{l} \text{Basis}_{\text{Sep 12}} = \mathbf{F}_{\text{Sep 12,Dec}} - \mathbf{S}_{\text{Sep 12}} = \mathbf{1.55} - \mathbf{1.60} = -.05 \text{ (5 points)} \\ \text{Basis}_{\text{Oct 29}} = \mathbf{F}_{\text{Oct 29,Dec}} - \mathbf{S}_{\text{Oct 29}} = \mathbf{1.50} - \mathbf{1.50} = \mathbf{0}. \end{array}$ Compared to previous Example, basis increased from -5 points to 0 points. (The basis has weakened from USD .05 to USD 0.) Date Long Position ("Buy") Dec Futures ("Sell") September 12 1,600,000 1,550,000 October 29 1,500,000 1,500,000 Gain -100,000 50,000 <u>Note</u>:  $(F_t - S_t) \uparrow$  $\Rightarrow$  Short hedger's profits loses (**USD -50,000**). Equal hedge is not perfect!

• Basis risk

• Recall IRPT. For the USD/GBP exchange rate, we have:

$$F_{t,T} = S_t * \frac{\left(1 + i_{d=USD} * \frac{T}{360}\right)}{\left(1 + i_{f=GBP} * \frac{T}{360}\right)}$$

Assume T = 360. After some algebra, we have:

$$F_{t,T} - S_t = S_t \frac{i_{USD} - i_{GBP}}{1 + i_{GBP}}$$

The basis is proportional to the interest rate differential.

 $\Rightarrow$  As interest rates change, the basis changes too.

• Derivation of the Optimal hedge ratio Additional notation:  $n_s$ : Number of units of foreign currency held.  $n_f$ : Number of futures foreign exchange units held.  $\Rightarrow$  Number of contracts =  $n_f$ /size of the contract  $\pi_{h,t}$ : Uncertain profit of the hedger at time t. h = Hedge ratio = $(n_f/n_s)$  = number of futures per spot in UP. We want to calculate  $h^*$  (optimal h): We minimize the variability of  $\pi_{h,t}$ .  $\pi_{h,T} = \Delta S_t \ n_s + \Delta F_{t,T} \ n_f$  (Or,  $\pi_{h,T} / n_s = \Delta S_t + h \Delta F_{t,T}$ .) We want to select h to minimize:  $Var(\pi_{h,T}/n_s) = Var(\Delta S_t) + h^2 Var(\Delta F_{t,T}) + 2 h Covar(\Delta S_t, \Delta F_{t,T})$   $= \sigma_S^2 + h^2 \sigma_F^2 + 2 h \sigma_{SF}$  $\Rightarrow h^* = -\sigma_{SF} / \sigma_F^2$  Optimal hedge ratio: h\* = -σ<sub>SF</sub>/σ<sub>F</sub><sup>2</sup>. (A covariance over a variance => A (OLS) regression estimate!)
<u>Remarks:</u>

h\* is the (negative) slope of an OLS regression of ΔS<sub>T</sub> against ΔF<sub>T</sub>: ΔS<sub>t</sub> = μ + θ ΔF<sub>t,T</sub> + ε<sub>t</sub> => h\* = -θ̂ (OLS estimate of)
Recall IRPT: F<sub>t,T</sub> = S<sub>t</sub> (1 + i<sub>d</sub>)/(1 + i<sub>f</sub>) = δ S<sub>t</sub>.
Calculating changes: ΔF<sub>t,T</sub> = δ ΔS<sub>t</sub> => ΔS<sub>t</sub> = (1/δ) ΔF<sub>t,T</sub>
Then, h\* = -θ̂ estimates (1/δ) in the IRPT equation: δ=1/h\*.

(3) Two cases regarding hedge ratios:

When F<sub>t,T</sub> is denominated in the same currency as the asset being hedged, we can use IRPT to get the hedge ratio, h\*.

When F<sub>t,T</sub> is denominated in a different currency than the asset being hedged (*cross-hedging*), OLS provides an estimate of h\*.

OLS Estimation of Optimal Hedge Ratio
Consider the following regression equation: ΔS<sub>t</sub> = μ + θ ΔF<sub>t,T</sub> + ε<sub>t</sub> ⇒ OLS produces θ\* = -σ<sub>SF</sub>/σ<sub>F</sub><sup>2</sup> = - h\*
A hedge is *perfect* only if ΔS<sub>t</sub> & ΔF<sub>t,T</sub> are perfectly correlated: R<sup>2</sup> = 1. (ε<sub>t</sub> has to be always zero.)
R<sup>2</sup> measures the efficiency of a hedge: The higher, the better. **Example**: We estimate a hedge ratio using OLS for UP **GBP 1M**: We use five years of monthly data for a total of 60 observations.

$$\Delta S_t = .001 + .92 \Delta F_{t,T}, \qquad R^2 = .95.$$
  
$$\Rightarrow h = -.92.$$

Q: How many contracts?

A:  $n_{\rm f}$ /size of the contract =  $h n_{\rm s}$  / size of the contract = = -.92 \* **GBP 1M** / **GBP 62,500** = -14.7 ≈ 15 contracts sold! ¶

• The high  $R^2$  points out the efficiency of the hedge:

⇒ Changes in futures USD/GBP prices are highly correlated with changes in USD/GBP spot prices.

<u>Note</u>: A different interpretation of the  $R^2$ : Hedging reduces the variance of the CF by an estimated 95%.

*Remark*: OLS estimates of the hedge ratio are based on historical data. The hedge we construct is for a future period. Problem!

• Time-varying hedge ratios:  $h_t^* = -\sigma_{SF,t} / \sigma_{F,t}^2$ 

There are many models that are used to forecast variances over time. Popular model: GARCH models.

GARCH models: The variance changes with the arrival of news (innovations) and past variances. These models accommodate the stylized fact that big (small) changes tend to be followed by big (small) changes.

Many GARCH models and specifications. The specification depends on the number of lagged errors ( $\varepsilon_t^2$ ) and lagged variances ( $\sigma_t^2$ ).

A GARCH(1,1) specification is a good approximation. For example, for changes in  $S_t$ :

$$\sigma_{S,t}^2 = \alpha_{S0} + \alpha_{S1} \epsilon_{S,t-1}^2 + \beta_{S1} \sigma_{S,t-1}^2$$

 $\varepsilon_{S,t-1}$  = forecasting error at t-1. (Under RWM,  $\varepsilon_{S,t-1}$  is the change in  $S_{t-1}$ .)  $\sigma_{S,t-1}^2$  = variance of changes in  $S_t$ . • Time-varying hedge ratios:  $h_t^* = -\sigma_{SF,t} / \sigma_{F,t}^2$ 

GARCH models accommodate two features of financial data:

- Large changes tend to be followed by large changes of either sign.

- Distribution is leptokurtic -i.e., fatter tails than a normal.

• Statistical packages that estimate GARCH models: SAS, E-views, R.

To estimate a time-varying hedge ratio, we need a model for the bivariate distribution of  $S_t$  and  $F_{t,T}$  (to get covariances). Things can get complicated quickly.

# **Hedging Strategies**

- Three problems associated with hedging in the futures market:
  - Contract size is fixed.
  - Expiration dates are also fixed.
  - Choice of underlying assets in the futures market is limited.

#### Imperfect hedges:

- Delta-hedge when the maturities do not match
- Cross-hedge when the currencies do not match.
- Another important consideration: Liquidity.

Contract Terms (Delta Hedging)
Major decision: Choice of contract terms.
Advantages of Short-term hedging:

Short-term *F<sub>t,T</sub>* closely follows *S<sub>t</sub>*.
Recall linearized IRPT :
 *F<sub>t,T</sub>* ≈ *S<sub>t</sub>* \* [1 + (i<sub>d</sub> - i<sub>t</sub>) \* T/360]
As T → 0, *F<sub>t,T</sub>* → *S<sub>t</sub>* (UP and HP will move closely)

Short-term *F<sub>t,T</sub>* has greater trading volume (more liquid).
Disadvantages of Short-term hedging:

Short-term hedges need to be rolled over: Cost!

# • Contract Terms (Delta Hedging)

- Short-term hedges are usually done with short-term contracts.
- Longer-term hedges are done using three basic contract terms:
- Short-term contracts, which must be rolled over at maturity;
- Contracts with a matching maturity (usually done with a forward);
- Longer-term contracts with a maturity beyond the hedging period.



#### • Short Term – Rollover

Rollover ("a *roll*") occurs when a trader closes out a position in an expiring contract ("the *front month*") and simultaneously reestablishes the same position in a future month. A *roll* extends the expiration of a position.

The gain or loss on the original contract will be settled by taking the difference between the price on the day the roll is executed and the previous day's mark-to-market.

**Example:** Trader is long GBP Dec futures (expiration Dec 16) On Dec 11, GBP Dec futures trades at  $F_{Dec \ 11, Dec} = 1.5530 \ USD/GBP$ . A roll: Close Dec position on Dec 11 at USD 1.5530.

Open (simultaneously) Mar position at  $F_{dec 11,Mar} = USD 1.5620$ .

On December 10, the Dec futures was mark-to-market at USD 1.5525. Then, the long side receives USD 31.250 (=.0005 \* 62,500) when the Dec futures position is closed.  $\P$ 

#### Long Term – Close FX Futures

A hedger decides to go for a longer maturity than the date of the UP (FC receivables/payables). The hedger closes the hedging position by taking an opposite forward position with the exact remaining maturity.

**Example:** It is June 2020. Six month ago, Goyco Corporation, sold a oneyear JPY forward contract at  $F_{Dec19,Dec20} = .0105 \text{ USD/JPY}$ .

Now, a 6-month forward contract trades at  $F_{June,Dec20} = .0102 \text{ USD/JPY}$ . Goyco closes its short Dec position by buying JPY forward at  $F_{Iune,Dec20}$ .

The CFs occur at expiration. That is, in Dec 2020, Goyco Corp. receives:

 $\mathbf{F}_{0,\text{one-year}} - \mathbf{F}_{\text{t=6-mo,6-mo}} = 12.5 \text{M} * (.0105 - .01025) = \text{USD 3,125}.$ 

Assume  $i_{USD} = 6\%$ . Then, today's value of the forward contract is:

 $\frac{\mathbf{F}_{0,\text{one-year}} - \mathbf{F}_{6-\text{mo},6-\text{mo}}}{[1 + \mathbf{i}_{\text{USD}} * (180/360)]} = \frac{\text{USD 3,125}}{[1 + .06 * (180/360)]} = \text{USD 3,034.} \P$ 

Different Currencies (Cross-Hedging)

Q: Under what circumstances do investors use cross-hedging?

• An investor may prefer a cross-hedge if:

(1) No available contract for the currency she wishes to hedge.
Futures contracts are actively traded for the major currencies (at the CME: GBP, JPY, EUR, CHF, MXN, CAD, BRR).
Example: Want to hedge a HUF position using CME futures: ⇒ you must cross-hedge.

(2) Cheaper and easier to use a different contract. Banks offer forward contracts for non-major currencies. These contracts may not be liquid (and expensive!).

• Empirical results:

(i) Optimal same-currency-hedge ratios are very effective.

(ii) Optimal cross-hedge ratios are quite unstable.

**Example**: Calculation of Cross-hedge ratios. Situation: - A U.S. firm has to pay HUF 10M in 180 days. - No futures contract on the HUF. - Liquid contracts on currencies highly correlated to the HUF. Solution: Cross-hedge using the EUR and the GBP. • Calculation of the appropriate OLS hedge ratios. USD/HUF changes ( $\Delta S_{USD/HUF}$ ) Dependent variable: USD/EUR 6-mo. futures changes ( $\Delta F_{USD/EUR}$ ) Independent variables: USD/GBP 6-mo. futures changes ( $\Delta F_{\text{USD/GBP}}$ ) Exchange rates: .0043 EUR/HUF .0020 GBP/HUF  $R^2 = 0.81$ .  $\Delta S_{\text{USD/HUF}} = \alpha + .84 \Delta F_{\text{USD/EUR}} + 0.76 \Delta F_{\text{USD/GBP}}$ Number of contracts bought is given by: EUR: (-10M \* .0043 /125,000) \* -0.84 = 2.89 ≈ 3 contracts. GBP:  $(-10M * .0020 / 62,500) * -0.76 = 3.20 \approx 3$  contracts.

# **Options: Brief Review**

# Terminology

Major types of option contracts:

- calls give the holder the right to buy the underlying asset
- *puts* give the holder the right to sell the underlying asset.

Terms of an option must specify:

- Exercise or strike price (X): Price at which the right is "exercised."
- Expiration date (T): Date when the right expires.
- *Type*. When the option can be exercised: Anytime (*American*)

At expiration (*European*).

The right to buy/sell an asset has a price: The premium, paid upfront.





# The Black-Scholes Formula

• Almost all financial securities have some characteristics of financial options, the Black-Scholes model can be widely applied.

• The variation for a call FX option is given by:

$$C_t = e^{-i_f(T-t)}S_t N(d1) - X e^{-i_d(T-t)}N(d2)$$

- The Black-Scholes formula is derived from a set of assumptions:
  - Risk-neutrality.
  - Perfect markets (no transactions costs, divisibility, etc.).
  - Log-normal distribution with constant moments.
  - Constant risk-free rate.
  - Costless to short assets.
  - Continuous pricing.

• According to the formula, FX premiums are affected by six factors:			
Euro Call	Euro Put	Amer. Call	Amer. Put
+	-	+	-
-	+	-	+
?	?	+	+
+	+	+	+
+	-	+	-
-	+	-	+
	to the formula, l Euro Call + - ? + + -	to the formula, FX premiums a <u>Euro Call</u> Euro Put + - - + ? ? + + + - - + - +	to the formula, FX premiums are affected by si <u>Euro Call Euro Put Amer. Call</u> + - + - + - ? ? + + + + + - + - + - - + -

- The Black–Scholes model does not fit the data. In general:
  - Overvalues deep OTM calls & undervalue deep ITM calls.
  - Misprices options that involve high-dividend stocks.
- The Black-Scholes formula is taken as a useful approximation.
- Limitations of the Black-Scholes Model
  - Trading is not cost-less: Liquidity risk (difficult to hedge)
  - No continuous trading: Gap risk (can be hedged)
  - Log-normal distribution: Not realistic (& cause of next limitations).
  - Underestimation of extreme moves: Left tail risk (can be hedged)
  - Constant moments: Volatility risk (can be hedged)

# **Black-Scholes for FX options**

• The Black-Scholes formula for pricing currency call options is given by:

$$C = \text{call premium} = e^{-\iota_f T} S_t N(d1) - X e^{-\iota_d T} N(d2)$$

d1 =  $[\ln(S_t/X) + (i_d - i_f + .5 \sigma^2) T]/(\sigma T^{1/2}),$ d2 =  $[\ln(S_t/X) + (i_d - i_f - .5 \sigma^2) T]/(\sigma T^{1/2}).$ 

Using put-call parity, we calculate the put premium:

$$P = \text{put premium} = C - e^{-i_f T} S_t + X e^{-i_d T}$$

<u>Note</u>:  $S_t$ , X, T,  $i_d$ , &  $i_f$  are observed.  $\sigma$  is *estimated*, not observed.

**Example**: Using the Black-Scholes formula to price FX options It is September 2008. We have the following data:  $S_{r} = 1.6186 \text{ USD}/\text{GBP}$ (observed) X = 1.62 USD/GBP(observed) T = 40/365 = .1096. (as a ratio of annual calendar) (observed)  $i_{d} = .0479$ . (annualized) (observed)  $i_{c} = .0583.$ (annualized) (observed)  $\sigma = .08.$ (annualized) (not observed, estimated!) (1) Calculate d1 and d2. d1 =  $[\ln(\frac{S_t}{X}) + (\frac{i_d}{I} - \frac{i_f}{I} + .5 \sigma^2) T]/(\sigma T^{1/2}) =$  $= [\ln(1.6186/1.62) + (.0479 - .0583 + .5.08^2) * .1096]/(.08*.1096^{1/2}) =$ = -0.062440. $d2 = [ln(S_t/X) + (i_d - i_f - .5 \sigma^2) T]/(\sigma T^{1/2}).$  $= [\ln(1.6186/1.62) + (.0479 - .0583 - .5.08^2) * .1096]/(.08*.1096^{1/2}) =$ = -0.088923.



 Empirical Check:

 From the WSJ quote (in USD cents):

 British Pound
 161.86

 10,000 British Pounds-European Style.

 161
 Sep
 32
 0.82
 ...

 162
 Oct
 32
 1.54
 ...
 0.01

<u>Note</u>: You can choose the volatility to match the observed premium. This is the "*implied volatility*."

# **Trading in Currency Options**

# • Markets for Foreign Currency options

(1) Interbank (OTC) market centered in London, NY, & Tokyo.

OTC options are tailor-made as to amount, maturity, and exercise price.

(2) Exchange-based markets centered in Philadelphia (PHLX, now NASDAQ), NY (ISE, now Eurex) and Chicago (CME Group).

- PHLX options are on spot amounts of 10,000 units of FC (MXN 100K, SEK 100K, JPY 1M).
- PHLX maturities: 1, 3, 6, and 12 months.
- PHLX expiration dates: March, June, Sept, Dec, plus 2 spot months.

- Exercise price of an option at the PHLX or CME is stated as the price in USD cents of a unit of foreign currency.

OPTIONS				
PHILADELPHIA EXCHANGE				
		Calls	Puts	
	Vol.	Last	Vol.	Last
Euro				$135.54 \leftarrow S_t = 1.3554$
10,000 Euro	o-cents	s per unit. ←		Size Size
132 Feb		0.01	3	0.38
132 Mar	3	2.74	90	0.15
134 Feb	3	(1.90) ←		$ P_{call} = USD .019$
134 Mar		0.01	25	1.70
136 Mar	8	1.85	12	2.83
138 Feb	75	0.43		0.01
142 Ma <u>r</u>	1	0.08	1	(7.81) ← P <sub>put</sub> = <b>USD .0781</b>
X=Strike	T=Ex	spiration		

Note on the value of Options
For the same maturity (T), we should have:
value of ITM options > value of ATM options > value of OTM options

ITM options are more expensive, the more ITM they are.
Example: Suppose S<sub>t</sub> = 1.3554 USD/EUR. We have two ITM Mar puts:
X<sub>put</sub> = 1.36 USD/EUR
X<sub>put</sub> = 1.42 USD/EUR.
premium (X=1.36) = USD 0.0170
premium (X=1.42) = USD 0.0781. ¶



• Iris Oil decides to use the $X = .84 \text{ USD/CAD}$ put $\Rightarrow$ Cost: USD 2.04M.						
A	At $T = t+90$ , there will be two scenarios:					
	Option is ITM $(exercised - i.e., S < X)$					
_	Option is OTM (not exercised)					
	Position	Initial CF	<b>S</b> <sub>t+90</sub> < .84 USD/CAD	<b>S</b> <sub>t+90</sub> ≥ .84 USD/CAD		
	Option (HP)	USD 2.04M	(.84 – S <sub>t+90</sub> ) * CAD 300M	0		
1	Underlying (UP)	0	S <sub>t+90</sub> * CAD 300M	S <sub>t+90</sub> * CAD 300M		
,	Total CF	USD 2.04M	USD 252M	S <sub>t+90</sub> * CAD 300M		
I L	Net CF in 90 days:         USD 252M - USD 2.04M = USD 249.96M       for $S_{t+90} < .84$ USD/CAD					
$S_{t+90} * CAD 300M - USD 2.04M$ for $S_{t+90} \ge .84 USD/CAD$						
Worst case scenario (floor): USD 249.96M (when put is exercised.)						
ŀ	<u>Remark</u> : The final CFs depend on $S_{t+90}!$					



• With options, there is a choice of strike prices (& premiums). A feature not available in forward/futures.				
• Suppose, Iris Oil also considers the $X = .82$ put $\Rightarrow$ Cost: USD 0.63M				
A	gain, at T = t+9	0, we will ha	ve two scenarios:	
	Position	Initial CF	<b>S</b> <sub>t+90</sub> < .82 USD/CAD	$S_{t+90} \ge .82$ USD/CAD
	Option (HP)	USD 0.63M	(.82 – S <sub>t+90</sub> ) * CAD 300M	0
	Underlying (UP)	0	S <sub>t+90</sub> * CAD 300M	S <sub>t+90</sub> * CAD 300M
	Total CF	USD 0.63M	USD 246M	S <sub>t+90</sub> * CAD 300M



I	Hedging with Currency Options
• Hedging (in <i>Situation 1</i> :	suring) with Options is Simple Underlying position: <i>long</i> in FC. Hedging position: long in FC <i>puts</i> .
Situation 2:	Underlying position: <i>short</i> in FC. Hedging position: long in FC <i>calls</i> .
OP = Und Value of C	lerlying position (UP) + Hedging position (HP-options) DP = Value of UP + Value of HP + Transactions Costs (TC)
Profit fron	$n OP = \Delta UP + \Delta HP \text{-options} + TC$

Advantage of options over futures:
⇒ Options expire if S<sub>t</sub> moves in a beneficial way.
Price of the asymmetric advantage: TC (insurance cost).
Q: What is the size of the hedging position with options?
A: Basic (Naive) Approach: Equal Size Modern (Dynamic) Approach: Optimal Hedge

• The Basic Approach: Equal Size Example: A U.S. investor is long GBP 1 million (=UP). She hedges using Dec put options with X= USD 1.60 (ATM). Underlying position:  $V_0 = GBP 1 M$ .  $S_{t=0} = 1.60 USD/GBP$ . Size of the PHLX contract: GBP 10,000. X = USD 1.60  $P_{t=0} = Premium of Dec put = USD .05$ . TC = Cost of Dec puts = 1 M \* USD .05 = USD 50,000. Number of contracts = GBP 1 M / GBP 10,000 = 100 contracts. On December  $S_{t=T} = 1.50 USD/GBP < X \Rightarrow$  option is ITM (exercise put)  $\Delta UP = V_0 * (S_t - S_0) = GBP 1M*(1.50 - 1.60) USD/GBP = USD -0.1M.$  $\Delta HP = V_0 * (X - S_1) = GBP 1M*(1.60 - 1.50) USD/GBP = USD 0.1M.$  **Example:** If at T,  $S_T = 1.80 \text{ USD/GBP} \Rightarrow \text{option is not exercised (put is OTM)}.$   $\Delta UP = \textbf{GBP 1M} * (1.80 - 1.60) \text{ USD/GBP} = \text{USD 0.2M}$   $\Delta HP = 0$  (No exercise)  $\Delta OP = \text{USD 200,000} - \text{USD 50,000} = \text{USD 150,000}.$  ¶ The price of this asymmetry is the premium: USD 50,000 (a sunk cost!).

Dynamic Hedging with Options (Optimal Hedge)
Listed options are continually traded.
⇒ Options positions are usually closed by reselling the options.
⇒ Part of the initial premium (TC) is recovered.
Profit from OP = ΔUP + Δ HP-options + TC<sub>1</sub> – TC<sub>0</sub>
Hedging is based on the relationship between ΔP<sub>t</sub> (or ΔC<sub>t</sub>) and ΔS<sub>t</sub>:
⇒ Goal: Get |ΔP<sub>t</sub>| (|ΔC<sub>t</sub>|) & |ΔS<sub>t</sub>| with similar changes.
⇒ Problem: The relation between ΔP<sub>t</sub> (ΔC<sub>t</sub>) & ΔS<sub>t</sub> is non-linear.





At A, S<sub>t</sub> changes from 1.60 to 1.59 USD/GBP. ⇒ P<sub>t</sub> = .015 + [USD -.01 \* (-0.5)] = USD .02
Q: What is a good hedge at A? If GBP depreciates by USD .01, each GBP put goes up by USD .005.
⇒ Buy 2 GBP puts for every GBP of British gilts (2 = -1/ -0.5)
⇒ That is, the optimal hedge ratio is h\* = -1/Δ. (Note: Negative sign to make h positive)
<u>Problem</u>: Δ-hedging only works for small changes of S<sub>t</sub>, where we can ignore the *approximation error*.

**Example**: At point **A** (S<sub>t</sub> = 1.60, P<sub>t</sub> = USD .015): *h* = Hedge ratio =  $(-1/\Delta) = -1/-0.5 = 2$ . *n* = 2 \* **1,000,000** = **2,000,000**. Number of contracts = **2,000,000**/10,000 = 200. Now, at **A'**, S<sub>t</sub> = 1.59 USD/GBP  $\Rightarrow$  P<sub>t</sub> = .02.  $\Delta$ HP = **2,000,000** \* (.02 - .015) = USD 10,000.  $\Delta$ UP = **1,000,000** \* (1.59 - 1.60) = USD -10,000. ¶ <u>Problem</u>: If the GBP depreciates, options protect the portfolio by its  $\Delta$  changes.



Example: Back to point B. Now,  $S_t = 1.55$  USD/GBP, with  $\Delta = -0.8$ . ⇒ New h = 1.25 (= -1/-0.8) New n = 1.25 \* 1M = 1,250,000. New Number of contracts = 1,250,000/10,000 = 125 contracts. No over hedging: The investor closes part of the initial position (200 put contracts). That is, the investor sells 75 put contracts and receives: USD .025 \* 10,000 \* 75 = USD 18,750 For a profit on the 75 put contracts closed: 10,000 \* (USD .025 - USD .015) = USD 100. ¶

- Summary of Problems associated with Delta Hedging
- Delta hedging only works for small changes of  $S_t$ .

-  $\Delta$  and *h* change with  $S_t \implies n$  must be adjusted continually.  $\implies$  this is expensive.

In practice, use periodical revisions in HP.

**Example**: *h* changes when there is a significant swing in  $S_t$  (2% or +). Between revisions, options offer usual asymmetric insurance.

# **Hedging Strategies**

• Hedging strategies with options can be more sophisticated:

 $\Rightarrow$  Investors can play with several exercise prices with options only.

**Example**: Hedgers can use:

- Out-of-the-money (least expensive)
- At-the-money (expensive)
- In-the-money options (most expensive)
- Same *trade-off* of car insurance:
  - Low premium (high deductible)/low floor or high cap: Cheap
  - High premium (low deductible)/high floor or low cap: Expensive

OPTIONS						
PHILADELPHIA EXCHANGE						
			Calls	Puts		
	١	/ol.	Last	Vol.	Last	
Euro					135.54	
10,000 Euro -cents per unit.						
132 Fe	b.		0.01	3	0.38	
132 Ma	ar 3	3	2.74	90	0.15	
134 Fe	eb 3	3	1.90			
134 Ma	ar		0.01	25	1.70	
136 Ma	ar 8	3	1.85	12	2.83	
138 Fe	b 7	'5	0.43		0.01	
142 Ma	ar 1		0.08	1	7.81	
Swedish Krona 15.37						
100,000 Swedish Krona -cents per unit.						

**Example:** It is February 2, 2011. UP = *Long* bond position EUR 1,000,000. HP = EUR Mar *put* options: X = 134 and X = 136.  $S_t = 1.3554$  USD/EUR. (A) OTM Mar X = 134 put, with premium = USD .0170. Total cost = USD .0170 \* 1,000,000 = USD 17,000 Floor = 1.34 USD/EUR \* EUR 1,000,000 = USD 1,340,000. Net Floor = USD 1.34M – USD .017M = USD 1.323M (B) ITM Mar X = 136 put, with premium = USD .0283. Total cost = USD .0283 \* 1,000,000 = USD 28,300 Floor = 1.36 USD/EUR \* EUR 1,000,000 = USD 1,360,000 Net Floor = USD 1.36M – USD .0283M = USD 1.3317M • As usual with options, under both instruments there is some uncertainty about the final cash flows. ¶



# Exotic Options Exotic options: Options with two or more option features. Example: A compound option (an option on an option). Two popular exotic options: Knock-outs & Knock-ins. Barrier Options: Knock-outs/Knock-ins Barrier options: The payoff depends on whether S<sub>t</sub> reaches a certain level during a certain period of time. Knock-out: A standard option with an "insurance rider" in the form of a second, out-of-the-money strike price. This "out-strike" is a stop-loss order: if the out-of-the-money X is crossed by S<sub>t</sub>, the option contract ceases to exist.

*Knock-ins:* the option contract does not exist unless and until S<sub>t</sub> crosses the out-of-the-money "in-strike" price. **Example:** Knock-out FX options
Consider the following European option:
1.65 USD/GBP March GBP call knock-out 1.75 USD/GBP. *S<sub>t</sub>* = 1.60 USD/GBP.
If in March *S<sub>t</sub>*= 1.70 USD/GBP, the option is exercised
⇒ writer profits: USD (1.65 – 1.70) + premium per GBP sold.
If in March S<sub>t</sub> ≥ 1.75 USD/GBP, the option is cancelled
⇒ writer profits are the premium. ¶
Q: Why would anybody buy one of these exotic options?
A: They are cheaper.



# FX Risk Management

# • Exposure

At the firm level, currency risk is called *exposure*.

# Three areas

(1) *Transaction exposure*: Risk of transactions denominated in FC with a payment date or maturity.

(2) *Economic exposure*: Degree to which a firm's expected cash flows are affected by unexpected changes in  $S_t$ .

(3) *Translation exposure*: Accounting-based changes in a firm's consolidated statements that result from a change in  $S_t$ . Translation rules create accounting gains/losses due to changes in  $S_t$ .

We say a firm is "exposed" or has exposure if it faces currency risk.

Q: How can FX changes affect the firm?

- Transaction Exposure

- Short-term CFs: Existing contract obligations.

- Economic Exposure

- Future CFs: Erosion of competitive position.

- Translation Exposure

- Revaluation of balance sheet (Book Value vs Market Value).

#### **Example**: Exposure.

A. Transaction exposure.

Swiss Cruises, a Swiss firm, sells cruise packages priced in USD to a broker. Payment in 30 days.

B. Economic exposure.

Swiss Cruises has 50% of its revenue denominated in USD and only 20% of its cost denominated in USD. A depreciation of the USD will affect future CHF cash flows.

C. Translation exposure.

Swiss Cruises obtains a USD loan from a U.S. bank. This liability has to be translated into CHF following Swiss accounting rules.

# **Measuring Transaction Exposure**

Transaction exposure (TE) is very easy to identify and measure:

TE = Value of a fixed future transaction in FC \* S<sub>t</sub>

For a MNC  $\Rightarrow$  TE: Consolidation of contractually fixed future FC inflows and outflows for all subsidiaries, by currency. (Net TE!)

**Example**: Swiss Cruises.

Sold cruise packages for USD 2.5 million. Payment: 30 days.

Bought fuel oil for USD 1.5 million. Payment: 30 days.

 $S_t = 1.45 \text{ CHF}/\text{USD}.$ 

Thus, the net transaction exposure in USD 30 days is:

Net TE = (USD 2.5M – USD 1.5M) \* 1.45 CHF/USD

= USD 1M \* 1.45 CHF/USD = CHF 1.45M. ¶

• Netting				
Firms take into acco	ount correlations to calculate Net TE			
	$\Rightarrow$ Portfolio Approach.			
NTE = Net	$TE = \sum_{j} TE_{j}$ $j = EUR, GBP, JPY, BRL, MXN,$			
Usually, NTE is rep	orted by maturity (up to 90 days; more than 90 days).			
Q: Why NTE?				
A: A U.S. MNC:	Subsidiary A with $CF(in EUR) > 0$			
	Subsidiary B with CF(in GBP) $< 0$			
	$\rho_{GBP,EUR}$ is very high and positive.			
	NTE may be very low for this MNC.			
Hedging decision	s are usually not made transaction by transaction; but			
based on the exposu	are of the portfolio.			

**Example:** Swiss Cruises.Net TE (in USD):**USD 1 million.** Due: 30 days.Loan repayment:CAD 1.50 million. Due: 30 days. $S_t = 1.47 \text{ CAD/USD.}$  $\rho_{CAD,USD} = .843$  (monthly from 1971 to 2017)Swiss Cruises considers NTE to be close to zero. ¶Note 1: Correlations vary a lot across currencies. In general, regionalcurrencies are highly correlated.From 2000-2017, $\rho_{GBP,NOK} = 0.58$  $\rho_{GBP,JPY} = 0.04$ Note 2: Correlations also vary over time.





#### • Q: How does TE affect a firm in the future?

Firms are interested in how TE will change in the future, say, in T days when transaction will be settled.

- Firms do not know  $S_{t+T}$ , they need to forecast  $S_{t+T} \implies E_t[S_{t+T}]$
- $E_t[S_{t+T}]$  has an associated standard error, which can be used to create a range (or interval) for  $S_{t+T}$  & TE.

- Risk management perspective:

How much DC can firm spend on account of a FC inflow in T days? How much DC will be needed to cover a FC outflow in T days?