Parity Conditions

PPP, IFE, EH & RW

(for private use, not to be posted/shared online)

- **Last Class**
- Central Bank Intervention
  - ◊ Effect on domestic MS & domestic interest rates
  - ◊ Sterilized Intervention

- Arbitrage in FX Markets.
  - ◊ Three elements:
    - \- Pricing Mistakes
    - \- No Risk
    - \- No Own Capital

  - ◊ Local (sets uniform rates across banks)
  - ◊ Triangular (sets cross rates)
  - ◊ Covered (sets forward rates)

• Last Class

• Covered Interest Rate (IRPT): It determines the forward rate,

$$F_{t,T} = S_t * \frac{\left(1 + i_d * \frac{T}{360}\right)}{\left(1 + i_f * \frac{T}{360}\right)}$$

If $F^{IRP}_{t,1-yr} \neq F^{A}_{t,1-yr}$ (Bank A's rate) ⇒ Arbitrage (& Capital Flows)

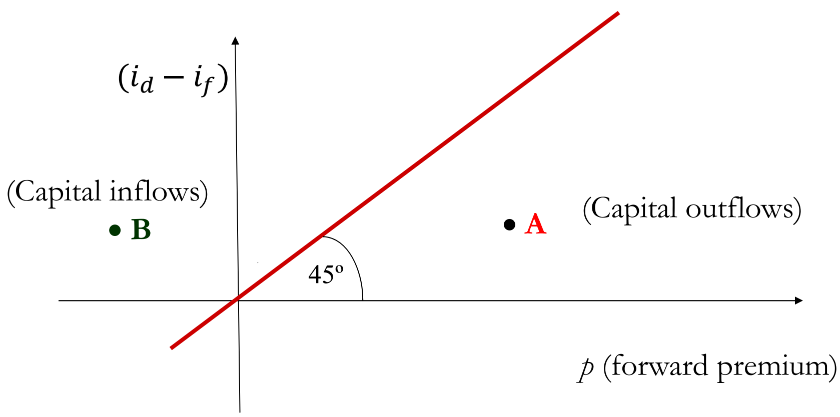
• If IRP is violated ⇒ Arbitrage

Steps of a Covered Arbitrage Strategy:

1) Borrow
2) Convert
3) Deposit
4) Cover

• Last Class

• In equilibrium, $p \approx (i_d - i_f)$



• Until 2008-2009 Financial Crisis, very strong evidence for IRP. Then, no evidence; maybe interest rates used no longer risk-free?

• This Class

• Explain and test theories of FX Determination –i.e., behavior of S_t

- ◊ PPP (based on Price and Inflation rates differentials)
- ◊ IFE (based on interest rates differentials)
- ◊ EH (based on uncovered IRP –i.e., no covered step)
- ◊ Macroeconomic Models
- ◊ RW

• Goal 1: Explain S_t with a theory, say T1. Then, $S_t^{T1} = f(.)$

Different theories can produce different $f(.)$'s.

Evaluation: How well a theory match the observed behavior of S_t.

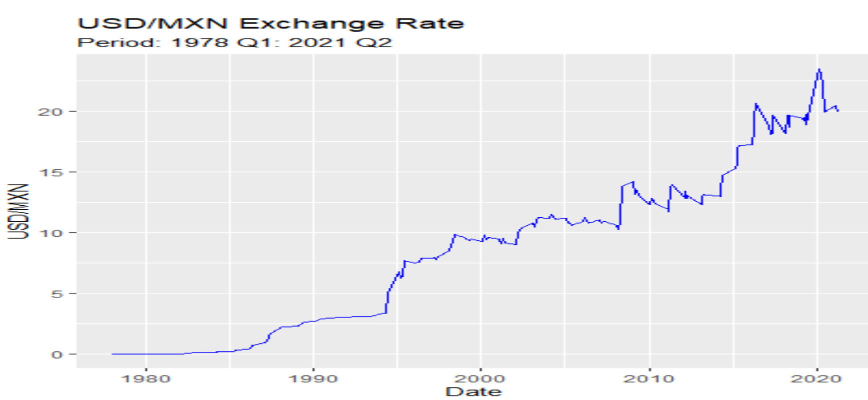
• Goal 2: Eventually, produce a formula to forecast $S_{t+T} = f(X_t) \Rightarrow E[S_{t+T}]$

• Review of Regression and Regression based Tests

• A Good Theory?

• Ideal situation: A theory perfectly matches observed S_t. $\Rightarrow$ Not realistic.

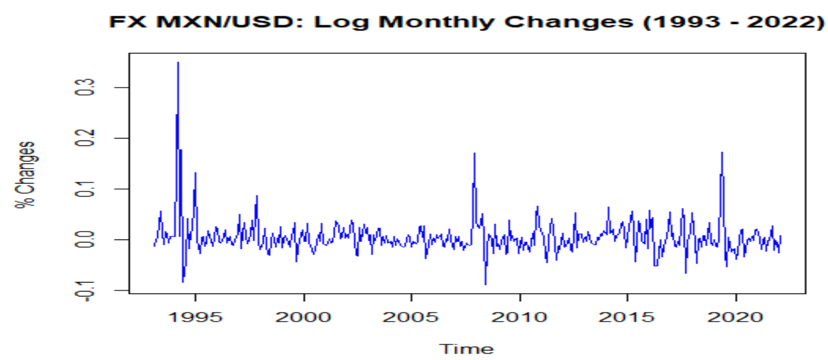
Q: On average, is $S_t \approx S_t^{T1}$? Or, alternatively, is $E[S_t] = E[S_t^{T1}]$?



• Like many macroeconomic series, S_t shows trends –in statistics the trends in macroeconomic series are called *stochastic trends.*

• It practice, it is better to try to match changes, not levels.

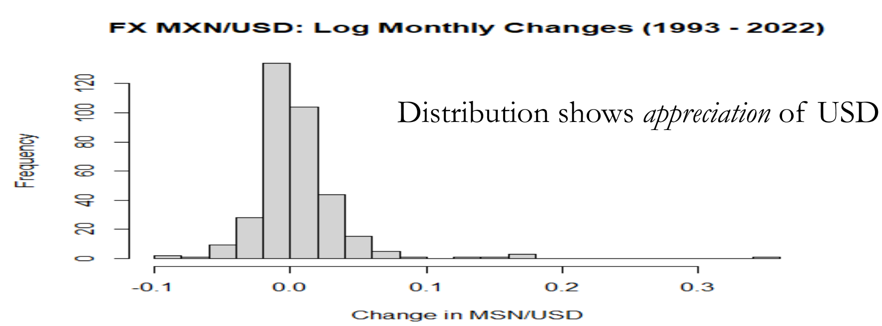
- Now, the trend is gone. Our goal is to explain s_t, the percentage change in S_t. (Notation: Many times s_t = $e_{f,t}$).



- Our goal is to explain s_t. Question for our models:

Does the model, say T1, match, on average, the observed behavior of s_t? For example, is E[s_t] = E[s_t^{T1}]?

- We will use statistics to formally tests theories.
- Data: Distribution of s_t for the MXN/USD –in this case, monthly percentage changes from 1993:Feb – 2022: Dec.



- Data:

  E[s_t] = Average monthly % change = **0.52%** (**6.3%**, annualized)
  SD[s_t] = **3.51%** (**12.2%** annualized).

- A good theory should predict, on average, an annualized change of **6.3%** for s_t. A better theory should also predict a **12%** annualized volatility.

• Descriptive stats for s_t for monthly JPY/USD and the MXN/USD.

	JPY/USD	*USD/MXN*
Mean	-0.0014	**0.0052**
Standard Error	0.0011	0.0019
Median	0.0002	0.0004
Standard Deviation	0.0262	**0.0351**
Sample Variance	0.0007	0.0021
Kurtosis	**4.0886**	**33.3631**
Skewness	**-0.4276**	**3.9122**
Minimum	-0.1052	-0.0887
Maximum	0.0807	0.3500
Count	577	350

• Developed currencies: less volatile, with smaller means/medians.

Purchasing Power Parity (PPP)

Purchasing Power Parity (PPP)

PPP is based on the law of one price (LOOP): Goods, once denominated in the same currency, should have the same price.

If they are not, then some form of arbitrage is possible.

Example: LOOP for Oil.

$P_{\text{oil-USA}}$ = **USD 60**

$P_{\text{oil-SWIT}}$ = **CHF 120**

$\Rightarrow$ $\mathbf{S_t^{LOOP}}$ = **USD 60** / **CHF 120** = **0.50 USD/CHF**.

If $\boldsymbol{S_t}$ **= 0.75 USD/CHF** $\Rightarrow$ Oil in Switzerland is more expensive (in USD) than in the US:

$P_{\text{oil-SWIT}}$ (USD) = **CHF 120** * **0.75 USD/CHF** = **USD 90** > $\mathbf{P_{oil\text{-}USA}}$

Example (continuation):

S_t = 0.75 USD/CHF > S_t^{LOOP} (LOOP is not holding)

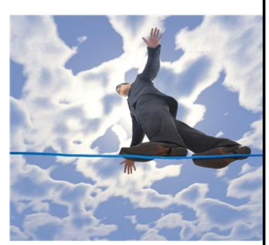
Trading strategy:
(1) Buy oil in the US at $P_{oil\text{-}USA}$ = **USD 60**.
(2) Export oil to Switzerland
(3) Sell US oil in Switzerland at $P_{oil\text{-}SWIT}$ = **CHF 120**.
(4) Sell CHF/buy USD at then S_t.

Strategy, exporting US of oil to Switzerland, will affect prices:

1) $P_{oil\text{-}USA}\uparrow$
2) $P_{oil\text{-}SWIT}\downarrow$ $\Rightarrow S_t^{LOOP}\uparrow$ (= $P_{oil\text{-}USA}\uparrow / P_{oil\text{-}SWIT}\downarrow$)
3) $S_t\downarrow$ $S_t \Leftrightarrow S_t^{LOOP}$ (convergence). ¶

Example (continuation):

LOOP Notes :



◊ LOOP gives an *equilibrium* exchange rate.
Equilibrium is achieved when there is no trade in oil.
(because of pricing mistakes): LOOP holds for oil!

◊ LOOP is telling what S_t *should be* (in equilibrium). Not what S_t *is* in the market today.

◊ Using the LOOP we have generated a model for S_t. When applied to many goods, we have the *PPP model.*

Problem with LOOP: There are many traded goods in the economy.

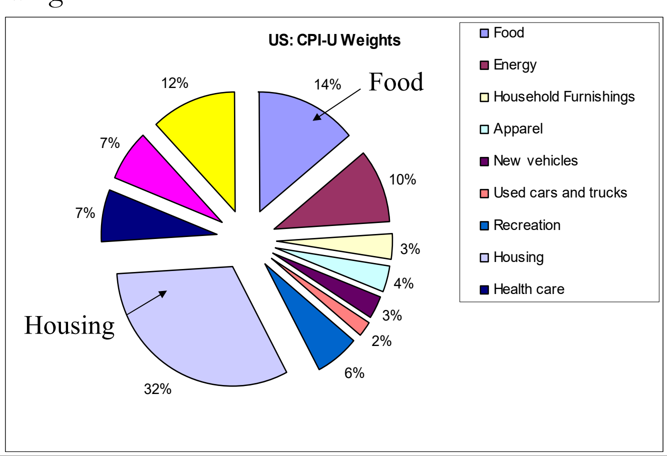
Solution: Use **baskets** of goods.

PPP: The price of a basket of goods should be the same across countries, once denominated in the same currency. That is, USD 1 should buy the same amounts of goods in the U.S. or in Colombia.

• A popular basket: The CPI basket.

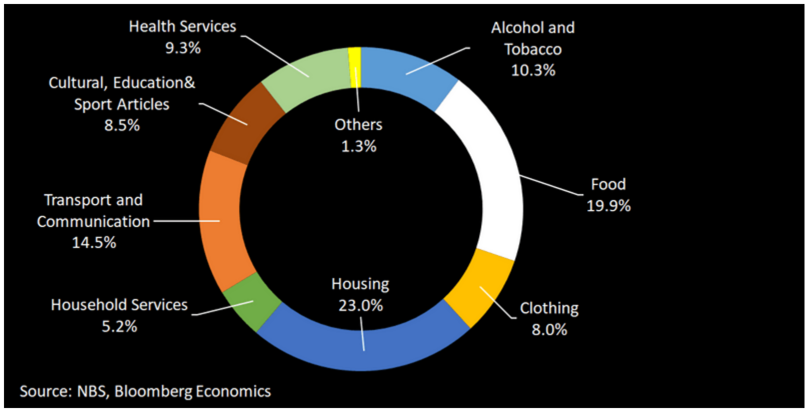
• In the U.S., the basket typically reported is the **CPI-U**. It represents the spending patterns of *all urban consumers and urban wage earners and clerical workers*. (**87%** of U.S. population).

• U.S. basket weights:



• Weights are different in different countries.

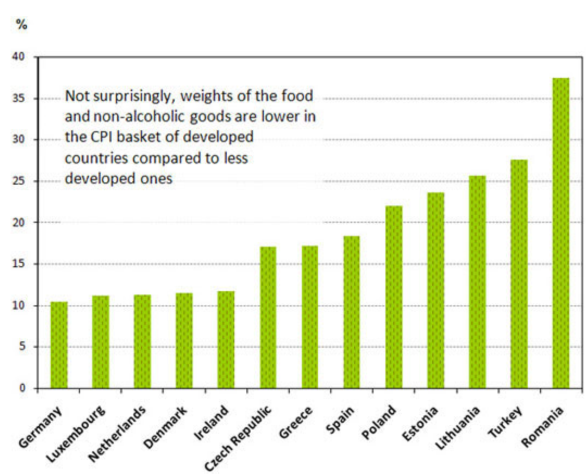
• China's basket weights:



• Relative to the U.S. weights, heavier weight given to Food & Clothing (Apparel, in the U.S.) and lower to Housing and Household Services (Energy, in the U.S.).

• The different weights is a problem when comparing CPI baskets: The composition of the index may vary widely across countries.

• For example, in Europe, the weight of the food category changes substantially as the income level increases.



Absolute version of PPP: The FX rate between two currencies is the ratio of the two countries' general price levels:

$$S_t^{PPP} = \text{Domestic Price level / Foreign Price level} = P_d / P_f$$

Example: LOOP for CPIs.

CPI-basket$_{USA}$ = P_{USA} = **USD 5,577**

CPI-basket$_{SWIT}$ = P_{SWIT} = **CHF 6,708**

$$\Rightarrow S_t^{PPP} = \text{USD } 5{,}577/\text{CHF } 6{,}708 = 0.\ 8314 \text{ USD/CHF}.$$

If $S_t \neq$ **0. 8314 USD/CHF**, there will be trade of the goods in the baskets.

Suppose S_t = 1.09 USD/CHF > S_t^{PPP}.

Then,

P_{SWIT} (in USD) = **CHF 6,708** * **1.09 USD/CHF**

= **USD 7,311.72** > P_{USA} = **USD 5,577**

Example (continuation): (disequilibrium: S_t= 1.09 USD/CHF > S_t^{PPP})

P_{SWIT} (in USD) = **CHF 6,708** * **1.09 USD/CHF**

= **USD 7,311.72** > P_{USA} = **USD 5,577**

Potential profit: **USD 7,311.72** – **USD 5,577** = **USD 1,734.72**

Traders will do the following *pseudo-arbitrage* strategy:

1) Borrow USD
2) Buy the CPI-basket in the U.S.
3) Sell the CPI-basket, purchased in the U.S., in Switzerland.
4) Sell the CHF/Buy USD
5) Repay the USD loan, keep the profits.

Note: "Equilibrium forces" at work:

2) $P_{USA}\uparrow$
3) $P_{SWIT}\downarrow$ } $(\Rightarrow S_t^{PPP}\uparrow = P_{USA}\uparrow / P_{SWIT}\downarrow)$

4) $S_t\downarrow$ $S_t \Leftrightarrow S_t^{PPP}$ (converge) ¶

• Real v. Nominal Exchange Rates

The absolute version of the PPP theory is expressed in terms of S_t, the *nominal exchange rate*.

We can write the absolute version of the PPP relationship in terms of the *real exchange rate*, R_t. That is,

$$R_t = S_t\, P_f / P_d = 1$$

R_t allows us to compare prices, translated to DC:

If $R_t > 1$, foreign prices (translated to DC) are more expensive

If $R_t = 1$, prices are equal in both countries –i.e., PPP holds!

If $R_t < 1$, foreign prices are cheaper

Economists associate $R_t > 1$ with a more efficient domestic economy.

Example: We have Big Mac ("the basket") prices in Switzerland & the US:

P_f = **CHF 6.70**

P_d = **USD 5.36**

S_t = **1. 0836 USD/CHF** ⇒ P_f (in USD) = USD 7.26 > P_d

$R_t = S_t\, P_{SWIT}/P_{US}$ = **1. 0836 USD/CHF** * **CHF 6.70/USD 5.36** = **1.3545**

Taking the Big Mac as our basket, the U.S. is more competitive than Switzerland. Swiss prices are **35.45%** higher than U.S. prices, after taking into account the nominal exchange rate.

To bring the economy to equilibrium –no trade in Big Macs-, we expect the USD to appreciate against the CHF.

According to PPP, the USD is *undervalued* against the CHF.

⇒ Trading Signal: Buy USD/Sell CHF. ¶

• The Big Mac ("Burgernomics," popularized by *The Economist*) has become a popular basket for PPP calculations. Why?

1) Standardized, common basket: beef, cheese, onion, lettuce, bread, pickles and special sauce. (CPI baskets, not standardized). Sold in 120+ countries.

Big Mac (Sydney)  Big Mac (Tokyo) 

2) Very easy to find out the price.

3) It turns out, it is correlated with more complicated common baskets.

• In theory, traders can exploit the price differentials in BMs.

The Economist's Big Mac Index

• In the previous example, Swiss traders can import US BMs.

From UH (US) to Rapperswill (CH)

• Not realistic. But, the components of a BM are internationally traded. LOOP suggests that prices of components should be similar in all markets.

The Economist reports the real exchange rate: $\mathbf{R_t} = S_t\, P_{BigMac,f}/P_{BigMac,d}$.

For example, in **Dec 2022**, for the British pound (GBP):

$\mathbf{R_t}$ = [**1.2318 USD/GBP** * GBP 3.79] / USD 5.36 = **0.87099**

⇒ (**12.90%** overvaluation)

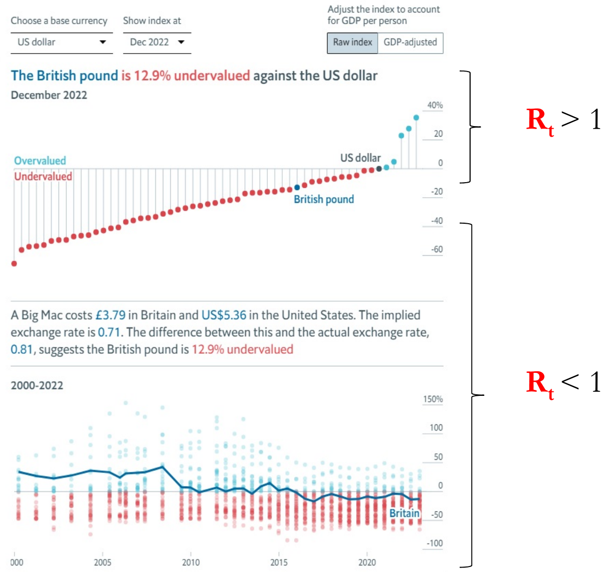
Example: (The Economist's) Big Mac Index in **Dec 2022**.

$S_t^{PPP} = P_{BigMac,d} \,/\, P_{BigMac,f}$

(The Economist reports $R_t - 1 = S_t \, P_{BigMac,f}/P_{BigMac,d=USD} - 1$).

The Big Mac index

Country		2000 —— 2022	Under/over valued, %
Switzerland	Franc		35.4
Uruguay	Peso		27.8
Norway	Krone		22.9
Sweden	Krona		4.8
Denmark	Krone		0.9
United States	US$		BASE CURRENCY
Argentina	Peso		-1.0
Euro area	Euro		-1.4
Australia	A$		-4.6
Saudi Arabia	Riyal		-5.6
Israel	Shekel		-5.7
Sri Lanka	Rupee		-6.9
Costa Rica	Colón		-7.4
UAE	Dirham		-8.6
New Zealand	NZ$		-9.0
Chile	Peso		-11.4
Britain	Pound		-12.9
Kuwait	Dinar		-14.5
Canada	C$		-14.7
Czech Rep.	Koruna		-15.8



Example: (The Economist's) Big Mac Index in **Dec 2022**.

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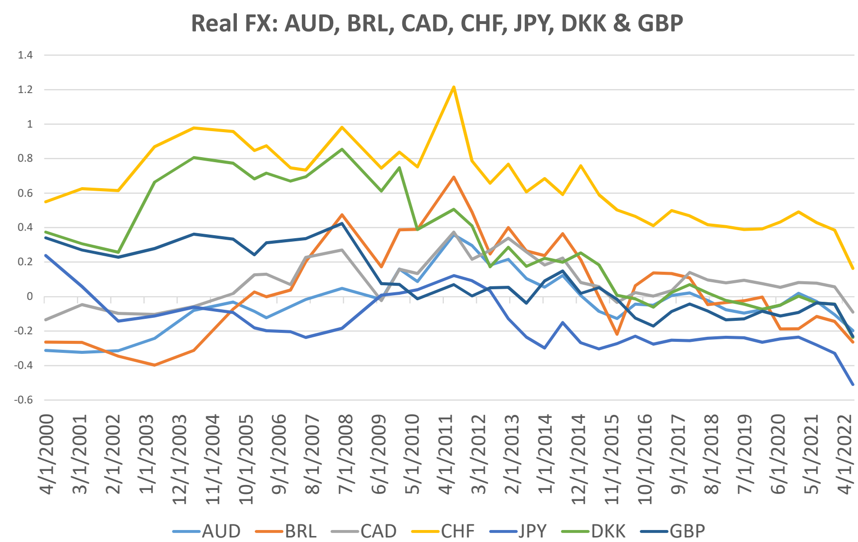
Argentina	Peso		-1.0
Euro area	Euro		-1.4
Australia	A$		-4.6
Saudi Arabia	Riyal		-5.6
Israel	Shekel		-5.7
Sri Lanka	Rupee		-6.9
Costa Rica	Colón		-7.4
UAE	Dirham		-8.6
New Zealand	NZ$		-9.0
Chile	Peso		-11.4
Britain	Pound		-12.9
Kuwait	Dinar		-14.5
Canada	C$		-14.7
Czech Rep.	Koruna		-15.8

Country		2000 —— 2022	Under/over valued, %
Bahrain	Dinar		-15.9
Lebanon	Pound		-16.5
Singapore	S$		-16.6
Brazil	Real		-17.2
Nicaragua	Córdoba		-21.2
Mexico	Peso		-21.8
Colombia	Peso		-22.4
Poland	Zloty		-23.6
Honduras	Lempira		-24.5
Turkey	Lira		-25.6
South Korea	Won		-26.0
Thailand	Baht		-27.2
Qatar	Riyal		-28.3
Hungary	Forint		-29.9
Oman	Rial		-31.2
Peru	Sol		-33.2
China	Yuan		-34.0
Jordan	Dinar		-34.3
Guatemala	Quetzal		-35.8
Pakistan	Rupee		-36.8

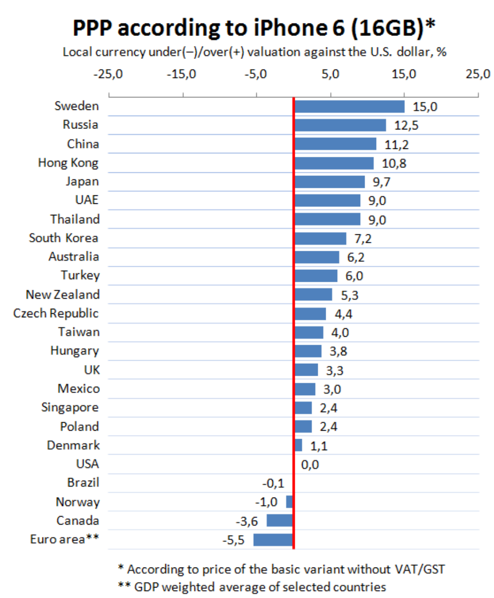
Example: Big Mac Index - (R_t - 1). Changes over time in **2000 - 2022**.



R_t does move over time. R_t departures from 1, can be very persistent.

Example: Iphone 6 (March 2015, taken from seekingalpha.com).

$R_t = S_t \, P_{IPhone,f}/P_{IPhone,d}$ (d=US) $\Rightarrow$ R_t=1 under Absolute PPP



• Empirical Evidence: Simple informal test:
Test: If Absolute PPP holds ⇒ $\mathbf{R_t} = 1$.
In the Big Mac example, PPP does not hold for the majority of countries.

⇒ Absolute PPP, in general, fails (especially, in the short-run).

• Absolute PPP: Qualifications

(1) *PPP emphasizes only trade and price levels*. Political/social factors, financial problems, etc. are ignored.

(2) Implicit assumption: *Absence of trade frictions* (tariffs, quotas, taxes, etc.).
Q: Realistic?
- On average, transportation costs add **7%** to the price of U.S. imports of meat and **16%** to the import price of vegetables.

- Many products are heavily protected, even in the U.S. For example, peanut imports are subject to a tariff as high as **163.8%.**

• Absolute PPP: Qualifications

Some everyday goods protected in the U.S.:
- Peanuts (shelled **131.8%**, and unshelled **163.8%**).
- Paper Clips (as high as **126.94%**)
- European Roquefort Cheese, cured ham, mineral water (**100%**)
- Japanese leather (**40%**)
- Sneakers (**48%** on certain sneakers)
- Chinese tires (**35%**)
- Canned Tuna (as high as **35%**)
- Synthetic fabrics (**32%**)
- Steel (**25%**)
- Indian wood furniture (**25%**)
- Italian footwear & eyeglasses (**25%**)
- Brooms (quotas and/or tariff of up to **32%**)
- Trucks (**25%**) & cars (**2.5%**)

• Absolute PPP: Qualifications

Some Japanese protected goods:
- Rice (**778%**)
- Sugar (**328%**)
- Powdered Milk (**218%**)
- Beef (38.5%, but can jump to 50% depending on volume).

Some European protected goods:
- Knitted Clothes (**100%**)
- Fresh Cheese (48.3%)
- Bovine Meat, boneless (41%)
- Fresh or dried grapefruit (25%)
- Atlantic Salmon (25%)

• Absolute PPP: Qualifications

(3) PPP is unlikely to hold if P_f and P_d represent *different baskets*. This is why the Big Mac is a popular choice.

(4) *Trade takes time* (contracts, information problems, etc.).

(5) *Internationally non-traded/non-tradable (NT) goods* –i.e. haircuts, home and car repairs, medical services, real estate. The NT good sector is big: **50%-60%** of consumption (big weight in CPI basket).

Then, in countries where NT goods are relatively expensive, the CPI basket will be relatively expensive. Thus, PPP will find these countries' currencies *overvalued* relative to currencies in low NT cost countries.

Note: In the short-run, cars will not be taken to Mexico to be repaired, but in the long-run, resources (capital, labor) will move.

⇒ Over-/under-valuation: An indicator of movement of resources.

• Absolute PPP: Qualifications
The NT sector also has an effect on the price of traded goods. For example, rent and utilities costs affect the price of a Big Mac: 25% of Big Mac due to NT goods.

• Empirical Fact
Price levels in richer countries are consistently higher than in poorer ones. This fact is called the *Penn effect*. Many explanations, the most popular: The *Balassa-Samuelson (BS) effect*.

• Borders Matter
You may look at the Big Mac Index and think: "No big deal: there is also a big dispersion in prices within the U.S., within Texas, and, even, within Houston!"

True. Prices vary within the U.S. For example, in **2015**, the price of a Big Mac (and Big Mac Meal) in New York was USD 5.23 (USD 7.45), in Texas as USD 4.39 (USD 6.26).

But, borders play a role, not just distance!

Engel and Rogers (1996) computed the variance of LOOP deviations for **city pairs** within the **U.S.**, within **Canada**, and **across the border**.

Conclusion: Distance between cities within a country matter, but the **border effect** is **significant**.

To explain the difference between prices across the border using the estimate distance effects within a country, they estimate the U.S.-Canada border should have a width of **75,000 miles**!

This huge estimate has been revised downward, but a large positive border effect remains.

• Balassa-Samuelson Effect

Labor costs affect all prices. We expect average prices to be cheaper in poor countries than in rich ones because **labor costs** are **lower**.

This is the *Balassa-Samuelson effect*: Rich countries have higher productivity and, thus, higher wages in the traded-goods sector than poor countries do. But, firms compete for workers.

Then, wages in NT goods and services are also higher

⇒ Overall prices are lower in poor countries.

• For example, in **2000**, a typical McDonald's worker in the U.S. made **USD 6.50/hour**, while in China made **USD 0.42/hour**.

In **2021**, the same numbers for a cashier are **USD 10/hour** and **USD 1.76**.

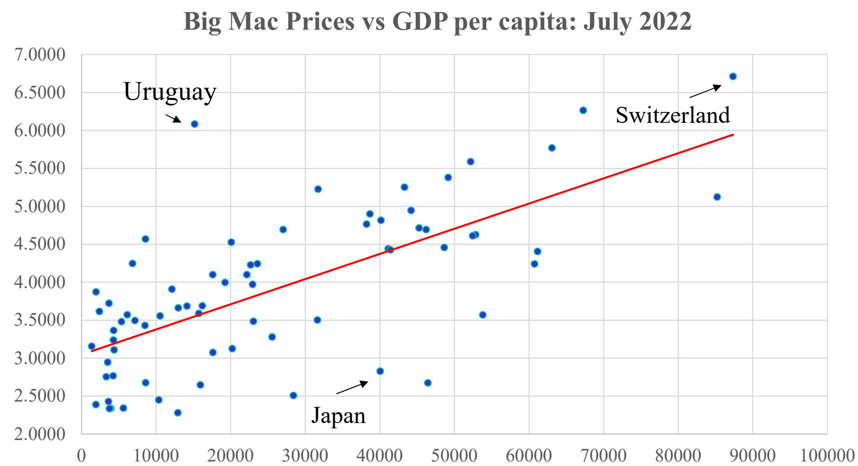


• Balassa-Samuelson effect: A positive correlation between *PPP exchange rates (overvaluation)* and high productivity countries.

Incorporating the Balassa-Samuelson effect into PPP:

1) Estimate a regression: Big Mac Prices against GDP per capita.

P_{BM} (in USD)$_t$ = $\alpha + \beta$ GDP_per_capita$_t$ + ε_t



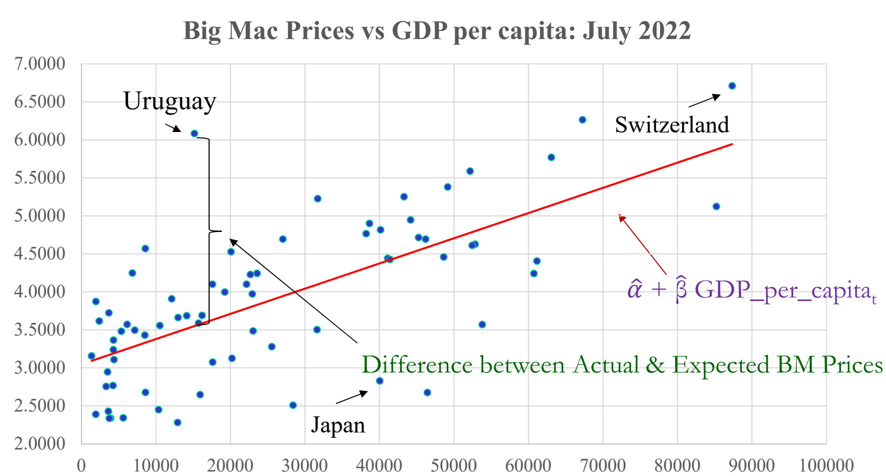
Points on Red line: *Fitted (Expected) Big Mac Prices*, given a *GDP per person*.

$\hat{P}_{BM,GDP\text{-}adj} = \hat{\alpha} + \hat{\beta}$ GDP_per_capita$_t$

Incorporating the Balassa-Samuelson effect into PPP:

2) Compute fitted values:

$\hat{P}_{BM,GDP\text{-}adj} = \hat{\alpha} + \hat{\beta}$ GDP_per_capita$_t$



GDP-adjusted over/under valuation: (BM Price/$\hat{P}_{BM,GDP\text{-}adjusted}$) – 1.

Incorporating the Balassa-Samuelson effect into PPP: Computations

Using data from The Economist for July 2022, we estimate the red line:

$$\hat{P}_{BM,GDP\text{-}adj} = 3.045895 + 0.0000332 * GDP\_per\_capita_t$$

Now, we can compute the "Expected BM prices, given the GDP of a given country." Let's compute the above value for Uruguay. Uruguay's GDP per capita in July 2022 was **USD 15,169.153**. Then,

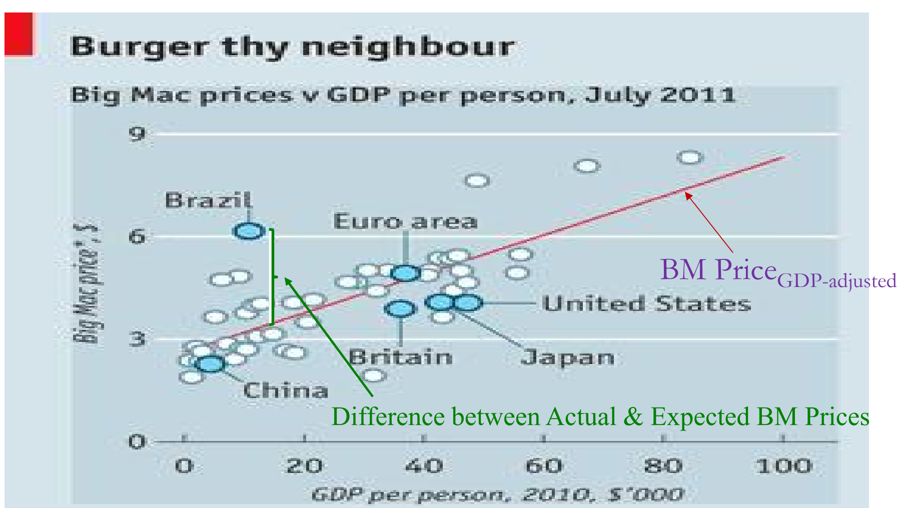
$$\hat{P}_{BM,GDP\text{-}adj}\ (Uruguay) = 3.045895 + 0.0000332 * \mathbf{15{,}169.153} = \mathbf{3.549511}$$

That is, the expected BM in Uruguay in July 2022, given its GDP per capita, was **USD 3.55**. Since the observed local BM price was UYU 255, which translates to **USD 6.08** (= UYU 255 * **41.91 USD/UYU**), then the *GDP-adjusted over/under valuation* was:

6.08 / **3.549511** – 1 = 71.29% (71.29% overvalued)

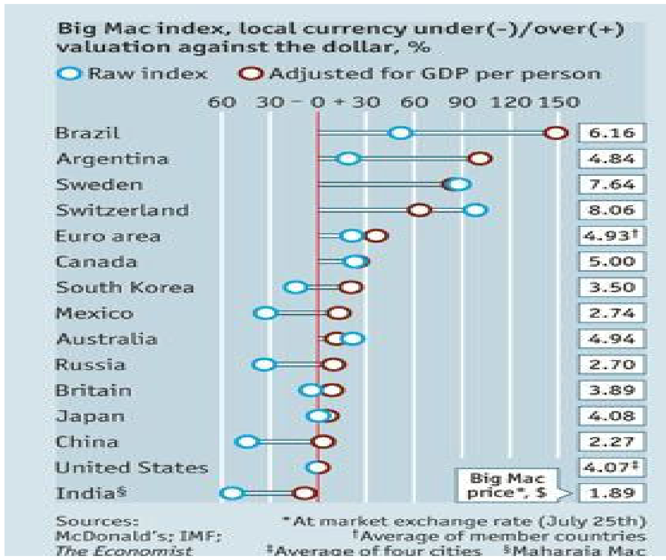
Incorporating the Balassa-Samuelson effect into PPP: July 2011

Same computation for **July 2011**.



Points on Red line: GDP-*adjusted Big Mac Prices* (BM $Price_{GDP\text{-}adjusted}$).

Incorporating the Balassa-Samuelson effect into PPP:
The GDP adjustment can make a difference.



Example: Raw vs GDP-Adjusted Big Mac Index in **Dec 2022**.

The Big Mac index

Country		2000 —— 2022	Under/over valued, %
Switzerland	Franc		35.4
Uruguay	Peso		27.8
Norway	Krone		22.9
Sweden	Krona		4.8
Denmark	Krone		0.9
United States	US$		BASE CURRENCY
Argentina	Peso		-1.0
Euro area	Euro		-1.4
Australia	A$		-4.6
Saudi Arabia	Riyal		-5.6
Israel	Shekel		-5.7
Sri Lanka	Rupee		-6.9
Costa Rica	Colón		-7.4
UAE	Dirham		-8.6
New Zealand	NZ$		-9.0
Chile	Peso		-11.4
Britain	Pound		-12.9
Kuwait	Dinar		-14.5
Canada	C$		-14.7
Czech Rep.	Koruna		-15.8

The Big Mac index

Country		2011 —— 2022	Under/over valued, %
Uruguay	Peso		52.7
Switzerland	Franc		37.3
Norway	Krone		23.6
Argentina	Peso		21.2
Sri Lanka	Rupee		16.6
Sweden	Krona		12.1
Costa Rica	Colón		11.0
Euro area	Euro		8.4
Saudi Arabia	Riyal		8.2
Chile	Peso		5.1
Denmark	Krone		3.3
Brazil	Real		1.1
United States	US$		BASE CURRENCY
Israel	Shekel		-0.9
Nicaragua	Córdoba		-1.2
UAE	Dirham		-2.3
New Zealand	NZ$		-2.8
Australia	A$		-3.1
Colombia	Peso		-4.3
Bahrain	Dinar		-4.9

• Pricing-to-Market

Krugman (1987): Positive relationship between GDP and price levels is caused by *Pricing-to-market* –i.e., price discrimination.

Producers discriminate: Same good is sold to rich countries at higher prices than to poorer countries.

Alessandria and Kaboski (2008): U.S. exporters, on average, charge the richest country a **48%** higher price than the poorest country.

But pricing-to-market struggles to explain why PPP does not hold among developed countries with similar incomes.

For example, Baxter and Landry (2012) report that IKEA prices deviate **16%** from the LOOP in Canada, but only **1%** in the U.S.

Main PPP criticism

Absolute PPP does not incorporate transaction costs and frictions. Relative PPP allows for fixed transaction costs/frictions (say, a fixed USD amount).

Relative PPP

The rate of change in the prices of products should be similar when measured in a common currency (as long as trade frictions are unchanged):

$$\mathrm{s}_{\mathrm{t,T}}^{\mathrm{PPP}} = \frac{\mathrm{S}_{\mathrm{t+T}}^{\mathrm{PPP}} - S_t}{S_t} = \frac{(1 + I_d)}{(1 + I_f)} - 1 \qquad \text{(Relative PPP)}$$

where,

I_f = foreign inflation rate from t to t+T.

I_d = domestic inflation rate from t to t+T.

Note: $\mathrm{s}_{\mathrm{t,T}}^{\mathrm{PPP}}$ is an expectation; what we expect to happen in equilibrium from t to t+T.

• Linear approximation: $\mathrm{s}_{\mathrm{t,T}}^{\mathrm{PPP}} \approx (I_d - I_f)$ ⇒ one-to-one relation

Relative PPP

- Linear approximation: $s^{PPP}_{t,T} \approx (I_d - I_f)$ ⇒ one-to-one relation

Example: From t=0 to t=1, prices increase **10%** in Mexico relative to prices in Switzerland. Then, S_t should also increase 10%.

If $S_{t=0}$ = 9 MXN/CHF ⇒ $\mathbf{S^{PPP}_{t=1}}$ = E[$S_{t=1}$] = **9.9 MXN/CHF**.

Suppose at t=1, S_t increases 13.33%. Then,

$S_{t=1}$= **10.2 MXN/CHF** > $\mathbf{S^{PPP}_{t=1}}$ **= 9.9 MXN/CHF**

⇒ According to Relative PPP, the CHF is overvalued. ¶

Notation: E[$S_{t=1}$] = Expected value of $S_{t=1}$ (model-based), a predicted value.

Example: Forecasting S_t (USD/ZAR) using PPP (ZAR=South Africa).

It's Dec 2022. You have the following information:

$CPI_{US,2022}$ = 104.5,

$CPI_{SA,2022}$ = 100.0,

$S_{t=2022}$ = **.2035 USD/ZAR**.

You are given the 2023 CPI's forecast for the U.S. and SA:

E[$CPI_{US,2023}$] = 110.8

E[$CPI_{SA,2023}$] = 102.5.

You want to forecast S_{2023} using the relative (linearized) version of PPP.

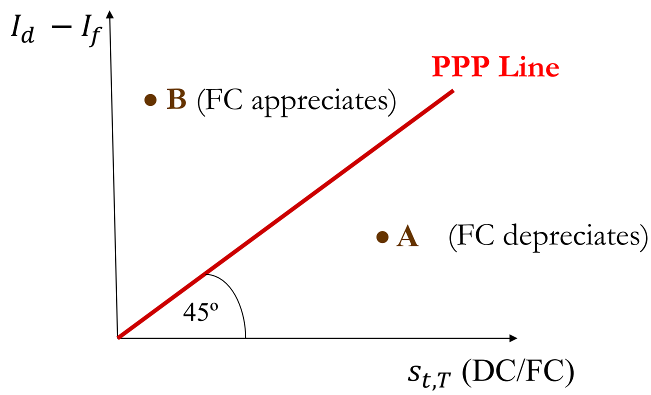
E[$I_{US,2023}$] = (110.8/104.5) - 1 = **.06029**

E[$I_{SA,2023}$] = (102.5/100) - 1 = **.025**

E[S_{2023}] = S_{2022} * (1 + $s^{PPP}_{t=2022,T=2023}$) = S_{2022} * (1 + E[I_{US}] - E[I_{SA}])

= **.2035 USD/ZAR** * (1 + **.06029** - **.025**) = .2107 USD/ZAR..

• Under the linear approximation, $s_{t,T}^{PPP} \approx (I_d - I_f)$, we have a PPP Line



Look at point **A**: $s_T > (I_d - I_f)$,

⇒ Priced in FC, the domestic basket is cheaper

⇒ pseudo-arbitrage (trade) against foreign basket ⇒ FC depreciates

• Relative PPP: Implications

(1) Under relative PPP, $\mathbf{R_t}$ remains constant (it can be different from 1!).

(2) Without relative price changes, an MNC faces no real operating FX risk (as long as the firm avoids fixed contracts denominated in FC).

• Relative PPP: Absolute versus Relative

- Absolute PPP compares price levels.

Under Absolute PPP, prices are equalized across countries:

"*A mattress costs* **GBP 200** *(=* **USD 320***) in the U.K. and* **BRL 800** *(=* **USD 320***) in Brazil.*"

- Relative PPP compares price changes.

Under Relative PPP, exchange rates change by the same amount as the inflation rate differential (original prices can be different):

"*U.K. inflation was 2% while Brazilian inflation was 8%. Meanwhile, the BRL depreciated 6% against the GBP. Then, relative cost comparison remains the same.*"

• Relative PPP is weaker than Absolute PPP: **R_t** can be different from 1.

• Relative PPP: Testing

Key: On average, what we expect to happen, $s_{t,T}^{PPP}$, should happen, $s_{t,T}$.

⇒ On average: $s_{t,T} \approx s_{t,T}^{PPP} \approx (I_d - I_f)$

or $E[s_{t,T}] = E[s_{t,T}^{PPP}] \approx E[(I_d - I_f)]$

A linear regression is a good framework to test theories. Recall,

$$s_{t,T} = \frac{S_{t+T} - S_t}{S_t} = \alpha + \beta\,(I_d - I_f)_{t+T} + \varepsilon_{t+T},$$

where ε_t: regression error. That is, $E[\varepsilon_{t+T}] = 0$.

Then, $E[s_{t,T}] = \alpha + \beta\, E[(I_d - I_f)_{t+T}] + E[\varepsilon_{t+T}] = \alpha + \beta\, E[s_{t,T}^{PPP}]$

⇒ $E[s_{t,T}] = \alpha + \beta\, E[s_{t,T}^{PPP}]$

⇒ For Relative PPP to hold, on average, we need **α=0** & **β=1**.

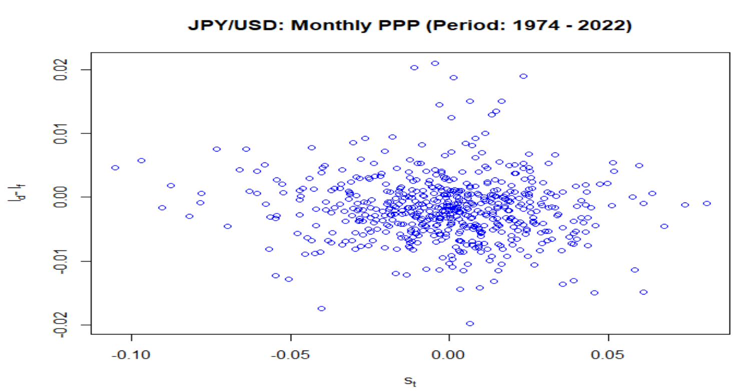
• Relative PPP: General Evidence

Under Relative PPP: $s_{t,T} \approx (I_d - I_f)$

1. Visual Evidence

Plot $(I_{JPY} - I_{USD})_t$ against s_t(JPY/USD), using monthly data **1975 - 2022**.

Test: Is there a 45° line?



No 45° line ⇒ Visual evidence rejects PPP.

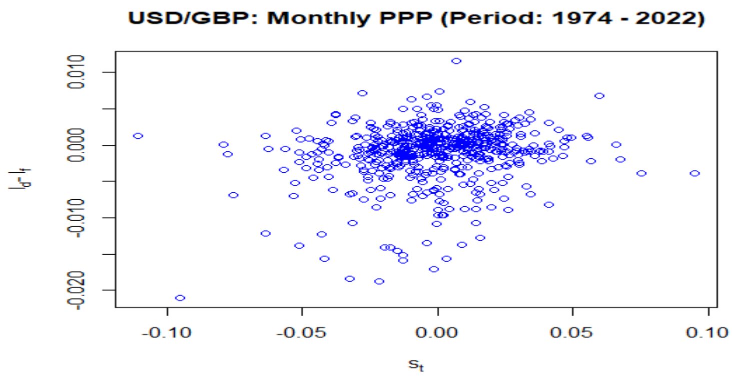
• Relative PPP: General Evidence

Under Relative PPP: $s_{t,T} \approx (I_d - I_f)$

1. Visual Evidence

Plot $(I_{USD} - I_{GBP})_t$ against s_t(USD/GBP), using monthly data **1975 - 2022**.

Test: Is there a 45° line?



• Relative PPP: General Evidence

Under Relative PPP: $s_{t,T} \approx (I_d - I_f)$

1. Visual Evidence

Plot $(I_{USD} - I_{GBP})$ against s_t(USD/GBP), using monthly data **1975 - 2022**.

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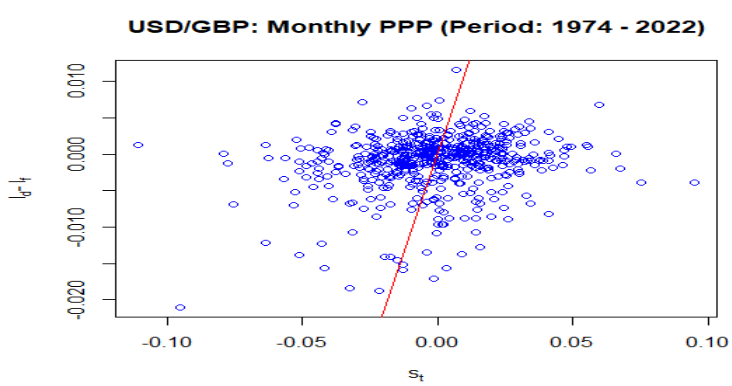
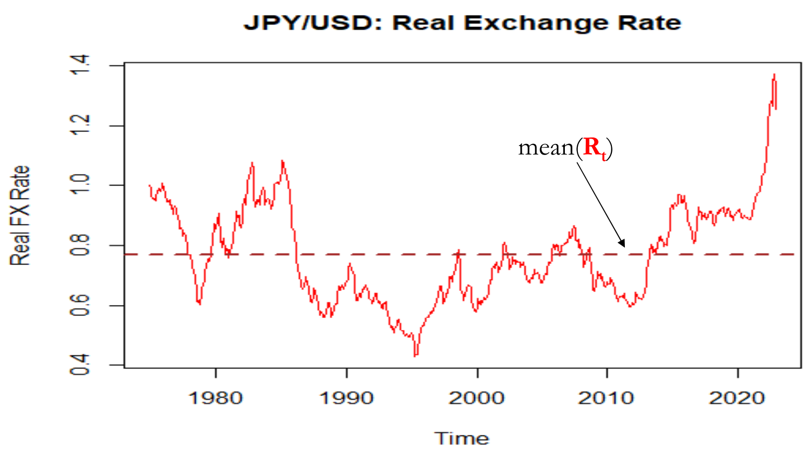


No 45° line $\Rightarrow$ Visual evidence rejects PPP.

• Relative PPP: General Evidence

1. Visual Evidence

Test: Is $\mathbf{R_t}$ ≈ Constant? (Under Absolute PPP ≈ 1)



Some evidence for mean reversion, though slow, for $\mathbf{R_t}$ (average = 0.77).

• Relative PPP: General Evidence (continuation)

In the long run, $\mathbf{R_t}$ moves around some mean number (long-run PPP parity?). But, the deviations from long-run parity are very *persistent*.

Economists report the number of years that a PPP deviation is expected to decay by 50%, the *half-life*. The half-life is in the range of **3 to 5 years** for developed currencies. Very slow!

• Descriptive Stats (**1975:Jan – 2022:Dec**)

Long-run, on average.

	I_{JPY}	I_{USD}	$I_{JPY} - I_{USD}$	$s_{t,T}$ (JPY/USD)
Mean	**0.00125**	**0.00303**	**-0.00179**	**-0.00139**
SD	**0.00485**	**0.00322**	**0.00502**	**0.02622**
Min	-0.01095	-0.01786	-0.01981	-0.08065
Median	0.00102	0.00266	-0.00184	0.00022
Max	0.02558	0.01420	0.02104	0.08066

Big difference in volatility.

2. Statistical Evidence

Formal test: Regression

$$s_{t,T} = \alpha + \beta\,(I_d - I_f)_{t+T} + \varepsilon_{t+T}, \qquad (\varepsilon_t\text{: error term, } E[\varepsilon_t] = 0).$$

The null hypothesis is: H_0 (Relative PPP true): **α=0** and **β=1**
H_1 (Relative PPP not true): **$\alpha \neq 0$** and/or **$\beta \neq 1$**

• **Tests:** *t-test* (individual tests on α and β) & *F-test* (joint test)

(1) Individual test: *t-test*

$$t\text{-test} = t_\theta = [\hat{\theta} - \theta_0]/\text{S.E.}(\hat{\theta})$$

where θ represents α or β $\Rightarrow$ (θ_0 = α or β evaluated under H_0).

Statistical distribution: $t_\theta \sim t_v$ ($v = N - K$ = degrees of freedom)

K = # parameters in model, & N = # of observations.

Rule: If $|t\text{-test}| > |t_{v,\alpha/2}|$, reject H_0 at the α level.
When $v = N - K > 30$, $t_{30+,.025} \approx$ **1.96** $\Rightarrow$ 2-sided C.I. α = .05 (5 %)

2. Statistical Evidence

(2) Joint Test: *F-test*

$$F = \frac{[\text{RSS}(H_0) - \text{RSS}(H_1)]/J}{\text{RSS}(H_1)/(N-K)}$$

Statistical distribution: $F \sim F_{J,N-K}$

J = # of restrictions in H_0 (under PPP, J=2: **α=0** & **β=1**)
K = # parameters in model (under PPP model, K=2: α & β)
N = # of observations

RSS = Residuals Sum of Squared, $\hat{\varepsilon}_t = e_t = s_t - [\hat{\alpha} + \hat{\beta}\,(I_{d,t} - I_{f,t})]$.

$$\text{RSS}(H_0) = \sum_{t=1}^{N} [s_t - (I_{d,t} - I_{f,t})]^2$$

$$\text{RSS}(H_1) = \sum_{t=1}^{N} (\hat{\varepsilon}_t)^2$$

Rule: If $F > F_{J,N-K,\alpha}$, reject at the α level. Usually, α = .05 (5 %)
When $N > 300$, $F_{J=2,300+,\alpha=.05} \approx$ **3**.

Example: Using monthly Japanese and U.S. data (**1975:Jan - 2022:Dec**), we fit the following regression (Observations = 576):

$$s_t \text{ (JPY/USD)} = (S_t - S_{t-1})/S_{t-1} = \alpha + \beta\,(I_{JAP} - I_{US})_t + \varepsilon_t.$$

$R^2 = 0.005621$

Standard Error (σ) = .02617

F-stat (slopes=0 –i.e., $\beta=0$) = 3.244 (*p-value* = 0.07219)

Observations (N) = 552

	Coefficient	Stand Err	t-Stat	P-value
Intercept ($\hat{\alpha}$)	-0.00209	0.001157	-1.804	0.0717
$(I_{JAP} - I_{US})$ ($\hat{\beta}$)	-0.39148	0.217343	-1.801	0.0722

We will test the H_0 (Relative PPP true): $\alpha=0$ & $\beta=1$

Two tests: (1) *t-tests* (individual tests)

(2) *F-test* (joint test)

Example: Using monthly Japanese and U.S. data (**1975:Jan - 2022:Dec**), we fit the following regression (Observations = 576):

$$s_t \text{ (JPY/USD)} = (S_t - S_{t-1})/S_{t-1} = \alpha + \beta\,(I_{JAP} - I_{US})_t + \varepsilon_t.$$

$R^2 = 0.005621$

Standard Error (σ) = .02617

F-stat (slopes=0 –i.e., $\beta=0$) = 3.244 (*p-value* = 0.07219)

F-test (H_0: $\alpha=0$ & $\beta=1$): **19.185** (*p-value*: < 0.00001) $\Rightarrow$ reject H_0 at 5% level ($F_{2,550,.05}$= **3.012**)

	Coefficient	Stand Err	t-Stat	P-value
Intercept ($\hat{\alpha}$)	-0.00209	0.001157	-1.804	0.0717
$(I_{JAP} - I_{US})$ ($\hat{\beta}$)	-0.39148	0.217343	-1.801	0.0722

Test H_0, using t-tests ($t_{574,.05}$ = **1.96** – <u>Note</u>: when N-K > 30, $t_{.05}$ = **1.96**):

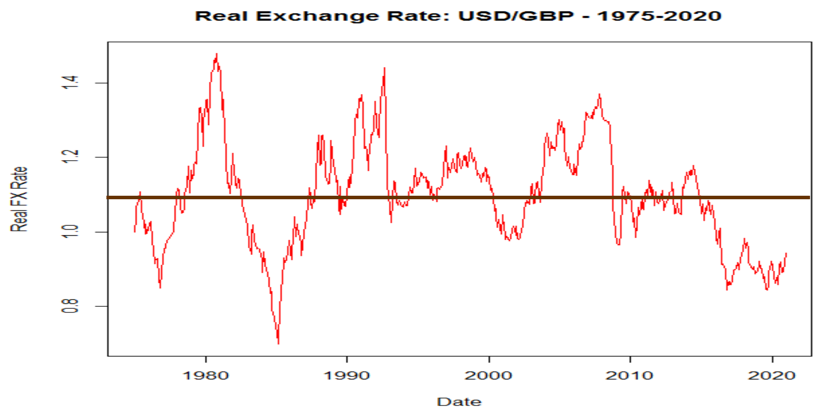
$t_{\alpha=0}$: (-0.00209 – 0)/0.001157 = -1.804 (*p-value* = .07) $\Rightarrow$ cannot reject H_0.

$t_{\beta=1}$: (-0.39148 – 1)/0.217343 = -6.402 (*p-value*: < .00001) $\Rightarrow$ reject H_0. ¶

• PPP Evidence:

◊ Relative PPP tends to be rejected in the short-run. In the long-run, there is debate about its validity: Currencies with high inflation rate differentials tend to depreciate.

◊ Some evidence for a mean reverting $\mathbf{R_t}$ (average $\mathbf{R_t}$ = 1.10). But deviations can last for years!

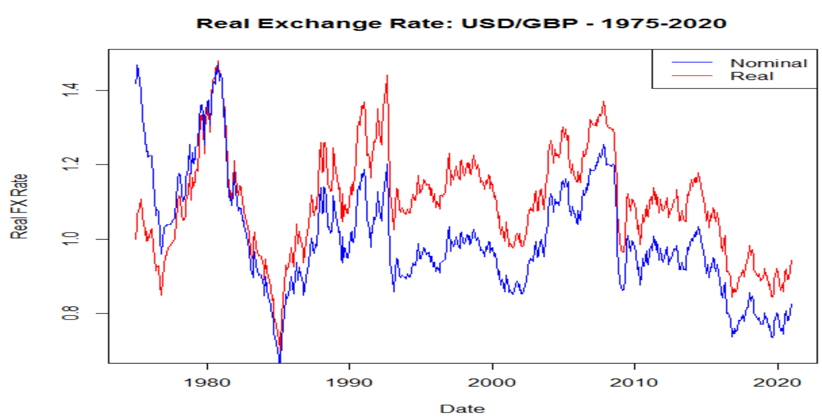


• PPP: R_t and S_t

Mussa (1986): $\mathbf{R_t}$ is more variable under a free float.

$\mathbf{R_t}$ variability is highly correlated with S_t variability.

Check Second Moments: Volatility (changes in $\mathbf{R_t}$) = 2.706% & Volatility (changes in S_t) = 2.622 (correlation = .983). Almost the same!



Implications: Price levels play a minor role in explaining the movements of **R_t** (prices are *sticky*).

Possible explanations:

(a) Contracts:

Prices cannot be continuously adjusted due to contracts.

(b) Mark-up adjustments:

Manufacturers and retailers moderate increases in their prices in order to keep market share. Changes in S_t are only partially transmitted or *pass-through* to import/export prices.

Average ERPT (exchange rate pass-through) is around **50%** over one quarter and **64%** over the long run for **OECD countries** (for the **U.S., 25%** in the short-run and **40%** over the long run).

(c) Repricing costs (*menu costs*)

Expensive to adjust continuously prices –a restaurant, re-printing the *menu*.

(d) Aggregation

Q: Is price rigidity a result of aggregation –i.e., the use of price index? Empirical work using **micro level data** –say, same good (exact UPC!) in Canadian and U.S. grocery stores– show that on average product-level **R_t** moves with S_t. But, evidence is not as solid.

• PPP: Puzzle

The fact that no single model of exchange rate determination can accommodate both the high persistent of PPP deviations and the high correlation between **R_t** and S_t has been called the "*PPP puzzle.*"

• PPP: Summary of Empirical Evidence

◊ R_t and S_t are highly correlated, P_d tends to be sticky.

◊ In the short run, PPP is a poor model to explain short-term S_t movements.

◊ PPP deviations are very persistent. They take years to disappear.

◊ In the long run, there is some evidence of mean reversion, though slow, for R_t. That is, S_t^{PPP} has long-run information:

Currencies that consistently have high inflation rate differentials tend to depreciate.

• The long-run interpretation is the one that economists like and use: S_t^{PPP} is seen as a benchmark.

• Calculating S_t^{PPP} (Long-Run FX Rate)

We want to calculate $S_t^{PPP} = P_{d,t} / P_{f,t}$ over time.

(1) Divide S_t^{PPP} by $S_{t=0}^{PPP}$ (t = 0 is our starting point).

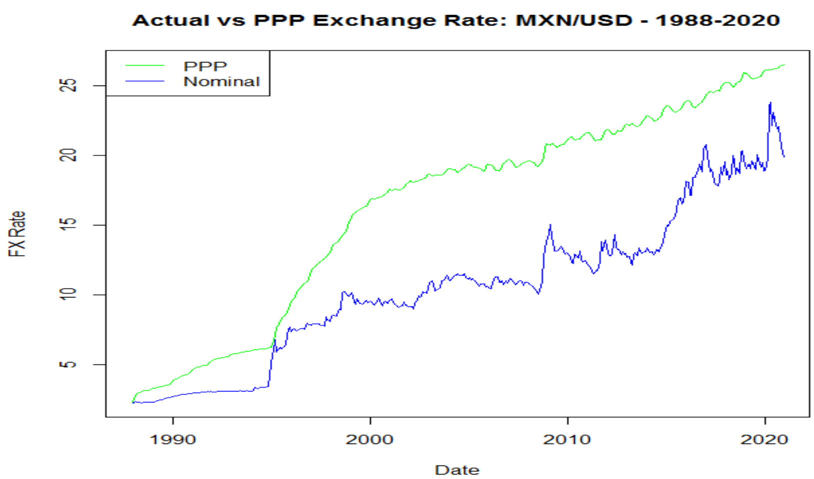
(2) After some algebra,

$$S_t^{PPP} = S_{t=0}^{PPP} * [P_{d,t} / P_{d,0}] * [P_{f,0}/P_{f,t}]$$

By assuming $S_{t=0}^{PPP} = S_0$, we plot S_t^{PPP} over time.

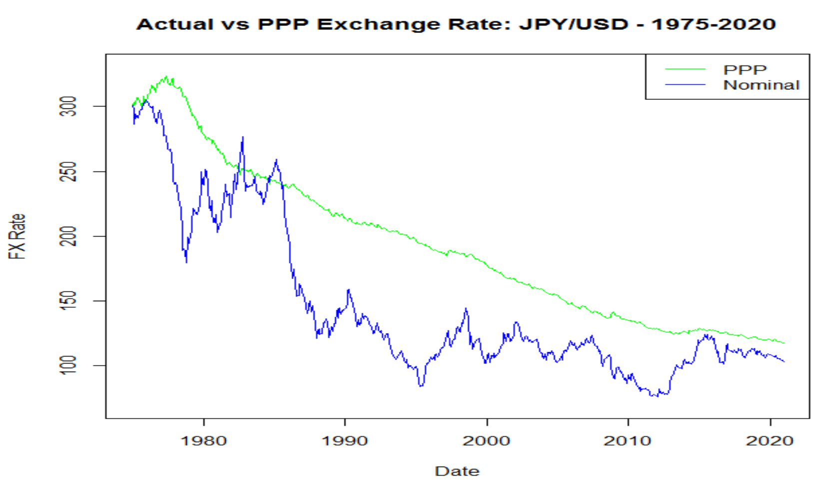
Note: $S_{t=0}^{PPP} = S_0$ assumes that at t=0, the economy was in *equilibrium*. This may not be true: Be careful when selecting a base year.

Let's look at the MXN/USD case.



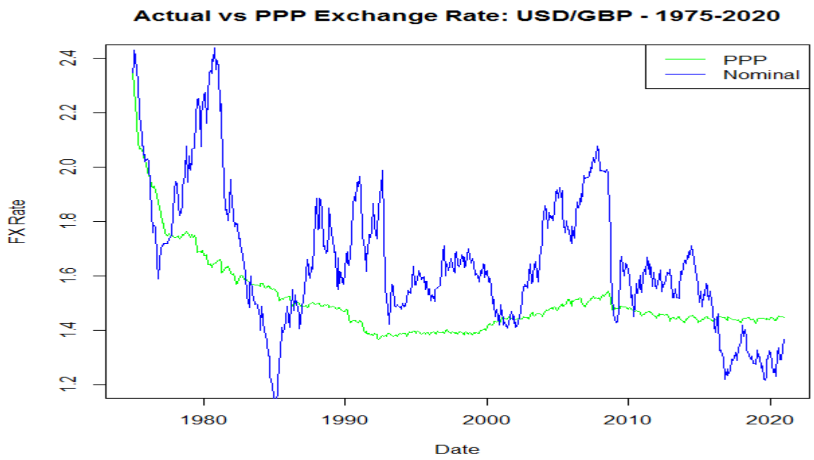
- In the short-run, $\mathbf{S_t^{PPP}}$ misses the target, S_t.
- But, in the long-run, $\mathbf{S_t^{PPP}}$ gets trend right, reflecting a consistent higher inflation in Mexico.

Another example, the JPY/USD case.



As predicted by PPP, since I_{US} has been consistently higher than I_{JAP}, in the long-run, the USD depreciates against the JPY.

Another example, the USD/GBP case.



As predicted by PPP, I_{US} was consistently lower than I_{UK} until the mid-90s, the USD appreciated against the GBP. Since then, it has been moving around a constant value.

• PPP Summary of Applications:

◊ Equilibrium ("*long-run*") exchange rates.

◊ Explanation of S_t movements.

◊ Indicator of competitiveness or under/over-valuation.

◊ International GDP comparisons: Instead of using S_t, $\mathbf{S_t^{PPP}}$ is used to translate local currencies to USD. For example, Chinese per capita GDP (World Bank figures, in 2017):

Nominal GDP per capita: **CNY 59,670.52**;
S_t = 0.14792 USD/CNY;
- Nominal GDP_cap (USD)= **CNY 59,670.52** * 0.1479 USD/CNY= **USD 8,827**

$\mathbf{S_t^{PPP}}$ = **0.2817 USD/CNY** ⇒ "U.S. is 90% more expensive"
- PPP GDP_cap (USD)= **CNY 59,670.52** * **0.2817 USD/CNY** = **USD 16,807**.

Country	GDP per capita (in USD) - 2017	
	Nominal	**PPP**
Luxembourg	104,103	103,745
USA	59,532	59,532
Japan	38,428	43,279
Italy	31,953	39,427
Czech Republic	20,368	36,504
Costa Rica	11,631	17,044
Brazil	9,821	15,484
China	**8,827**	**16,807**
Lebanon	8,524	14,676
Algeria	4,123	15,275
India	1,937	7,056
Ethiopia	767	1,899
Mozambique	416	1,247

Note: PPP GDP/Nominal GDP = **USD 16,807**/ **USD 8,827** = 1.9040

⇒ "U.S. is 90% more expensive." ¶

International Fisher Effect (IFE)

• IFE builds on the law of one price, but for financial transactions.

• Idea: The return to international investors who invest in money markets in their home country should be equal to the return they would get if they invest in foreign money markets once adjusted for currency fluctuations.

• Exchange rates are set in such a way that international investors cannot profit from interest rate differentials –i.e., no profits from *carry trades*.

Carry trade: A strategy that borrows the low interest currency to invest in the high interest currency.

That is, IFE determines $s_{t,T} = \frac{S_{t+T} - S_t}{S_t}$ that makes looking for the "extra yield" in international money markets not profitable.

The "*effective*" T-day return on a foreign bank deposit is:

$$r_f \text{ (in DC)} = \left(1 + i_f * \frac{T}{360}\right)(1 + s_{t,T}) - 1.$$

• While, the effective T-day return on a home bank deposit is:

$$r_d \text{ (in DC)} = i_d * T/360.$$

• Setting r_f (in DC) $= r_d$ and solving for $s_{t,T}$ $(= s_{t,T}^{IFE})$ we get:

$$s_{t,T}^{IFE} = \frac{\left(1 + i_d * \frac{T}{360}\right)}{\left(1 + i_f * \frac{T}{360}\right)} - 1 \qquad \text{(This is the IFE)}$$

• Using a linear approximation: $s_{t,T}^{IFE} \approx (i_d - i_f) * T/360.$

• $s_{t,T}^{IFE}$ represents an *expectation*: The expected change in S_t from t to t+T that makes looking for the "extra yield" in international money markets not profitable.

• Since IFE gives us an expectation for a future exchange rate, $S_{t,T}^{IFE}$, if we believe in IFE we can use this expectation as a forecast.

Example: Forecasting S_t using IFE.

It's 2022:I. You have the following information:

$S_{2022:I}$ = **1 .0659 USD/EUR**.

$i_{USD,2022:I}$ = 0.5%

$i_{EUR,2022:I}$ = 1.0%.

T = 1 semester = 180 days.

$$s_{t,T}^{IFE} = \frac{\left(1 + i_{d=USD,2022:I} * \frac{T}{360}\right)}{\left(1 + i_{f=EUR,2022:I} * \frac{T}{360}\right)} - 1 = \frac{\left(1 + .005 * \frac{180}{360}\right)}{\left(1 + .01 * \frac{180}{360}\right)} - 1 = \mathbf{-0.0024875}$$

$S_{t,2022:II}^{IFE} = S_{2022:I} * (1 + s_{t,2022:II}^{IFE})$ = **1.0659 USD/EUR** *(1 – **0.0024875**)

= **1.06325 USD/EUR**

⇒ IFE expects S_t to change to $S_{t,2022:II}^{IFE}$ = **1.06325 USD/EUR** to compensate for the lower US interest rates. ¶

Example (continuation):

$S^{IFE}_{t,2022:II} = S_{2022:I} * (1 + s^{IFE}_{t,2022:II})$
$= 1.0659 \text{ USD/EUR} * (1 - 0.0024875)$
$= 1.06325 \text{ USD/EUR}$

Suppose $S_{2022:II} = 1.08 \text{ USD/EUR} > S^{IFE}_{t,2022:II} = 1.06325 \text{ USD/EUR}$

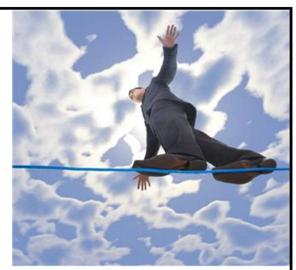
⇒ According to IFE, EUR is ***overvalued***.

⇒ Trading signal: Sell EUR/Buy USD.

Note: Same result by looking at the observed change:

$s_{2022:II} = 1.08/ 1.0659 - 1 = 0.01323 > s^{IFE}_{t,2022:II} = -0.0024875$.

⇒ According to IFE, EUR appreciated more than expected. That is, EUR is ***overvalued***. ¶



• Note: Like PPP, IFE also gives an *equilibrium* exchange rate. Equilibrium will be reached when there is no capital flows from one country to another to take advantage of interest rate differentials.

IFE: Implications

If IFE holds, the expected cost of borrowing funds is identical across currencies. Also, the expected return of lending is identical across currencies.

Carry trades –i.e., borrowing a low interest currency to invest in a high interest currency– should not be profitable.

If departures from IFE are consistent, investors can profit from them.

Example: Mexican peso depreciated 5% a year during the early 90s.
Annual interest rate differentials ($i_{MXN} - i_{USD}$) were between 7% and 16%.
Then, $E_t[\mathbf{s_{t,T}}]$ = -5% > $s_{t,T}^{IFE}$ = -7% ⇒ Pseudo-arbitrage is possible
(The MXN at t+T is ***overvalued***!)
Suppose we expect $E_t[\mathbf{s_{t,T}}] > s_{t,T}^{IFE}$ in next T days.

Carry Trade Strategy (USD = DC; we invest in the *overvalued* currency):
1) Borrow USD funds (at i_{USD}) for T days.
2) Convert to MXN at S_t
3) Invest in Mexican funds (at i_{MXN}) for T days.
4) *Wait until T*. Convert to USD at S_{t+T} –expect: $E[S_{t+T}] = S_t *(1+ E_t[\mathbf{s_{t,T}}])$.

Expected FX loss = 5% ($E_t[\mathbf{s_{t,T}}]$ = -5%)
Assume ($i_{USD} - i_{MXN}$) = -7%. (Say, i_{USD} = 6%; i_{MXN} = 13%.)
$E_t[\mathbf{s_{t,T}}]$ = -5% > $s_{t,T}^{IFE}$ = -7% ⇒ "On average," strategy (1)-(4) should work.

Example (continuation):
Expected USD return from MXN investment:

r_f (in DC) = $(1 + i_{MXN} * T/360) * (1 + E_t[\mathbf{s_{t,T}}]) - 1$
= (1 + .13) * (1 – **.05**) – 1 = 0.074

Payment for USD borrowing: $r_d = i_{d=USD} * T/360 = .06$

Expected Profit = E[Π] = 0.074 – .06 = .014 per year.

Overall expected profits ranged from: 1.4% to 11%. ¶

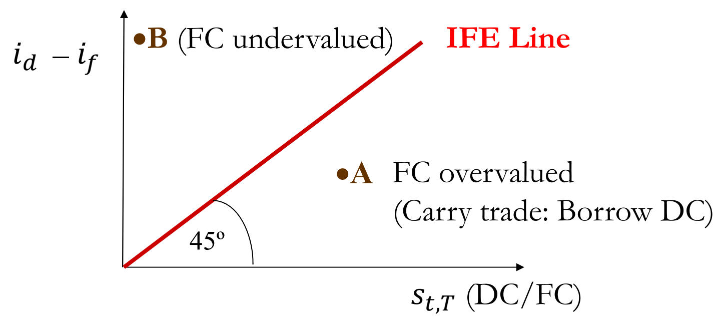
Note: A carry trade strategy is based on an expectation: $E_t[\mathbf{s_{t,T}}]$ = -5%. It may or may not occur every time. This is risky!

Example: Risk at work. Fidelity used this uncovered strategy during the early 90s. In Dec. 94, after the Tequila devaluation of the MXN against the USD (40% in a month), it lost everything it gained before. ¶

• An IFE driven carry trade differs from covered arbitrage in the final step. Step 4) involves no coverage. It's an *uncovered* strategy. IFE is also called *Uncovered Interest Rate Parity* (UIRP).

• UIRP is difficult to test since it involves an expectation (an *unobservable*). In general, we test UIRP assuming that on average what we expect occurs.

• Test: UIRP true (no carry trade profits) if $s_{t,T} \approx (i_d - i_f) * T/360$.

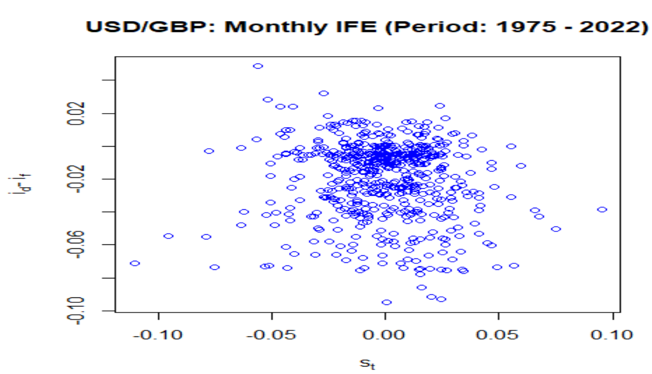


1. Visual evidence.

Based on linearized IFE: $s_{t,T} = \frac{S_{t+T} - S_t}{S_t} \approx (i_d - i_f) * T/360$

Expect a 45 degree line in a plot of $s_{t,T}$ against $(i_d - i_f)$

Example: Plot for the monthly USD/GBP exchange rate (**1975 - 2022**)



No 45° line ⇒ Visual evidence rejects IFE. ¶

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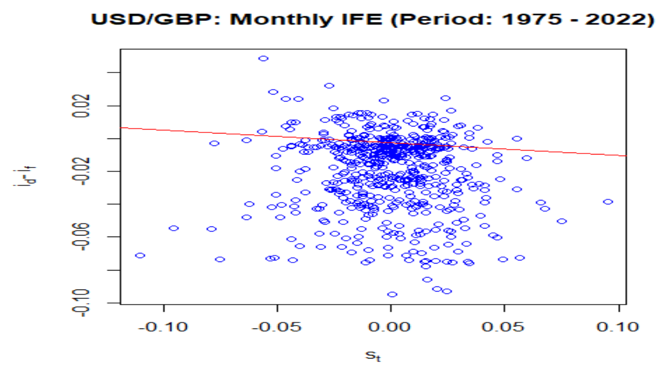

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2. Regression evidence

$$s_{t,T} = \alpha + \beta\,(i_d - i_f)_t + \varepsilon_t, \qquad (\varepsilon_t\text{: error term, } E[\varepsilon_t] = 0).$$

- The null hypothesis is: H_0 (IFE true): α=0 and β=1

  H_1 (IFE not true): α≠0 and/or β≠1

Example: Testing IFE for the USD/GBP with monthly data (**1975 - 2022**).

$R^2 = 0.00577$

Standard Error = 0.002377

F-statistic (slopes=0) = 3.33 (*p-value* = 0.0686)

F-test (α=0 and β=1) = **182.4331** (*p-value* = lower than 0.0001)

⇒ rejects H_0 at the 5% level ($F_{2,193,.05}$ = **3.05**)

Observations = 576

	Coefficients	Standard Error	t Stat	P-value
Intercept (α)	**-0.002676**	**0.001305**	-2.051	0.0408
$(i_d - i_f)_t$ (β)	**-0.077150**	**0.042590**	-1.825	0.0686

Let's test H_0, using t-tets ($t_{104,.05}$ = **1.96**) :

$t_{\alpha=0}$ (t-test for $\alpha = 0$): (**0.002676** – 0)/**0.00194** = **-2.051**

$\Rightarrow$ reject H_0 at the 5% level.

$t_{\beta=1}$ (t-test for $\beta = 1$): (**-0.077715** – 1)/**0.04259** = **-25.304**

$\Rightarrow$ reject H_0 at the 5% level.

Formally, IFE is rejected in the short-run (both the joint test and the t-tests reject H_0). Also, note that β is negative, not positive as IFE expects. ¶

- IFE is rejected. Then,

Q: Is a "carry trade" strategy profitable?

During the 1975-2022 period, the average monthly ($i_{USD} - i_{GBP}$) was:

-1.9947%/12= **-0.166%** $\Rightarrow$ s_t^{IFE} = **-0.166%** per month ($\neq$0, statistically)

Average monthly s_t(USD/GBP) was **-0.113%** ($\approx$**0**, statistically speaking)

$\Rightarrow$ **$E_t[s_t]$** = -**0.113%** > s_t^{IFE} = **-0.166%** (GBP overvalued!)

Note: Consistent deviations from IFE make carry trades profitable. During the 1975-2022 period, USD-GBP carry trades should have been profitable.

Carry trade strategy:

1) Borrow USD at i_{USD} for 30 days. (average i_{USD} = 4.28%)

2) Convert to GBP

3) Deposit BPG at i_{GBP} for 30 days. (average i_{GBP} = 6.27%)

4) Wait 30 days and convert back to USD (on average, 0% monthly change)

From 1) + 3), we make **0.166%** per month.

From 2) + 4), we lose **0.112%** per month.

Total carry trade gain over a year: **0.65%**.

$\Rightarrow$ Total gain over the whole period: **36.5%**. ¶

• IFE: Evidence

No short-run evidence ⇒ Carry trades work (on average).

Burnside (2008): The average excess return of an equally weighted carry trade strategy, executed monthly, over the period 1976–2007, was about **5% per year.** (Sharpe ratio twice as big as the S&P500, since annualized volatility of carry trade returns is much less than that for stocks).

Some long-run support:

"*Currencies with high interest rate differentials tend to depreciate.*"

(For example, the Mexican peso finally depreciated in Dec. 1994.)

Expectations Hypothesis (EH)

• According to the Expectations hypothesis (EH) of exchange rates:

$$E_t[S_{t+T}] = F_{t,T}.$$

⇒ On *average*, the future spot rate is equal to the forward rate.

Since expectations are involved, many times the equality will not hold. It will only hold on average.

Q: Why should this equality hold on average?

Suppose it does not hold. That means, what people expect to happen at time T is *consistently* different from the rate you can set for time T. A potential profit strategy can be developed that works, on average.

Example: Suppose that over time, investors violate EH.

Data: $F_{t,180}$ = **5.17 ZAR/USD.**

An investor expects: $E_t[S_{t+180}]$ = **5.34 ZAR/USD**. (A potential profit!)

Strategy for this investor:

1. Buy USD forward at **ZAR 5.17**
2. In 180 days, sell the USD for **ZAR 5.34.**

Now, suppose everybody expects $E_t[S_{t+180}]$ = **5.34 ZAR/USD**

⇒ *Disequilibrium*: Today, everybody buys USD forward. ($F_{t,180}$ ↑)

In 180 days, everybody will be selling USD. ($E_t[S_{t+180}]$↓)

⇒ Prices should adjust until EH holds.

Expectations are involved: Sometimes you will have a loss, but, on average, you profit from $E_t[S_{t+T}] \neq F_{t,T}$. ¶

Expectations Hypothesis: Implications

EH: $E_t[S_{t+T}] = F_{t,T}$ → On average, $F_{t,T}$ is an *unbiased* predictor of S_{t+T}.

Example: Today, it is 2014:II. A firm wants to forecast the quarterly S_t USD/GBP. You are given the **90-day** interest rate differential (in %) and S_t. Using IRP you calculate $F_{t,90}$:

$F_{t,90} = S_t * [1 + (i_{USD} - i_{GBP})_t * T/360]$. (⇒ S^{EH}_{t+90})

Data available:

$S_{t=2014:II}$ = **1.6883 USD/GBP**

$(i_{USD} - i_{GBP})_{t=2014:II}$ = -0.304%.

Then,

$F_{t,90}$ = **1.6883 USD/GBP** * [1 – 0.00304 * 90/360] = **1.68702 USD/GBP**

⇒ $S^{EH}_{t=2014:III}$ = **1.68702 USD/GBP**

According to EH, if a firm forecasts S_{t+T} using the forward rate, over time, will be right on average.

⇒ average forecast error $E_t[S_{t+T} - F_{t,T}] = 0$.

Expectations Hypothesis: Implications

Doing this forecasting exercise each period generates the following quarterly forecasts and forecasting errors, ε_t:

Quarter	$(i_{US} - i_{UK})$	S_t	$S^F_{t+90} = F_{t,90}$	$\varepsilon_T = S_{t+T} - S^F_{t+T}$
2014:II	-0.304	1.6883		
2014:III	-0.395	1.6889	**1.68702**	**0.0019**
2014:IV	-0.350	1.5999	**1.68723**	-0.0873
2015:I	-0.312	1.5026	**1.59850**	-0.0959
2015:II	-0.415	1.5328	**1.50143**	0.0314
2015:III	-0.495	1.5634	**1.53121**	0.0322
2015:IV		1.5445	**1.56146**	-0.0170

Calculation of the forecasting error for 2014:III:

$$\varepsilon_{t=2014:\text{III}} = 1.6889 - 1.68702 = 0.0019.$$ ¶

Note: Since $(S_{t+T} - F_{t,T})$ is unpredictable, expected cash flows associated with hedging or not hedging currency risk are the same.

Expectations Hypothesis: Evidence

Under EH, $E_t[S_{t+T}] = F_{t,T} \rightarrow E_t[S_{t+T} - F_{t,T}] = 0$

Empirical tests of the EH are based on a regression:

$$(S_{t+T} - F_{t,T})/S_t = \alpha + \beta Z_t + \varepsilon_t, \qquad \text{(where } E[\varepsilon_t]=0)$$

where Z_t represents any economic variable that might have power to explain S_t, for example, $(i_d - i_f)$.

H_0 (EH true): $\alpha = 0$ and $\beta = 0$. $((S_{t+T} - F_{t,T})$ should be unpredictable!)

H_1 (EH not true): $\alpha \neq 0$ and/or $\beta \neq 0$.

Usual result: $\beta < 0$ (and significant) when $Z_t = (i_d - i_f)$.
But, the R^2 is very low.

Expectations Hypothesis: IFE (UIRP) Revisited

EH: $E_t[S_{t+T}] = F_{t,T}$.

Replace $F_{t,T}$ by IRP, say, linearized version:

$E_t[S_{t+T}] \approx S_t * [1 + (i_d - i_f) * T/360]$.

A little bit of algebra gives:

$(E[S_{t+T}] - S_t)/S_t \approx (i_d - i_f) * T/360$ <= IFE linearized!

- EH can also be tested based on the Uncovered IRP (IFE) formulation:

$$(S_{t+T} - S_t)/S_t = s_{t,T} = \alpha + \beta\ (i_{US} - i_{UK})_t + \varepsilon_{t+T}.$$

The null hypothesis is H_0: α=0 and β=1.

Usual Result: $\beta < 0$ ⇒ when $(i_d - i_f)$ = 2%, the exchange rate appreciates by (β * .02), instead of depreciating by 2% as predicted by UIRPT!

- Risk Premium

The risk premium of a given security is defined as the return on this security, over and above the risk-free return.

- Q: Is a risk premium justified in the FX market?

  A: Only if exchange rate risk is not diversifiable.

After some simple algebra, we find that the expected excess return on the FX market is given by:

$$(E_t[S_{t+T}] - F_{t,T})/S_t = P_{t+T}.$$

A risk premium, P, in FX markets implies

$$E_t[S_{t+T}] = F_{t,T} + S_t\ P_{t+T}.$$

If P_{t+T} is consistently different from zero, markets will display a forward bias.

Example: Understanding the meaning of the FX Risk Premium.

Data: S_t = **1.58 USD/GBP**

$E_t[S_{t+6\text{-}mo}]$ = **1.60 USD/GBP**

$F_{t,6\text{-}mo}$ = **1.62 USD/GBP**.

- Expected change in S_t:

  ⇒ $E[s_{t+6\text{-}mo}] = (E_t[S_{t+6\text{-}mo}] - S_t)/S_t = (1.60 - 1.58)/1.58 = 0.0127$.

- 6-mo FX premium

  ⇒ $p_{6\text{-}mo} = (F_{t,6\text{-}mo} - S_t)/S_t = (1.62 - 1.58)/1.58 = 0.0253$.

- In the next 6-month period:

  The GBP is expected to appreciate against the USD by **1.27%**

  The forward premium suggests a GBP appreciation of **2.53%**.

  ⇒ $E[s_{t+6\text{-}mo}] < p_{6\text{-}mo}$ $(\approx (i_{d=USD} - i_{f=GBP})/2)$

⇒ Higher USD return from a USD deposit, than from a GBP deposit.

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E[Return from a GBP deposit] = GBP 1 * $(1 + i_{f=GBP}/2)$*1.60 USD/GBP

Return from a USD deposit = **1.58 USD/GBP** * $(1 + i_{d=USD}/2)$

• In the next 6-month period: $E[s_{t+6\text{-}mo}] \neq p_{6\text{-}mo}$

Discrepancy: The presence of a FX risk premium, $P_{t,t+6\text{-}mo}$, makes the forward rate a biased predictor of $S_{t+6\text{-}mo}$.

• The expected (USD) return from holding a GBP deposit will be less (different) than the USD return from holding a USD deposit.

Rational Investor: The lower return from holding a GBP deposit is necessary to induce investors to hold the riskier USD denominated investments. ¶