

Central Bank Intervention & Arbitrage in the FX Market

Dirty Float & IRP

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• Last Class

- S_t = Exchange rate = The Price of one currency in terms of another:

$$S_t = 1.0630 \text{ USD/EUR.}$$

- Determined by Supply & Demand in the Wholesale Tier (FX Market)

$$S_t = f(i_{\text{USD}} - i_{\text{FC}}, I_{\text{USD}} - I_{\text{FC}}, y_{\text{USD}} - y_{\text{FC}}, \text{other factors})$$

- Static Supply & Demand Effects

$(i_{\text{USD}} - i_{\text{FC}}) \uparrow \Rightarrow$ Foreign residents buy more U.S. T-bills (S of FC \uparrow) &
U.S. residents buy less FC T-bills (D for FC \downarrow)
 $\Rightarrow S_t \downarrow$ FC *depreciates* against USD.

$(I_{\text{USD}} - I_{\text{FC}}) \uparrow \Rightarrow$ Foreign residents buy less U.S. goods (S of FC \downarrow) &
U.S. residents buy more FC goods (D for FC \uparrow)
 $\Rightarrow S_t \uparrow$ FC *appreciates* against USD.

Uncertainty $\uparrow \Rightarrow S_t \uparrow$ FC haven currencies *appreciate* against USD.
 $\Rightarrow S_t \downarrow$ FC risky currencies *depreciate* against USD.

• **Last Class**

• FX Market

- ◊ Largest financial market: **USD 7.5T**
- ◊ Very liquid with small bid-ask spreads.
- ◊ Geographically dispersed: London (largest market), NY, HK, ...
- ◊ Open 24/7/365.
- ◊ OTC market organized around network of 2,000 banks & FX brokers.
- ◊ Three segments:
 - Spot (**30%**)
 - Forward (**15%**)
 - FX Swap (**50%**)
- ◊ Electronic settlement (currency never leaves country of emission).

• **Last Class**

• Central Bank & Monetary Policy

- ◊ CB is a “bank,” with assets (mainly, Treasuries and other assets) and liabilities (mainly, DC and bank deposits).
- ◊ Dual role in the economy:
 - Lender of last resort & Regulator of banking industry
 - Monetary policy.
- ◊ Today’s main function: Monetary policy, set i_d while balancing I_d & Y_d .
- ◊ Targets are conflicting: $i_d \uparrow \Rightarrow I_d \downarrow$, but $Y_d \downarrow$.
 $i_d \downarrow \Rightarrow I_d \uparrow$, but $Y_d \uparrow$.
- ◊ A Taylor rule approximates the empirical behavior of CB:
$$i_d = \omega + \lambda I_d + \theta Y_{\text{gap}}$$
Taylor rule says i_d was low in 2022 Q3: It should have been **7.42%**:
$$i_d = .01 + 1.5 * (.0469) + 0.5 * (-.0123) = .0742 \text{ (7.42\%)}$$

• **Last Class**

- Exchange Rate Regimes are defined by the role of CB:
 - ◊ Free Float or Flexible
 - ◊ Fixed (in practice, most popular).
- Main con of Fixed Regime: A CB gives up Monetary Policy.
- Trilemma: It is impossible for a country to have at the same time:
 - ◊ A fixed (stable) FX regime.
 - ◊ Free international capital mobility –i.e., no capital controls.
 - ◊ An autonomous (independent) monetary policy.
- A Currency crisis is the result of a CB's *inconsistent* monetary policy.
Typical example, a CB attempts to have an independent monetary policy under a Fixed Regime.

Potential problem: Big disparity between potential free float rate, S_f , & S^* .

• **Today's Class**

- Currency Crisis.
- Central Bank FX Intervention
 - ◊ Usual Mechanism (Non-sterilized)
 - ◊ Sterilized & Non-sterilized
 - ◊ Does CB Intervention work?
- Arbitrage in FX Markets
 - ◊ Local (sets exchange rates across FX dealers)
 - ◊ Triangular (sets cross-rates)
 - ◊ Covered (sets forward/futures rates)
- Behavior of Exchange Rates
 - ◊ Parity Conditions (PPP, IFE, EH)
 - ◊ Macroeconomic Models

Currency Crisis: Inconsistent Monetary Policy

An inconsistent monetary policy under a Fixed Regime creates a wedge between the potential free float rate S_t & S^* ($S_t > S^*$).

Notes:

- We think of free float S_t as the “*true equilibrium*” (or “*shadow*”) rate.
- The size of $(S_t - S^*)$ signals the magnitude of the inconsistency. It is also the size of the potential profit for speculators if CB abandons fixed parity.

Eventually, as inconsistency grows, a *speculative attack* on FC reserves occurs.

Speculators will attack the CB reserves when they have doubts that the CB will defend the parity. In these situations, we usually say a CB (or a country) faces a *currency crisis*.

- **CB Dilemma:** To Defend or Not To Defend parity?
A CB considers costs & benefits of defending fixed parity, S^* .

Usually, CBs defend S^* .

• Currency crisis: To Defend or Not To Defend?

Usually, CBs defend S^* . Tools:

- Sell FC reserves
- Borrow FC
- Substantially raise i_d
- Impose capital controls.

These actions may be costly & cause (or make worse) a recession.

- **Definite solution to a speculative attack:** Float the currency (abandon S^*).
- When a CB abandons S^* , a devaluation/depreciation occurs. Speculators gain!
- Speculators questions: Will the CB be able to defend the parity S^* ? Will the government bear the costs of defending it?
- **Currency Run:** Domestic residents *run* to banks to exchange DC for FC, before the devaluation occurs (or banks run out of FC!).

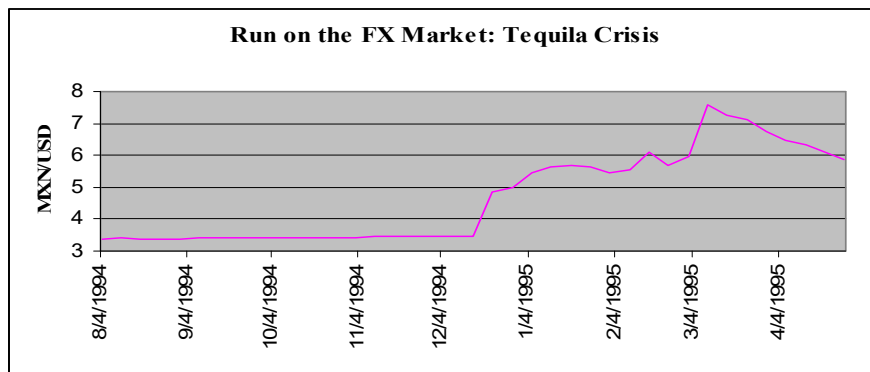
Currency Crisis: Devaluation

- Terminology

A *devaluation* (*revaluation*) occurs when the price of FC under a fixed exchange rate regime is increased (decreased) by the CB.

Note: The possibility of a currency crisis creates a risk: *devaluation risk*. The magnitude of this risk depends on the CB credibility –i.e., very credible CB, devaluation risk near zero.

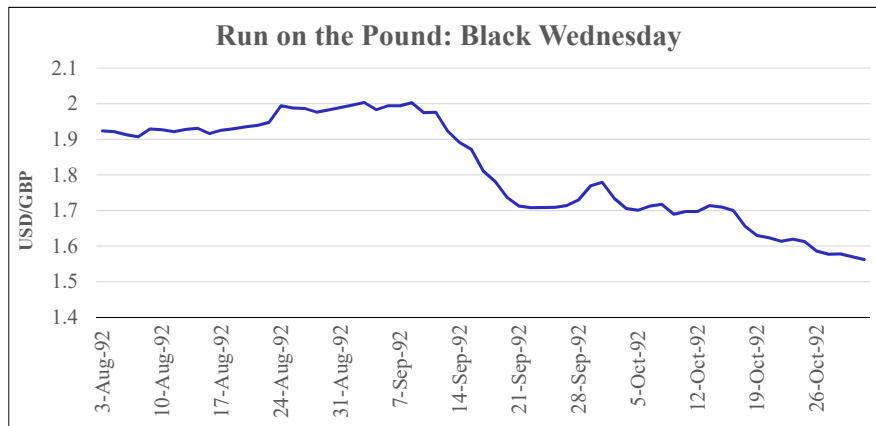
Currency Crisis: The “Tequila crisis” – Mexico Dec ‘94



Mexico had a crawling peg to the USD, but due to presidential elections, $MS_d \uparrow$. FC reserves went from **USD 18B** in **October 1994** to **USD 5B** in **December 1994**, when CB abandoned the fixed system.

Overall, Mexico spent **USD 25B** in FC reserves to defend the peso & also borrowed **USD 25B** (bailout funds from the U.S. Fed).

Currency Crisis: “Black Wednesday” – U.K. Sep 16 ‘92



U.K. was part of the ERM, with the GBP tied, implicitly, to the DEM at $S_t = 2.95 \text{ DEM/GBP}$, with $\pm 6\%$ band. But, when $i_{\text{DEM}} \uparrow$ to contain the spending due to German reunification, the BOE did not follow.

Overall, UK spent **GBP 30B** in FC reserves (and lost **GBP 3.3B**).

Currency Crisis: Big Devaluations

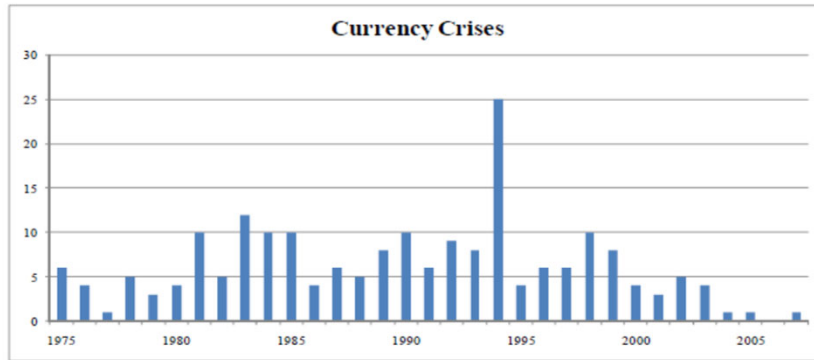
- On average, a currency crisis is followed by a **30%** devaluation of the DC. In many cases, there is a temporary higher drop (say, **50%**).

A very serious crisis: **75%+** (Indonesia '97, Argentina '01).

Examples: India '91, UK '92 (Black Wednesday), Mexico '94 (Tequila), Thailand '97 (Rice), Russia '98 (Vodka), Brazil '99 (Caipirinha), Argentina '01 (Tango), Uruguay '03, Iceland '08, Nigeria '16, Turkey '18, Lebanon '20 (ongoing).

Currency Crisis: Not Rare

Currency crisis are not rare. Figure below shows **208 successful** currency crises –defined as a **30% depreciation of DC** that is also, at least, a 10% increase from previous year. (Period: 1975 – 2008.)

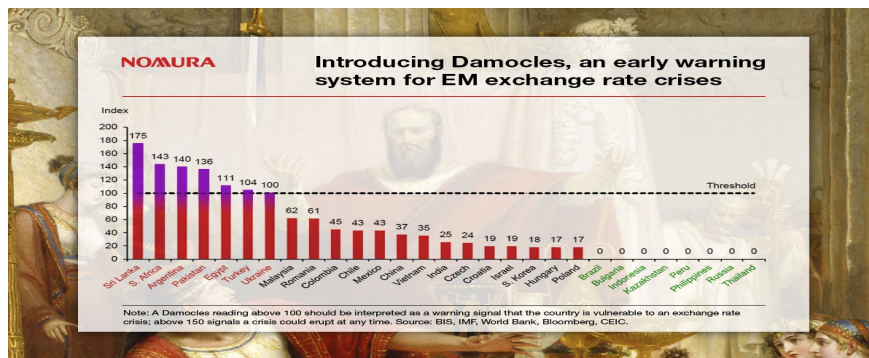


Note: Currency crisis is defined as a nominal depreciation of the currency of at least 30 percent that is also at least a 10 percent increase in the rate of depreciation compared to the year before. Five-year exclusion windows employed. The figure for 1994 is inflated by the devaluation of the 14 African members of the CFA zone against the French franc and the dollar.
Source: Laeven and Valencia (2008).

Currency Crisis: Predictors (“Early warning signals”)

Predictors of a currency crisis: Low FC reserves, high government deficits, low real exchange rate (DC overvalued, often due to high domestic inflation), weak financial system, high short-term debt, etc.

Many traders use an index to predict a currency crisis. A new one is the “*Damocles Index*,” used by *Nomura*. (Nomura claims 67% of past 54 EM currency crisis were predicted 12 months in advanced.)

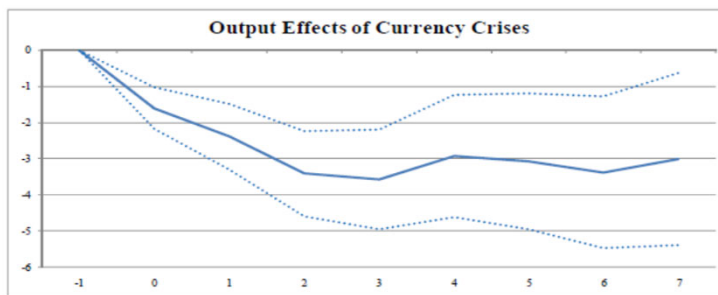


Devaluations Are Unpopular

- Economic Reasons:

- *Pass-through* to import prices (Domestic prices increase) \Rightarrow Inflation
- Real wages decrease
- Contractionary impact on the economy, especially in EM: **3%** average loss of GDP after 7 years!

The contraction is usually associated with balance sheet effects –i.e., a mismatch between currency of denomination of debt (mainly, in FC) and income (mainly, in DC)– in corporate and government sectors.



Devaluations Are Unpopular

- Politicians are run out of office.

- Cooper (1971) finds that heads of state lose their jobs twice as often within 1 year of devaluation:

30% as compared to **14%** in a non-devaluation control group.

- Frankel (2005), updated sample 1971 – 2003 and measured exit 6 months after devaluation:

23% (=23/109) as compared to **12%** in control group.

Twin Ds

- A currency crisis is a product of serious macro-economic problems: **Sovereign defaults** and/or **banking crisis** are not rare during these times.

In general, sovereign defaults are accompanied by large devaluations. These are the “*Twin Ds*”: *Default* and *Devaluation*.

- Reinhart (2002), looking at the period 1970 – 1999:
 - Prob[Devaluation | Default] = **84%**
 - Prob[Devaluation | No Default] = 17%

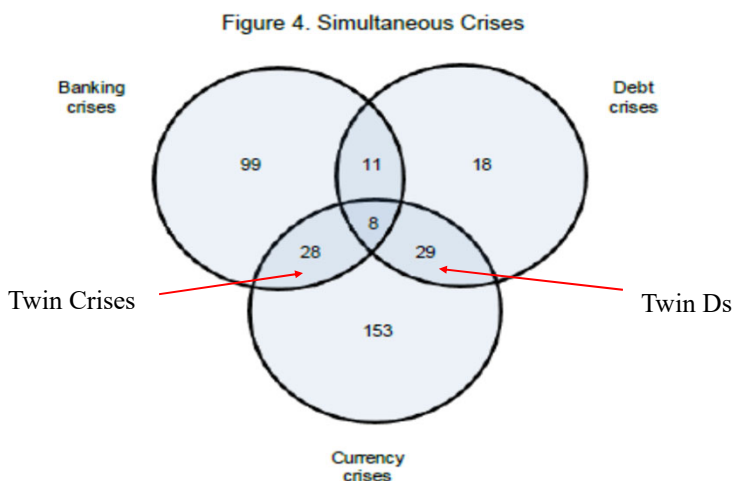
Na et al. (2017) expand sample to 2013: 84% is too high.

- Prob[Devaluation | Default] = **50%**

Laevan and Valencia (2012), using their own definitions of a currency crisis, find a similar probability: **56%** (=37/66).

Twin Ds

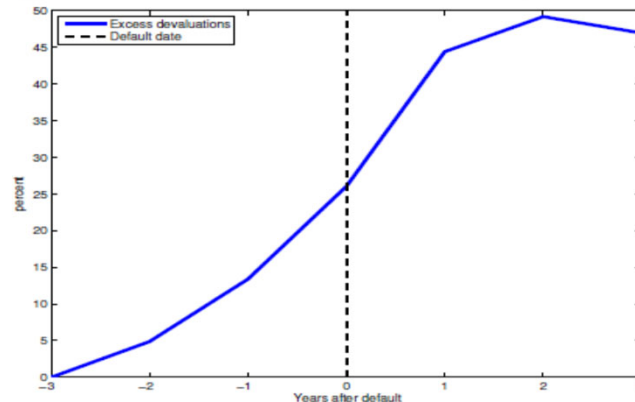
- Laevan and Valencia (2012) also report the following diagram with the Twin Ds and the *Twin Crisis* (simultaneous banking & currency crises).



Twin Ds

- Default is usually followed by a large devaluation: **45%** in a 6-year window around a default event.

Figure 1: Excess Devaluation Around Default, 1975-2013



Na et al. (2017) suggest that these large devaluations are needed to realign prices (real wages) to avoid an unemployment shock.

Other FX Regimes:

Managed Float

In practice, the FX rate system is a mixture: *Managed floating* or *dirty float*.

We see a free float, but the CB *intervenes* to buy & sell FC with the *intent* of changing the market determined S_t , every time the CB does not like S_t . CBs from EM countries tend to intervene much more than others.

Dual Systems

In some markets, S_t is fixed by the government. But, the government sells FC at the official S_t only for some transactions. For all the other transactions, a *black market* is created.

Example: By the end of 2022, Argentina had 10 (yes, “ten”) FX rates:

- 1) “Official”: **192 ARS/USD**, for official imports & some exports.
- 2) Black market (“Blue”): of **385 ARS/USD**.
- 3) Burse (MEP): **356 ARS/USD**, for buying/selling government debt.
- 4) Tourist: 30% tax on official rate + 70% extra as advanced income tax.
- 5) Cultural (“Coldplay”): 30% tax on official rate, for foreign artists.

Example (continuation): Argentina's multiple FX rates:



Range of Exchange Rate Regimes

Ranked in terms of (decreasing) flexibility for the CB:

- Free Float or Flexible
- Managed “*Dirty*” Float
- Crawling Peg
- Fixed
- Currency Board (Fixed + 100% FC reserves)
- Adopting a FC as legal tender, for example, “*dollarization*” (Panama, British Virgin Islands, El Salvador, Ecuador, Zimbabwe).
- In 2017, the IMF classifies:
 - 54% of currencies as “*anchored*” (fixed FX rate)
 - 20% as “*stabilized*” (anchored, but allowed to vary in some way)
 - 26% as “*floating*” (occasional CB Intervention OK).

Exchange Rate Regimes: Fixed or Flexible?		
Feature	Fixed	Flexible
	Cons	Pros
Adjustment to imbalances	Difficult	Easy
External shocks	Vulnerable	Less vulnerable
Support S_t	May need to raise i_d (or cause recession)	No need to do anything
Monetary policy	Ineffective	Effective
	Pros	Cons
FX Volatility	Stable S_t (good for trade & investments)	Volatile (P_d also volatile)
I_d : Control/Reduce	Good (with credibility)	Harder
Fiscal policy	Effective	Ineffective

Exchange Rate Regimes: Fixed or Flexible?
<ul style="list-style-type: none"> • Both regimes have pros and cons: No clear winner. • We observe: <ul style="list-style-type: none"> – Large economies with sound economic policies, good institutions & high credibility prefer a flexible regime. – Developed economies with bad economic policies, bad institutions & low credibility rely on a fixed regime. • <u>Aside Q</u>: If a CB decides to fix, which currency should be the anchor? <ul style="list-style-type: none"> Stable trade & investments advantage: Fix against currency of a large trading partner: <ul style="list-style-type: none"> – In Latin America, the USD is a good choice. – In Andorra (between Spain and France), the EUR should be the anchor.

Central Bank FX Intervention

• **Definition**

FX Intervention: CBs buys & sells FC with the *intent* to change S_t to a different S_t^E .

- CBs use models to determine S_t^E . Then, CB determines a range for S_t
 $\Rightarrow S_t$ should move between S_t^L and S_t^U .

If S_t is within the range ($S_t^L < S_t < S_t^U$), CB does nothing (Free float!)

If $S_t > S_t^U$, CB determines FC is overvalued \Rightarrow CB intervention

If $S_t < S_t^L$, CB determines FC is undervalued \Rightarrow CB intervention

$S_t > S_t^U$: Appreciating FC \Rightarrow CB sells FC.

$S_t < S_t^L$: Depreciating FC \Rightarrow CB buys FC.

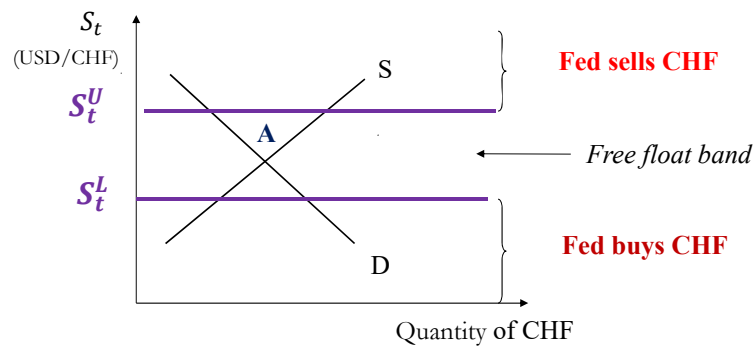
Situation: Suppose the US Fed follows the value of the CHF.

If **A** is within the range ($S_t^L < S_t < S_t^U$), Fed does nothing (*Free float*)

If $S_t > S_t^U$, Fed determines FC is overvalued \Rightarrow **Fed sells CHF**

If $S_t < S_t^L$, Fed determines FC is undervalued \Rightarrow **Fed buys CHF**

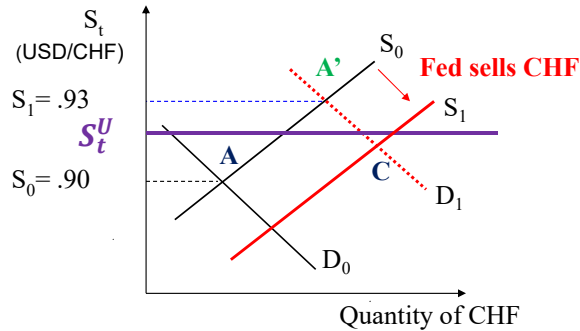
\Rightarrow The Fed acts like an FX speculator.



Example 1: USD depreciates against CHF (A to A').

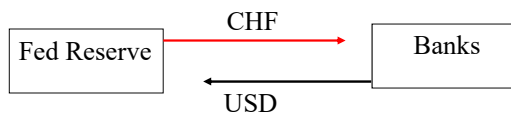
At $S_t = .93$ USD/CHF, the Fed determines CHF too expensive: $S_t > S_t^U$

⇒ CB intervention: **Fed sells CHF**



⇒ Fed sells CHF to bring S_t under S_t^U (A' to C).

FX Intervention



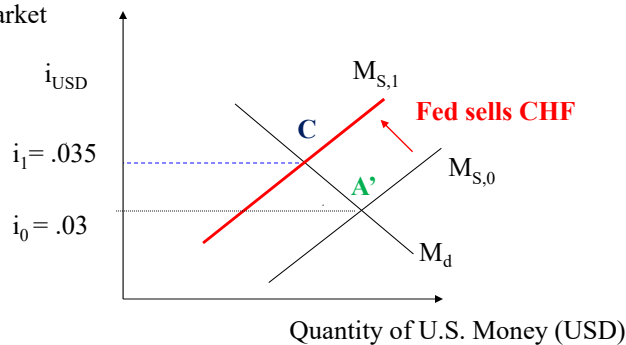
The Fed sells CHF and receives (“buys”) USD.

• CB FX intervention affects money supply:

When CB sells (buys) FC ⇒ Money supply decreases (increases)
(This is the Fixed Regime characteristic of the managed float.)

Example 1 (continuation): Fed intervenes to halt appreciation of CHF.

U.S. Money Market

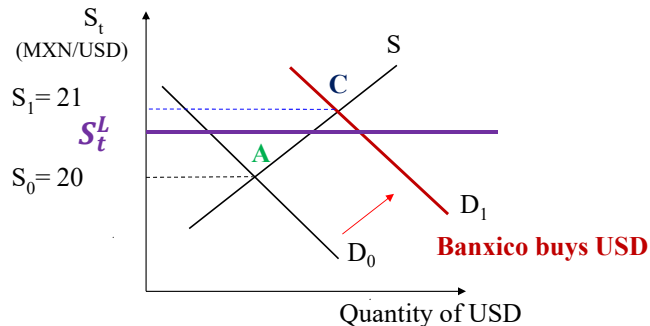


Process: **Fed sells CHF** ⇒ $M_S \downarrow$ ⇒ interest rates (i_{USD}) ↑

Example 2: Banco de Mexico (Banxico, Mexico's CB) FX Intervention

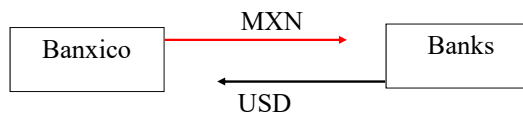
At **A**, Banxico considers the USD too inexpensive (*Undervalued*: $S_t < S_t^L$).

⇒ CB *intervenes*: **Banxico buys USD**



⇒ Banxico buys USD to bring S_t over S_t^L (**A** to **C**).

FX Intervention



Banxico buys USD and pays with MXN ⇒ $M_S \uparrow \Rightarrow i_{MXN} \downarrow$

• CB General Policy Objective for FX Intervention: Stabilization

Lean against the Wind:

- CB sells FC when it is appreciating.
- CB buys FC when it is depreciating.

• CB Intervention: Issues

(1) Implicit notion of “overvaluation/undervaluation” in FX market.

⇒ Q: Do CBs have "superior" information?

A: Mixed evidence: Some CBs have big losses; others profits.

(2) CB generates FX stability?

⇒ Uncertainty over CB actions increase FX volatility & risk.

Precisely, what a CB dislikes.

⇒ Q: But, do CBs succeed to reduce FX volatility? Not clear.

(3) Potential conflict with other countries. When a CB intervenes in the FX market ($S_t \uparrow$) to boost exports, trading partners will be affected.

⇒ *beggar-they-neighbor* devaluation. Popular in the 1930s.

• **CB Intervention: Details**

- CBs tend to deal with major domestic banks, but will also transact with major foreign banks.
- Size of intervention. The final size depends on the initial FX market reaction.
- How often do CBs intervene? In a 1999 BIS survey of CBs, CBs report intervening from **0.5%** to **40%** of business days (**4.5% median**).
- Disclosure of intervention? Most CBs intervene secretly. Why secrecy? Poor credibility, bad fundamentals.

• **CB Intervention: Data**

CB **do** intervene in FX markets.

Historically, the largest player by far is **Japan**. For example, between April 1991 and October 2021, the BOJ intervened in the FX Market:

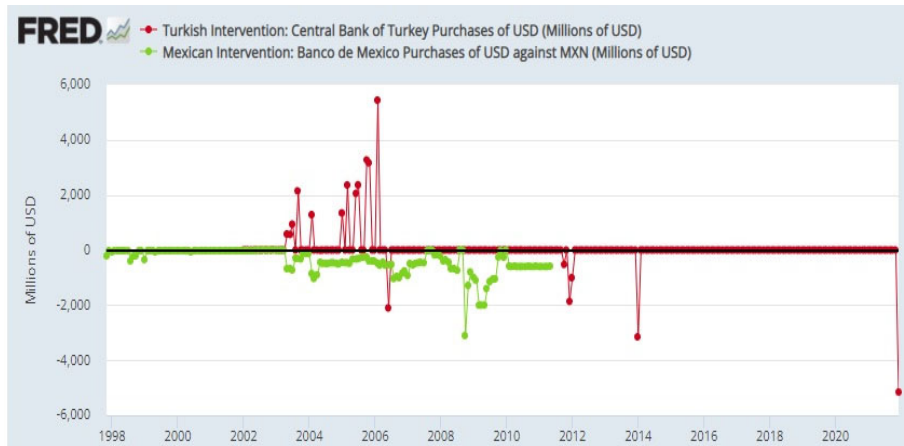
- Buying USD on 319 occasions for a total amount of USD 798B.
- Selling USD on 32 occasions for a total amount of USD 48B.

BOJ interventions exceeded U.S. interventions by a factor of more than 30.



• **CB Intervention: Data**

Example: Turkey and Mexico do CB intervention, in general to support their currency.



• **Other CB Interventions in the FX Market**

- CBs can buy foreign assets, instead of FC.

Example: The PBOC and the Bank of Japan may buy U.S. Treasuries to stop the decline of the USD against the CNY & the JPY, respectively.

- Other tools CBs can use:
 - Forward/option market, instead of the spot market.
 - Use taxes, capital controls, banking regulations, etc.
 - Coordinate with other CBs (*Concerted Intervention*).
 - Coordinate with other state agencies (sovereign funds, SOEs, etc.)
 - CB officials “Talk of under/overvaluation.”
- The last one is the most popular form of intervention, usually referred as *jawboning*. Here, the credibility of CBs plays a big role.

• **Sterilization**
 CB actions taken to neutralize the effects of intervention in Money Markets: Change in domestic interest rates.

Back to **Example 1**: Fed sells CHF (move from **A'** to **C**: $MS \downarrow$ & $i_{USD} \uparrow$)

FX Market

U.S. Money Market

- Suppose the Fed does not want a higher i_{USD} .
- CB tools to change MS: Open Market Operation (OMO), bank's RRR.

• **Sterilization**
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FX Market

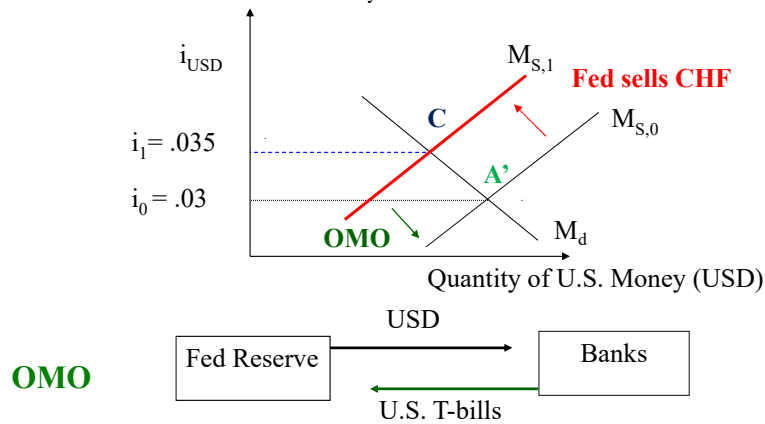
U.S. Money Market

- Suppose the Fed does not want a higher i_{USD} .
- CB tools to change MS: Open Market Operation (OMO), bank's RRR.

• **Sterilization in the U.S. with OMO**

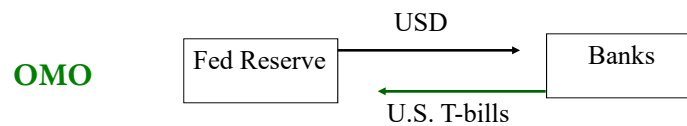
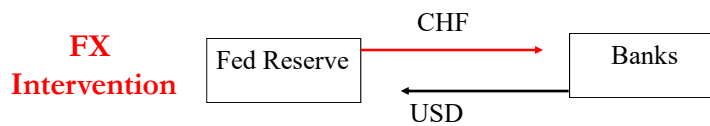
- When the Fed buys T-bills, exchanging USD for T-bills \Rightarrow US MS \uparrow
- When the Fed sells T-bills, exchanging USD for T-bills \Rightarrow US MS \downarrow

Example 1 (continuation): Back to previous example.
The Fed uses an OMO: Fed buys T-Bills to increase MS.

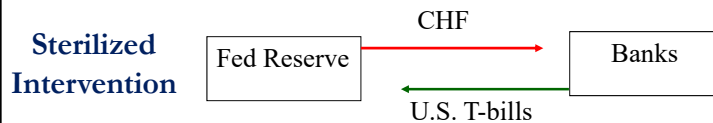


This CB intervention will be classified as *sterilized intervention*.

CB Intervention + Sterilization: Cash flows exchange:



Net effect: OMO + Fed Intervention

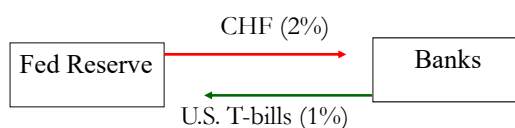


• **Sterilized Interventions: Side Effects**

- Sterilization changes the composition of the Fed's (and, in equilibrium, the public's) mix of DC & FC assets. This creates a *balance sheet effect*.

Depending on the rates of return of the assets involved, this effect can be positive or negative for the CB.

Example: If CHF T-bills pay 2% and U.S. T-bills pay 1%, the previous change in the Fed's mix has a negative effect.



• **Sterilized Interventions: Side Effects**

- Suppose the CB can keep for a while S_t artificially high/low and money markets out of sync with the FX Market.

Example: CB keeps S_t low (DC overvalued) to keep I_d low. Then, the CB forces the economy to subsidize the import sector (& domestic consumption) and leaves domestic producers in a tough situation.

For a short time, the side effects can be tolerated; for a long time, they can lead to a *resource allocation problem*.

- Banks do not like holding large amounts of government bonds and/or having high reserve-requirement ratios
⇒ A squeeze in bank's profits.

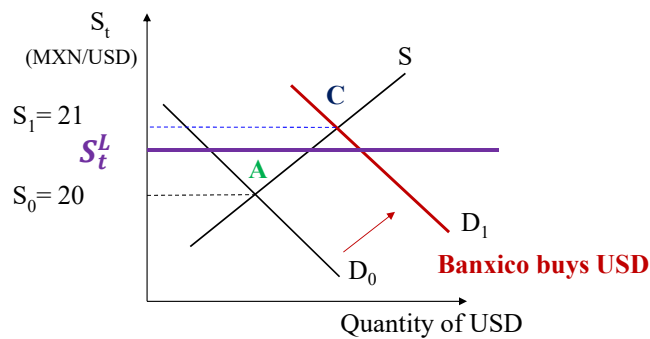
Example 2 (continuation): At **A**, Banxico considers the USD *undervalued* ($S_t < S_t^L$), with:

$$S_0 = 20 \text{ MXN/USD} \text{ \& } S_t^L = 20.6 \text{ MXN/USD}$$

Banxico decides to intervene, but does not want to affect i_{MXN} .

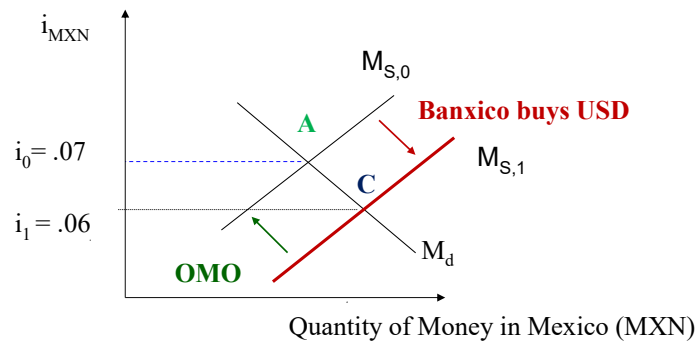
Original Situation: $S_0 = 20 \text{ USD/MXN}$ & $i_0 = 7\%$

Banxico FX intervention (Buy USD): $S_1 = 21 \text{ USD/MXN}$ & $i_1 = 6\%$



OMO: Buy MXN - Sell CETES: $S_1 = 21 \text{ USD/MXN}$ & $i_0 = 7\%$

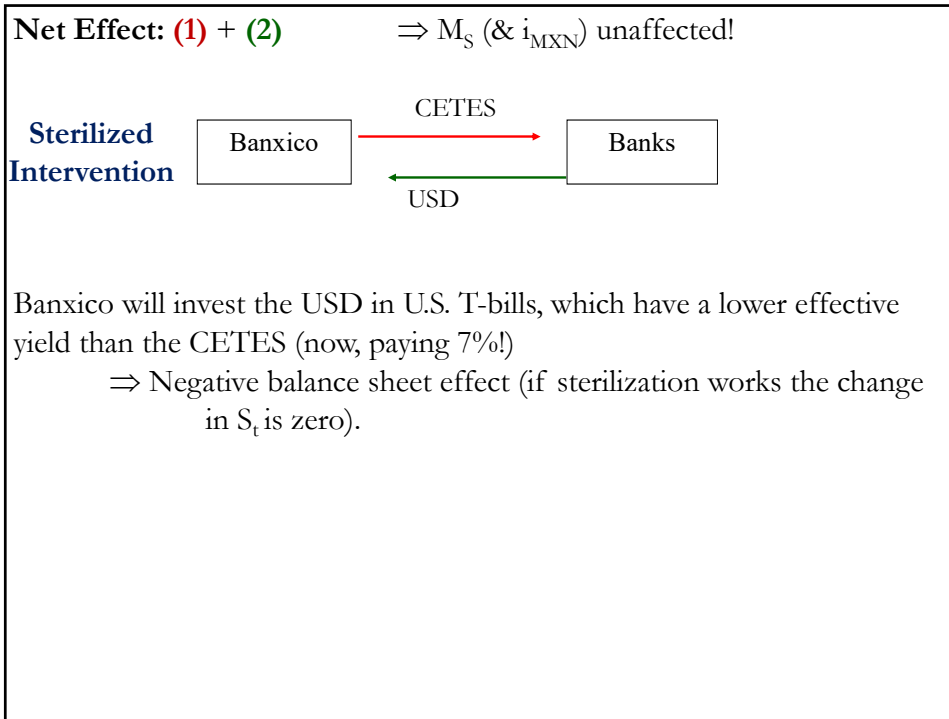
Mexican Money Market



Process:

- (1) **Banxico buys USD** $\Rightarrow M_S \uparrow \Rightarrow$ interest rates (i_{MXN}) \downarrow
- (2) **Banxico sells CETES** $\Rightarrow M_S \downarrow \Rightarrow$ interest rates (i_{MXN}) \uparrow

Net Effect: (1) + (2) $\Rightarrow M_S$ (& i_{MXN}) unaffected!



• **Sterilized Interventions: Do They Work?**

In the short-run, sterilizations tend to work, affecting S_t in the direction the CB wanted. But the evidence regarding lasting effects on S_t is mixed and it tends to be on the *negative side*, especially for major currencies.

Sustaining sterilizations can be costly, due to the balance sheet effects. Over time, these costs can be difficult to bear.

Mohanty and Turner (2005) report that, between 2000 and 2004, the CBs of Korea, the Czech Republic, and Israel issued currency-stabilizing bonds of values equivalent to 300%, 200% and, 150% of their respective reserve money for the purpose of sterilization operations.

\Rightarrow Interest payments, when domestic interest rates go up, render sterilization operations too costly to last.

• **Fear of Floating**

Central Banks report to the IMF their FX regime. Many CBs, especially from emerging markets, report a free float.

- However, from looking at the volatility of exchange rates, the actual regime looks more like a fixed regime. The volatility is small relative to the volatility of currencies with a free float (US, Australia, Japan, Europe).
- The data also shows relative high FC reserve volatility.
- That is, CBs in these countries they do intervene. This CB behavior is called “*fear of floating*.”

Arbitrage in FX Markets

Arbitrage Definition

It is an activity that takes advantages of *pricing mistakes* in financial assets in one or more markets. It involves *no risk* and *no capital of your own*.

Elements:

- Pricing mistake
- No Capital
- No Risk

- There are 3 types of arbitrage
 - (1) Local (sets uniform rates across banks)
 - (2) Triangular (sets cross rates)
 - (3) Covered (sets forward rates)

Elements of Arbitrage:

- Pricing mistake
- No Capital
- No Risk

Remark: The definition presents the ideal view of (riskless) arbitrage. “Arbitrage,” in the real world, involves some risk. We will call this arbitrage *pseudo arbitrage*.

- Big literature on the limitations of arbitrage in real world settings (“*limits of arbitrage*”).

Local Arbitrage (One good, one market)

Example: Suppose two banks have the following bid-ask FX quotes:

	Bank A		Bank B	
USD/GBP	1.50	1.51	1.53	1.55

Sketch of Local Arbitrage strategy:

- (1) Borrow USD 1.51
- (2) Buy GBP 1 from **Bank A** at **USD 1.51**
- (3) Sell GBP 1 to **Bank B** at **USD 1.53**
- (4) Return USD 1.51 & make a USD .02 profit (1.31% per USD borrowed)

Note I: All steps should be done simultaneously. Otherwise, there is risk!

Note II: **Bank A** and **Bank B** will notice a book imbalance

Bank A: All activity at $S_{A,ask}$ (buy GBP orders at 1.51)

Bank B: All activity at $S_{B,bid}$ (sell GBP orders at 1.53).

⇒ Both banks will adjust the quotes. Say,

Bank A adjusts $S_{A,ask} = 1.54$ USD/GBP ($S_{A,ask} \uparrow$). ¶

Triangular Arbitrage (Two related goods, one market)

Triangular arbitrage is a process where two related goods set a third price.

- In FX Markets, triangular arbitrage sets FX *cross rates*.
- Cross rates do not involve the USD. Most currencies are quoted against the USD. Thus, cross-rates are calculated from USD quotations.

Example: a JPY/GBP quote is derived from

$$S_{JPY/USD,t} \text{ (say, 100 JPY/USD)}$$

$$S_{USD/GBP,t} \text{ (say, 1.60 USD/GBP)}$$

- Cross-rates are calculated in a way that avoids triangular arbitrage. For example, using above quotes:

$$S_{JPY/GBP,t} = S_{JPY/USD,t} * S_{USD/GBP,t} = \mathbf{160 \text{ JPY/GBP}}$$

Example: Suppose Bank One gives the following quotes:

$$S_{JPY/USD,t} = 100 \text{ JPY/USD}$$

$$S_{USD/GBP,t} = 1.60 \text{ USD/GBP}$$

$$S_{JPY/GBP,t} = 140 \text{ JPY/GBP}$$

Taking the first two quotes \Rightarrow Implied (no-arbitrage) JPY/GBP:

$$S_{JPY/GBP,t}^I = S_{JPY/USD,t} * S_{USD/GBP,t} = \mathbf{160 \text{ JPY/GBP}} > S_{JPY/GBP,t}$$

At $S_t = 140 \text{ JPY/GBP}$, Bank One **undervalues** the GBP against the JPY (with respect to the first two quotes). \leq **Pricing mistake!**

Sketch of Triangular Arbitrage (Key: Buy undervalued GBP with the overvalued JPY):

(1) Borrow JPY 140

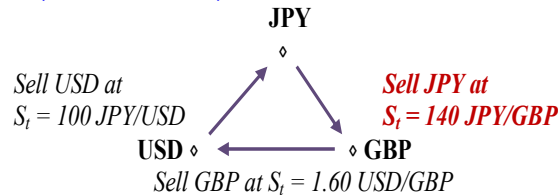
(2) **Sell the JPY/Buy GBP at $S_{JPY/GBP,t} = 140 \text{ JPY/GBP}$. Get GBP 1.**

(3) Sell GBP/Buy USD (at $S_{USD/GBP,t} = 1.60 \text{ USD/GBP}$). Get USD 1.60.

(4) Sell USD/Buy JPY (at $S_{JPY/USD,t} = 100 \text{ JPY/USD}$). Get JPY 160

$$\Rightarrow \text{Profit: } \Pi = \mathbf{\text{JPY } 20} \text{ (14.29\% per JPY borrowed).}$$

Example (continuation):



Note: Bank One will notice a book imbalance: All the activity involves **selling JPY for GBP**, **selling GBP for USD**, & **selling USD for JPY**.

Bank One will adjust quotes:

$$\left. \begin{array}{l} S_{JPY/GBP,t} \uparrow \quad (\text{say, } S_{JPY/GBP,t} = 145 \text{ JPY/GBP}). \\ S_{USD/GBP,t} \downarrow \quad (\text{say, } S_{USD/GBP,t} = 1.56 \text{ USD/GBP}) \\ S_{JPY/USD,t} \downarrow \quad (\text{say, } S_{JPY/USD,t} = 93 \text{ JPY/USD}). \end{array} \right\} \Rightarrow S_{JPY/GBP,t}^I \downarrow$$

There is convergence between $S_{JPY/GBP,t}^I$ & $S_{JPY/GBP,t}$:

$$S_{JPY/GBP,t}^I \downarrow (= S_{JPY/USD,t} \downarrow * S_{USD/GBP,t} \downarrow) \Leftrightarrow S_{JPY/GBP,t} \uparrow$$

Remark: Again, all steps should be done at the same time.

Example (continuation):

Note: It does not matter which currency you borrow in step (1). Recall the pricing mistake: Bank One *undervalues* the GBP against the JPY (with respect to the first two quotes):

$$S_{JPY/GBP,t}^I = 160 \text{ JPY/GBP} > S_{JPY/GBP,t} = 140 \text{ JPY/GBP}$$

Sketch of Triangular Arbitrage (Key: Buy undervalued GBP with the overvalued JPY). Simultaneously we do the following steps:

- (1) Borrow USD 1
- (2) Sell the USD/Buy JPY at $S_{JPY/USD,t} = 100 \text{ JPY/USD}$. Get JPY 100.
- (3) **Sell JPY/Buy GBP (at $S_{JPY/GBP,t} = 140 \text{ JPY/GBP}$).**
 \Rightarrow **Get GBP 0.7143**
- (4) Sell GBP/Buy USD (at $S_{USD/GBP,t} = 1.60 \text{ USD/GBP}$).
 \Rightarrow Get USD 1.1429

$$\text{Profit: } \Pi = \text{USD } 0.1429 \text{ (14.29\% per USD borrowed).}$$

Covered Interest Arbitrage (4 instruments: 2 goods per market & 2 markets)

From Bloomberg:

- Brazilian bonds yield **10%**
- Japanese bonds yield **1%**

Q: Why wouldn't capital flow to Brazil from Japan?

A: FX risk.

⇒ The only way to avoid FX risk is to be *covered* with a forward FX contract.

Intuition: Today, at $t=0$, we have the following data:

$i_{JPY} = 1\%$ for 1 year ($T=1$ year)

$i_{BRL} = 10\%$ for 1 year ($T=1$ year)

$S_t = .025$ BRL/JPY

Carry Trade: A speculative strategy to take “advantage” of interest rate differentials.

Today (time $t=0$), we do the following:

- (1) Borrow JPY 1,000 at **1%** for 1 year. (At $T=1$ year, repay JPY 1,010.)
- (2) Convert to BRL at **.025 BRL/JPY**. Get BRL 25.
- (3) Deposit BRL 25 at **10%** for 1 year. (At $T=1$ year, receive BRL 27.50.)

At time $T=1$ year, we do the final step:

- (4) Exchange BRL 27.50 for JPY at $S_{T=1-yr}$
⇒ $\Pi = \text{BRL } 27.50 / S_{T=1-yr} - \text{JPY } 1010$

Problem with this strategy: It is risky ⇒ today ($t=0$), $S_{T=1-yr}$ is unknown

Profits are a function of an unknown price at time $t=0$:

$$\Pi = \text{BRL } 27.50 / S_{T=1-yr} - \text{JPY } 1010$$

• Scenarios for $S_{T=1-year}$:

- $S_{T=1-yr} = 0.02 \text{ BRL/JPY}$. Then,

$$\Pi = \text{BRL } 27.50 / (0.02 \text{ BRL/JPY}) - \text{JPY } 1010 = \text{JPY } 365$$

- $S_{T=1-yr} = 0.03 \text{ BRL/JPY}$. Then,

$$\Pi = \text{BRL } 27.50 / (0.03 \text{ BRL/JPY}) - \text{JPY } 1010 = \text{JPY } -93.33$$

Note: The break-even $S_{T=1-yr}$ is

$$S_{T=1-yr}^{BE} = 27.50/1010 = 0.027227723 \text{ BRL/JPY.}$$

• We can cover ourselves and eliminate all uncertainty (& risk) with a forward contract.

• Carry trade with cover.

Suppose at $t=0$, a bank offers $F_{t,1-year} = .026 \text{ BRL/JPY}$.

Then, at time $T=1$ year, we do the final step:

(4') Exchange BRL 27.50 for JPY at $.026 \text{ BRL/JPY}$.

⇒ We get **JPY 1057.6923** (= BRL 27.50/.026 BRL/JPY).

⇒ $\Pi = \text{JPY } 1057.6923 - \text{JPY } 1010 = \text{JPY } 47.8$

or 4.78% per JPY borrowed.

Now, instead of borrowing **JPY 1,000**, we will try to borrow **JPY 10 billion** (and make a **JPY 478M profit**) or more.

Obviously, no bank will offer a $.026 \text{ BRL/JPY}$ forward contract!

⇒ Banks will offer $F_{t,1-year}$ contracts that produce $\Pi \leq 0$.

Interest Rate Parity Theorem (IRP)

Q: How do banks price FX forward contracts?

A: In a way that arbitrageurs cannot take advantage of their quotes.

To price a forward contract, banks consider covered arbitrage strategies.

Notation:

i_d = domestic nominal T days interest rate (annualized).

i_f = foreign nominal T days interest rate (annualized).

S_t = time t spot rate (direct quote, for example USD/GBP).

$F_{t,T}$ = forward rate for delivery at date T, at time t.

Note: In developed markets (like the US), *all* interest rates are quoted on annualized basis.

Now, consider the following (*covered*) strategy:

1. At $t=0$, borrow from a foreign bank FC 1 for T days.
 \Rightarrow At time T, We pay the foreign bank FC: $(1 + i_f * T/360)$.
2. At $t=0$, exchange FC 1 = DC S_t .
3. At $t=0$, deposit DC S_t in a domestic bank for T days.
 \Rightarrow At time T, receive DC: $S_t * (1 + i_d * T/360)$.
4. At $t=0$, buy a T-day forward contract to exchange DC for FC at a $F_{t,T}$.
 \Rightarrow At time T, exchange (in DC) $S_t * (1 + i_d * T/360)$ for FC, using $F_{t,T}$.
 \Rightarrow We get FC: $S_t * (1 + i_d * T/360) / F_{t,T}$.

This strategy will not be profitable if, at time T, what we receive in FC is less or equal to what we have to pay in FC. That is, arbitrage will force:

$$S_t * (1 + i_d * \frac{T}{360}) / F_{t,T} = (1 + i_f * \frac{T}{360}).$$

Solving for $F_{t,T}$, we get:
$$F_{t,T} = S_t * \frac{(1 + i_d * \frac{T}{360})}{(1 + i_f * \frac{T}{360})}$$

$$F_{t,T} = S_t * \frac{\left(1 + i_d * \frac{T}{360}\right)}{\left(1 + i_f * \frac{T}{360}\right)}$$

This equation represents the *Interest Rate Parity Theorem* (IRPT or just IRP).

It is common to use the following linear IRPT approximation:

$$F_{t,T} \approx S_t * \left[1 + (i_d - i_f) * \frac{T}{360}\right].$$

This linear approximation is very accurate for small differences in $(i_d - i_f)$.

Example: Using IRPT.

$S_t = 106 \text{ JPY/USD}$.

$i_{d=JPY} = .034$.

$i_{f=USD} = .050$.

T = 1 year

$$\Rightarrow F_{t,1-yr}^{IRP} = 106 \text{ JPY/USD} * (1+.034)/(1+.050) = 104.384 \text{ JPY/USD}.$$

Using the linear approximation:

$$F_{t,1-yr}^{IRP} \approx 106 \text{ JPY/USD} * (1 - .016) = 104.304 \text{ JPY/USD}.$$

Example 1: Violation of IRPT at work.

$S_t = 106 \text{ JPY/USD}$.

$i_{d=JPY} = .034$.

$i_{f=USD} = .050$.

$$F_{t,1-yr}^{IRP} = 106 \text{ JPY/USD} * (1 - .016) = 104.304 \text{ JPY/USD}.$$

Suppose Bank A offers: $F_{t,1-yr}^A = 100 \text{ JPY/USD}$.

$$F_{t,1-yr}^A = 100 \text{ JPY/USD} < F_{t,1-yr}^{IRP} \text{ (pricing mistake!)}$$

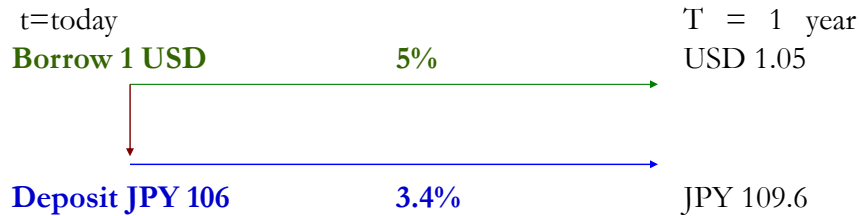
\Rightarrow Bank A *undervalues* the forward USD against the JPY.

We take advantage of Bank A's mistake: Buy USD/Sell JPY forward.

Sketch of a covered arbitrage strategy:

- (1) Borrow USD 1 from a U.S. bank for one year at 5%.
- (2) Exchange the USD for JPY at $S_t = 106 \text{ JPY/USD}$.
- (3) Deposit the JPY in a Japanese bank at 3.4%.
- (4) **Cover.** Buy USD forward/Sell forward JPY at $F_{t,1-yr}^A = 100 \text{ JPY/USD}$

Example 1 (continuation):



Cash flows at time $T = 1$ year,

(i) We get: $\text{JPY } 106 * (1 + .034) / (100 \text{ JPY/USD}) = \text{USD } 1.096$

(ii) We pay: $\text{USD } 1 * (1 + .05) = \text{USD } 1.05$

$$\Pi = \text{USD } 1.096 - \text{USD } 1.05 = \text{USD } .046$$

That is, after one year, the U.S. investor realizes a risk-free profit of USD .046 per USD borrowed (4.6% per unit borrowed).

Note: Arbitrage will force Bank A's quote to quickly converge to

$$F_{t,1-yr}^{IRP} = 104.304 \text{ JPY/USD. } \P$$

Example 2: Violation of IRPT 2.

Now, suppose Bank X offers: $F_{t,1-yr}^X = 110 \text{ JPY/USD}$.

$$F_{t,1-yr}^X = 110 \text{ JPY/USD} > F_{t,1-yr}^{IRP} \text{ (a pricing mistake!)}$$

\Rightarrow The forward USD is *overvalued* against the JPY.

We take advantage of Bank X's overvaluation: Sell USD forward.

Sketch of a covered arbitrage strategy:

- (1) Borrow JPY 1 for one year at 3.4%.
- (2) Exchange the JPY for USD at $S_t = 106 \text{ JPY/USD}$
- (3) Deposit the USD at 5% for one year.
- (4) **Cover.** Sell USD/Buy JPY forward at $F_{t,1-year}^X = 110 \text{ JPY/USD}$.

Cash flows at $T=1$ year:

(i) We get: $\text{USD } 1 / 106 * (1 + .05) * (110 \text{ JPY/USD}) = \text{JPY } 1.0896$

(ii) We pay: $\text{JPY } 1 * (1 + .034) = \text{JPY } 1.034$

$$\Pi = \text{JPY } 1.0896 - \text{JPY } 1.034 = \text{JPY } .0556 \text{ (or } 5.56\% \text{ per JPY borrowed)}$$

The Forward Premium and the IRPT

Reconsider the linearized IRPT. That is,

$$F_{t,T} \approx S_t * [1 + (i_d - i_f) * \frac{T}{360}]$$

A little algebra gives us:

$$\frac{F_{t,T} - S_t}{S_t} * \frac{360}{T} \approx (i_d - i_f)$$

Let $T = 360$. Then,

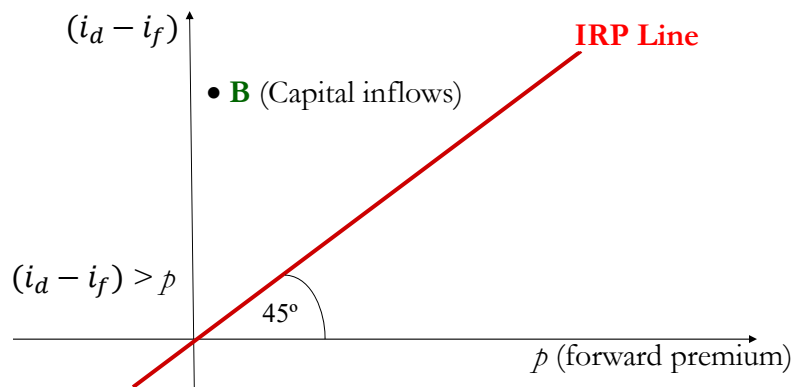
$$p \approx i_d - i_f$$

Note: p is the annualized % gain/loss of buying FC spot and selling it forward. Then,

- $p + i_f$: Annualized return from converting DC to FC & investing in FC (covered) for T days.
- i_d : Opportunity cost of borrowing DC (to buy FC at S_t).

Equilibrium: p exactly compensates $(i_d - i_f)$ → No arbitrage
 → No capital flows.

Equilibrium: $p \approx (i_d - i_f)$ ⇒ IRP Line

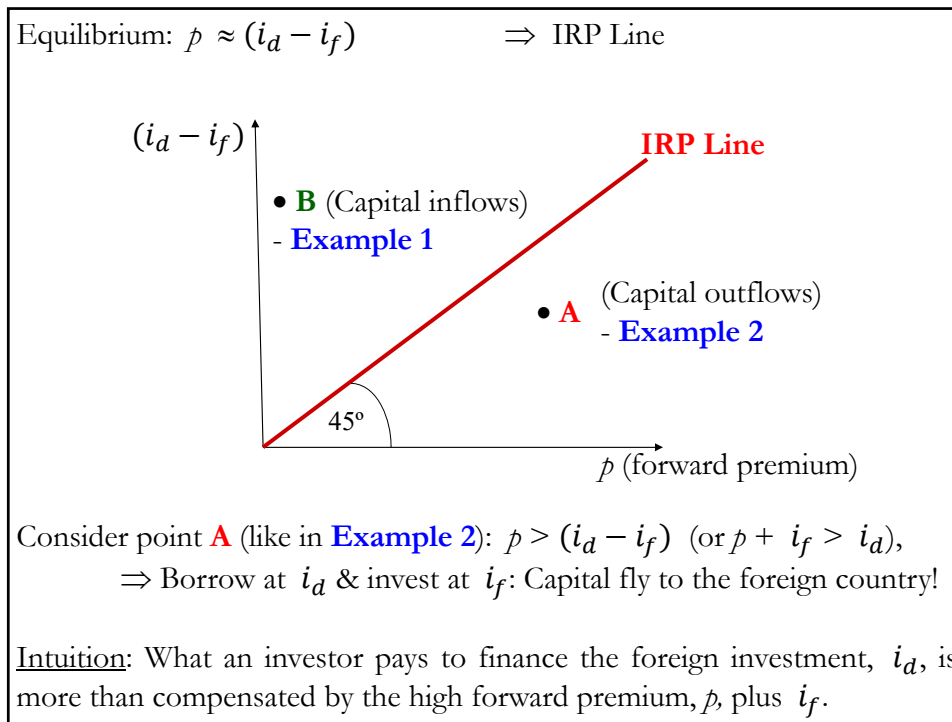


Example: Go back to **Example 1** (point B)

$$p = [(F_{t,T} - S_t)/S_t] * 360/T = [(100 - 106)/106] * 360/360 = -0.0566$$

$$p = -0.0566 < (i_d - i_f) = -0.016 \Rightarrow \text{Arbitrage (pricing mistake!)}$$

⇒ Capital flows to domestic country



IRPT: Assumptions

Behind steps (1) to (4), we have implicitly assumed:

- (1) *Funding is available*. Step (1) can be executed.
- (2) *Free capital mobility*. Step (2) and, later, Step (4) can be implemented.
- (3) *No default/country risk*. Steps (3) & (4) are safe.
- (4) *No significant frictions*. Typical examples: transaction costs & taxes. Small transactions costs are OK, as long as they do not impede arbitrage.

We are also implicitly assuming that the forward contract for the desired maturity T is available. This may not be true.

In general, the forward market is liquid for short maturities (up to 1 year).

For many currencies, say from emerging market, the forward market may be liquid for much shorter maturities (up to 30 days).

IRPT with Bid-Ask Spreads

FX rates & interest rates are quoted with bid-ask spreads.

Consider a trader in the interbank market:

She buys FC ($S_{ask,t}$, $F_{ask,t}$) or borrows at the other party's ask quote (i_{ask}).

She sells FC ($S_{bid,t}$, $F_{bid,t}$) or lends at the bid price (i_{bid}).

There are two roads to take for arbitrageurs:

- (1) Borrow domestic currency (at $i_{ask,d}$).
- (2) Borrow foreign currency (at $i_{ask,f}$).

• Bid's Bound: Borrow Domestic Currency

At time $t=0$, an arbitrageur simultaneously would do:

- (1) A trader borrows DC 1 at time $t=0$ at $i_{ask,d}$
- (2) Using the borrowed DC 1, she buys FC spot at $S_{ask,t}$, getting $(1/S_{ask,t})$
- (3) She deposits the FC at the foreign interest rate, $i_{bid,f}$.
- (4) She sells the FC forward for T days at $F_{bid,t,T}$

Note: The arbitrageur always gets the “worst” part of the bid-ask spread.

Cash flows (in DC) at time T:

- Arbitrageur repays: $1 + i_{ask,d} * T/360$.
- Arbitrageur gets: $(1/S_{ask,t}) (1 + i_{bid,f} * T/360) * F_{bid,t,T}$.

In equilibrium, this strategy should yield no profit. That is,

$$(1/S_{ask,t}) (1 + i_{bid,f} * T/360) * F_{bid,t,T} \leq (1 + i_{ask,d} * T/360).$$

Solving for $F_{bid,t,T}$:

$$F_{bid,t,T} \leq S_{ask,t} * \frac{(1 + i_{ask,d} * \frac{T}{360})}{(1 + i_{bid,f} * \frac{T}{360})} = U_{bid}$$

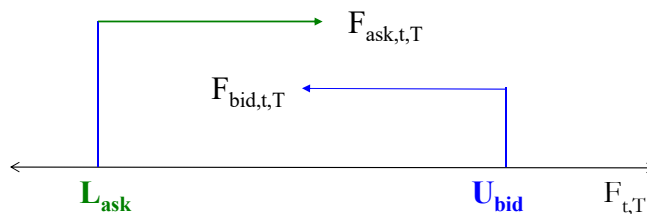
• Ask's Bound: Borrow Foreign Currency

- (1) The trader borrows FC 1 at time $t=0$ at $i_{ask,f}$.
- (2) Using the borrowed FC 1, she sells the FC spot for $S_{bid,t}$ units of DC.
- (3) She deposits the DC at the domestic interest rate, $i_{bid,d}$.
- (4) She buys the FC forward for T days at $F_{ask,t,T}$

Following a similar procedure as the one detailed above, we get:

$$F_{ask,t,T} \geq S_{bid,t} * \frac{\left(1 + i_{bid,d} * \frac{T}{360}\right)}{\left(1 + i_{ask,f} * \frac{T}{360}\right)} = L_{ask}$$

Exhibit III.2 - Trading bounds for $F_{t,T}$



Example: IRPT bounds at work.

Data: $S_t = 1.6540 - 1.6620$ USD/GBP

$i_{USD} = 7.25 - 7.50$

$i_{GBP} = 8.125 - 8.375$

$F_{t,1-yr} = 1.6400 - 1.6450$ USD/GBP.

Check if there is an arbitrage opportunity (we need to check the bid's bound and ask's bound).

Check if bid-ask forward quote is consistent with no arbitrage. That is, the forward quote has to be within the IRPT bounds. Check:

$$U_{bid} = S_{ask,t} * [(1 + i_{ask,d}) / (1 + i_{bid,f})] = 1.6620 * [(1 + .0750) / (1 + .08125)] \\ = 1.6524 \text{ USD/GBP} \geq F_{bid,t,T} = 1.6400 \text{ USD/GBP.}$$

$$L_{ask} = S_{bid,t} * [(1 + i_{bid,d}) / (1 + i_{ask,f})] = 1.6540 * [1.0725 / 1.08375] \\ = 1.6368 \text{ USD/GBP} \leq F_{ask,t,T} = 1.6450 \text{ USD/GBP.} \quad \blacksquare$$

Now, let's confirm that actual arbitrage does not work.

i) Bid's bound covered arbitrage strategy

Data: $S_t = 1.6540 - 1.6620$ USD/GBP

$i_{\text{USD}} = 7.25 - 7.50$

$i_{\text{GBP}} = 8.125 - 8.375$

$F_{t,1\text{-yr}} = 1.6400 - 1.6450$ USD/GBP.

• *Covered arbitrage strategy:*

- 1) Borrow USD 1 at **7.50%** for 1 year \Rightarrow Repay **USD 1.07500** in 1 year.
 - 2) Convert (Buy) to GBP & get USD $1 / (1.6620 \text{ USD/GBP}) = \text{GBP } 0.6017$
 - 3) Deposit GBP 0.6017 at **8.125%**
 - 4) Sell GBP forward at **1.64 USD/GBP**
- \Rightarrow we get: $\text{GBP } 0.6017 * (1 + .08125) * 1.64 \text{ USD/GBP} = \text{USD } 1.06694$

\Rightarrow No arbitrage: For each USD borrowed, we lose

$$\Pi = \text{USD } 1.06694 - \text{USD } 1.07500 = \text{USD } -.00806.$$

Example (continuation):

Data: $S_t = 1.6540 - 1.6620$ USD/GBP

$i_{\text{USD}} = 7.25 - 7.50$

$i_{\text{GBP}} = 8.125 - 8.375$

$F_{t,1\text{-yr}} = 1.6400 - 1.6450$ USD/GBP.

ii) Ask's bound covered arbitrage strategy:

- 1) Borrow GBP 1 at **8.375%** for 1 year \Rightarrow Repay **GBP 1.08375** in 1 year.
 - 2) Convert (sell GBP) to USD & get **USD 1.6540**
 - 3) Deposit **USD 1.6540** at **7.25%**
 - 4) Sell USD/Buy GBP forward at **1.645 USD/GBP**
- \Rightarrow get $\text{USD } 1.6540 * (1 + .0725) * (1 / 1.645 \text{ USD/GBP}) = \text{GBP } 1.07837$

\Rightarrow No arbitrage: For each GBP borrowed, we lose

$$\Pi = \text{USD } 1.07837 - \text{USD } 1.08375 = \text{GBP } -0.0054. \quad \uparrow$$

Synthetic Forward Rates

A trader is not able to find a specific forward currency contract.
This trader can replicate $F_{t,T}$ using a spot currency contract combined with borrowing and lending.
This replication is done using the IRP equation.

Example: Replicating a USD/GBP 10-year forward contract.

$$i_{\text{USD},10\text{-yr}} = 6\%$$

$$i_{\text{GBP},10\text{-yr}} = 8\%$$

$$S_t = 1.60 \text{ USD/GBP}$$

$$T = 10 \text{ years.}$$

Ignoring transactions costs, she creates a 10-year (*implicit quote*) forward quote:

- 1) Borrow USD 1 at 6% for 10 years
- 2) Convert to GBP at 1.60 USD/GBP
- 3) Invest in GBP at 8% for 10 years

Transactions to create a 10-year (implicit) forward quote:

- 1) Borrow USD 1 at 6% for 10 years.
- 2) Convert to GBP at **1.60 USD/GBP** (GBP 0.625)
- 3) Invest in GBP at 8% for 10 years.

Cash flows in 10 years:

- (1) Trader will receive **GBP 1.34933** ($= 1.08^{10}/1.60$)
- (2) Trader will have to repay **USD 1.79085** ($= 1.06^{10}$)

⇒ Implicit Exchange in 10 years: **GBP 1.34933** for **USD 1.79085**

We have created an implicit forward quote:

$$\text{USD } 1.79085 / \text{GBP } 1.34933 = 1.3272 \text{ USD/GBP. ¶}$$

Or

$$F_{t,10\text{-year}}^{\text{implicit}} = S_t * [(1 + i_{d,10\text{-yr}})/(1 + i_{f,10\text{-yr}})]^{10}$$

$$= 1.60 \text{ USD/GBP} * [1.06/1.08]^{10} = 1.3272 \text{ USD/GBP. ¶}$$

Synthetic forward contracts are very useful for exotic currencies.

IRPT: Evidence

Starting from Frenkel and Levich (1975), there is a lot of evidence that supports IRPT.

Taylor (1989): Strong support for IRP using **10'** intervals.

Akram, Rice and Sarno (2008, 2009): Short-lived (from **30'' up to 4'**) departures from IRP, with a profit range of 0.0002-0.0006 per unit.

Overall, we see a fairly efficient market, with data close to the IRPT line.

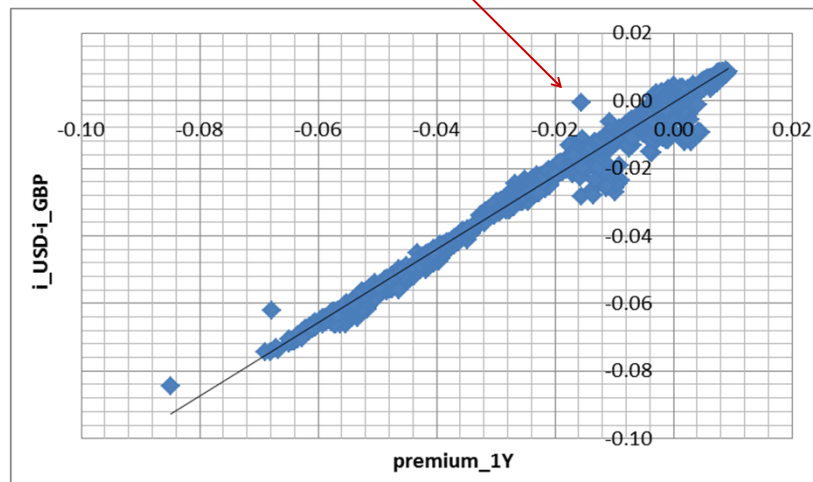
But, there are moments with significant deviations from the IRPT line. These situations reflect violations of IRPT's assumptions.

For example, during the 2008-2009 financial crisis. Probable cause: Funding constraints –Step (1) in trouble!

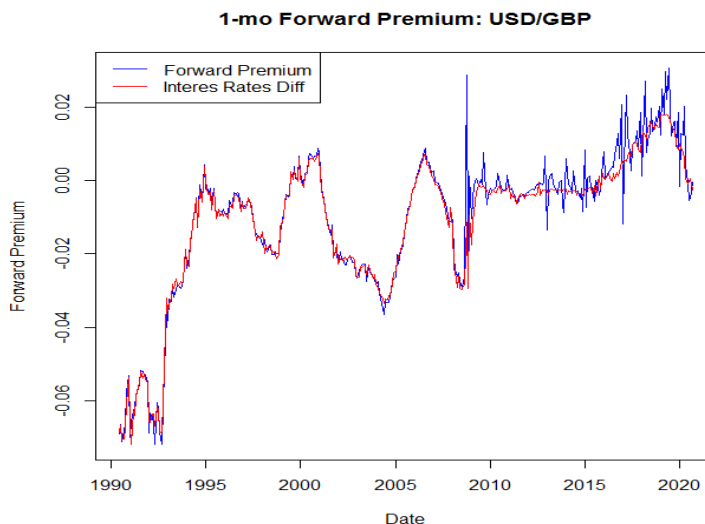
Some evidence that IRP since financial crisis is not holding as expected.

IRPT: Evidence

May 2009: (-.0154,-.0005).



IRPT: Evidence



Almost perfect fit until 2008. Since 2008-2009, IRPT the fit is not that good. One explanation, the interest rates used are no longer “risk-free.”

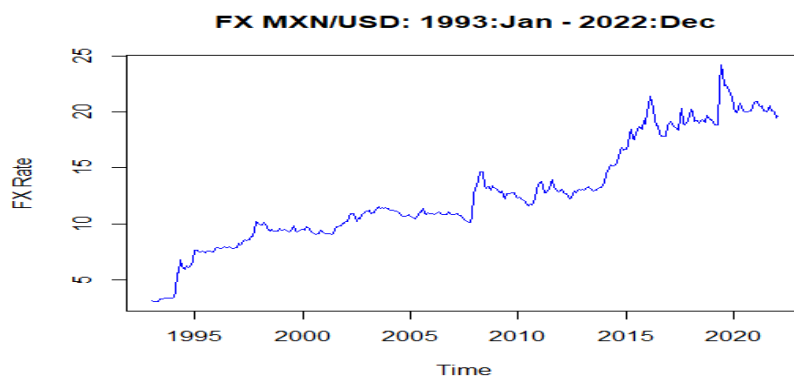
The Behavior of FX Rates

- Fundamentals that affect FX Rates:

- Inflation rates differentials ($I_{USD} - I_{FC}$)	PPP
- Interest rate differentials ($i_{USD} - i_{FC}$)	IFE
- Income growth rates ($y_{USD} - y_{FC}$)	Monetary Approach
- Trade flows	Balance of Trade
- Other: trade barriers, expectations, taxes, etc.	
- Goal 1: Explain S_t with a theory, say T1. Then, $S_t^{T1} = f(\cdot)$
 Different theories can produce different $f(\cdot)$'s.
 Evaluation: How well a theory match the observed behavior of S_t .
- Goal 2: Eventually, produce a formula to forecast $S_{t+T} = f(X_t) \Rightarrow E[S_{t+T}]$.

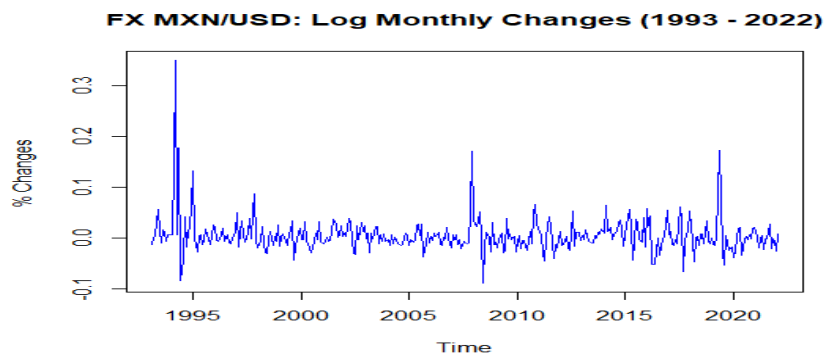
- We want a theory that matches observed S_t . But, not realistic to expect a perfect match.

Q: On average, is $S_t \approx S_t^{T1}$? Or, alternatively, is $E[S_t] = E[S_t^{T1}]$?



- Like many macroeconomic series, exchange rates have a trend –in statistics the trends in macroeconomic series are called *stochastic trends*. It is better to try to match changes, not levels.

- Now, the trend is gone. Our goal is to explain s_t , the percentage change in S_t . (Notation: Many times $s_t = e_{f,t}$).



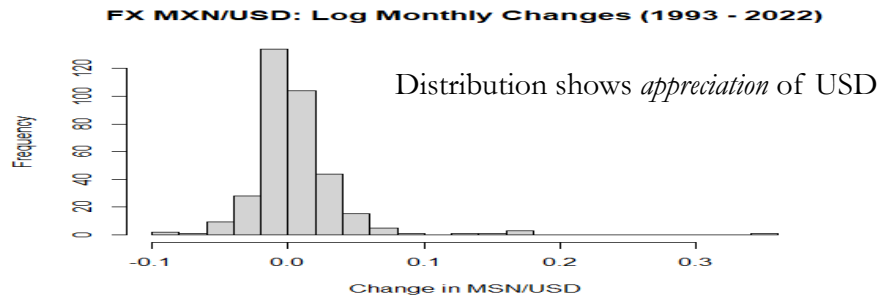
- The data will show us if the model we are using, say T1, matches, on average, the observed behavior of s_t .

Q for the data: Is $E[s_t] = E[s_t^{T1}]$?

- We will use statistics to formally tests theories.

• Data:

Distribution of MXN/USD monthly % changes, s_t (1993:Feb – 2022: Dec)



Descriptive Stats:

$E[s_t]$ = Average monthly % change = **0.52%** (**6.3%**, annualized)

$SD[s_t]$ = **3.51%** (**12.2%** annualized).

- A good theory should predict, on average, an annualized change of **6.3%** for s_t . A better theory should also predict a **12%** annualized volatility.

- Descriptive stats for s_t for monthly JPY/USD and the MXN/USD.

	<i>JPY/USD</i>	<i>USD/MXN</i>
Mean	-0.0014	0.0052
Standard Error	0.0011	0.0019
Median	0.0002	0.0004
Standard Deviation	0.0262	0.0351
Sample Variance	0.0007	0.0021
Kurtosis	4.0886	33.3631
Skewness	-0.4276	3.9122
Minimum	-0.1052	-0.0887
Maximum	0.0807	0.3500
Count	577	350

- Developed currencies tend to be less volatile, with smaller means/medians. They are not normal distributed, but closer to “normal.”