Eurocurrencies & Midterm 2 Review

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• **Last Class**

• Swaps (continuation)

◇ Commodity

◇ CDS (similar to insurance, but no need to own underlying asset).

◇ Financial Engineering: Combination of Swaps (Coffee roaster example)

• Eurocurrency futures

◇ Definition: "A futures contract on a **eurocurrency TD** having a principal value of **USD 1M** with **3-mo maturity**."

◇ It reflects **market expectations**, f, for future value of 3-mo SOFR.

◇ Eurostrip: A collection of consecutive maturities of 3-mo Eurocurrency futures.

◇ With Eurocurrency futures, a trader can go:

- **Long**: Assuring a yield for a **future 3-mo deposit**
- **Short**: Assuring a borrowing rate for a **future 3-mo loan**.

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- **Last Class (continuation)**
- Eurocurrency futures
  - ◊ Applications
    - - Hedging interest risk
    - - Pricing swap coupons (fixed-rate side)
    - - Gap risk management

- **This Class**
- FRAs ("*tailor made*" eurocurrency futures)
  - ◊ Definition & Differences with Eurocurrency futures
  - ◊ Applications

- Eurodollar futures Options
  - ◊ Definition & Differences with Eurocurrency futures
  - ◊ Caps, Floors & Collars
  - ◊ Valuation

- Midterm 2: Review

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Forward Rate Agreements (FRA)

FRA Contract

An FRA involves a buyer & a seller. The parties agree on fixing the interest rate at ***f***, *agreed rate*, on a nominal sum of money, ***N***, during a future period of time, the **FRA period**.

◦ Seller pays the buyer (increased interest cost) if ***i*** (market SOFR) > ***f***

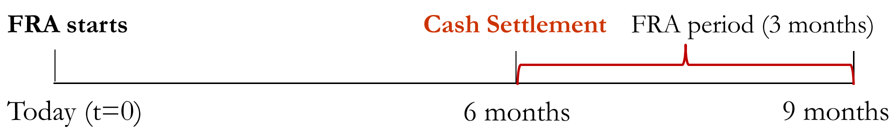
◦ Buyer pays the seller (decreased interest cost) if ***i*** < ***f***.

⇒ Buyer "*benefits*" when ***i*** ↑ (buyer is *short interest rate exposure*).

• Settlement: Cash, at the **beginning** of the FRA period. That is, an FRA is a cash-settled interbank forward contract on ***i***.

Terminology: An agreement on a 3-mo. interest rate for a 3-mo. period beginning 6-mo from now and terminating 9-mo from now ("**6x9**") .

⇒ this agreement is called "*six against nine*," or **6x9**.



FRA Contract Notation:

f = Agreed rate at t=0 ("Today"), expressed as a decimal.

S = Settlement rate (market rate, ***i***), observed at beginning of FRA period

N = Nominal contract amount,

fp = Days in the FRA period, and

yb = Year basis (360 or 365).

The FRA exchange payment (FRAP) is determined by:

$$\text{FRAP} = \frac{N * (i - f) * (fp/yb)}{[1 + i * (fp/yb)]}$$

• Then, if ***i*** > ***f*** (FRAP > 0), seller pays the buyer:
if ***i*** < ***f*** (FRAP < 0), buyer pays the seller.

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Note: Cash settlement is made at the beginning of the FRA period, then, the denominator discounts the payment back to that point.

Example: A bank *buys* a **3X6 FRA** for **USD 2M** with ***f*** = **7.5%**. (Bank pays if ***i*** < ***f***; gets paid if ***i*** > ***f***.) There are **91 days** in the FRA period. Suppose, in 3 months, at the beginning of the FRA period, ***i*** = **9%**.

Summary:

N = **USD 2M**,

fp = 91,

yb = 360,

f = **7.5%**.

i = **9%**. (***i*** > ***f*** ⇒ Bank gets paid.)

• Bank receives *cash* at the **beginning** of the FRA-period from the seller:

$$\text{FRAP} = \textbf{USD 2M} * \frac{(.09 - .075) * (91/360)}{[1 + .09 * (91/360)]} = \textbf{USD 7,414.65}$$

Example (continuation):

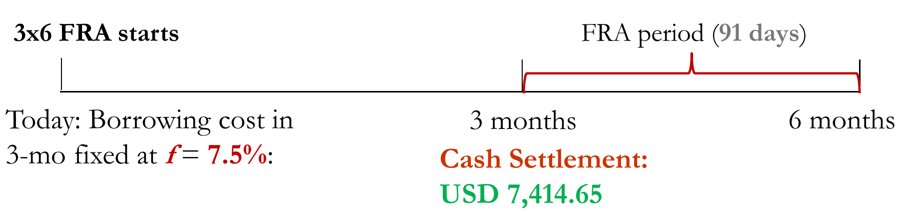
Check: The bank borrowing cost is ***f*** = **7.5%**:

USD 2M * **.075** * (91/360) = **USD 37,916.67**.

Bank's CFs at the end of the 6-mo (FRA) period:

• Net borrowing cost on USD 2M:

USD 2M * **.09** * (91/360)	= USD 45,500.00
minus (FRA adjustment)	
USD 7,414.65 * [1 + **.09** * (91/360)]	= **USD -7,583.33**
Net borrowing cost	= **USD 37,916.67** ¶



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FRA and Arbitrage

• An FRA is an interbank-traded equivalent of the implied forward rate.

• Consider how a bank would construct *FRA bid* & *ask* rates by reference to interbank bid & ask rates on **Eurodeposits** ("Cash").

Example: On Sep 24, a Eurobank wants **USD 100M** of **6-mo deposit**.

It is offered **USD 100M** of **9-mo deposit** at the bank's bid rate (.**105625**).

Current rates (bid=deposit, ask=borrow):

	Cash			FRA	
	bid	ask		bid	ask
6 months	**10.4375**	10.5625	6X9	**10.48**	10.58
9 months	**10.5625**	**10.6875**			

• Q: Should the bank take the 9-mo deposit?

The 9-mo deposit becomes a 6-mo deposit by selling a **6X9 FRA**. That is, the bank sells off (lends) the last 3-mo in the FRA market.

Example (continuation):

Days from September 26 to June 26 (**9-mo deposit**) = 273 days.

Days from March 26 to June 26 (**6X9 FRA**) = **92 days**.

• The interest paid at the end of nine months to the depositor is:

USD 100 M * (.**105625**) * (273/360) = **USD 8,009,895.83**.

• Interest earned on lending for 6-mo (**181 days**) in the interbank market, then another 3-mo at the FRA rate is:

USD 100M * [(1 + **.104375** * (**181**/360)) * (1 + **.1048** * (**92**/360)) – 1]

= **USD 8,066,511.50**.

At the end of nine months (273 days), there is a net profit:

USD 8,066,511.50 - **USD 8,009,895.83** = **USD 56,615.67**

⇒ Bank takes the 9-mo deposit at the bid's rate of **10.5625%**.

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Example (continuation):

Q: Is Arbitrage possible?

A: Check if **USD 8,066,511.50** > 9-mo borrowing cost in Cash market.

The bank would have to buy a deposit (borrow) for 9 months in the interbank (Cash) market at **10.6875%**:

$$\text{USD 100 M} * (.106875) * (273/360) = \text{USD 8,104,687.50}.$$

⇒ No arbitrage:

Interest paid on deposit (**USD 8,104,687.50**) > Interest earned on lending for 6-mo Cash & 3-mo FRA (**USD 8,066,511.50**). ¶

Eurodollar Futures Options and Other Derivatives

CME eurodollar put (call).

The put buyer pays a premium to acquire **the right to go short** (long) one CME eurodollar futures contract at the opening price given by the put's (call's) **strike price**.

- Expiration (T): **Last trade date** for the futures contract.
- Options premium is paid **today**.
- Settlement: Cash value of option payoff is **paid at option expiration**, which is the beginning of forward interest or FRA period.

Today (t=0) — T (Exercise?) — FRA period — Expiration of deposit/loan ⇒ CF

Option begins (Premium paid)

Option matures (Option settlement: CF, if exercised, ⇒ **Interest Rate is Fixed**)

Cash Settlement

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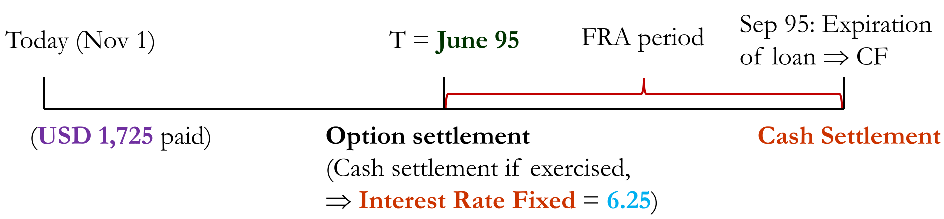
- CME Options are American.
- Strike prices are in intervals of .25 in terms of the CME index.
- Premium quotes: **in percentage points** (1 bp = **USD 25**).

Example: Today, November 1st, a dealer *buys a put* on June Eurodollar futures with a strike price $\mathbf{X_{Z,p}}$ = **93.75**. The dealer pays **0.69** per contract:

USD 25 * **69** = **USD 1,725**.

If exercised ($f \geq$ **X=6.25**), it gives the right to go *short* one **June eurodollar futures** contract at an opening price of **Z = 93.75**. ¶



Example: On Tuesday, **November 1, 1994**, the *WSJ* published the following quotes for eurodollar and LIBOR futures options.

EURODOLLAR (CME)

$ million; pts. of 100%

Strike	Calls-Settle			Puts-Settle		
Price	Dec	Mar	**June**	Dec	Mar	**June**
9350	0.56	0.29	**0.18**	0.01	0.18	0.53
9375	**0.33**	**0.16**	**0.10**	**0.03**	**0.30**	**0.69**
9400	0.14	0.07	0.05	0.09	0.45	0.89
9425	0.03	0.03	0.02	0.23	0.66	1.11
9450	0.00	0.01	0.10	0.45	0.89	1.36
9475	0.00	0.00	0.00	0.70	1.14	1.61

Est. vol. 56,820;

Fri vol. 80,063 calls; 72,272 puts

Op. Int. Fri 939,426 calls; 1,016,455 puts

- Premium quotes: in percentage points (1 bp = **USD 25**).

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Examples: **June** 19**95** put and call

(1) Consider the June 95 put, with a strike price of $X_{Z,p}$ = **93.75**. A price of **.69** would represent **USD 25** * **69** = **USD 1,725**.

(2) Consider the June 95 call, with a strike price of $X_{Z,c}$ = **93.50**. A price of **.18** would represent **USD 25** * **18** = **USD 450**.

Example: Buying insurance.

Underlying Position: *Short* a June 1995 eurodollar *futures* at **Z** = **93.99**.

UP's problem: Potential unlimited loss.

Solution: Buy insurance: Long a *June 1995 call* ($X_{Z,c}$= **93.50** & C= **.18**).

The spot (market) interest rate is 6%.

Hedging Position's (Long June 1995 call) cost:

- Call premium paid: **USD 25** * **18** = **USD 450**.
- Add 6% carrying cost: **USD 450** * [1 + .06 * (30/360)] = **USD 452.25**

◦ Simulate net payoffs for different **Z** in 30 days: 93.00; 93.50; & 94.50.

Scenario #1: In 30 days, **Z** = **93.00** (< $X_{Z,c}$, call no exercised)

- UP (futures) payoff: **93.99** – **93.00** = 0.99 or **USD 2,475** (=99***USD 25**)
- HP (not exercise): 0.

⇒ OP Net payoff: **USD 2,475** – **USD 452.25** = **USD 2,022.75.**

Scenario #2: In 30 days, **Z** = **93.50** (= $X_{Z,c}$, call no exercised/indifferent)

- UP payoff: **93.99** – **93.50** = .49 or **USD 1,225** (= 49 * **USD 25**)
- HP (no exercise): 0

⇒ OP Net payoff: **USD 1,225** – **USD 452.25** = **USD 772.75**.

Scenario #3: In 30 days, **Z** = **94.50** (> $X_{Z,c}$, call exercised)

- UP payoff: 93.99 – **94.50** = -.51 or **USD -1,275** (= -51 * **USD 25**)
- HP (exercise): **94.50** – **93.50** = 1.00 or **USD 2,500** (= 100 * **USD 25**)

⇒ OP Net payoff: **USD 1,225** – **USD 452.25** = **USD 772.75**.

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• **Payoff Matrix** (in 30 days) for possible Z prices: 93, 93.50, 94.50, 95.

Portfolio: UP: Short **June Z = 93.99** & HP: Long June call $X_{Z,c}$= **93.50**

Possible Z Prices	Future Payoff	Call Payoff	Option Cost	Carrying Cost	Total	Total (USD)
93.00	.99	0.00	.18	.0009	.8091	2022.75
93.50	.49	0.00	.18	.0009	.3091	772.75
94.50	-.51	1.00	.18	.0009	.3091	772.75
95.00	-1.01	1.50	.18	.0009	.3091	772.75

Note: Minimum payoff (floor): **USD 772.75** (= 30.91 * **USD 25**)

• By buying the call, the trader has limited his/her possible exposure on the future to .3091 basis points (or a minimum profit of **USD 772.75**).

⇒ This sum can be approximated: $Z - X_Z - C = 93.99 - \mathbf{93.50} - .18 = .31$.

Note: Usually a put establishes a *floor*. Here, the intuition is reversed.

Note: Q: Can a call establish a *floor*?

A: Recall that $Z = 100 - f$ ⇒ The cap is really a *floor* on *future* interest costs, given by f. Not on Z!

When $Z \leq 100 - X$ ⇒ $f \geq X$.

Thus a call on f, which pays off when $f > X$, is equivalent to a put on Z, which pays off when $Z < 100 - X$.

Example: Let $X_{\text{interest rate, call}} = 6.50$.

• A call on the forward rate f has a positive exercise value when $f > 6.50$.

• This is equivalent to an eurodollar futures price $Z < 100 - 6.50 = 93.50$.

⇒ Value of interest rate call with $X_{i,call} = 6.50$ = Value of eurodollar futures put with $X_{Z,put} = 93.50$. ¶

• Summary: The value of a call on f with strike price $X_{i,c}$ is equal to the value of a put on $Z = 100 - f$ with strike price $X_{Z,put} = 100 - X_{i,c}$.

Valuation of futures options

Q: How should eurodollar futures options be priced?

A: **Black-Scholes** formula.

• Underlying asset (uncertain variable): the *forward interest rate* (***f***).

Key: The forward interest rate, ***f***, embodied in the futures price.

• Value of a European call on ***f***:

$$c_t = B_t(T) * [f * N(d1) - X * N(d2)],$$

$$d1 = \frac{\ln(f/X) + .5\, v^2\, T}{v*\text{sqrt}(T)} \quad \text{and} \quad d2 = \frac{\ln(f/X) - .5\, v^2\, T}{v*\text{sqrt}(T)},$$

Inputs:

$B_t(T)$: Price of 1 future dollar at expiration date T, discounted

N(.): Cumulative normal distribution,

v^2: Variance of B_t.

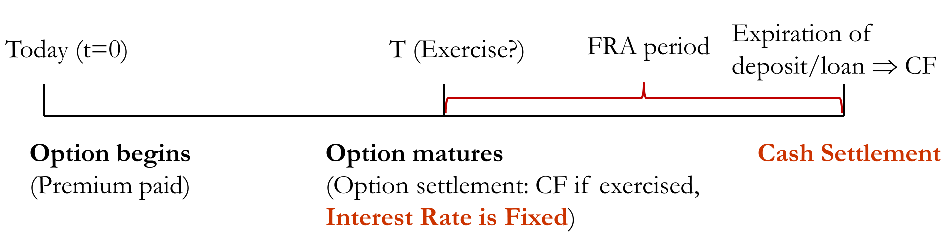
• The European put price is obtained from the put-call parity:

$$p = c + B_t(T) * (X - f).$$

• The European put and call will have equal values when the forward interest (or FRA) rate is equal to the strike price.

Timing of Cash flows associated with option:

- Option premium is paid **today**.
- Settlement: Cash value of option payoff is **paid at option expiration**, which is the beginning of the forward interest or FRA period.



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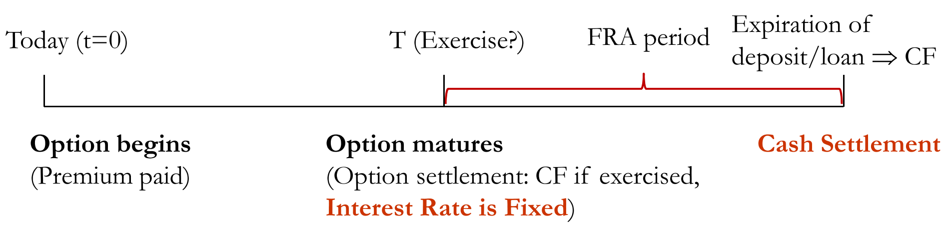

Example: Table XV.B (European options on interest rates).

• Assume v = .15.

• T = **90**/365 = **.2466**.

• Discount rate: 8% (B = 1/(1+.08/**4**) = **98.039**).

• Recall: Cash value of option payoff is **paid at option expiration** (start of FRA period).

Table XV.B

Value of European Options on Forward Interest Rates

		Call			Put		
	f:	**7.5**	8.0	8.5	**7.5**	8.0	8.5
X:	**7.0**	**.541**	.988	1.471	**.051**	.008	.001
	7.5	.218	.551	.992	.218	.060	.011
	8.0	.060	.233	.561	.551	.233	.071

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Example (continuation):

i. Calculations for the call and put option with X = 7 and *f* = 7.5

A. Call

Substituting into d1 and d2:

$$d1 = \frac{\ln(f/X) + .5\, v^2\, T}{v * \text{sqrt}(T)} \quad \text{and} \quad d2 = \frac{\ln(f/X) - .5\, v^2\, T}{v * \text{sqrt}(T)},$$

d1 = [ln(7.5/7) + .5 * $(.15)^2$ * .2466]/[.15 * $.2466^{.5}$] = .9635

d2 = [ln(7.5/7) – .5 * $(.15)^2$ * .2466]/[.15 * $.2466^{.5}$] = .8890

• Cumulative normal distribution at z =.9635: .3324.

Recall: since d1 is positive, we have to add .50%.

→ N(d1 = .9635) = .3324 + .50 = .8324

N(d2 = .8890) = .8130

Example (continuation):

i. Calculations for the call and put option with X = 7 and *f* = 7.5

N(d1 = .9635) = .8324

N(d2 = .8890) = .8130

c = $B_t(T)$ * [*f* * N(d1) – X * N(d2)] = .98039 * [7.5 * .8324 – 7 * .8130]

= .5408

B. Put

Substituting into put-call parity:

p = c + B * (X – *f*) = .5408 + .98039 * (7 – 7.5) = .050805. ¶

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Example (continuation):

Interpretation of option values:

In Table XV., let's pick: **X** = **7.0** & ***f*** = **7.5** ⇒ **c** = **.541**.

• Since **X** and ***f*** are in percent, the prices (**c** & **p**) is also stated in percent.

To translate this price to a dollar amount: we have to know the option size and the duration in days of the forward interest period.

- Suppose the option is based on **3-mo SOFR**.
- Nominal amount of **USD 10 million**.
- There are **92 days** in the 3-mo period.

• Then the dollar cost of the option is:

.541 * (1/100) * (**92**/360) * **USD 10,000,000** = **USD 13,825.56**.

Example (continuation):

• The values in Table XV.B also assume that the option premium is paid today, and that the cash in the option payoff is received at *expiration*, which is the beginning of the forward interest or FRA period.

For example, suppose the cash in the option payoff will not be received until the end of the forward interest period (92 days).

Then, the table value (for **X** = **7.0**, ***f*** = **7.5**) must be *discounted* by the forward interest rate ***f*** = **7.5** for 92 days:

.541 /[1 + **.075** * (92/360)] = .5308258.

This corresponds to an option premium of

.5308258 * (1/100) * (92/360) * **USD 10M** = **USD 13,565.55**. ¶

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• At the CME, Eurodollar options are *American*. To price CME Eurodollar options we use the **American option pricing equations**.

Example: The Eurodollar future is Z = **93.00**. We want to get the value of a future call with strike price of X_Z = **92.50**.

First, we calculate: f = 100 – **93.00** = 7.00

X = 100 – **92.50** = 7.50.

Table XV.C is the same as Table XV.B, but the eurodollar futures prices and strikes have been substituted for their interest rate equivalent, and the options are *American* instead of European.

		Call			Put		
X_Z(→)		91.50	92.00	**92.50**	91.50	92.00	**92.50**
Z(↓)	92.00	.071	.233	.555	.564	.233	.061
	92.50	.011	.061	.219	1.004	.555	.219
	93.00	.001	.008	.051	1.500	1.002	**.545**

Caps, Floors, and Collars

"*Cap*" on interest rates: *i* do not rise above some ceiling level.

"*Floor*" on interest rates: *i* do not fall too low.

Collar: A long cap and a floor.

• Motivation: Financial cost insurance.

Example: Collar on 6-mo SOFR

6-mo SOFR: 8.50%.

Two parties negotiate a collar: Cap 6-mo SOFR at 9%,

Floor 6-mo SOFR at **7.5%**. ¶

Note: If the cap level is low enough (say 8.25) and the floor level is high enough (say 8.25), one is left with a fixed-rate contract.

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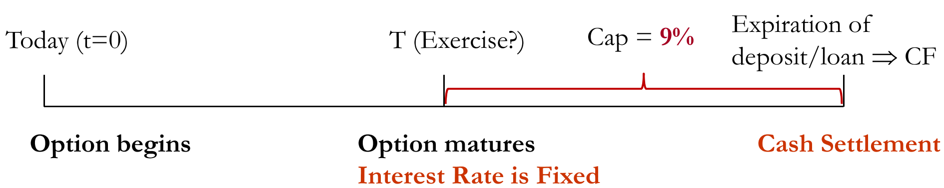
Incomplete Example: Cap.

A SOFR borrower buys an interest rate cap of **9%** on 6-mo. SOFR. The cap lasts T days.

Buyer of the cap: Pays an up-front price for the cap, a *premium,* at t=0.

When 6-mo. SOFR rises above **9%** in any loan period, the cap buyer (exercises option!) will be compensated for the increased interest cost.

Note: At t=0, the market interest rate on the first 6-mo. interval (say, from Jan 30 to July 30) is already known. It is typically *excluded* from the cap.

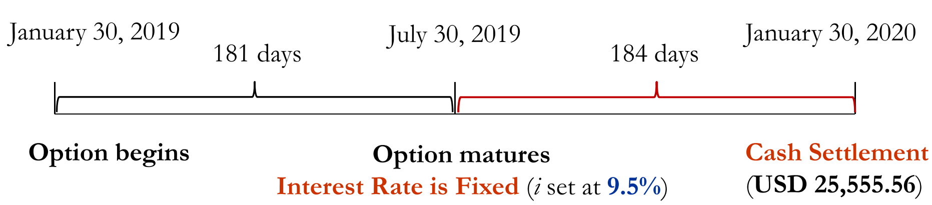


Example: A Cap.

• On December 17, 2018, a SOFR borrower buys a 2-yr interest rate cap of **9%**, with 6-mo. SOFR payments on January 30 and July 30.

• On Jan 30, 2019 the option to cap interest cost at **9%** begins. It covers the new 6-mo. interval starting on July 30, 2019 and extending to Jan 30.

• ***i*** (= 6-mo SOFR) for this period will be fixed on July 30, 2019, but interest will be paid on the following January 30, 2020.



• 6-mo SOFR is fixed at **9.5%** on July 30, 2019.

• On Jan 30, 2020 (184 days later) the cap writer will pay the cap buyer:

USD 10,000,000 * (**9.5** – **9**)/100 * (184/360) = **USD 25,555.56**.

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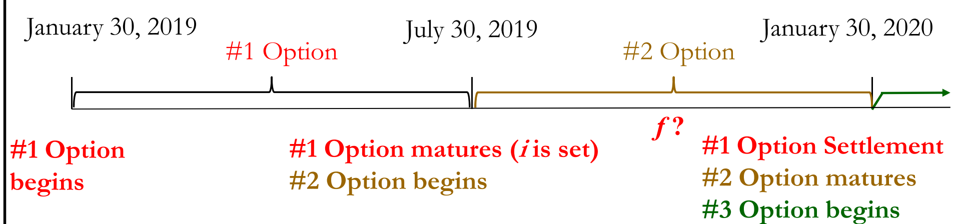
Example: A Cap.

⇒ The cap is a series of European call options on the interest rate, where the call strike price is the cap rate.

First option (#1) begins at the beginning of the cap period (Jan 30, 2019) & expires on first interest reset date. Underlying variable: 6-mo implied forward (or FRA) rate, ***f***, from July 30, 2019 to January 30, 2020.

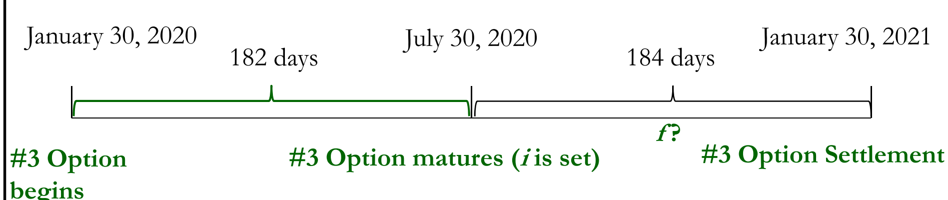


#1 Option expires on July 30, 2019 because the rate is set or determined on that date. But the cash value of the option will not be received until the following January 30, 2020.

Example (continuation):

#2 Option begins on July 30, 2019 and expires on following January 30, 2020 (second reset date). The underlying variable is ***f*** from January 30, 2020 to July 30, 2020 (182 days, leap year).

#3 Option begins on January 30, 2020 and expires on July 30, 2020 (third reset date). The underlying variable is ***f***. ¶



- Similarly, a floor is a series of European put options on the interest rate, where the put strike price is the interest floor.

- A collar is a combination of calls and puts.

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Valuation of a Cap

A cap is a series of European options. The value of the cap is equal to the sum of the value of all the options imbedded in the cap.

Example: Consider a 2-year interest rate cap of **9%** on 6-mo SOFR.

- Cap amount is **USD 10 million**.
- The cap trades on January 28 for effect on January 30.
- Reset dates: July 28 and January 28, and take effect two days later.
- There are 181 days from January 30 to July 30 (182 on leap year).

• At the time the cap is purchased, offered rates on time deposits are:

Period	Offered Rate	
6 month	**8.00**	← #1 Option
12 mo.	**8.50**	← #2 Option
18 mo.	8.65	← #3 Option
24 mo.	8.75	

Example (continuation):

• There are 3 options in the cap. Let's analyze the first one: Option #1.

• The first six months' rate of interest is already determined at 8%.

• Option #1 is thus written on the second six-month period.

• Underlying variable: The "6 against 12" FRA rate.

STEP 1

• Calculating the implied forward rate from the formula:

$$[1 + .085 * (365/360)] = [1 + .080 * (181/360)] * [1 + f * (184/360)]$$

$$\Rightarrow f = .08644.$$

• The option expires in six-months, but does not settle until the end of the second six-month period, which is one year from today.

STEP 2

• The discount rate on the option is **8.50%**. The discount factor is

$$[1 + .085 * (365/360)] = 1.08618.$$

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Example (continuation):

Note: Other forward (FRA) rates and discount factors may be calculated in a similar way.

Option #	Implied Forward Rate	Discount Factor
1	**8.644**	1.08618

STEP 3

- Impute volatilities to each time period. Based on recent activity in the market for caps, these are assumed to be 15 percent (v = .15).

STEP 4

- Calculate Call Value (**c**) and amount paid for #1 Option.
- Apply Black-Scholes: **c** = **.2029**.
- Since there are 365 – 181= **184 days** in the interest period, this corresponds to a USD amount of
- Amount paid = (**.2029**/100) * (184/360) * **USD 10M** = **USD 10,371.78**.

Example (continuation):

STEP 4 (continuation)

- Calculate Call Value (**c**) and total amount paid for the cap.

We now have the information needed to price each option and, thus, the cap:

Option #	T(365)	***f***	X	v	B (adjusted)	Call Value	USD Amount
1	181	**8.644**	9	.150	1/1.08618	**.2029**	**10,371.78**
2	184	**8.242**	9	.150	1/1.13119	**.1966**	**9,886.40**
3	181	**7.998**	9	.150	1/1.17743	**.2071**	**10,583.48**

Value of cap (USD) = **10,371.78** + **9,886.40** + **10,583.48** = **30,841.66**. ¶

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Cap Packaging

• Caps and floors are usually written by companies with existing floating rate borrowings, such as banks.

• Banks often hedge their option writing by borrowing funds at a variable rate with an interest cap.

Example: Bertoni Bank faces the following alternative operations:

• **Alternative 1**, with no cap:

a. Lend to company A at (SOFR + ⅞)

b. Borrow from investors at (SOFR + ⅛).

• **Alternative 2**, with a cap:

a. Lend money to company A at (SOFR + ⅞%).

b. Borrow money from investors at (SOFR + ⅜%) with a **cap at 10%.**

c. **Sell a cap option at 10%** to company B for ½% per year.

Q: Which alternative is more profitable?

Example (continuation):

Evaluation:

• **Alternative 1** (Standard banking operation)

Profit margin = ¾%.

• **Alternative 2**, with the cap. Bertoni Bank's net income is given by:

(SOFR + ⅞) – min(SOFR + ⅜, **10**) + ½ – max(0, SOFR – **10**).

Bertoni Bank's net income per year dependens on interest rates:

- If SOFR remains below **10%:**

(SOFR + ⅞) – (SOFR + ⅜) + ½ = 1%.

- If SOFR increases beyond **10%:**

(SOFR + ⅞) – (**10**) + ½ – (SOFR – **10**) = ⅞ + ½ = 1.375%.

Conclusion: Yes, packaging the cap is more profitable! ¶

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SOFR OPTIONS AND FRAs

Recall:

• In a previous example, we made an interest adjustment to the price of the zero-coupon or discount bond price B.

• The adjustment reflected the fact that each one of the series of call options involved in the interest rate cap expired at the beginning of the interest period.

• But the option payoff was only received at the end of the FRA period.

• If the number of days in the period is dtm, then, in the option formula we replace $B_t(T)$ with $B_t(T+dtm)$. At expiration,

$$B_t(T+dtm) = 1 / [1 + \boldsymbol{f} * (dtm/360)] \qquad \text{(XV.6)}$$

where f is the interest rate fixed at time t+T.

Thus, if $\boldsymbol{f} > X$, the call payoff is

$$(1 / [1 + \boldsymbol{f} * (dtm/360)]) * (\boldsymbol{f} - X). \qquad \text{(XV.7)}$$

Recall: If $\boldsymbol{f}>X$, the call payoff is: $(1/[1 + \boldsymbol{f} * (dtm/360)]) * (\boldsymbol{f} - X)$.

• Compare the above payoff with the value of an FRA: They are the same, provided the option strike price X is the rate agreed (A) in the FRA.

• Similarly, if $\boldsymbol{f} < X$, the call payoff will be zero, but the absolute value of (XV.7) will be the payoff to the corresponding put.

• Thus for SOFR options involved in a cap, floor, or collar, we may replace equation (XV.4), (XV.5)

$$\mathbf{c}_t = B_t(T+dtm) * [\boldsymbol{f} * N(d1) - X * N(d2)], \qquad \text{(XV.8)}$$

$$\mathbf{p} = \mathbf{c} + B_t(T+dtm) * (X - \boldsymbol{f}). \qquad \text{(XV.9)}$$

(The values of d1 and d2 remain unchanged.)

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• Then if I go long a call and short a put, $c_t - p_t$, each with strike price X corresponding to the agreed rate in an FRA, the payoff at option expiration will be:

$$c_t - p_t = (f - X) / [1 + f^* (dtm/360)],$$

(The payoff to the buyer of an FRA!)

• To summarize:

Long a SOFR call + Short a SOFR put = FRA bought.

Similarly,

Short a SOFR call + Long a SOFR put = FRA sold.

Note: the equivalence is in terms of *value*. But the cash flow on an FRA is received at the beginning of the FRA period, whereas the cash flow for the options is received at the end of the FRA period.

Example: Go back to previous Example.

• You want to buy an FRA with A = **7**, when f = **7.5**.

• From Table XV.B, we obtain **c** and **p** with X = **7.0** and f = **7.5**.

• Thus, the value of the FRA is .49 (= **.541** – **.051**).

Value of European Options on Forward Interest Rates

		Call			Put		
	f:	7.5	8.0	8.5	7.5	8.0	8.5
X:	**7.0**	**.541**	.988	1.471	**.051**	.008	.001
	7.5	.218	.551	.992	.218	.060	.011
	8.0	.060	.233	.561	.551	.233	.071

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