

Swaps & Eurocurrencies

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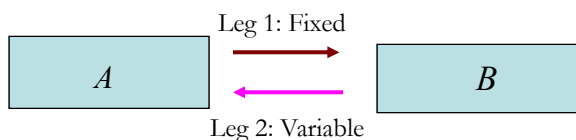
- **Last Class**

- International Bond Markets

- ◊ Valuation with Examples
- ◊ Brady Bonds Case

- Swaps

- ◊ Definition: A periodic exchange of CFs between 2 parties (one, a SD).



- ◊ Different Types, according to how legs are indexed: Interest Rate, Currency, Equity.
- ◊ Market Organization and Swap Dealers
- ◊ Uses: Change the profile of CFs, say, from Fixed to Variable.
- ◊ Valuation: Difference in the NPV of both legs.

- **This Class**
- Swaps (continuation)
 - ◊ Commodity & CDS.
 - ◊ Financial Engineering: Combination of Swaps
- Eurocurrencies & FRAs
 - ◊ Definition
 - ◊ Differences
 - ◊ Applications
- **Review for Second Midterm**

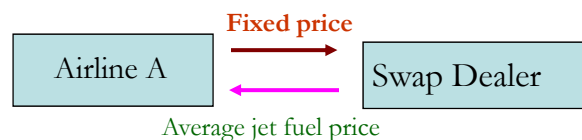
Commodity Swaps

Commodity swaps work like many other swaps: one leg involves a fixed commodity price and the other leg a (variable) commodity market price.

Unlike futures commodity contracts, *cash settlement* is the norm.

Example: Jet fuel oil swap.

Airline A enters into a 2-year jet-fuel oil swap. Every quarter, Airline A receives the **average market price** –based on a known price quote- & pays a **fixed price**.



Example: (continuation)

Cash settlement: If the average jet-fuel price paid is above (below) the fixed price, the SD will repay (receive from) the airline the difference in what it paid versus the fixed price. ¶

Note: There is no futures contract for jet fuel oil. A swap **completes** the **market**.

You can consider the 2-year swap as a **collection** of 8 **forward contracts**.

- **Q: Why commodity swaps?**

(1) *A commodity swap eliminates basis risk*

Southwest Airlines has used NYMEX crude oil and heating oil futures contracts to hedge jet fuel price risk. But, this introduces basis risk.

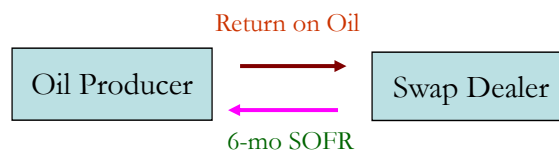
(2) *Expanded market*

Since there is cash settlement, market participants do not need to have the infrastructure to take delivery.

- **Commodity for interest swap**

They work like an equity swap: One leg pays a return on a commodity, the other leg pays an interest rate (say, SOFR plus or minus a spread).

Example: An oil producer enters into a 2-year swap. Every six months, the oil producer pays the **return on oil** –based on NYMEX Light Crude Oil– and receives **6-mo SOFR**.



- **Valuation of Commodity Swaps**

Commodity swaps are valued as a series of **commodity forwards**, each priced at inception with zero value.

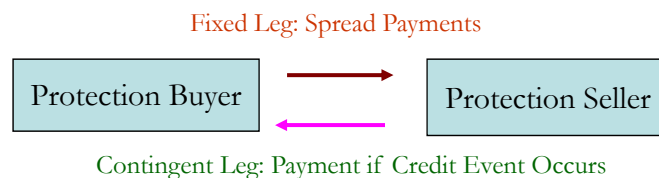
The fixed coupon payment is a weighted average of commodity forward prices.

Credit Default Swaps (CDS)

- A CDS is an agreement between two parties. One party buys *protection against specific risks associated with credit events* –i.e., defaults, bankruptcy, restructuring, or credit rating downgrades. Cash settlement is allowed.
- Facts:
 - Today, CDS is the most widely traded credit derivative product.
 - Outstanding amount: **USD 9.3 trillion** (November 2022).
 - Maturities range from 1 to 10 years (5 years is the most common).
 - Most CDS's are in the USD 10M to 20M range.
- CDS contracts are governed by the International Swaps and Derivatives Association (**ISDA**), which provides standardized definitions of CDS terms, including definitions of what constitutes a credit event.

CDS Mechanism

- One party *buys protection* –i.e., “sells” risk or “short credit exposure- and the counterparty *sells protection* –i.e., “buys” risk or credit exposure.
- Protection buyer pays a **periodic fee** (the *spread*) to protection seller.
- Protection seller agrees to pay the protection buyer a set amount **if there is a credit event** (usually, default).
- Usually, collateral rules apply to seller; following a **5-day 99% VaR**.



Though it is not necessary to buy a CDS, the protection buyer **tends to own the underlying asset** subject to risk.

Note: The spread is positively related to the likelihood of credit event.

CDS Benefits

Besides **hedging event risk**, the CDS provides the following benefits:

- A short positioning vehicle that does not require an initial cash outlay.
- Access to maturity exposures not available in the cash market.
- Access to credit risk not available in the cash market due to a limited supply of the underlying bonds.
- Investments in foreign credits without currency risk.
- Ability to effectively 'exit' credit positions in periods of low liquidity.

CDS: Not Insurance

- In car insurance, you need to own the car and show damage to receive compensation from a claim. In a CDS contract, the protection buyer does **not need to own the underlying** credit exposure.
- Protection seller is **not necessarily regulated**. No reserves are required.
- CDS's are **mark-to-market** (in the US).

Typical CDS Quote

A 5-year CDS quote for Bertoni Bank (on April 17, 2015)

Notional amount = **USD 10 million** (= Czech Rep Eurobond holdings)

Premium or Spread: **160 bps** (related to risk of Czech Republic)

Maturity: 5 years

Frequency: Quarterly Payments

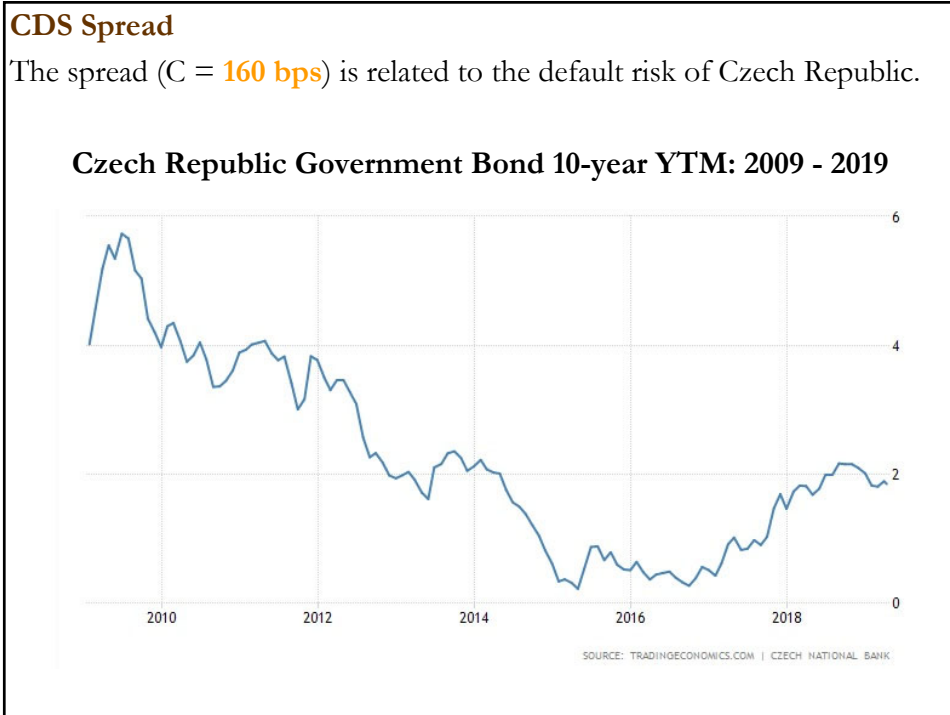
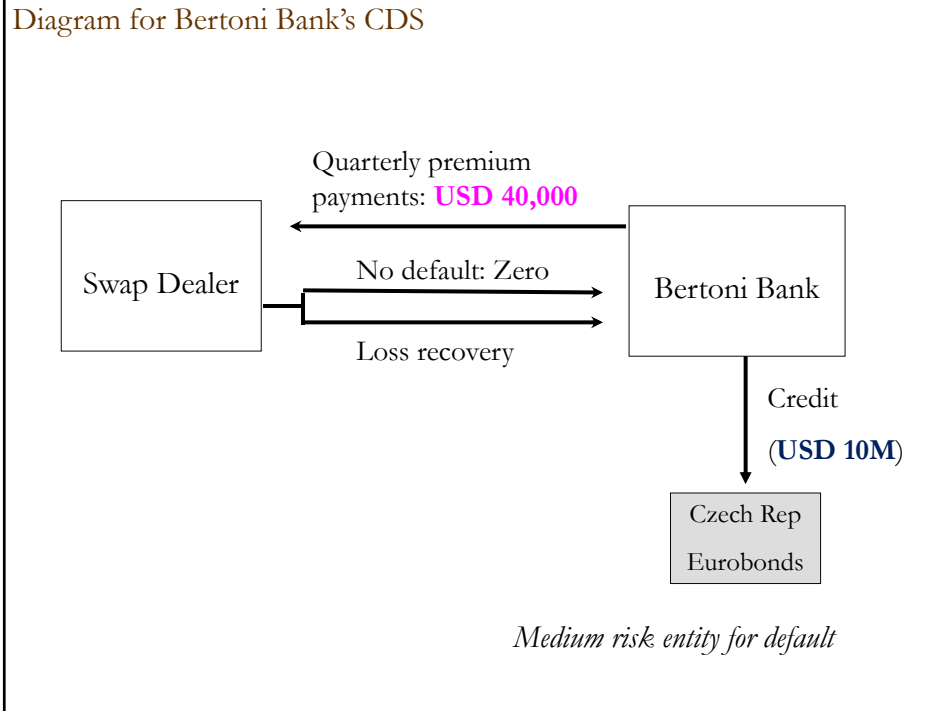
Credit event: Default

- Calculation of the Spread

Q; How much Bertoni Bank pays for protection?

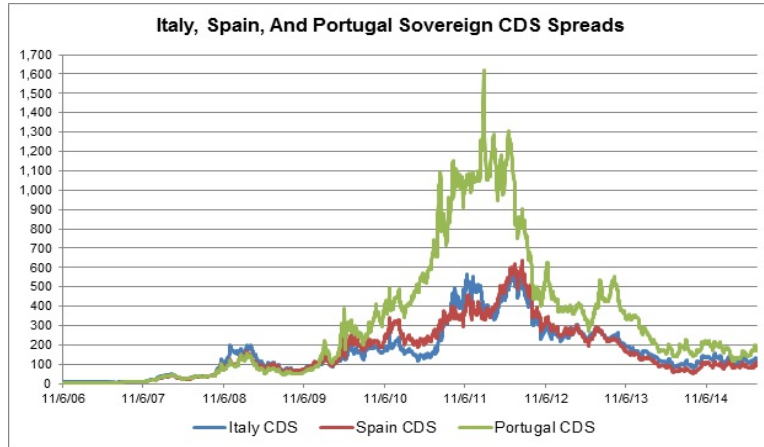
$(0.0160/4) * \text{USD } 10\text{M} = \text{USD } 40,000$ (every quarter as a premium for protection against company default)

If the Czech Republic (Eurobond issuer) defaults, the CDS covers the notional **USD 10M**.



• **CDS Spreads Do Reflect Default Risk.**

Since we look at CDSs as insurance for bondholders; an increase in CDS premiums indicates that investors are becoming worried about the safety of their investments.



Higher country risk ⇒ Higher CDS premiums

• **CDS Spreads Do Reflect Default Risk.**

Similar behavior seen in the table (2023) below, from Damodaran (NYU):

https://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/ctrypem.html

| Country | Adj. Default Spread | Equity Risk Premium | Country Risk Premium | Corporate Tax Rate | Moody's rating |
|-------------------|---------------------|---------------------|----------------------|--------------------|----------------|
| Abu Dhabi | 0.60% | 6.79% | 0.85% | 15.00% | Aa2 |
| Albania | 5.51% | 13.71% | 7.77% | 15.00% | B1 |
| Algeria | 3.68% | 11.13% | 5.19% | 26.00% | NR |
| Andorra | 2.33% | 9.23% | 3.29% | 18.98% | Baa2 |
| Angola | 7.95% | 17.16% | 11.22% | 25.00% | B3 |
| Anguilla | 7.93% | 17.13% | 11.19% | 25.63% | NR |
| Antigua & Barbuda | 7.93% | 17.13% | 11.19% | 25.63% | NR |
| Argentina | 14.68% | 26.65% | 20.71% | 35.00% | Ca |
| Armenia | 4.40% | 12.15% | 6.21% | 18.00% | Ba3 |
| Australia | 0.00% | 5.94% | 0.00% | 30.00% | Aaa |
| Austria | 0.49% | 6.63% | 0.69% | 24.00% | Aa1 |

CDS: Settlement

- Settlement can either be physical or cash.

(1) *Cash Settlement*

- Protection seller pays buyer **the difference** between **par value** (N) & **market price** of a debt obligation of the reference entity (say, market price of similar Czech Republic Eurobonds, R). That is, seller pays:

$$(100 - R)\% \text{ of Notional}$$

- To establish value of reference obligation ($R\%$), a dealer poll/auction can be used.

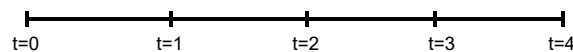
(2) *Physical Settlement*

- Protection seller pays buyer **par value** (N) & receives an **acceptable obligation** (say, Czech Republic Eurobonds) worth R , from buyer.
- Buyer of protection can choose, within certain limits, what obligation to deliver. Puts buyer in a CDB-like situation (pick lower R possible).

Note: Think of what the seller gets in the event of default as *recovery*, R .

CDS: Pricing

Suppose we want to price a 1-year CDS. Thus, there are 4 events.



Effective date

Nominal amount = N

Premium = C (annualized)

Quarterly payment = $N * (C/4)$

There are 5 possible outcomes in this CDS contract:

- No default (4 premium payments are made by bank to investor until the maturity date)
- Default occurs on t_1 , t_2 , t_3 , or t_4

Steps

- 1) Assign probability to each event –i.e., default at t_1 , default at t_2 , etc.
- 2) Calculate PV of payoff for each outcome (assuming $\delta_i = [1/(1+r_i)]^i$ as the discount rate for period i):
 - Seller’s net default payment is $N^* (1 - R)$
 - Buyer’s payments is $N^* C/4$
- 3) Expected NPV of CDS = Sum of PV of five payoffs multiplied by their probability of occurrence.
 - $E[\text{NPV}_{\text{CDS}}] \approx 0 \Rightarrow$ Determine *fair* C such that $E[\text{NPV}_{\text{CDS}}] \approx 0$.

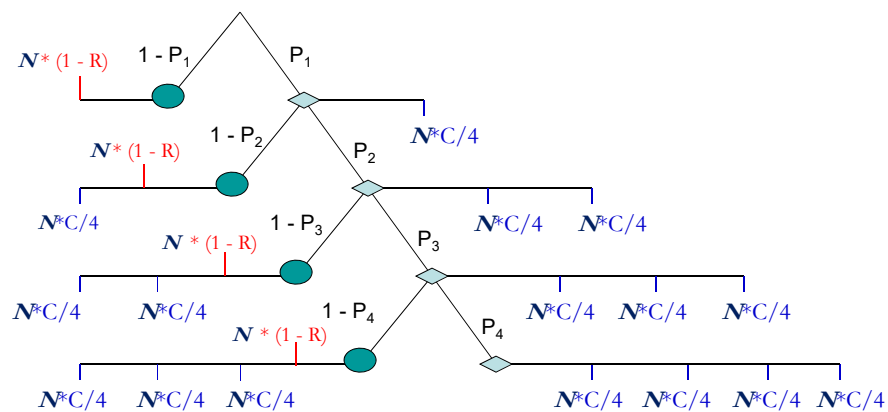
Notes: We think of the P_i 's as “*survival*” probabilities over an interval:

- Probabilities $\Rightarrow P_i$ [No default at t_i – issuer still alive at time t_i]
- $\Rightarrow 1 - P_i$ [Default at t_i – issuer is “dead” at time t_i]

Technically, P_i is the probability of surviving over interval $[t_i - t_{i-1}]$.

CDS: Payoff Diagram

CFs from the seller’s point of view (red=outflows, blue=inflows):



| Summary of Events and Payoffs | | | |
|-------------------------------|---|--------------------------|-------------------------------|
| Description | Premium Payment PV | Default Payment PV | Probability |
| Default at t_1 | 0 | $N * (1 - R) * \delta_1$ | $(1 - P_1)$ |
| Default at t_2 | $N * C/4 * \delta_1$ | $N * (1 - R) * \delta_2$ | $P_1 * (1 - P_2)$ |
| Default at t_3 | $N * C/4 * (\delta_1 + \delta_2)$ | $N * (1 - R) * \delta_3$ | $P_1 * P_2 * (1 - P_3)$ |
| Default at t_4 | $N * C/4 * (\delta_1 + \delta_2 + \delta_3)$ | $N * (1 - R) * \delta_4$ | $P_1 * P_2 * P_3 * (1 - P_4)$ |
| No default | $N * C/4 * (\delta_1 + \delta_2 + \delta_3 + \delta_4)$ | 0 | $P_1 * P_2 * P_3 * P_4$ |

- $E[\text{NPV}_{\text{Seller}}] = \text{NPV}\{\text{Premium payments}\} - \text{NPV}\{\text{Default payments}\}$
- To calculate the $E[\text{NPV of CDS}]$, we need as inputs:
 - Known: N , C (determined in the contract)
 - Undetermined/Unknown: $P_i = \{P_1, P_2, P_3, P_4\}$; R ; and the discount rate for period i , $\delta_i = 1/(1 + r_i)^i$.

CDS: Pricing and Inputs

- There are two popular ways to get P_i 's:
 - (1) **Assume a probability distribution**, for example, the *exponential*. The higher the risk (and spread), the higher the decay in the survival probability.
 - (2) **Use no-arbitrage model**. After some assumption, we can use market prices of similar bonds (ideally, from same issuer), B_j , and T-bond prices, B_{RF} , to compute expected PV of cost of default loss as $(B_j - B_{\text{RF}})$. Then, we “extract” the implied P_i 's. Which one?
- R is calculated from **historical** (“average”) recovery rates. In many situations, $R = 40\%$ is used as the default input. Constant?
- We use $\delta_i = 1/(1+r_i)^i$ from term structure. Under assumptions, we use same discount rate for $C/4$ & $N*(1 - R)$. But, we can use different discount rates for the *defaultable* part (N) and *non-defaultable* parts (C). Realistic?

CSD: Calculation of PV of CDS and Pricing

$$\begin{aligned}
\text{Expected Present Value of Credit Default Swap} &= E[\text{NPV}_{\text{CDS}}] = \\
&= (P_1 * P_2 * P_3 * P_4) * [N * C/4 * (\delta_1 + \delta_2 + \delta_3 + \delta_4)] \\
&\quad - (1 - P_1) * N * (1 - R) * \delta_1 \\
&\quad - P_1 * (1 - P_2) * [N * (1 - R) * \delta_2 - N * C/4 * \delta_1] \\
&\quad - P_1 * P_2 * (1 - P_3) * [N * (1 - R) * \delta_3 - N * C/4 * (\delta_1 + \delta_2)] \\
&\quad - P_1 * P_2 * P_3 * (1 - P_4) * [N * (1 - R) * \delta_4 - N * C/4 * (\delta_1 + \delta_2 + \delta_3)]
\end{aligned}$$

Recall that using this formula, we price the CDS –i.e., determine *fair* C. At $t = 0$, $E[\text{NPV}_{\text{CDS}}] = 0$ (or, ≈ 0) \Rightarrow get C such that $E[\text{NPV}] \approx 0$.

$$\begin{aligned}
&N * (1 - R) * \{(1 - P_1) * \delta_1 + P_1 * (1 - P_2) * \delta_2 + P_1 * P_2 * (1 - P_3) * \delta_3 + P_1 * P_2 * P_3 * (1 - P_4) * \delta_4\} \\
&= N * C/4 * \{P_1 * (1 - P_2) * \delta_1 + P_1 * P_2 * (1 - P_3) * (\delta_1 + \delta_2) + P_1 * P_2 * P_3 * (1 - P_4) * (\delta_1 + \delta_2 + \delta_3) \\
&\quad + (P_1 * P_2 * P_3 * P_4) * (\delta_1 + \delta_2 + \delta_3 + \delta_4)\}
\end{aligned}$$

CSD: Calculation of PV of CDS and Pricing

$$\begin{aligned}
&N * (1 - R) * \{(1 - P_1) * \delta_1 + P_1 * (1 - P_2) * \delta_2 + P_1 * P_2 * (1 - P_3) * \delta_3 + P_1 * P_2 * P_3 * (1 - P_4) * \delta_4\} \\
&= N * C/4 * \{P_1 * (1 - P_2) * \delta_1 + P_1 * P_2 * (1 - P_3) * (\delta_1 + \delta_2) + P_1 * P_2 * P_3 * (1 - P_4) * (\delta_1 + \delta_2 + \delta_3) + \\
&\quad (P_1 * P_2 * P_3 * P_4) * (\delta_1 + \delta_2 + \delta_3 + \delta_4)\}
\end{aligned}$$

Then,

$$\begin{aligned}
C &= 4 * (1 - R) * \{(1 - P_1) * \delta_1 + (1 - P_2) * \delta_2 + P_2 * (1 - P_3) * \delta_3 + P_2 * P_3 * (1 - P_4) * \delta_4\} \\
&\quad \{(1 - P_2) * \delta_1 + P_2 * (1 - P_3) * (\delta_1 + \delta_2) + P_2 * P_3 * (1 - P_4) * (\delta_1 + \delta_2 + \delta_3) + (P_2 * P_3 * P_4) * (\delta_1 + \delta_2 + \delta_3 + \delta_4)\}
\end{aligned}$$

\Rightarrow C is the *fair* CDS spread.

Example: Expected NPV for Bertoni Bank's CDS, with **2 payments left**

Notional amount = **USD 10 million** (Czech Republic Eurobonds)

Premium or Spread = **C = 160 bps**

Maturity: 6 months (2 payments left)

Frequency: Quarterly Payments

Credit event: Default

Discount rates: 3-mo = **0.035**; & 6-mo = **0.037**.

Recovery Rate = $(1 - R) = 60\%$

Quarterly payments = $N * C/4 = \text{USD } 10M * (0.0160/4) = \text{USD } 40,000$

Probability of Default: $P_1 = .99$; & $P_2 = .985$.

$E[\text{NPV of Credit Default Swap (in USD M)}] =$

$$\begin{aligned}
 &= (.99 * .985) * [.040 * \{1/(1 + .035/4)^1 + 1/(1 + .037/4)^2\}] \\
 &- (1 - .99) * 10 * .60 * 1/(1 + .035/4)^1 \\
 &- .99 * .015 * [10 * .60 * 1/(1 + .037/4)^2 - .040 * 1/(1 + .035/4)^1] = -0.0694
 \end{aligned}$$

Note: Like swaps, at inception the PV of CDS ≈ 0 . In this case, we call the spread (or premium) *fair*.

Example (continuation): Pricing CDS

Today, we want to price a similar CDS to the Bertoni Bank's CDS. Then, we set **C** such that $E[\text{NPV}] \approx 0$. That is, if a similar CDS is issued today with 2 payments left, the *fair spread* is: **C = 303.19 bps**, since:

$E[\text{NPV of Credit Default Swap (in USD M)}] =$

$$\begin{aligned}
 &= (.99 * .985) * [.075796 * \{1/(1 + .035/4)^1 + 1/(1 + .037/4)^2\}] \\
 &- (1 - .99) * 10 * .60 * 1/(1 + .035/4)^1 \\
 &- .99 * .015 * [10 * .60 * 1/(1 + .037/4)^2 - .075796 * 1/(1 + .035/4)^1] \\
 &\approx 0.
 \end{aligned}$$

Then, the quarterly payments are:

$$= N * C/4 = \text{USD } 10M * (0.030319/4) = \text{USD } 75,796. \quad \blacktriangleleft$$

CDS: Risks

- The main risk is *counterparty risk* –i.e., the seller defaults. If a major counterparty (say, AIG, Lehman) defaults a large number of market participants are left un-hedged.
- If a large seller defaults, network domino effects are possible.
- Collateral & margin can spiral out of control. Asset values are correlated with CDS protection sold & the economy. To post more collateral, firms have to de-leverage (sell assets at worst time: *fire sale*.)
- Modeling CDS spreads is complicated:
 - Market is illiquid –i.e., difficult to trust observed market prices.
 - P_i 's are not easy to determine.
 - Fat-tailed and left-skew distributions.
 - Difficult to aggregate risks (hard to measure default correlations).

CDS: Summary

- CDS are bilateral contracts, often sold and resold among parties.
- Large market, due to netting, the notional size of the CDS market is approximately 1/10th the size of the gross notional market.
- Due to its protection nature CDS market represents over one-half of the global credit derivative market.
- CDS allows a party who buys protection to trade and manage credit risks in much the same way as market risks.

Example: AIG

- In 2005 and early 2006, AIG sold protection on $N = \text{USD } 500\text{B}$ in assets, including **USD 78B** on collateralized debt obligations –backed by debt payments from mortgages, home equity loans, etc. At inception:
 - Probability of default were set **very low**.
 - Default correlations were well **not incorporated** into model.
 - Given AIG status, **no collateral was required**. It was required only under certain events (AIG's credit rating falls below AA-).
- As write-downs in real estate grew in 2007-08, AIG rating was lowered below AA-. By September 2008, **margin calls** reached **USD 32B**.
 - Started off **write-downs** (asset prices down) & faced **more margin calls**.
 - Eventually, margin calls rose to **USD 50B**.
- Aside: AIG did not help itself by investing the collateral cash received from shorts (AIG was big in lending securities) in subprime mortgage paper. As shorts returned stock, AIG could not give the collateral back.

Combination of Swaps

- Recall that swaps change the profile of cash flows.
- Swaps solve problems: *Financial Engineering*.

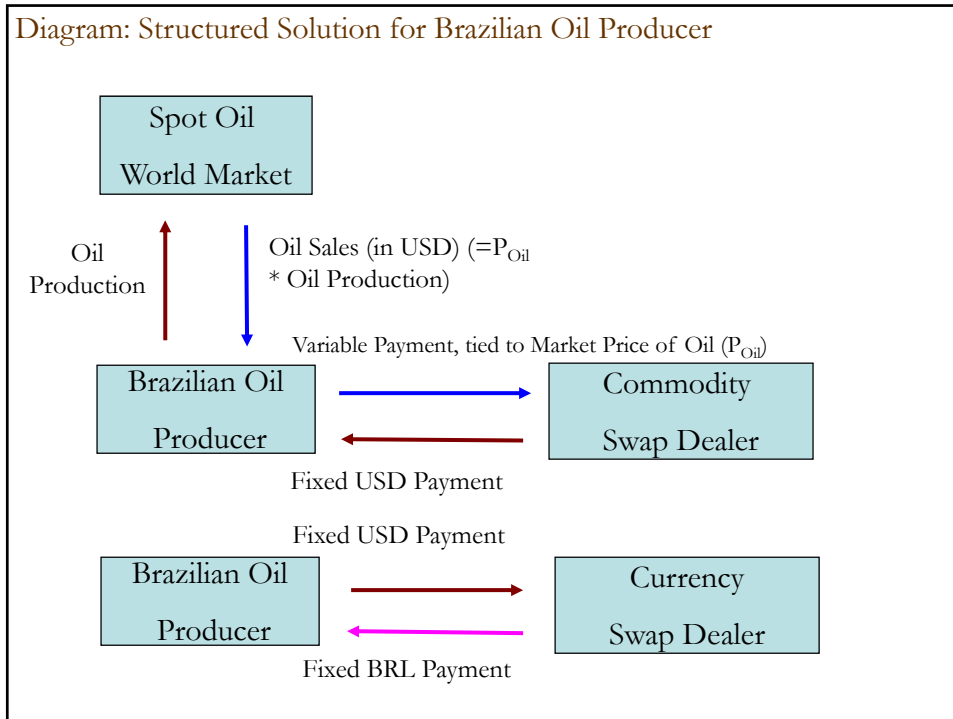
Example: A Brazilian oil producer is exposed to two forms of price risk:

- P_{Oil} (priced in USD/barrel of oil) \Rightarrow commodity price risk.
- S_t (BRL/USD) \Rightarrow FX risk.

Situation: Since expenses are in BRL, the Brazilian oil producer wants to **fix P_{Oil} in BRL/barrel of oil**.

Solution: *Financial Engineering*, a combination of swaps can do it!

Note: This is a typical problem for commodity producers and buyers from non-USD zones: Commodities are priced in USD.



Example: Combining Swaps.

- Belabu, a Lithuanian coffee roaster, uses **500,000 pounds** of Colombian coffee every 6-months. Colombian coffee trades spot at **P_{Coffee}** (USD).
- Belabu has contracts to sell its output at a fixed price for **4 years**.

The diagram shows Belabu's coffee supply chain. At the top, the 'Spot Coffee World Market' is shown. An upward red arrow indicates 'P_{Coffee} * 500,000 pounds' of coffee being sold to Belabu. A downward blue arrow indicates '500,000 pounds of Coffee' being purchased from Belabu. Below this, Belabu is connected to 'Lithuanian Coffee Shops'. A red arrow points from the shops to Belabu, labeled 'Fixed LTT Price'.

- **S_t = 4.74 LTT/USD**
- **P_{Coffee} = 1.95 USD/pound.**
- **Goal:** Belabu wants to **fix the price of coffee in LTT/pound.**
 - ⇒ Belabu, simultaneously, enters **three swaps:** A commodity swap, an interest rate swap, & a currency swap.

(1) Commodity swap dealer:
 Belabu receives: Average market price of coffee (P_{Coffee}).
 Belabu pays: A fixed-price of **USD 2.05 per pound**

Current mid-price quote for a 4-yr coffee swap is **USD 1.99 per pound**.
 (Dealer subtracts/adds **USD .06** to its mid-price.)

(2) Interest rate swap dealer:
 Belabu receives: USD fixed rate amount
 Belabu pays: USD floating rate amount
 4-yr swap interest rate quote: **8.2%** against 6-mo. SOFR.

(3) Currency swap dealer:
 Belabu receives: a USD floating rate amount
 Belabu pays: a LTT fixed-rate amount
 4-year LTT-for-USD currency swap quote: **7.8%** against 6-mo. SOFR.

Details

1. Determine the number of USD Belabu will need every six months:
 $500,000 \text{ pounds} * \text{USD } 2.05/\text{pound} = \text{USD } 1,025,000$

2. Determine notional principal required on a USD interest rate swap for the fixed-rate side to generate **USD 1,025,000** every 6-mo (**8.2%** rate)
 $\text{USD } 1,025,000 / .041 = \text{USD } 25,000,000$

Details (continuation)

3. Calculate the present value of the cash flows on the fixed-rate side of the interest rate swap using the current **8.2%**:

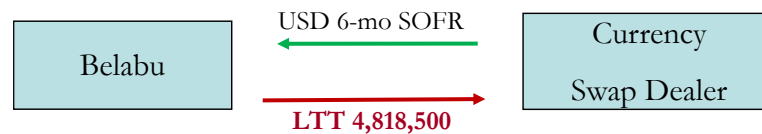
$$\text{PV}(\text{USD } 1,025,000; .041, 8 \text{ periods}) = \text{USD } 6,872,600$$

4. Translate the PV of the USD cash flows to its LTT equivalent:

$$4.74 \text{ LTT/USD} * \text{USD } 6,872,600 = \text{LTT } 32,576,124$$

5. Determine the LTT CFs on the fixed-rate side of the LTT-for-USD currency swap having a NPV of **LTT 32,576,124** at **7.8%** current rate.

$$\text{Coupon}(\text{PV} = \text{LTT } 32,576,124; .039, 8 \text{ periods}) = \text{LTT } 4,818,500$$

**Details (continuation)**

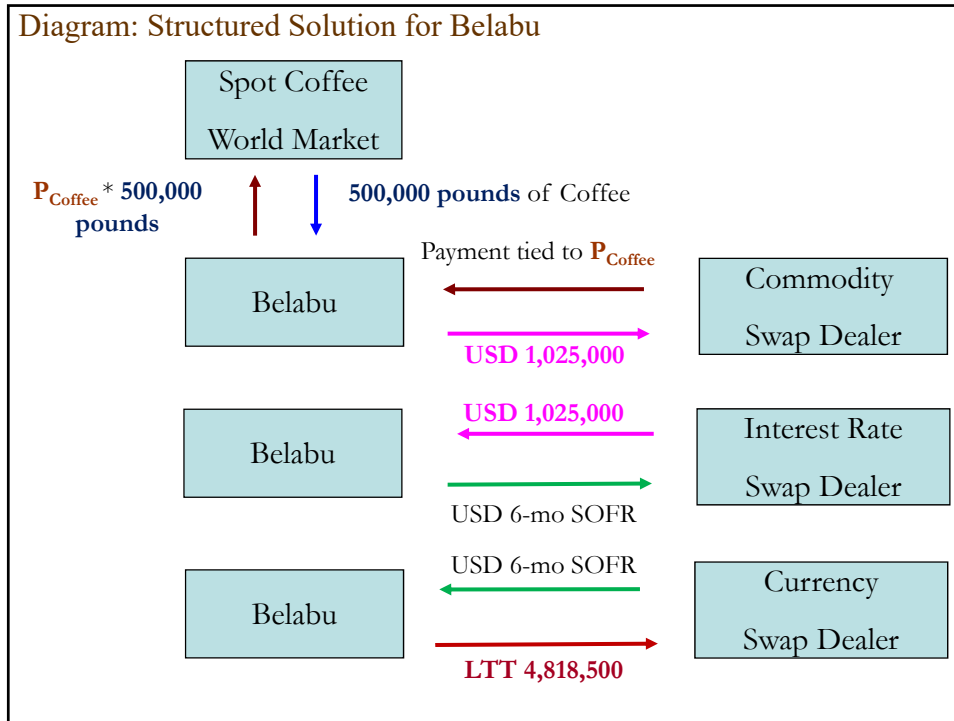
6. Determine the LTT notional principal that would generate the semiannual payments of **LTT 4,818,500** at **7.8%**:

$$\text{LTT } 4,818,500 / .039 = \text{LTT } 123,551,282.10$$

⇒ The structured solution has fixed the price of coffee for four years:

$$\text{LTT } 4,818,500 / 500,000 \text{ pounds} = 9.637 \text{ LTT/pound of coffee.}$$

That is, Belabu pays **9.637 LTT/pound of coffee** for 4 years.



Synthetic Instruments

Synthetic instruments are not securities at all.

- CF streams formed by combining the CF streams from one set of instruments to replicate the CF streams of another set of instruments.
- When combined with appropriate cash positions, it is possible to use swaps to replicate the CF stream associated with virtually any instrument.

Example: Dual currency bond.

Situation: Bertoni Bank issued dual currency bonds for **USD 1M**.

Coupon payments: JPY 6.5 M.

Frequency of payments: semiannual.

Maturity: 5 years.

Features: Sold and redeemed in USD. Interest payments in JPY.

$S_t = 100 \text{ JPY/USD}$

CF for BB: At issue ($t = 0$): BB receives **USD 1M**.

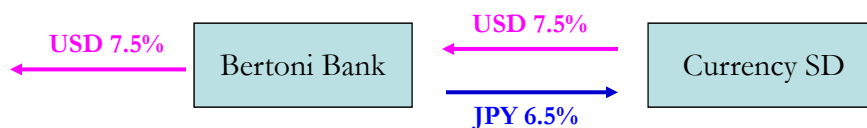
Every 6-mo: BB pays **JPY 6.5 M**

At maturity ($t = 5$): BB pays **USD 1M**.

- Bertoni Bank's CF can be synthesized using:
 - A Corporate USD straight bond.
 - A fixed-for-fixed currency swap.

Example (continuation):

- Instruments:
 - 7.5% Chase Bonds with (at least) 5 years to maturity.
 - A currency SD offers **USD 7.5%** against **JPY 6.5%** (SD pays USD)
- At $t = 0$, BB sells short **USD 1M** worth of Chase bonds
 - ⇒ BB receives **USD 1M** and makes **USD 7.5%** coupon payments.
- BB also enters a fixed-for-fixed currency swap & make JPY payments.
 - ⇒ BB pays **JPY 6.5%** & receives **USD 7.5%**.
- At $t = 5$, BB buys back the Chase bonds for **USD 1M**.



Example: Synthetic EquitySituation:

Goyco Corp. has **USD 3M** to invest for two years.

Goyco is bullish on the Japanese market in the near future.

Goyco decides to invest in synthetic Japanese equity using as tools:

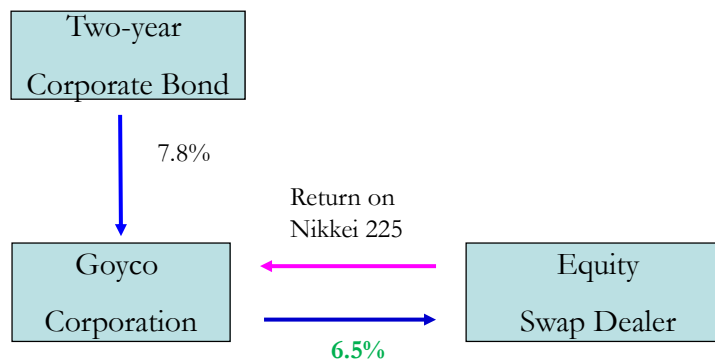
- A fixed-rate note
- An equity swap.

- Goyco buys a 7.8% FV = **USD 3M**, 2-yr bond, trading at par.
- At the same time, Goyco enters into a 2-yr equity swap:
 - Goyco pays the swap dealer **6.5%** annually and receives the return on the Nikkei 225 (r_{Nikkei}).
 - Goyco pays the swap dealer when $r_{\text{Nikkei}} < 0$.
(in addition to the fixed-rate payment of **6.5%**.)
 - Notional: **USD 3 million**

Example (continuation):

- Goyco's position has a return equal to the Nikkei 225 plus 130 bps.
- Net effect: Creation of a (synthetic) equity position for Goyco.

⇒ In some countries, swaps are **off-balance sheet** items: Goyco only shows its 2-year corporate bond on its balance sheet.



CASE 7 – WTI (Cash Management Swaps)

- World Tours, Inc. (WTI) plans on issuing debt.
- WTI has **seasonal cash flows** (higher in Q1 & Q3, lower in Q2& Q4).
- WTI can sell debt at a lower cost if it can reduce cash flow volatility.
- WTI uses a **statistical decomposition** of its revenues:

$$\text{Revenue} = \text{Trend} + \text{Seasonal} + \text{FX} + \text{Random}$$

$$\text{Trend} = 1200 + 200 * t$$

$$\text{Seasonal} = 0.2 * D * \text{Trend}$$

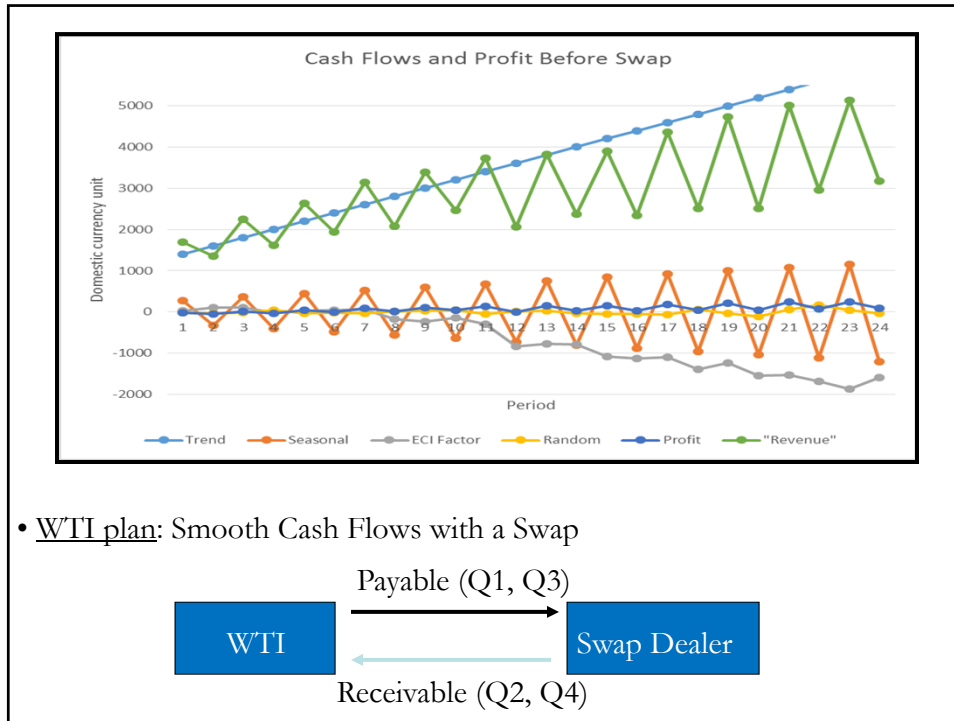
$$\text{FX (ECI)} = 0.6 * (S_t - 2) * \text{Trend}$$

t (quarters) = 1, 2, 3, ..., 24

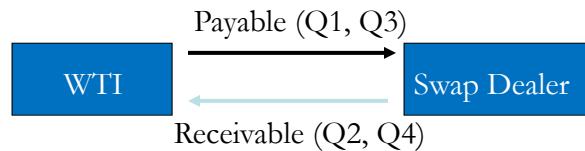
D: Seasonal dummy = +1 for quarters 1 & 3
 -1 for quarters 2 & 4

| Period | Revenue | Profit | FX Rate | D | Trend | Seasonal Factor | FX Factor | Predicted Revenue |
|--------|---------|----------|---------|----|-------|-----------------|-----------|-------------------|
| 1 | 1700.93 | -23.9256 | 2.04 | 1 | 1400 | 280 | 33.6 | 1714.6 |
| 2 | 1354.72 | -51.6224 | 2.111 | -1 | 1600 | -320 | 106.56 | 1385.56 |
| 3 | 2250.09 | 20.0072 | 2.103 | 1 | 1800 | 360 | 111.24 | 2272.24 |
| 4 | 1622.8 | -30.176 | 1.989 | -1 | 2000 | -400 | -13.2 | 1585.8 |
| 5 | 2629.77 | 50.3816 | 2.022 | 1 | 2200 | 440 | 29.04 | 2670.04 |
| 6 | 1941.6 | -4.672 | 2.026 | -1 | 2400 | -480 | 37.44 | 1956.44 |
| 7 | 3143.01 | 91.4408 | 2.033 | 1 | 2600 | 520 | 51.48 | 3172.48 |
| 8 | 2077.74 | 6.2192 | 1.899 | -1 | 2800 | -560 | -169.68 | 2069.32 |
| 9 | 3397.65 | 111.812 | 1.874 | 1 | 3000 | 600 | -226.8 | 3374.2 |
| 10 | 2464.48 | 37.1584 | 1.924 | -1 | 3200 | -640 | -145.92 | 2413.08 |
| 11 | 3735.75 | 138.86 | 1.856 | 1 | 3400 | 680 | -293.76 | 3787.24 |
| 12 | 2060.46 | 4.8368 | 1.614 | -1 | 3600 | -720 | -833.76 | 2045.24 |
| 13 | 3817.29 | 145.3832 | 1.663 | 1 | 3800 | 760 | -768.36 | 3792.64 |
| 14 | 2379.8 | 30.384 | 1.671 | -1 | 4000 | -800 | -789.6 | 2409.4 |
| 15 | 3895.08 | 151.6064 | 1.568 | 1 | 4200 | 840 | -1088.64 | 3952.36 |
| 16 | 2335.96 | 26.8768 | 1.57 | -1 | 4400 | -880 | -1135.2 | 2383.8 |
| 17 | 4360.8 | 188.864 | 1.605 | 1 | 4600 | 920 | -1090.2 | 4430.8 |
| 18 | 2506.56 | 40.5248 | 1.518 | -1 | 4800 | -960 | -1388.16 | 2450.84 |
| 19 | 4726.5 | 218.12 | 1.586 | 1 | 5000 | 1000 | -1242 | 4759 |
| 20 | 2507.7 | 40.616 | 1.504 | -1 | 5200 | -1040 | -1547.52 | 2611.48 |
| 21 | 5002.83 | 240.2264 | 1.528 | 1 | 5400 | 1080 | -1529.28 | 4951.72 |
| 22 | 2950.92 | 76.0736 | 1.497 | -1 | 5600 | -1120 | -1690.08 | 2788.92 |
| 23 | 5129.23 | 250.3384 | 1.463 | 1 | 5800 | 1160 | -1868.76 | 5092.24 |
| 24 | 3176.1 | 94.088 | 1.558 | -1 | 6000 | -1200 | -1591.2 | 3207.8 |

Revenue = Trend + Seasonal + ECI Factor + Random



- WTI plan: Smooth Cash Flows with a Swap



- Two parts
 - **Group assignment** (WTI)
 - **Class assignment**

- **Group assignment**

- ◆ Determine best method to calculate increasing yearly swap notional amount (accreting fixed-for-fixed interest rate swap)
 - Swap will be fixed-for-fixed (10% s.a.), with mismatched payments – i.e., pay Q1 & Q3, receive Q2 & Q4.
 - Retroactively apply swap values and generate quarterly cash flows
 - Graph combined outcomes
- ◆ Design a currency swap to remove FX exposure.
- ◆ Adjust swap to compensate dealer.

Eurocurrency Futures

- **Eurocurrency time deposit**

Euro- $\text{\$}$: The **currency of denomination** of the $\text{\$}$ instrument is **not** the official currency of the country where the $\text{\$}$ instrument is traded.

Example: Euro-deposit ($\text{\$}$ = a deposit)

A Mexican firm deposits USD in a Mexican bank. This deposit qualifies as a **Eurodollar deposit**. ¶

The interest rate paid on Eurocurrency deposits is called SOFR.

Eurodeposits tend to be *short-term*: 1 or 7 days; or 1, 3, or 6 months.

Typical Eurodeposit instruments:

Time deposit: Non-negotiable, registered instrument.

Certificate of deposit: Negotiable and often bearer.

Note I: Eurocurrency deposits are direct obligations of commercial banks accepting the deposits and are not guaranteed by any government. They are *low-risk* investments, but Eurodollar deposits are *not risk-free*.

Note II: Eurocurrency deposits play a major role in the international capital market. They serve as a benchmark interest rate for corporate funding.

- Eurocurrency time deposits are the *underlying asset* in Eurodollar currency futures.

- **Eurocurrency futures contract**

A Eurocurrency futures contract calls for the **delivery of a 3-mo Eurocurrency time USD 1M deposit** at a given interest rate (**SOFR**).

Similar to any other futures a trader can go *long* (a promise to make a future 3-mo deposit) or *short* (a promise to take a future 3-mo. loan).

With Eurocurrency futures, a trader can go:

- *Long*: Assuring a yield for a future USD 1M **3-mo deposit**
- *Short*: Assuring a borrowing rate for a future USD 1M **3-mo loan**.

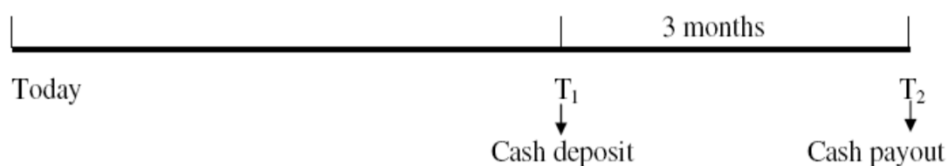
The *Eurodollar futures* contract should reflect the **market expectation** for the future value of SOFR for a 3-mo deposit.

- Q: How does a Eurocurrency futures work?

Think of a futures contract on a time deposit (TD), where the expiration day, T_1 , of the futures precedes the maturity date T_2 of the TD.

Typically, $T_2 - T_1$: 3-months.

Such a futures contract locks you in a 3-mo. interest rate at time T_1 .



Example: In June you agree to buy in mid-Sep a TD that expires in mid-Dec.

Value of the TD (you receive in mid-Dec) = USD 100.

Price you pay in mid-Sep = USD 99.

⇒ 3-mo return on mid-Dec $(100-99)/99 = 1.01\%$ (or 4.04% annually.)

- **Eurocurrency futures** work in the same way as the **TD futures**:

“A Eurocurrency futures represents a futures contract on a eurocurrency TD having a principal value of USD 1M with a 3-mo maturity.”

- Traded at exchanges around the world. Each market has its own reset rate: SOFR (USD), ESTR (EUR), SARON (CHF), SONIA (GBP), etc.
- Eurodollar futures price is based on **3-mo. SOFR**.
- Eurodollar deposits have a face value of USD 1,000,000.
- Delivery dates: March, June, September, & December.
- Delivery is only "in cash" –i.e., no physical delivery.
- The (*forward*) interest rate on a 3-mo. CD is quoted at an annual rate. The *eurocurrency futures price* is quoted as:

100 – the interest rate of a 3-mo. euro-USD deposit for forward delivery

Example: The interest rate on the forward 3-mo. deposit is 6.43%

⇒ The Eurocurrency futures price is 93.57. ¶

Note: If interest rates go up, the Eurocurrency futures price goes down, so the short side of the futures contract makes money.

- Minimum Tick: **USD 25**.

Since the face value of the Eurodollar contract is **USD 1M**

⇒ one basis point has a value of USD 100 for a 360-day deposit.

For a 3-month deposit, the value of 1 bp is **USD 25** (= USD100/4).

Example:

Eurodollar futures Apr 20: 93.57

Eurodollar futures Apr 21: 93.54 (forward interest rate up 0.03%)

⇒ Short side gains USD 75 = 3 x **USD 25**. ¶

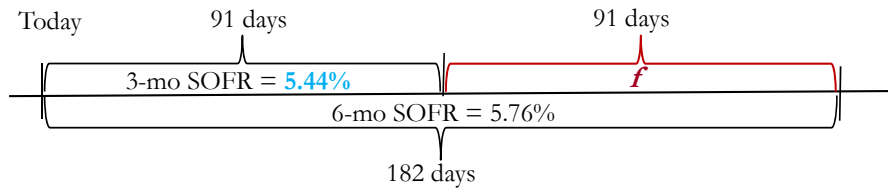
• Calculation of forward 3-mo SOFR

A: Eurodollar futures reflect market expectations of forward 3-month rates. An *implied forward rate* (f) indicates approximately where short-term rates may be expected to be sometime in the future.

Example: 3-month SOFR spot rate = **5.44%** (91 day period)

6-month SOFR spot rate = **5.76%** (182 day period)

3-month forward rate = f



$$(1 + .0576 * 182/360) = (1 + .0544 * 91/360) * (1 + f * 91/360)$$

$$\Rightarrow f = [(1 + 0.0576 * 182/360)/(1 + 0.0544 * 91/360) - 1] * (360/91)$$

$$f = 0.059975 \text{ (6.00\%)}$$

Example: From the WSJ (Oct. 24, 1994) Eurodollar contracts quotes:

EURODOLLAR (CME) - \$ 1 million; pts of 100%

| | Open | High | Low | Settle | Chg | Yield | Chg | Open Interest |
|------|-------|-------|-------|--------|------|-------|------|---------------|
| Dec | 94.00 | 94.03 | 93.97 | 94.00 | | 6.00 | | 447,913 |
| Mr95 | 93.56 | 93.60 | 93.53 | 93.57 | +.01 | 6.43 | -.01 | 402,624 |
| June | 93.11 | 93.15 | 93.07 | 93.12 | +.01 | 6.88 | -.01 | 302,119 |
| Sept | 92.77 | 92.80 | 92.73 | 92.77 | | 7.23 | | 243,103 |
| Dec | 92.45 | 92.49 | 92.42 | 92.46 | | 7.54 | | 176,045 |
| Mr96 | 92.38 | 92.41 | 92.34 | 92.38 | | 7.62 | | 152,827 |
| June | 92.26 | 92.28 | 92.24 | 92.25 | -.01 | 7.75 | +.01 | 124,984 |
| Sept | 92.16 | 92.18 | 92.12 | 92.15 | -.01 | 7.85 | +.01 | 117,214 |
| Dec | 92.04 | 92.06 | 92.01 | 92.03 | -.01 | 7.97 | +.01 | 95,555 |
| Mr97 | 92.05 | 92.06 | 92.01 | 92.04 | -.01 | 7.96 | +.01 | 83,127 |
| June | 91.98 | 92.01 | 91.95 | 91.97 | -.01 | 8.03 | +.01 | 69,593 |
| Sept | 91.92 | 91.94 | 91.89 | 91.91 | -.01 | 8.09 | +.01 | 55,103 |
| Dec | 91.81 | 91.82 | 91.77 | 91.79 | -.01 | 8.21 | +.01 | 53,103 |
| Mr98 | 91.83 | 91.84 | 91.79 | 91.81 | -.01 | 8.19 | +.01 | 43,738 |
| June | 91.77 | 91.78 | 91.73 | 91.75 | -.01 | 8.25 | +.01 | 37,785 |
| Sept | 91.71 | 91.73 | 91.67 | 91.69 | -.01 | 8.31 | +.01 | 26,751 |
| Dec | 91.60 | 91.61 | 91.56 | 91.58 | -.01 | 8.42 | +.01 | 24,137 |
| Mr99 | 91.62 | 91.63 | 91.58 | 91.60 | -.01 | 8.40 | +.01 | 21,890 |
| June | 91.56 | 91.57 | 91.52 | 91.54 | -.01 | 8.46 | +.01 | 15,989 |
| Sept | 91.50 | 91.51 | 91.46 | 91.48 | -.01 | 8.52 | +.01 | 10,263 |
| Dec | 91.40 | 91.41 | 91.36 | 91.37 | -.01 | 8.63 | +.01 | 7,354 |
| Mr00 | 91.44 | 91.44 | 91.40 | 91.41 | -.01 | 8.59 | +.01 | 7,536 |
| June | 91.39 | 91.40 | 91.35 | 91.36 | -.01 | 8.64 | +.01 | 4,971 |
| Sept | 91.34 | 91.35 | 91.30 | 91.31 | -.01 | 8.69 | +.01 | 7,691 |
| Dec | 91.23 | 91.24 | 91.19 | 91.21 | | 8.79 | | 6,897 |
| Mr01 | 91.28 | 91.28 | 91.24 | 91.26 | | 8.74 | | 7,312 |
| June | 91.24 | 91.24 | 91.20 | 91.22 | | 8.78 | | 5,582 |
| Sept | 91.21 | 91.20 | 91.17 | 91.19 | | 8.81 | | 5,040 |
| Dec | | | | 91.09 | | 8.91 | | 3,845 |
| Mr02 | 91.13 | 91.13 | 91.13 | 91.14 | | 8.86 | | 2,689 |
| June | | | | 91.10 | | 8.90 | | 2,704 |
| Sept | 91.06 | 91.06 | 91.06 | 91.08 | | 8.92 | | 2,016 |
| Dec | 90.97 | 90.97 | 90.97 | 91.00 | | 9.00 | | 1,341 |
| Mr03 | 91.01 | 91.03 | 91.01 | 91.04 | | 8.96 | | 1,589 |
| June | 90.97 | 90.97 | 90.97 | 91.00 | | 9.00 | | 1,367 |
| Sept | 90.85 | 90.98 | 90.95 | 90.98 | | 9.02 | | 1,354 |
| Dec | 90.85 | 90.88 | 90.85 | 90.89 | -.01 | 9.11 | +.01 | 1,227 |

Est vol 437,328; vol Thur 615,913; open int 2,576,727. +17,451.

Terminology

- *Amount:* A Eurodollar futures involves a face amount of **USD 1M**.

⇒ To hedge USD 10M, we need 10 futures contracts.

- *Duration:* Duration measures the time at which cash flows take place.

For money market instruments, all cash flows generally take place at the maturity of the instrument.

A 6-mo. deposit has approximately twice the duration of a 3-mo. deposit.

⇒ Value of 1 bp for 6-mo. is approximately **USD 50**.

Hedge a USD 1 million six-month deposit beginning in March with:

- (1) 2 March Eurodollar futures (*stack hedge*).
- (2) 1 March Eurodollar futures and 1 June Eurodollar futures (*strip hedge*).

Slope: Eurodollar contracts are used to hedge other interest rate instruments. The rates on these underlying instruments may not be expected to change one-for-one with Eurodollar interest rates.

If we define f as the interest rate in an Eurodollar futures contract, then

$$\text{slope} = \Delta \text{ underlying interest rate} / \Delta f \quad (\text{think of } \textit{delta})$$

If T-bill rates have a slope of .9, then we would only need 9 Eurodollar futures contracts to hedge USD 10M of 3-mo T-bill.

Notation:

F_A : Face amount of the underlying asset to be hedged

D_A : Duration of the underlying asset to be hedged.

n : Number of Eurodollar futures needed to hedge underlying position:

$$n = (F_A/1,000,000) * (D_A/90) * \text{slope}.$$

Notation:

F_A : Face amount of the underlying asset to be hedged

D_A : Duration of the underlying asset to be hedged.

n : Number of eurodollar futures needed to hedge underlying position

$$n = (F_A / 1,000,000) * (D_A / 90) * slope.$$

Example: To hedge **USD 10M** of 270-day commercial paper with a slope of .935 would require approximately **28 contracts**:

$$n = (F_A / 1M) * (D_A / 90) * slope = (10M / 1M) * (270 / 90) * .935 = 28.05$$

● Q: Who uses Eurocurrency futures?

A: Speculators and Hedgers.

• Hedging

Short-term interest rates futures can be used to hedge interest rate risk:

- You can lock future investment yields (*Long Hedge*).
- You can lock future borrowing costs (*Short Hedge*)

Example:

(1) **Long Hedge** (a promise to make a future 3-mo deposit) .

Bank A is offered a 3-mo **USD 1M** deposit in 2-mo. *Buying* eurocurrency futures allow Bank A to lock a profit on the *future deposit*.

(2) **Short Hedge** (a promise to take a future 3-mo. loan).

A company wants to borrow for 6-mo from Bank A in 1-mo. *Selling* eurocurrency futures allows Bank A to lock a profit on the *future loan*.

Eurodollar Strip Yield Curve and the CME (IMM) Swap

Typical quote of 4 successive Eurodollar futures:

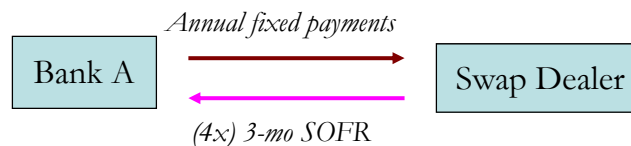
| | Price | Yield | Days and Period Covered |
|--------|-------|-------------|---------------------------------|
| Mar 95 | 93.57 | 6.43 | 92 = March 95 – June 95 |
| Jun 95 | 93.12 | 6.88 | 92 = June 95 – September 95 |
| Sep 95 | 92.77 | 7.23 | 91 = September 95 – December 95 |
| Dec 95 | 92.46 | 7.56 | 91 = December 95 – March 96 |

- Successive eurodollar futures give rise to a *strip yield curve*:
 - March future involves a 3-mo. rate: begins in March and ends in June.
 - June future involves a 3-mo. rate: begins in June and ends in Sep.
 - Etc...

⇒ This **strip yield curve** is called *Eurostrip*.

Note: Compounding the interest rates (yield) for 4 successive eurodollar contracts defines a one-year rate implied from four 3-mo. rates.

- A *CME swap* involves a trade whereby one party receives one-year fixed interest and makes floating payments of the three-months SOFR.



CME swap payments dates: Same as Eurodollar futures expiration dates.

Example: On August 15, a trader does a Sep-Sep swap.

Floating-rate payer makes payments on the third Wed. in Dec, & on the third Wed. of the following Mar, June, and Sep.

Fixed-rate payer makes a single payment on the third Wed. in Sep. ¶

Note: **Arbitrage** ensures that the one-year fixed rate of interest in the CME swap is similar to the one-year rate constructed from the Eurostrip.

Pricing Short-Dated Swaps

Swap coupons are routinely **priced off the Eurostrip**.

Key to pricing swaps: The swap coupon is set to **equate** the **present values** of the fixed-rate side and the floating-rate side of the swap.

- Eurodollar futures contracts provide a way to do that.
- The estimation of the fair mid-rate is complicated a bit by:
 - (a) the convention is to quote swap coupons for generic swaps on a s.a. bond basis, and
 - (b) the floating side, if pegged to SOFR, is usually quoted money market basis.

Pricing Short-Dated Swaps

Notation: If the swap has a tenor of m months and is priced off 3-mo Eurodollar futures, then pricing will require n sequential futures series, where $n=m/3$.

Example: If the swap is a 6-mo swap ($m=6$) \Rightarrow we need 2 Eurodollar futures contracts. ¶

- Procedure to price a swap coupon involves three steps:
 - i. **Calculate the implied effective annual SOFR** for the full duration (*full-tenor*) of the swap from the Eurodollar strip.
 - ii. **Convert** the full-tenor SOFR (quoted on money market basis) **to its fixed-rate equivalent $FRE_{0,3n}$** (quoted on annual bond basis).
 - iii. **Restate the fixed-rate equivalent** on the same payment frequency as the floating side of the swap. The result is the swap coupon **SC**.

Pricing Short-Dated Swaps: Details

• Three steps:

i. Calculate the **implied effective annual SOFR** for the full duration (*full-tenor*) of the swap from the Eurodollar strip:

$$r_{0,3n} = \prod_{t=1}^n \left[1 + r_{3(t-1),3t} \frac{A(t)}{360} \right]^{\tau} - 1, \quad \tau = 360 / \sum A(t)$$

ii. **Convert** the *full-tenor* SOFR, which is quoted on money market basis, to its **fixed-rate equivalent FRE_{0,3n}**, which is stated as an annual effective annual rate (annual bond basis):

$$\text{FRE}_{0,3n} = r_{0,3n} * (365/360).$$

iii. **Restate the fixed-rate equivalent** on the same payment frequency as the floating side of the swap. The result is the swap coupon **SC**. This adjustment is given by

$$\text{SC} = [(1 + \text{FRE}_{0,3n})^{1/k} - 1] * k, \quad k = \text{frequency of payments.}$$

Example:

Situation: It's October 24, 1994. H Bank wants to price a one-year fixed-for-floating interest rate swap against 3-mo SOFR starting on Dec 94.

Fixed rate will be paid quarterly (quoted quarterly bond basis).

Eurodollar Futures, Settlement Prices (October 24, 1994)

| | Price | Implied 3-mo. SOFR | Notation | Number of Days ($A(t)$) |
|--------|-------|-----------------------|----------|------------------------------|
| Dec 94 | 94.00 | 6.00 | 0 x 3 | 90 |
| Mar 95 | 93.57 | 6.43 | 3 x 6 | 92 |
| Jun 95 | 93.12 | 6.88 | 6 x 9 | 92 |
| Sep 95 | 92.77 | 7.23 | 9 x 12 | 91 |
| Dec 95 | 92.46 | 7.56 | 12 x 15 | 91 |

Housemann Bank wants to find the fixed rate that has the same present value as four successive 3-mo. SOFR payments.

(1) Calculate implied SOFR rate using (i).

Swap is for twelve months, $n = 4$.

$$f_{0,12} = [(1 + .06 * (90/360)) * (1 + .0643 * (92/360)) * (1 + .0688 * (92/360)) * (1 + .0723 * (91/360))]^{360/365} - 1 = .06760814. \text{ (money mkt basis)}$$

(2) Convert this money market rate to its effective equivalent stated on an annual bond basis.

$$FRE_{0,12} = .06760814 * (365/360) = .068547144. \text{ (bond basis)}$$

(3) Coupon payments are *quarterly*, $k = 4$. Restate this effective annual rate on an equivalent quarterly bond basis.

$$SC = [(1 + .068547144)^{1/4} - 1] * 4 = .0668524 \text{ (quarterly bond basis)}$$

⇒ The swap coupon mid-rate is 6.68524%.

Example: Now, Housemann Bank wants to price a one-year swap with *semiannual* ($k = 2$) fixed-rate payments against 6-month SOFR.

The swap coupon mid-rate is calculated to be:

$$SC = [(1 + .068547144)^{1/2} - 1] * 2 = .06741108 \text{ (s.a. bond basis).¶}$$

☞ A dealer can quote swaps having tenors out to the limit of the liquidity of Eurodollar futures on any payment frequency desired.

Gap Risk Management

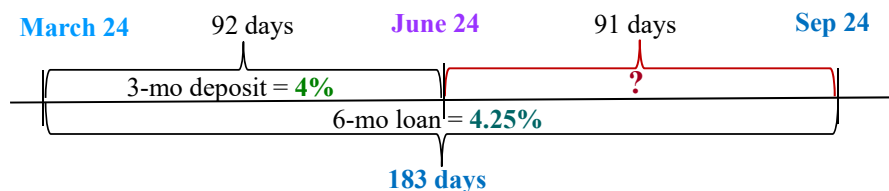
Gap risk: Assets and liabilities have **different maturities**.

Eurocurrency futures are used to hedge gap risk.

Example: Gap Risk Management

Situation: It's **March 20**.

- A bank can lend a 6-mo Euro-EUR deposit at **4.25%**, with a value date on **March 24** and maturity date on **September 24** (**183 days**).
- A Swiss bank observes a rate of **4%** on 3-mo euro-EUR deposits, with a value date of March 24. The deposit matures on **June 24** (92 days).
- June Euro-EUR futures are trading at **96.13** (or, yield = **3.87%**).

**Gap Risk Management****Example (continuation):**

- *Gap risk:* The bank receives a 3-mo deposit and lends for 6-mo.
 - ⇒ Risk: The interbank deposit interest rate on June 24 is uncertain.
 - ⇒ *Gap risk:* It can be managed using Jun Euro-EUR futures.
- Bank considers lending a 6-mo deposit at **4.25%**, funded by two 3-mo deposits: the 1st at **4%**; the 2nd one at the June Euro-EUR rate.

Q: Is it profitable for the bank?

Yes, if bank can get a 3-mo deposit starting in June at a lower rate than *f*.

Gap Risk Management

Calculations: We calculate f & compare it with the June Euro-EUR rate.

Implied forward rate, f (break even):

$$[1 + .0425 * (183/360)] = [1 + .04 * (92/360)] * [1 + f * (91/360)]$$
$$\Rightarrow f = 4.457\%$$

- As long as the bank can ensure that it will pay a rate less than **4.457%** for the 2nd 3-mo. period, the bank will make a profit.
- June Euro-EUR are at **3.87%** < $f = 4.457\%$.
 \Rightarrow *Shorting* one June Euro-EUR at **96.13**, makes the bank a profit.