International Bond Markets & Swaps

- **Last Class**
- International Equity Markets:
  - Why International Diversification pays? Low correlations
- Returns in International Equity Markets
  - Huge dispersion in ERP across markets, not easy to explain.
  - ERP measured with a lot of noise
  - Many proposed factors to explain ERPs.
- International Bond Markets
  - ◊ Eurobond + Foreign Bond Market = International Bond Market (30%)
  - ◊ Eurobond: Same structure as a domestic bond, but bearer securities are OK and face little regulations
  - ◊ Many Instruments in Eurobond Markets
  - ◊ Valuation with Examples

- **This Class**
- International Bond Markets
  - Selection of a Eurobond instrument: Example

- **CASE 6: Brady Bonds (due next Thursday, April 21)**

- Swaps
  - Definition
  - Different Types: Interest Rate, Currency, Equity, Commodity & CDS.
  - Market Organization and Swap Dealers
  - Uses and Valuation
  - Combination of Swaps

Case Study I: Merotex

Pricing a New Straight Bond: Merotex

The Borrower

- Leading construction firm, based in Gorizia, Italy.
- Recently bought two U.S. construction companies.
- Financed by bank loans: USD 250 million

Borrowing requirements

- Amount: **USD 250 million**
- Currency of exposure: **USD**
- Maturity: *Medium-term* (**5 to 7 years, preferred 7 years**) USD debt.
- Preference: *Simple straight bond* with no early call options.

• **Information**

Market conditions:

- **Good** for a USD Eurobond issue.
- U.S. economic conditions are above expectations
- USD is currently very strong.
- **Recent successful placement** of 10-year Euro-USD issue by Fica, a competitor.

Merotex's Perception:

– Merotex has issued **GBP Eurobonds**: obtained *best terms*.

– Merotex has **no outstanding Euro-USD issues**.

• Perception of similar international borrowers ("Benchmarking")

(1) Comenti: **Italian construction company**

- Several Eurodollar issues.
- Last issue has 6 years of remaining life.
- Currently trading at **40 bps** over 6-yr U.S. Treasuries.
- Excellent reputation in Euromarkets

(2) Fix Constructions (FC): **major U.S. competitor** in Florida.

- Launched a 10-yr Eurodollar issue five years ago.
- It has a *call option* two years from now.
- Currently trading at a **65 bps** over 5-year U.S. Treasuries.
- Well-regarded but *performance* has been *just average*.

(3) **Other large Italian companies**:

- Many Euro-USD bonds with 5-year maturity
- Currently trading within a range of ***40-70 bps***.

Evaluation

• Merotex's track record is limited but **very good**.

– Merotex's GBP bonds have been **well received** in the market.

– Merotex plans to include one **UK** house in management group.

⇒ **Size:** sufficient to promote liquidity; but not so much as to make the placement process difficult. Proposed size: **USD 200 million**.

⇒ **Maturity:** Merotex is a first-timer on USD-Eurobond segment: For first timers shorter maturities are better: ***5 years***.

• Concern: The FC issue is trading at a **relative high spread**. But,

– Issue might suffer from poor design.

– Deterioration of FC's perception

– Call provision.

⇒ **Yield:** Lead manager suggests setting **spread** on the low-end of range ("aggressive spread"): ***40-70 bps***.

Proposed Issue

Amount:

– Proposed size: **USD 200 million**, with a possible increase.

Maturity:

– Shorter maturity than preferred: ***5 years***.

Yield spread:

– Aggressive spread = **40 bps** over 5-yr U.S. Treasuries.

– First-time issue: Add a small premium: ***Spread = 45 bps***.

The lead manager is able to formulate a pricing scheme:

U.S. Treasury: 6.915% s.a. (semiannual)

Merotex spread: **0.45% s.a.**

Merotex yield (**YTM**): **7.365% s.a.**, or **7.501% p.a.** (annual)

⇒ Terms for investors: a 5-year Eurobond at a price to yield **7.50% p.a**.

Fees

Selling concession: ¾%	(Sellers buys the issue at 99¼).
Underwriting allowance: ¾%	(Underwriters pays 98½)
Managing fee: ¼%	(Lead manager pays (98¼)
Total fees: **1¾% (= USD 3.5M)**	

Final terms:

Competitive bidding: Issuing house sells the issue at **99.24**

Coupon required to yield **7½%** is lower.

Assuming **YTM** = **7½**, T = 5, P = **99.24**, and FV = 100, solve for **C**

⇒ **C** = **7.3113%.**

Rounding up, the coupon rate is set at **7 (5/16)**.

Total coupon payment = (7 + **5/16**) * **200 M** = **USD 14.625 M**

The issue is priced *at the selling concession.*

Expenses

1.- Paying Agency:	**100,000 bonds** in **USD 1,000 denominations**
	10,000 bonds in **USD 10,000 denominations.**
Total number of bonds:	**110,000**.
Coupon charge p.a.:	USD .07 per coupon payment (USD 7,700)
Redemption charge:	USD .70 **per bond** or USD 77,000
Authentication:	USD 4,000 on delivery of bonds.
Administration:	USD 2,000 (p.a.).
2.- Listing:	USD 20,000 payable in advance.
3.- Trustee:	USD 8,000 (p.a.) payable in advance.
4.- Other expenses:	USD 80,000.

Pro Forma of the Issue

Borrower:	Merotex C.A.
Guarantor:	None
Amount:	**USD 200 million**
Maturity:	**5 years**
Coupon:	**7 (5/16)** (= **7.3125%**)
Issue price:	100%
Amortization:	Bullet repayment on final maturity date
Issuer's call option:	None
Listing:	London
Denominations:	**USD 1,000** and **USD 10,000**
Form:	Bearer securities
.....	
Commissions:	**1¾% flat**
Yield:	**7.3125%** (at issue price), **7.5% p.a.** (at 99.24%)

Cash Flows of Merotex C.A. (in USD million):

Year	**0**	**1**	**2**	**3**	**4**	**5**
Principal	**200**	-	-	-	-	**-200**
Interest	-	**-14.625**	**-14.625**	**-14.625**	**-14.625**	**-14.625**
Commissions	**-3.500**	-	-	-	-	-
Paying Agency	-	-0.0077	-0.0077	-0.0077	-0.0077	-0.0847
Auth. & Adm.	-0.004	-0.002	-0.002	-0.002	-0.002	-0.002
Listing	-0.020	-	-	-	-	-
Trustee	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008
Reimburs. exp.	-0.080	-	-	-	-	-
Cash Flow	**196.39**	**-14.6427**	**-14.6427**	**-14.6427**	**-14.6427**	**-214.7117**

⇒ Cost of funds (IRR) = **7.7778% p.a.**

Note: Sometimes, IRR is calculated by *excluding* annual & minor expenses (listing, trustee, authentication, etc.). Under this method,

IRR = **7.7580%**.

Cost of Funds Exclusive Annual and Minor Expenses: Details

• This figure takes account:

- Coupon payments (**USD 14.625 M**)
- Commissions of **1¾%** flat on the issue amount (**USD 3.5 M**)
- Reimbursable managers' expenses (USD 80,000)

The issuer receives the net proceeds of:

USD 200,000,000 - USD 3,580,000 = **USD 196,420,000** (or 98.21%)

• All-in cost: IRR of a 5-year project:

- Positive cash flow of **USD 196.42 M** in year zero.
- Negative cash flows of **USD 14.625 M** every year.
- Negative cash flow of **USD 200 M** in year 5.

IRR = **7.7580%**. (Merotex obtains financing at a cost of 7.7580% p.a.)

⇒ Small difference between both IRRs.

Cost of Funds Inclusive Annual and Minor Expenses: Details

This figure takes account:

- Coupon payments
- Commissions of **1¾%** flat on the issue amount
- Reimbursable managers' expenses
- Commissions and Expenses

⇒ IRR = **7.7778% p.a.** (Merotex obtains financing at an annual cost of 7.7778%.)

Equity Warrants in Eurobonds

• *Equity warrant issues* have different components:
- A standard fixed-rate Eurobond issue
- A detachable equity warrant.

Equity warrant: A **call option** on the stock of the issuer.
⇒ Tend to have **longer maturities** than standard options.

The equity warrant is typically **detached**.

• Terminology
Exercise ratio: **Number of shares** a warrant buys.
Equity content: Amount (in currency) of **equity added** if warrants exercise. Usually expressed as a **percentage** relative to the size of bond issue.

• The **bond** is **priced as normal**.
• Warrant is priced as a function of: warrant premium & equity content.

• There is a unique price:

(1) **100 percent**: the bond price will be at a discount to par and the coupon at a below-market level (*discount bond*);

(2) **in excess of 100 percent**: the bond is issued at a normal market price, that is, close to 100 percent (*full coupon bond*).

• Black-Scholes formula is used to price warrant.

Traders make adjustments to the BS formula based on:

(1) Prices of other warrants

(2) Market perception on the company

(3) Stock market expectations.

• The ratio of equity raised to the issue size is the *equity ratio*.
Usual range: 100% to 200% of the nominal amount of the bond issue.

• Q: Why do firms use equity warrants?
A: For raising equity capital at a lower IRR.

Case Study II: VOMF

• **Information**
• New Zealand company in the uranium business, with good reputation.
• VOMF wants to refinance debt for **USD 100 million**. Seeking maturity in the range **3-7 years maturity** (the longer, the better).

• Market conditions:
- **Uranium prices are up**.
- New Zealand economy is **coming strong** out of a recession.
- Inflation is low.
- The stock market is **expected to do well** in the near future.

VOMF's situation:

- VOMF's **share price is up** because of exploration agreements.
- Shareholders might **increase capital** in the next assembly.
- VOMF has **outstanding debt in the Euro-USD** segment. It is currently paying a spread of **9 bps** over US Treasuries.

An investment bank suggests an **equity-linked financing**

⇒ A **straight bond** with **equity warrants attached**.

Data available	
US Treasury yields:	3-year 6.530% (s.a.)
	5-year 6.915% (s.a.)
	7-year 7.135% (s.a.)
NZ interest rate:	1-year 4.52% (p.a.)
VOMF Euro-USD bond yield:	U.S. Treasuries + **9 bps**
VOMF's share price (P_0):	**NZD 120**
Historic dividend yield:	5.50%
Historic stock price volatility:	**3-year 19.90%**
	5-year 21.40%
	7-year 25.50%.
Outstanding warrants	
Outstanding life:	3½ years
Current price (W_0):	USD 10-10.80 (**NZD 15.625 - 17**)
Exercise price (***X***):	**NZD 145**
Current exchange rate:	**.64 USD/NZD** (1.5625 NZD/USD).

• **Evaluation**

- Overall market conditions are **good**

  ⇒ Amount to be raised: **USD 100M** as borrower wants.

  ⇒ Long end of desired maturity: **7-years**

- Bond trading often in secondary market –i.e., **spread informative**.

  ⇒ Use current spread (**9 bps**) to price new issue.

- Prospect (& potential demand) for VOMF equity is also **good**.

  ⇒ Equity content: ***150%***

- Outstanding warrants also trade often –i.e., **premium** (& **global premium**, **GP**) **informative**. Last trade at **NZD 16**, with a **GP**:

  GP = (***X*** + W_0)/P_0 = (**145** + **16**)/**120** = 1.3417. (or 34.17%)

  ⇒ Use strike price & maturity close to outstanding issue:

  Maturity = **3-years**

  Strike Price = **X** = **NZD 150** (to minimize competition)

Proposed Issue

Terms for the bond

Amount:

- Same as refinancing need: **USD 100 million**
- In denomination of USD 1,000 (**100,000** bonds)

Maturity:

- Market conditions indicate a *7-year* bond.

Yield spread:

The lead manager is able to formulate a pricing scheme:

U.S. Treasury: **7.135% (s.a.)** (semiannual)

VOMF spread: **.09% (s.a.)**

VOMF yield (**YTM**): 7.225% s.a. (or **7.3555% p.a.** (annual))

Fees: 2% (**USD 2 M**)

Proposed Issue (continuation)

Terms for the warrants

Amount:

Equity content: ***150%*** (good market conditions). Exercise ratio: 1

Maturity: ***3 years*** (close to the life of outstanding issue).

Pricing warrant:

Investment bank proposes 3-year equity warrants with a **GP ≈ 35%** & ***X*** = **NZD 150**

Warrant price = **GP** * P_0 – ***X*** = **1.35** * **NZD 120** – **NZD 150** = **NZD 12**

⇒ At this price, the implied volatility is equal to **19.63%**.

Black-Scholes formula (theoretical price): **NZD 12.23**.

Proposed Issue

Final terms for the warrants

(Assume a conversion exchange rate = **.64 USD/NZD**)

Amount of equity raised:

USD 100M * ***1.50*** = USD 150M = **NZD 234,375,000**

Number of shares created on exercise:

NZD 234,375,000/ **NZD 150** = 1,562,500

Exercise ratio:	**1**
Number of warrants:	1,562,500/**1**
Number of bonds:	**100,000**
Number of warrants per bond:	1,562,500/**100,000** = **15.625**
Value of the warrants attached to each bond of USD 1,000:	**15.625** * **NZD 12** = **NZD 187.50** = **USD 120** (12% of nominal amount)

Terms for the bond with warrants

Final terms:

The bonds are offered at ***98.78 percent*** (competitive pressures)

VOMF's coupon is reduced to **7⅛% p.a.**

Full-coupon bond which trades better in the secondary market.

Issue price (bond + warrants): 112%

Cost of funds (based on total issue price less commissions of **2%**): **5.372% p.a.** or **5.302% s.a.**

⇒ IRR: **183 bps** (s.a.) below the yield on 7-year U.S. Treasuries:

1.83% (s.a.) = **7.135% (s.a.)** - **5.302% (s.a.)**

Case Study III: Bioneth

• **Selecting a Particular Bond**

Simple process:

(1) Compute cost of funds of different bonds under different scenarios

(2) Based on risk tolerance, a firm decides on the best instrument.

• We present an example showing how a Portuguese firm, Bioneth Engineering, selects a Eurobond issue with currency options attached

Eurobonds with Currency Options Attached

Attached to a Eurobond issue, a currency option is securitized as a tailor-made listed warrant.

• Advantages over standard currency options

For Buyers:

- Loophole: In some countries, rules may prevent investors from buying currency options per se.
- Smaller denominations or contract sizes
- Longer exercise periods.

For Issuers:

- Adding a securitized option reduces the cost of the borrowing.

• Disadvantage for the issuer: FX exposure. But, it can be hedged.

Example: Bioneth Engineering is a firm based in Portugal.

Situation:

- Bioneth has **GBP 100 million** of short-term debt.
- Refinance the GBP debt with a straight ***7-year 8%*** Euro-GBP bond.
- Commissions paid to ***1¾%*** (or **GBP 1.75M**).

An **investment bank** approaches Bioneth and **offers**:

A similar straight ***7-year 8%*** Euro-GBP bond, but with a ***3-year*** *currency warrants* attached giving entitlement to an American GBP-put/EUR call option with:

X_p = **1.50 EUR/GBP** (or X_c = **.6667 GBP/EUR**)

Size = **EUR 1,600**.

Note: The currency warrant is a **standard put** –i.e., exercised only when:

$$(X_p - S_t) > 0.$$

Terms of the bond

Amount:	**GBP 100 million**.
Maturity:	***7 years***.
Issue price:	100%
Denominations:	GBP 1,000 (⇒ **100,000 bonds**)
Interest:	***8%* p.a.** payable annually in arrears.
Early redemption:	None.
Redemption price:	100%
Issuance commissions:	***1¾%*** (**GBP 1.75M**).
Listing:	London
Cost of funds (including only commissions):	**8.34%**

Terms of the currency warrants

Exercise price:	**1.50 EUR/GBP** (**.6667 GBP/EUR**)
Exercise period:	At any time.
Current exchange rate:	**1.60 EUR/GBP** (.6250 GBP/EUR)
Structure:	Each bond has a warrant giving the right to receive the difference (in GBP) between: (1) **EUR 1,600** at X_p = **1.50 EUR/GBP**, and (2) **EUR 1,600** at S_t.
Warrant price:	**EUR 0.04935** per GBP. At S_t, **GBP 0.0308** or GBP 30.80 per bond (**3.08%**).
Premium of X/S:	.6667/.6250 = 1.0667 or 6.67%

Issue price (bond & warrants): 100% + **3.08%** = 103.08%

Note: Warrant will be exercised when:

$(X_p - S_t) > 0$ (like a put!).

Implication of currency warrants

• At expiration, two scenarios for GBP put:

if $S_{t+3-yrs} > X_p$ = **1.50 EUR/GBP** ⇒ No exercise.

if $S_{t+3-yrs} < X_p$ = **1.50 EUR/GBP** ⇒ Exercise.

Example: If $S_{t+3-yrs}$ = **1.40 EUR/GBP** (**.7143 GBP/EUR**), exercise.

⇒ receive, per bond:

EUR 1,600 * **.7143 GBP/EUR** – **EUR 1,600** * **.6667 GBP/EUR** =

= **GBP 76.19**.

• Bioneth is exposed to currency risk.

If $S_{t+3-yrs}$ = **1.40 EUR/GBP**, Bioneth has an additional cash flow of

GBP 76.19 * **100,000** = **GBP 7,619,000**.

⇒ Total CFs at $t + 3$ = GBP -8M + **GBP -7.619M** = **GBP -15.619M**

Implication of currency warrants

• To hedge, the investment bank offers Bioneth an identical currency option at **EUR 0.04** per GBP or **GBP 25** per GBP 1,000 bond.

That is, Bioneth can buy FX insurance for an upfront cost of

GBP 25/bond * **100,000 bonds** = **GBP 2.5M**.

⇒ At inception, Bioneth receives

GBP 100M – **GBP 1.75M** + **GBP 3.08M** – **GBP 2.5M** = **GBP 98.83M**.

• Suppose the investment bank considers likely $S_{t+3-yrs}$ = **1.40 EUR/GBP**. Bioneth compares CFs under different alternative scenarios:

Date	Str. Bond	CO Bond (NH/NExercised)	CO Bond (NH/Exercised)	CO Bond (Hedged)
0	98.250	101.330	101.330	**98.830**
1	-8.000	-8.000	-8.000	-8.000
2	-8.000	-8.000	-8.000	-8.000
3	-8.000	-8.000	**-15.619**	-8.000
4	-8.000	-8.000	-8.000	-8.000
5	-8.000	-8.000	-8.000	-8.000
6	-8.000	-8.000	-8.000	-8.000
7	-108.000	-108.000	-108.000	-108.000
IRR:	**8.340%**	**7.747%**	**8.904%**	**8.227%**

⇒ Bioneth issues bond with currency options attached and also hedge.

Case Study IV: Brady Bonds

Brady Bonds (BBs)

• Created to bring EM (Mexico, Brazil, etc.) out of the 1980s default.

• USD 180 billion market in its heyday.

• Idea:

Banks voluntarily reduce their claims in return for credit enhancements on their remaining exposure: Collateral accounts to guarantee the principal and/or interest in a bond exchange in the context of buybacks.

• Mexico, Costa Rica, and Venezuela were the first three countries to issue bonds as part of the Brady plan.

• Issued two *Brady bonds* for debt conversion:

- A *par bond* (fixed-rate)
- A *discount bond* (floating-rate)

• Brady Bonds: Mexican BBs

• Mexican Brady bonds were issued in March 1990, with **T = 30 years**.

- Principal: Guaranteed by **30-year** U.S. Treasury zero-coupon bonds.

- **Rolling interest guarantee** (RG) provided by a pool of collateral sufficient to cover **18-mo of** coupon payments –3 semester payments– at an assumed coupon rate of 10%.

- Banks have a choice of BBs to exchange for defaulted debt:

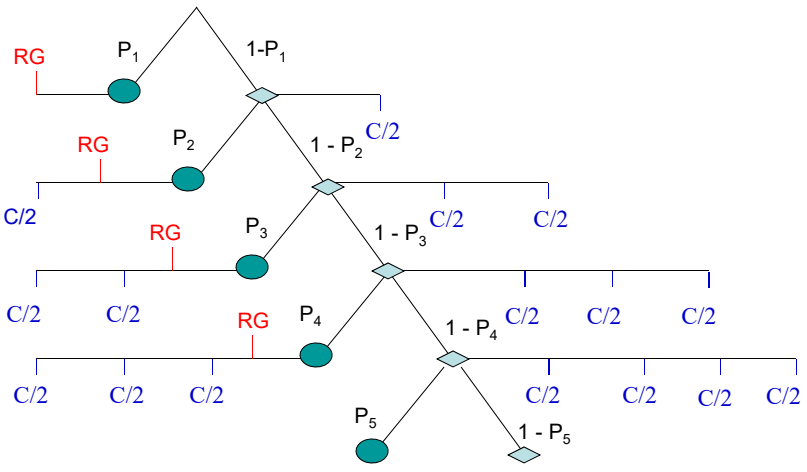
  - *Par Bond*: C = **6.25%**. Bank debt exchanged for the par bond with principal equal to the original face value of the debt.

  - *Discount Bond*: C = **LIBOR + 13/16**. But, bank debt exchanged at the discount 65% ratio.

- Both bonds include an oil price recapture clause that pays off if oil prices rise in 1997 and beyond.

Brady Bonds: Cash Flows

The coupon CFs follow a simple binomial tree, where P_t is the probability of default at time t, with $t = 1, 2, \ldots, 60$.



• NPV: Discount CFs with appropriate YTM (local YTM for C and US YTM for RG). We also need a model for default!

Brady Bonds: Cash Flows and Discount Rates

∘ Principal: Guaranteed with US T-bonds. At maturity, it will be repaid.

⇒ NPV of Principal: Use 30-year US YTM as discount rate.

∘ Coupon payments involve risk. Default can happen. There is uncertainty regarding the amount of coupon payments.

- As a minimum, a bondholders receives RG: RG kicks in immediately after default. It involves 3 C/2 payments.

⇒ NPV of RG: Use appropriate US YTM to discount CFs.

- The risky CFs are the ones not covered by guarantees: All coupon payments, beyond the RG.

⇒ NPV of coupons: Use appropriate Mexican YTM.

Brady Bonds: Cash Flows and Default Probabilities

∘ We need the probability associated with each final coupon CF at the each branch of the binomial tree.

Suppose there is default at $t = 3$ –i.e., after 2 coupon payments. There are 5 total payments ⇒ CFs for the bondholder are: {C/2, C/2, RG}.

The probability of receiving 5 coupon payments only –i.e., default occurs after $t = 3$– is given by:

$$(1 - P_1) * (1 - P_2) * P_3$$

∘ Multiply each final coupon CFs at each branch by its probability and add them up to compute an expected NPV of coupon CFs, or

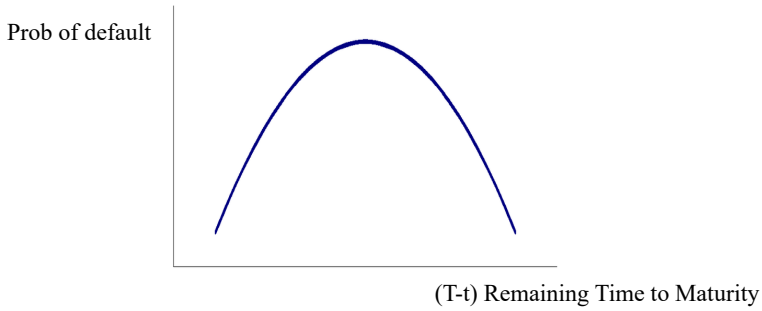
$$E[NPV_{Coupons}].$$

∘ $E[NPV_{Coupons}]$ + NPV of Principal (no uncertainty) = P_{BB}.

• Brady Bonds: Default Probabilities

• To calculate E[NPV], we need a model for the probability of default. Many ways to approach this problem.

Example: Use inverted U shape:



• Alternatively, we can buy a *credit default swap* or CDS –i.e., insurance – to cover the event of default of coupon payments (more on this next chapter). Although, there was no CDS market at the time.

SWAPS: Definition and Types

Definition

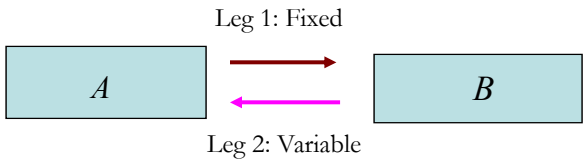
A *swap* is a contract between two parties to deliver one sum of money against another sum of money at periodic intervals.

• Obviously, the sums exchanged should be **different**:
  – Different **amounts** (say, one fixed & the other variable)
  – Different **currencies** (say, USD vs EUR)

• The two payments are the *legs* or sides of the swap.
  - Usually, one leg is *fixed* and one leg is *floating* (a market price).

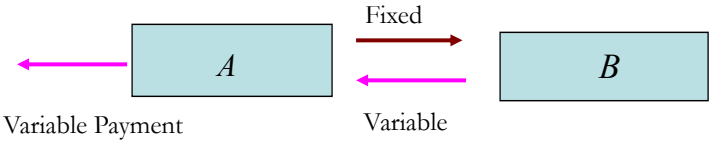
• The swap terms specify the duration and frequency of payments.

Example: Two parties (A & *B*) enter into a swap agreement. The agreement lasts for 3 years. The payments will be made semi-annually. Every six months, A and B will exchange payments.



• Swaps can be used to change the profile of a firm's cash flows.

If a swap is **combined** with an **underlying position**, one of the (or both) parties can change the profile of their cash flows (and risk exposure). For example, *A* can change its cash flows from variable to fixed.



• **Types**

Popular swaps:

- Interest Rate Swap (one leg floats with market **interest rates**)
- Currency Swap (one leg in one **currency**, other leg in another)
- Equity Swap (one leg floats with market **equity returns**)
- Commodity Swap (one leg floats with market **commodity prices**)
- CDS (one leg is paid if **credit event occurs**)

Most common swap: **fixed-for-floating** interest rate swap.

- Payments are based on hypothetical quantities called *notionals*.
- The fixed rate is called the *swap coupon*.
- Usually, only the *interest differential* needs to be exchanged.

• Usually, one of the parties is a **Swap Dealer**, also called *Swap Bank*.

Example: Interest Rate Swap (inception date: April)

Bank A (**fixed-rate payer**) *buys* an 8% swap

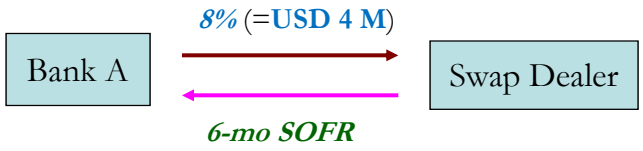
Notional: **USD 100 M**

Swap coupon (Fixed-rate): **8% (s.a.)**.

Floating-rate: **6-mo. SOFR** (= Secured Overnight Financing Rate).

Payment frequency: semiannual (April and October).

Maturity = Swap term or Swap tenor = 3 years.



Every six months Bank A (fixed-rate payer) pays:

USD 100 M * **.08**/2 = **USD 4 M**

Every six months Swap Dealer (floating-rate payer) pays:

USD 100 M * **6-mo SOFR**/2

Example: (continuation)

First payment exchange is in October. (The floating rate has already been fixed in April: **7.6%**.) Then, the Swap Dealer pays:

⇒ **USD 100 M** * **.076**/2 = **USD 3.8 M**

Bank A (fixed-rate payer) pays **USD 0.2 M** to the floating-rate payer.

Note: In October, the floating rate will be fixed for the second payment (in April of following year). ¶

Market Organization

• Most swaps are tailor-made contracts.
- Swaps trade in an **OTC** type environment.
- **Swap specialists** fill the role of broker and/or market maker.
- Brokers/market makers are usually **large banks**.
- Prices are quoted with respect to a standard, or generic, swap.

• *All-in-cost*: Price of the swap (quoted as the rate the fixed-rate side will pay to the floating-rate side)

• It is quoted on a **semiannual basis** (s.a.):
- absolute level ("9% fixed against six-month *SOFR flat*")
- bp spread over the U.S. Treasury yield curve ("the Treasury yield plus 57 bps against 6-mo SOFR flat").

"SOFR flat" = SOFR is quoted without a premium or discount.

• The fixed-rate payer is said to be "long" or to have "bought" the swap.

Example: Houseman Bank's *indicative swap pricing schedule*.

Maturity	HB Receives Fixed	HB Pays Fixed
1 year	1-yr TN sa + 44 bps	2-yr TN sa + 39 bps
2 years	2-yr TN sa + 50 bps	2-yr TN sa + 45 bps
3 years	**3-yr TN sa + 54 bps**	**3-yr TN sa + 48 bps**
4 years	4-yr TN sa + 55 bps	4-yr TN sa + 49 bps
5 years	5-yr TN sa + 60 bps	5-yr TN sa + 53 bps

• Consider the 3-year swap quote:

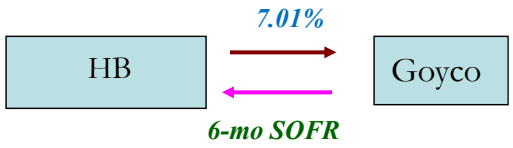
HB attempts to **sell** a 3- year swap to receive the offered spread of **54 bps** and **buy it back** to pay the bid spread of **48 bps**. HB's profit: **6 bps**.

Example: Goyco, a HB's client, wants to receive fixed-rate payments rather than pay fixed-rate for 3 years.

The current ("*on the run*") 3-yr Treasury Note rate is **6.53%**.

Goyco enters into a 3-yr swap.

Calculation of fixed rate: HB will pay ***7.01%*** (6.53 + **.48**) s.a. ¶



Note: SOFR will be reset at (T-2) for the next 6-mo period.

• Q: Are swaps riskless?

A: No! Default risk is always present.

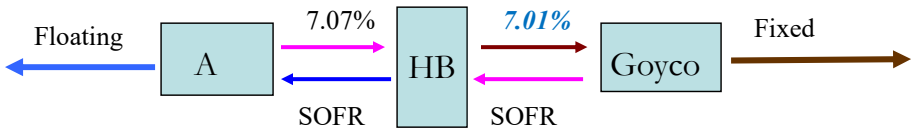
• The Dealer's Perspective

A swap dealer intermediating (making a market in) swaps makes a living out of the spread between the two sides of the swap.

Example: Houseman Bank entered into a 3-year swap with Goyco.

3-year quote: 3-yr TN sa + **48 bps** & 3-yr TN sa + **54 bps**

The *on the run* 3-yr Treasury Note rate is **6.53%**.



HB finds a counterparty A ⇒ HB gets rid of the swap exposure. ¶

Note: The difference between the rate paid by the fixed-rate payer over the rate of the *on the run* Treasuries with the same maturity as the swap is called the *swap spread*. In this example, the swap spread is **54 bps**.

Warehousing

When the SD matches the two sides (the buyer and the seller) of a swap is called **back-to-back transaction**, or "**matched book**" transaction.

In practice, a SD may not be able to find an immediate off-setting swap.

Most SD will **warehouse** the swap and **use interest rate derivatives** to hedge their risk exposure until they can find an off-setting swap.

In practice, it is **not always possible to find a second swap** with the same maturity and notional principal as the first swap, implying that the institution making a market in swaps has a residual exposure.

The relatively narrow bid/ask spread in the interest rate swap market implies that to make a profit, effective interest rate risk management is essential.

Dealer's Risk

- *Credit Risk*

  This is the major concern for a swap dealer: the risk that a counter party will default on its end of the swap.

- *Mismatch Risk*

  It is difficult to find a counterparty that wants to borrow the exact amount of money for the exact amount of time.

- *Sovereign Risk*

  The risk that a country will impose exchange rate restrictions that will interfere with performance on the swap.

• Market Size

Notional amount outstanding (Nov 2022):	**USD 463.0 trillion.**
- Interest rate swaps:	**USD 414.2 trillion**
- Currency swaps:	**USD 30.3 trillion** (≈ 7%)
- Equity-linked contracts (includes forwards):	**USD 6.9 trillion**
- Commodity contracts (includes forwards):	**USD 2.3 trillion**
- CDS market:	**USD 9.3 trillion** (≈ 2%)

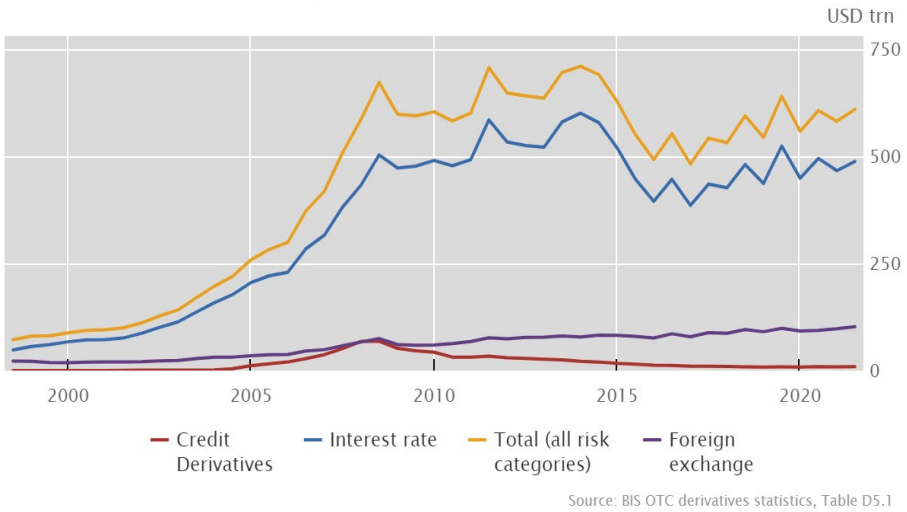
- Gross market value: **USD 16.35 trillion**

Interest rate swaps is a very popular derivative: It represents **60%** of the Global OTC Derivatives Market.

Interest rate swaps also shows big growth from early 1990s.

Growth of OTC Derivatives (Swap Market) - 1998-2021

Figure XIII.1 - Growth of Swap Market



Source: BIS (2021). ¶

Swaps: Why Use Them?

• Using Swaps

- Manage risks (change profile of cash flows)
- Arbitrage (take advantage of price differentials)
- Enter new markets (firms can indirectly create new exposures)
- Create new instruments (no forward contract exist, a swap completes the market)

(1) Change profile of cash flow

Goyco's underlying situation: Fixed payments to bondholders, but wants floating debt.

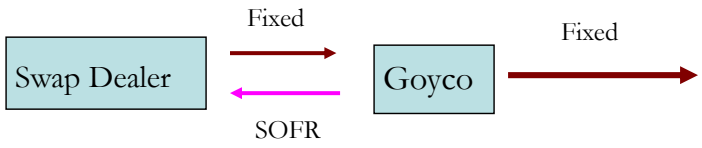
Solution: A fixed-for-floating debt solves Goyco's problem.

Example: Goyco enters into a swap agreement with a Swap Dealer.

Terms:

Duration: 3-years.

Goyco makes floating payments (indexed by SOFR) and receives fixed payments from the Swap Dealer.



Q: Why would Goyco enter into this swap?

A: To **change the profile** of its cash flow: From fixed to floating. ¶

• Swaps are derivative instruments (derived valued from value of legs!).

(2) Arbitrage: Comparative Advantage

A has a *comparative advantage* in borrowing in USD.

B has a *comparative advantage* in borrowing in GBP.

If they borrow according to their comparative advantage and then swap, there will be gains for both parties.

Example: Nakatomi, a Japanese company, wants USD debt.

There are at least two ways to get USD debt:

- Issue USD debt.
- Issue JPY debt and swap it for USD debt. ¶

Q: Why might a Japanese company take the second route?

A: It may be a cheaper way of getting USD debt -*comparative advantage*.

Note: From an economic point of view, there are two motives for entering into swaps:

- *Risk Sharing* (two firms share interest risk through a swap)
- *Comparative Advantage/Arbitrage*

Measuring comparative advantage

AAA companies routinely **have absolute advantage** in debt markets over all the other companies, due to their different credit-worthiness. A swap, however, takes advantage of **comparative** (or relative) **advantages**.

The *Quality Spread Differential* (**QSD**) measures comparative advantage

- **QSD** = Difference between the interest rates of debt obligations offered by two parties of different creditworthiness that engage in the swap.

- The **QSD** is the key to a swap. It is what can be shared between the parties.

Example: Comparative Advantage.

Nakatomi, a Japanese company, wants to finance a US project in USD.

HAL, a US company, wants to finance a Japanese project in JPY.

Both companies face the following borrowing terms.

	USD rate	JPY rate
HAL	**9%**	**4%**
Nakatomi	**8%**	**2%**

Nakatomi is the more credit-worthy: It has an **absolute advantage**.

- HAL pays 1% more to borrow in USD than Nakatomi.
- HAL pays 2% more to borrow in JPY than Nakatomi.

⇒ HAL has a **comparative advantage** in the USD rate market.
The **QSD** is 1% (different than zero!).

If they borrow according to their comparative advantage and then swap, there will be gains for both parties.

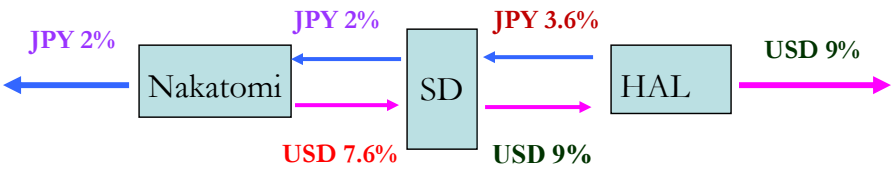
A Swap Dealer proposes the following swap:

- Nakatomi pays **7.6%** in USD and receives **2% in JPY**
- HAL pays **3.6%** in JPY and receives **9% in USD**

Swap + Domestic borrowing produces the following cost of borrowing:

Nakatomi: **2% in JPY** - **2% in JPY** + **7.6%** in USD = **7.6% in USD** < **8%**

HAL: **9% in USD** - **9% in USD** + **3.6%** in JPY = **3.6% in JPY** < **4%**



QSD is equally divided (**0.4%** savings each).

Note: The QSD is not usually divided equally. In general, the company with the worse credit gets the smaller share of the QSD to compensate the SD for a higher credit risk. ¶

Why the Growth of Swap Markets?

• Swap contracts have many **similarities with futures** contracts.

- Trade-off: Customization vs. liquidity.

Futures markets offer a high degree of **liquidity**, but contracts are more standardized.

Swaps offer additional flexibility since the they are **tailor-made**.

- **Settlement** is in **cash**.

There is no need to take physical delivery to participate in the market.

Interest Rate Swaps

• Most common swap: fixed-for-floating (*plain vanilla swap*)

- Used to **change profile of cash flows** (a firm can go from paying floating debt to paying fixed debt).
- Used to **lower debt costs**.

• Basis swap: floating-for-floating (*basis swaps*)

- Floating rates should be **different**, say 1-mo **SONIA** vs. 3-mo SONIA (Sterling Overnight Index Average) or USD T-bill vs **SOFR** (the alternative to USD LIBOR, Secured Overnight Financing Rate)
- Floating-for-floating currency swaps (also called *cross currency basis swaps*) are especial cases of interest rate basis swaps.

• Interest rates swaps have **very low** bid-ask spreads, lower than corporate bonds and, sometimes, government bonds.

Example: Plain Vanilla Swap

Underlying situation for Ardiles Co:

- **USD 70 M** floating debt at: **6-mo SOFR + 1%.**
- Ardiles Co. wants to change to fixed-rate USD debt.
- Currently, fixed-rate debt trades at **9.2%** (s.a.).

A Swap Dealer (Bertoni Bank) offers **8% (s.a)** against **6-mo SOFR**.

Terms: - Notional amount: **USD 70 M**
- Frequency: semi-annual
- Swap term or tenor (Duration): 4-years
- Fixed Coupon: **8% (s.a)** ⇒ **USD 70 M** * **.08**/2 = **USD 2.8M**

Ardiles Co. and SD only exchange the **net payment** (difference between the two legs of the swap: Ardiles pays SD if 6-mo SOFR < **8%**).

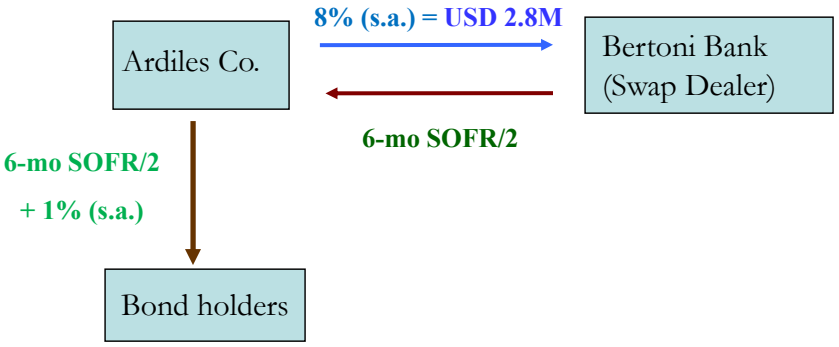
Notionals, obviously, will not be exchanged at maturity (year 4).

Example (continuation):

- Ardiles pays **8% (s.a.)** against **6-mo SOFR**.

Ardiles Co. reduces its cost of borrowing to:

(**6-mo SOFR + 1%**) + **8%** – **6-mo SOFR** = 9% (s.a.) (< **9.2%**)



Now, Ardiles has eliminated floating rate (**6-mo SOFR**) exposures. ¶

Day Count Convention

In the previous examples we have ignored the **day count conventions** on the short term rates.

For example, the floating payment refers to a **money market rate**, the 6-mo SOFR, which is quoted on an ***Actual/360* basis**. Suppose 6-mo SOFR was fixed at **8%,** the notional principal is USD 70M and assume there are **183 days** between payments.

Then, the actual payment should be

USD 70M * (**0.08**) * (**183**/360) = **USD 2.846667M**.

The fixed side must also be adjusted and as a result the **payments may not be equal** on each payment date.

Note: If the fixed rate is based on a different instrument, say a T-bond, then a different day count should be used for the fixed-rate side. In the **T-bond case**, it will be based on ***Actual/Actual***.

Swap Curve

Ardiles will observe the SD's indicative swap pricing schedule. The set of swap rates at different maturities is called the **swap curve**.

It is the equivalent of the **yield curve**.

• As we will see later, the swap curve will be **consistent** with the interest rate curve implied by the **Eurodollar futures contract**, which is used to hedge interest rate swaps that cannot be matched.

• It is easy to construct for **usual maturities** –i.e., **1-mo**, **3-mo**, **6-mo**, etc.– where there is **liquid Eurodollar futures** contracts and/or other similar market instruments (FRAs).

Interpolation techniques (linear, cubic spline, etc.) are used to complete the curve.

Swap Spreads

Swap spread: Interest rate paid by fixed-rate payer – Interest rate on the run treasury (with same maturity).

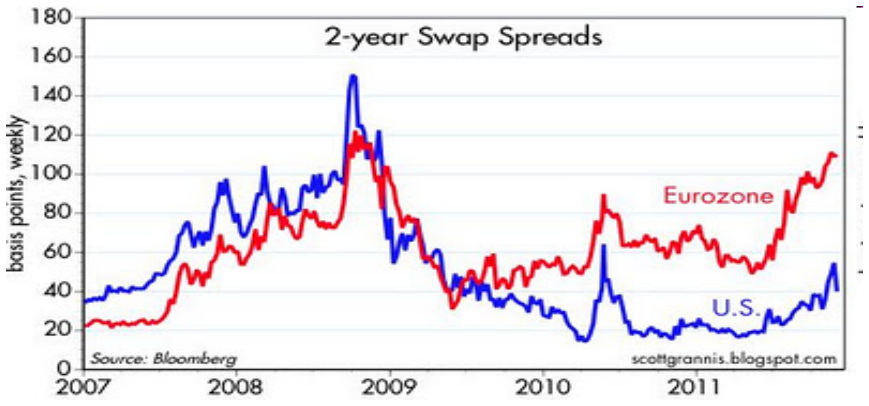
Example: In a 3-year swap, counterparty A pays the SD 7.07%, while the respective (3-year) on the run treasury rate is **6.53%**, then:

3-year swap spread = 7.07% - **6.53%** = **54 bps**. ¶

We expect to observe **positive swap spreads** since a negative spread, in theory, signals that banks or counterparties (say, counterparty A above) are viewed as safer than the government.

But, under unusual circumstances, for example, the 2008-2009 Financial Crisis, a **negative spread may occur**.

• Swap spreads are used as an indicator of a country's credit conditions.



Valuation of an Interest Rate Swap

Assume **no** possibility of **default**. (Credit risk zero!)

• An interest rate swap can be valued:

- As a **portfolio of bonds**: long position in one bond and a short position in another bond
- As a **portfolio of forward contracts**.

• We will value a swap as a **portfolio of bonds**. Define:

V: Value of swap

$\mathbf{B_{Fixed}}$: NPV of fixed-rate bond underlying the swap

$\mathbf{B_{Float}}$: NPV of floating-rate bond underlying the swap

$\Rightarrow$ Value to the fixed-rate payer (Ardiles Co.) = $\mathbf{V} = \mathbf{B_{Float}} - \mathbf{B_{Fixed}}$.

Note: At inception, $\mathbf{V \approx 0}$. The swap has to be *fair*. That is, the fixed coupon is set in a way that the NPV of both sides is approximately equal.

• The discount rates should reflect the level of risk of the cash flows:

An **appropriate discount rate** is given by the **floating-rate** underlying the swap agreement. In previous example, **6-mo. SOFR**.

• Since the discount rate is equal to the floating-rate payment, the value of the floating side payments ($\mathbf{B_{Float}}$) is equal to par value.

$\Rightarrow$ **V** changes when $\mathbf{B_{Fixed}}$ changes -the NPV of fixed-rate payments.

• If coupon (fixed-rate) payment is higher than discount rate, then:

$\mathbf{B_{Fixed}} > \mathbf{B_{Float}}$ $\Rightarrow$ fixed-rate payer has a **negative** swap valuation ($\mathbf{V}<0$)

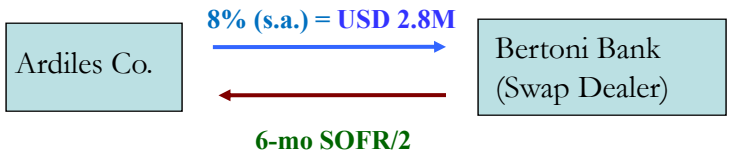
Technical Note: To use this approach to value a swap, we need to add an exchange of principals (in practice, it does not occur).

Example: Back to Ardiles' swap. Suppose the swap has **2 years left**.

Relevant SOFR rates: 6-mo: 6.00%; 12-mo: 6.25%; 18-mo: 6.25%; & 24-mo: 6.50%

Notional amount: **USD 70 M**

Ardiles pays **8%** (s.a.) fixed.



B_{Fixed} = 2.8M /[1 + .06 * (181/360)] + 2.8M /[1 + .0625 * (365/360)] +
+ 2.8M /[1 + .0625 * (546/360)] + 72.8M /[1 + .065 * (730/360)] =
= **USD 72,521,371.94**

B_{Float} = 2.1M/[1 + .06 * (181/360)] + 2.1875M/[1 + .0625 * (365/360)] +
+ 2.1875M/[1+.0625 * (546/360)] + 72.275M/[1+.065 * (730/360)] =
= **USD 69,951,000.36** (B_{Float} ≈ Q. Why?)

Example (continuation): We used an **Actual/360 day count**:

B_{Fixed} = **USD 72,521,371.94**

B_{Float} = **USD 69,951,000.36**

Value of the swap to Ardiles Co. (the fixed-rate payer):

V = **USD 69,951,000.36** - **USD 72,521,371.94** = **USD -2,570,368.38**

Interpretation:

- Ardiles can **pay USD 2,570,368.38** the SD **to close the swap**.
- Alternatively, SD can **sell** the swap –i.e., the CF– for **USD 2,570,368.38**.

Note: Today, a similar swap, with T = **2 years**, would have a fixed coupon = **6.26% (s.a.)**; with a s.a. payment of **USD 2.191M**. Check:

B_{Fixed} = 2.191M/[1 + .06*(181/360)] + 2.191M/[1 + .0625*(365/360)] +
+ 2.191M/[1 + .0625*(546/360)] + 72.191M/[1 + .065*(730/360)]
= **USD 69,972,490** ⇒ At inception, V ≈ 0! ¶

Euromarkets and interest rate swaps

Recall: **Knowledge of derivatives** is very important to select a lead manager of a Eurobond issue.

Arbitrage opportunities in Eurobond markets may exist: **Swaps** can be used to take advantage of them.

Case Study: Merotex (continuation)

Merotex issued 5-year **7(5/16)%** Eurobonds for **USD 200 M**.

Merotex's debt cost = **IRR** = **7.7479%** (p.a.)

But, Merotex wants USD floating rate debt.

Lead manager obtains a swap quotation from a swap dealer:

6-month SOFR v. U.S. Treasuries plus 77/**71** in 5 years.

⇒ Merotex gets U.S. Treasuries plus **71** & pays **6-mo SOFR**.

Case Study: Merotex (continuation)

Merotex receives U.S. Treasuries + **71** and pays 6-mo SOFR.

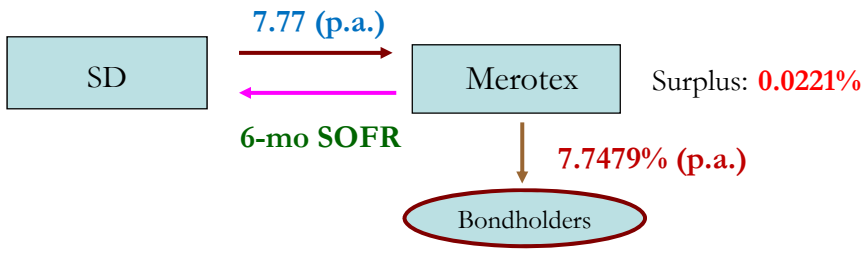
5-year U.S. Treasuries yield: **6.915% (s.a.)**

The fixed-rate coupon received by Merotex under the swap is:

6.915% (s.a.) + **.71% (s.a.)** = 7.625% (s.a.),

⇒ $[(1+.07615/2)^2 - 1] * 100 =$ **7.7700 (p.a.)** annual.

The notional principal of the swap is **USD 200M** (same as in USD Eurobond issue).



Swap fixed-rate coupon receipts > Eurobond's fixed-rate payments:

Receipts: **7.7700%** (p.a.)

Payments: **7.7479%** (p.a.)

Surplus **0.0221%** annual or 2.21 bps per annum (bond basis).

Recall: SOFR payments are calculated on a money market basis.

Merotex converts the surplus (p.a. bond terms) into money market terms:

7.7700% (p.a.) = 7.6250% (s.a.)

7.7479% (p.a.) = 7.6033% (s.a.)

Surplus = **0.0217%** (s.a.) or **2.17** basis points semi annual.

Surplus in s.a. (bond basis) terms is converted to **money-market**:

2.17 x (360/365) = **2.14 bps**.

The cost of floating-rate funding is: **6-mo SOFR** - **2.14 bps**.

Currency Swaps

- Also called *Cross currency swaps* (XCCY).
- The legs of the swap are denominated in **different currencies**.
- Currency swaps change the **profile of cash flows**.
- Many possibilities for the CF exchanges: fixed-fixed, fixed-floating *(Circus swap)* & floating-floating (*XCCY basis swap*).
- Reference rates were **IBOR**, usually USD LIBOR, Euribor (EUR IBOR), JPY TIBOR. They have been **replaced** by **SOFR** & Ameribor (USD), **€STR** or Euro Short-Term Rate (EUR), **TONAR** (JPY), etc.

Example:

Situation: ExxonMobil has USD debt, but wants to increase EUR debt.

Solution: A swap.

ExxonMobil pays EUR. A Swap Dealer pays USD.

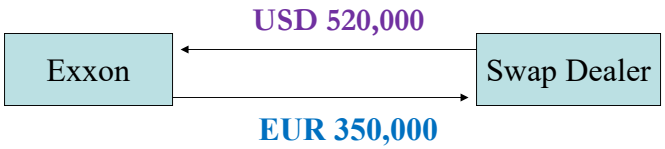
Example (continuation):

ExxonMobil pays EUR. A Swap Dealer pays USD.

• Swap Details:

- ExxonMobil pays **3.5%** in EUR, with a Notional principal: **EUR 20 M**
- Swap Dealer pays **4%** in USD, with a Notional principal **USD 26 M**
- Frequency of payments = 6-mo (**s.a.**)
- Duration = 4 years
- S_t = 1.30 USD/EUR.

Every six month, Exxon pays **EUR 350,000** & receives **USD 520,000**.



Note that Exxon and SD have implicitly fixed S_t for 4 years at:

S_{t+j}= **USD 520,000**/**EUR 350,000** = 1.485714 USD/EUR j=6,12,.,48. ¶

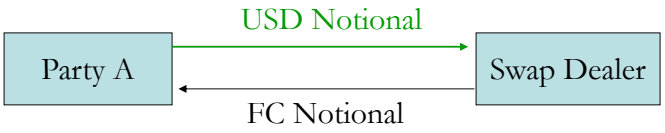
Usual CFs in a XCCY swap

In currency swaps the notional principals are **usually exchanged**. There are three sets of cash flows:

• At inception

• Periodic interest payments

• At end of term



Note: Similar to an exchange of bonds. At inception, Party A receives USD "funding" and pays interest during the duration ("tenor") of swap.

Currency Swaps: Variations

Key: Both legs are different currencies. **Different Instruments**:

1. Fixed-Fixed

Example: Exxon-Mobile example.

2. Fixed-Floating (also called *Circus swap* = Combined Interest Rate & Currency Swap)

Example: IBM pays 3-mo Ameribor in USD and receives 5% in EUR. ¶

3. Floating-Floating (also called *cross currency basis swap,* if initial exchange of notionals occurs)

Example: IBM pays 3-mo Ameribor in USD and receives 3-mo ESTR – **30 bps**. This EUR/USD XCCY swap is quoted "**-30 bps.**"

Note: **-30 bps** is the *spread* in EUR. The *spread* could be zero (IRP holds), positive or negative. ¶

Coss-currency Basis Swaps

The **difference** between the **two floating rates** in a currency swap is called the *basis swap spread*, usually quoted against USD Ameribor (unsecured) flat. For example, in the IBM example above:

EUR/USD *basis swap spread* = (3-mo ESTR – **30 bps**) – (3-mo Ameribor)

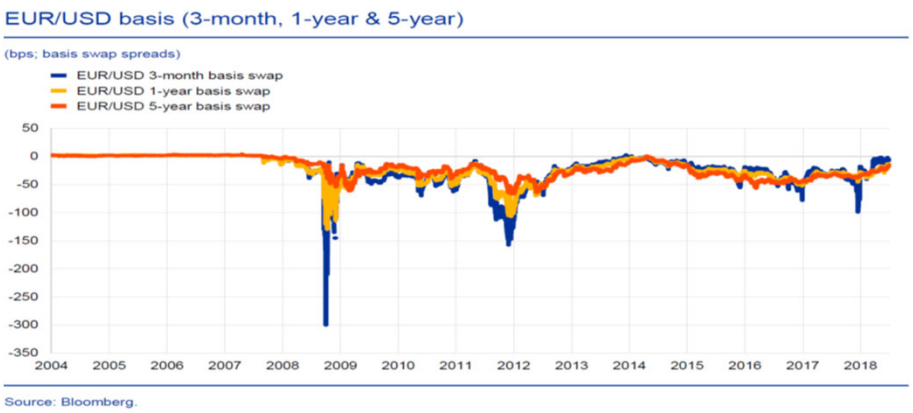
Note: Theoretically, IRP arbitrage should ensure that a XCCY basis swap trades without a spread.

• Who participates in a XCCY swap?

- The EUR/USD XCCY is used by **European banks to fund USD** assets if other USD funding sources become inaccessible. The typical **other side** of this swap are **European** issuers (in particular, agencies, international bodies, and sovereigns), which swap USD debt issues into EUR.

- European firms **issue USD bonds and swap proceeds into EUR** to **diversify** into other funding sources and, potentially, get **cheaper** funding.

• *Euro-USD Basis Swaps Spreads*: **2004-2019**



• After the financial crisis, the EUR-USD *XCCY basis swap spread* became negative. That is, a party seeking to switch from EUR to USD holdings is required to pay a premium for borrowing USD in the XCCY swap market).

The XCCY spread is taken as an indicator of funding conditions. For example, during the 2008-2009 period there was a shortage of USD.

Valuation of Currency Swaps

A currency swap can be decomposed into a position in two bonds:

- A domestic bond (or foreign currency 1 bond)
- A foreign bond (or foreign currency 2 bond)

V = Value of Swap (to DC payer) = NPV of FC bond – NPV of DC bond

In previous example the swap value to ExxonMobil is:

$$\mathbf{V} = B_D - S_t\, B_F$$

B_F: Value of FC denominated bond underlying the swap.
B_D: Value of DC denominated bond underlying the swap.
S_t: Spot exchange rate.

Note: For the Swap Dealer, the swap value (in DC) is:

$$\mathbf{V} = S_t\, B_F - B_D$$

Example FI:

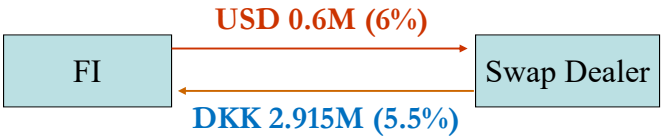
A U.S. financial institution (FI) is involved in a currency swap:

Terms:

- Frequency of payment: Annual.
- Notional principals: **DKK 53 million** & **USD 10 million**.
- FI get **5.5% p.a.** in DKK against **6% p.a.** in USD
- Payments: FI pays **USD 0.6M** & receives **DKK 2.915M**
- Swap will last for another three years (T = 3 years).

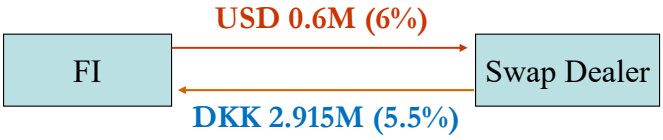
S_t = **0.18868 USD/DKK**.

Term structure in Denmark & U.S. is *flat,* with i_{DKK} = **5%** & i_{USA} = **6.5%**.



Note: At maturity, the principals will also be exchanged.

Example FI (continuation):



These periodic exchanges set an implicit forward contract (the swap forward rate), fixing S_t for each year at:

S_{t+j} = USD 0.6M / DKK 2.915M = **0.2058319 USD/DKK**, j =1, 2, 3.

There are 3 *long* forward contract for FI:
Maturity (T) = 1 year, 2 years, & 3 years.
Size: **DKK 2.915M**
Swap forward FX rate: **0.2058319 USD/DKK**

Note: Potentially, like in all long forward contracts, the FI profits if, at any T, the market forward rate, $\boldsymbol{F_{t,t_j}}$, is higher than **0.2058319 USD/DKK**.

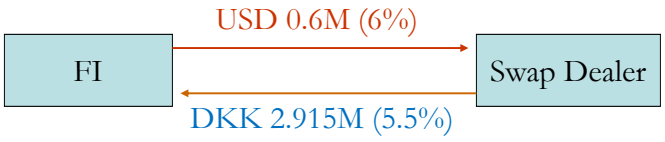
Example FI (continuation):

Discount rates: i_{DKK} = 5% & i_{USA} = 6.5%.

Coupons: **DKK 2.915M** & **USD 0.6M**

T = 3 years.

S_t = **0.18868 USD/DKK.**



$$B_D = \frac{.6M}{(1+.065)} + \frac{.6M}{(1+.065)^2} + \frac{.6M}{(1+.065)^3} + \frac{10M}{(1+.065)^3} = \text{USD } 9{,}867{,}577$$

$$B_F = \frac{2.915M}{(1+.05)} + \frac{2.915M}{(1+.05)^2} + \frac{2.915M}{(1+.05)^3} + \frac{55.915M}{(1+.05)^3} = \text{DKK } 53{,}721{,}661$$

$V_{US\ FI}$ = (53,721,661) * (.18868) – 9,867,577 = **USD 268,585.45**.

V_{SD} (paying DKK and receiving USD) = USD -268,585.45. ¶

Decomposition into Forward Contracts

The CFs of currency swap can be **valued** as a **series of forward contracts**, which are set by the exchanges of interest payments & principals.

Recall the value of a long forward contract is the present value of the amount by which the forward price exceeds the delivery price.

Example FI (continuation):

Annual exchanges: **DKK 2,915,000 = USD 600,000**

At maturity, final exchange: **DKK 53 M = USD 10 M**

⇒ Each of these payments represents an implicit forward contract.

- Swap forward rate fixed by the annual exchanges of interest payments: **USD 0.6M/ DKK 2,915,000 = 0.2058319 USD/DKK**.

- Swap forward rate fixed by the last exchange of principals at T = 3 years: **USD 10M/ DKK 53M = 0.1886792 USD/DKK**. ¶

• We value the swap forward rate relative to the IRPT forward rate, $\boldsymbol{F_{t,T}}$:

$$\boldsymbol{F_{t,T}} = S_t * \frac{\left(1 + i_d * \frac{T}{360}\right)}{\left(1 + i_f * \frac{T}{360}\right)}$$

Suppose in the swap, we are long the FC (the FI is long DKK). Then, the PV, using i_d as the discount rate, of each annual payment j is:

$$(\boldsymbol{F_{t,t_j}} - \text{Swap forward rate at time } t_j) * \frac{\text{Amount of FC}}{(1 + i_{d,j})^{t_j}}$$

Example FI (continuation):

FI's value of the exchange of principals at T = 3 years ($\text{Value}_{\text{FI,Principals}}$).

$\boldsymbol{F_{t,T=3-yr}}$ = **.18868 USD/DKK** $* \frac{(1 + .065)^3}{(1 + .05)^3}$ = **.19688 USD/DKK**

Swap forward rate = **USD 10M**/**DKK 53M** = **0.1886792 USD/DKK**.

$\text{Value}_{\text{FI,Principals}}$ = (**.19688** – **0.1886792**) $* \frac{\textbf{53M}}{(1 + .065)^3}$ = **USD 0.35982M**

Note: We can do the same for each exchange of CFs. ¶

• Alternatively, we can value the CFs in terms of forward DC.

Notation:

t_j: time of the jth settlement date

$i_{d,j}$: domestic interest rate applicable to time t_j

$\boldsymbol{F_{t,t_j}}$: forward exchange rate applicable to time t_j, calculated by IRPT.

• PV to the FI of the swap forward contract set by the corresponding exchange of payments at time t_j:

$$(\textbf{DKK 2,915,000} * \boldsymbol{F_{t,t_j}} - \textbf{USD 0.6M}) * \frac{1}{(1 + i_{d,j})^{t_j}}$$

• PV to the FI of the swap forward contract set by the exchange of principal payments at time T:

$$(\textbf{DKK 53M} * \boldsymbol{F_{t,T}} - \textbf{USD 10M}) * \frac{1}{(1 + i_{d,T})^{T}}.$$

$\Rightarrow$ The value of a currency swap can be calculated from the term structure of forward rates and the term structure of $i_{d,j}$.

Example (continuation): Reconsider FI Example.

S_t = **.18868 USD/DKK.**

i_{DKK} = **5%**

i_{USA} = **6.5%**.

Using IRPT, the one-, two- and three-year forward exchange rates are:

$$\mathbf{F_{t,T=1\text{-}yr}} = \mathbf{.18868\ USD/DKK} * \frac{(1+.065)}{(1+.05)} = \mathbf{.19137\ USD/DKK}$$

$$\mathbf{F_{t,T=2\text{-}yr}} = \mathbf{.18868\ USD/DKK} * \frac{(1+.065)^2}{(1+.05)^2} = \mathbf{.19411\ USD/DKK}$$

$$\mathbf{F_{t,T=3\text{-}yr}} = \mathbf{.18868\ USD/DKK} * \frac{(1+.065)^3}{(1+.05)^3} = \mathbf{.19688\ USD/DKK}$$

Example (continuation): Reconsider FI Example.

• The value of the implicit swap forward contracts corresponding to the exchange of interest are therefore (in millions of USD):

$$(\mathbf{DKK\ 2.915} * \mathbf{.19137\ USD/DKK} - \mathbf{USD\ .6}) * \frac{1}{(1+.065)} = \text{USD } -.03957\text{M}$$

$$(\mathbf{DKK\ 2.915} * \mathbf{.19411\ USD/DKK} - \mathbf{USD\ .6}) * \frac{1}{(1+.065)^2} = \text{USD } -.03013\text{M}$$

$$(\mathbf{DKK\ 2.915} * \mathbf{.19688\ USD/DKK} - \mathbf{USD\ .6}) * \frac{1}{(1+.065)^3} = \text{USD } -.02160\text{M}$$

• The final exchange of principal involves receiving **DKK 53M** & paying **USD 10M**. The value of the forward contract is:

$$(\mathbf{DKK\ 53M} * \mathbf{.19688\ USD/DKK} - \mathbf{USD\ 10M}) * \frac{1}{(1+.065)^3} = \mathbf{USD\ 359{,}816}$$

• Then, the total value of the swap is (in USD):

359,816 – 39,570 – 30,130 – 21,600 = **USD 268,516**.

$\Rightarrow$ FI would be willing to sell this swap for **USD 268,516**. ¶

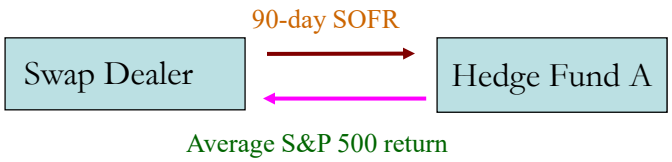
Equity Swaps

Equity swaps have two legs: One of the legs is pegged to the return of a stock or market index. Usually, the other is pegged to a floating rate (SOFR or ESTR plus or minus a spread).

Terms include notional principal, duration and frequency of payments.

Example: Stock returns against a floating rate.

On April 1, Hedge Fund A enters into a 3-year equity swap. Every quarter, Hedge Fund A pays the average S&P 500 return in exchange of 90-day SOFR (count 30/360) .



Example: (continuation)

Notional principal = **USD 40 million**.

Data at inception (April 1):

S&P500 index = 4100

90-day SOFR = **3%**.

On July 1, Hedge Fund A will pay (or receive if sum is negative):

USD 40 M * [**S&P 500 return (04/01 to 07/01)** – **0.03** * 90/360].

If on July 1, S&P 500 = **4153** ⇒ **Return** = 4153/4100 – 1 = **.0130**.

Then the payment will be:

USD 40M * [**.0130** – **0.03** * 90/360] = **USD 0.22M**.

On July 1, SOFR is set for the next 90-day period (07/01 to 10/01). ¶

• **Variations**

- Equity return against a **fixed rate** (S&P500 against 2%)
- Equity return against **another equity return** (S&P500 against NASDAQ)
- Equity return against a **foreign equity return** (S&P500 against FTSE)
- Equity swaps **with changing notional** ("reinvested") principals

• **Q: Why equity swaps?**

(1) Avoid **transaction costs** and **taxes**.
(2) Avoid **legal limits** (margins, capital controls) and **institutional rules**.
(3) Keep equity positions (and voting shares) **without equity risk**.

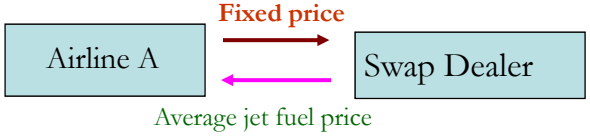
Commodity Swaps

Commodity swaps work like any other swap: one legs involves a fixed commodity price and the other leg a (variable) commodity market price.

Unlike futures commodity contracts, *cash settlement* is the norm.

Example: Jet fuel oil swap.

Airline A enters into a 2-year jet-fuel oil swap. Every quarter, Airline A receives the average market price –based on a known price quote- & pays a fixed price.



Example: (continuation)

Cash settlement: If the average jet-fuel price paid is above (below) the fixed price, the SD will repay (receive from) the airline the difference in what it paid versus the fixed price. ¶

Note: There is no futures contract for jet fuel oil. A swap **completes** the **market**.

You can consider the 2-year swap as a **collection** of 8 **forward contracts**.

- **Q: Why commodity swaps?**

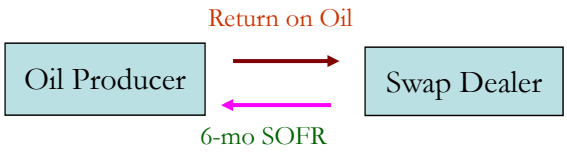
(1) *A commodity swap eliminates basis risk*
Southwest Airlines has used NYMEX crude oil and heating oil futures contracts to hedge jet fuel price risk. But, this introduces basis risk.

(2) *Expanded market*
Since there is cash settlement, market participants do not need to have the infrastructure to take delivery.

- Commodity for interest swap

They work like an equity swap: One leg pays a return on a commodity, the other leg pays an interest rate (say, SOFR plus or minus a spread).

Example: An oil producer enters into a 2-year swap. Every six month, the oil producer pays the return on oil –based on NYMEX Light Crude Oil– and receives 6-mo SOFR.



- Valuation of Commodity Swaps

Commodity swaps are valued as a series of **commodity forwards**, each priced at inception with zero value.
The fixed coupon payment is a weighted average of commodity forward prices.

