SWAPS

SWAPS: Definition and Types

Definition
A swap is a contract between two parties to deliver one sum of money against another sum of money at periodic intervals.

• Obviously, the sums exchanged should be different:
  – Different amounts (say, one fixed & the other variable)
  – Different currencies (say, USD vs EUR)

• The two payments are the legs or sides of the swap.
  - Usually, one leg is fixed and one leg is floating (a market price).

• The swap terms specify the duration and frequency of payments.
Example: Two parties (A & B) enter into a swap agreement. The agreement lasts for 3 years. The payments will be made semi-annually. Every six months, A and B will exchange payments.

- Swaps can be used to change the profile of cash flows.

If a swap is combined with an underlying position, one of the (or both) parties can change the profile of their cash flows (and risk exposure). For example, A can change its cash flows from variable to fixed.

• Types

Popular swaps:
- Interest Rate Swap (one leg floats with market interest rates)
- Currency Swap (one leg in one currency, other leg in another)
- Equity Swap (one leg floats with market equity returns)
- Commodity Swap (one leg floats with market commodity prices)
- CDS (one leg is paid if credit event occurs)

Most common swap: fixed-for-floating interest rate swap.
- Payments are based on hypothetical quantities called notional.
- The fixed rate is called the swap coupon.
- Usually, only the interest differential needs to be exchanged.

• Usually, one of the parties is a Swap Dealer, also called Swap Bank (a large bank).
Example: Interest Rate Swap (inception date: April)
Bank A (fixed-rate payer) buys an 8% swap
Notional: USD 100 M
Swap coupon (Fixed-rate): 8% (s.a.).
Floating-rate: 6-mo. LIBOR. (April's 6-mo. LIBOR: 7.6%)
Payment frequency: semiannual (April and October).
Maturity = Swap term or Swap tenor = 3 years.

Bank A
8% (=USD 4 M)
6-mo LIBOR
Swap Dealer

Every six months Bank A (fixed-rate payer) pays:
USD 100 M x .08/2 = USD 4 M

Every six months Swap Dealer (floating-rate payer) pays:
USD 100 M x 6-mo LIBOR/2

Example: (continuation)
First payment exchange is in October. (The floating rate has already been fixed in April: 7.6%.) Then, the Swap Dealer pays:

⇒ USD 100 M * .76/2 = USD 3.8 M

Bank A (fixed-rate payer) pays USD 0.2 M to the floating-rate payer.

Note: In October, the floating rate will be fixed for the second payment (in April of following year). ¶


Market Organization

- Most swaps are tailor-made contracts.
  - Swaps trade in an OTC type environment.
  - Swap specialists fill the role of broker and/or market maker.
  - Brokers/market makers are usually large banks.
  - Prices are quoted with respect to a standard, or generic, swap.

- *All-in-cost:* Price of the swap (quoted as the rate the fixed-rate side will pay to the floating-rate side)

- It is quoted on a semiannual basis:
  - absolute level ("9% fixed against six-month LIBOR flat")
  - bp spread over the U.S. Treasury yield curve ("the Treasury yield plus 57 bps against 6-mo LIBOR flat").

"LIBOR flat" = LIBOR is quoted without a premium or discount.

- The fixed-rate payer is said to be "long" or to have "bought" the swap.

**Example:** Houseman Bank's *indicative swap pricing schedule*.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>HB Receives Fixed</th>
<th>HB Pays Fixed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>1-yr TN sa + 44 bps</td>
<td>2-yr TN sa + 39 bps</td>
</tr>
<tr>
<td>2 years</td>
<td>2-yr TN sa + 50 bps</td>
<td>2-yr TN sa + 45 bps</td>
</tr>
<tr>
<td>3 years</td>
<td>3-yr TN sa + 54 bps</td>
<td>3-yr TN sa + 48 bps</td>
</tr>
<tr>
<td>4 years</td>
<td>4-yr TN sa + 55 bps</td>
<td>4-yr TN sa + 49 bps</td>
</tr>
<tr>
<td>5 years</td>
<td>5-yr TN sa + 60 bps</td>
<td>5-yr TN sa + 53 bps</td>
</tr>
</tbody>
</table>

- Consider the 3-year swap quote: Housemann Bank attempts to sell a 3-year swap to receive the offered spread of **54 bps** and buy it back to pay the bid spread of **48 bps.** HB’s profit: **6 bps.**
Example: Goyco wants to receive fixed-rate payments rather than pay fixed-rate for 3 years. The current (“on the run”) 3-yr Treasury Note rate is 6.53%. Goyco decides to buy a 3-yr swap from Housemann Bank.

Calculation of fixed rate: HB will pay 7.01% (6.53 + .48) s.a. ¶

- Note: LIBOR will be reset at (T-2) for the next 6-mo period.

* Q: Are swaps riskless?  
A: No! Default risk is always present.

The Dealer’s Perspective
A swap dealer intermediating (making a market in) swaps makes a living out of the spread between the two sides of the swap.

Example: Houseman Bank entered into a 3-year swap with Goyco.  
3-year quote: 3-yr TN sa + 48 bps & 3-yr TN sa + 54 bps  
The on the run 3-yr Treasury Note rate is 6.53%.

Note: The difference between the rate paid by the fixed-rate payer over the rate of the on the run Treasuries with the same maturity as the swap is called the swap spread. In this example, the swap spread is 54 bps.
Warehousing
When the SD matches the two sides (the buyer and the seller) of a swap is called *back-to-back transaction* (or “matched book” transaction).

In practice, a SD may not be able to find an immediate off-setting swap.

Most SD will warehouse the swap and use interest rate derivatives to hedge their risk exposure until they can find an off-setting swap.

In practice, it is not always possible to find a second swap with the same maturity and notional principal as the first swap, implying that the institution making a market in swaps has a residual exposure.

The relatively narrow bid/ask spread in the interest rate swap market implies that to make a profit, effective interest rate risk management is essential.

Dealer’s Risk

- *Credit Risk*
  
  This is the major concern for a swap dealer: the risk that a counterparty will default on its end of the swap.

- *Mismatch Risk*
  
  It is difficult to find a counterparty that wants to borrow the exact amount of money for the exact amount of time.

- *Sovereign Risk*
  
  The risk that a country will impose exchange rate restrictions that will interfere with performance on the swap.
• **Market Size**

Notional amount outstanding (Dec 2016):
- Interest rate swaps: USD 306.1 trillion
- Currency swaps: USD 22.2 trillion (≈ 6%)
- Equity-linked contracts (includes forwards): USD 2.8 trillion
- Commodity contracts (includes forwards): USD 1.4 trillion
- CDS market: USD 9.6 trillion (≈ 3%)

- Gross market value: **USD 16.35 trillion**

Interest rate swaps is a very popular derivative: It represents 60% of the Global OTC Derivatives Market.

Interest rate swaps also shows big growth from early 1990s.

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**Growth of OTC Derivatives (Swap Market) - 1998-2017**

Source: BIS (2017)
Global OTC Derivatives: Amounts Outstanding (BIS, Dec 2017)

Growth of Swap Market: Interest Derivatives (BIS 2017)
Swaps: Why Use Them?

- **Using Swaps**
  - Manage risks (change profile of cash flows)
  - Arbitrage (take advantage of price differentials)
  - Enter new markets (firms can indirectly create new exposures)
  - Create new instruments (no forward contract exist, a swap completes the market)

1. **Change profile of cash flow**
Goyco’s underlying situation: Fixed payments to bondholders, but wants floating debt.

Solution: A fixed-for-floating debt solves Goyco’s problem.
Example: Goyco enters into a swap agreement with a Swap Dealer.  

Terms:  
Duration: 3-years.  
Goyco makes floating payments (indexed by LIBOR) and receives fixed payments from the Swap Dealer.  

Q: Why would Goyco enter into this swap?  
A: To change the profile of its cash flow: From fixed to floating.

- Swaps are derivative instruments (derived valued from value of legs!).

(2) Arbitrage: Comparative Advantage

A has a **comparative advantage** in borrowing in USD.  
B has a **comparative advantage** in borrowing in GBP.  
If they borrow according to their comparative advantage and then swap, there will be gains for both parties.

Example: Nakatomi, a Japanese company, wants USD debt.  
There are at least two ways to get USD debt:  
- Issue USD debt  
- Issue JPY debt and swap it for USD debt.

Q: Why might a Japanese company take the second route?  
A: It may be a cheaper way of getting USD debt -**comparative advantage**.
Note: From an economic point of view, there are two motives for entering into swaps:
- *Risk Sharing* (two firms share interest risk through a swap)
- *Comparative Advantage/Arbitrage*

**Measuring comparative advantage**
AAA companies routinely have absolute advantage in debt markets over all the other companies, due to their different credit-worthiness. A swap, however, takes advantage of comparative (or relative) advantages.

The *Quality Spread Differential* (QSD) measures comparative advantage
- QSD = Difference between the interest rates of debt obligations offered by two parties of different creditworthiness that engage in the swap.
- The QSD is the key to a swap. It is what can be shared between the parties.

**Example**: Comparative Advantage.
Nakatomi, a Japanese company, wants to finance a US project in USD. HAL, a US company, wants to finance a Japanese project in JPY.
Both companies face the following borrowing terms.

<table>
<thead>
<tr>
<th></th>
<th>USD rate</th>
<th>JPY rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>HAL</td>
<td>9%</td>
<td>4%</td>
</tr>
<tr>
<td>Nakatomi</td>
<td>8%</td>
<td>2%</td>
</tr>
</tbody>
</table>

Nakatomi is the more credit-worthy: It has an absolute advantage.
- HAL pays 1% more to borrow in USD than Nakatomi.
- HAL pays 2% more to borrow in JPY than Nakatomi.
  ⇒ HAL has a *comparative advantage* in the USD rate market.
  The QSD is 1% (different than zero!).

If they borrow according to their comparative advantage and then swap, there will be gains for both parties.
A Swap Dealer proposes the following swap:
- Nakatomi pays 7.6% in USD and receives 2% in JPY
- HAL pays 3.6% in JPY and receives 9% in USD
Swap + Domestic borrowing produces the following cost of borrowing:
Nakatomi: 2% in JPY - 2% in JPY + 7.6% in USD = 7.6% in USD < 8%
HAL: 9% in USD - 9% in USD + 3.6% in JPY = 3.6% in JPY < 4%

Note: The Quality Spread Differential (QSD) is not usually divided equally. In general, the company with the worse credit gets the smaller share of the QSD to compensate the SD for a higher credit risk.

Why the Growth of Swap Markets?
• Swap contracts have many similarities with futures contracts.
  - Trade-off: Customization vs. liquidity.
  Futures markets offer a high degree of liquidity, but contracts are more standardized. Swaps offer additional flexibility since they are tailor-made.

  - Settlement is in cash.
  There is no need to take physical delivery to participate in the market.
**Interest Rate Swaps**

- Most common swap: fixed-for-floating *(plain vanilla swap)*
  - Used to change profile of cash flows (a firm can go from paying floating debt to paying fixed debt).
  - Used to lower debt costs.

- Basis swap: floating-for-floating *(basis swaps)*
  - Floating rates should be different, say 1-mo Euribor vs. 3-mo Euribor or USD LIBOR vs. US T-bill.
  - Floating-for-floating currency swaps (also called *cross currency basis swaps*) are especial cases of interest rate basis swaps.

- Interest rates swaps have very low bid-ask spreads, lower than corporate bonds and, sometimes, government bonds.

**Example**: Plain Vanilla Swap

Underlying situation for Ardiles Co:
- USD 70 M floating debt at: 6-mo LIBOR + 1%.
- Ardiles Co. wants to change to fixed-rate USD debt.
- Currently, fixed-rate debt trades at 9.2% (s.a.).

A Swap Dealer (Bertoni Bank) offers 8% (s.a) against 6-mo LIBOR.

Terms:
- Notional amount: USD 70 M
- Frequency: semi-annual
- Swap term or tenor (Duration): 4-years

Ardiles Co. and SD only exchange the net payment (difference between the two legs of the swap: Ardiles pays SD if 6-mo LIBOR < 8%). Notionals, obviously, will not be exchanged at maturity (year 4).
Example (continuation):
- Ardiles pays 8% (s.a.) against 6-mo LIBOR.

Ardiles Co. reduces its cost of borrowing to:

\[(6\text{-mo LIBOR} + 1\%) + 8\% - 6\text{-mo LIBOR} = 9\% \text{ (s.a.)} \quad (< 9.2\%)
\]

Now, Ardiles has eliminated floating rate (6-mo LIBOR) exposures.

Day Count Convention
In the previous examples we have ignored the day count conventions on the short term rates.

For example, the floating payment refers to a money market rate, the 6-mo LIBOR, which is quoted on an Actual/360 basis. Suppose 6-mo LIBOR was fixed at 8%, the notional principal is USD 70M and assume there are 183 days between payments. Then, the actual payment should be

\[
\text{USD } 70\text{M} \times (0.08) \times (183/360) = \text{USD } 2.846667\text{M}.
\]

The fixed side must also be adjusted and as a result the payments may not be equal on each payment date.

Note: If the fixed rate is based on a different instrument, say a T-bond, then a different day count should be used for the fixed-rate side. In the T-bond case, it will be based on Actual/Actual.
**Swap Curve**

Ardiles will observe the SD’s indicative swap pricing schedule. The set of swap rates at different maturities is called the *swap curve*.

It is the equivalent of the yield curve.

- As we will see later, the swap curve will be consistent with the interest rate curve implied by the Eurodollar futures contract, which is used to hedge interest rate swaps that cannot be matched.

- It is easy to construct for the usual maturities –i.e., 1-mo, 3-mo, 6-mo, etc.– where there is liquid Eurodollar futures contracts and/or other similar market instruments (FRAs).

Interpolation techniques (linear, cubic spline, etc.) are used to complete the curve.

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**Swap Spreads**

*Swap spread*: Interest rate paid by fixed-rate payer – Interest rate on the run treasury (with same maturity).

**Example**: A pays the SD 7.07%, while the respective (3-year) on the run treasury rate is 6.53%, then the 3-year swap spread is 54 bps.

We expect to observe positive swap spreads since a negative spread, in theory, signals that banks are viewed as safer than the government.

During the 2008-2009 Financial Crisis, swap rates declined to the level of on the run Treasuries (the swap spread was negative for the 30-yr maturity). This negative swap spread was considered an aberration (average swap spread 1997-2007 was 60 bps).

But, in September 2015, we observed a similar movement.
But, in September 2015, a similar situation occurred. What is going on?

- Regulations: Less market-making by banks, providing less liquidity.
- China’s surprise devaluation: PBOC sold U.S. treasuries to stabilize the CNY. Then, dealer’s holdings of US government debt increase, driving repo-rates, which is used to price Treasuries.
- Heavy issuing of debt: Companies are in a hurry to lock low rates (usually, they swap the debt from fixed to floating, which causes the swap spreads to tighten).

• Swap spreads are used as an indicator of a country’s credit conditions.
Valuation of an Interest Rate Swap
Assume no possibility of default. (Credit risk zero!)

- An interest rate swap can be valued:
  - As a long position in one bond and a short position in another bond
  - As a portfolio of forward contracts.

Define:
V: Value of swap
B_{Fixed}: NPV of fixed-rate bond underlying the swap
B_{Float}: NPV of floating-rate bond underlying the swap

\[ \Rightarrow \text{Value to the fixed-rate payer (Ardiles Co.)} = V = B_{\text{Float}} - B_{\text{Fixed}}. \]

Note: At inception, V ≈ 0. The swap has to be fair. That is, the fixed coupon is set in a way that the NPV of both sides is approximately equal.

- The discount rates should reflect the level of risk of the cash flows:
  An appropriate discount rate is given by the floating-rate underlying the swap agreement. In previous Example, 6-mo. LIBOR.

- Since the discount rate is equal to the floating-rate payment, the value of the floating side payments (B_{Float}) is equal to par value.

- That is, the value of the swap will change according to changes in the value of the coupon (fixed-rate) payments (B_{Fixed}).

- If the coupon payment is higher than the discount rate \[ B_{\text{Fixed}} > B_{\text{Float}} \]
  Then, a fixed-rate payer will have a negative swap valuation.
Example: Back to Ardiles’ swap. Suppose the swap has 2 years left.
Relevant LIBOR rates: 6-mo: 6.00%; 12-mo: 6.25%; 18-mo: 6.25%;
& 24-mo: 6.50%
Notional amount: USD 70 M
Ardiles pays 8% (s.a.) fixed.

Using an actual/360 day count:

\[
B_{\text{Fixed}} = 2.8\text{M}/[1 + .06 \times (181/360)] + 2.8\text{M}/[1 + .0625 \times (365/360)] + \\
+ 2.8\text{M}/[1 + .0625 \times (546/360)] + 72.8\text{M}/[1 + .065 \times (730/360)] = \\
= \textbf{USD 72,521,371.94}
\]

\[
B_{\text{Float}} = 2.1\text{M}/[1 + .06 \times (181/360)] + 2.1875\text{M}/[1 + .0625 \times (365/360)] + \\
+ 2.1875\text{M}/[1 + .0625 \times (546/360)] + 72.275\text{M}/[1 + .065 \times (730/360)] = \\
= \textbf{USD 69,951,000.36} \quad (B_{\text{Float}} \approx Q. \text{ Why?})
\]

The value of the swap to Ardiles Co. (the fixed-rate payer) is

\[
V = \textbf{USD 69,951,000.36} - \textbf{USD 72,521,371.94} = \textbf{USD -2,570,368.38}
\]

Example (continuation):

The value of the swap to Ardiles Co. (the fixed-rate payer) is

\[
V = \textbf{USD 69,951,000.36} - \textbf{USD 72,521,371.94} = \textbf{USD -2,570,368.38}
\]

Ardiles has to pay \textbf{USD 2,570,368.38} to Bertoni Bank -the Swap Dealer- to close the swap.

Alternatively, Bertoni Bank can sell the swap –i.e., the cash flows for \textbf{USD 2,570,368.38}.

Note: Today, a similar swap, with T = 2-year, would have a fixed coupon = 6.26% (s.a.); with a s.a. payment of \textbf{USD 2.191M}. Check:

\[
B_{\text{Fixed}} = 2.191\text{M}/[1 + .06*(181/360)] + 2.191\text{M}/[1 + .0625*(365/360)] + \\
+ 2.191\text{M}/[1 + .0625*(546/360)] + 72.191\text{M}/[1 + .065*(730/360)] \\
= \textbf{USD 69,972,490} \quad \Rightarrow \text{At inception, } V \approx 0!\]
• Many banks and investors own Eurobonds with a higher YTM than the current swap rate. An interest rate swap can offer profit opportunities.

**Example:** A Pension Fund owns a 4% USD 50M 3-year Eurobond, with \( P_{Dec\ 10,\ 2008} = 90.776508 \) (YTM of 7.55% p.a.)

• An investment bank proposal:
  1. Buy the bonds at par (\( P = 100 \)) and to pay the 4% coupon annually.  
     \( \Rightarrow \) Pension Fund receives **USD 50 M** + USD 2 M annually.
  2. Pension Fund pays 12-mo LIBOR + 25 bps.
  3. 3-yr swap against 12-mo LIBOR: coupon = 6.423% s.a. + 54 bps  
     \( \Rightarrow \) swap coupon: **7.0226% p.a.** Payment: USD 3,511,300 annually.  
     \( \Rightarrow \) Pension Fund pays net to bank 25 bps: USD 125,000 annually.

• The Pension Fund (II) accepts the proposal if the NPV of all CFs > 0.

**Diagram: Structured Offer for Pension Fund**

Original situation: Pension Fund owns 4% p.a. (coupon=USD 2 M).
Example:
(1) Annual CFs (Bond coupon payment + swap CF) for the II:

<table>
<thead>
<tr>
<th>Date</th>
<th>Pay to SD</th>
<th>Pay 25 bps to Bank</th>
<th>Receive 4% Coupon</th>
<th>Net</th>
</tr>
</thead>
<tbody>
<tr>
<td>12/10/09</td>
<td>-3,511,300</td>
<td>-125,000</td>
<td>2,000,000</td>
<td>-1,636,300</td>
</tr>
<tr>
<td>12/10/10</td>
<td>-3,511,300</td>
<td>-125,000</td>
<td>2,000,000</td>
<td>-1,636,300</td>
</tr>
<tr>
<td>12/10/11</td>
<td>-3,511,300</td>
<td>-125,000</td>
<td>2,000,000</td>
<td>-1,636,300</td>
</tr>
</tbody>
</table>

We need to discount rate for these CFs to calculate the NPV:
Discount rate = YTM of the 4% Eurobond = 7.55%
⇒ NPV of swap losses is USD 4,251,640.

(2) Bond Principal CF at time of proposal (12/10/08):
Par value - Market valuation = USD 50M - .90776508 * USD 50 M =
= USD 50M - USD 45,388,254.
= USD 4,611,746.

Example (continuation):
(1) Bond Coupon payment + Swap CF
⇒ NPV of swap losses is USD 4,251,640.

(2) Bond Principal payment
Bank buys the bonds at 100% = USD 50 M
Market value of Eurobonds: .90776508 * USD 50 M = USD 45,388,254.
⇒ Cash gain = USD 4,611,746.

(3) Difference: USD 4,611,746 - USD 4,251,640 = USD 360,106. (Yes!)
Note: This difference at a discount rate of 7.55% p.a. is equivalent to an annual payment of USD 138,600, or roughly 27.72 bps.

What has happened in this situation?

Market conditions have generated a profit margin:

\[
\begin{align*}
\text{YTM}_\text{BOND} &= 7.55 \text{ pa} \\
\text{Swap Rate} &= 7.0226 \text{ pa} \\
\text{Difference} &= 0.5274 \text{ pa}
\end{align*}
\]

The margin of 52.74 is split: 25 bps to the bank 27.74 bps to the Pension Fund.

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Euromarkets and interest rate swaps

Recall: Knowledge of derivatives is very important to select a lead manager of a Eurobond issue.

Arbitrage opportunities in Eurobond markets may exist → swaps.

Case Study: Merotex (continuation)

Merotex issued 5-year 7(5/16)% Eurobonds for USD 200 M.

Merotex’s debt cost = IRR = 7.7479% (p.a.)

But, Merotex wants USD floating rate debt.

Lead manager obtains a swap quotation from a swap dealer:

6-month LIBOR v. U.S. Treasuries plus 77/71 in 5 years.

⇒ Merotex gets U.S. Treasuries plus 71 & pays 6-mo LIBOR.
Case Study: Merotex (continuation)
Merotex receives U.S. Treasuries + 71 and pays 6-mo LIBOR. 5-year U.S. Treasuries yield: $6.915\%$ (s.a.)

The fixed-rate coupon received by Merotex under the swap is:

$6.915\%$ (s.a.) + $0.71\%$ (s.a.) = $7.625\%$ (s.a.),

$\Rightarrow [(1+.07615/2)^2 - 1] \times 100 = 7.7700$ (p.a.) annual.

The notional principal of the swap is USD 200 million (same amount as in the Euro-USD bond issue).

Without taking into account expenses, the issuer's cash flows are:

<table>
<thead>
<tr>
<th>Year</th>
<th>Bond</th>
<th>Swap</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>196,500,000</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>-14,625,000</td>
<td>15,540,000</td>
</tr>
<tr>
<td>2</td>
<td>-14,625,000</td>
<td>15,540,000</td>
</tr>
<tr>
<td>3</td>
<td>-14,625,000</td>
<td>15,540,000</td>
</tr>
<tr>
<td>4</td>
<td>-14,625,000</td>
<td>15,540,000</td>
</tr>
<tr>
<td>5</td>
<td>-214,625,000</td>
<td>15,540,000</td>
</tr>
</tbody>
</table>

IRR (bond issue alone): $7.7479\%$ (p.a.)

Swap fixed-rate coupon receipts > Eurobond's fixed-rate payments:

Receipts: $7.7700\%$ (p.a.)

Payments: $7.7479\%$ (p.a.)

Surplus $0.0221\%$ annual or 2.21 bps per annum (bond basis).
Recall: LIBOR payments are calculated on a money market basis.

Merotex converts the surplus (p.a. bond terms) into money market terms:

\[
\begin{align*}
7.7700\% \text{ (p.a.)} & = 7.6250\% \text{ (s.a.)} \\
7.7479\% \text{ (p.a.)} & = 7.6033\% \text{ (s.a.)} \\
\text{Surplus} & = 0.0217\% \text{ (s.a.) or 2.17 basis points semi annual.}
\end{align*}
\]

Surplus in s.a. terms is converted to money-market:

\[
2.17 \times \frac{360}{365} = 2.14 \text{ bps.}
\]

The cost of floating-rate funding is: 6-mo LIBOR - 2.14 bps.

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**Variations on Vanilla Interest Rate Swaps**

- Notional principals can be different on both sides or changing over time (decreasing = amortizing swap; increasing = accreting swap).
- Frequency of payment can be different on both sides
- Interest rate swaps can be floating-for-floating (6-mo LIBOR against 6-mo Commercial Paper). These are called *basis swaps*.

**Example**: Bank Two has a commercial debt portfolio which receives floating rate payments based on the prime rate. The bank usually funds this portfolio with short-term deposits, where the deposit rate is based on 3-mo LIBOR.

Suppose that Bank Two is concerned that the spread between LIBOR and prime may shrink, a basis swap can be used to hedge this risk.
Currency Swaps

- Also called *Cross currency swaps* (XCCY).
- The legs of the swap are denominated in different currencies.
- Currency swaps change the profile of cash flows.
- Many possibilities for the CF exchanges: fixed-fixed, fixed-floating (*Circus swap*) & floating-floating (*XCCY basis swap*).
- Reference rates are IBOR, usually USD LIBOR, Euribor (EUR IBOR), JPY LIBOR/TIBOR.

**Example:**
Situation: ExxonMobil has USD debt, but wants to increase EUR debt.
Solution: A swap.

ExxonMobil pays EUR. A Swap Dealer pays USD.

**Example (continuation):**
ExxonMobil pays EUR. A Swap Dealer pays USD.
- Swap Details:
  - ExxonMobil pays 3.5% in EUR, with a Notional principal: EUR 2 M
  - Swap Dealer pays 4.75% in USD, with a Notional principal USD 2 M
  - Frequency of payments = 6-mo.
  - Duration = 4 years
  - $S_t = 1.31$ USD/EUR.

Every six month, Exxon pays **EUR 35,000** & receives **USD 47,500**.

Note that Exxon and SD have implicitly fixed $S_t$ for 4 years at:

$$S_{t,j} = \frac{\text{USD 47,500}}{\text{EUR 35,000}} = 1.357143 \text{ USD/EUR} \quad j=6,12,..,48.$$
Usual CFs in a XCCY swap
Since in currency swaps the notional principals are usually exchanged. There are three sets of cash flows:

- At inception
  - Party A
  - Swap Dealer
  - USD Notional
  - FC Notional

- Periodic interest payments
  - Party A
  - Swap Dealer
  - USD interest
  - FC interest

- At end of term
  - Party A
  - Swap Dealer
  - USD Notional
  - FC Notional

Note: Similar to an exchange of bonds.

Variations
The key is that both legs are denominated in different currencies. There are different possibilities here:

1. Fixed-Fixed
   Example: Exxon-Mobile example.

2. Fixed-Floating (also called Circus swap)
   Example: IBM pays LIBOR in USD and receives 5% in EUR.

3. Floating-Floating (also called cross currency basis swap, if initial exchange of notionals occurs)
   Example: IBM pays LIBOR in USD and receives LIBOR in EUR.

The difference between the two floating rates in a currency swap is called the basis swap spread, usually quoted against USD LIBOR flat.
**Euro-USD Basis Swaps Spreads: 2008-2015**

The XCCY basis swap spread has not been positive since September 2007. That is, institutions have been willing to receive fewer interest rate payments on funds lent in non-USD currencies (in exchange for USD) since Sep 2007. The XCCY spread is taken as an indicator of funding conditions.

---

**Valuation of Currency Swaps**

A currency swap can be decomposed into a position in two bonds.

\[ V = \text{Value of a Swap} = \text{NPV of FC bond} - \text{NPV of DC bond} \]

In previous example the swap value to ExxonMobil is:

\[ V = B_D - S_t B_F \]

- \(B_F\): value of the foreign denominated bond underlying the swap.
- \(B_D\): the value of the domestic currency bond underlying the swap.
- \(S_t\): spot exchange rate.
Example FI:
Term structure in Denmark & U.S. is flat, with \( i_{\text{DKK}} = 5\% \) & \( i_{\text{USD}} = 6.5\% \). A U.S. financial institution (FI) is involved in a currency swap:

**Terms:**
- Frequency of payment: Annual.
- Notional principals: DKK 53 million and USD 10 million.
- FI get 5.5% p.a. in DKK (\( \text{DKK} \ 2.915\text{M} \)) against 6% p.a. in USD (\( \text{USD} \ 0.6\text{M} \))

Swap will last for another three years (\( T = 3 \) years).
\( S_t = 0.18868 \text{ USD/DKK} \).

Note: These exchanges set a forward contract, fixing \( S_t \) for 3 years at:
\( S_{t+j} = \text{USD} \ 0.6\text{M} / \text{DKK} \ 2.915\text{M} = 0.2058319 \text{ USD/DKK} \), \( j = 1, 2, 3 \).

Example FI (continuation):
Discount rates: \( i_{\text{DKK}} = 5\% \) & \( i_{\text{USD}} = 6.5\% \).
Coupons: \( \text{DKK} \ 2.915\text{M} \) & \( \text{USD} \ 0.6\text{M} \)
\( T = 3 \) years.
\( S_t = 0.18868 \text{ USD/DKK} \).

\[
B_D = \frac{.6\text{M}}{1+0.065} + \frac{.6\text{M}}{(1+0.065)^2} + \frac{10.6\text{M}}{(1+0.065)^3} = \text{USD} \ 9,867,577
\]
\[
B_F = \text{DKK} \ 2.915\text{M}/(1+0.05) + \text{DKK} \ 2.915\text{M}/(1+0.05)^2 + \frac{55.915\text{M}}{(1+0.05)^3} = \text{DKK} \ 53,721,661
\]
\[
V_{\text{US FI}} = (53,721,661)*(.18868) - 9,867,577 = \text{USD} \ 268,585.45
\]
\[
V_{\text{SD}} (\text{paying DKK and receiving USD}) = \text{USD} -268,585.45
\]
Decomposition into Forward Contracts

The CFs of currency swap can be valued as a series of forward contracts, which are set by the exchanges of interest payments & principals.

Recall the value of a long forward contract is the present value of the amount by which the forward price exceeds the delivery price.

**Example FI (continuation):**

Annual exchanges: \( \text{DKK 2,915,000} = \text{USD 600,000} \)

At maturity, final exchange: \( \text{DKK 53 M} = \text{USD 10 M} \)

\( \Rightarrow \) Each of these payments represents an implicit forward contract.

- Swap forward rate fixed by the annual exchanges of interest payments: \( \text{USD 0.6M/ DKK 2,915,000} = 0.2058319 \text{USD/DKK} \).
- Swap forward rate fixed by the last exchange of principals at T=3 yrs: \( \text{USD 10M/ DKK 53M} = 0.1886792 \text{USD/DKK} \).

• We can value the implicit swap forward rate relative to the forward rate determined by IRPT, \( F_{t,T} \). Using our usual notation, \( F_{t,T} \) is given by:

\[
F_{t,T} = S_t \frac{(1 + i_d \times T/360)}{(1 + i_f \times T/360)}.
\]

Suppose in the swap, we are long the FC (for the FI, DKK). Then, the PV, using \( i_d \) as the discount rate, of each annual payment \( j \) is:

\[
(F_{t,j} - \text{Swap forward rate at time } j) \times \text{Amount of FC}/(1+i_d)^j
\]

**Example FI (continuation):**

FI’s value of the exchange of principals at T=3 years (Value\(_{FI,\text{Principals}}\)).

\( F_{t,T=3-yr} = .18868 \text{ USD/DKK} \times (1+.065)^3/(1+.05)^3 = .19688 \text{ USD/DKK} \)

Swap forward rate = USD 10M/DKK 53M = 0.1886792 USD/DKK.

\( \text{Value}_{FI,\text{Principals}} = (0.19688 - 0.1886792) \times 53M/(1+.065)^3 = \text{USD 0.35982M} \)

Note: We can do the same for each exchange of CFs.
• Alternatively, we can value the CFs in terms of forward DC.

Notation:
t\_j: time of the jth settlement date
i\_d\_j: domestic interest rate applicable to time t\_j
F\_t\_j: forward exchange rate applicable to time t\_j, calculated by IRPT.

• PV to the FI of the swap forward contract set by the corresponding exchange of payments at time t\_j:
  \[(\text{DKK 2,915,000} \times F\_t\_j - \text{USD 0.6M})/(1+i\_d\_j)^{t\_j}\]

• PV to the FI of the swap forward contract set by the exchange of principal payments at time T:
  \[(\text{DKK 53M} \times F\_t\_T - \text{USD 10M})/(1+i\_d\_T)^{T}\].

⇒ The value of a currency swap can be calculated from the term structure of forward rates and the term structure of i\_d.

**Example (continuation):** Reconsider FI Example.

\[S_t = \text{.18868 USD/DKK}.\]
i\_USD = 6.5% per year.
i\_DKK = 5% per year.

Using IRPT, the one-, two- and three-year forward exchange rates are:

\[.18868 \text{ USD/DKK} \times (1+.065)/(1+.05) = .19137 \text{ USD/DKK}\]
\[.18868 \text{ USD/DKK} \times (1+.065)^2/(1+.05)^2 = .19411 \text{ USD/DKK}\]
\[.18868 \text{ USD/DKK} \times (1+.065)^3/(1+.05)^3 = .19688 \text{ USD/DKK}\]
Example (continuation): Reconsider FI Example.

- The value of the implicit swap forward contracts corresponding to the exchange of interest are therefore (in millions of USD):
  
  \[
  \frac{DKK \times 2.915 \times 1.19137 USD/DKK - USD \times 0.6}{1+0.065} = USD \times 0.03957M
  \]
  
  \[
  \frac{DKK \times 2.915 \times 1.19411 USD/DKK - USD \times 0.6}{(1+0.065)^2} = USD \times 0.03013M
  \]
  
  \[
  \frac{DKK \times 2.915 \times 1.19688 USD/DKK - USD \times 0.6}{(1+0.065)^3} = USD \times 0.02160M
  \]

- The final exchange of principal involves receiving DKK 53 million and paying USD 10 million. The value of the forward contract is:
  
  \[
  \frac{DKK \times 53M \times 1.19688 USD/DKK - USD \times 10M}{(1+0.065)^3} = USD 359,816
  \]

- The total value of the swap is (in USD):
  
  \[
  359,816 - 39,570 - 30,130 - 21,600 = USD 268,516.
  \]

⇒ FI would be willing to sell this swap for USD 268,516.

---

**Equity Swaps**

Equity swaps have two legs: an interest rate leg (usually LIBOR) and an equity leg, pegged to the return of a stock or market index.

Terms include notional principal, duration and frequency of payments.

**Example:** Equity swap: Stock returns against a floating rate.

On April 1, Hedge Fund A enters into a 3-year equity swap. Every quarter, Hedge Fund A pays the average S&P 500 return in exchange of 90-day LIBOR (count 30/360).
Example: (continuation)
Notional principal = USD 40 million.

On April 1 (at inception), the S&P500 index is 1150 and 90-day LIBOR is 1.50%. On July 1, Hedge Fund A will pay (or receive if sum is negative):

$$\text{USD 40 M} \times [\text{S&P 500 return (04/01 to 07/01)} - 0.0150 \times 90/360].$$

If on July 1, the S&P 500 is 1165 (return = 1165/1150 – 1 = 0.0130).

Then the payment will be:

$$\text{USD 40M} \times [0.0130 - 0.0150 \times 90/360] = \text{USD 0.37M}.$$ 

On July 1, LIBOR is set for the next 90-day period (07/01 to 10/01).

• Variations
- Equity return against a fixed rate (S&P500 against 2%)
- Equity return against another equity return (S&P500 against NASDAQ)
- Equity return against a foreign equity return (S&P500 against FTSE)
- Equity swaps with changing notional (“reinvested”) principals

• Q: Why equity swaps?
(1) Avoid transaction costs and taxes.
(2) Avoid legal limits (margins, capital controls) and institutional rules.
(3) Keep equity positions (and voting shares) without equity risk.
Commodity Swaps

Commodity swaps work like any other swap: one leg involves a fixed commodity price and the other leg a (variable) commodity market price.

Unlike futures commodity contracts, cash settlement is the norm.

**Example:** Jet fuel oil swap.
Airline A enters into a 2-year jet-fuel oil swap. Every quarter, Airline A receives the average market price –based on a known price quote- and pays a fixed price.

**Example:** (continuation)
Settlement is in cash: If the average jet-fuel price paid is above (below) the fixed price, the SD will repay (receive from) the airline the difference in what it paid versus the fixed price. ¶

**Note:** There is no futures contract for jet fuel oil. A swap completes the market.

You can consider the 2-year swap as a collection of 8 forward contracts.

**Q: Why commodity swaps?**
(1) A commodity swap eliminates basis risk
Southwest Airlines has used NYMEX crude oil and heating oil futures contracts to hedge jet fuel price risk. But, this introduces basis risk.

(2) Expanded market
Since there is cash settlement, market participants do not need to have the infrastructure to take delivery.
• **Commodity for interest swap**
  They work like an equity swap: One leg pays a return on a commodity, the other leg pays an interest rate (say, LIBOR plus or minus a spread).

**Example:** An oil producer enters into a 2-year swap. Every six months, the oil producer pays the return on oil—based on NYMEX Light Crude Oil—and receives 6-mo LIBOR.

<table>
<thead>
<tr>
<th>Return on Oil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil Producer</td>
</tr>
<tr>
<td>Swap Dealer</td>
</tr>
<tr>
<td>6-mo LIBOR</td>
</tr>
</tbody>
</table>

• **Valuation of Commodity Swaps**
  Commodity swap are valued as a series of commodity forwards, each priced at inception with zero value. The fixed coupon payment is a weighted average of commodity forward prices.

---

**Credit Default Swaps (CDS)**

• A CDS is an agreement between two parties. One party buys *protection against specific risks associated with credit events*—i.e., defaults, bankruptcy, restructuring, or credit rating downgrades. Cash settlement is allowed.

• Facts:
  - Today, CDS is the most widely traded credit derivative product.
  - Outstanding amount: USD 9.6 trillion (December 2017).
  - Maturities range from 1 to 10 years (5 years is the most common).
  - Most CDS’s are in the USD 10M to 20M range.

• CDS contracts are governed by the International Swaps and Derivatives Association (ISDA), which provides standardized definitions of CDS terms, including definitions of what constitutes a credit event.
**CDS Mechanism**
- One party *buys protection* — i.e., “sells” risk or “short credit exposure— and the counterparty *sells protection* — i.e., “buys” risk or credit exposure.
- Protection buyer pays a periodic fee (the *spread*) to protection seller.
- In return, the protection seller agrees to pay the protection buyer a set amount if there is a credit event (usually, default).
- Usually, collateral rules apply to seller; following a 5-day 99% VaR.

### Fixed Leg: Spread Payments

![Diagram](image1.png)

**Protection Buyer** → **Protection Seller**

**Contingent Leg: Payment if Credit Event Occurs**

Though it is not necessary to buy a CDS, the protection buyer tends to own the underlying asset subject to risk.

**Note:** The spread is positively related to the likelihood of credit event.

**CDS Quotes**
Below we show a snapshot from a *Bloomberg* terminal (from window for “Par CDS spread”). Ford has multiple CDS contracts outstanding, each based on a different bond. The first one, is a CDS based on the 5-year senior bond (the most liquid CDS contract).
CDS Quotes
Below we show another snapshot from a Bloomberg terminal, showing historical prices (=CDS spreads). We show the last price of each day. CDS spreads do vary.

<table>
<thead>
<tr>
<th>Date</th>
<th>Last Price</th>
<th>Date</th>
<th>Last Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fr 06/20/15</td>
<td>107.500</td>
<td>Fr 04/22/15</td>
<td>107.500</td>
</tr>
<tr>
<td>Th 06/22/15</td>
<td>113.500</td>
<td>Th 04/11/15</td>
<td>107.700</td>
</tr>
<tr>
<td>We 06/23/15</td>
<td>112.500</td>
<td>We 04/14/15</td>
<td>107.500</td>
</tr>
<tr>
<td>Tu 06/24/15</td>
<td>108.300</td>
<td>Tu 04/19/15</td>
<td>107.500</td>
</tr>
<tr>
<td>Mo 06/29/15</td>
<td>108.300</td>
<td>Mo 04/19/15</td>
<td>108.300</td>
</tr>
</tbody>
</table>

CDS Benefits
Besides hedging event risk, the CDS provides the following benefits:
- A short positioning vehicle that does not require an initial cash outlay.
- Access to maturity exposures not available in the cash market.
- Access to credit risk not available in the cash market due to a limited supply of the underlying bonds.
- Investments in foreign credits without currency risk.
- Ability to effectively ‘exit’ credit positions in periods of low liquidity.

CDS: Not Insurance
- In a car insurance, you need to own the car and show damage to receive compensation from a claim. In a CDS contract, the protection buyer does not need to own the underlying credit exposure.
- Protection seller is not necessarily regulated. No reserves are required.
- CDS’s are mark-to-market (in the US).
**Typical CDS Quote**
A 5-year CDS quote for Bertoni Bank (on April 17, 2015)
Notional amount = **USD 10 million** (= Czech Rep Eurobond holdings)
Premium or Spread: **160 bps** (related to risk of Czech Republic)
Maturity: 5 years
Frequency: Quarterly Payments
Credit event: Default

**• Calculation of the Spread**
Q: How much Bertoni Bank pays for protection?
(0.0160/4) * USD 10M = **USD 40,000** (every quarter as a premium for protection against company default)

If the Czech Republic (Eurobond issuer) defaults, the CDS covers the notional **USD 10M**.

---

**Diagram for Bertoni Bank’s CDS**

Swap Dealer

Quarterly premium payments: **USD 40,000**

Bertoni Bank

No default: Zero

Loss recovery

Credit (USD 10M)

Czech Rep Eurobonds

Medium risk entity for default
CDS Spread
The spread (C = \textbf{160 bps}) is related to the default risk of Czech Republic.

Czech Republic Government Bond 10-year YTM: 2009 - 2019

• CDS Spreads Do Reflect Default Risk.
Since we look at CDSs as insurance for bondholders; an increase in CDS premiums indicates that investors are becoming worried about the safety of their investments.

Higher country risk $\Rightarrow$ Higher CDS premiums
CDS: Settlement

• Settlement can either be physical or cash.

(1) Physical Settlement
– Protection seller pays buyer par value \( N \) and receives an acceptable obligation (say, Czech Republic Eurobonds) worth \( R \), from buyer.
– Buyer of protection can choose, within certain limits, what obligation to deliver. Puts buyer in a CDB-like situation (pick lower \( R \) possible).

(2) Cash Settlement
– Protection seller pays buyer the difference between par value \( N \) and market price of a debt obligation of the reference entity (say, market price of similar Czech Republic Eurobonds). That is, seller pays:
\[
(100 - R)\% \text{ of Notional}
\]
– To establish value of reference obligation \( R\% \), a dealer poll/auction can be used.

Note: Think of what the seller gets in the event of default as recovery, \( R \).

CDS: Pricing

Suppose we want to price a 1-year CDS. Thus, there are 4 events.

\[ \begin{array}{ccccccc}
    t=0 & t=1 & t=2 & t=3 & t=4
\end{array} \]

\text{Effective date}

Nominal amount = \( N \)
Premium = \( C \) (annualized)
Quarterly payment = \( N \times (C/4) \)

There are 5 possible outcomes in this CDS contract:
– No default (4 premium payments are made by bank to investor until the maturity date)
– Default occurs on \( t_1, t_2, t_3, \) or \( t_4 \)
Steps
1) Assign probability to each event – i.e., default at t₁, default at t₂, etc.

2) Calculate PV of payoff for each outcome (assuming \( \delta_i = 1/(1+r_i)^i \) as the discount rate for period \( i \)):
   - Seller’s net default payment is \( N \ast (1 - R) \)
   - Buyer’s payments is \( N \ast \frac{C}{4} \)

3) Expected NPV of CDS = Sum of PV of five payoffs multiplied by their probability of occurrence.
   \[ E[NPV_{CDS}] \approx 0 \Rightarrow \text{Determine fair } C \text{ such that } E[NPV_{CDS}] \approx 0. \]

Notes: We think of the \( P_i \)'s as “survival” probabilities over an interval:
- Probabilities \( \Rightarrow P_i \) [No default at \( t_i \) – issuer still alive at time \( t_i \)]
- \( \Rightarrow 1 - P_i \) [Default at \( t_i \) – issuer is “dead” at time \( t_i \)]
  Technically, \( P_i \) is the probability of surviving over interval \([t_i - t_{i-1}]\).

CDS: Payoff Diagram

CFs from the seller’s point of view (red=outflows, blue=inflows):
Summary of Events and Payoffs

<table>
<thead>
<tr>
<th>Description</th>
<th>Premium Payment PV</th>
<th>Default Payment PV</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default at $t_1$</td>
<td>0</td>
<td>$N \times (1 - R) \times \delta_1$</td>
<td>$(1 - P_1)$</td>
</tr>
<tr>
<td>Default at $t_2$</td>
<td>$N \times C/4 \times \delta_1$</td>
<td>$N \times (1 - R) \times \delta_2$</td>
<td>$P_1 \times (1 - P_2)$</td>
</tr>
<tr>
<td>Default at $t_3$</td>
<td>$N \times C/4 \times (\delta_1 + \delta_2)$</td>
<td>$N \times (1 - R) \times \delta_3$</td>
<td>$P_1 \times P_2 \times (1 - P_3)$</td>
</tr>
<tr>
<td>Default at $t_4$</td>
<td>$N \times C/4 \times (\delta_1 + \delta_2 + \delta_3)$</td>
<td>$N \times (1 - R) \times \delta_4$</td>
<td>$P_1 \times P_2 \times P_3 \times (1 - P_4)$</td>
</tr>
<tr>
<td>No default</td>
<td>$N \times C/4 \times (\delta_1 + \delta_2 + \delta_3 + \delta_4)$</td>
<td>0</td>
<td>$P_1 \times P_2 \times P_3 \times P_4$</td>
</tr>
</tbody>
</table>

• $E[\text{NPV}_{\text{Seller}}] = \text{NPV}\{\text{Premium payments}\} - \text{NPV}\{\text{Default payments}\}$

• To calculate the $E[\text{NPV of CDS}]$, we need as inputs:
  – Known: $N$, $C$ (determined in the contract)
  – Undetermined/Unknown: $P_i = \{P_1, P_2, P_3, P_4\}$; $R$; and the discount rate for period $i$, $\delta_i = 1/(1+r_i)^i$.

CDS: Pricing and Inputs

• There are two popular ways to get $P_i$’s:
  (1) Assume a probability distribution, for example, the exponential. The higher the risk (and spread), the higher the decay in the survival probability.
  (2) Use no-arbitrage model. After some assumption, we can use market prices of similar bonds (ideally, from same issuer), $B_J$, and T-bond prices, $B_{RF}$, to compute expected PV of cost of default loss as $(B_J - B_{RF})$. Then, we “extract” the implied $P_i$’s. Which one?

• $R$ is calculated from historical (“average) recovery rates. In many situations, $R = 40\%$ is used as the default input. Constant?

• We use $\delta_i = 1/(1+r_i)^i$ from term structure. Under assumptions, we use same discount rate for $C/4$ & $N \times (1 - R)$. But, we can use different discount rates for the defaultable part ($N$) and non-defaultable parts ($C$). Realistic?
**CSD: Calculation of PV of CDS and Pricing**

Expected Present Value of Credit Default Swap = \( E[\text{NPV}_{\text{CDS}}] = \)

\[
= (P_1 \times P_2 \times P_3 \times P_4) \times [\frac{N \times C}{4} \times (\delta_1 + \delta_2 + \delta_3 + \delta_4)] \\
- (1 - P_1) \times N \times (1 - R) \times \delta_1 \\
- P_1 \times (1 - P_2) \times [\frac{N \times (1 - R)}{4} \times \delta_2 - \frac{N \times C}{4} \times \delta_1] \\
- P_1 \times P_2 \times (1 - P_3) \times [\frac{N \times (1 - R)}{4} \times \delta_3 - \frac{N \times C}{4} \times (\delta_1 + \delta_2)] \\
- P_1 \times P_2 \times P_3 \times (1 - P_4) \times [\frac{N \times (1 - R)}{4} \times \delta_4 - \frac{N \times C}{4} \times (\delta_1 + \delta_2 + \delta_3)]
\]

Recall that using this formula, we price the CDS –i.e., determine **fair C**.

At \( t = 0 \), \( E[\text{NPV}_{\text{CDS}}] = 0 \) (or, \( \approx 0 \)) \( \Rightarrow \) get \( C \) such that \( E[\text{NPV}] \approx 0 \).

\[
N \times (1 - R) \times \{ (1 - P_1) \times \delta_1 + P_1 \times (1 - P_2) \times \delta_2 + \frac{P_1 \times P_2 \times (1 - P_3) \times \delta_3 + P_1 \times P_2 \times P_3 \times (1 - P_4) \times \delta_4}{P_1} \}
\]

\begin{align*}
N(1 - R) & \times \{(1 - P_1) \times \delta_1 + P_1 \times (1 - P_2) \times \delta_2 + \frac{P_1 \times P_2 \times (1 - P_3) \times \delta_3 + P_1 \times P_2 \times P_3 \times (1 - P_4) \times \delta_4}{P_1} \}
\end{align*}

Then,

\[
C = \frac{4 \times (1 - R) \times \{(1 - P_1) / P_1 \times \delta_1 + (1 - P_2) / P_2 \times \delta_2 + (1 - P_3) / P_3 \times \delta_3 + (1 - P_4) / P_4 \times \delta_4 \}}{\{(1 - P_2) / P_2 \times (1 - P_3) / P_3 \times (1 - P_4) / P_4 \times \delta_1 + \delta_2 + \delta_3 \}}
\]

\( \Rightarrow C \) is the fair CDS spread.
**Example**: Expected NPV for Bertoni Bank’s CDS, with 2 payments left

Notional amount = **USD 10 million** (Czech Republic Eurobonds)

Premium or Spread = \( C = 160 \text{ bps} \)

Maturity: 6 months (2 payments left)

Frequency: Quarterly Payments

Credit event: Default

Discount rates: 3-mo = 0.035; & 6-mo = 0.037.

Recovery Rate = \((1 - R) = 60\%\)

Quarterly payments = \( N \times C / 4 = \text{USD 10M} \times (0.0160 / 4) = \text{USD 40,000} \)

Probability of Default: \( P_1 = .99; \) & \( P_2 = .985 \)

\[
\text{E[NPV of Credit Default Swap (in USD M)]=} \\
= (.99 \times .985) \times [.040 \times \{1/(1+0.035/4)^1 + 1/(1+0.037/4)^2\}] \\
- (1 - .99) \times 10 \times .60 \times 1/(1+0.035/4)^1 \\
- .99 \times .015 \times [10 \times .60 \times 1/(1+0.037/4)^2 - .040 \times 1/(1+0.035/4)^1] = -0.0694
\]

**Note**: Like swaps, at inception the PV of CDS \( \approx 0 \). In this case, we call the spread (or premium) *fair*.

**Example (continuation)**: Pricing CDS

Today, we want to price a similar CDS to the Bertoni Bank’s CDS. Then, we set \( C \) such that \( \text{E[NPV]} \approx 0 \). That is, if a similar CDS is issued today with 2 payments left, the *fair spread* is: \( C = 303.19 \text{ bps} \), since:

\[
\text{E[NPV of Credit Default Swap (in USD M)] =} \\
= (.99 \times .985) \times [.075796 \times \{1/(1+0.035/4)^1 + 1/(1+0.037/4)^2\}] \\
- (1 - .99) \times 10 \times .60 \times 1/(1+0.035/4)^1 \\
- .99 \times .015 \times [10 \times .60 \times 1/(1+0.037/4)^2 - .075796 \times 1/(1+0.035/4)^1] \\
\approx 0.
\]

Then, the quarterly payments are:

\[
= N \times C / 4 = \text{USD 10M} \times (0.030319 / 4) = \text{USD 75,796}.
\]
**CDS: Risks**

- The main risk is *counterparty risk* – i.e., the seller defaults. If a major counterparty (say, AIG, Lehman) defaults a large number of market participants are left un-hedged.

- If a large seller defaults, network domino effects are possible.

- Collateral & margin can spiral out of control. Asset values are correlated with CDS protection sold & the economy. To post more collateral, firms have to de-leverage (sell assets at worst time: *fire sale*.)

- Modeling CDS spreads is complicated:
  - Market is illiquid – i.e., difficult to trust observed market prices.
  - $P_i$’s are not easy to determine.
  - Fat-tailed and left-skew distributions.
  - Difficult to aggregate risks (hard to measure default correlations).

**CDS: Summary**

- CDS are bilateral contracts, often sold and resold among parties.

- Large market, due to netting, the notional size of the CDS market is approximately 1/10th the size of the gross notional market.

- Due to its protection nature CDS market represents over one-half of the global credit derivative market.

- CDS allows a party who buys protection to trade and manage credit risks in much the same way as market risks.
Example: AIG
- In 2005 and early 2006, AIG sold protection on $N = \text{USD 500B}$ in assets, including $\text{USD 78B}$ on collateralized debt obligations—backed by debt payments from mortgages, home equity loans, etc. At inception:
  - Probability of default were set very low.
  - Default correlations were well not incorporated into model.
  - Given AIG status, no collateral was required. It was required only under certain events (AIG’s credit rating falls below AA-).

- As write-downs in real estate grew in 2007-08, AIG rating was lowered below AA-. By September 2008, margin calls reached $\text{USD 32B}$.
  - Started off write-downs (asset prices down) & faced more margin calls.
  - Eventually, margin calls rose to $\text{USD 50B}$.

- Aside: AIG did not help itself by investing the collateral cash received from shorts (AIG was big in lending securities) in subprime mortgage paper. As shorts returned stock, AIG could not give the collateral back.

Combination of Swaps

- Recall that swaps change the profile of cash flows.
- Swaps solve problems: *Financial Engineering*.

Example: A Brazilian oil producer is exposed to two forms of price risk:
- $P_{\text{Oil}}$ (priced in USD/barrel of oil) ⇒ commodity price risk.
- $S_t$ (BRL/USD) ⇒ FX risk.

Situation: Since expenses are in BRL, the Brazilian oil producer wants to fix $P_{\text{Oil}}$ in BRL/barrel of oil.
Solution: *Financial Engineering*, a combination of swaps can do it!

Note: This is a typical problem for commodity producers and buyers from non-USD zones: Commodities are priced in USD.
Diagram: Structured Solution for Brazilian Oil Producer

**Example:** Combining Swaps.
- Belabu, a Lituanian coffee refiner, uses 500,000 pounds of Colombian coffee every 6-months. Colombian coffee trades spot at $P_{\text{Coffee}}$ (USD).
- Belabu has contracts to sell its output at a fixed price for 4 years.

- $S_t = 4.74 \text{ LTT/USD}$
- $P_{\text{Coffee}} = 1.95 \text{ USD/pound}$.
- **Goal:** Belabu wants to fix the price of coffee in LTT/pound.
  - $\Rightarrow$ Belabu simultaneously enters three swap: A commodity swap, an interest rate swap, & a currency swap.
(1) Commodity swap dealer:
   Belabu pays: A fixed-price of **USD 2.05 per pound**
   Belabu receives: The average market price of coffee ($P_{Coffee}$).

Current mid-price quote for a 4-yr coffee swap is USD 1.99 per pound.
   (Dealer adds USD .06 to its mid-price.)

(2) Interest rate swap dealer:
   Belabu pays: USD floating rate amount
   Belabu receives: USD fixed rate amount

4-yr swap interest rate quote: **8.2%** against 6-mo. LIBOR.

(3) Currency swap dealer:
   Belabu pays: a LTT fixed-rate amount
   Belabu receives: a USD floating rate amount

4-year LTT-for-USD currency swap quote: **7.8%** against 6-mo. LIBOR.

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**Details**

1. Determine the number of USD Belabu will need every six months:
   
   500,000 pounds * **USD 2.05/pound** = **USD 1,025,000**

2. Determine notional principal required on a USD interest rate swap for
   the fixed-rate side to generate **USD 1,025,000** every 6-mo (**8.2%** rate)
   
   **USD 1,025,000 / .0416** = USD 25,000,000
Details (continuation)

3. Calculate the present value of the cash flows on the fixed-rate side of the interest rate swap using the current 8.2%:
   \[
   PV(\text{USD 1,025,000}; .041, 8 \text{ periods}) = \text{USD 6,872,600}
   \]

4. Translate the PV of the USD cash flows to its LTT equivalent:
   \[
   4.74 \text{ LTT/USD} * \text{USD 6,872,600} = \text{LTT 32,576,124}
   \]

5. Determine the LTT CFs on the fixed-rate side of the LTT-for-USD currency swap having a NPV of LTT 32,576,124 at 7.8% current rate.
   Coupon(PV = \text{LTT 32,576,124}; .039, 8 periods) = \text{LTT 4,818,500}

\[
\begin{array}{c}
\text{Belabu} \\
\text{USD 6-mo LIBOR} \\
\downarrow \text{LTT 4,818,500} \\
\text{Currency Swap Dealer}
\end{array}
\]

Details (continuation)

6. Determine the LTT notional principal that would generate the semiannual payments of LTT 4,818,500 at 7.8%:
   \[
   \text{LTT 4,818,500} / .039 = \text{LTT 123,551,282.10}
   \]

\[\Rightarrow\text{ The structured solution has fixed the price of coffee for four years:}
\]

\[
\text{LTT 4,818,500} / 500,000 \text{ pounds} = 9.637 \text{ LTT/pound of coffee.}
\]
Diagram: Structured Solution for Belabu

- **Spot Coffee World Market**: $P_{\text{Coffee}} \times 500,000$ pounds
- **Payment tied to $P_{\text{Coffee}}$**
  - USD 1,025,000
- **Belabu**
- **Commodity Swap Dealer**: USD 1,025,000
- **Belabu**
- **Interest Rate Swap Dealer**: USD 1,025,000
- **Belabu**
- **Currency Swap Dealer**: USD 6-mo LIBOR
- **USD 6-mo LIBOR**
- **Belabu**
- **USD 6-mo LIBOR**
- **Belabu**
- **LTT 4,818,500**

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**Synthetic Instruments**

*Synthetic instruments* are not securities at all.

- CF streams formed by combining the CF streams from one set of instruments to replicate the CF streams of another set of instruments.

- When combined with appropriate cash positions, it is possible to use swaps to replicate the CF stream associated with virtually any instrument.
Example: Dual currency bond.
Situation: Bertoni Bank issues 1,000 dual currency bonds for USD 1M.
   - Coupon payments: JPY 6.5 M.
   - Frequency of payments: semiannual.
   - Maturity: 5 years.
   - Features: Sold and redeemed in USD. Interest payments in JPY.
   - $S_t = 100 \text{ JPY/USD}$

CF for BB:
- At issue (t=0): BB receives USD 1 M.
- Every 6-mo: BB pays JPY 6.5 M
- At maturity (t=5): BB pays USD 1 M.

• Bertoni Bank's CF can be synthesized using:
  - A Corporate USD straight bond.
  - A fixed-for-fixed currency swap.

Example (continuation):
• Instruments:
  - 7.5% Chase Bonds with (at least) 5 years to maturity.
  - A currency SD offers USD 7.5% against JPY 6.5% (SD pays USD)

• At t=0, BB sells short USD 1M worth of Chase bonds
  ⇒ BB receives USD 1M and makes USD 7.5% coupon payments.
• BB also enters a fixed-for-fixed currency swap & make JPY payments.
  ⇒ BB pays SD JPY 6.5% and receives from SD USD 7.5%.

• At t=5, BB buys back the Chase bonds for USD 1M.
Example: Synthetic Equity
Situation:
Goyco Corp. has USD 3M to invest for two years.
Goyco is bullish on the Japanese market in the near future.
Goyco decides to invest in synthetic Japanese equity using as tools:

- A fixed-rate note
- An equity swap.

• Goyco buys a 7.8% FV=USD 3M, 2-yr bond, trading at par.
• At the same time, Goyco enters into a 2-yr equity swap:
  - Goyco pays the swap dealer 6.5% annually and receives the return on the Nikkei 225 (r_{Nikkei}).
  - Goyco pays the swap dealer when r_{Nikkei} < 0.
    (in addition to the fixed-rate payment of 6.5%.)
  - Notional: USD 3 million

Example (continuation):
• Goyco's position has a return equal to the Nikkei 225 plus 130 bps.
• Net effect: Creation of a (synthetic) equity position for Goyco.

⇒ In some countries, swaps are off-balance sheet items: Goyco only shows its 2-year corporate bond on its balance sheet.