Forecasting FX Rates

Fundamental and Technical Models

Forecasting Exchange Rates

Model Needed
A forecast needs a model, which specifies a function for $S_t$:

$$ S_t = f(X_t) $$

- The model can be based on
  - Economic Theory (say, PPP: $X_t = (I_{d,t} - I_{f,t}) \Rightarrow f(X_t) = I_{d,t} - I_{f,t}$)
  - Technical Analysis (say, past trends)
  - Statistics
  - Experience of forecaster
  - Combination of all of the above
Forecasting: Basics

• A forecast is an expectation – i.e., what we expect on average:
  \[ E_t[S_{t+T}] \Rightarrow \text{Expectation of } S_{t+T} \text{ taken at time } t. \]

• Easier to predict changes \( \Rightarrow E_t[s_{t+T}]. \)

Note: From \( E_t[s_{t,t+1}] \), we get \( E_t[S_{t+T}] \Rightarrow E_t[S_{t+T}] = S_t \ast (1 + E_t[s_{t+1}]) \)

• Based on a model for \( S_t \), we generate \( E_t[S_{t+T}]: \)
  \[ S_t = f(X_t) \Rightarrow E_t[S_{t+T}] = E_t[f(X_{t+T})] \]

Assumptions needed for \( X_{t-t} \)

Today, we do not know \( X_{t+T} \). We will make assumptions to get \( X_{t+T} \).

Example: \( X_{t+T} = h(Z_t) \), \( -Z_t: \) data available today.

\[ \Rightarrow \text{We’ll use } Z_t \text{ to forecast the future } S_{t+T}: E_t[S_{t+T}] = g(Z_t) \]

Example: What is \( g(Z_t) \)?

Suppose we are interest in forecasting USD/GBP changes using PPP:

1. Model for \( S_t \)
   \[ E_t[s_{t+1}] = s_{t+1}^F = \frac{s_{t+1}^F}{s_t} - 1 \approx I_{d,t+1} - I_{t+1} \]

Now, once we have \( s_{t+1}^F \), we can forecast the level \( S_{t+1} \)
   \[ E_t[S_{t+1}] = S_t \ast [1 + s_{t+1}^F] = S_t \ast [1 + (I_{US,t+1} - I_{UK,t+1})] \]

2. Assumption for \( I_{t+1} \Rightarrow I_{t+1} = h(Z_t) \)
   - \( I_{US,t+1} = \alpha_0^{US} + \alpha_1^{US} I_{US,t} \)
   - \( I_{UK,t+1} = \alpha_0^{UK} + \alpha_1^{UK} I_{UK,t} \)

3. \( E_t[S_{t+1}] = g(Z_t) \)
   - \( E_t[S_{t+1}] = g(I_{US,t}, I_{UK,t}) \)
     \[ = S_t \ast [1 + \alpha_0^{US} + \alpha_1^{US} I_{US,t} - \alpha_0^{UK} - \alpha_1^{UK} I_{UK,t}] \]
• There are two forecasts: *in-sample* and *out-of-sample*.
  - *In-sample*: It uses sample info to forecast sample values. Not really forecasting, it can be used to evaluate the fit of a model.
  - *Out-of-sample*: It uses the sample info to forecast values outside the sample. In time series, it forecasts into the future.

**Two Pure Approaches to Forecasting**
Based on the “driving” variables $X_t$, we have:
- Fundamental Approach (based on data considered fundamental).
- Technical Analysis or TA (based on data that incorporates only past prices: $P_{t-1}, P_{t-2}, P_{t-3}, \ldots$).

**Fundamental Approach**

**Economic Model**
Generate $E_t[S_{t+T}] = E_t[f(X_{t+T})] = g(X_t)$, where $X_t$ is a dataset of fundamental economic variables:
- GNP growth rate,
- Current Account,
- Interest rates,
- Inflation rates, etc.

• Fundamental variables: Taken from *economic models* (PPP, IFE, etc.)
  $\Rightarrow$ The model says how the fundamental data relates to $S_t$.
  That is, the model specifies $f(X_t)$—for PPP, $f(X_t) = I_{dt} - I_{et}$
• The model usually incorporates:
  – Statistical characteristics of the data
  – Experience of the forecaster
    ⇒ Mixture of art and science.

Fundamental Forecasting: Steps
(1) Selection of Model (say, PPP model).
(2) Get Data: $S_t$ & $X_t$ (for PPP: $S$ & CPI data.)
(3) Estimation of model, if needed.
(4) Generation of forecasts. Assumptions about $X_{t+T}$ may be needed.
(5) Evaluation of forecasts. If forecasts are bad, model must be changed.
    Popular evaluation metrics:
    ⇒ MSE (Mean Square Error) = $\frac{\sum_{j=1}^{Q} (S_{t+j} - S_{t+j}^F)^2}{Q}$
    ⇒ MAE (Mean Absolute Error) = $\frac{\sum_{j=1}^{Q} |S_{t+j} - S_{t+j}^F|}{Q}$
1) Select a (long) part of the sample to estimate the parameters of the selected model.
2) Keep a (short) part of the sample to check the model’s forecasting skills. This is the validation step. You can calculate true MSE or MAE.
3) Forecast out-of-sample.
**Out-of-Sample Forecasting: Estimation and Validation Period**

1. **Practice**
   - Model
   - Data
   - Estimation
   - Test Model
   - Pass?
   - Forecast
   - Evaluation
   - Out-of-sample

2. **Theory**
   - Modify/Change
   - Model Test
   - Model
   - Data
   - Validation
   - Period

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**Example: In-sample PPP forecasting of USD/GBP**

PPP equation for USD/GBP changes:

\[
E_t[s_{t+1}] = s^{F}_{t+1} \approx I_{US,t+1} - I_{UK,t+1} \quad \Rightarrow \quad E_t[s_{t+1}] = S_{t+1} = S_t \cdot [1 + s^{F}_{t+1}]
\]


- P\textsubscript{d,US}: US-CPI: 149.4, 150.2, 151.3
- P\textsubscript{d,UK}: UK-CPI: 167.4, 170.0, 170.4
- \(S_{1996:1} = 1.5262\) USD/GBP.
- \(S_{1996:2} = 1.5529\) USD/GBP.

1. **Forecast \(S^{F}_{1996:2}\)**

\[
I_{US,1996:2} = (US\textsubscript{1996:2}/US\textsubscript{1996:1}) - 1 = (150.2/149.4) - 1 = .00535.
I_{UK,1996:2} = (UK\textsubscript{1996:2}/UK\textsubscript{1996:1}) - 1 = (170.0/167.4) - 1 = .01553
S^{F}_{1996:2} = S^{F}_{1996:1} \cdot [1 + s^{F}_{1996:2}] = 1.5262\ USD/GBP \cdot [1 + (-.01018)] = 1.5107\ USD/GBP.

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• Example (continuation): *In-sample* PPP forecasting

\[ S_{1996:2}^F = 1.5107 \text{ USD/GBP}. \]

2. Forecast evaluation (Forecast error: \( S_{1996:2}^F - S_{1996:2} \))

\[ \varepsilon_{1996:2} = S_{1996:2}^F - S_{1996:2} = 1.5107 - 1.5529 = -0.0422. \]

For the whole sample:

\[
\text{MSE: } \frac{(-0.0422)^2 + (-0.0471)^2 + \ldots + (-0.1259)^2}{6} = 0.017063278
\]

Note: Not a true forecasting model. At time \( t \), \( I_{t+1} \) is unknown. We need \( E_t[I_{t+1}] \).

**Example 1: Out-of-sample** Forecast: \( E_t[S_{t+T}] \)

• Simple forecasting model: Naive forecast (\( E_t[I_{t+1}] = I_t \))

\[ E_t[S_{t+1}] = S_{t+1}^F = (E_t[S_{t+1}]/S_t) - 1 \approx I_{d,t} - I_{f,t}. \]

Using the above information we can predict \( S_{1996:3} \):

1. Forecast \( S_{1996:3}^F \)

\[ S_{1996:3}^F = I_{US,1996:2} - I_{UK,1996:2} = .00535 - .01553 = -0.01018. \]

\[ S_{1996:3}^F = S_{1996:2} * [1 + s_{1996:3}^F] = 1.5529 * [1 + (-0.01018)] = 1.53709 \]

2. Forecast Evaluation

\[ \varepsilon_{1996:3} = S_{1996:3}^F - S_{1996:3} = 1.53709 - 1.5653 = -0.028210. \]
More sophisticated forecasts can be achieved by estimating models for I, survey data on expectations of I, etc.

**Example 1A**: AR(1) model for inflation,

\[ I_{US,t} = \alpha_0^{US} + \alpha_1^{US} I_{US,t-1} + \varepsilon_{US,t} \]

\[ I_{UK,t} = \alpha_0^{UK} + \alpha_1^{UK} I_{UK,t-1} + \varepsilon_{UK,t} \]

Suppose we estimate both equations. The estimated coefficients (a’s) are:

- \( a_0^{US} = .0036 \), \( a_1^{US} = .64 \), \( a_0^{UK} = .0069 \), & \( a_1^{UK} = .43 \).

\[ \Rightarrow I_{US,1996:3}^F = .0036 + .64 \times (.00535) = .007024 \]

\[ \Rightarrow I_{UK,1996:3}^F = .0069 + .43 \times (.01553) = .013578. \]

\[ s^F_{1996:3} = I_{US,1996:3}^F - I_{UK,1996:3}^F = .007024 - .013578 = -.00655. \]

\[ s^F_{1996:3} = 1.5529 \text{ USD/GBP} \times [1 + (-.00655)] = 1.5427 \text{ USD/GBP}. \]

\[ \varepsilon_{1996:3} = s^F_{1996:3} - s^F_{1996:3} = 1.5427 - 1.5653 = -0.0226. \]

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**Example 1A (continuation)**: Exchange Rate Forecasts

US Excel Regression Results for US Inflation Forecasts \( I_{US,t+1} \):

<table>
<thead>
<tr>
<th>Regression Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple R</td>
</tr>
<tr>
<td>R Square</td>
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<tr>
<td>Adjusted R Square</td>
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<tr>
<td>Standard Error</td>
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<tr>
<td>Observations</td>
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</tbody>
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<table>
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<tr>
<th>ANOVA</th>
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</thead>
<tbody>
<tr>
<td>df</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>Regression</td>
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<tr>
<td>Residual</td>
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<tr>
<td>Total</td>
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<table>
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<tr>
<th>Coefficients</th>
<th>Standard Error</th>
<th>t Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.00366</td>
<td></td>
<td>0.000123</td>
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<tr>
<td>X Variable 1</td>
<td>0.640707</td>
<td>0.071274</td>
<td>8.9893</td>
</tr>
</tbody>
</table>
Example 1A (continuation): $I_{UK,t+1}$ - UK Regression Results:

**SUMMARY OUTPUT**

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Multiple R</td>
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$I_{UK,t+1}$ explains 18.10% of the variability of $I_{UK,t}$

$t$-stat is significant at the 5% level ($|t|>1.96$)

$=>$ Lagged Inflation explains current Inflation

**ANOVA**

<table>
<thead>
<tr>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>Significance F</th>
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<tr>
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<td>0.003473</td>
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<tr>
<td>Residual</td>
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<td>0.015719</td>
<td>0.000128</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>124</td>
<td>0.019192</td>
<td></td>
<td></td>
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**Coefficients**

<table>
<thead>
<tr>
<th>Standard Error</th>
<th>t Stat</th>
<th>P-value</th>
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</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.006918</td>
<td>4.932637</td>
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<tr>
<td>X Variable 1</td>
<td>0.428132</td>
<td>5.213237</td>
</tr>
</tbody>
</table>

Example 2: *Out-of-sample* Forecasting FX with an Ad-hoc Model

Forecast monthly MYR/USD changes with the following model:

$$s_{MYR/USD,t} = \alpha_0 + \alpha_1 (I_{MYR} - I_{USD})_t + \alpha_2 (y_{MYR} - y_{USD})_t + \epsilon_t$$

Excel Regression Results:

**SUMMARY OUTPUT**

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**ANOVA**

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<th>MS</th>
<th>F</th>
<th>Significance F</th>
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</thead>
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<td>0.000258</td>
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<tr>
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<tr>
<td>Total</td>
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<td>0.294195</td>
<td></td>
<td></td>
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</table>

**Coefficients**

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<tr>
<th>Standard Error</th>
<th>t Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.006934</td>
<td>1.339903</td>
</tr>
<tr>
<td>X Variable 1 $(I_{MYR} - I_{USD})_t$</td>
<td>0.215927</td>
<td>2.040435</td>
</tr>
<tr>
<td>X Variable 2 $(y_{MYR} - y_{USD})_t$</td>
<td>0.091592</td>
<td>1.772428</td>
</tr>
</tbody>
</table>
Example 2 (continuation): Out-of-sample Forecasting w/Ad-hoc Model

\[ s_{MYR/USD,t} = \alpha_0 + \alpha_1 (I_{MYR} - I_{USD},_t) + \alpha_2 (y_{MYR} - y_{USD},_t) + \varepsilon, \]

0. Model Evaluation

Estimated coefficient: \( \alpha_0 = .0069, \alpha_1 = .2159, \) and \( \alpha_2 = .0915. \)

\[ t-stats: t_{\alpha_1} = |2.040435| > 1.96 \) (reject \( H_0); \) \( t_{\alpha_2} = |1.772428| < 1.96 \) (can’t reject \( H_0) \)

Do the signs make sense? \( \alpha_1 = .2159 > 0 \) \( \Rightarrow \) PPP  
\( \alpha_2 = .0915 > 0 \Rightarrow \) Trade Balance

1. Forecast \( S_{t+1}^F \)

\[ E[s_{MYR/USD,t}] = .0069 + .2159 \) \( (I_{MYR} - I_{USD},_t) + .0915 \) \( (y_{MYR} - y_{USD},_t) \)

Forecasts for next month (\( t+1 \)): \( E_t[INF_{t+1}] = 3\% \) & \( E_t[INC_{t+1}] = 2\% \).

\[ E_t[s_{MYR/USD,t+1}] = .0069 + .2159 \ast (0.03) + .09157 \ast (0.02) = .0152. \]

The MYR is predicted to depreciate 1.52% against the USD next month.

Example 2 (continuation): Out-of-sample Forecasting w/Ad-hoc Model

1. Forecast \( S_{t+1}^F \) (continuation)

\[ E_t[s_{MYR/USD,t+1}] = .0152. \]

Suppose \( S_t = 3.1021 \) MYR/USD

\[ S_{t+1}^F = 3.1021 \) MYR/USD \( \ast (1 + .0152) = 3.1493 \) MYR/USD.

2. Forecast Evaluation

Suppose \( S_{t+1} = 3.0670 \) MYR/USD.

\[ \varepsilon_{t+1} = S_{t+1}^F - S_{t+1} = 3.1493 - 3.0670 = 0.0823. \]
• **Practical Issues in Fundamental Forecasting**

Issues:
- Are we using the "right model?"
- Estimation of the model.
- Some explanatory variables (\(Z_{t+T}\)) are contemporaneous.
  \[ \Rightarrow \text{We also need a model to forecast the } Z_{t+T} \text{ variables.} \]

• **Does Forecasting Work?**

RW models beat structural (and other) models: Lower MSE, MAE.

• **Right Evaluation Metric?**

Richard Levich compared forecasting services to the free forward rate. He found that forecasting services may have some ability to predict direction (appreciation or depreciation).

For some investors, the direction is what really matters, not the error.

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**Example**: Two forecasts: Forward Rate and Forecasting Service (FS)

\[ F_{t,1\text{-month}} = .7335 \text{ USD/CAD} \]
\[ E_{FS}[S_{t+1\text{-month}}] = .7342 \text{ USD/CAD}. \]

(Sternin’s strategy: buy CAD forward if FS forecasts CAD appreciation.)

Based on the FS forecast, Sternin buys CAD forward at \( F_{t,1\text{-month}} = .7335 \).

(A) Suppose that the CAD appreciates to \( .7390 \text{ USD/CAD} \).

\[ \text{MAE}_{FS} = .7390 - .7342 = .0052 \text{ USD/CAD}. \]

Sternin makes a profit of \( .7390 - .7335 = .055 \text{ USD/CAD} \).

(B) Suppose that the CAD depreciates to \( .7315 \text{ USD/CAD} \).

\[ \text{MAE}_{FS} = |.7315 - .7342| = .0027 \text{ USD/CAD} \, \text{(smaller!)} \]

Sternin takes a loss of \( .7315 - .7335 = -.0020 \text{ USD/CAD}. \)
Technical Analysis Approach

• Based on a small set of the available data: Past price information.

Q: Why ignore fundamentals, say, (I_{d,t} – I_{f,t})?
EMH: FX market “discounts” public information regarding fundamentals

• TA looks for the repetition of specific price patterns.

• TA attempts to generate signals: trends and turning points.

• TA models range from very simple (say, looking at price charts) or very sophisticated, incorporating neural networks and genetic algorithms.

Technical Analysis Approach

• Popular models:
  - Moving Averages (MA)
  - Filters
  - Momentum indicators.
  - Bollinger Bands (MA + SD, used to create “bands” for MA)
  - Relative Strength Index, RSI (it determines “over/under-sold”)
  - Fibonacci Retracements, “Fibs” (Fibonacci ratios determine potential retracements from a high)
(1) **MA models**

MA model: Smooth erratic daily swings of $S_t$.
Define $S_t^{MA}$ as SMA:

$$S_t^{MA} = \frac{(S_t + S_{t-1} + S_{t-2} + \ldots + S_{t-(Q-1)})}{Q}$$

*Double MA system* uses two SMAs: Long-run MA, with $Q=Q_L$
Short-run MA, with $Q=Q_S$, $Q_L > Q_S$.

LRMA always lag a SRMA (gives smaller weights to recent $S_t$).

Buy FC signal: When SRMA crosses LRMA from below.
Sell FC signal: When SRMA crosses LRMA from above.

**Example**: $S_t$ (USD/GBP) Double MA - $Q_L=30$ days (red); & $Q_S=150$ days (green).
Filter models

Filter = X, a percentage that helps to spot a trend.

Buy signal: When $S_t$ rises X% above its most recent trough.
Sell signal: When $S_t$ falls X% below the previous peak.

Idea:

When $S_t$ reaches a peak $\Rightarrow$ Sell FC
When $S_t$ reaches a trough $\Rightarrow$ Buy FC.

Key: Identifying a peak/trough. The filter does it:

When $S_t$ moves X% above (below) its most recent peak (trough), we have a trading signal.

Example: $X = 1\%$, $S_t$ (CHF/USD)

Peak = 1.486 CHF/USD ($X = CHF \cdot 0.01486$)
$\Rightarrow$ When $S_t$ crosses 1.47114 CHF/USD, Sell USD

Trough = 1.349 CHF/USD ($X = CHF \cdot 0.01349$)
$\Rightarrow$ When $S_t$ crosses 1.36249 CHF/USD, Buy USD
Example: $X = 1\%$, $S_t$ (CHF/USD)

Peak = 1.486 CHF/USD ($X = \text{CHF} .01486$)  
$\Rightarrow$ When $S_t$ crosses 1.47114 CHF/USD, Sell USD

(3) Momentum models
They determine the strength of an asset by examining the change in velocity of asset prices’ movements.

We are looking at the second derivative (a change in the slope).

Buy signal: When $S_t$ climbs at increasing speed.
Sell signal: When $S_t$ decreases at increasing speed.
• **TA Newer Models:**
In MA and filter models, we need to select a parameter (Q & X). Subjective selection: Two TA practitioners using the same model may generate different signals.

Newer TA methods rely on more sophisticated formulas to determine when to buy/sell, without the subjective selection of parameters.

Clements (2010, *Technical Analysis in FX Markets*) describes four of these methods: Relative strength indicator (RSI), Exponentially weighted moving average (EWMA), Moving average convergence divergence (MACD) and (iv) Rate of change (ROC).

• **TA Summary:**
  TA models monitor the derivative (slope) of a time series graph. Signals are generated when the slope varies significantly.

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• **Technical Approach: Evidence**
  - Against TA:
    - RW model: A good forecasting model.
    - Economists have a negative view of TA: TA runs against EMH.

  - For TA:
    - Informal evidence: FX Mkt is full of TA newsletters & traders (30%).
    - Formal (academic) support:
      - In general, in-sample results tend to be good (profitable). But, not out-of-sample.
      - LeBaron (1999) speculates that the apparent success of TA in the FX market is influenced by CB intervention.
      - Lo (2004): Markets are adaptive efficient: TA may work for a while.
      - Ohlson (2004): Even in-sample, profitability has declined (≈ 0 profits by the 1990s).