Project: Testing PPP and Forecasting Exchange Rates

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Review - Purchasing Power Parity (PPP)

• Notation and definitions:

The exchange rate is a price: The relative price of two currencies.

Example: On October 30, 2024, the price of a euro (EUR) in terms of USD was USD 1.09 per EUR $\Rightarrow$ EUR 1 = USD 1.09

Notation: S_t = Exchange rate = 1.09 USD/EUR.

We use the "direct notation" for S_t:

S_t = Units of domestic currency per unit of foreign currency.

The percentage change at time t of S_t is $e_{f,t}$. As usual, we will define $e_{f,t}$ with the log change of S_t.

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Review - Purchasing Power Parity (PPP)

• **PPP** is based on the **law of one price** (LOOP): Baskets of goods, once denominated in the same currency, should have the same price. If they are not, then traders can profit with pseudo-arbitrage strategies.

Absolute version of PPP:

The FX rate between two currencies is simply the ratio of the two countries' general price levels:

$$S_t^{PPP} = \text{Domestic Price level / Foreign Price level} = P_d/P_f$$

Relative PPP

The rate of change in the prices of products should be similar when measured in a common currency (as long as trade frictions are unchanged):

$$e_{f,t} \approx e_{f,t}^{PPP} \approx I_d - I_f$$

PPP: Equilibrium Exchange Rate

PPP Notes :

- PPP gives an *equilibrium* exchange rate. Equilibrium will be reached when there is no trade in the basket (because of mispricing). That is, when the PPP holds for the same basket.
- PPP is telling what S_t *should be* (in equilibrium). It is not telling what S_t *is* in the market today.

Testing PPP:

Absolute version: Easily Rejected. The assumptions behind Relative PPP are seen as questionable, especially, no trade frictions.

Relative version: This is what the first part of the project does.

- Visual tests (check scatter plot of $e_{f,t}$ vs $(I_d - I_f)$ & plot R_t over time)
- Statistical tests

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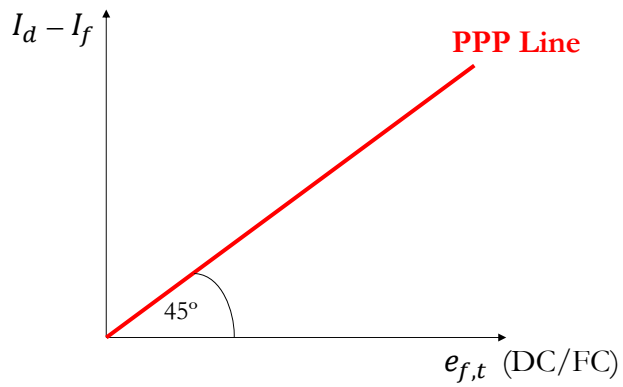
PPP: Relative Version

- If Relative PPP holds, we should see a 45° line (PPP Line) when we plot $e_{f,t}$ & $I_d - I_f$.

Recall Relative PPP: $e_{f,t}^{PPP} \approx I_d - I_f$



PPP: Relative Version

- We define the Real exchange rate, R_t:

$$R_t = \frac{S_t * P_f}{P_d}$$

If $R_t = 1$, foreign goods, once translated to domestic currency, have the same price as domestic goods. If $R_t > 1$ ($R_t < 1$), foreign goods, once translated to DC, are more (less) expensive than domestic goods.

- Under relative PPP, R_t should be constant. (Under Absolute PPP, $R_t = 1$).

We usually associate R_t with under/over-valuation:

- If R_t is over the mean, we consider the foreign currency overvalued (under Absolute PPP, if $R_t > 1$).

- If R_t is under the mean, we consider the domestic currency overvalued (under Absolute PPP, if $R_t < 1$).

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PPP: Relative Version – Visual Evidence

• Data: Monthly Swedish & U.S. data (1/1973 - 7/2024): CPI and S_t

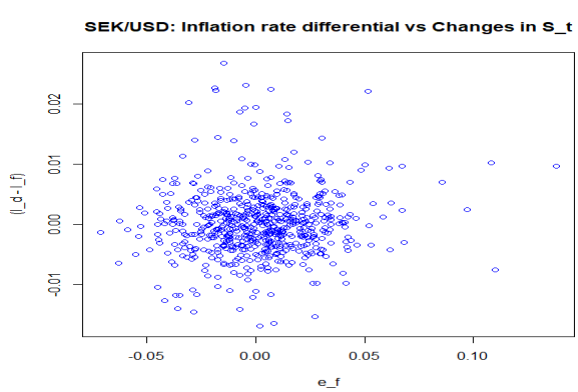
```
FMX_da <- read.csv("http://www.bauer.uh.edu/rsusmel/4397/ppp_2020_m.csv", head='TRUE, sep=",")
x_date <- FMX_da$Date
us_CPI <- FMX_da$US_CPI
swed_CPI <- FMX_da$SWED_CPI
S_sek <- FMX_da$SEK_USD

T <- length(us_CPI)
us_I <- log(us_CPI[-1]/us_CPI[-T])
swed_I <- log(swed_CPI[-1]/swed_CPI[-T])
e_sek <- log(S_sek[-1]/S_sek[-T])
inf_d <- swed_I - us_I

plot(e_sek, inf_d, col="blue", ylab ="(I_d - I_f)", xlab ="e_f")
title("SEK/USD: Inflation rate differential vs Changes in S_t")
```

PPP: Relative Version – Visual Evidence

1. Visual Evidence

Plot $(I_{SEK} - I_{USD})$ against $e_{f,t}$ (SEK/USD), using monthly data 1973-2024.

Check to see if there is a 45° PPP line: $e_{f,t} \approx I_d - I_f$



Conclusion: Plot shows a line. Maybe a 45° line ⇒ Visual evidence OK?.

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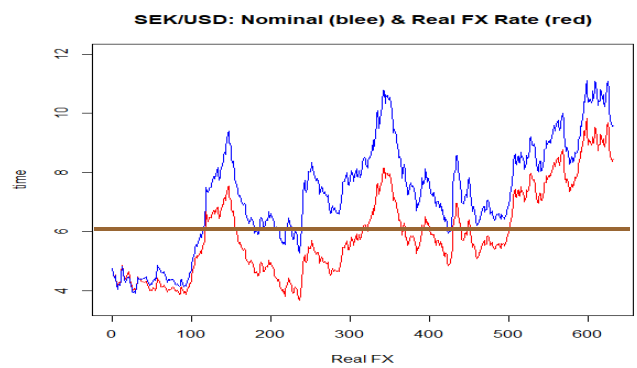
PPP: Relative Version – Visual Evidence

• Relative PPP: General Evidence

1. Visual Evidence

Is $\mathbf{R_t}$ close to being constant?



In general, we have some evidence for mean reversion, though slow, for $\mathbf{R_t}$.

PPP: Relative Version – Statistical Evidence

2. Statistical Evidence

We use a regression. Recall that, on average, $e_{f,t} \approx e_{f,t}^{PPP} \approx I_d - I_f$

$$e_{f,t} = (S_{t+T} - S_t)/S_t = \alpha + \beta\,(I_d - I_f)_t + \varepsilon_t,$$

The null hypothesis is: H_0 (Relative PPP true): $\alpha=0$ and $\beta=1$

H_1 (Relative PPP not true): $\alpha\neq0$ and/or $\beta\neq1$

Tests:

- *t*-test (individual tests: H_0: $\alpha=0$ and H_0: $\beta=1$)
- *Wald*-test (joint test: H_0 (Relative PPP true): $\alpha=0$ and $\beta=1$)

• Adequacy of Model:

But before testing PPP, we check the adequacy of model: outliers, multicollinearity, normality of residuals, heteroscedasticity, autocorrelation, & structural change.

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PPP: Relative Version – Regression

Example: SEK/USD

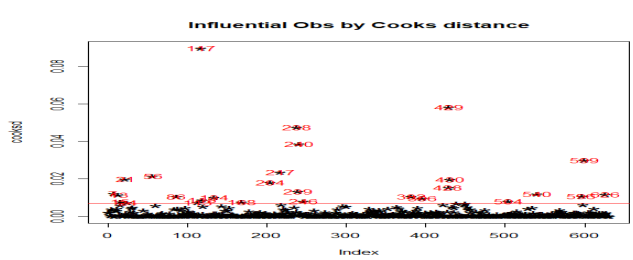
```
fit_ppp <- lm(e_sek ~ inf_d)                    # PPP-based regression
> summary(fit_ppp)

Coefficients:
            Estimate    Std. Error t value Pr(>|t|)
(Intercept) 0.0010342  0.0009983  1.036   0.3006
inf_d       0.4027145  0.1813419  2.221   0.0267 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.02506 on 629 degrees of freedom
Multiple R-squared:  0.00778, Adjusted R-squared:  0.006202
F-statistic: 4.932 on 1 and 629 DF,  p-value: 0.02672
```

Conclusion: Low R^2, high F-stat and t-stats. Model seems just OK.

PPP: Relative Version – Outliers

Example: Calculate and Plot Cook's D

```
dat_xy <- data.frame(e_sek, inf_d)              #R data frame used to show influential obs
cooksd <- cooks.distance(fit_ppp)               # Cook's distance
# plot cook's distance
plot(cooksd, pch="*", cex=2, main="Influential Obs by Cooks distance")
# add cutoff line
abline(h = 4*mean(cooksd, na.rm=T), col="red")  # add cutoff line
# add labels
text(x=1:length(cooksd)+1,  y=cooksd,  labels=ifelse(cooksd>4*mean(cooksd,  na.rm=T),
    names(cooksd),""), col="red")  # add labels
```



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PPP: Relative Version – Outliers

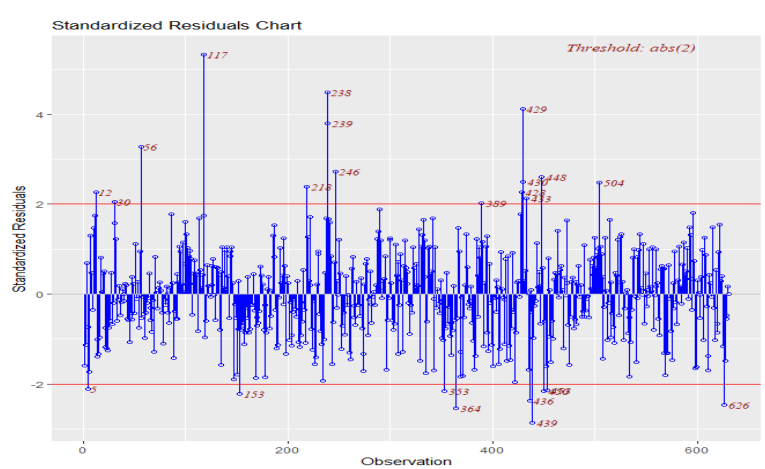
Example: Calculate Rule of Thumbs

```
library(olsrr)                          # need to install package olsrr
x_resid <- residuals(fit_ppp)           # extract residuals from lm (mod)
x_stand_resid <- x_resid/sd(x_resid)    # standardized residuals
> sum(x_stand_resid > 2)                # Rule of thumb count (5% is OK)
[1] 15                                  ⇒ very low number 15/631 = 0.0238
 x_lev <- ols_leverage(fit_ppp)         # leverage residuals
> sum(x_lev > (2*k+2)/T)                # Rule of thumb count (5% is OK)
[1] 24                                  ⇒ low number 24/631 = 0.0380
> sum(cooksd > 4/T)                     # Rule of thumb count (5% is OK)
[1] 30                                  ⇒ in the margin: 30/631 = 0.0475

# plots
ols_plot_resid_stand(fit_ppp)           # Plot standardized residuals
ols_plot_dffits(fit_ppp)                # Plot Difference in fitted values
```

PPP: Relative Version – Outliers

Example: Plot standardized residuals & Dffits:



Conclusion: Overall, some evidence, but not overwhelming at 5% level.

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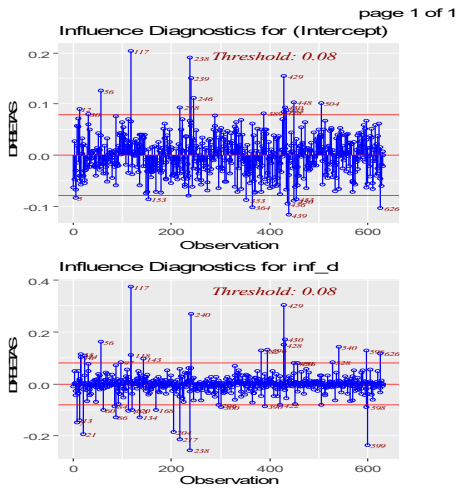
PPP: Relative Version – Outliers

Example: Plot standardized residuals & Dffits:



Conclusion: Overall, some evidence, but not overwhelming at 5% level.

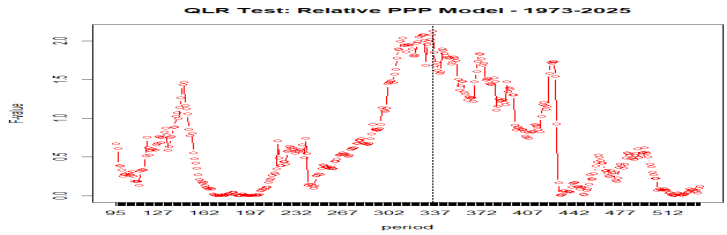
PPP: Relative Version – Structural Change

Example:

```
library(desk)
pie <- .15
T0 <- round(T * pie)
T1 <- round(T *(1-pie))
my.qlr <- qlr.test(fit_ppp, from = T0, to = T1, sig.level = 0.05, details = TRUE)
> max(my.qlr$f.stats)
[1] 2.127649
```

⇒ low sup-F test, Andrew's critical value = 5.86. Reject H_0.

```
>plot(my.qlr, col = "red", main = "QLR Test: Relative PPP Model - 1973-2024") # Plot test results
```



Conclusion: No evidence of structural break ("breaking point:" 10/2000).

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PPP: Relative Version – Normality of Residuals

Example: I will use the *jarque.bera.test* from the *tseries* package

```
library(tseries)                    # Do not forget to install package tseries
e_s <- fit_ppp$residuals
jarque.bera.test(e_s)

        Jarque Bera Test

data:  e_s
X-squared = 140.42, df = 2, p-value < 2.2e-16
```

⇒ reject normality at 5% level.

Conclusion: Residuals are not normal. Tests should be done using asymptotic distributions or, if possible, bootstraps.

PPP: Relative Version – Heteroscedasticity Tests

Example: GQ and White LM test for heteroscedasticitys

```
> gqtest(fit_ppp, fraction = .20)

    Goldfeld-Quandt test

data:  fit_ppp
GQ = 0.91723, df1 = 251, df2 = 250, p-value = 0.7527
```

⇒ cannot reject homoscedasticity (5% level)

```
> inf_d2 <- inf_d^2
> bptest(formula = fit_ppp, varformula = ~ inf_d + inf_d2)

    studentized Breusch-Pagan test

data:  fit_ppp
BP = 3.9043, df = 2, p-value = 0.142
```

⇒ cannot reject homoscedasticity at 5% level.

Conclusion: There is no evidence of structural change in the variance (GQ test) and/or general heteroscedasticity (White test).

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PPP: Relative Version – Heteroscedasticity Tests

Example: LB test for squared residuals

```
> Box.test(e_s2, lag=4, type="Ljung-Box")

        Box-Ljung test

data:  e_s2
X-squared = 33,599, df = 4, p-value = 9.005e-07 ⇒ reject homoscedasticity at 5% level.

> Box.test(e_s2, lag=12, type="Ljung-Box")

        Box-Ljung test

data:  e_s2
X-squared = 36.738, df = 12, p-value = 0.0002463  ⇒ reject homoscedasticity at 5% level.
```

Conclusion: There is evidence of time-varying heteroscedasticity (LB test with 4 & 12 lags).

PPP: Relative Version – Autocorrelation Tests

Example: LM BG test for residuals with 4 and 12 lags

```
> bgtest(fit_ppp, order=4)

        Breusch-Godfrey test for serial correlation of order up to 4

data:  fit_ppp
LM test = 96.263, df = 4, p-value < 2.2e-16          ⇒ reject no autocorrelation at 5% level.

> bgtest(fit_ppp, order=12)

        Breusch-Godfrey test for serial correlation of order up to 12

data:  fit_ppp
LM test = 103.82, df = 12, p-value = 2.2e-16          ⇒ reject no autocorrelation at 5% level.
```

Conclusion: There is strogn evidence for autocorrelation using 4 and 12 lags.

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PPP: Relative Version – DW Tests

Example: DW test for first-order --AR(1)– autocorrelation

```
> dwtest(fit_ppp)

        Durbin-Watson test

data:  fit_ppp
DW = 1.2686, p-value = 2.2e-16
alternative hypothesis: true autocorrelation is greater than 0
```

⇒ reject no autocorrelation at 5% level

Conclusion: There is evidence for AR(1) autocorrelation.

PPP: Relative Version – Individual tests

Example: Individual tests: 2 t-tests

```
fit_ppp <- lm(e_sek ~ inf_d)
> summary(fit_ppp)

Coefficients:
            Estimate   Std. Error  t value Pr(>|t|)
(Intercept) 0.0010342  0.0009983   1.036   0.3006
inf_d       0.4027145  0.1813419   2.221   0.0267 *

Residual standard error: 0.03084 on 594 degrees of freedom
Multiple R-squared:  3.883e-06, Adjusted R-squared:  -0.00168
F-statistic: 0.002307 on 1 and 594 DF,  p-value: 0.9617
```

⇒ α is not significant at 5% level

⇒ t= (0.4027145 – 1)/ 0.1813 = -3.2945

Conclusion: Reject at 5% level only H_0: β=1. Relative PPP is rejected for SEK/USD by the individual tests.

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PPP: Relative Version – Joint Tests

Example: Joint test: Wald-tests using library car

```
library(car)
> linearHypothesis(fit_ppp,c("(Intercept) = 0","inf_d = 1"),test="F")          # Exact F test
Linear hypothesis test

Hypothesis:
(Intercept) = 0
inf_d = 1

Model 1: restricted model
Model 2: e_sek ~ inf_d

  Res.Df     RSS Df Sum of Sq     F    Pr(>F)
1    631 0.40235
2    629 0.39501  2 0.0073413 11.69   0.002894 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Conclusion from joint test: Reject H_0: **α=0** & **β=1** at 5% level.

PPP: Relative Version – Joint Tests

Example:

Conclusion from individual tests: Cannot reject H_0: **α=0**; but reject H_0: **β=1** at 5% level.

Conclusion from joint tests: Reject H_0: **α=0** & **β=1** at 5% level.

From both tests, we reject Relative PPP for the SEK/USD exchange rate. This is the usual result, especially in the short-run. In the long-run, there is a debate about its validity. Researchers find that currencies with high inflation rate differentials tend to depreciate.

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PPP: Relative Version – Individual Tests: NW SE

Example:

```
library(sandwich)NW <- NeweyWest(fit_ppp, lag = 4, prewhite = FALSE)
> coeftest(fit_ppp, vcov =NW)
t test of coefficients:

             Estimate   Std. Error t value Pr(>|t|)
(Intercept) 0.0010342  0.0012738  0.8118  0.41719
inf_d       0.4027145  0.2018636  1.9950  0.04648 *
```

Conclusion from individual tests: No change in our results. Using SE that correct for autocorrelation and heteroscedasticy, we reject reject H_0: β=1 at 5% level.

PPP: Relative Version – Model and RW MSEs

Example:

```
T <- length(e_s2)
MSE_mod <- sum(e_s2[2:T])/(T-1)
> MSE_mod
[1] 0.0006244687

> e_RW <- e_sek[2:T]                    # Any change is a surprise under RWM
> MSE_RW <- sum(e_RW^2)/(T-1)
> MSE_RW
[1] 0.0006308117
```

Conclusion: The model does better than slightly the RW model in sample.

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PPP: Relative Version – Augmented PPP model

Example: We augment the PPP Model with the FF factors, RF, and log changes in USD Index, crude oil prices and gold prices.

```
fit_ppp_aug <- lm(e_sek ~ inf_d  + Mkt_RF + SMB + HML + RF + oil + gold + cons)
summary(fit_ppp_aug)
```

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	0.0011782	0.0015798	0.746	0.45607	
inf_d	0.3557359	0.1763612	**2.017**	0.04412	*
Mkt_RF	-0.0010098	0.0002240	**-4.509**	7.80e-06	***
SMB	0.0005324	0.0003319	1.604	0.10918	
HML	-0.0001775	0.0003219	-0.551	0.58158	
RF	0.0031096	0.0034344	0.905	0.36558	
oil	-0.0330431	0.0105355	**-3.136**	0.00179	**
gold	-0.0766564	0.0171783	**-4.462**	9.62e-06	***
cons	0.0361448	0.0191314	1.889	0.05932	.

Residual standard error: 0.002623 on 597 degrees of freedom

Multiple R-squared: **0.8078**, Adjusted R-squared: 0.8052

F-statistic: 313.6 on 8 and 597 DF, p-value: < 2.2e-16

PPP: Relative Version – Augmented PPP model

Example: We use NW SE to draw inferences and select driving variables:

```
> NW <- NeweyWest(fit_ppp_aug, lag = 12, prewhite = FALSE)
> coeftest(fit_ppp_aug, vcov =NW)
```

t test of coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	0.00117823	0.00217760	0.5411	0.588655	
inf_d	0.35573592	0.20466202	1.7382	0.082677	.
Mkt_RF	-0.00100975	0.00035792	**-2.8212**	0.004938	**
SMB	0.00053239	0.00030037	1.7725	0.076809	.
HML	-0.00017747	0.00039352	-0.4510	0.652168	
RF	0.00310963	0.00456654	0.6810	0.496150	
oil	-0.03304310	0.02031665	-1.6264	0.104370	
gold	-0.07665639	0.01672743	**-4.5827**	5.549e-06	***
cons	0.03614476	0.02443669	1.4791	0.139615	

<u>Conclusion</u>: Now, Mkt_RF and gold are the drivers at the 5% level. We decide to keep SMB and inf_d, since they're significant at 10%.

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PPP: Relative Version – Specific Model

Example: We estimate the reduced augmented (specific) model

```
fit_ppp_red <- lm(e_sek ~ inf_d + Mkt_RF + SMB + gold)
> summary(fit_ppp_red)
Coefficients:
              Estimate     Std. Error t value Pr(>|t|)
(Intercept)   0.0020817  0.0009800   2.124   0.0340 *
inf_d         0.4474543  0.1756582   2.547   0.0111 *
Mkt_RF       -0.0009741  0.0002169  -4.491 8.45e-06 ***
SMB           0.0005403  0.0003295   1.640   0.1015
gold         -0.0846981  0.0171251  -4.946 9.75e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.02424 on 626 degrees of freedom
Multiple R-squared:  0.07585,  Adjusted R-squared:  0.0699
```

⇒ MSE = **005826 < MSE(RW) = 0.00631**

Note: Now, only SMB is not significant at 5% level.

Relative PPP: Specific Model - Estimation Period

Example: Estimation period estimation (1973:Feb - 2019:Dec)

```
y <- e_sek
xx <- cbind(inf_d, Mkt_RF, SMB, gold)
T0 <- 1
T1 <- 552                          # End of Estimation Period (Dec 2019)
T2 <- T1+1                         # Start of Validation Period (Jan 2020)
y1 <- y[T0:T1]
x1 <- xx[T0:T1,]
fit_ppp_est <- lm(y1~ x1)          # Estimation Period Regression
b_est <- fit_ppp_est$coefficients  # Extract OLS coefficients from regression
summary(fit_ppp_est)
```

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Relative PPP: Specific Model - Estimation Period

Example: Estimation period estimation (1973:Feb - 2019:Dec)

```
> summary(fit_ppp_est)

Coefficients:
              Estimate     Std. Error t value Pr(>|t|)
(Intercept)   0.0020058   0.0010419   1.925  0.05471 .
x1inf_d       0.5308196   0.1898041   2.797  0.00534 **
x1Mkt_RF     -0.0010554 0.0002351  -4.489 8.71e-06 ***
x1SMB         0.0007016   0.0003553   1.974  0.04882 *
x1gold       -0.0800852  0.0177286  -4.517 7.65e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.002545 on 547 degrees of freedom
Multiple R-squared:  0.8148,    Adjusted R-squared:  0.8135
F-statistic: 601.7 on 4 and 547 DF,  p-value: < 2.2e-16
```

Relative PPP: Specific Model – AR Model for X_1

Example: AR(1) for Independent Variables - $(\boldsymbol{I_d} - \boldsymbol{I_f})_t$

```
x1_1 <- xx[1:(T1-1),1]                    # Estimation period data: (I_d - I_f)_t-1
x1_0 <- xx[2:T1,1]                        # Estimation period data: (I_d - I_f)_t
fit_1 <-lm(x1_0 ~ x1_1)                   # AR(1) for (I_d - I_f)_t
b_1 <- fit_1$coefficients                 # Extract AR(1) coefficients
> summary(fit_1)
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.0002116  0.0002285   0.926  0.35492
x1_1        0.1138200  0.0419829   2.711  0.00691 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.005412 on 560 degrees of freedom
Multiple R-squared:  0.01296,  Adjusted R-squared:  0.01119
F-statistic:  7.35 on 1 and 560 DF,  p-value: 0.006912
```

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Relative PPP: Specific Model – AR Model for X_2

Example: AR(1) for Independent Variables - Mkt_RF_t

```
x2_1 <- xx[1:(T1-1),2]                    # Estimation period data: Mkt_RF_t-1
x2_0 <- xx[2:T1,2]                        # Estimation period data: Mkt_RF_t
fit_2 <- lm(x2_0 ~ x2_1)                  # AR(1) for Mkt_RF_t
b_2 <- fit_2$coefficients
> summary(fit_2)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.55311    0.19174   2.885  0.00407 **
x2_1        0.05300    0.04215   1.257  0.20919
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.51 on 560 degrees of freedom
Multiple R-squared:  0.002815, Adjusted R-squared:  0.001034
F-statistic: 1.581 on 1 and 560 DF,  p-value: 0.2092
```

Relative PPP: Specific Model – AR Model for X_3

Example: AR(1) for Independent Variables – SMB_t

```
x3_1 <- xx[1:(T1-1),3]                    # Estimation period data: (RF)_t-1
x3_0 <- xx[2:T1,3]                        # Estimation period data: (RF)_t
fit_3 <-lm(x3_0 ~ x3_1)                   # AR(1) for (RF)_t
b_3 <- fit_3$coefficients
> summary(fit_3)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.199690   0.126007   1.585    0.114
x3_1       -0.005775   0.042187  -0.137    0.891

Residual standard error: 2.981 on 560 degrees of freedom
Multiple R-squared:  3.346e-05,        Adjusted R-squared:  -0.001752
F-statistic: 0.01874 on 1 and 560 DF,  p-value: 0.8912
```

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Relative PPP: Specific Model – AR Model for X_4

Example: AR(1) for Independent Variables - $gold_t$

```
x4_1 <- xx[1:(T1-1),4]                    # Estimation period data: oil_{t-1}
x4_0 <- xx[2:T1,4]                        # Estimation period data: oil_t
fit_4 <-lm(x4_0 ~ x4_1)                   # AR(1) for oil_t
b_4 <- fit_4$coefficients
> summary(fit_4)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.005234   0.002447   2.139   0.0329 *
x4_1        -0.006956  0.041946  -0.166   0.8683
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.05775 on 560 degrees of freedom
Multiple R-squared:  4.911e-05,        Adjusted R-squared:  -0.001737
F-statistic: 0.0275 on 1 and 560 DF,  p-value: 0.8683
```

Relative PPP: Specific Model – Forecasts

Example: Validation period forecast for S_t

```
T_val <- T1 + 1                                   # Start of Validation period
xx_cons <- rep(1,T-T_val+1)
xx1_0 <- cbind(xx_cons,xx[T_val:T,1]) %*% b_1
xx2_0 <- cbind(xx_cons,xx[T_val:T,2]) %*% b_2
xx3_0 <- cbind(xx_cons,xx[T_val:T,3]) %*% b_3
xx4_0 <- cbind(xx_cons,xx[T_val:T,4]) %*% b_4

k_for <- T - T_val+1                              # Number of forecasts
e_sek_mod_0 <- cbind(xx_cons,xx1_0,xx2_0,xx3_0,xx4_0)%*%b_est  # Forecast for e_{f,t}
S_mod_f0 <- S[T1:(T-1)] * (1 + e_sek_mod_0)       # Forecast for S_t
e_mod_f0 <- S[T_val:T] - S_mod_f0                 # Forecasat error_{Mod,t} = S_t – Ŝ_t
mse_e_f0 <- sum(e_mod_f0^2)/k_for                 # MSE
> mse_e_f0
[1] 0.05644962
```

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Relative PPP: AR(1) Model – Estimation Period

Example: AR(1) for $e_{f,t}$

```
y_1 <- y[1:(T1-1)]    # Estimation period data: e_{f,t-1}
y_0 <- y[2:T1]                                  # Estimation period data: e_{f,t}
fit_y <- lm(y_0 ~ y_1)                          # AR(1) for e_{f,t}
b_y <- fit_y$coefficients
> summary(fit_y)
Coefficients:
            Estimate     Std. Error t value Pr(>|t|)
(Intercept) 0.0008138  0.0009887   0.823    0.411
y_1         0.3791174  0.0390361   9.712    <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.02341 on 560 degrees of freedom
Multiple R-squared:  0.1442,    Adjusted R-squared:  0.1426
F-statistic: 94.32 on 1 and 560 DF,  p-value: < 2.2e-16
```

Relative PPP: AR(1) Model – Forecasts

Example: AR(1) for $e_{f,t}$

```
y_f0 <-  cbind(xx_cons,y[T_val:T])%*% b_y          # Forecast for e_{f,t}
S_ar1_f0 <- S[T1:(T-1)] * (1 + y_f0)               # Forecast for S_t
e_ar1_f0 <- S[T_val:T] - S_ar1_f0                  # Forecasat error e_{AR,t} = S_t – Ŝ_t
mse_e_ar1_f0 <- sum(e_ar1_f0^2)/k_for              # MSE
> mse_e_ar1_f0
[1] 0.02182967
```

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Relative PPP: Random Walk Model – Forecasts

Example: Random Walk Model for $e_{f,t}$

```
e_rw_f0 <- S[T_val:T] - S[T1:(T-1)]                    # Error for RW model e_RW,t = S_t – S_t-1
mse_e_rw_f0 <- sum(e_rw_f0^2)/k_for
> mse_e_rw_f0
[1] 0.05669124
```

Relative PPP: MGN/HLN Tests

Example: Testing accuracy of forecasts.

1) Mod vs RW

```
z_mgn <- e_rw_f0 + e_mod_f0
x_mgn <- e_rw_f0 - e_mod_f0
fit_mgn <- lm(z_mgn ~ x_mgn)
> summary(fit_mgn)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.1479     0.1424  -1.039    0.303
x_mgn       11.7484    10.8941   1.078    0.285

Residual standard error: 0.4784 on 66 degrees of freedom
Multiple R-squared:  0.01732, Adjusted R-squared:  0.002427
F-statistic: 1.163 on 1 and 66 DF,  p-value: 0.2848
```

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Relative PPP: MGN/HLN Tests

Example: Testing accuracy of forecasts.

1) AR(1) vs RW

```
z_mgn <- e_rw_f0 + e_ar1_f0
x_mgn <- e_rw_f0 - e_ar1_f0
fit_mgn <- lm(z_mgn ~ x_mgn)
> summary(fit_mgn)

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.0365283  0.0008821  -41.41   <2e-16 ***
x_mgn        4.2631360  0.0097155  438.80   <2e-16 ***        ⇒ significant: AR(1)'s MSE is lower!
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.007248 on 66 degrees of freedom
Multiple R-squared:  0.9997, Adjusted R-squared:  0.9997
F-statistic: 1.925e+05 on 1 and 66 DF,  p-value: < 2.2e-16
```

Relative PPP: MGN/HLN Tests

Example: Testing accuracy of forecasts.

3) Mod vs AR(1)

```
z_mgn <- e_mod_f0 + e_ar1_f0
x_mgn <- e_mod_f0 - e_ar1_f0
fit_mgn <- lm(z_mgn ~ x_mgn)
> summary(fit_mgn)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.002487   0.002332   1.066     0.29
x_mgn       4.279208   0.025926 165.058   <2e-16 ***      ⇒ significant: AR(1)'s MSE is lower!
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.01921 on 66 degrees of freedom
Multiple R-squared:  0.9976, Adjusted R-squared:  0.9975
F-statistic: 2.724e+04 on 1 and 66 DF,  p-value: < 2.2e-16
```

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