Project: Testing PPP and Forecasting Exchange Rates

Review - Purchasing Power Parity (PPP)

• Notation and definitions:
The exchange rate is a price: The relative price of two currencies.

Example: On October 30, 2024, the price of a euro (EUR) in terms of USD was USD 1.09 per EUR $\Rightarrow$ EUR 1 = USD 1.09

Notation: S_t = Exchange rate = 1.09 USD/EUR.

We use the "direct notation" for S_t:
S_t = Units of domestic currency per unit of foreign currency.

The percentage change at time t of S_t is $e_{f,t}$. As usual, we will define $e_{f,t}$ with the log change of S_t.

Review - Purchasing Power Parity (PPP)

• **PPP** is based on the **law of one price** (LOOP): Baskets of goods, once denominated in the same currency, should have the same price. If they are not, then traders can profit with pseudo-arbitrage strategies.

Absolute version of PPP:

The FX rate between two currencies is simply the ratio of the two countries' general price levels:

$$S_t^{PPP} = \text{Domestic Price level / Foreign Price level} = P_d/P_f$$

Relative PPP

The rate of change in the prices of products should be similar when measured in a common currency (as long as trade frictions are unchanged):

$$e_{f,t} \approx e_{f,t}^{PPP} \approx I_d - I_f$$

PPP: Equilibrium Exchange Rate

PPP Notes :

◊ PPP gives an *equilibrium* exchange rate. Equilibrium will be reached when there is no trade in the basket (because of mispricing). That is, when the PPP holds for the same basket.

◊ PPP is telling what S_t *should be* (in equilibrium). It is not telling what S_t *is* in the market today.

Testing PPP:

Absolute version: Easily Rejected. The assumptions behind Relative PPP are seen as questionable, especially, no trade frictions.

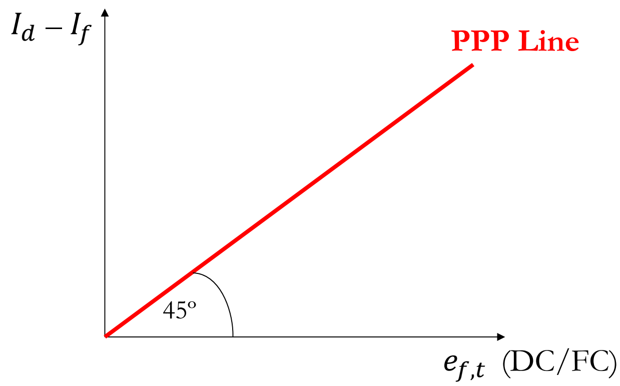
Relative version: This is what the first part of the project does.

- Visual tests (check scatter plot of $e_{f,t}$ vs $(I_d - I_f)$ & plot R_t over time)
- Statistical tests

PPP: Relative Version

- If Relative PPP holds, we should see a 45º line (PPP Line) when we plot $e_{f,t}$ & $I_d - I_f$.

Recall Relative PPP: $e_{f,t}^{PPP} \approx I_d - I_f$



PPP: Relative Version

- We define the Real exchange rate, R_t:

$$R_t = \frac{S_t * P_f}{P_d}$$

If $R_t = 1$, foreign goods, once translated to domestic currency, have the same price as domestic goods. If $R_t > 1$ ($R_t < 1$), foreign goods, once translated to DC, are more (less) expensive than domestic goods.

- Under relative PPP, R_t should be constant. (Under Absolute PPP, $R_t = 1$).

We usually associate R_t with under/over-valuation:

- If R_t is over the mean, we consider the foreign currency overvalued (under Absolute PPP, if $R_t > 1$).

- If R_t is under the mean, we consider the domestic currency overvalued (under Absolute PPP, if $R_t < 1$).

PPP: Relative Version – Visual Evidence

• Data: Monthly Swedish & U.S. data (1/1973 - 7/2024): CPI and S_t

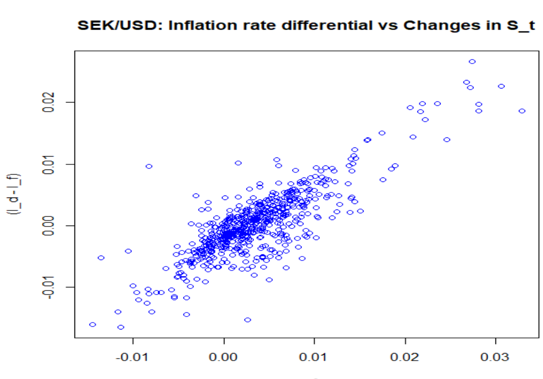
```
FMX_da <- read.csv("http://www.bauer.uh.edu/rsusmel/4397/ppp_2020_m.csv", head=TRUE, sep=",")
x_date <- FMX_da$Date
us_CPI <- FMX_da$US_CPI
swed_CPI <- FMX_da$SWED_CPI
S_sek <- FMX_da$SEK_USD

T <- length(us_CPI)
us_I <- log(us_CPI[-1]/us_CPI[-T])
swed_I <- log(swed_CPI[-1]/swed_CPI[-T])
e_sek <- log(S_sek[-1]/S_sek[-T])
inf_d <- swed_I - us_I

plot(e_sek, inf_d, col="blue", ylab ="(I_d - I_f)", xlab ="e_f")
title("SEK/USD: Inflation rate differential vs Changes in S_t")
```

PPP: Relative Version – Visual Evidence

1. Visual Evidence

Plot $(I_{SEK} - I_{USD})$ against $e_{f,t}$ (SEK/USD), using monthly data 1973-2024.

Check to see if there is a 45° PPP line: $e_{f,t} \approx I_d - I_f$



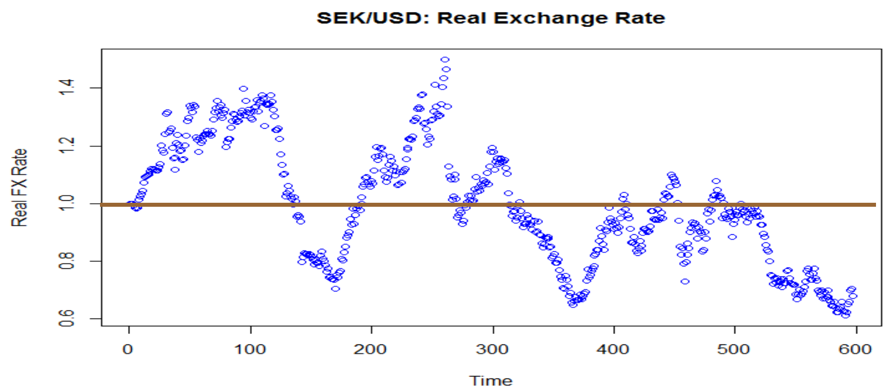
Conclusion: Plot shows a line. Maybe a 45° line ⇒ Visual evidence OK?.

PPP: Relative Version – Visual Evidence

- Relative PPP: General Evidence

1. Visual Evidence

Is $\mathbf{R}_t$ close to being constant?



In general, we have some evidence for mean reversion, though slow, for $\mathbf{R}_t$.

PPP: Relative Version – Statistical Evidence

2. Statistical Evidence

We use a regression. Recall that, on average, $e_{f,t} \approx e_{f,t}^{PPP} \approx I_d - I_f$

$$e_{f,t} = (S_{t+T} - S_t)/S_t = \alpha + \beta\,(I_d - I_f)_t + \varepsilon_t,$$

The null hypothesis is: H_0 (Relative PPP true): $\alpha=0$ and $\beta=1$
H_1 (Relative PPP not true): $\alpha\neq0$ and/or $\beta\neq1$

Tests:

- *t*-test (individual tests: H_0: $\alpha=0$ and H_0: $\beta=1$)
- *Wald*-test (joint test: H_0 (Relative PPP true): $\alpha=0$ and $\beta=1$)

- Adequacy of Model:

But before testing PPP, we check the adequacy of model: outliers, multicollinearity, normality of residuals, heteroscedasticity, autocorrelation, & structural change.

PPP: Relative Version – Regression

Example: SEK/USD

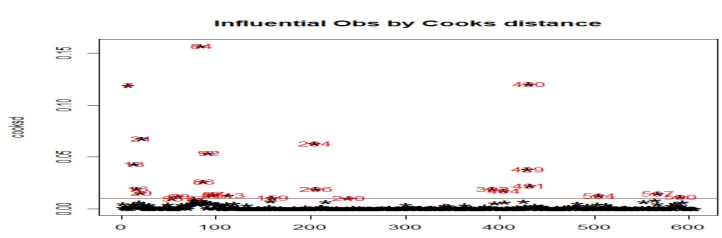
```
fit_ppp <- lm(e_sek ~ inf_d)                    # PPP-based regression
> summary(fit_ppp)

Coefficients:
             Estimate   Std. Error t value Pr(>|t|)
(Intercept) 0.0032676  0.0001351   24.19   <2e-16 ***
inf_d       0.8960097  0.0245413   36.51   <2e-16 ***

Residual standard error: 0.003321 on 604 degrees of freedom
Multiple R-squared:  0.6882, Adjusted R-squared:  0.6877
F-statistic:  1333 on 1 and 604 DF,  p-value: < 2.2e-16
```

Conclusion: High R^2, F-stat and t-stats. A good sign for the Relative PPP model.

PPP: Relative Version – Outliers

Example: Calculate and Plot Cook's D

```
dat_xy <- data.frame(e_sek, inf_d)              #R data frame used to show influential obs
cooksd <- cooks.distance(fit_ppp)               # Cook's distance
# plot cook's distance
plot(cooksd, pch="*", cex=2, main="Influential Obs by Cooks distance")
# add cutoff line
abline(h = 4*mean(cooksd, na.rm=T), col="red")   # add cutoff line
# add labels
text(x=1:length(cooksd)+1, y=cooksd, labels=ifelse(cooksd>4*mean(cooksd, na.rm=T),
   names(cooksd),""), col="red")  # add labels
```



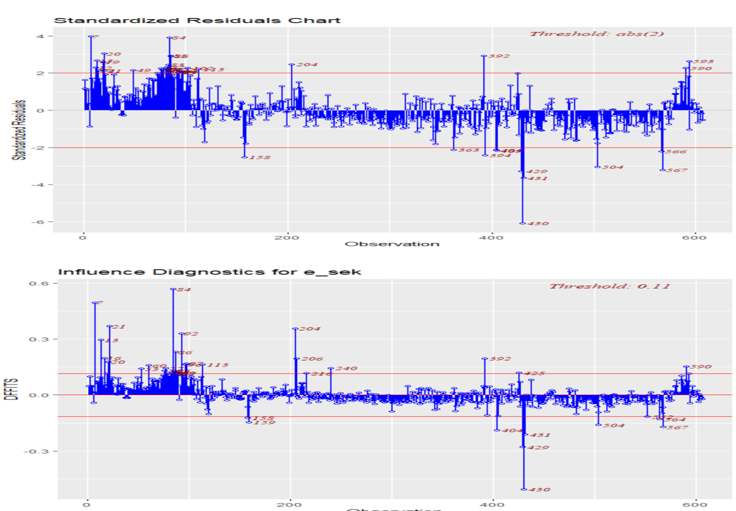
PPP: Relative Version – Outliers

Example: Calculate Rule of Thumbs

```
library(olsrr)                                  # need to install package olsrr
x_resid <- residuals(fit_ppp)                   # extract residuals from lm (mod)
x_stand_resid <- x_resid/sd(x_resid)            # standardized residuals
> sum(x_stand_resid > 2)                        # Rule of thumb count (5% is OK)
[1] 30                                ⇒ very low number 17/596 = 0.04942339
 x_lev <- ols_leverage(fit_ppp)                 # leverage residuals
> sum(x_lev > (2*k+2)/T)                        # Rule of thumb count (5% is OK)
[1] 25                                ⇒ low number 26/596 = 0.0118616
> sum(cooksd > 4/T)                             # Rule of thumb count (5% is OK)
[1] 37                                ⇒ in the margin: 33/596 = 0.06095552

# plots
ols_plot_resid_stand(fit_ppp)                   # Plot standardized residuals
ols_plot_dffits(fit_ppp)                        # Plot Difference in fitted values
```

PPP: Relative Version – Outliers

Example: Plot standardized residuals & Dffits:



Conclusion: Overall, some evidence, but not overwhelming at 5% level.

PPP: Relative Version – Structural Change

Example:

library(desk)

pie <- .15

T0 <- round(T * pie)

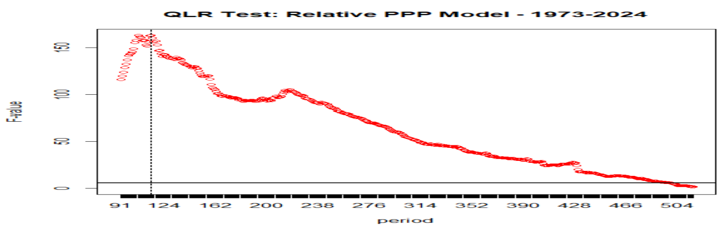
T1 <- round(T *(1-pie))

my.qlr <- qlr.test(fit_ppp, from = T0, to = T1, sig.level = 0.05, details = TRUE)

> max(my.qlr$f.stats)

[1] 162.9835 ⇒ high sup-F test, Andrew's critical value = 5.86. Reject H_0.

>plot(my.qlr, col = "red", main = "QLR Test: Relative PPP Model - 1973-2024") # Plot test results



Conclusion: Strong evidence of structural break.

PPP: Relative Version – Normality of Residuals

Example: I will use the *jarque.bera.test* from the *tseries* package

library(tseries) # Do not forget to install package tseries

e_s <- fit_ppp$residuals

jarque.bera.test(e_s)

Jarque Bera Test

data: e_s

X-squared = **304.98**, df = 2, p-value < 2.2e-16 ⇒ reject normality at 5% level.

Conclusion: Residuals are not normal. Tests should be done using asymptotic distributions or, if possible, bootstraps.

PPP: Relative Version – Heteroscedasticity Tests

Example: LB test for squared residuals

```
> Box.test(e_s2, lag=4, type="Ljung-Box")

	Box-Ljung test

data:  e_s2
X-squared = 11.476, df = 4, p-value = 0.02171
```

⇒ reject homoscedasticity at 5% level.

```
> Box.test(e_s2, lag=12, type="Ljung-Box")

	Box-Ljung test

data:  e_s2
X-squared = 15.417, df = 12, p-value = 0.2194
```

⇒ cannot reject homoscedasticity at 5% level.

Conclusion: There is some evidence of structural change in the variance (GQ test) and time-varying heteroscedasticity (LB test with 4 lags).

PPP: Relative Version – Heteroscedasticity Tests

Example:. White Test

```
e_s2 <- e_s^2                              # Step 2 – squared residuals
inf_d2 <- inf_d^2
fit_W <- lm (e_s2 ~ inf_d + inf_d2)        # Step 2 – Auxiliary regression
b_W <- fit_W$coefficients
m_df <- length(b_W) – 1                    # degrees of freedom
Re_2 <- summary(fit_W)$r.squared           # Step 2 – Keep R^2 from Auxiliary reg
LM_W_test <- Re_2 * T                      # Step 3 – Compute LM Test: R^2 * T
LM_W_test
16.01068
p_val <- 1 - pchisq(LM_W_test, df = m_df)  # p-value of LM_test
p_val
[1] 0.0003336764
```

⇒ reject homoscedasticity at 5% level.

PPP: Relative Version – Heteroscedasticity Tests

Example: BP Test & GQ Test

```
library(lmtest)
> bptest(fit_ppp)

	studentized Breusch-Pagan test

data:  fit_ppp
BP = 2.834, df = 1, p-value = 0.09229
```

⇒ Reject homoscedasticity at 5% level.

```
> gqtest(fit_ppp, fraction = .20)

	Goldfeld-Quandt test

data:  fit_ppp
GQ = 0.8636, df1 = 241, df2 = 240, p-value = 0.8719
```

⇒ cannot reject H_0 at 5% level.

```
alternative hypothesis: variance increases from segment 1 to 2
```

PPP: Relative Version – Autocorrelation Tests

Example: LM BG test for residuals with 4 and 12 lags

```
> bgtest(fit_ppp, order=4)

	Breusch-Godfrey test for serial correlation of order up to 4

data:  fit_ppp
LM test = 273.75, df = 4, p-value < 2.2e-16
```

⇒ cannot reject no autocorrelation at 5% level.

```
> bgtest(fit_ppp, order=12)

	Breusch-Godfrey test for serial correlation of order up to 12

data:  fit_ppp
LM test = 298.2, df = 12, p-value = 2.2e-16
```

⇒ cannot reject no autocorrelation at 5% level.

Conclusion: There is stogn evidence for autocorrelation using 4 and 12 lags.

PPP: Relative Version – DW Tests

Example: DW test for first-order --AR(1)– autocorrelation

```
> dwtest(fit_ppp)

        Durbin-Watson test

data:  fit_ppp
DW = 0.70747, p-value = 2.2e-16          ⇒ reject no autocorrelation at 5% level
alternative hypothesis: true autocorrelation is greater than 0
```

Conclusion: There is evidence for AR(1) autocorrelation.

PPP: Relative Version – Individual tests

Example: Individual tests: 2 t-tests

```
fit_ppp <- lm(e_sek ~ inf_d)
> summary(fit_ppp)

Coefficients:
              Estimate   Std. Error t value Pr(>|t|)
(Intercept) 0.0032676  0.0001351   24.19   <2e-16 ***    ⇒ α is significant at 5% level
inf_d        0.8960097  0.0245413   36.51   <2e-16 ***    ⇒ t= (0.8960097 – 1)/ 0.0245413 = -4.237

Residual standard error: 0.03084 on 594 degrees of freedom
Multiple R-squared:  3.883e-06, Adjusted R-squared:  -0.00168
F-statistic: 0.002307 on 1 and 594 DF,  p-value: 0.9617
```

Conclusion: Reject both H_0 at 5% level. Reject H_0: **$\alpha=0$** & reject H_0: **$\beta=1$**. Relative PPP is rejected for SEK/USD by the individual tests.

PPP: Relative Version – Joint Tests

Example: Joint test: Wald-tests using library car

```
library(car)
> linearHypothesis(fit_ppp,c("(Intercept) = 0","inf_d = 1"),test="F")        # Exact F test
Linear hypothesis test

Hypothesis:
(Intercept) = 0
inf_d = 1

Model 1: restricted model
Model 2: e_sek ~ inf_d

  Res.Df       RSS Df Sum of Sq      F    Pr(>F)
1    606 0.0132250
2    604 0.0066619  2  0.006563 595.03 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Conclusion from joint test: Reject H_0: **α=0** & **β=1** at 5% level.

PPP: Relative Version – Joint Tests

Example:

Conclusion from individual tests: Cannot reject H_0: **α=0**; but reject H_0: **β=1** at 5% level.

Conclusion from joint tests: Reject H_0: **α=0** & **β=1** at 5% level.

From both tests, we reject Relative PPP for the SEK/USD exchange rate. This is the usual result, especially in the short-run. In the long-run, there is a debate about its validity. Researchers find that currencies with high inflation rate differentials tend to depreciate.

PPP: Relative Version – Individual Tests: NW SE

Example:

```
library(sandwich)
NW <- NeweyWest(fit_ppp, lag = 12, prewhite = FALSE)
SE_NW <- diag(sqrt(abs(NW)))
t_NW <- b_sek/SE_NW
> SE_NW
(Intercept)      inf_d
0.001412006 0.242248013
> t_NW
(Intercept)      inf_d
 9.369341   26.553783
```

Conclusion from individual tests: No change in our results. Using SE that correct for autocorrelation and heteroscedasticy, we reject H_0: **α=0** and reject H_0: **β=1** at 5% level.

PPP: Relative Version – Model and RW MSEs

Example:

```
T <- length(e_s2)
MSE_mod <- sum(e_s2[2:T])/(T-1)
> MSE_mod
[1] 1.098698e-05

e_RW <- e_sek[2:T]                    # Any change is a "surprise" for the RW model
MSE_RW <- sum(e_RW^2)/(T-1)
> MSE_RW
[1] 4.746273e-05
```

Conclusion: The model does better than the RW model in sample.

PPP: Relative Version – Augmented PPP model

Example: We augment the PPP Model with the FF factors, RF, and log changes in USD Index, crude oil prices and gold prices.

```
fit_ppp_aug <- lm(e_sek ~ inf_d + e_usd + Mkt_RF + SMB + HML + RF + oil + gold)
> summary(fit_ppp_aug)
             Estimate   Std. Error t value Pr(>|t|)
(Intercept)  1.297e-03  1.731e-04   7.491 2.46e-13 ***
inf_d        8.944e-01  1.975e-02  45.298  < 2e-16 ***
e_usd       -1.938e-02  8.640e-03  -2.244  0.02523 *
Mkt_RF      -6.547e-05  2.473e-05  -2.647  0.00833 **
SMB          2.328e-05  3.668e-05   0.635  0.52592
HML          3.229e-05  3.539e-05   0.912  0.36190
RF           5.463e-03  3.795e-04  14.394  < 2e-16 ***
oil          1.219e-02  1.189e-03  10.253  < 2e-16 ***
gold         4.031e-03  1.898e-03   2.125  0.03403 *
---
Residual standard error: 0.002623 on 597 degrees of freedom
Multiple R-squared:  0.8078,	Adjusted R-squared:  0.8052
F-statistic: 313.6 on 8 and 597 DF,  p-value: < 2.2e-16
```

PPP: Relative Version – Augmented PPP model

Example: We use NW SE to draw inferences and select driving variables:

```
NW <- NeweyWest(fit_ppp_aug, lag = 4, prewhite = FALSE)
> SE_NW
 (Intercept)     inf_d        e_usd       Mkt_RF        SMB          HML          RF
 3.546764e-04 2.776912e-02 1.055079e-02 2.538963e-05 3.936517e-05 3.358719e-05 9.693329e-04
oil          gold
1.952399e-03 2.218349e-03 > t_NW
(Intercept)    inf_d       e_usd     Mkt_RF      SMB        HML       RF
 3.6571464  32.2100566  -1.8371615  -2.5786851  0.5912692  0.9614890  5.6359480
oil          gold
 6.2441329   1.8173024
```

Conclusion: Now, USD Index and gold are not longer significant using NW SE.

PPP: Relative Version – Specific Model

Example: We estimate the reduced augmented (specific) model

```
fit_ppp_red <- lm(e_sek ~ inf_d + Mkt_RF + RF + oil)
> summary(fit_ppp_red)
Coefficients:
              Estimate    Std. Error t value Pr(>|t|)
(Intercept)   1.334e-03  1.733e-04   7.699 5.66e-14 ***
inf_d         8.908e-01  1.970e-02  45.232  < 2e-16 ***
Mkt_RF       -5.937e-05  2.332e-05  -2.546   0.0111 *
RF            5.313e-03  3.766e-04  14.108  < 2e-16 ***
oil           1.327e-02  1.147e-03  11.569  < 2e-16 ***
---
Residual standard error: 0.002638 on 601 degrees of freedom
Multiple R-squared:  0.8042,    Adjusted R-squared:  0.8029
F-statistic:   617 on 4 and 601 DF,  p-value: < 2.2e-16
```

⇒ MSE = **46.89701e-06 < MSE(RW)**

Note: Only the Market factor is negatively related to e_sek.

Relative PPP: Specific Model - Estimation Period

Example: Estimation period estimation (1973:Feb - 2019:Dec)

```
y <- e_sek
xx <- cbind(inf_d, Mkt_RF, RF, oil)
T0 <- 1
T1 <- 552                                # End of Estimation Period (Dec 2019)
T2 <- T1+1                               # Start of Validation Period (Jan 2020)
y1 <- y[T0:T1]
x1 <- xx[T0:T1,]
fit_ppp_est <- lm(y1~ x1)                # Estimation Period Regression
b_est <- fit_ppp_est$coefficients        # Extract OLS coefficients from regression
summary(fit_ppp_est)
```


Relative PPP: Specific Model - Estimation Period

Example: Estimation period estimation (1973:Feb - 2019:Dec)

> summary(fit_ppp_est)

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)	
((Intercept)	9.264e-04	1.820e-04	5.092	4.89e-07	***
x1inf_d	8.862e-01	2.023e-02	43.814	< 2e-16	***
x1Mkt_RF	-4.813e-05	2.407e-05	-2.000	0.046	*
x1RF	5.980e-03	3.804e-04	15.719	< 2e-16	***
x1oil	1.373e-02	1.272e-03	10.793	< 2e-16	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.002545 on 547 degrees of freedom
Multiple R-squared: 0.8148, Adjusted R-squared: 0.8135
F-statistic: 601.7 on 4 and 547 DF, p-value: < 2.2e-16

Relative PPP: Specific Model – AR Model for X_1

Example: AR(1) for Independent Variables - $(\boldsymbol{I_d} - \boldsymbol{I_f})_t$

```
x1_1 <- xx[1:(T1-1),1]          # Estimation period data: (Id - If )t-1
x1_0 <- xx[2:T1,1]              # Estimation period data: (Id - If )t
fit_1 <-lm(x1_0 ~ x1_1)         # AR(1) for (Id - If)t
b_1 <- fit_1$coefficients       # Extract AR(1) coefficients
```

> summary(fit_1)

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	0.0002010	0.0002294	0.876	0.38133	
x1_1	0.1363256	0.0424153	**3.214**	**0.00139**	******

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.005378 on 549 degrees of freedom
Multiple R-squared: 0.01847, Adjusted R-squared: 0.01668
F-statistic: 10.33 on 1 and 549 DF, p-value: 0.001386

Relative PPP: Specific Model – AR Model for X_2

Example: AR(1) for Independent Variables - Mkt_RF_t

```
x2_1 <- xx[1:(T1-1),2]          # Estimation period data: Mkt_RF_{t-1}
x2_0 <- xx[2:T1,2]              # Estimation period data: Mkt_RF_t
fit_2 <- lm(x2_0 ~ x2_1)        # AR(1) for Mkt_RF_t
b_2 <- fit_2$coefficients
> summary(fit_2)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.52978    0.19429   2.727   0.0066 **
x2_1         0.05756    0.04267   1.349   0.1779 ---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.529 on 549 degrees of freedom
Multiple R-squared:  0.003304, Adjusted R-squared:  0.001488
F-statistic:  1.82 on 1 and 549 DF,  p-value: 0.1779
```

Relative PPP: Specific Model – AR Model for X_3

Example: AR(1) for Independent Variables – RF_t

```
x3_1 <- xx[1:(T1-1),3]          # Estimation period data: (RF)_{t-1}
x3_0 <- xx[2:T1,3]              # Estimation period data: (RF)_t
fit_3 <-lm(x3_0 ~ x3_1)         # AR(1) for (RF)_t
b_3 <- fit_3$coefficients
> summary(fit_3)

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.009319   0.004595   2.028    0.043 *
x3_1        0.974648   0.009609 101.430   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.06491 on 549 degrees of freedom
Multiple R-squared:  0.9493,    Adjusted R-squared:  0.9492
F-statistic: 1.029e+04 on 1 and 549 DF,  p-value: < 2.2e-16
```


Relative PPP: Specific Model – AR Model for X_4

Example: AR(1) for Independent Variables - oil_t

```
x4_1 <- xx[1:(T1-1),4]                 # Estimation period data: oil_{t-1}
x4_0 <- xx[2:T1,4]                     # Estimation period data: oil_t
fit_4 <-lm(x4_0 ~ x4_1)                # AR(1) for oil_t
b_4 <- fit_4$coefficients
> summary(fit_4)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.003697   0.003557   1.039    0.299
x4_1        0.240336   0.041433   5.801 1.12e-08 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.08338 on 549 degrees of freedom
Multiple R-squared:  0.05775,   Adjusted R-squared:  0.05603
F-statistic: 33.65 on 1 and 549 DF,  p-value: 1.117e-08
```

Relative PPP: Specific Model – Forecasts

Example: Validation period forecast for S_t

```
T_val <- T1 + 1                                        # Start of Validation period
xx_cons <- rep(1,T-T_val+1)
xx1_0 <- cbind(xx_cons,xx[T_val:T,1]) %*% b_1
xx2_0 <- cbind(xx_cons,xx[T_val:T,2]) %*% b_2
xx3_0 <- cbind(xx_cons,xx[T_val:T,3]) %*% b_3
xx4_0 <- cbind(xx_cons,xx[T_val:T,4]) %*% b_4

k_for <- T - T_val+1                                   # Number of forecasts
e_sek_mod_0 <- cbind(xx_cons,xx1_0,xx2_0,xx3_0,xx4_0)%*%b_est  # Forecast for e_{f,t}
S_mod_f0 <- S[T1:(T-1)] * (1 + e_sek_mod_0)            # Forecast for S_t
e_mod_f0 <- S[T_val:T] - S_mod_f0                      # Forecasat errore_{Mod,t} = S_t – Ŝ_t
mse_e_f0 <- sum(e_mod_f0^2)/k_for                      # MSE
> mse_e_f0
[1] 0.6500819
```

Relative PPP: AR(1) Model – Estimation Period

Example: AR(1) for $e_{f,t}$

```
y_1 <- y[1:(T1-1)]    # Estimation period data: e_f,t-1
y_0 <- y[2:T1]                                # Estimation period data: e_f,t
fit_y <- lm(y_0 ~ y_1)                        # AR(1) for e_f,t
b_y <- fit_y$coefficients
> summary(fit_y)
Coefficients:
              Estimate    Std. Error t value Pr(>|t|)
(Intercept) 0.0024118  0.0002791   8.640  < 2e-16 ***
y_1         0.2981197  0.0409273   7.284 1.13e-12 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.005634 on 549 degrees of freedom
Multiple R-squared:  0.08813,   Adjusted R-squared:  0.08647
F-statistic: 53.06 on 1 and 549 DF,  p-value: 1.131e-12
```

Relative PPP: AR(1) Model – Forecasts

Example: AR(1) for $e_{f,t}$

```
y_f0 <-  cbind(xx_cons,y[T_val:T])%*% b_y          # Forecast for e_f,t
S_ar1_f0 <- S[T1:(T-1)] * (1 + y_f0)               # Forecast for S_t
e_ar1_f0 <- S[T_val:T] - S_ar1_f0                  # Forecasat error e_AR,t = S_t – Ŝ_t
mse_e_ar1_f0 <- sum(e_ar1_f0^2)/k_for              # MSE
> mse_e_ar1_f0
[1] 0.2793324
```


Relative PPP: Random Walk Model – Forecasts

Example: Random Walk Model for $e_{f,t}$

```
e_rw_f0 <- S[T_val:T] - S[T1:(T-1)]                    # Error for RW model e_RW,t = S_t – S_t-1
mse_e_rw_f0 <- sum(e_rw_f0^2)/k_for
> mse_e_rw_f0
[1] 0.7792084
```

Relative PPP: MGN/HLN Tests

Example: Testing accuracy of forecasts.

1) Mod vs RW

```
z_mgn <- e_rw_f0 + e_mod_f0
x_mgn <- e_rw_f0 - e_mod_f0
fit_mgn <- lm(z_mgn ~ x_mgn)
> summary(fit_mgn)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.9134     0.3709   2.463   0.0171 *
x_mgn        -1.0089     1.3920  -0.725   0.4718          ⇒ not significant: similar MSEs!
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.541 on 52 degrees of freedom
Multiple R-squared:   0.01,        Adjusted R-squared:  -0.009038
F-statistic: 0.5253 on 1 and 52 DF,  p-value: 0.4718
```

Relative PPP: MGN/HLN Tests

Example: Testing accuracy of forecasts.

1) AR(1) vs RW

```
z_mgn <- e_rw_f0 + e_ar1_f0
x_mgn <- e_rw_f0 - e_ar1_f0
fit_mgn <- lm(z_mgn ~ x_mgn)
> summary(fit_mgn)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -1.74250    0.03013  -57.82   <2e-16 ***
x_mgn        5.53830    0.06455   85.80   <2e-16 ***        ⇒ significant: AR(1)'s MSE is lower!
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1094 on 52 degrees of freedom
Multiple R-squared:  0.993,                    Adjusted R-squared:  0.9929
F-statistic:  7362 on 1 and 52 DF,  p-value: < 2.2e-16
```

Relative PPP: MGN/HLN Tests

Example: Testing accuracy of forecasts.

3) Mod vs AR(1)

```
z_mgn <- e_mod_f0 + e_ar1_f0
x_mgn <- e_mod_f0 - e_ar1_f0
fit_mgn <- lm(z_mgn ~ x_mgn)
> summary(fit_mgn)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.53069    0.08875  -5.979 2.08e-07 ***
x_mgn        4.38672    0.27129  16.170  < 2e-16 ***      ⇒ significant: AR(1)'s MSE is lower!
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.5364 on 52 degrees of freedom
Multiple R-squared:  0.8341,                   Adjusted R-squared:  0.8309
F-statistic: 261.5 on 1 and 52 DF,  p-value: < 2.2e-16
```