Project: Testing PPP and Forecasting Exchange Rates

Purchasing Power Parity (PPP)

We are going to derive and, then, test a very popular model for exchange rates, the PPP Model. For us, the exchange rate, S_t, is the direct quote: units of domestic currency per one unit of foreign currency. For example, S_t = 0.75 USD/CHF means USD 0.75 = CHF 1. (USD=DC, CHF=FC.)

We say the domestic currency *appreciates* (*depreciates*) when S_t ↓(↑). That is, it cost less domestic currency to buy 1 unit of the foreign currency.

Purchasing Power Parity (PPP)

PPP is based on the law of one price (LOOP): Goods, once denominated in the same currency, should have the same price.

If they are not, then some form of arbitrage is possible.

Purchasing Power Parity (PPP)

Example: LOOP for Oil.

$P_{oil\text{-}USA}$ = **USD 80**.

$P_{oil\text{-}SWIT}$ = **CHF 160**.

$\Rightarrow$ **S_t^{LOOP}** = **USD 80** / **CHF 160** = 0.50 USD/CHF.

If S_t = 0.75 USD/CHF, then a barrel of oil in Switzerland is more expensive -once denominated in USD- than in the US:

$P_{oil\text{-}SWIT}$ (USD) = **CHF 160** * 0.75 USD/CHF = USD 120 > $P_{oil\text{-}USA}$

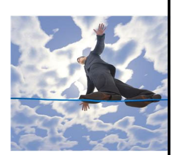
Traders will buy oil in the US (& export it to Switzerland) and sell US oil in Switzerland. Then, at the end, traders will sell CHF/buy USD.

This movement of oil from the U.S. to Switzerland will affect prices:

$P_{oil\text{-}USA}\uparrow$; $P_{oil\text{-}SWIT}\downarrow$; & $S_t\downarrow \Rightarrow$ **S_t^{LOOP}** $\uparrow$ (S_t & **S_t^{LOOP}** converge) ¶

PPP: Equilibrium Exchange Rate

LOOP Notes :



- LOOP gives an *equilibrium* exchange rate. Equilibrium will be reached when there is no trade in oil (because of pricing mistakes). That is, when the LOOP holds for oil.
- LOOP is telling what S_t *should be* (in equilibrium). It is not telling what S_t *is* in the market today.
- We have a model for S_t. This model, when applied to many goods, is the (absolute) *PPP model.*

Problem with **S_t^{LOOP}** : There are many traded goods in the economy.

Solution: Use baskets of goods.

PPP: The price of a basket of goods should be the same across countries, once denominated in the same currency. That is, USD 1 should buy the same amounts of goods here (in the U.S.) or in Colombia.

PPP: Absolute Version

Absolute version of PPP: The FX rate between two currencies is simply the ratio of the two countries' general price levels:

$$S_t^{PPP} = \text{Domestic Price level / Foreign Price level} = P_d / P_f$$

Example: Law of one price for CPIs.

CPI-basket$_{USA}$ = P_{USA} = **USD 755.3**

CPI-basket$_{SWIT}$ = P_{SWIT} = **CHF 1241.2**

$\Rightarrow S_t^{PPP}$ = **USD 755.3**/**CHF 1241.2** = 0.6085 USD/CHF.

If $S_t \neq$ 0.6085 USD/CHF, there will be trade of the goods in the basket between Switzerland and US.

Suppose S_t = 0.70 USD/CHF > S_t^{PPP}.

Then, P_{SWIT} (in USD) = **CHF 1241.2** * 0.70 USD/CHF

= **USD 868.70** > P_{USA} = **USD 755.3**

PPP: Absolute Version

Example (continuation):

P_{SWIT} (in USD) = **CHF 1241.2** * 0.70 USD/CHF

= **USD 868.70** > P_{USA} = **USD 755.3**

Potential profit: **USD 868.70** – **USD 755.3** = USD 93.40

Traders will do the following *pseudo-arbitrage* strategy:

1) Borrow USD
2) Buy the CPI-basket in the US
3) Sell the CPI-basket, purchased in the US, in Switzerland.
4) Sell the CHF/Buy USD
5) Repay the USD loan, keep the profits. ¶

Note: "Equilibrium forces" at work: 2) $P_{USA}\uparrow$ & 3) $P_{SWIT}\downarrow$ ($\Rightarrow S_t^{PPP}\uparrow$)

4) $S_t\downarrow$. ($S_t \leftrightarrow S_t^{PPP}$)

PPP: Real and Nominal Exchange Rates

• Real v. Nominal Exchange Rates

The absolute version of the PPP theory is expressed in terms of S_t, the *nominal exchange rate*.

We can modify the absolute version of the PPP relationship in terms of the *real exchange rate*, R_t. That is,

$$\mathbf{R_t} = S_t \, P_f \,/\, P_d.$$

$\mathbf{R_t}$ allows us to compare prices, translated to DC:

If $\mathbf{R_t} > 1$, foreign prices (translated to DC) are more expensive

If $\mathbf{R_t} = 1$, prices are equal in both countries –i.e., PPP holds!

If $\mathbf{R_t} < 1$, foreign prices are cheaper

Economists associate $\mathbf{R_t} > 1$ with a more efficient domestic economy.

PPP: Real and Nominal Exchange Rates

Example: Suppose a basket –the Big Mac- cost in Switzerland and in the U.S. is CHF 6.23 and USD 3.58, respectively.

P_f = **CHF 6.23**

P_d = **USD 3.58**

S_t = **1.012 USD/CHF** ⇒ P_f (in USD) = **USD 6.3048**

$\mathbf{R_t} = S_t \, P_{SWIT} \,/\, P_{US}$ = **1.012 USD/CHF** * **CHF 6.23**/**USD 3.58** = **1.7611**.

Taking the Big Mac as our basket, the U.S. is more competitive than Switzerland. Swiss prices are **76.11%** higher than U.S. prices.

To bring the economy to equilibrium –no trade in Big Macs–, we expect the USD to appreciate against the CHF.

According to PPP, the USD is *undervalued* against the CHF.

⇒ Trading signal: Buy USD/Sell CHF. ¶

PPP: Real and Nominal Exchange Rates

• The Big Mac ("Burgernomics," popularized by *The Economist*) has become a popular basket for PPP calculations. Why?

1) It is a standardized, common basket: beef, cheese, onion, lettuce, bread, pickles and special sauce. It is sold in over 120 countries.

Big Mac (Sydney)  Big Mac (Tokyo) 

2) It is very easy to find out the price.

3) It turns out, it is correlated with more complicated common baskets.

Using the CPI basket may not work well for absolute PPP. The CPI baskets can be substantially different. In theory, traders can exploit the price differentials in BMs.

PPP: Real and Nominal Exchange Rates

• In the previous example, Swiss traders can import US BMs.

From UH (US) to Rapperswill (CH)

• This is not realistic. But, the components of a BM are internationally traded. The LOP suggests that prices of the components should be the same in all markets.

The Economist reports the real exchange rate: $\mathbf{R_t} = S_t P_{BigMac,f}/P_{BigMac,d}$.

For example, for Swiss Frank (CHF): $\mathbf{R_t}$ = **6.30**/**3.58** = 1.7598

⇒ (**75.98**% overvaluation)

PPP: Absolute Version – Evidence

Example: (The Economist's) Big Mac Index (January 2011)

$R_t = S_t P_{BigMac,f} / P_{BigMac,d}$ (US=domestic) $\Rightarrow$ R_t=1 under Absolute PPP

PPP: Absolute Version – Evidence

Example (continuation): (The Economist's) Big Mac Index (June 2020)

The Big Mac index

Country		2000 —— 2020	Under/over valued, %
Switzerland	Franc		20.9
Lebanon	Pound		4.2
Sweden	Krona		0.8
United States	US$		BASE CURRENCY
Norway	Krone		-2.8
Canada	C$		-11.1
Israel	Shekel		-13.4
Euro area	Euro		-16.2
Denmark	Krone		-19.8
Australia	A$		-19.8
New Zealand	NZ$		-23.8
Uruguay	Peso		-24.2
Britain	Pound		-25.1
Singapore	S$		-25.5
Thailand	Baht		-28.6
Costa Rica	Colón		-29.3
UAE	Dirham		-29.7
Brazil	Real		-31.5
Czech Rep.	Koruna		-33.4
South Korea	Won		-34.4
Saudi Arabia	Riyal		-34.6
Kuwait	Dinar		-34.6
Bahrain	Dinar		-35.0
Sri Lanka	Rupee		-35.9
Japan	Yen		-36.3
Qatar	Riyal		-37.5
Honduras	Lempira		-38.3
Argentina	Peso		-38.5
Nicaragua	Córdoba		-38.9
Chile	Peso		-39.1
Peru	Sol		-40.4
Croatia	Kuna		-41.9

$\Rightarrow$ R_t changes over time!

$\Rightarrow$ Developed countries tend to be on top, developing countries on the bottom.

PPP: Absolute Version – Evidence

• Empirical Evidence: Simple informal test:
Test: If Absolute PPP holds $\Rightarrow$ $\mathbf{R_t} = 1$.
In the Big Mac example, PPP does not hold for the majority of countries.

Several tests of the absolute version have been performed: Absolute version of PPP, in general, fails (especially, in the short run).

• Absolute PPP: Qualifications
(1) *PPP emphasizes only trade and price levels*. Political/social factors (instability, wars), financial problems (debt crisis), etc. are ignored.

(2) Implicit assumption: *Absence of trade frictions* (tariffs, quotas, transactions costs, taxes, etc.).
Q: Realistic? On average, transportation costs add 7% to the price of U.S. imports of meat and 16% to the import price of vegetables. Many products are heavily protected, even in the U.S

PPP: Absolute Version – Qualifications

• Absolute PPP: Qualifications
Some everyday goods protected in the U.S.:
- European Roquefort Cheese, cured ham, mineral water (100%)
- Paper Clips (as high as 126.94%)
- Canned Tuna (as high as 35%)
- Synthetic fabrics (32%)
- Japanese leather (40%)
- Peanuts (shelled 131.8%, and unshelled 163.8%).
- Brooms (quotas and/or tariff of up to 32%)
- Chinese tires (35%)
- Trucks (25%) & cars (2.5%)

Some Japanese protected goods:
- Rice (778%)
- Beef (38.5%, but can jump to 50% depending on volume).
- Sugar (328%)

PPP: Absolute Version – Qualifications

• Absolute PPP: Qualifications

(3) PPP is unlikely to hold if P_f and P_d represent *different baskets*. This is why the Big Mac is a popular choice.

(4) *Trade takes time* (contracts, information problems, etc.).

(5) Internationally *non-traded (NT) goods* –i.e. haircuts, home and car repairs, hotels, restaurants, medical services, real estate. The NT good sector is big: 50%-60% of GDP (big weight in CPI basket).

Then, in countries where NT goods are relatively high, the CPI basket will also be relatively expensive. Thus, PPP will find these countries' currencies *overvalued* relative to currencies in low NT cost countries.

Note: The NT sector also has an effect on the price of T goods. For example, rent, utilities costs affect the BM price (25% is due to NT goods.)

PPP: Relative Version

Relative PPP

The rate of change in the prices of products should be similar when measured in a common currency (as long as trade frictions are unchanged):

$$e_{f,T}^{PPP} = \frac{S_{t+T} - S_t}{S_t} = \frac{(1 + I_d)}{(1 + I_f)} - 1 \qquad \text{(Relative PPP)}$$

where,

I_f = foreign inflation rate from t to t+T;

I_d = domestic inflation rate from t to t+T.

Note: $e_{f,t}^{PPP}$ is an expectation; what we expect to happen in equilibrium.

• Log linear approximation: $e_{f,t}^{PPP} \approx (I_d - I_f)_t$ $\Rightarrow$ one-to-one relation.

PPP: Relative Version

Example: From t=0 to t=1, prices increase 10% in Mexico relative to Switzerland. Then, S_t should increase 10%; say, from $S_{t=0}$=15 MXN/CHF to S_1=16.5 MXN/CHF.

If $S_{t=1}$> 16.5 MXN/CHF, then, according to PPP the CHF is overvalued. ¶

• Relative PPP: Absolute versus Relative

- Absolute PPP compares price levels.

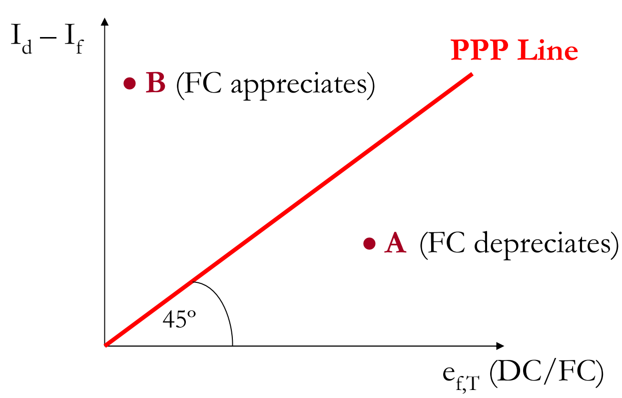
Under Absolute PPP, prices are equalized across countries: "*A mattress costs GBP 200 (= USD 320) in the U.K. and BRL 800 (=USD 320) in Brazil.*"

- Relative PPP compares price changes.

Under Relative PPP, FX rates change by the same amount as the $(I_d – I_f)_t$: "*U.K. inflation was 2% while Brazilian inflation was 8%. Meanwhile, the BRL depreciated 6% against the GBP. Then, relative cost comparison remains the same.*"

PPP: Relative Version

Under the log linear approximation, we have PPP Line



Look at point **A**: $e_{f,T} > I_d – I_f$,

⇒ Priced in FC, the domestic basket is cheaper

⇒ pseudo-arbitrage against foreign basket ⇒ FC depreciates

PPP: Relative Version – Testing

Key: On average, what we expect to happen, $e_{f,T}^{PPP}$, should happen, $e_{f,T}$.

$\Rightarrow$ On average: $e_{f,T} \approx e_{f,t}^{PPP} \approx I_d - I_f$

or $E[e_{f,T}] = E[e_{f,t}^{PPP}] \approx E[I_d - I_f]$

We use a regression to test Relative PPP:

$e_{f,T} = (S_{t+T} - S_t)/S_t = \alpha + \beta (I_d - I_f)_{t+T} + \varepsilon_{t+T}$.

Then, $E[e_{f,T}] = \alpha + \beta E[(I_d - I_f)_{t+T}] + E[\varepsilon_{t+T}] = \alpha + \beta E[e_{f,t}^{PPP}]$

$\Rightarrow E[e_{f,T}] = \alpha + \beta E[e_{f,t}^{PPP}]$

$\Rightarrow$ For Relative PPP to hold, on average, we need α=0 & β=1.

PPP: Relative Version – Visual Evidence

• Data: Monthly Swedish & U.S. data (1/1971 - 9/2020): CPI and S_t

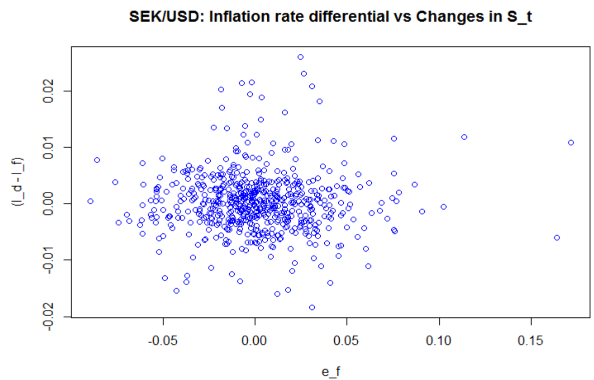
```
FMX_da <- read.csv("http://www.bauer.uh.edu/rsusmel/4397/ppp_2020_m.csv", head=TRUE, sep=",")
x_date <- FMX_da$Date
us_CPI <- FMX_da$US_CPI
swed_CPI <- FMX_da$SWED_CPI
S_sek <- FMX_da$SEK_USD

T <- length(us_CPI)
us_I <- log(us_CPI[-1]/us_CPI[-T])
swed_I <- log(swed_CPI[-1]/swed_CPI[-T])
e_sek <- log(S_sek[-1]/S_sek[-T])
inf_d <- swed_I - us_I

plot(e_sek, inf_d, col="blue", ylab ="(I_d - I_f)", xlab ="e_f")
title("SEK/USD: Inflation rate differential vs Changes in S_t")
```

PPP: Relative Version – Visual Evidence

1. Visual Evidence

Plot (I_{SEK} - I_{USD}) against $e_{f,t}$(SEK/USD), using monthly data 1971-2020.

Check to see if there is a 45° PPP line: $e_{f,T} \approx I_d - I_f$



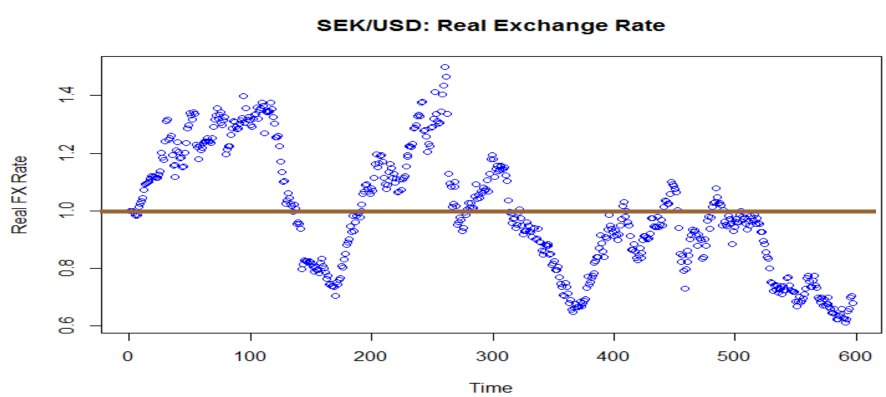
Conclusion: No PPP (45°) line ⇒ Visual evidence rejects PPP.

PPP: Relative Version – Visual Evidence

• Relative PPP: General Evidence

1. Visual Evidence

Is $\mathbf{R_t}$ close to being constant?



In general, we have some evidence for mean reversion, though slow, for $\mathbf{R_t}$.

PPP: Relative Version – Statistical Evidence

2. Statistical Evidence

We use a regression:

$e_{f,T} = (S_{t+T} - S_t)/S_t = \alpha + \beta (I_d - I_f)_{t+T} + \varepsilon_{t+T},$

The null hypothesis is: H_0 (Relative PPP true): $\alpha=0$ and $\beta=1$

H_1 (Relative PPP not true): $\alpha\neq0$ and/or $\beta\neq1$

Tests:

- *t*-test (individual tests: H_0: $\alpha=0$ and H_0: $\beta=1$)
- *Wald*-test (joint test: H_0 (Relative PPP true): $\alpha=0$ and $\beta=1$)

• Adequacy of Model:

But before testing PPP, we check the adequacy of model: outliers, multicollinearity, normality of residuals, heteroscedasticity, autocorrelation, & structural change.

PPP: Relative Version – Regression

Example: SEK/USD

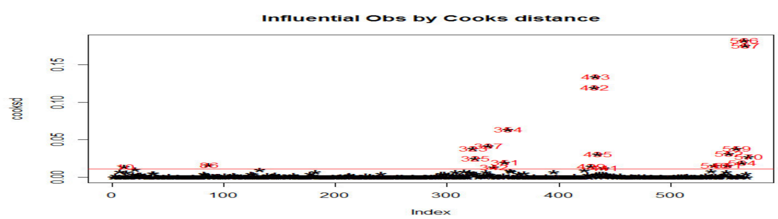
```
fit_ppp <- lm(e_sek ~ inf_d)                    # PPP-based regression
> summary(fit_ppp)

Coefficients:
              Estimate   Std. Error t value Pr(>|t|)
(Intercept) 0.0009209  0.0012648  0.728   0.467
inf_d        0.0111926  0.2330375  0.048   0.962

Residual standard error: 0.03084 on 594 degrees of freedom
Multiple R-squared:  3.883e-06, Adjusted R-squared:  -0.00168
F-statistic: 0.002307 on 1 and 594 DF,  p-value: 0.9617
```

Conclusion: Very low R^2, F-stat and t-stats. Not a good sign for the Relative PPP model.

PPP: Relative Version – Outliers

Example: Calculate and Plot Cook's D

```
dat_xy <- data.frame(e_sek, inf_d)                    #R data frame used to show influential obs
cooksd <- cooks.distance(fit_ppp)                     # Cook's distance
# plot cook's distance
plot(cooksd, pch="*", cex=2, main="Influential Obs by Cooks distance")
# add cutoff line
abline(h = 4*mean(cooksd, na.rm=T), col="red")        # add cutoff line
# add labels
text(x=1:length(cooksd)+1, y=cooksd, labels=ifelse(cooksd>4*mean(cooksd, na.rm=T),
     names(cooksd),""), col="red")  # add labels
```



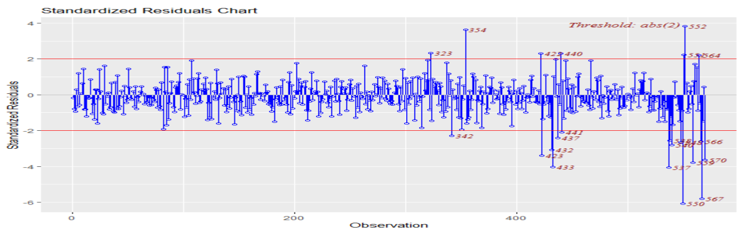
PPP: Relative Version – Outliers

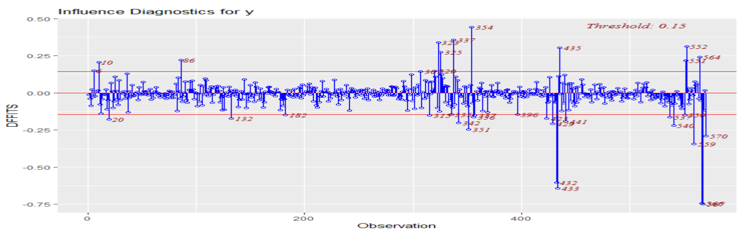
Example: Calculate Rule of Thumbs

```
library(olsrr)                                  # need to install package olsrr
x_resid <- residuals(fit_ppp)                   # extract residuals from lm (mod)
x_stand_resid <- x_resid/sd(x_resid)            # standardized residuals
> sum(x_stand_resid > 2)                        # Rule of thumb count (5% is OK)
[1] 17                                ⇒ very low number 17/596 = 0.0285234
 x_lev <- ols_leverage(fit_ppp)                 # leverage residuals
> sum(x_lev > (2*k+2)/T)                        # Rule of thumb count (5% is OK)
[1] 26                                ⇒ low number 26/596 = 0.04355109
> sum(cooksd > 4/T)                             # Rule of thumb count (5% is OK)
[1] 33                                ⇒ in the margin: 33/596 = 0.055369

# plots
ols_plot_resid_stand(fit_ppp)                   # Plot standardized residuals
ols_plot_dffits(fit_ppp)                        # Plot Difference in fitted values
```

PPP: Relative Version – Outliers

Example: Plot standardized residuals & Dffits:





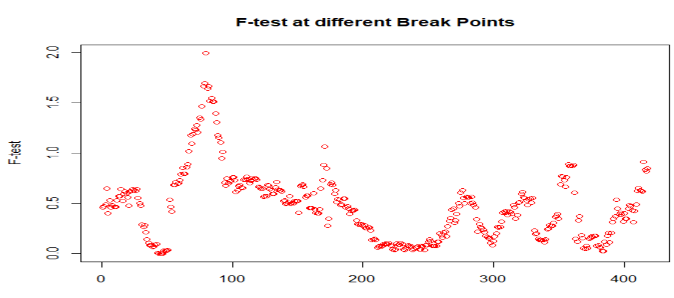
Conclusion: Overall, not a lot of evidence for outliers.

PPP: Relative Version – Structural Change

Example:

```
> plot(All_F, col="red",ylab ="F-test", xlab ="Break Point")
> title("F-test at different Break Points")
> F_max <- max(All_F)
> F_max
[1] 1.99271
```

$\Rightarrow$ low sup-F test, Andrew's critical value = 8.85. Cannot reject H_0.



Conclusion: No evidence of structural break.

PPP: Relative Version – Normality of Residuals

Example: I will use the *jarque.bera.test* from the *tseries* package

```
library(tseries)                    # Do not forget to install package tseries
jarque.bera.test(e_s)

        Jarque Bera Test

data:  e_s
X-squared = 256.77, df = 2, p-value < 2.2e-16
```

⇒ reject normality at 5% level.

Conclusion: Residuals are not normal. Tests should be done using asymptotic distributions or, if possible, bootstraps.

PPP: Relative Version – Heteroscedasticity Tests

Example:. White Test, BP Test and GQ tests

```
e_s2 <- e_s^2                              # Step 2 – squared residuals
inf_d2 <- inf_d^2
fit_W <- lm (e_s2 ~ inf_d + inf_d2)        # Step 2 – Auxiliary regression
b_W <- fit_W$coefficients
m_df <- length(b_W) – 1                    # degrees of freedom
Re_2 <- summary(fit_W)$r.squared           # Step 2 – Keep R^2 from Auxiliary reg
LM_W_test <- Re_2 * T                      # Step 3 – Compute LM Test: R^2 * T
LM_W_test
[1] 0.6002829
p_val <- 1 - pchisq(LM_W_test, df = m_df)  # p-value of LM_test
p_val
[1] 0.7407134
```

⇒ cannot reject homoscedasticity at 5% level.

PPP: Relative Version – Heteroscedasticity Tests

Example:

```
library(lmtest)
> bptest(fit_ppp)

	studentized Breusch-Pagan test

data:  fit_ppp
BP = 0.2307, df = 1, p-value = 0.631
```

⇒ cannot reject homoscedasticity at 5% level.

```
> gqtest(fit_ge, fraction = .20)

	Goldfeld-Quandt test

data:  fit_ge
GQ = 2.6911, df1 = 224, df2 = 224, p-value = 1.983e-13
```

⇒ reject homoscedasticity at 5% level.

```
alternative hypothesis: variance increases from segment 1 to 2
```

PPP: Relative Version – Heteroscedasticity Tests

Example: LB test for squared residuals

```
> Box.test(e_s2, lag=4, type="Ljung-Box")

	Box-Ljung test

data:  e_s2
X-squared = 11.476, df = 4, p-value = 0.02171
```

⇒ reject homoscedasticity at 5% level.

```
> Box.test(e_s2, lag=12, type="Ljung-Box")

	Box-Ljung test

data:  e_s2
X-squared = 15.417, df = 12, p-value = 0.2194
```

⇒ cannot reject homoscedasticity at 5% level.

Conclusion: There is some evidence of structural change in the variance (GQ test) and time-varying heteroscedasticity (LB test with 4 lags).

PPP: Relative Version – Autocorrelation Tests

Example: LM BG test for residuals with 4 and 12 lags

```
> bgtest(fit_ppp, order=4)

    Breusch-Godfrey test for serial correlation of order up to 4

data:  fit_ppp
LM test = 6.6801, df = 4, p-value = 0.1538
```

⇒ cannot reject no autocorrelation at 5% level.

```
> bgtest(fit_ppp, order=12)

    Breusch-Godfrey test for serial correlation of order up to 12

data:  fit_ppp
LM test = 9.8585, df = 12, p-value = 0.6284
```

⇒ cannot reject no autocorrelation at 5% level.

Conclusion: There is no evidence for autocorrelation using 4 and 12 lags.

PPP: Relative Version – DW Tests

Example: DW test for first-order --AR(1)– autocorrelation

```
> dwtest(fit_ppp)

    Durbin-Watson test

data:  fit_ppp
DW = 1.8283, p-value = 0.0177
alternative hypothesis: true autocorrelation is greater than 0
```

⇒ reject no autocorrelation at 5% level

Conclusion: There is evidence for AR(1) autocorrelation, which is overwhelmed by the lack of autocorrelation of higher order, in the test with 4 and 12 lags.

```
> bgtest(fit_ppp, order=1)

    Breusch-Godfrey test for serial correlation of order up to 1

data:  fit_ppp
LM test = 4.3734, df = 1, p-value = 0.0365
```

PPP: Relative Version – Individual tests

Example: Individual tests: 2 t-tests

fit_ppp <- lm(e_sek ~ inf_d)

> summary(fit_ppp)

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	0.0009209	0.0012648	**0.728**	0.467	⇒ α non significant:
inf_d	**0.0111926**	**0.2330375**	0.048	0.962	⇒ t= (**0.0111926** – **1**)/ **0.2330375** = **-4.24**

Residual standard error: 0.03084 on 594 degrees of freedom

Multiple R-squared: 3.883e-06, Adjusted R-squared: -0.00168

F-statistic: 0.002307 on 1 and 594 DF, p-value: 0.9617

Conclusion: Cannot reject H_0: **α=0**; but reject H_0: **β=1** at 5% level. Relative PPP is rejected for SEK/USD by the individual tests.

PPP: Relative Version – Joint Tests

Example: Joint test: Wald-tests using library car

library(car)

> linearHypothesis(fit_ppp,c("(Intercept) = 0","inf_d = 1"),test="F") # Exact F test

Linear hypothesis test

Hypothesis:

(Intercept) = 0

inf_d = 1

Model 1: restricted model

Model 2: e_sek ~ inf_d

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)	
1	596	0.58230					
2	594	0.56492	2	0.017378	**9.136**	**0.0001236**	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Conclusion from joint test: Reject H_0: **α=0** & **β=1** at 5% level.

PPP: Relative Version – Joint Tests

Example: Joint test: Wald-tests

```
Var_b <- vcov(fit_ppp)
b <- fit_ppp$coefficients
J <- 2                                       # number of restriction
R <- matrix(c(1,0,0,1), nrow=2)              # matrix of restrictions
q <- c(0, 1)                                 # hypothesized values
m <- R%*%b - q                               # m = Estimated R*Beta - q
Var_m <- R %*% Var_b %*% t(R)                # Variance of m
F_t <- W/J                                   # F-test statistic
> F_t
[1,] 9.136004
qf(.95, df1=J, df2=(T - k))                  # exact distribution (F-dist) if errors normal
[1] 3.010917
> p_val
[1,] 0.0001236473
qchisq(.95, df=J)
```

PPP: Relative Version – Joint Tests

Example:

```
> p_val
[1,] 0.0001236473
qchisq(.95, df=J)
p_val <- 1 - pchisq(F_t, df=J)               # p-value(F_t) under asymptotic distribution
> p_val
[1,] 0.01037867
```

Conclusion from individual tests: Cannot reject H_0: **$\alpha=0$**; but reject H_0: **$\beta=1$** at 5% level.

Conclusion from joint tests: Reject H_0: **$\alpha=0$** & **$\beta=1$** at 5% level.

From both tests, we reject Relative PPP for the SEK/USD exchange rate. This is the usual result, especially in the short-run. In the long-run, there is a debate about its validity. Researchers find that currencies with high inflation rate differentials tend to depreciate.

PPP: Relative Version – Individual Tests: NW SE

Example:

```
library(sandwich)
NW <- NeweyWest(fit_ppp, lag = 12, prewhite = FALSE)
SE_NW <- diag(sqrt(abs(NW)))
t_NW <- b_sek/SE_NW
> SE_NW
(Intercept)      inf_d
0.001412006 0.242248013
> t_NW
(Intercept)      inf_d
 0.65222140  0.04620308
```

Conclusion from individual tests: No change in our results. Using SE that correct for autocorrelation and heteroscedasticy, we cannot reject H_0: $\alpha=0$; but reject H_0: $\beta=1$ at 5% level.

PPP: Relative Version – Model and RW MSEs

Example:

```
T <- length(e_s2)
MSE_mod <- sum(e_s2[2:T])/(T-1)
> MSE_mod
[1] 0.0009494476

e_RW <- e_sek[2:T]                    # Any change is a "surprise" for the RW model
MSE_RW <- sum(e_RW^2)/(T-1)
> MSE_RW
[1] 0.0009503091
```

Conclusion: The model does barely better than the RW model in sample. But, given that the model is not a good one, the RW is doing also doing a poor job in sample.

PPP: Relative Version – Augmented PPP model

Example: We augment the PPP Model with the FF factors, RF, and log changes in USD Index, crude oil prices and gold prices.

```
fit_ppp_aug <- lm(e_sek ~ inf_d + e_usd + Mkt_RF + SMB + HML + RF + oil + gold)
> summary(fit_ppp_aug)
             Estimate   Std. Error t value Pr(>|t|)
(Intercept)  0.0016950  0.0019091   0.888 0.374995
inf_d       -0.0776827  0.2132871  -0.364 0.715827
e_usd        0.6417020  0.0843317   7.609 1.10e-13 ***      => keep it
Mkt_RF      -0.0010034  0.0002748  -3.651 0.000285 ***      => keep it
SMB          0.0006799  0.0003947   1.723 0.085502 .        => maybe keep it?
HML         -0.0002397  0.0003978  -0.603 0.546987
RF          -0.0020845  0.0041301  -0.505 0.613958
oil         -0.0134775  0.0120054  -1.123 0.262059
gold        -0.1210773  0.0200747  -6.031 2.88e-09 ***      => keep it
---
Residual standard error: 0.02767 on 587 degrees of freedom
Multiple R-squared:  0.2047,    Adjusted R-squared:  0.1938
F-statistic: 18.88 on 8 and 587 DF,  p-value: < 2.2e-16
```

PPP: Relative Version – Augmented PPP model

Example: We use NW SE to draw inferences and select driving variables:

```
NW <- NeweyWest(fit_ppp_aug, lag = 4, prewhite = FALSE)
> SE_NW
 (Intercept)        inf_d        e_usd       Mkt_RF          SMB
0.0017661384 0.2214059333 0.1141467525 0.0003820641 0.0004087771
         HML           RF          oil         gold
0.0004246454 0.0035049813 0.0116392470 0.0222801934
> t_NW
(Intercept)      inf_d      e_usd     Mkt_RF        SMB        HML
  0.9597143 -0.3508610  5.6217279 -2.6263633  1.6633559 -0.5644970
         RF        oil       gold
 -0.5947218 -1.1579348 -5.4343034
```

Conclusion: Besides the inflation rate differential (inf_d), we keep changes in the USD Index (e_usd), the Market factor (Mkt_RF) and changes in gold prices (gold).

PPP: Relative Version – Specific Model

Example: We estimate the reduced augmented (specific) model + inf_d

```
fit_ppp_red <- lm(e_sek ~ inf_d + e_usd + Mkt_RF + gold)
> summary(fit_ppp_red)
Coefficients:
             Estimate    Std. Error t value Pr(>|t|)
(Intercept)  0.0008115   0.0011741   0.691   0.4898
inf_d        -0.0683382  0.2119518  -0.322   0.7472
e_usd        0.6499965   0.0827284   7.857  1.87e-14 ***
Mkt_RF       -0.0008479  0.0002586  -3.279   0.0011 **
gold         -0.1210779  0.0199805  -6.060  2.43e-09 ***
---
Residual standard error: 0.02769 on 591 degrees of freedom    ⇒ MSE = 4.645973e-05>MSE(RW)
Multiple R-squared:  0.1978,    Adjusted R-squared:  0.1924
F-statistic: 36.43 on 4 and 591 DF,  p-value: < 2.2e-16
```

Note: An increase in the Market factor and an appreciation of gold decreases appreciates the SEK against the USD –i.e., e_sek ↓.

Relative PPP: Specific Model - Estimation Period

Example: Estimation period estimation (1971:Feb-2017:Dec)

```
y <- e_sek
xx <- cbind(inf_d, e_usd, Mkt_RF, gold)
T0 <- 1
T1 <- 563                          # End of Estimation Period (Dec 2017)
T2 <- T1+1                         # Start of Validation Period (Jan 2018)
y1 <- y[T0:T1]
x1 <- xx[T0:T1,]
fit_red_est <- lm(y1~ x1)          # Estimation Period Regression
b_est <- fit_red_est$coefficients  # Extract OLS coefficients from regression
summary(fit_red_est)
```

Relative PPP: Specific Model - Estimation Period

Example: Estimation period estimation (1971:Feb-2017:Dec)

```
> summary(fit_red_est)

Coefficients:
              Estimate    Std. Error t value Pr(>|t|)
(Intercept)   0.0007168   0.0012232   0.586 0.558088
x1inf_d       0.0219868   0.2219083   0.099 0.921110
x1e_usd       0.6209707   0.0863378   7.192 2.06e-12 ***
x1Mkt_RF     -0.0009126   0.0002725  -3.349 0.000867 ***
x1gold       -0.1209382   0.0204624  -5.910 5.95e-09 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.02809 on 558 degrees of freedom
Multiple R-squared:  0.1857,    Adjusted R-squared:  0.1799
F-statistic: 31.81 on 4 and 558 DF,  p-value: < 2.2e-16
```

Relative PPP: Specific Model – AR Model for X_1

Example: AR(1) r Independent Variables - $(I_d - I_f)_t$

```
x1_1 <- xx[1:(T1-1),1]          # Estimation period data: (I_d - I_f)_{t-1}
x1_0 <- xx[2:T1,1]              # Estimation period data: (I_d - I_f)_t
fit_1 <-lm(x1_0 ~ x1_1)         # AR(1) for (I_d - I_f)_t
b_1 <- fit_1$coefficients       # Extract AR(1) coefficients
> summary(fit_1)
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.0002799  0.0002273   1.231  0.21867
x1_1        0.1374646  0.0418671   3.283  0.00109 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.005379 on 560 degrees of freedom
Multiple R-squared:  0.01889,   Adjusted R-squared:  0.01714
F-statistic: 10.78 on 1 and 560 DF,  p-value: 0.00109
```

Relative PPP: Specific Model – AR Model for X_2

Example: AR(1) r Independent Variables - e_usd_t

```
x2_1 <- xx[1:(T1-1),2]                    # Estimation period data: e_usd_t-1
x2_0 <- xx[2:T1,2]                        # Estimation period data:  e_usd_t
fit_2 <- lm(x2_0 ~ x2_1)                  # AR(1) for e_usd_t
b_2 <- fit_2$coefficients
> summary(fit_2)

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.0016621  0.0005936   2.800  0.00529 **
x2_1        0.2365131  0.0410553   5.761 1.38e-08 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.01391 on 560 degrees of freedom
Multiple R-squared:  0.05595,   Adjusted R-squared:  0.05426
F-statistic: 33.19 on 1 and 560 DF,  p-value: 1.382e-08
```

Relative PPP: Specific Model – AR Model for X_3

Example: AR(1) r Independent Variables – Mkt_RF_t

```
x3_1 <- xx[1:(T1-1),3]                    # Estimation period data: (Mkt_RF)_t-1
x3_0 <- xx[2:T1,3]                        # Estimation period data: (Mkt_RF)_t
fit_3 <-lm(x3_0 ~ x3_1)                   # AR(1) for (Mkt_RF)_t
b_3 <- fit_3$coefficients
> summary(fit_3)

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.52372    0.19015   2.754  0.00607 **
x3_1         0.06836    0.04216   1.621  0.10548
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.472 on 560 degrees of freedom
Multiple R-squared:  0.004673,  Adjusted R-squared:  0.002896
F-statistic: 2.629 on 1 and 560 DF,  p-value: 0.1055
```

Relative PPP: Specific Model – AR Model for X_4

Example: AR(1) for Independent Variables - $gold_t$

```
x4_1 <- xx[1:(T1-1),4]                    # Estimation period data: gold_t-1
x4_0 <- xx[2:T1,4]                        # Estimation period data: gold_t
fit_4 <-lm(x4_0 ~ x4_1)                   # AR(1) for gold_f,t
b_4 <- fit_4$coefficients
> summary(fit_4)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.006216   0.002492   2.495   0.0129 *
x4_1        0.006312   0.042257   0.149   0.8813
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.05873 on 560 degrees of freedom
Multiple R-squared:  3.985e-05, Adjusted R-squared:  -0.001746
F-statistic: 0.02231 on 1 and 560 DF,  p-value: 0.8813
```

Relative PPP: Specific Model – Forecasts

Example: Validation period forecast for S_t

```
T_val <- T1 + 1                                        # Start of Validation period
xx_cons <- rep(1,T-T_val+1)
xx1_0 <- cbind(xx_cons,xx[T_val:T,1]) %*% b_1
xx2_0 <- cbind(xx_cons,xx[T_val:T,2]) %*% b_2
xx3_0 <- cbind(xx_cons,xx[T_val:T,3]) %*% b_3
xx4_0 <- cbind(xx_cons,xx[T_val:T,4]) %*% b_4

k_for <- T - T_val+1                                   # Number of forecasts
e_sek_mod_0 <- cbind(xx_cons,xx1_0,xx2_0,xx3_0,xx4_0)%*%b_est  # Forecast for e_f,t
S_mod_f0 <- S[T1:(T-1)] * (1 + e_sek_mod_0)            # Forecast for S_t
e_mod_f0 <- S[T_val:T] - S_mod_f0                      # Forecasat error_Mod,t = S_t – Ŝ_t
mse_e_f0 <- sum(e_mod_f0^2)/k_for                      # MSE
> mse_e_f0
[1] 2.476651
```

Relative PPP: AR(1) Model – Estimation Period

Example: AR(1) for $e_{f,t}$

```
y_1 <- y[1:(T1-1)]                    # Estimation period data: e_{f,t-1}
y_0 <- y[2:T1]                        # Estimation period data: e_{f,t}
fit_y <- lm(y_0 ~ y_1)                # AR(1) for e_{f,t}
b_y <- fit_y$coefficients
> summary(fit_y)
Coefficients:
             Estimate   Std. Error t value Pr(>|t|)
(Intercept) 0.0007388  0.0013058   0.566   0.5718
y_1         0.0908950  0.0421043   2.159   0.0313 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.03095 on 560 degrees of freedom
Multiple R-squared:  0.008254,  Adjusted R-squared:  0.006483
F-statistic:  4.66 on 1 and 560 DF,  p-value: 0.03129
```

Relative PPP: AR(1) Model – Forecasts

Example: AR(1) for $e_{f,t}$

```
y_f0 <-  cbind(xx_cons,y[T_val:T])%*% b_y      # Forecast for e_{f,t}
S_ar1_f0 <- S[T1:(T-1)] * (1 + y_f0)           # Forecast for S_t
e_ar1_f0 <- S[T_val:T] - S_ar1_f0              # Forecasat error e_{AR,t} = S_t – Ŝ_t
mse_e_ar1_f0 <- sum(e_ar1_f0^2)/k_for          # MSE
> mse_e_ar1_f0
[1] 2.815829
```

Relative PPP: Random Walk Model – Forecasts

Example: Random Walk Model for $e_{f,t}$

```
e_rw_f0 <- S[T_val:T] - S[T1:(T-1)]                # Error for RW model e_RW,t = S_t – S_t-1
mse_e_rw_f0 <- sum(e_rw_f0^2)/k_for
> mse_e_rw_f0
[1] 3.53209
```

Relative PPP: MGN/HLN Tests

Example: Testing accuracy of forecasts.

1) Mod vs RW

```
z_mgn <- e_rw_f0 + e_mod_f0
x_mgn <- e_rw_f0 - e_mod_f0
fit_mgn <- lm(z_mgn ~ x_mgn)
> summary(fit_mgn)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.58011    0.07311  -7.935 5.87e-09 ***
x_mgn       11.34504    0.23442  48.396  < 2e-16 ***      ⇒ significant: Mod forecasts better!
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4049 on 31 degrees of freedom
Multiple R-squared:  0.9869,    Adjusted R-squared:  0.9865
F-statistic:  2342 on 1 and 31 DF,  p-value: < 2.2e-16
```

Relative PPP: MGN/HLN Tests

Example: Testing accuracy of forecasts.

1) AR(1) vs RW

```
z_mgn <- e_rw_f0 + e_ar1_f0
x_mgn <- e_rw_f0 - e_ar1_f0
fit_mgn <- lm(z_mgn ~ x_mgn)
> summary(fit_mgn)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -0.6119    0.4962  -1.233    0.227
x_mgn         8.6295    1.6471   5.239 1.08e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.656 on 31 degrees of freedom
Multiple R-squared:  0.4696,    Adjusted R-squared:  0.4525
F-statistic: 27.45 on 1 and 31 DF,  p-value: 1.081e-05
```

⇒ significant: AR(1) forecasts better!

Relative PPP: MGN/HLN Tests

Example: Testing accuracy of forecasts.

3) Mod vs AR(1)

```
z_mgn <- e_ar1_f0 + e_mod_f0
x_mgn <- e_ar1_f0 - e_mod_f0
fit_mgn <- lm(z_mgn ~ x_mgn)
> summary(fit_mgn)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.4352    0.5133   0.848  0.40309
x_mgn         6.9960    2.2926   3.052  0.00464 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.928 on 31 degrees of freedom
Multiple R-squared:  0.231,     Adjusted R-squared:  0.2062
F-statistic: 9.312 on 1 and 31 DF,  p-value: 0.004638
```

⇒ significant: Mod forecasts better!