Lecture 5
Testing in the CLM

Brooks (4th edition): Chapters 3 & 4

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Review: Bootstrapping in the CLM Var[b]

- We use a bootstrap to estimate **b**, Var[**b**], t-stats, and C.I. for **b**.

- Steps to bootstrap **b** in the CLM and get t-stats for **b**:
  1. Estimate CLM using full sample (of size *T*) ⇒ get **b**
  2. Repeat *B* times:
     - Draw *T* observations from the sample, *with replacement*
     - Do OLS to compute bootstrapped $\boldsymbol{b}_r$
     - Estimate β with mean of bootstrapped **b**'s: $\bar{\boldsymbol{b}}_r = \frac{\sum_{r=1}^{B} \boldsymbol{b}_r}{B}$
  3. Estimate variance with: $\mathbf{V}_{\text{boot}} = (1/B)\, [\boldsymbol{b}_r - \mathbf{b}][\boldsymbol{b}_r - \mathbf{b}]'$
     (Square root along the diagonal of $\mathbf{V}_{\text{boot}}$ gives SE[$\boldsymbol{b}_r$]).
  4. Estimate t-stats with: $t = \boldsymbol{b}_r / \text{SE}[\boldsymbol{b}_r]$

Review: Bootstrapping in the CLM

- Comparing OLS and Bootstrap Estimation for the FF 3-factor model for IBM returns:

	OLS		Bootstrap		Bias
	Coeff. (1)	S.E.	Coeff. (2)	S.E.	(2)-(1)
x	-0.00509	0.00249	-0.00501	0.00249	8.0765e-05
xMkt_RF	**0.90829**	**0.05672**	**0.90684**	**0.06132**	-0.0014571
xSMB	**-0.21246**	**0.08411**	**-0.21245**	**0.11080**	1.9914e-06
xHML	**-0.17150**	**0.08468**	**-0.17099**	**0.09730**	0.0005133

- Higher SE for the bootstrap: More conservative tests (less rejections of H_0).When in doubt, always use more conservative tests.

Review – OLS with Restrictions

- CLM Assumptions

(**A1**) DGP: $\boldsymbol{y} = \mathbf{X}\,\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ is correctly specified.

(**A2**) $E[\boldsymbol{\varepsilon}|\mathbf{X}] = 0$

(**A3**) $Var[\boldsymbol{\varepsilon}|\mathbf{X}] = \sigma^2\,\mathbf{I}_T$

(**A4**) $\mathbf{X}$ has full column rank –rank($\mathbf{X}$)=k-, where $T \geq k$.

- Now, we impose a linear restriction to the DGP (**A1**): $\mathbf{R}\boldsymbol{\beta} = \mathbf{q}$

Dimensions:

$\mathbf{R}$: Jxk - J = # of restrictions & k = # of pars.

$\boldsymbol{\beta}$: kx1

$\mathbf{q}$: kx1

We do LS imposing restriction $\mathbf{R}\boldsymbol{\beta} = \mathbf{q}$: Get a restricted estimator: $\mathbf{b}^*$

Review – Restricted Estimation: Properties

- Restricted estimation produces $\mathbf{b}^*$

$$\mathbf{b}^* = \mathbf{b} - (\mathbf{X}'\mathbf{X})^{-1}\mathbf{R}'[\mathbf{R}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{R}']^{-1}(\mathbf{Rb} - \mathbf{q})$$
$$= \mathbf{b} + \text{correction}$$

- Properties:

1. Unbiased?
   - Yes, if Theory is correct: $E[\mathbf{b}^* | \mathbf{X}] = E[\mathbf{b} | \mathbf{X}] = \beta$
   - No, if Theory is incorrect: $E[\mathbf{b}^* | \mathbf{X}] \neq \beta$
2. Efficiency? Yes. $Var[\mathbf{b}^* | \mathbf{X}] < Var[\mathbf{b} | \mathbf{X}]$
3. We can show that RSS never decreases with restrictions:

$$\boldsymbol{e}'\boldsymbol{e} = (\boldsymbol{y} - \mathbf{Xb})'(\boldsymbol{y} - \mathbf{Xb}) \leq \boldsymbol{e}^{*\prime}\boldsymbol{e}^* = (\boldsymbol{y} - \mathbf{Xb}^*)'(\boldsymbol{y} - \mathbf{Xb}^*)$$

$\Rightarrow$ Restrictions cannot increase R^2 $\Rightarrow R^2 \geq R^{2*}$

Review – Testing Restrictions: Wald Statistic

- Q: How do we test joint restrictions in the context of OLS?

A: We use Wald tests & F-tests.

- Wald statistic:

Let $\mathbf{z}$ = (vector of estimators – hypothesized value) be the distance

$W = \mathbf{z}' [Var(\mathbf{z})]^{-1} \mathbf{z}$ -a quadratic form, produces a number

Example: Let $\mathbf{z} = \mathbf{Rb} - \mathbf{q}$, which under (**A5**) & H_0: $\mathbf{R}\beta = \mathbf{q}$

$\mathbf{z} \sim N(\mathbf{0}, Var[\mathbf{z}])$ $Var[\mathbf{z}] = \mathbf{R}\,[Var[\mathbf{b} | \mathbf{X}]]^{-1}\,\mathbf{R}'$

Then, if H_0 is correct, W should be a small number, ideally close to 0.
Distribution of W?

– If $\mathbf{z}$ is normal, $W \sim F_{J,T-k}$

– If $\mathbf{z}$ is not normal, using asymptotic theory, $W \xrightarrow{d} \chi^2_{rank[Var(z)]}$

Review – Testing H_0: $\mathbf{R}\beta - \mathbf{q} = 0$ with W

- Q: Is $\mathbf{Rb} - \mathbf{q}$ close to $\mathbf{0}$? Two different approaches to this question.

Approach (1): We base the answer on the discrepancy vector:

$$\mathbf{m} = \mathbf{Rb} - \mathbf{q} \qquad \text{(this is } \mathbf{z} \text{ above).}$$

Then, we construct a Wald statistic:

$$W = \mathbf{m}' \,(\text{Var}[\mathbf{m}\,|\,\mathbf{X}])^{-1}\, \mathbf{m}$$

to test if $\mathbf{m}$ is different from 0.

Test H_0 with

$$W^* = (\mathbf{Rb} - \mathbf{q})' \, \{\mathbf{R}[s^2(\mathbf{X'X})^{-1}]\mathbf{R}\}^{-1} \,(\mathbf{Rb} - \mathbf{q})$$

$$\text{F} = W^*/J \sim F_{J,T-k}.$$

If (**A5**) is not assumed, the results are only asymptotic: $J * F \xrightarrow{d} \chi^2_J$

Review – Testing H_0: $\mathbf{R}\beta - \mathbf{q} = 0$ with W

Example: We test in the 3 FF factor model for IBM returns (T=569). Steps

1. H_0: $\beta_{SMB} = 0.2$ and $\beta_{HML} = 0.6$.
   H_1: $\beta_{SMB} \neq 0.2$ and/or $\beta_{HML} \neq 0.6$. $\Rightarrow J = 2$

We define $\mathbf{R}$ (2x4) below and write $\mathbf{m} = \mathbf{R}\beta - \mathbf{q} = \mathbf{0}$:

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \beta_1 \\ \beta_{Mkt} \\ \beta_{SMB} \\ \beta_{HML} \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.6 \end{bmatrix}$$

2. Test-statistic: $\text{F} = W^*/J = (\mathbf{Rb} - \mathbf{q})' \, \{\mathbf{R}[s^2(\mathbf{X'X})^{-1}]\mathbf{R'}\}^{-1} \,(\mathbf{Rb} - \mathbf{q})$
   Distribution under H_0: $\text{F} = W^*/2 \sim F_{2,T-4}$
   (or asymptotic, $2 * F \xrightarrow{d} \chi^2_2$)

Review – Testing H_0: $R\beta - q = 0$ with *W*

Example (continuation): We use the R package *car* to test H_0.

```
library(car)
linearHypothesis(fit_ibm_ff3, c("SMB = 0.2","HML = 0.6"), test="F") # "F": exact test

Linear hypothesis test

Hypothesis:
SMB = 0.2
HML = 0.6

Model 1: restricted model
Model 2: ibm_x ~ Mkt_RF + SMB + HML

Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1    567 2.2691
2    565 1.9324  2   0.33667 49.217 < 2.2e-16 ***
```

⇒ reject H_0 at 5% level

Review – Testing H_0: $R\beta - q = 0$ with *W*

Example (continuation): The asymptotic test uses test="Chisq".

```
> linearHypothesis(fit_ibm_ff3, c("SMB = 0.2","HML = 0.6"), test="Chisq")      # Asymptotic F
Linear hypothesis test

Hypothesis:
SMB = 0.2
HML = 0.6

Model 1: restricted model
Model 2: ibm_x ~ Mkt_RF + SMB + HML

  Res.Df    RSS Df Sum of Sq  Chisq Pr(>Chisq)
1    567 2.2691
2    565 1.9324  2   0.33667 98.433  < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

⇒ reject H_0 at 5% level

```
qf(.95, df1=J, df2=(T - k))          # asymptotic distribution (Chi-square-distribution)
[1] 5.991465
```

F_t_asym > **5.991465** ⇒ reject H_0 at 5% level

Review – Testing H_0: $R\beta - q = 0$ with F

Approach(2): We know that imposing the restrictions leads to a loss of fit: RSS must go up, R^2 must go down. Does R^2 go down a lot? – i.e., significantly?

We based the test on:

$$e^{*\prime}e^{*} - e'e = (\mathbf{Rb} - \mathbf{q})'[\mathbf{R}(\mathbf{X'X})^{-1}\mathbf{R'}]^{-1}(\mathbf{Rb} - \mathbf{q})$$

$$\Rightarrow F = \frac{(e^{*\prime}e^{*} - e'e)/J}{[e'e/(T-k)]} \sim F_{J,T-k}.$$

- We can write the F-test in terms of R^2's. Let
  R^2 = unrestricted model = 1 – RSS/TSS
  R^{*2} = restricted model fit = 1 – RSS*/TSS

After some algebra: $F = \frac{(R^2 - R^{*2})/J}{(1-R^2)/(T-k)} \sim F_{J,T-k}.$

The F-Test: Goodness-of-Fit Test (with *F*)

- In the linear model, with a constant ($\mathbf{X}_1 = \boldsymbol{i}$):

$$\boldsymbol{y} = \mathbf{X}\beta + \varepsilon = \beta_1 + \mathbf{X}_2\beta_2 + \mathbf{X}_3\beta_3 + \dots + \mathbf{X}_k\beta_k + \varepsilon$$

- We want to test if the slopes of $\mathbf{X}_2, \dots, \mathbf{X}_k$ are equal to zero. That is,
  H_0: $\beta_2 = \cdots = \beta_k = 0$
  H_1: at least one $\beta_k \neq 0$ $\Rightarrow J = k - 1$

- We can write H_0: $\mathbf{R}\beta - \mathbf{q} = \mathbf{0}$ $\Rightarrow \begin{bmatrix} 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \dots \\ \beta_k \end{bmatrix} = \begin{bmatrix} 0 \\ \dots \\ 0 \end{bmatrix}$

- We have $J = k - 1$. Then,
  $F = \{ (R^2 - R^{2*})/(k-1) \} / [(1 - R^2)/(T-k)] \sim F_{k-1,T-k}.$

- For the restricted model, $R^{2*} = 0$.

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The F-Test: Goodness-of-Fit Test (with *F*)

Then, $F = \frac{R^2/(k-1)}{(1-R^2)/(T-k)} \sim F_{k-1,T-k}$.

- This test statistic is called the *F-test of goodness of fit*.
- It is reported in all regression packages as part of the regression output. In R, the lm function reports it as "*F-statistic*."

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The F Test: H_0: F-test of Goodness of Fit

Example: We want to test if all the FF factors (Market, SMB, HML) are significant (J=3), using monthly data 1973 – 2020 (T=569).

```
T <- length(ibm_x)
k <- 4
e <- fit_ibm_ff3$residuals                    # Extract residuals
y <- ibm_x - mean(ibm_x)
RSS <- sum(e^2)
R2 <-  1 - RSS/sum(y^2)                 #R-squared
> R2
[1] 0.338985
> F_goodfit <- (R2/(k - 1))/((1 - R2)/(T - k))                # F-test of goodness of fit.
> F_goodfit
[1] 96.58204
```

⇒ F_goodfit > $F_{3,565,.05}$ = **2.62068** ⇒ Reject H_0.

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The F Test: General Case – Example

- In the linear model

$$\boldsymbol{y} = \mathbf{X}\,\beta + \varepsilon = \beta_1 + \mathbf{X}_2\,\beta_2 + \mathbf{X}_3\,\beta_3 + \mathbf{X}_4\,\beta_4 + \varepsilon$$

- We want to test if the slopes of $\mathbf{X}_3$, $\mathbf{X}_4$ are equal to zero. That is,

H_0: $\beta_3 = \beta_4 = \mathbf{0}$

H_1: $\beta_3 \neq \mathbf{0}$ or $\beta_4 \neq \mathbf{0}$ or both β_3 and $\beta_4 \neq \mathbf{0}$

- We use, $F = \dfrac{(\boldsymbol{e}^{*\prime}\boldsymbol{e}^{*} - \boldsymbol{e}'\boldsymbol{e})/J}{[\boldsymbol{e}'\boldsymbol{e}/(T-k)]} \sim F_{J,T-k}$.

- Define

$$\boldsymbol{y} = \mathbf{X}\,\beta + \varepsilon = \beta_1 + \mathbf{X}_2\,\beta_2 + \varepsilon \qquad (RSS_R)$$

$$\boldsymbol{y} = \beta_1 + \mathbf{X}_2\,\beta_2 + \mathbf{X}_3\,\beta_3 + \mathbf{X}_4\,\beta_4 + \varepsilon \qquad (RSS_U)$$

$$F(k_U - k_R, T-k) = \frac{\frac{RSS_R - RSS_U}{(k_U - k_R)}}{\frac{RSS_U}{(T - k_U)}}$$

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The F Test: Are SMB and HML Priced Factors?

Example: We want to test if the additional FF factors (SMB, HML) are significant, using monthly data 1973 – 2020 (T=569).

Unrestricted Model:

(U) $(r_{IBM,t} - r_f) = \beta_0 + \beta_1\,(r_{m,t} - r_f) + \beta_2\,SMB_t + \beta_3\,HML_t + \varepsilon_t$

Hypothesis: H_0: $\beta_2 = \beta_3 = 0$

H_1: $\beta_2 \neq 0$ and/or $\beta_3 \neq 0$

Then, the Restricted Model:

(R) $(r_{IBM,t} - r_f) = \beta_0 + \beta_1\,(r_{m,t} - r_f) + \varepsilon_t$

Test: $F = \dfrac{(RSS_R - RSS_U)/J}{RSS_U/(T-k_u)} \sim F_{J,T-k}$, with $J = k_U - k_R = 4 - 2 = 2$

The F Test: Are SMB and HML Priced Factors?

Example (continuation): The unrestricted 3-factor FF model was already estimated (**fit_ibm_ff3**). Same for the restricted model (**fit_ibm_capm**):

```
e_u <- fit_ibm_ff3$residuals              # Unrestricted residuals
e_r <- fit_ibm_capm$residuals             # Restricted residuals
T <- length(ibm_x)
k <- 4
k_r <- 2

RSS <- sum(e_u^2)                         # RSS_U
RSS_r <- sum(e_r^2)                       # RSS_R
> RSS = 1.932442                          > RSS2 = 1.964844

J <- k – k_r                              # J = degrees of freedom numerator
F_test <- ((RSS_r - RSS)/J)/(RSS/(T-k))
```

The F Test: Are SMB and HML Priced Factors?

Example (continuation):

```
> F_test <- ((RSS2 - RSS)/J)/(RSS/(T-k))
> F_test
[1] 4.736834
> qf(.95, df1=J, df2=(T-k))               # F_{2,565,.05} value (≈ 3)
[1] 3.011672                              ⇒ Reject H0.
> p_val <- 1 - pf(F_test, df1=J, df2=(T-k))   # p-value of F_test
> p_val
[1] 0.009117494                           ⇒ p-value is small ⇒ Reject H0.
```

The F Test: Are SMB and HML Priced Factors?

Example (continuation):

There is package in R, *lmtest,* that performs this test, *waldtest,* (and many others, used in this class). You need to install it first.

Note: The models need to be nested. For the *waldtest*, the default reports the *F-test* with the F distribution.

```
library(lmtest)
fit_wU <- lm (ibm_x ~ Mkt_RF + SMB + HML)          # fit_ibm_ff3
fit_wR <- lm (ibm_x ~ Mkt_RF)                      # fit_ibm_capm
waldtest(fit_wU, fit_wR)

Wald test

Model 1: ibm_x ~ Mkt_RF + SMB + HML
Model 2: ibm_x ~ Mkt_RF
  Res.Df Df      F   Pr(>F)
1    565
2    567 -2 4.7368 0.009117 **
```

⇒ p-value is small: Reject H_0

F-test: Two Categories & The Chow Test

- Suppose we are interested in the effect of gender on CEO's compensation. We have data on CEO's compensation (y) and CEO's gender, along with CEO's experience (X_1), sales of the CEO's company (X_2), and profitability (X_3).

- We hypothesize that gender matter. Then, we estimate two models, one for each gender:

M1 $\quad y_i = \beta_0^1 + \beta_1^1 X_{1,I} + \beta_2^1 X_{2,i} + \beta_3^1 X_{3,i} + \varepsilon_i$ for i = *Male*

M2 $\quad y_i = \beta_0^2 + \beta_1^2 X_{1,I} + \beta_2^2 X_{2,i} + \beta_3^2 X_{3,i} + \varepsilon_i$ for $i \neq$ *Female*

- Alternatively, we estimate only one model ("*pooling*"). That is, gender does not affect a CEO's compensation. Then, we estimate:

Pooled $\quad y_i = \beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + \beta_3 X_{3,i} + \varepsilon_i \quad$ for all i

Q: Which model should we use?

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F-test: Two Categories & The Chow Test

• We test H_0 (No *gender differences*): $\beta_0^1 = \beta_0^2 = \beta_0$
$\beta_1^1 = \beta_1^2 = \beta_1$
$\beta_2^1 = \beta_2^2 = \beta_2$
$\beta_3^1 = \beta_3^2 = \beta_3$

H_1 (*gender differences*): For at least k (= 0, 1, 2, 3): $\beta_k^1 \neq \beta_k^2$

• An F-Test can be used to test H_0:

- The pooled estimation is the Restricted estimation
- The two estimations (by gender) are the Unrestricted estimation.

• The F-test constructed using a variable that can divide the data into 2 categories to compute RSS_R & RSS_U is usually referred as *Chow test*.

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F-test: Two Categories & The Chow Test

• A Chow Test is used to test if a variable that can divide the data into 2 categories matters. That is, a Chow test checks if we need only one model ("*pooling*") for both categories or not.

• Chow Test (an F-test) –Chow (1960, *Econometrica*):

(1) Run OLS with all the data, with no distinction between categories. (Pooled regression or Restricted regression). Keep RSS_R.

(2) Run two separate OLS, one for each category (Unrestricted regression). Keep RSS_1 and RSS_2 $\Rightarrow RSS_U = RSS_1 + RSS_2$.

(3) Run a standard F-test (testing Restricted vs. Unrestricted models):

$$F = \frac{(RSS_R - RSS_U)/(k_U - k_R)}{(RSS_U)/(T - k_U)} = \frac{(RSS_R - [RSS_1 + RSS_2])/k}{(RSS_1 + RSS_2)/(T - 2k)}$$

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Chow Test: Males or Females visit doctors more?

German Health Care Usage Data, 7,293 Individuals, Varying Numbers of Periods
Variables in the file are
Data downloaded from Journal of Applied Econometrics Archive. This is an unbalanced panel with 7,293 individuals. There are altogether **27,322** observations. The number of observations ranges from 1 to 7 per family. (Frequencies are: 1=1525, 2=2158, 3=825, 4=926, 5=1051, 6=1000, 7=987). The dependent variable of interest is

DOCVIS = number of visits to the doctor in the observation period
GENDER_F = gender (1 = female)
HHNINC = household nominal monthly net income in German marks / 10000.
(4 observations with income=0 were dropped)
HHKIDS = children under age 16 in the household = 1; otherwise = 0
EDUC = years of schooling
AGE = age in years
MARRIED= marital status (1 = if married)
WHITEC = 1 if has "white collar" job

Chow Test: Males or Females visit doctors more?

```
Health_Da <-
read.csv("https://www.bauer.uh.edu/rsusmel/4397/german_health.csv",
head=TRUE, sep=",")

x_fem <- Health_Da$Gender_F
 x_age <- Health_Da$age
 x_edu <- Health_Da$educ
 x_hhinc <- Health_Da$hhninc/100
 x_hhkids <- Health_Da$hhkids
 x_married <- Health_Da$married
 x_white_col <- Health_Da$whitecollar
 x_docvis <- Health_Da$docvis

fit_doc_vis <- lm (x_docvis ~  x_age +  x_edu + x_married + x_white_col +
x_hhkids + x_hhinc)
summary(fit_doc_vis)
```

Chow Test: Males or Females visit doctors more?

• OLS Estimation for **ALL**. Keep RSS_{ALL} = **858,435** (= **5.606**^2 * **27,315**)

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   2.683700   0.249282  10.766  < 2e-16 ***
x_age         0.061810   0.003444  17.947  < 2e-16 ***
x_edu        -0.118858   0.015573  -7.632 2.38e-14 ***
x_married    -0.090716   0.089056  -1.019    0.308
x_white_col  -0.115412   0.076540  -1.508    0.132
x_hhkids     -0.492028   0.080014  -6.149 7.89e-10 ***
x_hhinc      -0.015429   0.002046  -7.539 4.87e-14 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 5.606 on 27,315  degrees of freedom
Multiple R-squared:  0.02949,  Adjusted R-squared:  0.02928
F-statistic: 138.3 on 6 and 27315 DF,  p-value: < 2.2e-16
```

Note: We compute RSS_R, we impose there is no gender effect on the coefficients.

$$RSS_R = s^2 * (T - k) = 5.606\text{^}2 * 27{,}315 = 858{,}435$$

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Chow Test: Males or Females visit doctors more?

Run a regression with only **Women** data. Use Allgen to collect relevant data for women only. We will do a for loop and keep data if x_fem is greater than 0.

```
xx <- cbind(x_fem, x_docvis, x_age,  x_edu, x_married, x_white_col, x_hhkids, x_hhinc)
Allgen = NULL         # Initialize empty (to collect variables by one sex (f/m) only)
i <- 1
T <- length(x_fem)
k <- ncol(xx)

for (i in 1:T) {
  if (xx[i,1] > 0) {
    Allgen = rbind(Allgen, xx[i,2:k])
  }
}

y_g <- Allgen[,1]             # Dependent variable: doctor's visits by women only
x_g <- Allgen[,2:(k-1)]
```

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Chow Test: Males or Females visit doctors more?

```
fit_doc_vis_f <- lm (y_g ~  x_g)
summary(fit_doc_vis_f)
```

• OLS Estimation for **Women** only. Keep RSS_W = **478,894.2**

```
Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)      2.999559   0.453506   6.614 3.88e-11 ***
x_gx_age         0.049366   0.005719   8.632  < 2e-16 ***
x_gx_edu        -0.048141   0.027011  -1.782   0.0747 .
x_gx_married    -0.119853   0.133846  -0.895   0.3706
x_gx_white_col  -0.006734   0.124768  -0.054   0.9570
x_gx_hhkids     -0.636619   0.128844  -4.941 7.87e-07 ***
x_gx_hhinc      -0.015651   0.003174  -4.932 8.25e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 6.052 on 13075 degrees of freedom
Multiple R-squared:  0.01984,     Adjusted R-squared:  0.01939
F-statistic: 44.11 on 6 and 13075 DF,  p-value: < 2.2e-16
```

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Chow Test: Males or Females visit doctors more?

```
# Use above code, but  change for loop (now, keep data if  x_fem less than 1)
for (i in 1:T) {
  if (xx[i,1] < 1) {
    Allgen = rbind(Allgen, xx[i,2:k])
  }
}

y_g <- Allgen[,1]             # Dependent variable: doctor's visits by women only
x_g <- Allgen[,2:(k-1)]

fit_doc_vis_m <- lm (y_g ~  x_g)
summary(fit_doc_vis_m)
```

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Chow Test: Males or Females visit doctors more?

• OLS Estimation for **Men** only. Keep RSS_M = **379.8470**

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	1.801539	0.290792	6.195	5.98e-10	***
x_gx_age	0.067656	0.004421	15.302	< 2e-16	***
x_gx_edu	-0.105462	0.018814	-5.605	2.12e-08	***
x_gx_married	0.022278	0.121467	0.183	0.854480	
x_gx_white_col	-0.367075	0.096300	-3.812	0.000139	***
x_gx_hhkids	-0.428916	0.102070	-4.202	2.66e-05	***
x_gx_hhinc	-0.015438	0.002629	-5.872	4.40e-09	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 5.118 on 14233 degrees of freedom

Multiple R-squared: 0.03602, Adjusted R-squared: 0.03561

• Chow Test:

$$F = \frac{(RSS_R - [RSS_1 + RSS_2])/k}{(RSS_1 + RSS_2)/(T-2k)} = \frac{(858{,}435 - [372{,}818.1 + 478{,}894.2])/7}{(372{,}818.1 + 478{,}894.2)/(27{,}323 - 14)}$$

= 31.1178 ⇒ since F(7, 27309) = 2.009925 ⇒ reject H_0 at 5% level.

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F-Test: Structural Change & Chow Test

• Suppose there is an event that we think had a big effect on the behaviour of our model. Suppose the event occurred at time T_{SB}. We think that the before and after behaviour of the model is significantly different. For example, the parameters are different before and after T_{SB}. That is,

$$y_i = \beta_0^1 + \beta_1^1 X_{1,I} + \beta_2^1 X_{2,i} + \beta_3^1 X_{3,i} + \varepsilon_i \qquad \text{for } i \le T_{SB}$$
$$y_i = \beta_0^2 + \beta_1^2 X_{1,I} + \beta_2^2 X_{2,i} + \beta_3^2 X_{3,i} + \varepsilon_i \qquad \text{for } i > T_{SB}$$

The event caused *structural change* in the model. T_{SB} separates the behaviour of the model in two regimes/categories ("*before*" & "*after*".)

• A Chow test tests if one model applies to both regimes:

$$y_i = \beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + \beta_3 X_{3,i} + \varepsilon_i \qquad \text{for all } i$$

• Under H_0 (No *structural change*), the parameters are the same for all i.

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F-Test: Structural Change & Chow Test

• We test H_0 (No *structural change*): $\beta_0^1 = \beta_0^2 = \beta_0$
$\beta_1^1 = \beta_1^2 = \beta_1$
$\beta_2^1 = \beta_2^2 = \beta_2$
$\beta_3^1 = \beta_3^2 = \beta_3$

H_1 (*structural change*): For at least k (= 0, 1, 2, 3): $\beta_k^1 \neq \beta_k^2$

• What events may have this effect on a model? A financial crisis, a big recession, an oil shock, Covid-19, etc.

• Testing for structural change is the more popular use of Chow tests.

• Chow tests have many interpretations: tests for structural breaks, pooling groups, parameter stability, predictive power, etc.

• One important consideration: T may not be large enough.

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F-Test: Structural Change & Chow Test

• We structure the Chow test to test H_0 (No *structural change*), as usual.

• Steps for Chow (Structural Change) Test:

(1) Run OLS with all the data, with no distinction between regimes. (Restricted or pooled model). Keep RSS_R.

(2) Run two separate OLS, one for each regime (Unrestricted model):

- Before Date T_{SB}. Keep RSS_1.
- After Date T_{SB}. Keep RSS_2. $\Rightarrow RSS_U = RSS_1 + RSS_2$.

(3) Run a standard F-test (testing Restricted vs. Unrestricted models):

$$F = \frac{(RSS_R - RSS_U)/(k_U - k_R)}{(RSS_U)/(T - k_U)} = \frac{(RSS_R - [RSS_1 + RSS_2])/k}{(RSS_1 + RSS_2)/(T - 2k)}$$

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F-Test: Structural Change & Chow Test

Example: 3 Factor Fama-French Model for IBM (continuation)

Q: Did the dot.com bubble (end of 2001, T_{SB} = 348) affect the structure of the FF Model? Sample: January 1973 – December 2023 (T = 611).

Pooled RSS = **2.0826**

Jan 1973 – Dec 2001 RSS = RSS_1 = **1.3576** (T = 348)

Jan 2002 – June 2020 RSS = RSS_2 = **0.7076** (T = 263)

$$F = \frac{[RSS_R - (RSS_1 + RSS_2)]/J}{(RSS_1 + RSS_2)/(T-k)} = \frac{[1.9324 - (1.3576 + 0.7076)]/4}{(1.3576 + 0.7076)/(611 - 2*4)} = 1.2744$$

$\Rightarrow$ Since $F_{4,611,.05}$ = **2.39**, we cannot reject H_0

	Constant	Mkt – rf	SMB	HML	RSS	T
1973-2020	-0.0049*	0.8857*	-0.2281*	-0.0582	**2.0826**	611
1973-2001	-0.0032	0.7994*	-0.2196*	-0.2294*	**1.3576**	342
2002 – 2023	-0.0054	0.9414*	-0.2611*	-0.0309&	**0.7076**	227

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F-Test: Structural Change & Chow Test

Example (continuation): The R package *sctrucchange* estimates the Chow test. (As usual, you need to install package first.)

```
>library(sctrucchange)
> t_s <- 348
> sctest(ibm_x ~ Mkt_RF + SMB + HML, type = "Chow", point = t_s)

        Chow test

data:  ibm_x ~ Mkt_RF + SMB + HML
F = 1.2744, p-value = 0.2787
```

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F-Test: Structural Change & Chow Test

Example: We test if the Oct 1973 oil shock in quarterly GDP growth rates had an structural change on the GDP growth rate model.

We model GDP the growth rate with an AR(1) model, that is, GDP growth rate depends only on its own lagged growth rate:

$$y_t = \beta_0 + \beta_1 y_{t-1} + \varepsilon_t$$

```
GDP_da <- read.csv("http://www.bauer.uh.edu/rsusmel/4397/GDP_q.csv", head=TRUE,
sep=",")
x_date <- GDP_da$DATE
x_gdp <- GDP_da$GDP
x_dummy <- GDP_da$D73
T <- length(x_gdp)
t_s <- 108                    # T_SB = Oct 1973

lr_gdp <- log(x_gdp[-1]/x_gdp[-T])
T <- length(lr_gdp)
lr_gdp0 <- lr_gdp[-1]
lr_gdp1 <- lr_gdp[-T]
t_s <- t_s -1                 # Adjust t_s (we lost the first observation)
```

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F-Test: Structural Change & Chow Test

Example (continuation):

```
y <- lr_gdp0
x1 <-  lr_gdp1
T <- length(y)
x0 <- matrix(1,T,1)
x <- cbind(x0,x1)
k <- ncol(x)

# Restricted Model (Pooling all data)
fit_ar1 <- lm(lr_gdp0 ~ lr_gdp1)          # Fitting AR(1) (Restricted) Model
e_R <- fit_ar1 $residuals                # regression residuals, e
RSS_R <- sum(e_R^2)                      # RSS Restricted

> summary(fit_ar1)

Coefficients:
             Estimate   Std. Error t value Pr(>|t|)
(Intercept) 0.011565   0.001145  10.097  < 2e-16 ***
lr_gdp1     0.244846   0.056687   4.319 2.14e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.01296 on 294 degrees of freedom
```

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F-Test: Structural Change & Chow Test

Example (continuation):

```
# Unrestricted Model (Two regimes)

y_1 <- y[1:t_s]
x_u1 <- x[1:t_s,]
fit_ar1_1 <- lm(y_1 ~ x_u1 - 1)                    # AR(1) Regime 1
e1 <- fit_ar1_1$residuals                           # Regime 1 regression residuals, e
RSS1 <- sum(e1^2)                                   # RSS Regime 1

kk = t_s+1                                          # Starting date for Regime 2
y_2 <- y[kk:T]
x_u2 <- x[kk:T,]
fit_ar1_2 <- lm(y_2 ~ x_u2 - 1)                    # AR(1) Regime 2
e2 <- fit_ar1_2$residuals                           # Regime 2 regression residuals, e
RSS2 <- sum(e2^2)                                   #  RSS Regime 2

F <- ((RSS_R - (RSS1+RSS2))/k)/((RSS1+RSS2)/(T - 2*k))
> F
[1] 4.877371
p_val <- 1 - pf(F, df1 = 2, df2 = T - 2*k)  # p-value of F_test
> p_val
[1] 0.00824892
```

⇒ small p-values: Reject H_0 (No *structural change*).

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