Chapter 10 - Measuring FX Exposure

At the firm level, currency risk is called FX exposure. Recall that currency risk describes how the value of an asset/liability fluctuates due to changes in $S_e$.

◆ From Kellogg’s 2014 Annual Report

Almost everyone knows Kellogg, but the general public may not be aware that they do more than cereal. Kellogg is the world's largest cereal company; second largest producer of cookies and crackers; a major producer of snacks; and a major North American frozen foods company.

“Our operations face significant foreign currency exchange rate exposure and currency restrictions which could negatively impact our operating results.

We hold assets and incur liabilities, earn revenue and pay expenses in a variety of currencies other than the U.S. dollar, including the euro, British pound, Australian dollar, Canadian dollar, Mexican peso, Venezuelan bolivar fuerte and Russian ruble. Because our consolidated financial statements are presented in U.S. dollars, we must translate our assets, liabilities, revenue and expenses into U.S. dollars at then-applicable exchange rates. Consequently, changes in the value of the U.S. dollar may unpredictably and negatively affect the value of these items in our consolidated financial statements, even if their value has not changed in their original currency”

Three areas of FX exposure:
(1) **Transaction exposure**: Risk of transactions denominated in FX. The transaction should have a settlement or maturity date, $T$ (today, 60 days, 90 days, 1 year, etc.).
(2) **Economic exposure**: Degree to which a firm's expected cash flows are affected by unexpected changes in $S_e$.
(3) **Translation exposure**: Accounting-based changes in a firm's consolidated statements that result from a change in $S_e$.

**Example**: The different FX exposures.
A. Transaction exposure.
Swiss Cruises (SC), a Swiss firm, sells cruise packages to U.S. customers priced in USD. SC also has several U.S. suppliers that price in USD.

B. Economic exposure.
SC has the majority of its costs denominated in CHF. Almost 50% of its revenue is in USD. The CHF appreciates against the USD. SC cannot increase the USD prices of its cruise packages (competitive business). SC’s net CHF cash flows will be affected.

C. Translation exposure.
SC has inventories in USD and a USD loan from a U.S. bank of equal USD amounts. These balance sheet items will be translated to CHF. Due to Swiss accounting rules, different exchange
rates are used to translate USD inventories and the USD loan to CHF. Thus, an accounting
gain/loss will be generated.

Q: How can FX changes affect the firm?
• Transaction Exposure
  Short-term CFs: Existing contractual obligations

• Economic Exposure
  Future CFs: Erosion of competitive position

• Translation Exposure
  Revaluation of balance sheet (book value vs market value)

Q #1: How do we measure these FX exposures?
Q #2: How do we use these measures to manage FX exposures?

1. Measuring TE
TE is easy to identify and measure, especially in the short-run, when firms can forecast future CF
with high accuracy.

TE represents today’s value of a future, certain transaction denominated in FC translated to the
DC:

\[ TE = \text{Value of transaction denominated in FC} \times S_t \]

**Example:** Swiss Cruises has sold cruise packages to a U.S. wholesaler for USD 2.5 million.
Payment is due in 30 days.
\( S_t = 1.45 \text{ CHF/USD} \).

\[ TE = \text{USD 2.5 M} \times 1.45 \text{ CHF/USD} = \text{USD 3.625 M}. \]

MNCs measure Net TE. If a subsidiary has CF>0 in EUR and another subsidiary has CF<0 in
EUR, net TE might be very low.

**Example:** Swiss Cruises.
SC has sold cruise packages to a U.S. wholesaler for USD 2.5 million.
SC has bought fuel oil for USD 1.5 million.
Both cash flows are going to occur in \( T=30 \) days.
\( S_t = 1.45 \text{ CHF/USD} \).

\[ \text{TE}_{\text{EUR}} \text{ (in USD)}: (\text{USD 2,500,000} - \text{USD 1,500,000}) \times 1.45 \text{ CHF/USD} = \text{CHF 1,450,000}. \]

An MNC, like GE or MSFT, has many transactions denominated in FC, say EUR, GBP, JPY,
MXN, etc. Since all net TEs by currencies are translated to DC, TE is easy to aggregate in a single
number, the company’s overall NTE:

\[ \text{NTE} = \text{TE}_{\text{EUR}} + \text{TE}_{\text{GBP}} + \text{TE}_{\text{JPY}} + \text{TE}_{\text{MXN}} + ... \]
Usually, companies aggregate and report the overall NTE by maturity date, say “less than 90 days,” and “more than 90 days.”

MNCs measure Net TE. If a subsidiary has CF>0 in EUR and another subsidiary has CF<0 in EUR, NTE might be very low.

If we believe in the RW model, this is what we expect to happen at time T. Now, we know St will likely change at time T. We want to know how TE will be affected by changes in St. That is, we want to measure the FX risk involved with the transaction.

To do this, we need to say something about the variability (volatility) of St.

**Range Estimates of Transaction Exposure**

Exchange rates are volatile, difficult to forecast. A range estimate of NTE will provide a more useful number for risk managers. The smaller the range, the lower the sensitivity of the NTE⇒ The lower the FX risk.

Three popular methods for estimating a range for transaction exposure:
1. **Ad-hoc Rule**
   - Usually assuming a change in $e_t$, for example ± 0.10.
2. **Simulation/Sensitivity Analysis**
   - We use the empirical distribution (ED) of St to simulate St+T.
3. **Assuming a statistical distribution for exchange rates.**

**1. Ad-hoc Rule**

Based on past experiences, a firm assumes that St can change (in either direction) by a fixed percentage. Then, it calculates the range of NTE under the assumed percentage.

**Example:** SC wants to estimates the sensitivity of NTE to changes in the St. They use the ±10% rule ($e_t = ±0.10$). (Recall: receivable USD 1 M due in 30 days.)

⇒ if St changes by ±10%, then NTE changes by CHF ±145,000. ¶

**Note:** This example presents a range for NTE. $NTE \in [\text{USD 1.305M}, \text{USD 1.595 M}]$. The wider the range, the riskier an exposure is.

The ± percentage used depends on the volatility of the portfolio of currencies in NTE:

- Higher volatility in the currencies in the portfolio ⇒ A higher ± percentage.

**2. Simulation/Sensitivity Analysis**

**Goal:** Measure the sensitivity of TE to different exchange rates.

There are different ways to approach sensitivity analysis. Popular approaches: Look at the empirical distribution (ED), do a simulation.

**Examples:** Sensitivity of TE to extreme forecasts of St.

Sensitivity of TE to randomly simulated thousands of St. Then, draw a histogram to analyze the empirical distribution of TE.
Example ED-I: Using the ED for SC’s Net TE (CHF/USD) over one month.
Statistics for the Empirical Distribution (ED) of monthly \(e_{ft}\) over the past 27 years (1990-2017), for 325 \(e_{ft}\)’s:

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>-0.12260</td>
</tr>
<tr>
<td>0.25</td>
<td>-0.02066</td>
</tr>
<tr>
<td>Median</td>
<td>-0.00086</td>
</tr>
<tr>
<td>0.75</td>
<td>0.00192</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.12570</td>
</tr>
<tr>
<td>Average ((\mu))</td>
<td>-0.000886</td>
</tr>
<tr>
<td>Standard Deviation ((\sigma))</td>
<td>0.03151</td>
</tr>
</tbody>
</table>

\(\Rightarrow\) Extremes: \(15.09\)% (on September 2011) and \(-12.26\)% (on December 2008).

(A) Best case scenario: largest appreciation of USD: 0.1509
NTE: USD 1M x 1.45 CHF/USD x (1 + 0.12570) = CHF 1,632,265.

(B) Worst case scenario: largest depreciation of USD: -0.1162
NTE: USD 1M x 1.45 CHF/USD x (1 + (-0.12260)) = CHF 1,272,230.

Based on these extremes, we estimate a range for TE
\(\Rightarrow\) TE \(\in\) [CHF 1,272,230, CHF 1,632,265]

Practical Application: If SC is counting on the USD 1M to cover CHF expenses, from a risk management perspective, the expenses to cover should not exceed CHF 1,281,510. ¶

Note: Some firms may feel that this range, based on extremes, is too conservative. After all, the probability of the worst case scenario to happen is very low (only once in 248 months!). Under more likely scenarios, we may be able to cover more expenses with the lower bound.

Simulating TE from Empirical Distribution
A different range can be constructed through sampling from the ED. Typical simulation:

(i) Randomly draw one scenarios from the ED -say, \(e_{ft}=e_{ft}=\text{Jun 1999}\).
(ii) Calculate quantity of interest using simulated scenario -say, \(TE = \text{USD 1M} \times S_{t+30} (1 + e_{ft=\text{Jun 1999}})\).
(iii) Repeat (i)-(ii) \(R\) times. This is your simulated distribution. Analyze it as usual (calculate mean, SD, (1-\(\alpha\))% C.I., etc.)

Example ED-II: Simulation for SC’s Net TE (CHF/USD) over one month.
Based on the ED, we will draw \(R = 1,000\) \(e_{ft}\) realizations (past monthly \(e_{ft}\).). Then, we calculate 1,000 TE for each scenario drawn. Steps:

(i) Randomly draw \(e_{ft} = e_{sim,1}\) from ED: Observation 19: \(e_{ft} = 0.0034\).
(ii) Calculate \(S_{sim,1} = 1.45 \text{ CHF/USD} \times (1 + 0.0034) = 1.4549\)
(iii) Calculate \(TE_{sim,1} = \text{USD 1M} \times S_{t+30} = 1,454.937.57\)
(iv) Repeat (i)-(iii) 1,000 times. Plot the 1,000 TEs in a histogram. (This is your simulated TE distribution.)
Using the excel functions Randbetween and Vlookup, below we have the first 9 draws:

<table>
<thead>
<tr>
<th>Lookup cell</th>
<th>$e_{t,t}$</th>
<th>Random Draw with Randbetween</th>
<th>Draw $e_{sim}$ with Vlookup</th>
<th>$S_{sim}$</th>
<th>TE(sim)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.0025</td>
<td>19</td>
<td><strong>0.0034</strong></td>
<td>1.4549</td>
<td><strong>1,454,937.57</strong></td>
</tr>
<tr>
<td>3</td>
<td>-0.0027</td>
<td>147</td>
<td>-0.0104</td>
<td>1.4349</td>
<td>1,434,895.83</td>
</tr>
<tr>
<td>4</td>
<td><strong>0.0001</strong></td>
<td>99</td>
<td>0.0125</td>
<td>1.4682</td>
<td>1,468,189.96</td>
</tr>
<tr>
<td>5</td>
<td>-0.0443</td>
<td>203</td>
<td>-0.0584</td>
<td>1.3653</td>
<td>1,365,272.92</td>
</tr>
<tr>
<td>6</td>
<td>-0.0017</td>
<td>82</td>
<td>-0.0727</td>
<td>1.3446</td>
<td>1,344,597.25</td>
</tr>
<tr>
<td>7</td>
<td>-0.0031</td>
<td><strong>4</strong></td>
<td><strong>0.0001</strong></td>
<td>1.4502</td>
<td>1,450,168.79</td>
</tr>
<tr>
<td>8</td>
<td>-0.0227</td>
<td>67</td>
<td>-0.0226</td>
<td>1.4172</td>
<td>1,417,218.22</td>
</tr>
<tr>
<td>9</td>
<td>-0.0099</td>
<td>136</td>
<td>0.0095</td>
<td>1.4638</td>
<td>1,463,838.02</td>
</tr>
<tr>
<td>10</td>
<td>0.0098</td>
<td>232</td>
<td>0.0191</td>
<td>1.4777</td>
<td>1,477,749.46</td>
</tr>
</tbody>
</table>

Note: You can find this example in my homepage: www.bauer.uh.edu/rsusmel/4386/chfusd-sim.xls

Based on this simulated distribution, we can estimate a 95% range (leaving 2.5% observations to the left and 2.5% observations to the right)

$$\Rightarrow \ TE \in [\text{CHF 1.356775 M, CHF 1.52688 M}]$$

Practical Application: If SC expects to cover expenses with this USD inflow, the maximum amount in CHF to cover, using this 95% CI, should be **CHF 1,356,775.**
3. **Assuming a Distribution**

Confidence intervals (CI) based on a distribution provide a range for TE. We will assume a normal distribution to construct a CI.

**Example:** A firm can assume that \( S_t \) changes (\( e_{t,t} \)) follow a normal distribution and based on this distribution construct a \((1-\alpha)\)% confidence interval. Then, a 95\% (\( \alpha=0.05 \)) CI is given by 
\[
[\mu \pm 1.96 \sigma]. \quad \text{(Instead of 1.96, you can use 2.)} \\
\]

**Example:** CI range based on a Normal distribution.

Swiss Cruises believes that CHF/USD monthly changes (\( e_{t,t} \)) follow a normal distribution. SC estimates the mean and the variance from the empirical distribution from past 20 years.
\[
\mu = \text{Monthly mean} \approx -0.00886 \approx -0.09\% \\
\sigma^2 = \text{Monthly variance} = 0.000992 \quad (\Rightarrow \sigma = 0.03151, \text{ or } 3.15\%) \\
e_{t,t} \sim N(-0.00152, 0.03151^2).
\]

SC decides to construct a 95\% CI for \( e_{t,t} \). Then, a 95\% confidence interval is given by:
\[
[-0.000886 \pm 1.96 \times 0.03151] = [-0.06365; 0.06087].
\]

Thus, \( e_{t,t} \) is expected to be between -0.06365 and 0.006087 (with 95\% confidence).

Based on this range for \( e_{t,t} \), we derive bounds for the net TE:

(A) Upper bound
NTE: USD 1M x 1.45 CHF/USD x (1 + 0.06087) = CHF 1,538,267.

(B) Lower bound
NTE: USD 1M x 1.45 CHF/USD x (1 - 0.06365) = CHF 1,359,164.

\( \Rightarrow \) **TE** \( \in \) [CHF 1,359,164, CHF 1,538,267]

**Interpretation of CI:** If we have 100 transactions, with the same terms, we expect that in 95 of those transactions the TE will lie within the estimated 95\% confidence interval.

**Note 1:** VaR interpretation (VaR measures the worst case scenario within a one-sided CI):

**CHF 1,359,164** is the minimum revenue to be received by SC in the next 30 days, within a 97.5\% CI. **Notation:** VaR(97.5\%) = CHF 1,359,164.

If SC expects to cover expenses with this USD inflow, the maximum amount in CHF to cover, within a 97.5\% CI, should be **CHF 1,359,164**.

The VaR is often expressed as a potential loss, relative to today’s value of the cash flow at risk (or expected value, using the RWM!). In this case, we call it VaR(mean):

\[
\text{VaR(mean)} = \text{CHF 1,359,164} - \text{CHF 1,450,000} = \text{CHF -90,836}. 
\]
Note 2: For a given data set, using logarithmic (geometric) returns, it is easy to extend the results to different maturities. Suppose we are interested in building a 95% C.I. for TE in $q$ months, using data based on frequency $n$ (say, $n=1$ month). Approximation formulas for mean and SD:

\[
q\text{-mo mean} = n\text{-mo mean} \times \frac{q}{n}
\]

\[
q\text{-mo SD} = n\text{-mo SD} \times \sqrt{\left(\frac{q}{n}\right)}
\]

Suppose, in the above example, the maturity of the cash flows is extended to 3 months. Using the monthly data, we create a 95% C.I. for 3-mo TE. Then, $q=3$ and $n=1$:

3-mo mean = \text{-0.00152} \times 3 = -0.00456

3-mo SD = \text{0.03184} \times \sqrt{3} = 0.05515

95% C.I. for $e_t$:

\[
[-0.00456 \pm 1.96*0.05515] = [-0.11265; 0.10353]
\]

(A) Upper bound

NTE: USD 1M x 1.45 CHF/USD x (1 + 0.10353) = CHF 1,600,120.

(B) Lower bound

NTE: USD 1M x 1.45 CHF/USD x (1 - 0.11265) = CHF 1,286,660.

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**Real World Example: Transaction Exposure: The Case of Kellogg**

![VaR using a 97.5% one-sided CI](image)

VaR(97.5%):

Minimum revenue within a 97.5% C.I.

2.5% CHF -90,836

CHF 1.359M - CHF 1.45M

VaR: Minimum revenue within a 97.5% C.I.

2.5% CHF -90,836

CHF 1.359M - CHF 1.45M

95% C.I. for $e_t$:

\[
[-0.00456 \pm 1.96*0.05515] = [-0.11265; 0.10353]
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(A) Upper bound

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(B) Lower bound

NTE: USD 1M x 1.45 CHF/USD x (1 - 0.11265) = CHF 1,286,660.
The Kellogg Company is the world’s largest cereal company; second largest producer of cookies and crackers; and a major producer of snacks and frozen foods. The principal markets for these products include the U.S. and Europe. Kellogg’s operations are managed in two major divisions—U.S. and International— with International further delineated into Europe, Latin America, Canada, Australia, and Asia. Operating profits for fiscal year 2014 were USD 1,024 million on sales of USD 14.58 billion.

Primary exposures include the USD versus the EUR, GBP, MXN, AUD, CAD, VEF, and RUB, and in the case of inter-subsidiary transactions, the British pound versus the euro.

The total notional amount of foreign currency derivative instruments at year-end 2014 was USD 764 million, representing a settlement receivable of USD 23 million. The total notional amount of foreign currency derivative instruments at year-end 2013 was USD 517 million, representing a settlement obligation of USD 1 million. All of these derivatives were hedges of anticipated transactions, translational exposure, or existing assets or liabilities, and mature within 18 months. Assuming an unfavorable 10% change in year-end exchange rates, the settlement receivable would have become a settlement obligation of USD 53 million at year-end 2014 and the settlement obligation at year-end 2013 would have increased by approximately USD 52 million.


CHAPTER 10 - BONUS COVERAGE I: TE Simulation in R

Excel is not a very flexible way to construct a simulated C.I. We can use R to do; for this, you need to install R in your computer (go to https://www.r-project.org to download the latest version).

• Code in R (cut and paste; play with parameters R & alpha):

```r
chfusd <- read.csv("http://www.bauer.uh.edu/rsusmel/4386/chfusd.csv",sep="","") # Data
summary(chfusd) # Distribution of Data (S_t)
names(chfusd) # Repeats Header info ("DATA" & "CHF_USD"
S <- chfusd$CHF_USD # Defines S_t as the CHF_USD column of the data
T <- length(S) # Check total T (1971:1 to 2017:1)
Tstart <- 229 # Start of sample period: 1990:1
e <- matrix(0,T-1,1) # create vector of 0 to put the e_f,t
Val <- 1 # Value of transaction in FC (in M)
S_0 <- 1.45 # Today's S_t

for (i in 2:T){ # loop to calculate log (or arithmetic) returns
  # e[i] <- (S[i]/S[i-1]-1)
  e[i] <- log(S[i])-log(S[i-1])
}

Tsart <- Tstart-1 # adjust Tstart (we lost one observation in the above loop
summary(e) # summary stats for the whole sample of e
summary(e[Tstart:T-1]) # summary stats for sample of interest of e
sd(e[Tstart:T-1]) # Standard Deviation of e during sample of interest

# simulation
```

Excel is not a very flexible way to construct a simulated C.I. We can use R to do; for this, you need to install R in your computer (go to https://www.r-project.org to download the latest version).
R = 1000  # Number of simulations
alpha = .05  # Specify (1-alpha)% Confidence Interval

R_low <- R*alpha/2  
R_high <- R*(1-alpha/2)  

x_sam <- sample(e[Tstart:T-1], size=R, replace=TRUE)  # generate R e_f draws from ED

TE <- Val*S_0*(1+x_sam)  # calculate R TE values

# Sort and Histogram
STE <- sort(TE)  # sort TE (from lowest to highest)
print(STE[R_low])  # print TE_lower bound
print(STE[R_high])  # print TE_upper bound

hist(TE,main="Histogram for TE",
     xlab="TE (in CHF)", breaks=30)  # TE histogram

• You should get something similar to:

> print(STE[R_low])  # print TE_lower bound
[1] 1.356775
> print(STE[R_high])  # print TE_upper bound
[1] 1.52688

> hist(TE,main="Histogram for TE",
+     xlab="TE (in CHF)", breaks=30)  # TE histogram

CHAPTER 10 - BONUS COVERAGE II: Value-at-Risk (VaR)
VaR provides a number, which measures the market risk exposure of a portfolio of a firm over a given length of time. VaR measures the maximum expected loss in a given time interval, within a (one-sided) confidence interval. (This is what we call before VaR(mean).)

Note: To calculate the VaR of a portfolio, we need to specify a time interval and the significance level for the confidence interval.

**Interpretation of VaR**: VaR of an FX portfolio.
- Time interval: 1 day
- Level of significance ($\alpha$): 5% ($z_\alpha=1.645$, 95% C.I.)
- VaR of FX portfolio: USD 10,000.

This VaR amount (USD 10,000) represents the potential loss of the FX portfolio in about one every twenty days within a 95% one-sided C.I.

Banks report the VaR for their trading desk. They use $\alpha=.01\%$ ($z_\alpha=2.33$, 99% C.I.) and $T=10$ days. Regulators use the reported VaR to calculate capital requirements (capital charge). Say,

Capital charge = VaR $\times k$, $k>3$.

**Example**: Microsoft uses a VaR computation, within a 97.5% confidence interval, to estimate the maximum potential 20-day loss in the fair value of its foreign currency denominated investments and account receivables, interest-sensitive investments and equity securities. At the end of June 2001, Microsoft calculated a VaR of negligible for foreign currency instruments, USD 363 million for interest sensitive instruments, and USD 520 million for equity investments.

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**CHAPTER 10 - BONUS COVERAGE III: The Normal Distribution**

Suppose the random variable $X$ has a probability distribution function (pdf) given by:

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}, \quad -\infty < x < \infty,$$

where $\mu$ and $\sigma^2$ are any number such that $-\infty < \mu < \infty$, and $0 < \sigma^2 < \infty$. Then $X$ is said to follow a *normal distribution*. We will use the following notation: $X \sim N(\mu, \sigma^2)$. The pdf has the following bell shape:
If $Z$ follows a standard normal distribution then $Z \sim N(0,1)$, that is the green function in the above graph. Using the formula of the pdf normal distribution, tables for the cdf of the standard normal distribution have been tabulated.

1.B.1 Useful Results of the Normal Distribution
Let $X \sim N(\mu, \sigma^2)$. Then,

(i) $E[X] = \mu$ (the mean, $\mu$, is also the mode and the median)
(ii) $\text{Var}(X) = E[(X-\mu)^2] = \sigma^2$
(iii) The shape of the pdf is a symmetric bell-shaped curve centered on the mean.
(iv) Let $Z = (X-\mu)/\sigma$. Then, $Z \sim N(0,1)$.
(v) If $\delta$ and $\beta$ are any numbers with $\delta < \beta$, then

$$P[\delta < X < \beta] = P[(\delta-\mu)/\sigma < Z < (\beta-\mu)/\sigma] = F_Z[(\beta-\mu)/\sigma] - F_Z[(\delta-\mu)/\sigma],$$

where $F_Z$ represents the cdf of $Z$.

**Example**: Let $X$ be the annual stock returns (in percentage points) in the U.S. Assume that $X \sim N(11.44,16.22^2)$. (The mean and variance of $X$ have been obtained from annualizing the 1980-1990 U.S. weekly mean return and variance.) Suppose you are the manager of a portfolio that tracks the U.S. Index. You want to find the probability that your portfolio's return ($X$) is lower than -30% next year (i.e., a market crash).


That is, the probability that next year stock return is lower than 30% is 1.22%.

**Example**: Go back to the previous example. Now, suppose you want to determine a minimum return, say $\tau\%$, with probability .95. That is, you want to find the probability that your portfolio's return exceeds a level $\tau$ with probability .05. That is,
\[0.95 = P[X > \tau] = P[Z > (\tau - 11.44)/16.22] = 1 - F_Z((\tau - 11.44)/16.22).\]

From the Normal Table, we obtain that if \(F_Z(z) = 0.05\), \(z = -1.645\).

Then, \((\tau - 11.44)/16.22 = -1.645 \implies \tau = 11.44 - 1.645 (16.22) = -15.323\).

That is, there is a 95% probability that next year's portfolio return will be bigger than -15.323%.

Using the above properties, it is very easy to construct confidence intervals for the random variable \(X\), which is normally distributed with mean \(\mu\) and variance \(\sigma^2\). The key to construct confidence intervals is to select an appropriate \(z\) value, such that

\[X \in [\mu \pm z_{\alpha/2} \sigma] \text{ with a probability } (1-\alpha).\]

For a 99% confidence interval (i.e, \(\alpha = 0.01\)) \(z\) is equal to 2.58.
For a 98% confidence interval (i.e, \(\alpha = 0.02\)) \(z\) is equal to 2.33.
For a 95% confidence interval (i.e, \(\alpha = 0.05\)) \(z\) is equal to 1.96 (\(\approx\)2).
For a 90% confidence interval (i.e, \(\alpha = 0.10\)) \(z\) is equal to 1.645.