Lying to Speak the Truth: Selective Manipulation and Improved Information Transmission

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ABSTRACT

We analyze a principal-agent model in which an effort-averse agent can manipulate a publicly observable performance report. The principal cannot observe the agent's cost of effort, her effort choice, and whether she manipulated the report. An optimal contract links compensation to the realized output and the (possibly manipulated) report. Manipulation can be beneficial to the principal because it can make the report more informative about the agent's effort choice, thereby reducing the agent's information rent. This is achieved through a contract that incentivizes the agent to selectively engage in manipulation based on her effort choice.

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Financial reporting allows investors to monitor the performance of firms in which they invest. However, financial reporting is noisy, which adds frictions to the design of incentive compensation and may lead to suboptimal decisions. Some have argued that investors may benefit from allowing executives discretion in "managing" financial reports, if this would reduce the noise in their reporting (e.g., Subramanyam (1996)). But such discretion can be abused by managers if their compensation depends on the perceived performance of their firms, and a large literature on agency problems (discussed below) views the manipulation of financial reports as undesirable.[1]

We show that these two views are not necessarily incompatible. Using an optimal contracting model in which performance reports are noisy and managers have the ability to manipulate them, we find that even if manipulation can easily be prevented, shareholders may not find it optimal to do so. Instead, shareholders may benefit from allowing managers who expect their firms to perform well in the long run to manipulate an unfavorable short-term report. Such *selective manipulation* can make the report more informative about the manager's performance, decreasing the incentive compensation required to motivate the manager to exert costly effort.

However, this improved informativeness of the report comes at a cost. First, a manager's willingness to exert effort will be reduced if she anticipates that she may subsequently have to bear the disutility of manipulation. This would make it more costly for the firm to incentivize effort. Second, by diverting resources from more productive uses, manipulation may lower the firm's cash flow. Shareholders need to trade off these costs against the informational benefits of selective manipulation. We show that the benefits outweigh the costs when managerial effort is only moderately productive and the expected reduction in cash flow due to manipulation is not too large. In contrast, when effort is highly productive or the expected reduction in cash flow is large, the optimal contract prevents all manipulation.

The survey results in Graham, Harvey, and Rajgopal (2005) and De Jong et al. (2014) show that it is common for Chief Financial Officers (CFOs) to manipulate financial reports.[2] Importantly, it seems that CFOs regard such manipulation as in their firms' best interest. The view that manipulation is benign is also evident from an episode described in Jack Welch's memoir (Welch and Byrne (2003)), in which he complains about the managers of one division of General Electric (GE) that were unwilling to "pitch in" to make up for an unexpected earnings shortfall.[3] It is also consistent with the increasing use of non-

[1] Different terms are used in the literature to describe various forms of manipulation, such as fraud, irregularities, misconduct, misreporting, or misrepresentation. See Amiram et al. (2018) for an overview.

[2] See Hobson and Stirnkorb (2020) for experimental evidence.

[3] After a negative earnings surprise of \$350m was discovered, Welch was pleased by the GE division

GAAP measures in financial reporting, which firms adopt to make their performance look more appealing to investors (e.g., Doyle, Jennings, and Soliman (2013), Curtis, McVay, and Whipple (2014), Laurion (2020)).

We demonstrate the optimality of selective manipulation in a simple principal-agent model in which a manager exerts costly effort and shareholders do not observe the manager's effort cost or effort choice. Effort increases the firm's chances of earning a high terminal cash flow. Prior to the realization of the cash flow, the firm's accounting system generates a public report that is a noisy signal of the firm's performance. Both the report and the cash flow are verifiable and can be used to incentivize effort. However, the manager can manipulate the report before it is released: at a personal utility cost, she can convert an unfavorable report into a favorable one. The optimal incentive contract determines whether the manager exerts effort (depending on her effort cost) and whether she manipulates an unfavorable report. Asymmetric information about the cost of effort leads to an adverse selection problem that enables the manager to earn an information rent. The possible manipulation of the report complicates the firm's optimization problem.

A critical assumption of our model is that shareholders can influence the manager's cost of manipulating the performance report through their choice of corporate governance structures. For example, the board of directors may decide to implement a more elaborate internal control system or appoint more financial experts to the audit committee, thereby making it more difficult or costly for the manager to manipulate the firm's financial reports. For ease of exposition, we assume that the board's choice of governance arrangements—and hence the manager's manipulation cost—has no impact on the firm's cash flow. In other words, we assume that it is equally costly to the firm to have relaxed reporting standards as it is to have strict standards that prevent manipulation. Arguably, stricter standards are likely to be more costly to implement, which could make manipulation optimal at the margin. However, abstracting from this allows us to isolate the informational benefits of manipulation.

Our analysis shows that shareholders may tolerate the manipulation of performance reports even when it could easily be prevented. To understand this result, which may seem counterintuitive, it is important to note that the optimal contract condones manipulation only when the manager exerted high effort—never when she exerted low effort. This is achieved by making the manager's compensation increase in both the reported performance

managers' offers to "pitch in": *"The response of our business leaders to the crisis was typical of the GE culture. [...] many immediately offered to pitch in [...]. Some said they could find an extra $10 million, $20 million, and even $30 million from their businesses to offset the surprise. [...] their willingness to help was a dramatic contrast to the excuses I had been hearing from the Kidder people."* (Welch and Byrne (2003), ch. 15).

and the realized cash flow in such a way that the manager's marginal benefit from manipulating an unfavorable report increases in the level of effort that she chooses. Shareholders can therefore set the manipulation cost so that the manager's expected benefit of manipulation outweighs her manipulation cost only when she exerts high effort. The resulting *selective-manipulation strategy* makes the firm's report more informative about the manager's effort choice. Performance manipulation may therefore be not only unavoidable, as the literature argues, but actually desirable: allowing the manager to overstate firm performance enables shareholders to design a more efficient contract.

However, the improved informativeness of the report comes at a cost. The expected manipulation cost that the manager incurs under a selective-manipulation contract effectively increases her disutility from exerting high effort, which makes inducing managerial effort more costly to shareholders. In addition, by diverting resources from more productive uses, manipulation may lower the firm's cash flow.

We show that the benefits of a more informative report outweigh the costs associated with manipulation when managerial effort is only moderately productive and the expected reduction in cash flow due to manipulation is not too large. When the productivity of effort is limited, the optimal contract provides less powerful incentives. The manager is therefore unlikely to exert high effort (she will do so only if her effort cost is low), which means that she is also unlikely to incur the manipulation cost and the firm is unlikely to experience a reduction in cash flow due to manipulation. The manipulation costs borne by shareholders are further reduced when manipulation has a smaller effect on the firm's expected cash flow. In contrast, the optimal contract prevents all manipulation when managerial effort is highly productive or when the expected reduction in cash flow due to manipulation is large. This is because a highly productive manager is likely to be incentivized to exert high effort and hence to manipulate an unfavorable report. If the expected reduction in cash flow due to manipulation is large, the expected cost of manipulation borne by shareholders more than offsets the informational benefits of selective manipulation.

Besides offering a novel explanation for why firms may tolerate the manipulation of performance reports, our model may also help explain the growing use of performance-vesting stock and stock option grants in executive compensation packages (e.g., Bettis et al. (2018)). As we discuss in more detail in Section III.B, such grants vest when certain performance targets are met. These targets often include accounting performance measures, which can be manipulated. Our model shows that such contracts are beneficial not *despite* a manager's ability to manipulate compensation-relevant reports, but *because* of it.

A variety of explanations for the presence of manipulation have been offered in the literature. First, numerous authors argue that manipulation is an unavoidable feature of incentive compensation: incentivizing effort provision necessarily also creates incentives for manipulating performance measures (e.g., Stein (1989), Fischer and Verrecchia (2000), Guttman, Kadan, and Kandel (2006)). Some manipulation is thus tolerated under the optimal compensation contract because preventing it would destroy a manager's incentive to exert effort and hence would be too costly for the firm.[4] This is in stark contrast to our analysis, which shows that some manipulation by the manager may be beneficial to shareholders, even when it could easily be prevented.

Second, if there are limits to communication, contractibility, or commitment, then it may be optimal to let an agent manipulate information (Dye (1988), Arya, Glover, and Sunder (1998), Demski (1998)). Our results do not rely on such constraints, as the results are driven by asymmetric information.

Third, current shareholders in a firm may benefit from manipulation if it allows the firm to raise funds from third parties at favorable rates (e.g., Bar-Gill and Bebchuk (2003), Povel, Singh, and Winton (2007), Strobl (2013)). This is different from our model, since there is no second period in which funds need to be raised.

Fourth, firms may rely on information generated by investors (and revealed through market prices) when making decisions, in which case it can be optimal to allow for some manipulation if doing so strengthens the incentive to generate such information (e.g., Gao and Liang (2013)). There is no such effect in our model.

The remainder of the paper is organized as follows. Section I introduces the model. Section II solves for the equilibrium contract. The implications of the model are discussed in Section III. Section IV investigates the robustness of the model's results in various extensions and alternative setups. Finally, Section V summarizes our contribution and concludes. All proofs are contained in the Appendix.

I. The Model

We study an agency model with two risk-neutral parties—a board of directors and a manager—that takes place over times 0, 1, 2, and 3. At time 0, the board (the principal) chooses the firm's governance system (explained below) and hires a manager (the agent) to run the firm. The board represents the interests of shareholders and offers the manager a

[4] Demski, Frimor, and Sappington (2004), Goldman and Slezak (2006), Crocker and Slemrod (2007), Beyer, Guttman, and Marinovic (2014), and Bertomeu, Darrough, and Xue (2017) analyze the optimal compensation contract under the possibility of manipulation in a static setting, whereas Edmans et al. (2012), Zhu (2018), and Marinovic and Varas (2019) study the optimal dynamic contract.

contract that maximizes the value of the firm, net of the cost of managerial compensation.[5] At time 1, the manager exerts unobservable effort to enhance the value of the firm. At time 2, the firm's accounting system produces a public report about the terminal cash flow that will be realized at time 3. A key feature of our model is that this report can be manipulated by the manager.

The firm's cash flow is either high ($v = v_h$) or low ($v = v_\ell < v_h$). The distribution of v depends on the manager's effort choice and manipulation decision. For ease of exposition, we separate the effects that these two managerial actions have on the cash flow distribution and denote by $\tilde{v} \in \{v_h, v_\ell\}$ the preliminary cash flow prior to the manager's manipulation decision that depends only on the manager's effort choice: if the manager exerts effort $e \in \{0, 1\}$, then $\tilde{v} = v_h$ with probability λ_e and $\tilde{v} = v_\ell$ with probability $1 - \lambda_e$, where $0 < \lambda_0 < \lambda_1 < 1$. The preliminary cash flow $\tilde{v}$ is unobservable to all parties. The distribution of the final cash flow v conditional on the preliminary cash flow $\tilde{v}$ is assumed to be independent of the manager's effort choice e and depends only on the manipulation decision (discussed below).

The manager's private utility cost of exerting high effort ($e = 1$), denoted by c, is drawn from a uniform distribution over the interval $[0, \bar{c}]$; the cost of low effort ($e = 0$) is normalized to zero. The manager's effort choice e and effort cost c are her private information and hence cannot be used for contracting purposes. To make the problem interesting, we assume that $\bar{c} > (\lambda_1 - \lambda_0)(v_h - v_\ell)$, which ensures that inducing high effort is suboptimal when a high cost of effort c is realized. We also assume that incentivizing high effort is optimal for shareholders when the manager's cost of effort c is low. As will become clear in Section II.D below, in the absence of manipulation this requires that $\lambda_1 > 2\lambda_0$.

Prior to the realization of the cash flow, the firm's accounting system generates a preliminary report $\tilde{r}$. This report can take on one of two values, r_h or r_ℓ, and is correlated with the firm's preliminary cash flow $\tilde{v}$ as follows:

$$\text{prob}\,[\tilde{r} = r_h \,|\, \tilde{v} = v_h] = \text{prob}\,[\tilde{r} = r_\ell \,|\, \tilde{v} = v_\ell] = \delta, \tag{1}$$

where $\delta \in \left(\frac{1}{2}, 1\right)$. The parameter δ captures the quality of the firm's accounting system and is taken as exogenous in our analysis. It represents various accounting standards and conventions in the economy as well as firm- and auditor-specific factors such as the transparency of the firm's operations and the auditor's experience in the industry. The manager privately observes the preliminary report $\tilde{r}$ before it is publicly released and, at a cost, can alter its outcome—for example, by exploiting any leeway in accounting rules or by hiding information from the auditor. The publicly released report, which we denote by $r \in \{r_h, r_\ell\}$,

[5]We therefore use the terms "shareholders" and "board of directors" synonymously in our analysis.

may thus differ from the preliminary report $\tilde{r}$. Specifically, we assume that by incurring a utility cost g, the manager can turn an unfavorable preliminary report $\tilde{r} = r_\ell$ into a favorable publicly observable report $r = r_h$. Such an intervention by the manager, which we refer to as manipulation, is not observable to shareholders (and hence cannot be used for contracting purposes). Absent manipulation, the publicly released report r is identical to the preliminary report $\tilde{r}$. We allow for the possibility of mixed-strategy equilibria and denote by $m \in [0, 1]$ the probability with which the manager engages in manipulation.

It is important to note that the preliminary (unmanipulated) report $\tilde{r}$ does not reveal any information about the manager's effort choice beyond that contained in the firm's cash flow. In the absence of manipulation, the cash flow v is a sufficient statistic for the pair (r, v) with respect to the manager's effort choice e: e affects the probability of receiving a high report r_h only through its effect on the cash flow distribution. This implies that the public report r may be useful for contracting purposes not *despite* the fact that it may have been manipulated by the manager, but *because* of it: the report conveys incremental information about the manager's effort choice only if the manager's manipulation decision is contingent on her effort choice.[6]

The ability to manipulate information (at a cost) is found in many "costly state falsification" models (e.g., Dye (1988), Stein (1989), Lacker and Weinberg (1989), Maggi and Rodríguez-Clare (1995), Fischer and Verrecchia (2000), Guttman, Kadan, and Kandel (2006), Crocker and Slemrod (2007), Kartik (2009), Kartik, Ottaviani, and Squintani (2007), Beyer and Guttman (2012), Dutta and Fan (2014), Marinovic and Povel (2017)). The manipulation cost g may reflect the time spent coming up with creative ways to manage the firm's earnings or the effort involved in convincing an auditor to sign off on a biased report; it may also reflect the chances that the manager is subsequently caught and punished (which includes losing her job).[7] This cost is influenced by the legal system in which the firm operates, but firm-specific factors are also relevant, such as the rigor of the firm's accounting system and internal controls, the skills and independence of the firm's accounting and internal audit teams, the independence and experience of the board's audit committee, the choice of external auditors, etc. The firm commits to its governance system before the manager signs the contract.

To focus our analysis on the informational benefits of manipulation, we make the follow-

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[6] In Section IV.D, we discuss the implications of an alternative signal structure in which the report is a noisy signal of the manager's effort choice rather than the firm's cash flow.

[7] We could model the manipulation cost g as the product of the probability that the manager's manipulation activity is detected and the penalty that the manager would face in this case. However, doing so complicates the algebra without generating any new economic insights.

ing two assumptions. First, we assume that the board of directors can improve the firm's governance—and hence increase the manager's manipulation cost—at no cost to the firm. That is, at time 0 the board can choose any $g \geq 0$ without having to spend any resources.[8] Second, the board can discourage manipulation only through its choice of manipulation cost g and compensation scheme (described below). That is, the board cannot impose an outcome-dependent nonpecuniary penalty on the manager.[9]

Manipulation also imposes a cost on the firm. Arguably, the manipulation of financial reports may lead to a waste of resources or to suboptimal decisions based on inaccurate information, thereby reducing the firm's cash flow. We capture this cost by assuming that manipulation may lower the firm's cash flow: in the case of a high preliminary cash flow $\tilde{v} = v_h$, manipulation leads to a low final cash flow $v = v_\ell$ with probability $1 - \theta$, where $\theta \in (0, 1)$; with probability θ, the cash flow remains unchanged at $v = \tilde{v} = v_h$.[10] In practice, manipulation may also waste resources if the firm's cash flow is low, but we abstract from that case in the interest of tractability: if $\tilde{v} = v_\ell$, manipulation has no effect on the firm's cash flow (i.e., $v = \tilde{v} = v_\ell$).

The board chooses the firm's governance system and the manager's contract to maximize the value of the firm, net of the cost of managerial compensation. A contract specifies the manager's compensation as a function of the public report r and the terminal cash flow v. The manager is risk neutral, has no wealth, and is protected by limited liability, so that all payments must be nonnegative. Her reservation level of utility is normalized to zero.

The contractual frictions in our model are created by asymmetric information: the board faces an adverse selection problem (the manager's cost of effort is unobservable) and two moral hazard problems (the manager's effort choice and manipulation decision are unobservable). There is no signaling in our model since the manager's actions are all unobservable. The firm could achieve the first-best outcome if the cost of effort, c, and the chosen effort level, e, were verifiable. Under symmetric information, it would be optimal for the board to elicit high effort if and only if $\lambda_1 v_h + (1 - \lambda_1) v_\ell - c \geq \lambda_0 v_h + (1 - \lambda_0) v_\ell$; thus, the first-best effort level is $e_{FB} = 1$ if $c \leq (\lambda_1 - \lambda_0)(v_h - v_\ell)$, and $e_{FB} = 0$ otherwise.

[8]For empirical evidence that boards can affect the likelihood of manipulation by changing the firm's governance, see, for example, Beasley (1996), Dechow, Sloan, and Sweeney (1996), Fich and Shivdasani (2007), and Zhao and Chen (2008).

[9]Any penalty contingent on the report r and the cash flow v would effectively relax the nonnegativity constraints of the compensation payments and hence make incentivizing (selective) manipulation more beneficial for shareholders, but it would not alter our results qualitatively.

[10]The assumption that $\theta < 1$ is not essential for the structure of the optimal no-manipulation and selective-manipulation contract described in Propositions 1 and 2, respectively. However, as will become clear, it is necessary for selective manipulation to be suboptimal in some cases.

II. Equilibrium Analysis

In this section, we solve for the optimal incentive contract. Our specification of the set of available contracts is without loss of generality in the sense that it is fully consistent with the revelation principle. We can therefore restrict attention to truthful direct revelation mechanisms. It is important to note that this does not imply that the board will induce the manager to abstain from manipulating an unfavorable report: the manager's decision to manipulate the report r is an action and not a message. Instead, it implies that any allocation that can be achieved through a contract that is contingent on the report and the firm's cash flow can also be achieved through a truthful direct mechanism.

In the ensuing analysis, let $w(r, v|c)$ denote the compensation scheme under the direct mechanism. The fact that the manager has no wealth means that all compensation payments must be nonnegative. This implies that the manager's participation constraint is trivially satisfied: by choosing to exert zero effort and to not manipulate the report, the manager can always achieve a nonnegative payoff.

A. Preliminary Results

We first show that under the optimal contract, the manager's effort choice is characterized by a cost threshold $\hat{c}$ such that the manager exerts high effort if and only if $c < \hat{c}$. This follows immediately from incentive compatibility considerations. Suppose a manager with a cost of effort c finds it optimal to choose the high effort level. A manager with a strictly smaller cost $c' < c$ faces exactly the same feasible actions and continuation payoffs as the manager with cost c: if she also chooses the high effort level, then the continuation payoffs for each feasible action are identical for c and c', but the payoff of the manager with the lower cost c' is larger because her cost of effort is smaller. The continuation payoffs after choosing the low effort level are identical for the two managers, because the cost of exerting low effort is zero. It must therefore be optimal for a manager with a cost $c' < c$ to also choose the high effort level. Conversely, if a manager with a cost of effort c finds it optimal to choose the low effort level, then a manager with a strictly higher cost $c'' > c$ must also find it optimal to choose the low effort level.

LEMMA 1: *There exists a threshold $\hat{c} \in [0, \bar{c}]$ such that the optimal contract induces high managerial effort (i.e., $e = 1$) for all $c < \hat{c}$ and low managerial effort (i.e., $e = 0$) for all $c > \hat{c}$.*

Note that the manager is never indifferent between the high and low effort levels, except when her cost of effort is exactly at the threshold, $c = \hat{c}$. In equilibrium, under an optimal contract shareholders are also indifferent between inducing high and low managerial effort when $c = \hat{c}$, but not for any other realizations of c.[11] Since both shareholders and the manager are indifferent if and only if the zero-probability event $c = \hat{c}$ occurs, we can ignore mixed strategies concerning the manager's effort choice e. (The results in this section hold if the *manipulation* decision is randomized; however, in Proposition 3 below we show that the optimal contract never induces a mixed manipulation strategy.)

Our next result concerns the manipulation decision that the optimal contract induces the manager to take. We demonstrate that this decision depends on the manager's cost of effort only through its effect on the manager's effort choice e. This is not surprising because the cost c has no *direct* effect (besides its effect on effort choice) on the manipulation decision that the firm wants to induce: for a given effort choice e, the cost c does not affect the firm's cash flow v or report r and hence has no impact on shareholders' expected payoff.

LEMMA 2: *Suppose that an optimal contract induces the same effort choice e for two different effort costs c and c'. Then if the manipulation decision m is optimal when the manager's effort cost is c, it must also be optimal when her effort cost is c'.*

Lemma 1 shows that under the optimal contract, the manager's effort choice is identical for all realizations of the cost parameter c below the threshold $\hat{c}$ and for all realizations above the threshold $\hat{c}$. Together with the result in Lemma 2, this implies that any allocation resulting from an optimal direct mechanism can be implemented through a menu of contracts that pools all managers of type $c < \hat{c}$ and of type $c > \hat{c}$.

LEMMA 3: *The optimal mechanism can be implemented by offering the manager a menu of contracts that pools all types $c \in [0, \hat{c})$ and all types $c \in (\hat{c}, \bar{c}]$.*

Without loss of generality, we can thus set $w(r, v|c) = w_1(r, v)$ for all $c \in [0, \hat{c})$ and $w(r, v|c) = w_0(r, v)$ for all $c \in (\hat{c}, \bar{c}]$, where the subscript 1 (respectively, 0) indicates the region of parameter values c over which the optimal contract induces high (respectively, low) managerial effort. The optimal compensation scheme can therefore be characterized by the menu $\mathcal{W} = \{\boldsymbol{w}_0, \boldsymbol{w}_1\}$, where $\boldsymbol{w}_e = (w_e(r_h, v_h), w_e(r_\ell, v_h), w_e(r_h, v_\ell), w_e(r_\ell, v_\ell))$. For notational convenience, we also define the *manipulation schedule* $\mathcal{M} = (m_0, m_1) \in [0, 1]^2$ as the manipulation choices that the board wants to induce, where m_e is the desired manipulation choice for a given effort choice e.

[11] We analyze the optimal choice of the cost threshold $\hat{c}$ in Proposition 6 below.

B. The Principal's Problem

An optimal contract $\mathcal{C}$ consists of a compensation scheme $\mathcal{W}$ and manipulation cost g that maximize shareholders' expected payoff (i.e., the firm's expected cash flow net of the manager's expected compensation). Using the results in Lemmas 1 to 3, we can decompose this optimization problem into two separate problems. First, we characterize the contract that minimizes the firm's expected cost of compensation subject to implementing a specific manipulation schedule $\mathcal{M}$ and cost threshold $\hat{c}$ (i.e., inducing the manager to exert high effort if and only if $c \leq \hat{c}$). Second, we solve for the optimal contract by optimizing over all possible manipulation schedules and cost thresholds. We begin our analysis by discussing the incentive compatibility constraints that a contract has to satisfy to induce the manager to exert high effort if and only if $c \leq \hat{c}$ and to follow the desired manipulation schedule.

To simplify the notation, let $\pi_{e,m_e}(r,v)$ denote the probability that a report $r \in \{r_h, r_\ell\}$ and cash flow $v \in \{v_h, v_\ell\}$ are realized if the manager chooses an effort level $e \in \{0,1\}$ and makes the manipulation decision $m_e \in [0,1]$ (if a preliminary report $\tilde{r} = r_\ell$ is realized). For example, if the manager exerts high effort ($e = 1$) and chooses not to manipulate ($m_1 = 0$), the firm generates a high cash flow and a high report with probability $\lambda_1\delta$; if the manager chooses to manipulate ($m_1 = 1$) a low preliminary report $\tilde{r} = r_\ell$, the probability of this outcome increases to $\lambda_1[\delta + (1-\delta)\theta]$. Thus, we have $\pi_{1,m_1}(r_h, v_h) = \lambda_1[\delta + (1-\delta)\theta m_1]$. The probabilities of the other possible outcomes are defined analogously (see the proof of Proposition 1). Using the results in Lemmas 1 to 3, we can then express the expected cost of compensation as

$$\frac{\hat{c}}{\bar{c}} \sum_{r,v} \pi_{1,m_1}(r,v)\, w_1(r,v) + \left(1 - \frac{\hat{c}}{\bar{c}}\right) \sum_{r,v} \pi_{0,m_0}(r,v)\, w_0(r,v). \tag{2}$$

For a given cost threshold $\hat{c}$ and manipulation schedule $\mathcal{M}$, the optimal contract $\mathcal{C} = (\mathcal{W}, g)$ minimizes the expected payment to the manager subject to the nonnegativity constraints

$$g \geq 0, \quad w_0(r,v) \geq 0, \quad w_1(r,v) \geq 0, \qquad \forall r \in \{r_h, r_\ell\}, v \in \{v_h, v_\ell\}, \tag{3}$$

and the following incentive compatibility (IC) constraints that ensure that the manager takes the desired actions. First, for the manager to exert high effort when $c < \hat{c}$ and low effort when $c > \hat{c}$, we must have

$$\sum_{r,v} \pi_{1,m_1}(r,v)\, w_1(r,v) - c - [\lambda_1(1-\delta) + (1-\lambda_1)\delta]\, g m_1$$

$$\geq \max_{m\in[0,1]} \sum_{r,v} \pi_{0,m}(r,v)\, w_1(r,v) - [\lambda_0(1-\delta) + (1-\lambda_0)\delta]\, gm, \quad \forall c \in [0,\hat{c}), \quad (4)$$

and

$$\sum_{r,v} \pi_{0,m_0}(r,v)\, w_0(r,v) - [\lambda_0(1-\delta) + (1-\lambda_0)\delta]\, gm_0$$
$$\geq \max_{m\in[0,1]} \sum_{r,v} \pi_{1,m}(r,v)\, w_0(r,v) - c - [\lambda_1(1-\delta) + (1-\lambda_1)\delta]\, gm, \quad \forall c \in (\hat{c},\bar{c}]. \quad (5)$$

The IC constraints above take into account the fact that the manager's effort choice affects the distribution of the firm's report and hence the likelihood that the manager will incur the manipulation cost g. The term $\lambda_1(1-\delta) + (1-\lambda_1)\delta$ (respectively, $\lambda_0(1-\delta) + (1-\lambda_0)\delta$) captures the probability of a low preliminary report $\tilde{r} = r_\ell$ if the manager chooses to exert high (respectively, low) effort.

Second, the contract needs to induce the manager to follow the manipulation schedule $\mathcal{M} = (m_0, m_1)$. The expected change in the manager's compensation if she manipulates a low preliminary report $\tilde{r} = r_\ell$ (as opposed to not manipulating it) is equal to

$$\begin{aligned} &\mathbb{E}w_e(\tilde{r} = r_\ell, m = 1) - \mathbb{E}w_e(\tilde{r} = r_\ell, m = 0) \\ &\quad = \lambda_e^\ell \left[\theta\big(w_e(r_h, v_h) - w_e(r_\ell, v_h)\big) + (1-\theta)\big(w_e(r_h, v_\ell) - w_e(r_\ell, v_h)\big)\right] \\ &\qquad + \left(1 - \lambda_e^\ell\right)\big(w_e(r_h, v_\ell) - w_e(r_\ell, v_\ell)\big), \quad (6) \end{aligned}$$

where λ_e^ℓ denotes the pre-manipulation probability of receiving a high cash flow $\tilde{v} = v_h$ conditional on observing a low preliminary report $\tilde{r} = r_\ell$, that is,

$$\lambda_e^\ell = \text{prob}\,[\tilde{v} = v_h \,|\, \tilde{r} = r_\ell, e] = \frac{\lambda_e(1-\delta)}{\lambda_e(1-\delta) + (1-\lambda_e)\delta}. \quad (7)$$

To induce the manager to follow the manipulation schedule $\mathcal{M} = (m_0, m_1)$, the compensation scheme has to satisfy the constraints

$$\mathbb{E}w_e(\tilde{r}{=}r_\ell, m{=}1) - \mathbb{E}w_e(\tilde{r}{=}r_\ell, m{=}0) \;-\; g \begin{cases} \leq 0 & \text{if } m_e = 0, \\ = 0 & \text{if } m_e \in (0,1), \\ \geq 0 & \text{if } m_e = 1, \end{cases} \qquad \forall e \in \{0,1\}. \quad (8)$$

These constraints ensure that the manager's expected benefit of manipulation (i.e., the expected increase in the manager's compensation when the report is r_h rather than r_ℓ)

outweighs (respectively, does not outweigh) her cost of manipulation when $m_e = 1$ (respectively, $m_e = 0$). The manager's cost of manipulation includes her utility cost g as well as the expected reduction in her compensation due to a potential decline in the firm's cash flow from v_h to v_ℓ, which happens with probability $1-\theta$ if $\tilde{v} = v_h$ and the manager engages in manipulation.

Finally, to ensure that the manager truthfully reports her effort cost c, it must be the case that

$$\sum_{r,v} \pi_{1,m_1}(r,v)\, w_1(r,v) - c - [\lambda_1(1-\delta) + (1-\lambda_1)\delta]\, g m_1$$
$$\geq \max_{e,m} \sum_{r,v} \pi_{e,m}(r,v)\, w_0(r,v) - ec - [\lambda_e(1-\delta) + (1-\lambda_e)\delta]\, gm, \quad \forall c \in [0,\hat{c}), \quad (9)$$

and

$$\sum_{r,v} \pi_{0,m_0}(r,v)\, w_0(r,v) - [\lambda_0(1-\delta) + (1-\lambda_0)\delta]\, g m_0$$
$$\geq \max_{e,m} \sum_{r,v} \pi_{e,m}(r,v)\, w_1(r,v) - ec - [\lambda_e(1-\delta) + (1-\lambda_e)\delta]\, gm, \quad \forall c \in (\hat{c},\bar{c}]. \quad (10)$$

For a given cost threshold $\hat{c}$ and manipulation schedule $\mathcal{M} = (m_0, m_1)$, the principal's optimization problem is thus to choose a compensation scheme $\mathcal{W} = \{\boldsymbol{w}_0, \boldsymbol{w}_1\}$ and manipulation cost g that minimize the expected cost of compensation in (2), subject to the constraints in (3) to (10). We derive the solution to this optimization problem in Section II.C below. In Section II.D, we then solve for the optimal contract by determining the cost threshold $\hat{c}$ and manipulation schedule $\mathcal{M} = (m_0, m_1)$ that maximize the expected value of the firm net of the expected cost of compensation.

C. No Manipulation versus Selective Manipulation

In this section, we derive the optimal contract for various manipulation schedules $\mathcal{M} \in [0,1]^2$, taking the cost threshold $\hat{c}$ as given. We show that incentivizing manipulation is never optimal when the manager exerts low effort, but it may be optimal when she exerts high effort.

We begin our analysis by characterizing the optimal no-manipulation contract $\mathcal{C}^n = (\boldsymbol{w}_0^n, \boldsymbol{w}_1^n, g^n)$, that is, the optimal contract that induces the manager to never manipulate the report, irrespective of her chosen effort level. Shareholders can prevent manipulation by setting a sufficiently high manipulation cost g^n. Alternatively, shareholders can design a compensation scheme that does not depend on the public report r, thus removing any incentive to manipulate the report. The following proposition shows that such a contract that is not contingent on the report is indeed optimal.

PROPOSITION 1 (No-manipulation contract): *For any cost threshold $\hat{c} \in [0, \bar{c}]$, the compensation scheme*

$$w_0^n(r,v) = w_1^n(r,v) = \begin{cases} \frac{\hat{c}}{\lambda_1 - \lambda_0} & \text{if } v = v_h, \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

and manipulation cost $g^n \geq 0$ induce the manager to exert high effort if $c \leq \hat{c}$ and low effort if $c > \hat{c}$, and to follow the manipulation schedule $m_0 = m_1 = 0$ (i.e., to never engage in manipulation), at minimum cost.

The optimal no-manipulation contract ignores the report r because the report does not improve the board's information about the manager's effort choice e compared to that conveyed by the firm's cash flow v. As discussed in Section I, in the absence of manipulation, the cash flow v is a sufficient statistic for the pair (r, v) with respect to the manager's effort choice e. Thus, the (unmanipulated) report r adds nothing to the power of inference and hence does not enable the board to design a more efficient contract. It is important to note that this is not a restriction imposed on the contract, but rather a feature of the optimal contract itself.

Despite the fact that the manager has private information about her cost of effort c, shareholders cannot benefit from offering the manager a menu of type-specific contracts with different compensation schemes depending on the (truthfully reported) cost of effort. The reason is that both the principal and the agent are risk neutral in our setting: both parties care only about the expected value of payments, contingent on the manager's actions e and m. This explains why setting the compensation scheme $\boldsymbol{w}_0$ equal to $\boldsymbol{w}_1$ is optimal: this choice of $\boldsymbol{w}_0$ (i) incentivizes a manager with cost $c > \hat{c}$ to exert low effort and (ii) ensures that the expected compensation of a low-effort manager is equal to the minimum amount required by the truth-telling constraint in (10). Intuitively, there are no real effects if a manager with cost $c > \hat{c}$ falsely reports a cost below $\hat{c}$, as long as she then chooses the desired effort level $e = 0$ and does not manipulate.

Risk neutrality is also the reason why the optimal no-manipulation contract is not unique. Any compensation scheme that leads to the same expected contingent payments as that in (11) is optimal, as long as it satisfies the IC constraints. For example, the contract could offer the manager the option to receive a fixed payment of $w_0^n = \mathbb{E}\left[w_1^n \mid e = 0\right] = \frac{\lambda_0 \hat{c}}{\lambda_1 - \lambda_0}$ instead of a cash flow–contingent payment since such an option would be attractive only to managers with a high cost of effort $c > \hat{c}$. Alternatively, the contract could include lotteries with an expected value of zero. A special case of such a lottery is a payment that is contingent on both the cash flow v and the report r (as long as the manipulation cost g^n is sufficiently high to prevent manipulation). The proof of Proposition 1 characterizes such a contract that is equivalent in terms of outcomes to that in Proposition 1.

We next turn to the optimal contract that implements the manipulation schedule $m_0 = 0$ and $m_1 = 1$, that is, that induces the manager to manipulate a low preliminary report if she exerts high effort, but not if she exerts low effort. We refer to such a contract as a selective-manipulation contract, $\mathcal{C}^s = (\boldsymbol{w}_0^s, \boldsymbol{w}_1^s, g^s)$.

PROPOSITION 2 (Selective-manipulation contract): *For any cost threshold* $\hat{c} \in [0, \bar{c}]$, *the compensation scheme*

$$w_0^s(r,v) = w_1^s(r,v) = \begin{cases} \frac{\hat{c}}{(\lambda_1-\lambda_0)\delta}\,\frac{\lambda_0(1-\delta)+(1-\lambda_0)\delta}{\lambda_0(1-\delta)+(1-\lambda_0)\delta+(1-\delta)\theta} & \text{if } r = r_h \text{ and } v = v_h, \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

and manipulation cost $g^s = \lambda_0^\ell \theta\, w_1^s(r_h, v_h)$ *induce the manager to exert high effort if* $c \leq \hat{c}$ *and low effort if* $c > \hat{c}$, *and to follow the manipulation schedule* $m_0 = 0$ *and* $m_1 = 1$ *(i.e., to engage in manipulation when* $e = 1$, *but not when* $e = 0$*), at minimum cost.*

In contrast to the no-manipulation case, under selective manipulation it is (strictly) optimal to make the compensation scheme contingent on the report r. The reason is that the firm's cash flow v is no longer a sufficient statistic for the pair (r, v) with respect to the manager's effort choice e: by manipulating an unfavorable preliminary report $\tilde{r} = r_\ell$ when she exerts high effort, the manager provides additional information about her effort choice through the report r that is not contained in the cash flow v. The board can use this additional information to design a more efficient contract, because the realization of the doubly favorable outcome $r = r_h$ and $v = v_h$ allows for a stronger inference that the manager exerted high effort than merely observing a favorable cash flow $v = v_h$. Importantly, when the board offers a selective-manipulation contract, it benefits from contracting on the report r not *despite* the fact that the signal may have been manipulated by the manager, but *because* it may have been manipulated by the manager.

The cost of manipulation, g^s, is chosen such that only a manager who exerts high effort has an incentive to manipulate a low preliminary report $\tilde{r} = r_\ell$. It should be set to the lowest possible value because a manager who exerts high effort anticipates that she may have to incur the cost g^s, and this makes her less willing to choose the high effort level. A higher compensation payment is then required to incentivize effort provision by the manager. To avoid unnecessary costs, the board therefore sets g^s equal to $\lambda_0^\ell \theta\, w_1^s(r_h, v_h)$, the minimum value required to prevent a manager who exerts low effort from manipulating a low preliminary report. Recall from (7) that λ_0^ℓ is the pre-manipulation probability of realizing a high cash flow $\tilde{v} = v_h$ if the manager chooses $e = 0$ and then observes a preliminary report $\tilde{r} = r_\ell$. Manipulation changes the publicly observed report from $r = r_\ell$ to $r = r_h$, but at the same time reduces the probability of realizing a high cash flow $v = v_h$ from λ_0^ℓ to $\lambda_0^\ell \theta$. The cost g^s thus makes a manager who exerts low effort and observes a preliminary report $\tilde{r} = r_\ell$ indifferent between manipulating and not manipulating the report: manipulation increases her expected compensation from zero to $\lambda_0^\ell \theta\, w_1^s(r_h, v_h)$, but also causes a utility loss of $g^s = \lambda_0^\ell \theta\, w_1^s(r_h, v_h)$. Since $\lambda_1^\ell > \lambda_0^\ell$, a manager who exerts high effort strictly prefers to manipulate an unfavorable preliminary report: her expected benefit of manipulation, $\lambda_1^\ell \theta\, w_1^s(r_h, v_h)$, exceeds her manipulation cost, g^s.

This difference in the expected benefit of manipulation between a high-effort manager and a low-effort manager is imperative for the feasibility of a selective-manipulation schedule: unless the difference in payments following a high report and a low report, $w^s(r_h, \cdot) - w^s(r_\ell, \cdot)$, increases in a variable that is positively correlated with managerial effort (such as the cash flow v), it is impossible to incentivize selective manipulation. This means that an "additive" payment scheme of the form $w(r, v) = w_r(r) + w_v(v)$ cannot be used to induce selective manipulation: in this case, if the high-effort manager has an incentive to manipulate an unfavorable report, so does the low-effort manager.[12]

Our next result shows that there do not exist other potentially optimal contracts: it is never optimal for shareholders to incentivize the manager to manipulate a low preliminary report when she exerts low effort or to play a mixed manipulation strategy when she exerts high effort.

PROPOSITION 3: *An optimal contract (i) does not induce manipulation after low effort and (ii) does not induce a randomized manipulation decision after high effort.*

A contract that incentivizes a manager who exerts high effort to use mixed strategies when making her manipulation decision cannot be optimal for two reasons. First, compared to the selective-manipulation contract $\mathcal{C}^s$, which always induces manipulation of a low preliminary report $\tilde{r} = r_\ell$ after high effort, inducing such behavior with a probability of less than one makes the public report r less informative about the manager's effort choice. Second, inducing $m_1 \in (0, 1)$ requires a higher manipulation cost $g > g^s$ because the cost must make a manager who chose the *high* effort level (and hence expects a high cash flow $v = v_h$ with probability $\lambda_1^\ell > \lambda_0^\ell$) indifferent between manipulating and not manipulating. In contrast, under the selective-manipulation contract $\mathcal{C}^s$, a manager who exerts *low* effort

[12] Section III.B below discusses potential implementations of a selective-manipulation contract.

is kept indifferent, whereas a manager who exerts high effort strictly prefers manipulation. Inducing mixed strategies over the choice of m_1 thus leads to two inefficiencies for shareholders: the link between effort and compensation is weakened, and the required increase in the manipulation cost g makes it more costly for shareholders to incentivize high managerial effort. Similarly, incentivizing manipulation by a manager who exerts low effort has a negative effect: it reduces the informativeness of the report about the manager's effort choice and hence increases the compensation payment required to induce high managerial effort.

Proposition 3 implies that for any desired cost threshold $\hat{c}$, the optimal contract is either the no-manipulation contract $\mathcal{C}^n$ defined in Proposition 1 (which prevents manipulation entirely) or the selective-manipulation contract $\mathcal{C}^s$ defined in Proposition 2 (which permits manipulation only after high effort). The following proposition compares the manager's expected compensation under these two contracts (taking the threshold $\hat{c}$ as given).

PROPOSITION 4: *For any cost threshold $\hat{c} \in (0, \bar{c}]$ that the board wants to implement and for any effort cost $c \in [0, \bar{c}]$, the manager's expected compensation is strictly higher under the no-manipulation contract $\mathcal{C}^n$ (defined in Proposition 1) than under the selective-manipulation contract $\mathcal{C}^s$ (defined in Proposition 2).*

Selective manipulation has two effects. First, it makes the report r more informative about the manager's effort choice: it increases the likelihood that a high-effort manager generates a high cash flow $v = v_h$ and a high report $r = r_h$ from $\lambda_1\delta$ to $\lambda_1[\delta + (1-\delta)\theta]$, while leaving the likelihood that a low-effort manager produces such an outcome unchanged (at $\lambda_0\delta$). This improved informativeness allows for a more efficient contract: it reduces the expected compensation required to induce a high level of effort.

Second, selective manipulation makes effort provision more costly for the manager. To prevent a manager who exerts low effort from manipulating, shareholders must set a sufficiently high cost of manipulation g^s. This cost must be borne by a manager who exerts high effort and, due to bad luck, generates an unfavorable preliminary report $\tilde{r} = r_\ell$. Anticipating this possibility, a manager with a low cost of effort $c < \hat{c}$ becomes less willing to choose the high effort level: manipulating a low report selectively when $e = 1$ effectively increases the manager's cost of exerting high effort by the amount of her expected manipulation cost, $[\lambda_1(1-\delta) + (1-\lambda_1)\delta]\, g^s$. The promised payment $w(r_h, v_h)$ must therefore be raised to restore the manager's incentives to exert high effort.

The increase in the payment $w(r_h, v_h)$ necessary to restore the effort partly undoes, but never outweighs, the reduction in the expected compensation made possible by the improved informativeness of the report r. The reason is that, under the selective-manipulation contract $\mathcal{C}^s$, a manager who exerts high effort is "more than willing" to manipulate a low preliminary report $\tilde{r} = r_\ell$ (i.e., the manipulation IC constraint in (8) is slack when $e = 1$). The contract's focus is not on incentivizing a high-effort manager to manipulate, but rather on preventing a *low*-effort manager from manipulating: the manipulation IC constraint of a low-effort manager is binding under the selective-manipulation contract, making the manager indifferent between manipulating and not manipulating when $\tilde{r} = r_\ell$. As both types of manager bear the same manipulation cost, a high-effort manager is therefore strictly better off manipulating a low report because she has better prospects of generating a high cash flow $v = v_h$ and hence of receiving the payment $w(r_h, v_h)$. The possibility of increasing her payoff through manipulation thus provides an additional incentive for the manager to exert high effort: the manager profits from manipulation only if she exerts high effort, which makes her more willing to exert high effort. This relaxes the effort IC constraint in (4) and hence lowers the payment $w(r_h, v_h)$ required to induce high managerial effort.[13]

The result that the manager's expected compensation is lower under the selective-manipulation contract $\mathcal{C}^s$ than under the no-manipulation contract $\mathcal{C}^n$ immediately implies that the manager prefers the latter to the former (for a given $\hat{c}$): the manager not only earns higher compensation under the no-manipulation contract, but also avoids having to bear the cost of manipulation g^s if she is confronted with a low preliminary report $\tilde{r} = r_\ell$ after having exerted high effort.

Although selective manipulation reduces a firm's cost of compensation, shareholders are not necessarily better off with such a selective-manipulation contract because manipulation also imposes a cost on the firm: it reduces a high cash flow v_h to a low cash flow v_ℓ with probability $1 - \theta$. If θ is sufficiently small or if the likelihood of manipulation is sufficiently large (which is the case if the cost threshold $\hat{c}$ is sufficiently large), then the reduction in the firm's expected cash flow may undo the benefit of the reduced expected cost of compensation, as the following proposition shows.

PROPOSITION 5: *There exists a threshold $\hat{c}^*$ (defined in (A63) in the Appendix) such that, for any cost threshold $\hat{c} \in (0, \bar{c}]$, the optimal contract is the selective-manipulation contract $\mathcal{C}^s$ if $\hat{c} < \hat{c}^*$ and the no-manipulation contract $\mathcal{C}^n$ if $\hat{c} > \hat{c}^*$.*

Furthermore, there exist thresholds $\bar{\theta}$ and $\underline{\theta}$, with $0 < \underline{\theta} < \bar{\theta} < 1$, such that $\hat{c}^* > \bar{c}$ for all $\theta > \bar{\theta}$ (making the contract $\mathcal{C}^s$ optimal for all $\hat{c} \in (0, \bar{c}]$) and $\hat{c}^* < 0$ for all $\theta < \underline{\theta}$ (making the contract $\mathcal{C}^n$ optimal for all $\hat{c} \in (0, \bar{c}]$).

Proposition 5 shows that if the expected loss in cash flow due to manipulation is small

[13]Substituting the optimal value of g^s into the effort IC constraint in (4) shows that the left-hand side of the constraint is increasing in the probability of manipulation, m_1, which means that an increase in m_1 relaxes the constraint.

(i.e., if θ is large), the board always chooses to offer the manager a selective-manipulation contract. In contrast, if the expected loss due to manipulation is large (i.e., if θ is small), the board prefers a no-manipulation contract. For intermediate values of θ, the board offers a selective-manipulation contract if it wants to implement a low cost threshold $\hat{c}$ and a no-manipulation contract if it wants to induce a high cost threshold $\hat{c}$. If $\hat{c}$ is small, the manager is unlikely to exert high effort (which happens only if $c < \hat{c}$) and hence to manipulate the report. In this case, shareholders prefer the selective-manipulation contract, because the expected loss in cash flow due to manipulation is small compared to the reduction in the expected compensation due to improved information transmission. In contrast, if $\hat{c}$ is large, the manager is likely to exert high effort and hence to manipulate the report. Shareholders thus prefer the no-manipulation contract, because the expected loss in cash flow due to selective manipulation outweighs the reduction in the expected compensation due to improved information transmission.

Inspection of $\hat{c}^*$, the maximum value of the cost threshold $\hat{c}$ for which shareholders prefer the selective-manipulation contract to the no-manipulation contract, shows that it increases in θ. This is intuitive. If manipulation has a smaller chance of destroying cash flow (i.e., a lower probability of reducing a high cash flow v_h to a low cash flow v_ℓ), a selective-manipulation contract becomes more attractive.

D. *Optimal Contract*

Our analysis in Section II.C shows that the optimal contract to implement a given cost threshold $\hat{c}$ is either a no-manipulation contract or a selective-manipulation contract. We now endogenize the board's choice of threshold $\hat{c}$ and analyze which of these two contracts will be offered in equilibrium. We demonstrate that the board's decision depends on the expected reduction in cash flow due to manipulation (which is inversely related to θ) and the (normalized) productivity of managerial effort, $\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}}$.[14] We show that when the expected reduction in cash flow due to manipulation is small and effort is only moderately productive, the board chooses a low threshold $\hat{c}$ and implements it using a selective-manipulation contract. In contrast, when the expected reduction in cash flow due to manipulation is large or effort is highly productive, the board implements a higher threshold $\hat{c}$ using a no-manipulation contract. As a first step, we derive the optimal threshold $\hat{c}$ for each type of contract.

[14] The numerator, $(\lambda_1 - \lambda_0)(v_h - v_\ell)$, is the expected value of the incremental cash flow when the manager exerts high rather than low effort. It is divided by $\bar{c}$, which captures the average cost of effort (since c is uniformly distributed over the interval $[0, \bar{c}]$).

PROPOSITION 6: *Under the no-manipulation contract $\mathcal{C}^n$, firm value is maximized at a cost threshold of*

$$\hat{c}_n = \max\left\{\frac{1}{2}\left((\lambda_1-\lambda_0)(v_h-v_\ell)-\frac{\lambda_0\bar{c}}{\lambda_1-\lambda_0}\right),0\right\}. \tag{13}$$

Under the selective-manipulation contract $\mathcal{C}^s$, firm value is maximized at a cost threshold of

$$\hat{c}_s = \max\left\{\frac{1}{2}\left(\frac{[\lambda_1-\lambda_0-\lambda_1(1-\delta)(1-\theta)](v_h-v_\ell)}{1+\frac{\lambda_0}{\lambda_1-\lambda_0}\frac{1-\delta}{\delta}\frac{[\lambda_1(1-\delta)+(1-\lambda_1)\delta]\theta}{\lambda_0(1-\delta)+(1-\lambda_0)\delta+(1-\delta)\theta}}-\frac{\lambda_0\delta\bar{c}}{(\lambda_1-\lambda_0)\delta+\lambda_1(1-\delta)\theta}\right),0\right\}. \tag{14}$$

Furthermore, there exists a threshold $\hat{\theta}\in(0,1)$ such that $\hat{c}_n\geq\hat{c}_s$ if $\theta<\hat{\theta}$ and $\hat{c}_n\leq\hat{c}_s$ if $\theta>\hat{\theta}$; these inequalities are strict if $\hat{c}_n>0$.

Under both types of contract, the board may optimally choose not to incentivize effort provision: if the expected value-added of high effort, $(\lambda_1-\lambda_0)(v_h-v_\ell)$, is small compared to the expected compensation payment, the optimal cost threshold $\hat{c}$ is equal to zero. The board implements a positive cost threshold $\hat{c}>0$ (and hence induces high effort provision by a manager with a cost $c<\hat{c}$) only if effort is sufficiently productive. The following result follows immediately from (13) and (14).

COROLLARY 1: *The optimal contract implements a cost threshold $\hat{c}>0$ (i.e., incentivizes high effort with a strictly positive probability) if and only if*

$$\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} > \min\left\{\frac{\lambda_0}{\lambda_1-\lambda_0},\frac{\lambda_0}{\lambda_1[\delta+(1-\delta)\theta]-\lambda_0}\,\frac{\lambda_0(1-\delta)+(1-\lambda_0)\delta}{\lambda_0(1-\delta)+(1-\lambda_0)\delta+(1-\delta)\theta}\right\}. \tag{15}$$

If the condition in (15) is not satisfied, the optimal contract does not incentivize effort provision (i.e., $\hat{c}_n=\hat{c}_s=0$). In this case, all compensation payments are set to zero and the two contracts are identical. If the condition in (15) is satisfied, at least one of the cost thresholds $\hat{c}_n$ and $\hat{c}_s$ is strictly positive and the optimal contract incentivizes effort provision if the manager's cost of effort is sufficiently low. Whether the board chooses the no-manipulation contract $\mathcal{C}^n$ (with cost threshold $\hat{c}=\hat{c}_n$) or the selective-manipulation contract $\mathcal{C}^s$ (with cost threshold $\hat{c}=\hat{c}_s$) in this case depends on which of these two contracts leads to a higher firm value. This is the object of the following proposition.

PROPOSITION 7: *Suppose that the condition in (15) is satisfied. Then there exists a threshold Γ (defined in (A85) in the Appendix) such that the optimal contract is the selective-*

manipulation contract $\mathcal{C}^s$ with $\hat{c} = \hat{c}_s$ if $\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} < \Gamma$ and the no-manipulation contract $\mathcal{C}^n$ with $\hat{c} = \hat{c}_n$ if $\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} > \Gamma$.

Furthermore, there exist thresholds $\bar{\theta}$ and $\underline{\theta}$, with $0 < \underline{\theta} < \bar{\theta} < 1$, such that the selective-manipulation contract $\mathcal{C}^s$ is optimal if $\theta > \bar{\theta}$ and the no-manipulation contract $\mathcal{C}^n$ is optimal if $\theta < \underline{\theta}$.

Proposition 7 characterizes the optimal contract in terms of two key parameters, the (normalized) productivity of managerial effort, $\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}}$, and the expected reduction in cash flow due to manipulation, which is inversely related to the parameter θ. Not surprisingly, if manipulation has little effect on the firm's cash flow (i.e., if θ is close to one), the board prefers the selective-manipulation contract $\mathcal{C}^s$ to induce managerial effort (i.e., to implement a cost threshold $\hat{c} > 0$). This is intuitive: selective manipulation reduces the firm's expected cost of compensation for any $\hat{c} > 0$, as shown in Proposition 4. Thus, if the expected reduction in cash flow due to manipulation is small (because θ is large), the board prefers the selective-manipulation contract. In contrast, if manipulation is likely to reduce the firm's cash flow to v_ℓ (i.e., if θ is close to zero), the board prefers the no-manipulation contract $\mathcal{C}^n$. This happens for two reasons. First, manipulation causes a significant reduction in the firm's expected cash flow. Second, a low θ makes the firm's cash flow less informative about the manager's effort choice, which reduces the contracting benefits of a more informative report r due to selective manipulation. For values of θ close to zero, the expected loss in cash flow under the selective-manipulation contract thus outweighs the reduction in agency costs due to improved information transmission.

For intermediate values of θ, the board's choice of contract depends on the productivity of managerial effort. The more productive effort is, the more attractive it is for shareholders to induce a high level of effort (i.e., to implement a high cost threshold $\hat{c}$), and hence the more likely it is for the manager to engage in manipulation under the selective-manipulation contract (and thus to potentially reduce the firm's cash flow). Consistent with Proposition 5, the board therefore prefers the selective-manipulation contract if effort is only moderately productive (i.e., if $\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} < \Gamma$) and thus it chooses to implement a low cost threshold $\hat{c}_s < \hat{c}^*$, because in this case the expected loss in cash flow due to manipulation is small compared to the reduction in expected compensation due to improved information transmission. In contrast, if effort is highly productive (i.e., if $\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} > \Gamma$), the board adopts the no-manipulation contract to implement a high cost threshold $\hat{c}^n > \hat{c}^*$, because any reduction in the manager's expected compensation due to selective manipulation would be more than offset by the expected loss in cash flow caused by a high likelihood of manipulation (manipulation is more likely when $\hat{c}$ is high because the manager is more likely to exert high effort in this case).

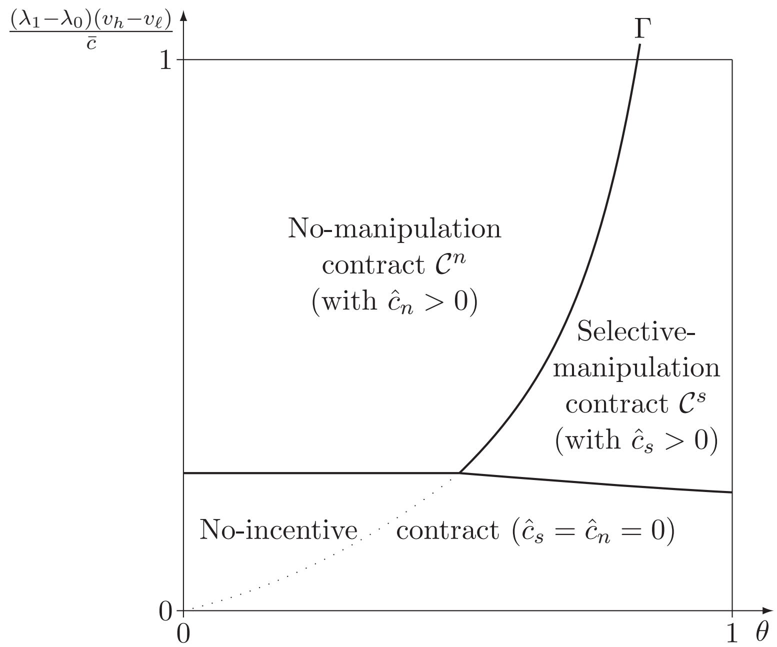

Figure 1. Optimal contract. This figure illustrates how the board's choice of contract depends on $\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}}$, the (normalized) productivity of managerial effort, and on θ, the likelihood that manipulation does not cause a reduction in the firm's cash flow from v_h to v_ℓ. The parameter values in this numerical example are $\lambda_0 = \frac{1}{6}$, $\lambda_1 = \frac{5}{6}$, and $\delta = \frac{7}{8}$.

Figure 1 provides a graphical representation of the results in Corollary 1 and Proposition 7. Recall that, by assumption, $\theta \in [0,1]$ and $\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} \in (0,1)$. The upward-sloping line is Γ, the productivity of effort that makes the board indifferent between offering the no-manipulation and the selective-manipulation contract (with optimally chosen cost thresholds $\hat{c}_n$ and $\hat{c}_s$, respectively). The horizontal line and the downward-sloping line are the lower bounds for $\hat{c}_n > 0$ and $\hat{c}_s > 0$, respectively, as defined in (15). At the intersection of these lower bounds, the value of the firm is identical under the no-manipulation and the selective-manipulation contract (because $\hat{c}_n = \hat{c}_s = 0$), which implies that they also intersect with Γ at this point.

To understand why Γ increases in θ, consider the effect that a small increase in the productivity of effort has on the value of the firm under the two contracts $\mathcal{C}^n$ and $\mathcal{C}^s$. Under both contracts, an increase in $\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}}$ makes effort provision more valuable and the board therefore increases the thresholds $\hat{c}_n$ and $\hat{c}_s$ in response. However, as argued above, the selective-manipulation contract becomes relatively less attractive at higher levels of $\hat{c}$ due to the increased loss in expected cash flow caused by manipulation. Thus, the increase in firm value is larger under the no-manipulation contract than under the selective-manipulation contract. This means that a reduction in the manipulation cost borne by the firm (i.e., an increase in θ) is needed to keep the board indifferent between the two contracts: increasing θ raises the firm's value under the selective-manipulation contract, but does not affect its value under the no-manipulation contract. Hence, Γ must be increasing in θ.

III. Implications

We now discuss additional implications of our model. We start in Section III.A by arguing that, although in equilibrium manipulation is positively related to the manager's effort choice, the discovery that a report was manipulated (which is not part of our model) can convey bad news to shareholders about the firm's future cash flow. In Section III.B, we examine how certain contractual arrangements that are frequently part of executive compensation packages can be used to implement a selective-manipulation strategy. Finally, in Section III.C, we summarize our model's predictions and relate them to a few empirical studies.

A. Manipulation: Good News or Bad News?

Since the board has full control over the manager's cost of manipulation in our model, the manager engages in manipulation only if the contract gives her an incentive to do so: in equilibrium, manipulation occurs only when shareholders choose the selective-manipulation contract, and it is limited to managers who exert high effort. One might therefore conclude that manipulation is good news for a firm's shareholders, as it suggests that the manager exerted high effort. Consequently, if investors were to discover that a report they just observed was in fact manipulated (which is not part of our model), they should raise their expectations about the firm's cash flow, and the firm's share price should increase. Such an effect would go against the traditional view that the discovery of manipulation is bad news and causes share prices to drop (e.g., Burns and Kedia (2006), Karpoff, Lee, and Martin (2008)).[15]

The above argument, however, ignores that the discovery of manipulation has a second effect on shareholders' beliefs: a manager engages in manipulation only if she observes a low preliminary report $\tilde{r} = r_\ell$. If the report's precision is sufficiently high (i.e., δ is sufficiently large), the observation of such a report indicates that the firm's cash flow is likely to be low ($v = v_\ell$). Moreover, manipulation itself may have reduced the firm's cash flow from v_h to v_ℓ (which happens with probability $1 - \theta$). Observing that a report was manipulated is thus

[15] Reputation effects also affect share prices. For more references, see Section 4.2.1 of Amiram et al. (2018).

both good news and bad news. It is good news for shareholders because it implies that the manager exerted high effort, and it is bad news because it reveals a low preliminary report $\tilde{r} = r_\ell$ and a possible cash flow reduction. Which effect dominates depends on the parameter values. If the firm's cash flow is more strongly correlated with the preliminary report than with the manager's effort choice (i.e., δ is large relative to λ_1 and $1 - \lambda_0$) or if manipulation is likely to reduce the firm's cash flow (i.e., θ is small), the negative effect dominates and the discovery that a high report was manipulated leads to a worsening of investors' expectations about the firm's cash flow, as the following proposition shows.

PROPOSITION 8: *Suppose that prior to the realization of the firm's cash flow v, shareholders can unexpectedly observe whether a favorable report r_h was manipulated by the manager. The discovery of manipulation lowers shareholders' expectations about the firm's cash flow if*

$$\frac{\lambda_1(1-\delta)\theta}{\lambda_1(1-\delta) + (1-\lambda_1)\delta} < \frac{\lambda_s\delta}{\lambda_s\delta + (1-\lambda_s)(1-\delta)}, \tag{16}$$

where $\lambda_s = \lambda_0 + \frac{\hat{c}_s}{\bar{c}}(\lambda_1 - \lambda_0)$. A sufficient condition for this inequality to be satisfied is that $\delta > \left(1 + \sqrt{\frac{1-\lambda_1}{\lambda_1}\frac{\lambda_0}{1-\lambda_0}}\right)^{-1}$.

The fact that the (unmodeled and unexpected) discovery that a high report was manipulated reveals a high managerial effort choice—and hence may lead to an upward revision of shareholders' expectations about the firm's cash flow—is driven by our assumption that the board has full control over the manager's cost of manipulation and thus can prevent any undesirable manipulation of the firm's report. This assumption, which simplifies the exposition, is not crucial to our results and can be easily relaxed. For example, we could extend our model by introducing an alternative manipulation technology that is less effective (in the sense that a manipulation attempt does not always succeed) but costless to the manager. In this case, the optimal selective-costly-manipulation contract would incentivize the manager to engage in costly manipulation when she exerts high effort, but could not prevent the manager from employing the costless manipulation technology when she exerts low effort. As a result, the discovery of manipulation would no longer be an unambiguous sign of high managerial effort.[16]

[16]Such an extension of the model complicates the analysis without generating any new insights into the costs and benefits of manipulation.

B. *Implementation of a Selective-Manipulation Strategy*

The compensation scheme of the selective-manipulation contract characterized in Proposition 2 can be implemented through various contractual arrangements that are frequently part of executive compensation packages. One such arrangement, for example, is a bonus scheme that includes clawback provisions. The selective-manipulation contract specifies the total compensation that the manager receives depending on the report r and the cash flow v, but does not restrict the timing of the payments. In particular, the contract does not preclude payments prior to the realization of the firm's cash flow. The board can thus induce selective manipulation through a bonus payment that is contingent on a favorable report $r = r_h$ in combination with a clawback provision that enables the board to reclaim the payment if the firm earns a low profit in the future. Such clawback provisions became quite common after the financial crisis of 2007 to 2009. Babenko et al. (2023) report that usage of such provisions among S&P 1,500 firms has increased from less than 1% in 2000 to over 60% by 2013.

Alternatively, a selective-manipulation contract's compensation scheme can be implemented through so-called performance-vesting (p-v) grants of stock or stock options. Instead of vesting over time at a pre-determined schedule, p-v stock and stock option grants vest only if certain performance targets (usually including accounting performance measures) are met. An executive thus enjoys the benefits of stock or stock option awards only if the accounting performance reports are favorable *and* the market's assessment of the firm's performance is sufficiently positive. Such p-v grants are standard in the United Kingdom (Kuang and Qin (2009)) and have become increasingly common in the United States: Bettis et al. (2018) study the executive compensation packages of the 750 largest U.S. public firms and report that the use of p-v grants increased from 20% in 1998 to almost 70% in 2012.

Our model offers a novel explanation for the popularity of p-v grants in executive compensation: by making payments contingent on accounting performance measures (which can easily be manipulated by executives) in addition to stock prices (which are less prone to manipulation), p-v grants incentivize a manager to selectively manipulate performance reports, thereby making these reports more informative about the manager's performance. Thus, p-v grants are beneficial to shareholders not *despite* a manager's ability to manipulate compensation-relevant reports, but *because* of it.[17]

[17] It seems plausible that managers can manipulate accounting measures more easily than stock prices, because changing investors' *beliefs* through financial reports is more difficult and because stock prices incorporate information generated independently by investors.

C. *Empirical Predictions*

The most obvious application of our model is to the manipulation of financial reports issued by companies. A key result is that firms may find it optimal to tolerate manipulation when the manager exerts high effort, but not when she exerts low effort. That is, manipulation is positively related to effort provision. Furthermore, since managers are induced to exert high effort only if the cost c is sufficiently low (i.e., below the threshold $\hat{c}$), manipulation is negatively related to the cost of effort.

Testing these predictions is difficult because doing so requires proxies for the manager's unobservable effort, which are not readily available. Most CEOs are dedicated to their jobs and spend every waking hour working. Models with unobservable effort are therefore often meant to capture a CEO's willingness to make difficult decisions or perform tedious tasks that add value, instead of pursuing more pleasant or exciting tasks that are not value-creating for the firm. Our model thus predicts that managers who are more focused on adding long-term value are more likely to manipulate financial reports than managers who spend time socializing or working on noncore activities. That is, managers who spend too much time on the golf course are less likely to manipulate.

The threshold $\hat{c}$ that a contract implements can be interpreted as the incentive power of the contract: a more high-powered contract induces high effort even if a manager has a higher cost of effort. With this interpretation, our model predicts that manipulation occurs with low-powered contracts, but not with high-powered contracts (see Proposition 5). This result contrasts with the prediction of models with unavoidable manipulation (e.g., Goldman and Slezak (2006)) in which *higher*-powered incentives to exert effort also incentivize a manager to manipulate the performance measure used to determine her compensation. Note, however, that this interpretation of incentive power differs from that typically found in the executive compensation literature. Empirical papers tend to focus on the *sensitivity* of a manager's compensation to the firm's performance. Although readily available, such a sensitivity measure is not meaningful in the context of our model because the optimal payment scheme under a no-manipulation contract is not unique (as explained in Section II.C) and may or may not be contingent on the published report r, which makes comparisons across contract types difficult.

Complicating matters further, the optimal contract induces low effort with certainty when the productivity of effort is sufficiently low (i.e., when the condition in (15) is violated). If we assume that this describes the situation in certain firms (where the CEO's effort is not crucial to their success), our model predicts a nonmonotonic relationship: manipulation should only be observed when contracts have low but positive incentive power. In cases in which the incentive power is high or there is zero incentive power, manipulation should not be observed. This nonmonotonic relationship might help reconcile conflicting empirical evidence. Some studies find a positive relationship for some (but not all) components of incentive compensation (Bergstresser and Philippon (2006), Burns and Kedia (2006)), while others find a negative relationship (Armstrong, Jagolinzer, and Larcker (2010)) or no relationship at all (Erickson, Hanlon, and Maydew (2006)).

As discussed in Section II.D, the optimality of selective manipulation depends on the (normalized) productivity of managerial effort, $\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}}$. The board prefers the selective-manipulation contract over the no-manipulation contract if this ratio does not exceed the threshold Γ but is still large enough to satisfy the condition in (15) (see Figure 1). Our model thus predicts that manipulation is more likely to occur when managerial effort is only moderately productive rather than highly productive. This result is immediate if the difference in the productivity of effort is due to a difference in $\frac{v_h-v_\ell}{\bar{c}}$, because Γ does not depend on v_h, v_ℓ, or $\bar{c}$. If the difference in the productivity of effort is due to a difference in $(\lambda_1 - \lambda_0)$, the interpretation is complicated by the fact that these parameters also influence Γ. However, numerical calculations show that Γ increases in λ_0 and decreases in λ_1.[18] This means that an increase in $(\lambda_1 - \lambda_0)$ reduces the likelihood of manipulation not only because such a change in parameter values increases the productivity of managerial effort, but also because it lowers the threshold Γ (and hence increases the set of values of the ratio $\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}}$ for which the no-manipulation contract is optimal). Unfortunately, testing the prediction that manipulation is less likely to occur when managerial effort is highly productive is challenging because existing empirical studies provide little guidance on how to proxy for the productivity of managerial effort.

Selective manipulation is beneficial for the firm because it improves the informativeness of the contracting variables and hence reduces the firm's expected cost of compensation. However, manipulation also imposes a cost on the firm: it reduces a high cash flow v_h to a low cash flow v_ℓ with probability $1-\theta$. A lower cost (i.e., higher θ) therefore makes selective manipulation more attractive to shareholders (see Proposition 7 and Figure 1). Hence, our model predicts that more manipulation will take place when manipulation is less damaging to a firm's cash flow (e.g., when manipulating financial reports requires less costly distortions of a firm's investment policy).

Our model also highlights the difficulty of drawing inferences from stock price reactions to the discovery of manipulation. The literature has interpreted negative stock price reactions as evidence that manipulation is detrimental to shareholders (e.g., Burns and Kedia (2006),

[18]Unfortunately, signing the derivative of Γ with respect to λ_1 and λ_0 is analytically intractable. Details can be found in the Internet Appendix. The Internet Appendix is available in the online version of the article on the *Journal of Finance* website.

Karpoff, Lee, and Martin (2008)). However, a negative ex-post reaction does not mean that manipulation was undesirable from an ex-ante perspective. In fact, as discussed in Section III.A, the discovery of manipulation may be good or bad news to shareholders in our model. Thus, a negative stock price reaction to the discovery of financial misreporting can not be taken as evidence that selective manipulation is value-destroying from an ex-ante perspective. Moreover, the sample of firms used in empirical studies is likely biased towards poorly performing firms. Identifying manipulation is difficult and often relies on investor lawsuits or regulatory action, events that are likely triggered by a firm's poor performance. This selection bias may lead to a further underestimation of the contracting benefits of selective manipulation.

IV. Robustness

In this section, we assess the robustness of our results. Specifically, we examine the implications of changing the timing of the contracting problem and argue that the optimality of the selective-manipulation contract is unaffected by this change. We also argue that our manipulation technology is standard and clarify that the benefit of selective manipulation is a feature of the optimal contract and not of the manipulation technology. Finally, we discuss an alternative model setup in which the report and the cash flow are conditionally independent signals of the manager's effort choice.

A. *Ex-Ante Contracting*

The manager knows her effort cost c at the contracting stage in our model. However, our results are robust to the alternative assumption that the contract is signed at the *ex-ante* stage before the manager discovers her cost of effort c.

Having the board of directors and the manager contract at the ex-ante stage would not change the incentive constraints, since these constraints are specific to the cost level c or the effort choice e, and hence need to be satisfied, unchanged, by any optimal direct mechanism, independent of the timing assumption. However, ex-ante contracting would change the manager's participation constraint: whereas the ex-post participation constraints in our model must be satisfied for any cost level c, an ex-ante participation constraint would require that the manager earn her reservation utility level of zero in expectation, given the distribution of possible cost levels. In some contracting models, replacing the ex-post participation constraints with an ex-ante participation constraint affects the principal's optimal trade-off between rent extraction and efficiency. In our setup, however, the participation constraints play no role because they are trivially satisfied: the manager can achieve a nonnegative payoff under any contract by not exerting effort and not manipulating. The limited liability constraints in (3) are sufficient to satisfy the participation constraints, and the participation constraints are therefore not part of the principal's optimization problem. The same is true in the case of ex-ante contracting. The limited liability constraints, which have to hold independent of the timing assumption, ensure that the ex-ante participation constraint is satisfied. Thus, our results would be unchanged if the contract were signed at the ex-ante stage.

B. *Ex-Ante Manipulation*

The manipulation technology employed by the manager enables her to turn an unfavorable preliminary report $\tilde{r} = r_\ell$ into a favorable public report $r = r_h$. This means that the manager can condition her manipulation decision on the preliminary report $\tilde{r}$. Our results remain qualitatively unchanged if the manager instead has to make her manipulation decision *before* observing the preliminary report: incentivizing selective manipulation is still feasible and, if the expected reduction in the firm's cash flow due to manipulation is not too large, still optimal with such an "ex-ante manipulation technology." The reason is that a manager who exerts high effort is more likely to generate a high cash flow $v = v_h$—and thus more likely to benefit from manipulation—than a manager who exerts low effort not only conditionally on having observed a low preliminary report (since $\lambda_1^\ell > \lambda_0^\ell$, where λ_e^ℓ is defined in (7)), but also unconditionally (since $\lambda_1 > \lambda_0$).

Suppose that by incurring a manipulation cost g', the manager can ensure that a favorable report $r = r_h$ will be published. Having to make her decision without (or prior to) observing the preliminary report $\tilde{r}$, the manager knows that manipulation is futile in some situations, namely when the firm's accounting system would have generated a favorable report even without manipulation. This affects the manager's IC constraints as follows. First, the manager's expected manipulation cost $[\lambda_e(1-\delta) + (1-\lambda_e)\delta]\, g$ has to be replaced by her ex-ante cost g' in the effort IC constraints in (4) and (5) and the truth-telling IC constraints in (9) and (10). Second, the manager's manipulation IC constraint in (8) becomes

$$\sum_{r,v} \big(\pi_{e,1}(r,v) - \pi_{e,0}(r,v)\big) w_e(r,v) \;-\; g' \begin{cases} \leq 0 & \text{if } m_e = 0, \\ = 0 & \text{if } m_e \in (0,1), \\ \geq 0 & \text{if } m_e = 1, \end{cases} \qquad \forall e \in \{0,1\}, \tag{17}$$

where, as in Section II, $\pi_{e,m_e}(r,v)$ denotes the probability that a report $r \in \{r_h, r_\ell\}$ and a cash flow $v \in \{v_h, v_\ell\}$ are realized if the manager chooses an effort level $e \in \{0,1\}$ and makes the manipulation decision $m_e \in [0,1]$ (see the proof of Proposition 1). These changes to the principal's optimization problem do not affect the optimal no-manipulation contract. However, they do affect the selective-manipulation contract: arguments analogous to those in the proof of Proposition 2 show that the optimal compensation scheme to induce selective manipulation is given by $w^s(r_h, v_h) = \frac{\hat{c}}{(\lambda_1-\lambda_0)[\delta+(1-\delta)\theta]}$ and $w^s(r_\ell, v_h) = w^s(r_h, v_\ell) = w^s(r_\ell, v_\ell) = 0$. Our result that selective manipulation reduces a firm's expected cost of compensation, meanwhile, still holds: the expected compensation of a low-effort manager under a selective-manipulation contract, which is given by $\pi_{0,0}(r_h, v_h) w^s(r_h, v_h) = \frac{\lambda_0 \delta \hat{c}}{(\lambda_1-\lambda_0)[\delta+(1-\delta)\theta]}$, is strictly lower than that under a no-manipulation contract (since $\theta > 0$), and the expected compensation of a high-effort manager, which is given by $\pi_{1,1}(r_h, v_h) w^s(r_h, v_h) = \frac{\lambda_1[\delta+(1-\delta)\theta]\hat{c}}{(\lambda_1-\lambda_0)[\delta+(1-\delta)\theta]}$, is identical to that under a no-manipulation contract. Thus, if the expected reduction in the firm's cash flow due to manipulation is not too large, it remains optimal for shareholders to tolerate some manipulation even when the manager cannot condition her manipulation decision on the preliminary report $\tilde{r}$.

C. *Manipulation and Information Quality*

Our model is a model of manipulation, although in equilibrium manipulation may (for some parameter values) become a tool to improve information transmission: selective manipulation makes the firm's report more informative about the manager's effort choice. We emphasize, however, that improved information transmission is a feature of the equilibrium contract, not of the manipulation technology itself.

Manipulation enables a manager to adjust a performance measure upwards, with the intention of improving her compensation. Intuitively, one might expect manipulation to make the possibly manipulated performance measure less informative for the principal. This is indeed the case in our model, depending on the manipulation schedule $\mathcal{M} = (m_0, m_1)$ being implemented. Conditional on observing a high report r_h, the probability that the manager exerted high effort $e = 1$ is given by

$$\begin{aligned} &\text{prob}\,[e = 1 \,|\, r = r_h, m_0, m_1] \\ &\quad = \frac{\frac{\hat{c}}{c}\Big[\lambda_1\Big(\delta+(1-\delta)m_1\Big)+(1-\lambda_1)\Big(1-\delta+\delta m_1\Big)\Big]}{\frac{\hat{c}}{c}\Big[\lambda_1\Big(\delta+(1-\delta)m_1\Big)+(1-\lambda_1)\Big(1-\delta+\delta m_1\Big)\Big]+\Big(1-\frac{\hat{c}}{c}\Big)\Big[\lambda_0\Big(\delta+(1-\delta)m_0\Big)+(1-\lambda_0)\Big(1-\delta+\delta m_0\Big)\Big]}. \end{aligned} \tag{18}$$

It is easily verified that

$$\begin{aligned}&\text{prob}\,[e=1\,|\,r=r_h,m_0=1,m_1=0] \;<\; \text{prob}\,[e=1\,|\,r=r_h,m_0=1,m_1=1]\\&\quad<\; \text{prob}\,[e=1\,|\,r=r_h,m_0=0,m_1=0] \;<\; \text{prob}\,[e=1\,|\,r=r_h,m_0=0,m_1=1]\,. \qquad (19)\end{aligned}$$

That is, compared with the no-manipulation case, a high report $r = r_h$ is a worse predictor of a high effort choice $e = 1$ when the manager always manipulates (i.e., when $m_0 = m_1 = 1$) or when she selectively manipulates after exerting low effort (i.e., when $m_0 = 1$ and $m_1 = 0$), while it is a better predictor when the manager selectively manipulates after exerting high effort (i.e., when $m_0 = 0$ and $m_1 = 1$). This result demonstrates that the manager can use the manipulation technology to make the report more or less informative about her effort choice. In equilibrium, the manager chooses to manipulate only if doing so increases her expected utility, and shareholders incentivize manipulation only if doing so improves the value of the firm. The result that manipulation improves the information quality of the report is therefore a feature of the equilibrium and reflects the optimality of the contract rather than any limitations of the manipulation technology. Our manipulation technology is standard, but the equilibrium contract uses this technology in a novel way.

D. *Reports as a Signal of Effort*

In our model, the preliminary report $\tilde{r}$ is a noisy signal of the preliminary cash flow $\tilde{v}$. Through this dependence, the preliminary report $\tilde{r}$ is indirectly also a noisy signal of the manager's effort choice. This signal is not incrementally informative when the board uses a no-manipulation contract: the cash flow v is a sufficient statistic for the pair (r, v) with respect to the manager's effort choice e in the absence of manipulation. However, under a selective-manipulation contract, the *observed* report r is useful because it contains information about the manager's effort choice that is not contained in the cash flow v (through the manager's effort-dependent manipulation decision).

Arguably, the manager's effort choice might also influence the report in a more direct way, not only through its effect on the firm's cash flow v. For example, the report could be interpreted as an intermediate performance report, and if the manager made good decisions in the recent past (i.e., exerted high effort), the probability of a favorable intermediate report could be higher. To assess the robustness of our findings to such an alternative signal structure, we analyze a version of the model in which the (preliminary) report and the cash flow are conditionally independent signals of the manager's effort choice. The results of this alternative model are largely the same: the board always offers either a selective-manipulation contract or a no-manipulation contract, and it incentivizes selective manipulation when managerial effort is only moderately productive and the board therefore prefers to implement a low cost threshold $\hat{c}$. However, since the report conveys additional information about the manager's effort choice that is not contained in the firm's cash flow even without manipulation, the optimal no-manipulation contract rewards the manager only when both the report and the cash flow signal a high effort choice (i.e., when $r = r_h$ and $v = v_h$).[19] This is in contrast to our baseline model analyzed in Section II, in which, in the absence of manipulation, the firm does not benefit from making the manager's compensation contingent on the report.

V. Conclusion

Practitioners have long argued that manipulation may be helpful in that it can eliminate some of the noise inherent in financial reports, particularly when unfavorable reports shed a wrong (negative) light on a firm's performance. In this paper, we formalize this argument and show that under an appropriately designed incentive contract, manipulation can indeed improve the informativeness of a performance report used to determine the manager's compensation, thereby reducing the manager's information rent. Of course, this benefit of manipulation must be traded off against the cost of manipulation: by diverting resources from more productive uses, manipulation may reduce the firm's cash flow. In addition, manipulation may be used opportunistically by managers to increase their own compensation.

We present a simple principal-agent model that captures this trade-off. We show that an optimally designed incentive contract may condone the manipulation of unfavorable reports by managers who exert a high level of effort (and hence expect their firms to perform well), but never by managers who exert a low level of effort. This type of selective manipulation makes the report more informative about the manager's effort choice and thus strengthens the link between effort choice and compensation. However, selective manipulation is not always optimal. When the expected reduction in the firm's cash flow due to manipulation is large or the manager is likely to engage in manipulation under a selective-manipulation contract (which is the case when managerial effort is highly productive and hence likely to be incentivized), the costs associated with manipulation outweigh the benefits of a more informative performance report and the firm finds it optimal to prevent all manipulation.

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[19] A detailed analysis of this alternative model setup can be found in the Internet Appendix.

Appendix

Proof of Lemma 1: We prove this result by contradiction. Suppose the result does not hold. Then there must exist a cost $c_0 > 0$ that induces effort choice $e = 0$ and a cost $c_1 > c_0$ that induces effort choice $e = 1$. Thus, letting $U(e, m, c)$ denote the manager's expected utility if she chooses effort e and manipulation strategy m when facing a cost of effort c (that she reports truthfully), we must have

$$U(0, m_0, c_0) \geq U(1, m_1, c_0), \tag{A1}$$

$$U(1, m_1, c_1) \geq U(0, m_0, c_1), \tag{A2}$$

where m_e denotes the manager's optimal manipulation choice for a given effort choice e. Furthermore, let $\hat{U}(e, m, c, c')$ denote a type-c manager's expected utility from choosing e and m when she mimics the behavior of a type-c' manager (i.e., claims to be of type c' and chooses e and m accordingly). Since a type-c_0 manager prefers not to mimic the behavior of a type-c_1 manager, we have

$$U(0, m_0, c_0) \geq \hat{U}(1, m_1, c_0, c_1) > U(1, m_1, c_1), \tag{A3}$$

where the last inequality follows from the fact that $c_1 > c_0$. Similarly, since a type-c_1 manager prefers not to mimic the behavior of a type-c_0 manager, we have

$$U(1, m_1, c_1) \geq \hat{U}(0, m_0, c_1, c_0) = U(0, m_0, c_0), \tag{A4}$$

where the equality follows from the fact that the effort cost does not directly affect the manager's expected utility if she chooses low effort $e = 0$. Clearly, the two inequalities in (A3) and (A4) are inconsistent with each other, proving that such a case cannot exist. The result must therefore be true. □

Proof of Lemma 2: For a given effort choice e, the manager's cost of effort does not affect the distribution of the firm's cash flow v or the report r. Thus, if the manager chooses the same effort level e when her effort cost is c or c', her continuation payoffs and hence her incentives to engage in manipulation are the same in both cases. Furthermore, since the firm's cash flow v depends on the manger's effort cost only through its effect on the manager's effort choice e, if shareholders find it optimal to induce the manager to manipulate the report with probability m when her effort cost is c, doing so must also be optimal when the manager's effort cost is c', as long as the manager's optimal effort choice is the same for c and c'. □

Proof of Lemma 3: From Lemma 1, it follows that all manager types $c \in [0, \hat{c})$ choose the same effort $e = 1$ and hence make the same manipulation decision m_1 (Lemma 2). Thus, these types face the same probability of generating outcome (r, v), for all $r \in \{r_h, r_\ell\}$ and $v \in \{v_h, v_\ell\}$. This means that under an incentive compatible mechanism, these types must all receive the same expected compensation. Otherwise, they would all report to be of the type that generates the highest expected compensation. Without loss of generality, we can therefore set $w(r, v|c) = w_1(r, v)$, for all $c \in [0, \hat{c})$. An analogous argument holds for all manager types $c \in (\hat{c}, \bar{c}]$, so without loss of generality we can set $w(r, v|c) = w_0(r, v)$, for all $c \in (\hat{c}, \bar{c}]$. $\square$

Proof of Proposition 1: We derive the optimal no-manipulation contract by first considering a simplified optimization problem. We then show that the solution to this simplified problem is also a solution to the full optimization problem in (2) to (10).

To simplify the notation, let $\pi_{e,m_e}(r, v)$ denote the probability that a report $r \in \{r_h, r_\ell\}$ and a cash flow $v \in \{v_h, v_\ell\}$ are produced when the manager chooses effort level $e \in \{0, 1\}$ and follows the manipulation schedule $m_e \in [0, 1]$, that is,

$$\pi_{e,m_e}(r_h, v_h) = \lambda_e[\delta + (1-\delta)\theta m_e], \tag{A5}$$

$$\pi_{e,m_e}(r_\ell, v_h) = \lambda_e(1-\delta)(1-m_e), \tag{A6}$$

$$\pi_{e,m_e}(r_h, v_\ell) = (1-\lambda_e)(1-\delta+\delta m_e) + \lambda_e(1-\delta)(1-\theta)m_e, \tag{A7}$$

$$\pi_{e,m_e}(r_\ell, v_\ell) = (1-\lambda_e)\delta(1-m_e). \tag{A8}$$

Also, define $\Delta\pi_{m_0,m_1}(r, v) = \pi_{1,m_1}(r, v) - \pi_{0,m_0}(r, v)$.

We begin by rewriting the expected cost of compensation in (2). Setting $e = 0$ and $m = m_0$ on the right-hand side of (9) yields

$$\sum_{r,v} \pi_{1,m_1}(r, v)\, w_1(r, v) \geq \sum_{r,v} \pi_{0,m_0}(r, v)\, w_0(r, v) + c + G(m_0, m_1), \tag{A9}$$

where $G(m_0, m_1)$ denotes the difference in the manager's expected manipulation cost when she exerts high rather than low effort, that is,

$$G(m_0, m_1) = [\lambda_1(1-\delta) + (1-\lambda_1)\delta]\, gm_1 - [\lambda_0(1-\delta) + (1-\lambda_0)\delta]\, gm_0. \tag{A10}$$

Similarly, setting $e = 1$ and $m = m_1$ on the right-hand side of (10), we have

$$\sum_{r,v} \pi_{0,m_0}(r, v)\, w_0(r, v) \geq \sum_{r,v} \pi_{1,m_1}(r, v)\, w_1(r, v) - c - G(m_0, m_1). \tag{A11}$$

Since (A9) must hold for any $c \in [0, \hat{c})$ and (A11) must hold for any $c \in (\hat{c}, \bar{c}]$, both constraints must be binding at $c = \hat{c}$. For a given cost threshold $\hat{c}$ and manipulation schedule $\mathcal{M} = (m_0, m_1)$, the principal's objective function can therefore be written as

$$\min_{\boldsymbol{w}_0, \boldsymbol{w}_1, g} \sum_{r,v} \pi_{1,m_1}(r,v)\, w_1(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right) \big(\hat{c} + G(m_0, m_1)\big). \tag{A12}$$

We next consider a simplified optimization problem. In particular, we solve for the optimal compensation scheme $\boldsymbol{w}_1$ that implements an effort choice characterized by the threshold $\hat{c} \in (0, \bar{c}]$ for a given manipulation schedule $\mathcal{M}^n = (0,0)$ and (temporarily) ignore the contracting variables $\boldsymbol{w}_0$ and g, the effort IC constraint in (5) (for the case in which $c > \hat{c}$), and the truth-telling constraints in (9) and (10). Since $G(m_0, m_1) = 0$ when $m_0 = m_1 = 0$, the simplified problem is given by

$$\min_{\boldsymbol{w}_1} \sum_{r,v} \pi_{1,0}(r,v)\, w_1(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right) \hat{c} \tag{A13}$$

$$\text{s.t.} \sum_{r,v} \Delta\pi_{0,0}(r,v)\, w_1(r,v) \geq \hat{c} \tag{A14}$$

$$w_1(r,v) \geq 0, \quad \forall r \in \{r_h, r_\ell\}, v \in \{v_h, v_\ell\}. \tag{A15}$$

Denoting the Lagrangian multiplier of the constraint in (A14) by ν and the respective multipliers of the limited liability constraints in (A15) by $\xi_{r,v}$, we derive the first-order condition of the optimization problem above with respect to $w_1(r,v)$ as

$$\pi_{1,0}(r,v) - \nu\, \Delta\pi_{0,0}(r,v) - \xi_{r,v} = 0, \tag{A16}$$

with the complementary slackness condition $\xi_{r,v} w_1(r,v) = 0$.

We first show that the effort IC constraint in (A14) must be binding. For the constraint to be satisfied for any $\hat{c} > 0$, at least one of the payments $w_1(r_h, v_h)$ and $w_1(r_\ell, v_h)$ must be strictly positive because $\Delta\pi_{0,0}(r_h, v_\ell) = -(\lambda_1 - \lambda_0)(1-\delta) < 0$ and $\Delta\pi_{0,0}(r_\ell, v_\ell) = -(\lambda_1 - \lambda_0)\delta < 0$. If the constraint in (A14) were not binding for any $\hat{c} > 0$, the expected compensation in (A13) could be reduced by lowering one of these positive payments without violating any constraints. Optimality thus requires that the IC constraint in (A14) be binding.

Since $\pi_{1,0}(r,v) > 0$ and $\Delta\pi_{0,0}(r,v) < 0$ for the two outcomes (r_h, v_ℓ) and (r_ℓ, v_ℓ) and since $\nu \geq 0$, the first-order condition in (A16) implies that $\xi_{r_h,v_\ell} > 0$ and $\xi_{r_\ell,v_\ell} > 0$. Thus, complementary slackness requires that $w_1(r_h, v_\ell) = w_1(r_\ell, v_\ell) = 0$. Furthermore, for the IC constraint in (A14) to hold for $\hat{c} > 0$, at least one of the two remaining payments, $w_1(r_h, v_h)$

and $w_1(r_\ell, v_h)$, must be positive. In fact, it is optimal for both of them to be positive: if the probabilities $\pi_{1,0}(r_h, v_h)$ and $\Delta\pi_{0,0}(r_h, v_h)$ satisfy the first-order condition in (A16) when $\xi_{r_h,v_h} = 0$, so do the probabilities $\pi_{1,0}(r_\ell, v_h)$ and $\Delta\pi_{0,0}(r_\ell, v_h)$ when $\xi_{r_\ell,v_h} = 0$, because

$$\frac{\pi_{1,0}(r_h, v_h)}{\Delta\pi_{0,0}(r_h, v_h)} = \frac{\lambda_1}{\lambda_1 - \lambda_0} = \frac{\pi_{1,0}(r_\ell, v_h)}{\Delta\pi_{0,0}(r_\ell, v_h)}. \tag{A17}$$

This implies that any nonnegative payments $w_1(r_h, v_h)$ and $w_1(r_\ell, v_h)$ that make the IC constraint in (A14) hold with equality are optimal, which includes the payments

$$w_1(r_h, v_h) = w_1(r_\ell, v_h) = \frac{\hat{c}}{\lambda_1 - \lambda_0}. \tag{A18}$$

Now consider the "no-manipulation" contract $\mathcal{C}^n = (\boldsymbol{w}_0^n, \boldsymbol{w}_1^n, g^n)$ with $w_1^n(r_h, v_h) = w_1^n(r_\ell, v_h) = \frac{\hat{c}}{\lambda_1 - \lambda_0}$ and $w_1^n(r_h, v_\ell) = w_1^n(r_\ell, v_\ell) = 0$ as above, $w_0^n(r, v) = w_1^n(r, v)$ for all $r \in \{r_h, r_\ell\}$ and $v \in \{v_h, v_\ell\}$, and $g^n \geq 0$. Since $\boldsymbol{w}_0$ and g are not part of the simplified problem, this contract is clearly a solution to the simplified problem in (A13) to (A15). Furthermore, since the objective functions in (A12) and (A13) are identical when $m_0 = m_1 = 0$ and since the constraints in (A14) and (A15) are implied by the constraints in (4) and (3), respectively, the contract $\mathcal{C}^n$ is also a solution to the full optimization problem characterized in Section II.B if it satisfies the additional constraints in (3) to (10).

The contract $\mathcal{C}^n$ clearly satisfies the nonnegativity constraints in (3). Furthermore, the compensation payments satisfy the manipulation IC constraint in (8) for any $g^n \geq 0$ when $m_0 = m_1 = 0$ because the manager's expected gain from manipulation is negative: $w_1^n(r_h, v) = w_1^n(r_\ell, v)$ for all $v \in \{v_h, v_\ell\}$ and $w_1^n(r_\ell, v_h) > w_1^n(r_h, v_\ell)$.

Since the manager's expected gain from manipulation is negative, the right-hand side of (4) is maximized by setting $m = 0$. The constraint in (4) then becomes identical to the constraint in (A14) and is binding. The right-hand side of (5) is also maximized by setting $m = 0$. Since $\boldsymbol{w}_0^n = \boldsymbol{w}_1^n$, this means that the expression on the right-hand side of (5) is identical to the expression on the left-hand side of (4) when $m_1 = 0$. Furthermore, the expression on the left-hand side of (5) is identical to the expression on the right-hand side of (4) when $m_0 = 0$ because the right-hand side of (4) is maximized by setting $m = 0$, as argued above. Thus, the result that (4) is binding implies that (5) is also binding.

The truth-telling constraint in (9) is identical to the constraint in (4) when $e = 0$ on the right-hand side of (9) because $\boldsymbol{w}_0^n = \boldsymbol{w}_1^n$. When $e = 1$, the constraint in (9) is more restrictive when $m = 0$ on the right-hand side because the manager's expected gain from manipulation is negative, as argued above. This means that the constraint is trivially satisfied when $e = 1$ because, for $m = 0$ (and $m_1 = 0$), the expression on the left-hand side equals the expression on the right-hand side. Similarly, the truth-telling constraint in (10) is identical to the constraint in (5) when $e = 1$ on the right-hand side of (10) because $\boldsymbol{w}_0^n = \boldsymbol{w}_1^n$. When $e = 0$, the constraint in (10) is more restrictive when $m = 0$ on the right-hand side because the manager's expected gain from manipulation is negative. This means that the constraint is trivially satisfied when $e = 0$ because, for $m = 0$ (and $m_0 = 0$), the expression on the left-hand side equals the expression on the right-hand side.

The optimal no-manipulation contract is not unique, however. As argued above, any nonnegative payments $w_1(r_h, v_h)$ and $w_1(r_\ell, v_h)$ that make the IC constraint in (A14) hold with equality are optimal. Thus, there exists a continuum of optimal compensation schemes satisfying

$$(\lambda_1 - \lambda_0)\left[\delta\, w_1(r_h, v_h) + (1-\delta)\, w_1(r_\ell, v_h)\right] = \hat{c}. \tag{A19}$$

As is easily verified, the expected cost of compensation in (A13) is the same for any non-negative $w_1(r_h, v_h)$ and $w_1(r_\ell, v_h)$ that satisfy this condition. If $w_1(r_\ell, v_h) \geq \frac{\theta}{\delta+(1-\delta)\theta}\frac{\hat{c}}{\lambda_1-\lambda_0}$, any $g \geq 0$ is sufficient to satisfy the manipulation IC constraint in (8), while for smaller $w_1(r_\ell, v_h)$ the board must set

$$g \geq \lambda_1^\ell \left[\theta \frac{\hat{c}}{(\lambda_1 - \lambda_0)\delta} - \left(1 + \frac{1-\delta}{\delta}\theta\right) w_1(r_\ell, v_h)\right]. \tag{A20}$$

A special case is the contract that pays positive compensation only if $r = r_h$ and $v = v_h$. In this case, the payment is $w_1(r_h, v_h) = \frac{\hat{c}}{(\lambda_1-\lambda_0)\delta}$ and the manipulation cost is $g = \lambda_1^\ell \theta \frac{\hat{c}}{(\lambda_1-\lambda_0)\delta}$. □

Proof of Proposition 2: The derivation of the optimal contract that induces manipulation by the manager if she exerts high effort but not if she exerts low effort (i.e., if and only if $c < \hat{c}$) is similar to that of the optimal no-manipulation contract. We again first consider a simplified optimization problem that minimizes the cost of implementing an effort choice characterized by the threshold $\hat{c}$ for a given manipulation schedule $\mathcal{M}^s = (0, 1)$. We then show that its solution is also a solution to the full optimization problem in (2) to (10). The simplified problem consists of (i) the objective function in (A12) (ignoring the contracting variable $\boldsymbol{w}_0$), which is equivalent to the objective function in (2) as demonstrated in the proof of Proposition 1, (ii) the effort-choice constraint in (4) for the case in which $c < \hat{c}$ (for both $m = 0$ and $m = 1$ on the right-hand side), and (iii) the nonnegativity constraint for $\boldsymbol{w}_1$ in (3). Since $G(m_0, m_1) = [\lambda_1(1-\delta) + (1-\lambda_1)\delta]\, g \geq 0$ when $m_0 = 0$ and $m_1 = 1$, the simplified problem is thus given by

$$\min_{\boldsymbol{w}_1, g} \sum_{r,v} \pi_{1,1}(r,v)\, w_1(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right) \left(\hat{c} + [\lambda_1(1-\delta) + (1-\lambda_1)\delta]\, g\right) \tag{A21}$$

$$\text{s.t.} \sum_{r,v} \Delta\pi_{0,1}(r,v)\, w_1(r,v) \geq \hat{c} + [\lambda_1(1-\delta) + (1-\lambda_1)\delta]\, g \tag{A22}$$

$$\sum_{r,v} \Delta\pi_{1,1}(r,v)\, w_1(r,v) \geq \hat{c} + (\lambda_1 - \lambda_0)(1-2\delta) g \tag{A23}$$

$$w_1(r,v) \geq 0, \quad \forall r \in \{r_h, r_\ell\}, v \in \{v_h, v_\ell\}. \tag{A24}$$

Denoting the Lagrangian multiplier of the constraint in (A22) by ν, the multiplier of the constraint in (A23) by μ, and the respective multipliers of the limited liability constraints in (A24) by $\xi_{r,v}$, we derive the first-order condition of the optimization problem above with respect to $w_1(r,v)$ as

$$\pi_{1,1}(r,v) - \nu\, \Delta\pi_{0,1}(r,v) - \mu\, \Delta\pi_{1,1}(r,v) - \xi_{r,v} = 0, \tag{A25}$$

with the complementary slackness condition $\xi_{r,v} w_1(r,v) = 0$, and the first-order condition with respect to g as

$$-\left(1 - \frac{\hat{c}}{\bar{c}}\right) [\lambda_1(1-\delta) + (1-\lambda_1)\delta] + \nu\, [\lambda_1(1-\delta) + (1-\lambda_1)\delta] + \mu(\lambda_1 - \lambda_0)(1-2\delta) = 0. \tag{A26}$$

We first show that the IC constraints in (A22) and (A23) must both be binding. Suppose this is not the case. If the constraint in (A22) is slack, we must have $\nu = 0$. The first-order condition in (A26) then implies that $\mu < 0$ (since $\lambda_1 > \lambda_0$ and $\delta > \frac{1}{2}$). But this violates the condition that the multiplier μ has to be nonnegative at the optimum. Thus, the constraint in (A22) must be binding. Similarly, if the constraint in (A23) is slack, we must have $\mu = 0$. Since a payment $w_1(r,v)$ can be strictly positive only if $\xi_{r,v} = 0$, the first-order condition in (A25) then implies that $\nu = \frac{\pi_{1,1}(r,v)}{\Delta\pi_{0,1}(r,v)} = \frac{\pi_{1,1}(r,v)}{\pi_{1,1}(r,v) - \pi_{0,0}(r,v)}$. However, since $\pi_{1,1}(r,v) \geq 0$ and $\pi_{0,0}(r,v) > 0$ for all (r,v), this expression either exceeds one (if $\pi_{1,1}(r,v) \geq \pi_{0,0}(r,v)$) or is nonpositive (if $\pi_{1,1}(r,v) < \pi_{0,0}(r,v)$). In both cases, it violates the first-order condition in (A26) when $\mu = 0$, which requires that $\nu = 1 - \frac{\hat{c}}{\bar{c}} \in [0,1]$. Thus, the constraint in (A23) must be binding.

Since both IC constraints in (A22) and (A23) must be binding at the optimum, we obtain the following expression for g by subtracting (A23) from (A22):

$$g = \frac{1}{\lambda_0(1-\delta) + (1-\lambda_0)\delta} \sum_{r,v} \left(\Delta\pi_{0,1}(r,v) - \Delta\pi_{1,1}(r,v)\right) w_1(r,v) \tag{A27}$$

$$= \frac{1}{\lambda_0(1-\delta)+(1-\lambda_0)\delta} \sum_{r,v} \big(\pi_{0,1}(r,v) - \pi_{0,0}(r,v)\big)\, w_1(r,v). \tag{A28}$$

Note that with this choice of g, the two IC constraints in (A22) and (A23) become identical. We can therefore drop one of the constraints. Substituting g into the objective function in (A21) and the constraint in (A22), we can rewrite the optimization problem as

$$\min_{\boldsymbol{w}_1} \sum_{r,v} \pi_{1,1}(r,v)\, w_1(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right) \left[\hat{c} + \kappa \sum_{r,v} \big(\pi_{0,1}(r,v) - \pi_{0,0}(r,v)\big)\, w_1(r,v)\right] \tag{A29}$$

$$\text{s.t.} \sum_{r,v} \Delta\pi_{0,1}(r,v)\, w_1(r,v) = \hat{c} + \kappa \sum_{r,v} \big(\pi_{0,1}(r,v) - \pi_{0,0}(r,v)\big)\, w_1(r,v) \tag{A30}$$

$$w_1(r,v) \geq 0, \quad \forall r \in \{r_h, r_\ell\}, v \in \{v_h, v_\ell\}, \tag{A31}$$

where

$$\kappa = \frac{\lambda_1(1-\delta)+(1-\lambda_1)\delta}{\lambda_0(1-\delta)+(1-\lambda_0)\delta} < 1. \tag{A32}$$

As before, denote the Lagrangian multiplier of the constraint in (A30) by ν and the respective multipliers of the limited liability constraints in (A31) by $\xi_{r,v}$. The first-order condition with respect to $w_1(r,v)$ is then given by

$$\pi_{1,1}(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right) \kappa\big(\pi_{0,1}(r,v) - \pi_{0,0}(r,v)\big)$$
$$- \nu \left[\Delta\pi_{0,1}(r,v) - \kappa\big(\pi_{0,1}(r,v) - \pi_{0,0}(r,v)\big)\right] - \xi_{r,v} = 0. \tag{A33}$$

For a payment $w_1(r,v)$ to be positive, complementary slackness requires that $\xi_{r,v} = 0$. The first-order condition in (A33) then implies that

$$\nu = \frac{\pi_{1,1}(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right) \kappa\big(\pi_{0,1}(r,v) - \pi_{0,0}(r,v)\big)}{\Delta\pi_{0,1}(r,v) - \kappa\big(\pi_{0,1}(r,v) - \pi_{0,0}(r,v)\big)}. \tag{A34}$$

First, consider the outcome (r_ℓ, v_ℓ). Since $\pi_{1,1}(r_\ell, v_\ell) = \pi_{0,1}(r_\ell, v_\ell) = 0$ and $\pi_{0,0}(r_\ell, v_\ell) = (1-\lambda_0)\delta > 0$, the numerator of (A34) is positive, whereas the denominator of (A34) is negative (because $\kappa < 1$). But this violates the condition that the multiplier ν has to be nonnegative at the optimum. Thus, we must have that $\xi_{r_\ell, v_\ell} > 0$ and hence $w_1(r_\ell, v_\ell) = 0$.

The same is true for the outcome (r_ℓ, v_h). Since $\pi_{1,1}(r_\ell, v_h) = \pi_{0,1}(r_\ell, v_h) = 0$ and $\pi_{0,0}(r_\ell, v_h) = \lambda_0(1-\delta) > 0$, the expression in (A34) is negative, again violating the condition that the multiplier ν has to be nonnegative at the optimum. We must therefore have that $\xi_{r_\ell, v_h} > 0$ and hence $w_1(r_\ell, v_h) = 0$.

Next, consider the outcome (r_h, v_ℓ). In this case, the denominator of (A34), which is given by

$$\begin{aligned}
&\Delta\pi_{0,1}(r_h, v_\ell) - \kappa\big(\pi_{0,1}(r_h, v_\ell) - \pi_{0,0}(r_h, v_\ell)\big) \\
&\quad = 1 - \lambda_1 + \lambda_1(1-\delta)(1-\theta) - (1-\lambda_0)(1-\delta) \\
&\qquad - \kappa\left[1 - \lambda_0 + \lambda_0(1-\delta)(1-\theta) - (1-\lambda_0)(1-\delta)\right] && \text{(A35)} \\
&\quad = -(\lambda_1 - \lambda_0)(1-\delta) + (1-\lambda_1)\delta + \lambda_1(1-\delta)(1-\theta) - \kappa\left[(1-\lambda_0)\delta + \lambda_0(1-\delta)(1-\theta)\right] && \text{(A36)} \\
&\quad = -(\lambda_1 - \lambda_0)(1-\delta) - (\lambda_1 - \kappa\lambda_0)(1-\delta)\theta, && \text{(A37)}
\end{aligned}$$

is negative because $\lambda_1 > \lambda_0$ and $\kappa < 1$. The numerator of (A34) can be positive or negative. If it is positive (which happens for values of $\hat{c}$ close to $\bar{c}$), this again violates the optimality condition that $\nu \geq 0$. Hence, $w_1(r_h, v_\ell) = 0$ in this case. If the numerator is negative (which happens for values of $\hat{c}$ close to zero), it follows from (A34) that $0 \leq \nu < 1$ (because $\pi_{0,0}(r_h, v_\ell) = (1-\lambda_0)(1-\delta) > 0$ and hence $\pi_{1,1}(r_h, v_\ell) > \Delta\pi_{0,1}(r_h, v_\ell)$, and because $\pi_{0,1}(r_h, v_\ell) > \pi_{0,0}(r_h, v_\ell)$). Thus, the payment $w_1(r_h, v_\ell)$ can be positive in this case. However, it cannot be the only positive payment, because if it were, the effort IC constraint in (A30) would be violated (since $\Delta\pi_{0,1}(r_h, v_\ell) - \kappa[\pi_{0,1}(r_h, v_\ell) - \pi_{0,0}(r_h, v_\ell)] < 0$, as shown above). Thus, $w_1(r_h, v_h)$ also has to be positive in this case. But if $w_1(r_h, v_h) > 0$ and hence $\xi_{r_h, v_h} = 0$, (A34) implies that $\nu > 1$ because

$$\pi_{1,1}(r_h, v_h) = \lambda_1[\delta + (1-\delta)\theta] > \pi_{0,1}(r_h, v_h) = \lambda_0[\delta + (1-\delta)\theta] > \pi_{0,0}(r_h, v_h) = \lambda_0\delta \tag{A38}$$

and $\kappa < 1$. Thus, since $w_1(r_h, v_\ell) > 0$ requires that $\nu < 1$, and since $w_1(r_h, v_h) > 0$ requires that $\nu > 1$, the two payments cannot both be positive. Together with the fact that the payment $w_1(r_h, v_\ell)$ cannot be the only positive payment, this implies that $w_1(r_h, v_\ell) = 0$.

For the IC constraint in (A30) to hold, the payment $w_1(r_h, v_h)$ must be equal to

$$\begin{aligned}
w_1(r_h, v_h) &= \frac{\hat{c}}{\Delta\pi_{0,1}(r_h, v_h) - \kappa\big(\pi_{0,1}(r_h, v_h) - \pi_{0,0}(r_h, v_h)\big)} && \text{(A39)} \\
&= \frac{\hat{c}}{(\lambda_1 - \lambda_0)\delta + (\lambda_1 - \kappa\lambda_0)(1-\delta)\theta}, && \text{(A40)}
\end{aligned}$$

which is positive since $\lambda_1 > \lambda_0$ and $\kappa < 1$. Using the definition of κ in (A32), we can write this payment as

$$w_1(r_h, v_h) = \frac{\hat{c}}{(\lambda_1 - \lambda_0)\delta} \frac{\lambda_0(1-\delta) + (1-\lambda_0)\delta}{\lambda_0(1-\delta) + (1-\lambda_0)\delta + (1-\delta)\theta}. \tag{A41}$$

Since $w_1(r_\ell, v_h) = w_1(r_h, v_\ell) = w_1(r_\ell, v_\ell) = 0$, it follows from (A28) that the optimal manipulation cost is given by $g = \lambda_0^\ell \theta\, w_1(r_h, v_h)$, where λ_0^ℓ is defined in (7).

Now consider the selective-manipulation contract $\mathcal{C}^s = (\boldsymbol{w}_0^s, \boldsymbol{w}_1^s, g^s)$ with $w_1^s(r_h, v_h) = \frac{\hat{c}}{(\lambda_1 - \lambda_0)\delta + (\lambda_1 - \kappa\lambda_0)(1-\delta)\theta}$, $w_1^s(r_\ell, v_h) = w_1^s(r_h, v_\ell) = w_1^s(r_\ell, v_\ell) = 0$, $g^s = \lambda_0^\ell \theta\, w_1^s(r_h, v_h)$ as above, and $w_0^s(r, v) = w_1^s(r, v)$ for all $r \in \{r_h, r_\ell\}$ and $v \in \{v_h, v_\ell\}$. Since $\boldsymbol{w}_0$ is not part of the simplified problem, this contract is clearly a solution to the simplified problem in (A21) to (A24). Furthermore, since the objective functions in (A12) and (A21) are identical when $m_0 = 0$ and $m_1 = 1$ and since the constraints in (A22), (A23), and (A24) are implied by the constraints in (3) and (4), the contract $\mathcal{C}^s$ is also a solution to the full optimization problem characterized in Section II.B if it satisfies the additional constraints in (3) to (10).

The contract $\mathcal{C}^s$ clearly satisfies the nonnegativity constraints in (3). Furthermore, the compensation payments satisfy the manipulation IC constraint in (8): since $g^s = \lambda_0^\ell \theta\, w_1(r_h, v_h)$, the manager's expected gain from manipulation conditional on a low (preliminary) report $\tilde{r} = r_\ell$, $\lambda_0^\ell \theta\, w_1(r_h, v_h)$, is equal to her manipulation cost when $e = 0$ (i.e., the constraint is binding when $e = 0$) and it exceeds her manipulation cost when $e = 1$ (i.e., the constraint is slack when $e = 1$) because $\lambda_1^\ell > \lambda_0^\ell$.

Since $g^s = \lambda_0^\ell \theta\, w_1(r_h, v_h)$, the right-hand side of (4) is the same for $m = 0$ and $m = 1$: the (ex ante) expected gain from manipulation, $\lambda_0(1-\delta)\theta\, w_1(r_h, v_h)$, is equal to the expected cost, $[\lambda_0(1-\delta) + (1-\lambda_0)\delta]\, g^s$. The constraint in (4) then becomes identical to the constraint in (A22) and is binding. The right-hand side of (5) is maximized by setting $m = 1$: the expected gain from manipulation, $\lambda_1(1-\delta)\theta\, w_1(r_h, v_h)$, exceeds the expected cost, $[\lambda_1(1-\delta) + (1-\lambda_1)\delta]\, g^s = \kappa\lambda_0(1-\delta)\theta\, w_1(r_h, v_h)$. Since $\boldsymbol{w}_0^s = \boldsymbol{w}_1^s$, this means that the expression on the right-hand side of (5) is identical to the expression on the left-hand side of (4) when $m_1 = 1$. Furthermore, the expression on the left-hand side of (5) is identical to the expression on the right-hand side of (4) when $m_0 = 0$ because the right-hand side of (4) is maximized by setting $m = 0$, as demonstrated above. Thus, the result that (4) is binding implies that (5) is also binding.

The truth-telling constraint in (9) is identical to the constraint in (4) when $e = 0$ on the right-hand side of (9) because $\boldsymbol{w}_0^s = \boldsymbol{w}_1^s$. When $e = 1$, the constraint in (9) is more restrictive when $m = 1$ on the right-hand side: the expected gain from manipulation is $\lambda_1(1-\delta)\theta\, w_0(r_h, v_h)$ and hence exceeds the expected cost of $[\lambda_1(1-\delta) + (1-\lambda_1)\delta]\, g^s = \kappa\lambda_0(1-\delta)\theta\, w_1(r_h, v_h)$. This means that the constraint is trivially satisfied when $e = 1$ because, for $m = 1$ (and $m_1 = 1$), the expression on the left-hand side equals the expression on the right-hand side. Similarly, the truth-telling constraint in (10) is identical to the constraint in (5) when $e = 1$ on the right-hand side of (10) because $\boldsymbol{w}_0^s = \boldsymbol{w}_1^s$. When $e = 0$, the constraint in (10) is (weakly) more restrictive when $m = 0$ on the right-hand side: the expected gain from manipulation is $\lambda_0(1-\delta)\theta\, w_1(r_h, v_h)$ and hence equals the expected cost of $[\lambda_0(1-\delta)+(1-\lambda_0)\delta]\, g^s = \lambda_0(1-\delta)\theta\, w_1(r_h, v_h)$. This means that the constraint is trivially satisfied when $e = 0$ because, for $m = 0$ (and $m_0 = 0$), the expression on the left-hand side equals the expression on the right-hand side. $\square$

Proof of Proposition 3: We prove this result by showing that (i) any contract that induces manipulation decisions $m_0 > 0$ and $m_1 = 0$ is dominated by the no-manipulation contract $\mathcal{C}^n$ derived in Proposition 1, (ii) any contract that induces manipulation decisions $m_0 > 0$ and $m_1 = 1$ is dominated by the selective-manipulation contract $\mathcal{C}^s$ derived in Proposition 2, and (iii) any contract that induces manipulation decisions $m_0 \geq 0$ and $m_1 \in (0, 1)$ is dominated by the no-manipulation contract $\mathcal{C}^n$ as well.

As shown in the proof of Proposition 1, the expected cost of compensation can be written as

$$\sum_{r,v} \pi_{1,m_1}(r, v)\, w_1(r, v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right) \big(\hat{c} + G(m_0, m_1)\big), \tag{A42}$$

where $G(m_0, m_1)$ is defined in (A10). Since $g \geq 0$ and hence $G(0, m_1) \geq G(m_0, m_1)$, the expected cost of compensation if $m_0 > 0$ can therefore not be lower than

$$\sum_{r,v} \pi_{1,m_1}(r, v)\, w_1(r, v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right) \big(\hat{c} + G(0, m_1)\big), \tag{A43}$$

the expected cost of compensation if $m_0 = 0$.

First, consider the case in which $m_0 > 0$ and $m_1 = 0$. If $m_1 = 0$, the IC constraint in (4) requires that

$$\sum_{r,v} \Delta\pi_{0,0}(r, v)\, w_1(r, v) \geq \hat{c} \tag{A44}$$

when m is set to zero on the right-hand side. This constraint is identical to the IC constraint in (A14) of the simplified problem analyzed in the proof of Proposition 1. Furthermore, the objective function in (A13) is identical to (A43) if $m_1 = 0$. The optimal no-manipulation contract $\mathcal{C}^n$ thus minimizes (the lower bound of) the expected cost of compensation in (A43) (with $m_1 = 0$) subject to the IC constraint in (A44) and the limited liability constraints $w_1(r, v) \geq 0$. But these constraints also have to be satisfied by any contract that implements the manipulation decisions $m_0 > 0$ and $m_1 = 0$. Furthermore, the additional constraints in (3) to (10) cannot reduce the expected cost of compensation. Hence, any contract that implements the cost threshold $\hat{c}$ and the manipulation decisions $m_0 > 0$ and $m_1 = 0$ is dominated by the no-manipulation contract $\mathcal{C}^n$.

Next, consider the case in which $m_0 > 0$ and $m_1 = 1$. If $m_1 = 1$, the IC constraint in (4) requires that

$$\sum_{r,v} \Delta\pi_{0,1}(r,v)\, w_1(r,v) \geq \hat{c} + [\lambda_1(1-\delta) + (1-\lambda_1)\delta]\, g \tag{A45}$$

when m is set to zero on the right-hand side, and that

$$\sum_{r,v} \Delta\pi_{1,1}(r,v)\, w_1(r,v) \geq \hat{c} + (\lambda_1 - \lambda_0)(1-2\delta) g \tag{A46}$$

when m is set to one on the right-hand side. These constraints are identical to the IC constraints in (A22) and (A23) of the simplified problem analyzed in the proof of Proposition 2. Furthermore, the objective function in (A21) is identical to (A43) if $m_1 = 1$. The optimal selective-manipulation contract $\mathcal{C}^s$ thus minimizes (the lower bound of) the expected cost of compensation in (A43) (with $m_1 = 1$) subject to the IC constraints in (A45) and (A46) and the limited liability constraints $w_1(r,v) \geq 0$. But these constraints also have to be satisfied by any contract that implements the manipulation decisions $m_0 > 0$ and $m_1 = 1$. Furthermore, the additional constraints in (3) to (10) cannot reduce the expected cost of compensation. Hence, any contract that implements the cost threshold $\hat{c}$ and the manipulation decisions $m_0 > 0$ and $m_1 = 1$ is dominated by the selective-manipulation contract $\mathcal{C}^s$.

Finally, consider the case in which $m_0 \geq 0$ and $m_1 \in (0,1)$. In this case, a manager who chose the high effort level must be indifferent between choosing $m_1 = 0$ and $m_1 = 1$. Thus, the IC constraints in (4) and (8) require that

$$\sum_{r,v} \Delta\pi_{0,m_1}(r,v)\, w_1(r,v) \geq \hat{c} + [\lambda_1(1-\delta) + (1-\lambda_1)\delta]\, g m_1 \tag{A47}$$

and

$$g = \lambda_1^\ell \big(\theta w_1(r_h,v_h) + (1-\theta) w_1(r_h,v_\ell) - w_1(r_\ell,v_h)\big) + \Big(1-\lambda_1^\ell\Big)\big(w_1(r_h,v_\ell) - w_1(r_\ell,v_\ell)\big), \tag{A48}$$

where λ_1^ℓ is defined in (7). Substituting (A48) into (A47) (and using the definitions of $\pi_{e,m_e}(r,v)$ in (A5) to (A8)) yields

$$\sum_{r,v} \Delta\pi_{0,0}(r,v)\, w_1(r,v) \geq \hat{c}, \tag{A49}$$

which is identical to the effort IC constraint in (A14) of the simplified problem considered in the proof of Proposition 1. Furthermore, using (A48) (and the definitions of $\pi_{e,m_e}(r,v)$ in (A5) to (A8)), we can write the lower bound of the expected cost of compensation in (A43) as

$$\sum_{r,v} \pi_{1,0}(r,v)\, w_1(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right)\hat{c} + \frac{\hat{c}}{\bar{c}}\left[\lambda_1(1-\delta) + (1-\lambda_1)\delta\right] g m_1. \tag{A50}$$

Since $g \geq 0$, the expected cost of compensation can therefore not be lower than

$$\sum_{r,v} \pi_{1,0}(r,v)\, w_1(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right)\hat{c}, \tag{A51}$$

the expected cost of compensation in the no-manipulation case given by (A13). The optimal no-manipulation contract $\mathcal{C}^n$ thus minimizes (the lower bound of) the expected cost of compensation subject to the IC constraint in (A49) and the limited liability constraints $w_1(r,v) \geq 0$. But these constraints also have to be satisfied by any contract that implements the manipulation decisions $m_0 \geq 0$ and $m_1 \in (0,1)$. Furthermore, the additional constraints in (3) to (10) cannot reduce the expected cost of compensation. Hence, any contract that implements the cost threshold $\hat{c}$ and the manipulation decisions $m_0 \geq 0$ and $m_1 \in (0,1)$ is dominated by the no-manipulation contract $\mathcal{C}^n$. □

Proof of Proposition 4: For a given cost threshold $\hat{c} > 0$, the expected compensation of a manager with a high cost of effort $c > \hat{c}$ equals $\sum_r \pi_{0,0}(r,v_h)\, w_1^n(r,v_h) = \frac{\lambda_0 \hat{c}}{\lambda_1 - \lambda_0}$ under the no-manipulation contract $\mathcal{C}^n$ defined in Proposition 1 and $\pi_{0,0}(r_h,v_h)\, w_1^s(r_h,v_h) = \frac{\lambda_0 \delta \hat{c}}{(\lambda_1-\lambda_0)\delta} \frac{\lambda_0(1-\delta)+(1-\lambda_0)\delta}{\lambda_0(1-\delta)+(1-\lambda_0)\delta+(1-\delta)\theta}$ under the selective-manipulation contract $\mathcal{C}^s$ defined in Proposition 2. The latter is clearly smaller (because $\theta > 0$).

For a given cost threshold $\hat{c} > 0$, the expected compensation of a manager with a low cost of effort $c < \hat{c}$ equals $\sum_r \pi_{1,0}(r,v_h)\, w_1^n(r,v_h) = \frac{\lambda_1 \hat{c}}{\lambda_1 - \lambda_0}$ under the no-manipulation contract $\mathcal{C}^n$ defined in Proposition 1 and $\pi_{1,0}(r_h,v_h)\, w_1^s(r_h,v_h) = \frac{\lambda_1[\delta+(1-\delta)\theta]\hat{c}}{(\lambda_1-\lambda_0)\delta} \frac{\lambda_0(1-\delta)+(1-\lambda_0)\delta}{\lambda_0(1-\delta)+(1-\lambda_0)\delta+(1-\delta)\theta}$ under the selective-manipulation contract $\mathcal{C}^s$ defined in Proposition 2. The latter is smaller because

$$\frac{\delta + (1-\delta)\theta}{\delta} \frac{\lambda_0(1-\delta) + (1-\lambda_0)\delta}{\lambda_0(1-\delta) + (1-\lambda_0)\delta + (1-\delta)\theta} < 1, \tag{A52}$$

or equivalently, $\lambda_0(1-\delta) + (1-\lambda_0)\delta < \delta$ (which holds since $\delta > \frac{1}{2}$). □

Proof of Proposition 5: From Proposition 3, we know that for any cost threshold $\hat{c}$, the optimal contract is either the no-manipulation contract $\mathcal{C}^n$ defined in Proposition 1 or the selective-manipulation contract $\mathcal{C}^s$ defined in Proposition 2.

Under the optimal no-manipulation contract $\mathcal{C}^n$ specified in Proposition 1, the value of the firm (net of the expected cost of compensation) is given by

$$V_n(\hat{c}) = \frac{\hat{c}}{\bar{c}}\left(\lambda_1 v_h + (1-\lambda_1)v_\ell\right) + \left(1 - \frac{\hat{c}}{\bar{c}}\right)\left(\lambda_0 v_h + (1-\lambda_0)v_\ell\right) - \mathbb{E}w^n(\hat{c}) \tag{A53}$$

$$= V_0 + \frac{\hat{c}}{\bar{c}}(\lambda_1 - \lambda_0)(v_h - v_\ell) - \mathbb{E}w^n(\hat{c}), \tag{A54}$$

where $V_0 = \lambda_0 v_h + (1-\lambda_0)v_\ell$. Using (A12), we can write the expected cost of compensation as

$$\mathbb{E}w^n(\hat{c}) = \sum_r \pi_{1,0}(r, v_h)\, w_1^n(r, v_h) - \left(1 - \frac{\hat{c}}{\bar{c}}\right)\hat{c} = \frac{\lambda_1}{\lambda_1 - \lambda_0}\,\hat{c} - \left(1 - \frac{\hat{c}}{\bar{c}}\right)\hat{c}. \tag{A55}$$

Similarly, under the optimal selective-manipulation contract $\mathcal{C}^s$ specified in Proposition 2, the value of the firm (net of the expected cost of compensation and net of the expected loss of cash flow directly due to manipulation) is given by

$$\begin{aligned} V_s(\hat{c}) = {} & \frac{\hat{c}}{\bar{c}}\left(\lambda_1[\delta + (1-\delta)\theta]v_h + [\lambda_1(1-\delta)(1-\theta) + 1 - \lambda_1]v_\ell\right) \\ & + \left(1 - \frac{\hat{c}}{\bar{c}}\right)\left(\lambda_0 v_h + (1-\lambda_0)v_\ell\right) - \mathbb{E}w^s(\hat{c}) \end{aligned} \tag{A56}$$

$$= V_0 + \frac{\hat{c}}{\bar{c}}(\lambda_1 - \lambda_0)(v_h - v_\ell) - \frac{\hat{c}}{\bar{c}}\lambda_1(1-\delta)(1-\theta)(v_h - v_\ell) - \mathbb{E}w^s(\hat{c}), \tag{A57}$$

where, as before, $V_0 = \lambda_0 v_h + (1-\lambda_0)v_\ell$. The expected cost of compensation $\mathbb{E}w^s(\hat{c})$ follows immediately from the objective function in (A12) and the compensation scheme in Proposition 2 (using the expression in (A40)):

$$\mathbb{E}w^s(\hat{c}) = \pi_{1,1}(r_h, v_h)\, w_1^s(r_h, v_h) - \left(1 - \frac{\hat{c}}{\bar{c}}\right)\left(\hat{c} + [\lambda_1(1-\delta) + (1-\lambda_1)\delta]\, g^s\right) \tag{A58}$$

$$= \frac{\lambda_1[\delta + (1-\delta)\theta]\,\hat{c}}{(\lambda_1 - \lambda_0)\delta + (\lambda_1 - \kappa\lambda_0)(1-\delta)\theta} - \left(1 - \frac{\hat{c}}{\bar{c}}\right)\left(\hat{c} + [\lambda_1(1-\delta) + (1-\lambda_1)\delta]\, g^s\right) \tag{A59}$$

$$= \frac{\lambda_1\delta + \left[\lambda_1 - \left(1 - \frac{\hat{c}}{\bar{c}}\right)\kappa\lambda_0\right](1-\delta)\theta}{(\lambda_1 - \lambda_0)\delta + (\lambda_1 - \kappa\lambda_0)(1-\delta)\theta}\,\hat{c} - \left(1 - \frac{\hat{c}}{\bar{c}}\right)\hat{c}, \tag{A60}$$

where κ is defined in (A32).

Shareholders (strictly) prefer the selective-manipulation contract over the no-manipulation contract if $V_s(\hat{c})$ (strictly) exceeds $V_n(\hat{c})$. For any cost threshold $\hat{c} > 0$, the expressions in (A54) and (A57) imply that $V_s(\hat{c}) \underset{(<)}{>} V_n(\hat{c})$ if and only if

$$\frac{\hat{c}}{\bar{c}}\lambda_1(1-\delta)(1-\theta)(v_h-v_\ell)+\frac{\lambda_1\delta+\left[\lambda_1-\left(1-\frac{\hat{c}}{\bar{c}}\right)\kappa\lambda_0\right](1-\delta)\theta}{(\lambda_1-\lambda_0)\delta+(\lambda_1-\kappa\lambda_0)(1-\delta)\theta}\,\hat{c} \underset{(>)}{<} \frac{\lambda_1}{\lambda_1-\lambda_0}\,\hat{c}, \tag{A61}$$

or equivalently, if and only if

$$\hat{c} \underset{(>)}{<} \frac{\left(\frac{\lambda_1}{\lambda_1-\lambda_0}-\frac{\lambda_1\delta+(\lambda_1-\kappa\lambda_0)(1-\delta)\theta}{(\lambda_1-\lambda_0)\delta+(\lambda_1-\kappa\lambda_0)(1-\delta)\theta}\right)\bar{c}-\lambda_1(1-\delta)(1-\theta)(v_h-v_\ell)}{\frac{\kappa\lambda_0(1-\delta)\theta}{(\lambda_1-\lambda_0)\delta+(\lambda_1-\kappa\lambda_0)(1-\delta)\theta}}. \tag{A62}$$

Using (A32) to replace κ, we can write this condition as

$$\hat{c} \underset{(>)}{<} \frac{\delta\left(\bar{c}-\frac{\lambda_1}{\lambda_0}\frac{1-\theta}{\theta}(\lambda_1-\lambda_0)(v_h-v_\ell)\left[\lambda_0(1-\delta)+(1-\lambda_0)\delta+(1-\delta)\theta\right]\right)}{\lambda_1(1-\delta)+(1-\lambda_1)\delta} \equiv \hat{c}^*. \tag{A63}$$

If $\theta=1$, we have $\hat{c}^*=\frac{\delta\bar{c}}{\lambda_1(1-\delta)+(1-\lambda_1)\delta}>\bar{c}$ (since $\delta>\frac{1}{2}$). If $\theta=0$, the term $\frac{1-\theta}{\theta}$ becomes infinitely large and hence $\hat{c}^*<0$. Since $\hat{c}^*$ is a continuous and increasing function of θ, there thus must exist thresholds $\bar{\theta}$ and $\underline{\theta}$, with $0<\underline{\theta}<\bar{\theta}<1$, such that $\hat{c}^*>\bar{c}$ for all $\theta>\bar{\theta}$ and $\hat{c}^*<0$ for all $\theta<\underline{\theta}$. $\square$

Proof of Proposition 6: For a given cost threshold $\hat{c}\in[0,\bar{c}]$, the value of the firm (net of the expected cost of compensation) under the optimal no-manipulation contract $\mathcal{C}^n$ specified in Proposition 1 is given by (A54). Substituting (A55) into (A54), we have

$$V_n(\hat{c})=V_0+\frac{\hat{c}}{\bar{c}}(\lambda_1-\lambda_0)(v_h-v_\ell)-\left(\frac{\lambda_0}{\lambda_1-\lambda_0}+\frac{\hat{c}}{\bar{c}}\right)\hat{c}, \tag{A64}$$

where, as before, $V_0=\lambda_0 v_h+(1-\lambda_0)v_\ell$. Note that V_n is a strictly concave function of $\hat{c}$. The derivative of V_n with respect to $\hat{c}$ evaluated at $\hat{c}=\bar{c}$ is given by $V_n'(\bar{c})=\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}}-\frac{\lambda_0}{\lambda_1-\lambda_0}-2$. This term is negative because, by assumption, $(\lambda_1-\lambda_0)(v_h-v_\ell)<\bar{c}$. Thus, if $V_n'(0)\geq 0$, the optimal cost threshold that maximizes V_n is uniquely determined by the first-order condition

$$\hat{c}_n=\frac{1}{2}\left((\lambda_1-\lambda_0)(v_h-v_\ell)-\frac{\lambda_0\bar{c}}{\lambda_1-\lambda_0}\right). \tag{A65}$$

If $V_n'(0)<0$, the expression above is negative and the optimal cost threshold is zero.

Similarly, the value of the firm (net of the expected cost of compensation and net of the expected loss of cash flow directly due to manipulation) under the optimal selective-manipulation contract $\mathcal{C}^s$ specified in Proposition 2 is given by equation (A57). Substituting

(A60) into (A57), we have

$$V_s(\hat{c}) = V_0 + \frac{\hat{c}}{\bar{c}}\left[\lambda_1 - \lambda_0 - \lambda_1(1-\delta)(1-\theta)\right](v_h - v_\ell) - \left(\frac{\lambda_0\delta + \frac{\hat{c}}{\bar{c}}\kappa\lambda_0(1-\delta)\theta}{(\lambda_1-\lambda_0)\delta + (\lambda_1 - \kappa\lambda_0)(1-\delta)\theta} + \frac{\hat{c}}{\bar{c}}\right)\hat{c}. \quad \text{(A66)}$$

Similarly to V_n, V_s is a strictly concave function of $\hat{c}$. Furthermore, the derivative of V_s with respect to $\hat{c}$ evaluated at $\hat{c} = \bar{c}$ satisfies $V_s'(\bar{c}) < (\lambda_1 - \lambda_0)(v_h - v_\ell)/\bar{c} - 2 < 0$ (where we ignore some of the negative terms). Thus, if $V_s'(0) \geq 0$, the optimal cost threshold that maximizes V_s is uniquely determined by the first-order condition

$$\hat{c}_s = \frac{1}{2}\left(\frac{[\lambda_1 - \lambda_0 - \lambda_1(1-\delta)(1-\theta)](v_h - v_\ell)}{1 + \frac{\kappa\lambda_0(1-\delta)\theta}{(\lambda_1-\lambda_0)\delta + (\lambda_1-\kappa\lambda_0)(1-\delta)\theta}} - \frac{\lambda_0\delta\bar{c}}{(\lambda_1-\lambda_0)\delta + \lambda_1(1-\delta)\theta}\right). \quad \text{(A67)}$$

Using (A32) to replace κ, we can write this condition as

$$\hat{c}_s = \frac{1}{2}\left(\frac{[\lambda_1 - \lambda_0 - \lambda_1(1-\delta)(1-\theta)](v_h - v_\ell)}{1 + \frac{\lambda_0}{\lambda_1-\lambda_0}\frac{1-\delta}{\delta}\frac{[\lambda_1(1-\delta)+(1-\lambda_1)\delta]\theta}{\lambda_0(1-\delta)+(1-\lambda_0)\delta+(1-\delta)\theta}} - \frac{\lambda_0\delta\bar{c}}{(\lambda_1-\lambda_0)\delta + \lambda_1(1-\delta)\theta}\right). \quad \text{(A68)}$$

If $V_s'(0) < 0$, the expression above is negative and the optimal cost threshold is zero.

The expression for $\hat{c}_s$ in (A67) is increasing in θ: the second term in parentheses on the right-hand side is clearly decreasing in θ. To see that the first term is increasing in θ, note that

$$\begin{aligned}
&\frac{d}{d\theta}\left(\frac{\lambda_1 - \lambda_0 - \lambda_1(1-\delta)(1-\theta)}{1 + \frac{\kappa\lambda_0(1-\delta)\theta}{(\lambda_1-\lambda_0)\delta + (\lambda_1-\kappa\lambda_0)(1-\delta)\theta}}\right) \\
&\quad = \frac{\lambda_1(1-\delta)\left(1 + \frac{\kappa\lambda_0(1-\delta)\theta}{(\lambda_1-\lambda_0)\delta+(\lambda_1-\kappa\lambda_0)(1-\delta)\theta}\right) - \frac{[\lambda_1-\lambda_0-\lambda_1(1-\delta)(1-\theta)]\,\kappa\lambda_0(\lambda_1-\lambda_0)\delta(1-\delta)}{[(\lambda_1-\lambda_0)\delta+(\lambda_1-\kappa\lambda_0)(1-\delta)\theta]^2}}{\left(1 + \frac{\kappa\lambda_0(1-\delta)\theta}{(\lambda_1-\lambda_0)\delta+(\lambda_1-\kappa\lambda_0)(1-\delta)\theta}\right)^2} \\
&\quad = \frac{\lambda_1(1-\delta)[\lambda_1(\delta + (1-\delta)\theta) - \lambda_0\delta] - \frac{[\lambda_1(\delta+(1-\delta)\theta)-\lambda_0]\,\kappa\lambda_0(\lambda_1-\lambda_0)\delta(1-\delta)}{(\lambda_1-\lambda_0)\delta+(\lambda_1-\kappa\lambda_0)(1-\delta)\theta}}{[(\lambda_1-\lambda_0)\delta + (\lambda_1-\kappa\lambda_0)(1-\delta)\theta]\left(1 + \frac{\kappa\lambda_0(1-\delta)\theta}{(\lambda_1-\lambda_0)\delta+(\lambda_1-\kappa\lambda_0)(1-\delta)\theta}\right)^2}. \quad \text{(A69)}
\end{aligned}$$

The denominator of this expression is strictly positive because $\kappa < 1$; the numerator is also strictly positive because

$$\lambda_1(\delta + (1-\delta)\theta) - \lambda_0\delta > \lambda_1(\delta + (1-\delta)\theta) - \lambda_0 \quad \text{(A70)}$$

and

$$\lambda_1(1-\delta)[(\lambda_1-\lambda_0)\delta+(\lambda_1-\kappa\lambda_0)(1-\delta)\theta] \geq \lambda_1(1-\delta)(\lambda_1-\lambda_0)\delta > \kappa\lambda_0(\lambda_1-\lambda_0)\delta(1-\delta). \quad \text{(A71)}$$

Thus, $\hat{c}_s$ increases in θ (strictly, if $\hat{c}_s > 0$). Now compare $\hat{c}_n$ and $\hat{c}_s$ for the lowest and highest possible value of θ, $\theta = 0$ and $\theta = 1$. If $\theta = 0$, we have from (A67) that

$$\hat{c}_s = \max\left\{\frac{1}{2}\left([\lambda_1 - \lambda_0 - \lambda_1(1-\delta)](v_h - v_\ell) - \frac{\lambda_0\bar{c}}{\lambda_1 - \lambda_0}\right), 0\right\}. \quad \text{(A72)}$$

Comparison of (A72) with (13) shows that if $\theta = 0$, then $\hat{c}_s \leq \hat{c}_n$ (the inequality is strict if $\hat{c}_n > 0$). If $\theta = 1$, we have from (A67) that

$$\hat{c}_s = \max\left\{\frac{1}{2}\left(\frac{(\lambda_1 - \lambda_0)(v_h - v_\ell)}{1 + \frac{\kappa\lambda_0(1-\delta)}{(\lambda_1-\lambda_0)\delta+(\lambda_1-\kappa\lambda_0)(1-\delta)}} - \frac{\lambda_0\delta\bar{c}}{\lambda_1 - \lambda_0\delta}\right), 0\right\}. \quad \text{(A73)}$$

Comparison of (A73) with (13) shows that if $\theta = 1$, then $\hat{c}_s \geq \hat{c}_n$ (the inequality is strict if $\hat{c}_n > 0$). This is because the inequality

$$\frac{(\lambda_1 - \lambda_0)(v_h - v_\ell)}{1 + \frac{\kappa\lambda_0(1-\delta)}{(\lambda_1-\lambda_0)\delta+(\lambda_1-\kappa\lambda_0)(1-\delta)}} - \frac{\lambda_0\delta\bar{c}}{\lambda_1 - \lambda_0\delta} > (\lambda_1 - \lambda_0)(v_h - v_\ell) - \frac{\lambda_0\bar{c}}{\lambda_1 - \lambda_0} \quad \text{(A74)}$$

is equivalent to

$$\frac{\bar{c}}{(\lambda_1 - \lambda_0)(v_h - v_\ell)}\left(\frac{1}{\lambda_1 - \lambda_0} - \frac{\delta}{\lambda_1 - \lambda_0\delta}\right) > \frac{\kappa(1-\delta)}{\lambda_1 - \lambda_0\delta}. \quad \text{(A75)}$$

Since $\kappa < 1$ and $(\lambda_1 - \lambda_0)(v_h - v_\ell) < \bar{c}$, a sufficient condition for this inequality to hold is that

$$\frac{1}{\lambda_1 - \lambda_0} - \frac{\delta}{\lambda_1 - \lambda_0\delta} > \frac{1-\delta}{\lambda_1 - \lambda_0\delta}, \quad \text{(A76)}$$

which is clearly satisfied because $\frac{1}{\lambda_1-\lambda_0} > \frac{1}{\lambda_1-\lambda_0\delta}$.

Since (i) $\hat{c}_s$ is a continuous and strictly increasing function of θ, (ii) $\hat{c}_s < \hat{c}_n$ if $\theta = 0$ (and $\hat{c}_n > 0$), and (iii) $\hat{c}_s > \hat{c}_n$ if $\theta = 1$ (and $\hat{c}_n > 0$), there must exist a $\hat{\theta} \in (0, 1)$ such that $\hat{c}_s < \hat{c}_n$ for all $\theta \in [0, \hat{\theta})$ and $\hat{c}_s > \hat{c}_n$ for all $\theta \in (\hat{\theta}, 1]$. □

Proof of Corollary 1: From Proposition 3, we know that for any cost threshold $\hat{c}$, the optimal contract is either the no-manipulation contract $\mathcal{C}^n$ defined in Proposition 1 or the selective-manipulation contract $\mathcal{C}^s$ defined in Proposition 2. Thus, a necessary and sufficient condition for the optimal contract to implement a cost threshold $\hat{c} > 0$ is that $\max\{\hat{c}_n, \hat{c}_s\} > 0$.

From (13), it immediately follows that the condition $\hat{c}_n > 0$ is equivalent to

$$\frac{(\lambda_1 - \lambda_0)(v_h - v_\ell)}{\bar{c}} > \frac{\lambda_0}{\lambda_1 - \lambda_0}. \tag{A77}$$

The denominator of the first term in parentheses on the right-hand side of (14) can be written as

$$\begin{aligned} 1 + \frac{\lambda_0}{\lambda_1 - \lambda_0} \frac{1-\delta}{\delta} \frac{[\lambda_1(1-\delta) + (1-\lambda_1)\delta]\,\theta}{\lambda_0(1-\delta) + (1-\lambda_0)\delta + (1-\delta)\theta} \\ = \frac{[(\lambda_1 - \lambda_0)\delta + \lambda_1(1-\delta)\theta][\lambda_0(1-\delta) + (1-\lambda_0)\delta]}{(\lambda_1 - \lambda_0)\delta[\lambda_0(1-\delta) + (1-\lambda_0)\delta + (1-\delta)\theta]}. \end{aligned} \tag{A78}$$

Thus, since $\lambda_1 > 2\lambda_0$ and hence $\lambda_1 - \lambda_0 - \lambda_1(1-\delta)(1-\theta) > \lambda_1\delta - \lambda_0 > \frac{\lambda_1}{2} - \lambda_0 > 0$, the condition $\hat{c}_s > 0$ is equivalent to

$$\frac{(\lambda_1 - \lambda_0)(v_h - v_\ell)}{\bar{c}} > \frac{\lambda_0}{\lambda_1[\delta + (1-\delta)\theta] - \lambda_0} \frac{\lambda_0(1-\delta) + (1-\lambda_0)\delta}{\lambda_0(1-\delta) + (1-\lambda_0)\delta + (1-\delta)\theta}. \tag{A79}$$

$\square$

Proof of Proposition 7: From Proposition 3, we know that for any cost threshold $\hat{c}$, the optimal contract is either the no-manipulation contract $\mathcal{C}^n$ defined in Proposition 1 or the selective-manipulation contract $\mathcal{C}^s$ defined in Proposition 2. Furthermore, Proposition 6 shows that the firm value under the no-manipulation contract (respectively, the selective-manipulation contract) is maximized at a cost threshold of $\hat{c}_n$ (respectively, $\hat{c}_s$). Thus, to prove the result that the selective-manipulation contract $\mathcal{C}^s$ (respectively, the no-manipulation contract $\mathcal{C}^n$) is optimal if $\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} < \Gamma$ (respectively, if $\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} > \Gamma$), we need to show that $V_s(\hat{c}_s) \underset{(<)}{>} V_n(\hat{c}_n)$ if $\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} \underset{(>)}{<} \Gamma$, where, as in the proof of Proposition 6, V_s denotes the firm value under the selective-manipulation contract and V_n the firm value under the no-manipulation contract.

Let Λ_n and Λ_s denote the right-hand side of the inequalities in (A77) and (A79), respectively. Condition (15) in Corollary 1 can then be written as $\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} > \min\{\Lambda_n, \Lambda_s\}$. We first consider the case in which $\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} > \max\{\Lambda_n, \Lambda_s\}$, which implies that the cost thresholds $\hat{c}_n$ and $\hat{c}_s$ are strictly positive and given by (A65) and (A67), respectively (see (13) and (14)). Substituting (A65) into (A64), we have

$$V_n(\hat{c}_n) = V_0 + \frac{\hat{c}_n^2}{\bar{c}}. \tag{A80}$$

Similarly, substituting (A67) into (A66) (and rearranging), we have

$$V_s(\hat{c}_s) = V_0 + \left(1 + \frac{\kappa\lambda_0(1-\delta)\theta}{(\lambda_1-\lambda_0)\delta + (\lambda_1-\kappa\lambda_0)(1-\delta)\theta}\right)\frac{\hat{c}_s^2}{\bar{c}}. \tag{A81}$$

Using (A32) to replace κ, we can write $V_s(\hat{c}_s)$ as

$$V_s(\hat{c}_s) = V_0 + \left(1 + \frac{\lambda_0}{\lambda_1-\lambda_0}\frac{1-\delta}{\delta}\frac{[\lambda_1(1-\delta)+(1-\lambda_1)\delta]\,\theta}{\lambda_0(1-\delta)+(1-\lambda_0)\delta+(1-\delta)\theta}\right)\frac{\hat{c}_s^2}{\bar{c}}. \tag{A82}$$

Thus, $V_s(\hat{c}_s) \underset{(<)}{>} V_n(\hat{c}_n)$ if and only if $\Omega\,\hat{c}_s^2 \underset{(<)}{>} \hat{c}_n^2$, where

$$\Omega = 1 + \frac{\lambda_0}{\lambda_1-\lambda_0}\frac{1-\delta}{\delta}\frac{[\lambda_1(1-\delta)+(1-\lambda_1)\delta]\,\theta}{\lambda_0(1-\delta)+(1-\lambda_0)\delta+(1-\delta)\theta}. \tag{A83}$$

Since the cost thresholds $\hat{c}_n$ and $\hat{c}_s$ are strictly positive and $\Omega > 0$, the condition $\Omega\,\hat{c}_s^2 \underset{(<)}{>} \hat{c}_n^2$ is equivalent to $\sqrt{\Omega}\,\hat{c}_s \underset{(<)}{>} \hat{c}_n$. Using (A65) and (A67), we can rewrite this condition as

$$\sqrt{\Omega}\left(\frac{\lambda_1-\lambda_0-\lambda_1(1-\delta)(1-\theta)}{(\lambda_1-\lambda_0)\Omega}\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} - \frac{\lambda_0\delta}{(\lambda_1-\lambda_0)\delta+\lambda_1(1-\delta)\theta}\right) \underset{(<)}{>} \frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} - \frac{\lambda_0}{\lambda_1-\lambda_0}, \tag{A84}$$

or, since $\Omega > 1$ and hence $\frac{\lambda_1-\lambda_0-\lambda_1(1-\delta)(1-\theta)}{(\lambda_1-\lambda_0)\sqrt{\Omega}} < 1$, as

$$\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} \underset{(>)}{<} \frac{\frac{\lambda_0}{\lambda_1-\lambda_0} - \frac{\lambda_0\delta\sqrt{\Omega}}{(\lambda_1-\lambda_0)\delta+\lambda_1(1-\delta)\theta}}{1 - \frac{\lambda_1-\lambda_0-\lambda_1(1-\delta)(1-\theta)}{(\lambda_1-\lambda_0)\sqrt{\Omega}}} \equiv \Gamma. \tag{A85}$$

Next, consider the case in which $\Lambda_s < \frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} \leq \Lambda_n$. In this case, the optimal contract is the selective-manipulation contract $\mathcal{C}^s$ because $\hat{c}_s > 0 = \hat{c}_n$ and hence $V_s(\hat{c}_s) > V_0 = V_n(\hat{c}_n)$. If we ignore the nonnegativity constraint in (13), then the fact that $\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} \leq \Lambda_n$ implies $\hat{c}_n \leq 0$. Thus, the inequality $\sqrt{\Omega}\,\hat{c}_s > \hat{c}_n$ holds. Based on the definition of Γ in (A85), this is equivalent to $\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} < \Gamma$.

Finally, consider the case in which $\Lambda_n < \frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} \leq \Lambda_s$. In this case, the optimal contract is the no-manipulation contract $\mathcal{C}^n$ because $\hat{c}_n > 0 = \hat{c}_s$ and hence $V_n(\hat{c}_n) > V_0 = V_s(\hat{c}_s)$. If we ignore the nonnegativity constraint in (14), then the fact that $\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} \leq \Lambda_s$ implies $\hat{c}_s \leq 0$. Thus, the inequality $\sqrt{\Omega}\,\hat{c}_s < \hat{c}_n$ holds. Based on the definition of Γ in (A85), this is equivalent to $\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} > \Gamma$.

The above arguments show that for any $\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} > \min\{\Lambda_n, \Lambda_s\}$, $V_s(\hat{c}_s) > V_n(\hat{c}_n)$ if

$\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} < \Gamma$ and $V_s(\hat{c}_s) < V_n(\hat{c}_n)$ if $\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} > \Gamma$, which concludes the proof of the first part of the proposition.

The result in the second part of the proposition follows immediately from Proposition 5. Suppose that $\theta > \bar{\theta}$ (where $\bar{\theta}$ is defined in Proposition 5). Proposition 5 then implies that $V_s(\hat{c}) > V_n(\hat{c})$ for all $\hat{c} \in (0, \bar{c}]$ and hence that $V_s(\hat{c}_n) > V_n(\hat{c}_n)$ if $\hat{c}_n > 0$. Furthermore, the optimality of $\hat{c}_s$ implies that $V_s(\hat{c}_s) \geq V_s(\hat{c}_n)$. Thus, $V_s(\hat{c}_s) > V_n(\hat{c}_n)$ if $\hat{c}_n > 0$. If $\hat{c}_n = 0$, then the fact that $\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} > \min\{\Lambda_n, \Lambda_s\}$ implies $\hat{c}_s > 0$ and hence $V_s(\hat{c}_s) > V_0 = V_n(\hat{c}_n)$. Thus, the selective-manipulation contract $\mathcal{C}^s$ is optimal for all $\theta > \bar{\theta}$.

Next, suppose that $\theta < \underline{\theta}$ (where $\underline{\theta}$ is defined in Proposition 5). Proposition 5 then implies that $V_s(\hat{c}) < V_n(\hat{c})$ for all $\hat{c} \in (0, \bar{c}]$ and hence that $V_s(\hat{c}_s) < V_n(\hat{c}_s)$ if $\hat{c}_s > 0$. Furthermore, the optimality of $\hat{c}_n$ implies that $V_n(\hat{c}_n) \geq V_n(\hat{c}_s)$. Thus, $V_s(\hat{c}_s) < V_n(\hat{c}_n)$ if $\hat{c}_s > 0$. If $\hat{c}_s = 0$, then the fact that $\frac{(\lambda_1-\lambda_0)(v_h-v_\ell)}{\bar{c}} > \min\{\Lambda_n, \Lambda_s\}$ implies $\hat{c}_n > 0$ and hence $V_s(\hat{c}_s) = V_0 < V_n(\hat{c}_n)$. Thus, the no-manipulation contract $\mathcal{C}^n$ is optimal for all $\theta < \underline{\theta}$. $\square$

Proof of Proposition 8: The firm's expected cash flow conditional on a favorable report $r = r_h$ is given by

$$\mathbb{E}[v \,|\, r = r_h] = v_\ell + \text{prob}\,[v = v_h \,|\, r = r_h]\,(v_h - v_\ell). \tag{A86}$$

The discovery that a high report r_h was manipulated therefore leads to a lowering of investors' expectations about the firm's cash flow if

$$\text{prob}\,[v = v_h \,|\, r = r_h, m = 1] < \text{prob}\,[v = v_h \,|\, r = r_h, m = 0]\,, \tag{A87}$$

where $m \in \{0, 1\}$ denotes the manager's manipulation decision.

Under the optimal selective-manipulation contract, the unconditional probability that the investment project will generate a high (pre-manipulation) cash flow $\tilde{v} = v_h$ is equal to $\lambda_s = \lambda_0 + \frac{\hat{c}_s}{\bar{c}}(\lambda_1 - \lambda_0)$. In the absence of manipulation, we thus have

$$\text{prob}\,[v = v_h \,|\, r = r_h, m = 0] = \text{prob}\,[\tilde{v} = v_h \,|\, \tilde{r} = r_h] = \frac{\lambda_s \delta}{\lambda_s \delta + (1 - \lambda_s)(1 - \delta)}. \tag{A88}$$

The observation of a manipulated report under the selective-manipulation contract reveals that the manager exerted high effort $e = 1$ and that she observed a low preliminary report $\tilde{r} = r_\ell$. Thus,

$$\text{prob}\,[v = v_h \,|\, r = r_h, m = 1] = \theta\,\text{prob}\,[\tilde{v} = v_h \,|\, \tilde{r} = r_\ell, e = 1] = \frac{\lambda_1(1 - \delta)\theta}{\lambda_1(1 - \delta) + (1 - \lambda_1)\delta}. \tag{A89}$$

The discovery that a high report r_h was manipulated therefore leads to a lowering of investors' expectations about the firm's cash flow if

$$\frac{\lambda_1(1-\delta)\theta}{\lambda_1(1-\delta)+(1-\lambda_1)\delta} < \frac{\lambda_s\delta}{\lambda_s\delta+(1-\lambda_s)(1-\delta)}. \tag{A90}$$

This condition is satisfied for sufficiently large values of δ and small values of θ, and it is violated for sufficiently small values of δ and large values of θ (since $\lambda_s < \lambda_1$). Since $\lambda_s > \lambda_0$, a sufficient condition for this inequality to hold for any $\theta \in (0,1)$ is that

$$\frac{\lambda_1(1-\delta)}{\lambda_1(1-\delta)+(1-\lambda_1)\delta} < \frac{\lambda_0\delta}{\lambda_0\delta+(1-\lambda_0)(1-\delta)}, \tag{A91}$$

which is equivalent to

$$\left(\frac{\delta}{1-\delta}\right)^2 > \frac{\lambda_1}{1-\lambda_1}\frac{1-\lambda_0}{\lambda_0} \tag{A92}$$

or, since $\delta > 0$, to

$$\delta > \left(1+\sqrt{\frac{1-\lambda_1}{\lambda_1}\frac{\lambda_0}{1-\lambda_0}}\right)^{-1}. \tag{A93}$$

$\square$

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