Takeover Contests with Asymmetric Bidders

Paul Povel
Carlson School of Management, University of Minnesota

Rajdeep Singh
Carlson School of Management, University of Minnesota

Target firms often face bidders that are not equally well informed, which reduces competition, because bidders with less information fear the winner's curse more. We analyze how targets should be sold in this situation. We show that a sequential procedure can extract the highest possible transaction price. The target first offers an exclusive deal to a better-informed bidder, without considering a less well-informed bidder. If rejected, the target offers either an exclusive deal to the less well-informed bidder, or a modified first-price auction. Deal protection devices can be used to enhance a target's commitment to the procedure. (*JEL* G34, K22, D44)

If a firm is to be sold, the seller's problem is to identify the buyer who will pay the highest price. Negotiations with potential buyers are one way to discover who this buyer is and at what price a transaction can take place; auctions are an alternative method. Selling firms by inviting competitive bids has become increasingly popular in practice, and the announcement that a firm is "evaluating strategic alternatives" is commonly regarded as an invitation to submit competitive bids.[1]

Bidders participating in an auction are not always equally well informed. Management bids are the clearest example: in many cases the target's management team (or a subset of senior managers) declares an interest in purchasing the target, and their privately known value estimate must be more reliable than any other potential buyer's. Similarly, a competitor should find it much easier to evaluate a target's prospects than a bidder with no experience in the target's line of business. Either way, a less well-informed bidder will be particularly worried about the

We would like to thank an anonymous referee and Bob McDonald (the editor) for very helpful comments. We are also indebted to Kerry Back, Espen Eckbo, Sander Onderstal, Sergio Parreiras, Andy Winton, and seminar participants at the University of Minnesota, University of Virginia, Washington University (St. Louis), and Michigan State University and to participants at the Western Finance Association meeting (2004), the European Finance Association meeting (2004), the ESSFM meeting in Gerzensee (2004), and the American Finance Association meeting (2005). Address correspondence to Department of Finance, Carlson School of Management, University of Minnesota, Minneapolis, MN 55455, or e-mail: povel@umn.edu and rajsingh@umn.edu.

[1] See Boone and Mulherin (2003) for a description of the process by which target firms solicit bids in a takeover auction. See also Boone and Mulherin (2004).

doi:10.1093/rfs/hhj034 Advance Access publication February 23, 2006

winner's curse (i.e., overpaying after beating a better-informed rival in a takeover contest because of an over-optimistic value estimate).

In this article we ask how a target should optimally be sold if bidders are not equally well informed about its value. We analyze this question in a simple auction setup, and we derive the optimal selling procedure. Unlike the existing literature (discussed below), our model allows for both private value and common value bidding environments. For example, trade buyers may be interested in a target because of possible synergies that are not available to other bidders; this situation can be modeled as a private values environment. Alternatively, all bidders may be able to exploit the same sources of gains (e.g., cost-cutting, financial restructuring), but their value is unknown; this situation with financial buyers can be modeled as a common values environment.

In our setting a sequential procedure is optimal (i.e., it can extract the highest expected transaction price). The sequential procedure has simple and realistic properties. Initially, the target communicates exclusively with the better-informed bidder. If she is willing to pay a sufficiently high price, a sale is concluded right away, without soliciting any additional bids. If the offered price is not sufficiently high, the target inquires whether the better-informed bidder is willing to bid at least a certain amount in a bidding contest that may follow. If not, then the target is offered to the less well-informed bidder at a price that she will not reject. If the better-informed bidder is willing to bid at least the minimum amount, then the target invites bids from any party and holds a modified first-price auction, in which the winner pays the price she bid.[2]

The key feature of the sequential procedure is that it treats the bidders asymmetrically. It inflates the better-informed bidder's chances of winning if her willingness to pay is high, and it inflates her chances of losing if the willingness to pay is low. This is most apparent in the exclusive deals that may be closed: if the better-informed bidder's valuation is above a certain threshold, she wins with certainty, while if it is below a different (lower) threshold, she loses with certainty. The aim is to induce high bids from the better-informed bidder; without the extra incentive, she would often buy the target at a low price, compared with her valuation.

Bidders participate in the sequential procedure because they hope to buy the target at a price lower than their estimated value (i.e., they expect to earn positive net payoffs, so-called rents). The better the bidder's value estimate (i.e., the more informative her signal is), the higher the rents that she expects to earn. The seller's aim is to extract as much as possible of the bidders' rents, and given that there is more scope for the better-informed

[2] Besides the strategic effects analyzed here, using a sequential procedure also has the advantage that the sale may take place after small-scale negotiations, with less disruption for management (and the operations) than, say, an outright auction.

bidder to earn large payoffs (since she can form more reliable value estimates), the seller should focus first on extracting them from that bidder. Consequently, the optimal selling procedure is more biased if the bidder asymmetry is larger, and it becomes more likely that the better-informed bidder will either enjoy an exclusive deal or will be excluded from bidding.

How the sequential procedure extracts larger payments from the better-informed bidder is similar to setting a reserve price. When an auctioneer sets a reserve price, the bidders know that if the bids remain below that price, the asset for sale will be withdrawn from the auction. In other words, they may be better off bidding above the reserve price if their valuation is higher, since that is the only way to earn a rent, even if a higher bid eats into that rent. In our setting, the better-informed bidder is forced to bid higher, but instead of using a reserve price, the target threatens to sell to the less well-informed bidder. The advantage over a reserve price is obvious: withdrawing the target from the sale (in the case of a reserve price) may be very inefficient, while the less well-informed bidder may pay a decent price if she wins (in the case of the sequential procedure).

The sequential procedure has realistic features. In practice, we do observe exclusive negotiations with one bidder. In these negotiations, the target may threaten to exclude this bidder from any further negotiations or bidding, if her offers seem low. These negotiations may end with a deal, and third-party offers are not solicited or considered (this is often the case in management buy-outs). Sometimes, bids from third parties may be invited or encouraged, and all bidders compete directly for the target. Finally, the bidder who enjoyed initial exclusive negotiations may be excluded from further bidding, say, if the requested and offered bids did not seem to be converging, and new bidders are invited to make offers.[3] These possible outcomes are consistent with the broad predictions of our model.

Our model also highlights some strategic stumbling blocks that targets and winning bidders may encounter. These appear if it is hard to commit to the rules of the sequential procedure. The ability to commit is crucial if the exclusive negotiations with the better-informed bidder fail, in the sense that she displays a low willingness to pay. If that happens, the target may decide to renegotiate the rules, instead of making the required exclusive offer to the less well-informed bidder. As we explain below, in practice targets seem to be able to credibly commit to the rules they devise; also, a target can strengthen the credibility of her rules by hiring an investment bank to structure the transaction, thereby "renting" the bank's reputation for never renegotiating.

[3] For example, in the sale of NCS Healthcare, Inc., described in Omnicare, Inc. v. NCS Healthcare, Inc., 818 A.2d 914 (Del. 2003).

A different commitment problem may arise after a winner has been declared. The loser's valuation of the target may be higher than the price that the winner is supposed to pay, so she may try to break the deal by offering a higher price. In this situation, commitment to the rules is needed from the target's board, but also from the shareholders, to whom a "hostile" bid may be addressed. As we discuss below, targets and bidders agree to use a variety of deal protection devices like lock-ups, termination fees, no-shopping clauses, or to the selective lifting of poison pills.[4] These devices make the target less attractive to rejected bidders, thereby reducing their incentive to top up the winning bid. The courts have been struggling with the tension between the target's ex-ante and ex-post incentives when assessing the legality of these devices. In a recent split decision, the Delaware Supreme Court ruled that an "absolute lock-up" was a breach of fiduciary duty (see note 3). Some earlier decisions upheld the use of lock-ups and similar devices. Our analysis shows that courts should focus more on the ex-ante benefits of using deal protection devices.

The popularity of deal protection devices in practice has attracted a lot of attention, from target shareholders, rejected bidders, the media, etc., and also from academics. Some authors analyze whether agency problems between managers and shareholders can explain the use of takeover defenses [see Baron (1983); Comment and Schwert (1995); Cotter, Shivdasani, and Zenner (1997); Agrawal and Jaffe (2003); see Becht, Bolton, and Röell (2003) for a survey of the literature]. Together, these studies suggest that bid resistance is not driven mainly by managerial entrenchment. This supports our modeling choice of abstracting from agency problems between managers and shareholders and focusing instead on the target board's possibilities to extract the highest possible transaction price from bidders in a realistic setting.

We are not the first to study takeovers as bidding contests. Fishman (1988) rationalized high initial-bid premia by modeling takeovers as a potential contest between two bidders. Daniel and Hirshleifer (1992), Bhattacharyya (1992), Burkart (1995), Singh (1998), Bulow, Huang, and Klemperer (1999), Ravid and Spiegel (1999), and Rhodes-Kropf and Viswanathan (2004) also model takeover contests in an applied auction setting. However, all these papers take the target as largely passive. In our paper, the target is active. It designs the selling procedure, and it thereby affects the behavior of the bidders. A similar approach is taken by Dasgupta and Tsui (1992), who show that, in a common value setup with bidder asymmetries, the target may benefit from using a "matching auction."

[4] In stock lock-ups, the favored bidder is given the option to purchase a certain number of shares, either from certain (typically large) shareholders, or out of treasury stock. This makes it harder for other bidders to take over a target, or more expensive. Termination fees are payable to the favored bidder if a takeover is not consummated. With no-shopping clauses, target boards commit not to actively encourage additional bids.

This article is related to Shleifer and Vishny (1986) and Berkovitch and Khanna (1990), who provide rationales for the frequent use of greenmail or value-reduction strategies by takeover targets. Their models are complementary to ours, since they analyze a takeover market in which firms wait for raiders to identify them as takeover targets, while we focus on a situation in which it is publicly known that a firm is up for sale, for example if a firm announces that it is selling a noncore division. Also, our approach is more general, since we impose fewer constraints on the target's maximization problem.[5]

Our article is also related to the literature on optimal auction design. Myerson (1981) was the first to use mechanism design to analyze optimal auctions. In an example, Myerson (1981) also shows that bidder asymmetry can lead to an asymmetric mechanism, but he does not analyze what impact this has on the expected transaction price, a bidder's chances of winning, etc. A series of papers has analyzed the revenue-generation performance of standard auctions (open auction, first-price auction, etc.) in the special case of pure private value settings. Maskin and Riley (2000) show why different standard auctions cannot be optimal in the presence of bidder asymmetry, but they do not analyze the optimal selling procedure itself and how it depends on the bidding environment (how asymmetric the bidders are, how the optimal selling procedure and the outcome change in a common value setup, etc.). Cantillon (2000) extends their analysis, by showing that an increase in bidder asymmetry hurts the seller if she uses standard auctions, since it decreases expected revenue. This underlines the relevance of designing an optimal selling procedure, in particular if bidders are not equally well informed. Finally, our model is related to that in Povel and Singh (2004), who analyze a simple pure common value setup with asymmetric bidders; their main result is that the expected transaction price increases if the bidders become more asymmetrically informed, a result that also holds in our model (see Proposition 5.1).

The rest of the article proceeds as follows. Section 1 introduces the auction model. In Section 2 we describe some key properties of the optimal selling procedure. We present the sequential procedure in Section 3. In Section 4 we discuss some implications, both theoretical and empirical. In Section 5 we discuss extensions of our model, focusing on the possibility that the bidders' signals can be low and the target may plan to abort the sale if the bids seem low, and on the possibility that bidders may misrepresent the quality of their information. Section 6 concludes. Some of the proofs are in the Appendix.

[5] Unlike the earlier studies, the target is not limited to a given set of selling procedures (there are no restrictions on the sequence of bids, the use of legal devices, etc.). Additionally, our model allows for both private value and common value components, while the earlier research allows for pure private values, only.

1. The Model

A target is for sale, and two imperfectly informed bidders, $i,j \in \{1,2\}$, are interested in buying it. We assume that a bidder's full information value of the target is a weighted average of two independent components, t_1 and t_2. Bidder 1 values the target at $\alpha t_1 + (1-\alpha)t_2$, and bidder 2 values the target at $\alpha t_2 + (1-\alpha)t_1$. We assume that $\alpha \in [\frac{1}{2}, 1]$. If $\alpha = 1$, we have pure private values; if $\alpha = \frac{1}{2}$, this is a model with pure common values; if $\alpha \in (\frac{1}{2}, 1)$, both private and common value components are present.[6] The valuation components t_1 and t_2 are independently and identically distributed on the support $[\underline{t}, \overline{t}]$, with density f and c.d.f. F. Denote the hazard rate by $H(t_i) = f(t_i)/(1-F(t_i))$. For tractability reasons, we assume that the hazard rate H is increasing in t_i. Also for tractability, we normalize to zero the target shareholders' valuation of the target, and we assume that $\underline{t}$ is sufficiently high: $\underline{t}H(\underline{t}) \geq 1$.[7]

Bidder i privately observes an imperfect signal s_i on component t_i:

$$s_1 = \begin{cases} t_1 \text{ with prob. } \varphi \\ \tau_1 \text{ with prob. } 1-\varphi, \end{cases} \quad , \quad s_2 = \begin{cases} t_2 \text{ with prob. } \delta\varphi \\ \tau_2 \text{ with prob. } 1-\delta\varphi, \end{cases} \tag{1}$$

where $\varphi, \delta \in [0,1)$, and τ_1 and τ_2 are i.i.d. random variables that have the same distributions as t_1 and t_2. Thus, with probability φ and $\delta\varphi$, the signals s_1 and s_2 are informative, and with probability $1-\varphi$ and $1-\delta\varphi$ they are pure noise. (The case $\delta = 1$ is analyzed in the auction literature; with symmetric bidders, a symmetric auction would be optimal.) If the bidders could observe *both* signals, they would use Bayes' law to update their priors, and the conditional expected value would be given by

$$v_1(s_1,s_2) = E[t] + \varphi\alpha(s_1 - E[t]) + \delta\varphi(1-\alpha)(s_2 - E[t]), \tag{2}$$

$$v_2(s_1,s_2) = E[t] + \varphi(1-\alpha)(s_1 - E[t]) + \delta\varphi\alpha(s_2 - E[t]), \tag{3}$$

where $E[t]$ is the unconditional expected value of t_i and s_i. Notice that variations in α, δ, or φ do not affect bidder i's unconditional expected value of the target, $E_{s_1,s_2}[v_i(s_1,s_2)]$.

[6] This valuation model is familiar from the auction literature, see, for example, Myerson (1981) or Bulow and Klemperer (2002). Bulow, Huang, and Klemperer (1999) use it to analyze takeover contests, assuming pure common values (the case $\alpha = \frac{1}{2}$ in our model). All other analyses of takeover contests are restricted to pure private value models (the case $\alpha = 1$ in our model); see, for example, Fishman (1988), Bhattacharyya (1992), Daniel and Hirshleifer (1992), Burkart (1995), Singh (1998), Ravid and Spiegel (1999), and Rhodes-Kropf and Viswanathan (2004).

[7] This simplifies the exposition by ensuring that the target finds it optimal to always conclude a sale. In Section 5.1 we analyze the effect of allowing for lower values of $\underline{t}$. We find that the target may withdraw from the sale altogether if the bids seem too low, by posting a reserve price.

The assumption that bidder 2 is less well informed ($\delta < 1$) captures a variety of situations in which different bidders have different expertise in evaluating a target. For example, bidders may specialize on different sides of a target (e.g., its operations, its optimal capital structure, its growth potential, its cash-generation potential), and their ideas about how to value other dimensions may be very vague. Alternatively, some bidders may have superior information because of their special relationship with the target.[8] Such bidders may be competitors, suppliers or customers, who know more than third parties about the target's strengths and weaknesses. Similarly, a management team offering a buy-out can be expected to have superior information.[9]

By varying the parameter α, our model captures a second dimension in which bidding environments may differ. A target may attract trade buyers (e.g., competitors, suppliers, customers), who may hope to realize individual synergies if they take over the target. If each bidder can realize synergies that are not available to others, a model with private values is appropriate. We can capture this situation by assuming that α is large. In other situations, bidders may be better described as financial buyers. For example, private equity funds may be attracted to a takeover contest if there are possibilities to add value that are not specific to certain bidders, for example, firing current management, selling off noncore assets, cutting costs, or changing financial leverage. Nevertheless, some bidders may have superior information about the potential for cost-cutting, etc. These situations are better modeled as common value environments [i.e., α should be small (close to $1/2$)].

In order to focus on how informational asymmetry affects bidding and the optimal selling scheme, we abstract from issues that have been analyzed elsewhere. We assume that bids are cash bids and financed internally, the target is all-equity financed, and we assume that bidders do not own any shares in the target.[10] Furthermore, the target (the target's board of directors) and the bidders maximize their expected profits. Finally, all players are risk-neutral, and all reservation payoffs are zero.

[8] The differences in the quality of information need not be caused by data availability problems; targets typically set up "data rooms", which bidders can access after signing a confidentiality agreement. Instead, different bidders may have unequal experience with the target or the industry in which it operates, so they are not equally able to interpret the data and form equally reliable value estimates.

[9] We assume that the identity of the better-informed bidder is common knowledge. This seems plausible in most cases, for example, if senior managers submit a bid, or if a close competitor, supplier, or customer competes with bidders who have less experience with the target's products, markets, etc. In those cases, it may be unavoidable for the bidder to be identified as having superior information. In Section 5.2, we analyze the bidders' incentive to misrepresent the quality of their information, and how doing so affects the expected transaction price and the bidders' expected payoffs.

[10] See Rhodes-Kropf and Viswanathan (2000) and (2005) for a discussion of how noncash bids or debt-financed bids affect bidding. Israel (1991) examines the bids for a target, in situations in which some of the increase in value of the target is captured by debt. For an analysis of toeholds see Burkart (1995), Singh (1998), and Bulow, Huang, and Klemperer (1999).

2. Properties of an Optimal Mechanism

To solve for the optimal selling procedure, we follow the approach in Bulow and Roberts (1989) and Bulow and Klemperer (1996), which simplifies the method developed in Myerson (1981). Using the revelation principle, Myerson (1981) shows that the seller's optimization problem is quite tractable, and that the bidder with the highest "virtual surplus" should win the target (if it is sufficiently high).[11] Bulow and Roberts (1989) and Bulow and Klemperer (1996) show that the maximization problem is algebraically equivalent to that of a monopolist serving many markets, in which the monopolist can charge different prices (i.e., third-degree price discrimination is possible). The benefit of using the analogy is that the equivalent of Myerson's "virtual surplus" is the well-understood concept of "marginal revenue": as long as marginal revenue is larger than marginal cost, it pays for the monopolist to increase supply, and if there are capacity constraints, the monopolist should sell more of its goods in the market with the highest marginal revenue, and less in markets with a lower marginal revenue.

The equivalence to the auction setup is based on defining "market," "quantity," and "price" appropriately. The equivalent to a market is a bidder. In a market, for any given price the demand schedule determines the output that can be sold. In the case of a bidder with an unknown willingness to pay, the seller knows, for a given price, the probability with which the bidder's willingness to pay is higher than that price (i.e., how likely it is that the bidder would buy the asset at that price). Thus, Bulow and Roberts (1989) and Bulow and Klemperer (1996) suggest using a bidder's valuation (given all signals) as the "price," and the probability that the willingness to pay is higher than a given value as the "quantity." The seller's problem then is to decide how likely it is that a specific bidder wins at any given price, given the signals that are revealed truthfully in equilibrium. The marginal cost is the seller's own valuation of the target, which we normalized to zero. If a bidder's marginal revenue is higher than another bidder's, that first bidder's "quantity" (i.e., the probability of winning) should be increased, just like in the case of the monopolist (who should sell more in the market with the highest marginal revenue and less in markets with lower marginal revenues).

Thus, for a given signal pair, we consider a bidder's valuation $v_i(s_1, s_2)$ as the "price," and the measure corresponding to "quantity" $q_i(s_i)$ is a

[11] The tractability comes from separating the allocation decision from the choice of transfer payments when analyzing the optimal mechanism. Optimality follows if the allocation rule satisfies certain conditions, and incentive compatibility then determines the transfers that the bidders make. The proofs based on the approach in Myerson (1981) are longer than those based on Bulow and Roberts (1989) and Bulow and Klemperer (1996), but they lead to exactly the same results; details are available in an earlier version of this paper.

bidder's probability of having a higher valuation, $(1 - F(s_i))$. Multiplying price and quantity yields the expected revenue from that bidder,

$$v_i(s_1, s_2) \cdot q_i(s_i) \equiv v_i(s_1, s_2) \cdot (1 - F(s_i)), \qquad i = 1, 2.$$

The seller then must compare the "marginal revenue" from raising either bidder's probability of winning, given the realized signals. Bidder i's marginal revenue is

$$\begin{aligned} MR_1(s_1, s_2) &= \frac{\partial}{\partial q(s_1)}[v_1(s_1, s_2) \cdot q(s_1)] = \frac{\frac{\partial}{\partial s_1}[v_1(s_1, s_2)(1 - F(s_1))]}{\frac{\partial}{\partial s_1}(1 - F(s_1))} \\ &= v_1(s_1, s_2) - \frac{\varphi\alpha}{H(s_1)} \end{aligned} \tag{4}$$

$$\begin{aligned} MR_2(s_1, s_2) &= \frac{\partial}{\partial q(s_2)}[v_2(s_1, s_2) \cdot q(s_2)] = \frac{\frac{\partial}{\partial s_2}[v_2(s_1, s_2)(1 - F(s_2))]}{\frac{\partial}{\partial s_2}(1 - F(s_2))} \\ &= v_2(s_1, s_2) - \frac{\delta\varphi\alpha}{H(s_2)} \end{aligned} \tag{5}$$

For expositional ease, define

$$\Psi(s_i) \equiv \frac{2\alpha - 1}{\alpha}(s_i - E[t]) - \frac{1}{H(s_i)}, \qquad i = 1, 2. \tag{6}$$

Substituting $v_i(s_1, s_2)$ in Equations (4)–(5) using Equations (2)–(3), and then rearranging using Equation (6), we obtain that bidder 1's marginal revenue is higher than bidder 2's if and only if

$$\begin{aligned} &E[t] + \varphi\alpha(s_1 - E[t]) + \delta\varphi(1 - \alpha)(s_2 - E[t]) - \frac{\varphi\alpha}{H(s_1)} \\ &> E[t] + \varphi(1 - \alpha)(s_1 - E[t]) + \delta\varphi\alpha(s_2 - E[t]) - \frac{\delta\varphi\alpha}{H(s_2)} \\ &\iff \Psi(s_1) > \delta\Psi(s_2). \end{aligned} \tag{7}$$

Since Ψ is monotonically increasing in s_i, it has an inverse, which we denote by Ψ^{-1}. We can then define a threshold signal $z_1(s_2)$ for bidder 1, such that bidder 1's marginal revenue is higher if and only if $s_1 \geq z_1(s_2)$, where

$$z_1(s_2) \equiv \Psi^{-1}(\delta\Psi(s_2)). \tag{8}$$

It is easily verified that $z_1(s_2) \in (\underline{t}, \overline{t}]$ for all s_2, so z_1 is well defined. The inverse of z_1, denoted by z_2, is the threshold function for bidder 2's signal:

$$z_2(s_1) = \begin{cases} \overline{t} & \text{if} \quad s_1 > z_1(\overline{t}), \\ \Psi^{-1}(\frac{1}{\delta}\Psi(s_1)) & \text{if} \quad z_1(\underline{t}) < s_1 \leq z_1(\overline{t}), \\ \underline{t} & \text{if} \quad s_1 \leq z_1(\underline{t}). \end{cases}$$

We denote an allocation rule by $p_i(s_1, s_2)$. It specifies the probability that bidder i wins the target, given signals s_1 and s_2.

Lemma 1. *The optimal allocation rule is* $p_1(s_1, s_2) = 1 - p_2(s_1, s_2) = 1$ *if and only if bidder 1's marginal revenue is (weakly) higher than bidder 2's, or, equivalently,*

$$p_1(s_1, s_2) = 1 - p_2(s_1, s_2) = \begin{cases} 1 & \text{if } s_1 \geq z_1(s_2), \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

Proof. Our assumption $\underline{t}H(\underline{t}) \geq 1$ ensures that the bidders' marginal revenue is never negative. The rest of the proof follows directly from Bulow and Klemperer (1996) and is therefore omitted. ■

To study the properties of the optimal allocation rule, it will be convenient to work with bidder 1's cut-off signal, $z_1(s_2)$, as defined in Equation (8). It is optimal to sell the target firm to bidder 1 if and only if her signal is at least weakly higher than $z_1(s_2)$. Analyzing the properties of z_1, thus, allows us to predict under what circumstances the target firm will be sold to a better-informed bidder.

Lemma 2. *The function z_1 is monotonically increasing. At $s_2 = \underline{t}$ it attains a value $z_1(\underline{t}) > \underline{t}$. If $\alpha > \frac{1}{2}$ then $z_1(\overline{t}) < \overline{t}$, and there exists exactly one signal $s_2 = \sigma$ such that $z_1(\sigma) = \sigma$; for $s_2 > \sigma$, we have $z_1(s_2) < s_2$, and for $s_2 < \sigma$, we have $z_1(s_2) > s_2$. If $\alpha = \frac{1}{2}$ (pure common values), then $\sigma = \overline{t}$, and $z_1(s_2)$ is never smaller than s_2.*

Proof. See the Appendix. ■

Figure 1 sketches the optimal allocation rule for the case $\alpha > \frac{1}{2}$ (i.e., if the bidders' valuations have a private value component). Bidder 2's signals are measured on the horizontal axis, and bidder 1's on the vertical axis. The upward sloping solid line is the threshold signal function $z_1(s_2)$: if $s_1 \geq z_1(s_2)$, bidder 1 wins; if $s_1 < z_1(s_2)$, bidder 2 wins. If bidder 1's signal is low enough, $s_1 < z_1(\underline{t})$, she certainly does not win the target. If bidder 1's signal is high enough, $s_1 \geq z_1(\overline{t})$, she is certain to win the target. If bidder 1 receives an intermediate signals, that is, if

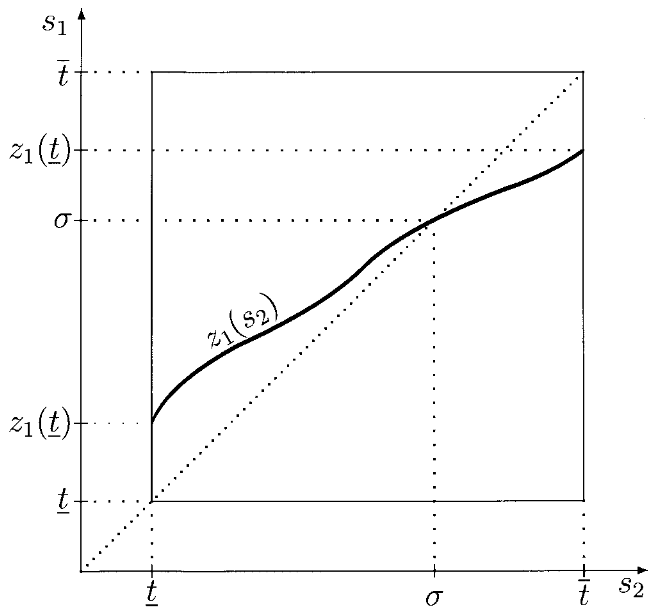

Figure 1
The optimal allocation rule for $\alpha > \frac{1}{2}$

$s_1 \in [z_1(\underline{t}), z_1(\bar{t}))$, either bidder may win, and a higher signal increases the likelihood of winning. The monotonicity of z_1 implies that bidder 1's likelihood of winning is nondecreasing in s_1 and nonincreasing in s_2, which are plausible properties of an optimal mechanism.

With pure common values (i.e., if $\alpha = \frac{1}{2}$), the picture is slightly different: the threshold $z_1(\bar{t})$ collapses in $\bar{t}$ [i.e., we have $z_1(s_2) > s_2$ for every $s_2 < \bar{t}$], and there is no signal s_1 that guarantees that bidder 1 wins the target. (But there always exist signals s_1 that guarantee that bidder 1 does *not* win, since $z_1(\underline{t}) > \underline{t}$.)

3. A Sequential Procedure That Is Optimal

We now describe a sequential selling procedure that has realistic features and is optimal. The target first communicates exclusively with one bidder at a time, possibly closing a deal right away. The key feature of this first stage is that there is no active bidding contest among the bidders. If the first stage does not end with an exclusive deal, the second stage starts, and the target invites competitive bids from both bidders. Their bids then determine the identity of the winner, and the winner's bid is the price that she must pay.

Stage I (Exclusive deals): The target (or her agent, an investment bank) commits to the sequential procedure. Then, bidder 1 is asked whether she is willing to purchase the target at a price of $\overline{b}_1$, or participate in competitive bidding (Stage II) with a minimum bid of $\underline{b}_1$, or pass up the acquisition opportunity. If bidder 1 passes, the target is offered to bidder 2 at a price $\underline{b}_2$.

Stage II (Competitive bidding): Bidders 1 and 2 submit sealed bids b_1 and b_2. Bidder 1 wins if and only if $b_1 > \widehat{z}_1(b_2)$, for a given function $\widehat{z}_1$; bidder 2 wins otherwise. The winner pays her bid, the loser pays nothing.

Just like a standard first-price auction, Stage II requires the winning bidder to pay her own bid, and the loser nothing. However, we call it a modified first-price auction, since the winning bidder did not necessarily submit the highest bid. (Also, bidder 1 is not allowed to bid below $\underline{b}_1$.)

Lemma 3. *The following describes a Bayesian Nash Equilibrium in the sequential procedure*. Bidder 1's strategy: *Accept the offer to pay $\overline{b}_1$ in Stage I if $s_1 \geq z_1(\overline{t})$; agree to participate in Stage II only if $s_1 \geq z_1(\underline{t})$; and in Stage II bid*

$$b_1(s_1) = E_{s_2 \in [\underline{t}, z_2(s_1)]}[v_1(s_1, s_2)] - \varphi\alpha \int_{z_1(\underline{t})}^{s_1} \frac{F(z_2(s))}{F(z_2(s_1))} ds. \tag{10}$$

Bidder 2's strategy: *Accept the offer to pay $\underline{b}_2$ in Stage I; and in Stage II bid*

$$b_2(s_2) = E_{s_1 \in [z_1(\underline{t}), z_1(s_2)]}[v_2(s_1, s_2)] - \delta\varphi\alpha \int_{\underline{t}}^{s_2} \frac{F(z_1(s)) - F(z_1(\underline{t}))}{F(z_1(s_2)) - F(z_1(\underline{t}))} ds. \tag{11}$$

These are equilibrium strategies if the target sets

$$\overline{b}_1 = v_1(z_1(\overline{t}), E[t]) - \varphi\alpha \int_{z_1(\underline{t})}^{z_1(\overline{t})} F(z_2(s)) ds, \tag{12}$$

$$\underline{b}_1 = v_1(z_1(\underline{t}), \underline{t}), \tag{13}$$

$$\underline{b}_2 = E_{s_1 \in [\underline{t}, z_1(\underline{t})]}[v_2(s_1, \underline{t})], \tag{14}$$

$$\hat{z}_1(b) = b_1(z_1(b_2^{-1}(b))), \tag{15}$$

where b_2^{-1} is the inverse of b_2.

Proof. See the Appendix. ■

The bidding functions b_1 and b_2 are reminiscent of the equilibrium bidding functions in a standard first-price auction. The first term is the expected value conditional on the bidder winning the auction (after having reached Stage II in the sequential procedure); the second term is the level of shading that maximizes the bidder's expected payoff. If we set $\delta = 1$, then the optimal selling procedure sets $z_1(s_2) = s_2$ (i.e., we have a symmetric allocation rule), and the bidding functions b_1 and b_2 are the standard first-price equilibrium bidding functions.

Proposition 1. *The sequential selling procedure is optimal.*

Proof. As in Bulow and Klemperer (1996), a selling procedure is optimal if it uses the optimal allocation rule (described in Lemma 1), and a bidder with the lowest possible signal realization earns her reservation payoff (which we assumed is zero). Both requirements are satisfied by construction: the allocation rules are the same, and either bidder's payoff is zero if her signal is $\underline{t}$ [bidder 1 does not win; bidder 2 pays the expected value of the target, conditional on $s_2 = \underline{t}$ and $s_1 \leq z_1(\underline{t})$.] ■

The target's goal is to extract as much value as possible from the bidders. The bidders' goal is to earn large rents (i.e., to buy the target at a price below their valuation). The more precise a bidder's value estimate, the higher the rent that she expects to earn. Thus, if the bidders are not equally well informed, the target finds it optimal to focus on extracting value from the better-informed bidder, since this bidder has more scope for earning large rents.

The optimal procedure is a stick-and-carrot mechanism, aimed at bidder 1. Compared with a standard auction, it promises bidder 1 a better chance of winning if she reveals a high signal and therefore a high valuation (the "carrot" side of the mechanism), and it threatens reduced chances of winning if bidder 1 claims that her signal and valuation are low (this is the "stick" side). This incentive scheme induces her to submit high bids: while submitting a low bid increases bidder 1's rent, it also reduces the likelihood that this larger rent will actually be earned.[12]

The downside of this stick-and-carrot mechanism is that sometimes the threat of selling to bidder 2 has to be executed. Selling to her may seem particulary costly if it happens in stage I, after bidder 1 declined the offer to pay $\overline{b}_1$ and also to compete with a bid of at least $\underline{b}_1$, since in that case the target is sold to bidder 2 at a fixed price, irrespective of bidder 2's valuation. However, if the bidders are asymmetric, executing the threat is not as costly as it may seem. First, bidder 2 is less well informed, so her value estimate will tend to be closer to the unconditional expected value of the target. Second, the more asymmetric the bidders, the higher is $z_1(\underline{t})$, and the more likely bidder 2 is to win in an exclusive deal. Consequently, bidder 2 learns less about bidder 1's signal from the mere

[12] This generalizes to settings with more than two bidders. With three or more asymmetrically informed bidders, the best informed bidder will be offered an exclusive deal first; and the least well informed bidder will be offered an exclusive deal if all other bidders declined to participate in competitive bidding (with certain minimum bids).

observation that bidder 1 was excluded from any further bidding, and her value estimate increases further [she learns that $s_1 < z_1(\underline{t})$, which increases as δ decreases].

Thus, when designing the selling procedure, the target trades off conflicting goals. Biasing the procedure helps extract more value from the better-informed bidder, but it extracts less value from the less well-informed bidder. Additionally, a biased allocation rule makes it more likely that the winning bidder is not the bidder with the highest valuation, so less value is potentially created, and therefore less value can potentially be extracted (a less asymmetric procedure extracts a smaller fraction of the value that the bidders expect, but if the value is higher, the expected extracted value may be higher).[13] The more asymmetric the bidders, the more it pays to use an asymmetric procedure, because the benefits exceed the possible costs. Conversely, in a conventional setting with symmetric bidders, the optimal selling procedure is symmetric [this follows immediately from Equation (8) and Lemma 1, setting $\delta = 1$]. While a threat not to sell to one of the bidders may potentially extract a higher price from her, the rival bidder may get a good deal when the threat is executed, so less value is extracted from that second bidder. If the bidders are symmetric, the costs of biasing the procedure outweigh the benefits, and a standard auction is optimal.

The ability to credibly commit to the procedure is, of course, central to its success in extracting the highest possible transaction price. This commitment may be threatened in many ways.[14] Each party (including the target) may have an incentive to act opportunistically and to deviate from the rules of the sequential procedure. We discuss some issues here and others in later sections.

The ability to commit is most critical when the target threatens to walk away from bidder 1, if she declined both offers in Stage I: to buy the target at a price $\overline{b}_1$ in an exclusive deal, and to bid at least $\underline{b}_1$ in Stage II. The target should then be offered to bidder 2 at a price $\underline{b}_2$. Commitment is critical here, since bidder 1 could make an uninvited bid slightly higher than $\underline{b}_2$, which the target should accept: after all, the only alternative is to sell to bidder 2 at the lower price $\underline{b}_2$.

The threat to exclude bidder 1 from the procedure is similar to the use of a reserve price in an auction. The only difference is that in the case of a reserve price, the sale is canceled if the bids seem low, while in our sequential procedure the target is instead sold to bidder 2.[15] The seller

[13] Notice that with pure common values (i.e., if $\alpha = \frac{1}{2}$), the value that is created does not depend on the identity of the winner, since both find the target equally valuable; the target must then only worry about extracting as much value as possible.

[14] We thank an anonymous referee for clarifying the role of commitment in the sequential procedure and for encouraging us to explore the issue of possible opportunistic behavior in more depth.

[15] The target finds it optimal not to post a reserve price in our model; we discuss an extension in which reserve prices may be optimal in Section 5.1.

faces a commitment problem when posting a reserve price: if the reserve is not met, the sale should be canceled, but the seller may prefer to start a second round of bidding, possibly posting a lower reserve price. Reserve prices are quite common in practice, despite those commitment problems. This suggests that, in practice, sellers have some ability to commit to the rules they devised. Given the similarity of the commitment problems, and the fact that executing the threat is less costly in the case of our sequential procedure (the sale is not called off, but instead the target is sold to bidder 2), targets should be able to commit to the rules of the sequential procedure.

In practice, targets seem to be able to credibly reject offers that are low but nevertheless financially more attractive than existing outside options. For example, in the case cited in note 3, NCS abandoned the negotiations with Omnicare after deciding that its bid was too low, starting a search for new bidders (with little hope of success: NCS was insolvent at the time). Some firms prepare for initial public offerings while on the auction block; that is, they threaten to sell to outside bidders at a low price (taking into consideration that most IPOs experience some degree of underpricing).[16]

There is also systematic evidence supporting the idea that managers can credibly commit to reject seemingly attractive bids. Firms routinely reject takeover offers that are much higher than their current share price, and in many cases no more bids materialize or a takeover is not consummated. The run-up and markup around bids that eventually fail erodes within six months, suggesting that target shareholders would, on average, benefit from accepting such bids [the combined run-up and markup is about 20%, see Schwert (1996, Figure 2); for similar evidence, see Betton and Eckbo (2000)]. Sometimes these bids are rejected outright, sometimes in favor of lower bids from another bidder.[17]

In practice, reputational considerations may be one reason why sellers seem to be able to commit to their rules.[18] Some sellers are in the market repeatedly, for example, private equity funds that regularly buy or sell firms, and it is well understood how reputation allows agents to commit to actions that hurt them in the short run. Other sellers are not in the

[16] Recent cases include the sale of Carrols Holdings, owned by Madison Dearborn Partners (June 2004); Alcan Inc. and Pechiney SA's sale of their jointly owned aluminum rolling mill in Ravenswood, W.Va. (September 2004); Alcan Inc.'s aluminum rolled products division Novelis, Montreal (November 2004); E.ON AG's Ruhrgas Industries metering business and Viterra real estate division (November 2004); Saga Group Ltd. (a British travel, financial, and health conglomerate; September 2004); Borden Chemical, owned by KKR, (April 2004); and Iasis Healthcare, owned by JLL Partners, (May 2004). Another case was the sale of Vivendi's U.S. entertainment assets in 2003.

[17] Such a bidder is sometimes called a "White Knight." A famous example is Paramount's rejection of an offer from QVC, which was much higher than an earlier offer from Viacom. Another famous example is Revlon's rejection of a bid from Pantry Pride in favor of a lower bid from Forstmann Little.

[18] In fact, experimental evidence on commitment and opportunistic behavior shows that reputation may not be needed as a commitment device; without any explicit commitment possibilities, human subjects are able to earn higher payoffs than what economic theory would predict for agents that have no commitment power and that choose sequentially rational actions [see Camerer and Thaler (1995) for an overview].

market regularly, but they typically rely on the services of financial and legal advisors who provide their services to a large number of firms throughout the year. Similar to firms planning an initial public offering, one-time sellers may "borrow" an investment bank's reputation and credibility, say, and the fees paid to financial and legal advisors (and the importance of league tables for advisor choice) give investment banks an incentive to build up a strong reputation. Since the mid-1980s, target boards must be able to prove that a transaction involving a change in control maximized shareholder value and the board fulfilled its "duty of care."[19] Hiring an investment bank is regarded as adding some impartiality if the investment bank advises the target's board and provides a fairness opinion to evaluate an offer. Not surprisingly, publicly held firms now routinely hire investment banks to assist in sale transactions.

A second type of commitment ability is required once a winner has been declared. As we show in Section 4.1, the losing bidder may top up the winner's bid after the procedure has ended. This may happen since the sequential procedure is asymmetric, so it may happen that the loser has a higher valuation than the winner. This problem is important in practice, since the losing bidder can make the offer directly to the target's shareholders, who at that stage (after the procedure has ended) have every incentive to act opportunistically. As we explain in Section 4.1, targets in practice use a variety of legal and contractual tools to protect a deal.

Finally, the winning bidder may turn around and offer the target to the losing bidder at a higher price; since the allocation rule is biased, it is possible that the loser's valuation is higher than the winner's. It can be shown that the optimal response is to change the allocation rule and pricing decision, such that bidder 2 is more likely to win, often selling the target on to bidder 1. Anticipating the possible profits of doing so, the target can extract these profits when selling to the winning bidder. A resale possibility thus requires a changed design, but it does not reduce the expected transaction price, and the identity of the final owner of the target is unchanged (given the realized signals).[20]

4. Implications

In this section we discuss the main determinants in the design of the optimal selling procedure. We also discuss some empirical implications, and we suggest how some of the key factors may be identified or measured in practice.

[19] Frequently cited court decisions include Revlon, Inc. v. MacAndrews & Forbes Holdings, Inc., 506 A.2d 173 (Del. 1985); Smith v. Van Gorkom, 488 A.2d 858 (Del. 1985); and Paramount Communications v. QVC Network, 637 A.2d 34 (Del. 1994).

[20] Details are available in an earlier version of this article.

4.1 The Use of Deal Protection Devices

We have assumed that the target commits not to change the rules of the sequential procedure once it has started. Without such a commitment, the procedure will not be incentive compatible. It is easy to show that both bidders may have an incentive to bypass the sequential procedure if they do not win the target, by offering to pay more than the winner.

Proposition 2. *If the sequential procedure ends in Stage I, the losing bidder's estimate of the target's value may be higher than the price that the winning bidder is supposed to pay, so the losing bidder can (profitably) offer to pay more.*

Proof. See the Appendix. ■

The loser may be willing to top the winner's price, since under the optimal allocation rule the winner does not necessarily have the highest valuation. If the bidders anticipate that the target may accept an unsolicited top-up bid, the incentive structure of the sequential procedure is undermined. The official winner may top up the loser's unsolicited bid, the loser may in turn raise the bid, etc. If the bidders anticipate this possibility, the procedure takes on features of a standard English auction, in which the bids are raised until one of the bidders drops out. This type of auction treats the bidders symmetrically, and the target loses the benefits of the optimal bias built into the sequential procedure.

Even if the target's board is committed not to renegotiate, the losing bidder can ex post contact the target's shareholders directly, urging them to remind the target's board of its fiduciary duty toward shareholders. This seems particularly effective if the sequential procedure ends in Stage I, where one bidder enjoys exclusivity and the other bidder is excluded from bidding, since to an outsider there seems to have been no real competition for the target.

In practice, targets can prevent unsolicited top-up bids by using a variety of tools that reduce the value of the target to the losing bidder, but not to the winner. In some cases, mere animosity toward unsolicited bids may keep unwanted bidders from speaking up (for example, the unsolicited bid may be termed "hostile," or the target may initiate antitrust proceedings). In other cases, targets will have to resort to legal devices that help them cement the deal with the winning bidder. Numerous deal protection devices are used in practice, most of which prevent unsolicited bids by increasing an unwanted bidder's cost of acquiring the target. One example are termination fees, payable to a bidder if she was promised a sale but the deal is not concluded. Another example are lock-up clauses, which give a certain bidder the right to buy shares or assets at a low price, a right that will be exercised if a third party takes over the target. No-shopping clauses can make it harder for third parties to prepare a bid, by restricting access to relevant information (however, they cannot prevent the target from *considering* unsolicited bids).

Finally, a target may have poison pills in place that it promises to lift after concluding a sale to the preferred bidder, but not otherwise.

These deal protection devices are widely used in practice,[21] but their use rarely goes unchallenged. The courts have upheld the use of these devices in some cases and ruled them unacceptable in other cases. Our results show how important it is to distinguish the ex ante and ex post benefits and costs of deal protection devices. Clearly, after a deal has been concluded, it is in the interest of target shareholders if the target's board behaves opportunistically and accepts a late bid from a losing bidder. However, this undermines the ability of the board to extract a higher price by designing an optimal procedure. It is thus in the interest of the shareholders of all future takeover targets to allow the use of deal protection devices, and courts should therefore dismiss shareholder lawsuits if a target's board can show that its asymmetric treatment of potential bidders happened in response to informational asymmetries between the bidders.[22]

4.2 Bidders' Chances of Winning

We now analyze how changes in the degree of bidder asymmetry affect the bidders' chances of winning.

Proposition 3. *An decrease in δ, that is, an increase in bidder asymmetry, results in an increased likelihood of an exclusive deal and a decreased likelihood of a bidding contest. Both bidders' chances of closing an exclusive deal increase.*

Proof. See the Appendix. ■

Thus, if the weak bidder gets weaker, the target's optimal response is to accentuate the stick-and-carrot policy. Depending on the realized signals, the optimal procedure selectively makes it more or less likely that either bidder wins. As bidder 1's signal becomes *relatively* more informative, distortions of the allocation become more effective, but at the same time, such distortions become more costly for high signals s_1. Consequently, under the optimal selling procedure it is more likely that bidder 1 wins if her signal is high, and less likely if her signal is low. Unavoidably, the likelihood of enjoying exclusivity increases for both bidders (except if $\alpha = \frac{1}{2}$, cf. Lemma 2).

[21] Coates and Subramanian (2000) find that 37.3% of the firms in their samples sign termination fees, and 12.7% grant lock-up options. Bates and Lemmon (2003) find that 37% of the firms in their sample agree to target termination fees, and 17% grant lock-up options. They find that the use of termination fees has increased from 2% in 1989 to 60% in 1998. Officer (2003) finds that more than 40% of the firms in his sample include target termination fees.

[22] Some practitioners seem to share this opinion: "Deal makers say the inability to lock up key shareholders could actually result in lower prices being paid because prospective buyers won't put their best offer on the table if they can't be certain the transaction will be completed." (Cited from "Merger Business Faces New Order With Court Ruling on Lock-ups", *Wall Street Journal*, April 7, 2003, p. C4.) The dissenting opinion in Omnicare, Inc. v. NCS Healthcare, Inc., 818 A.2d 914 (Del. 2003) also emphasizes the benefits of deal protection devices, recognizing that without them, bidders may never come forward with their (best) offers.

The use of deal protection devices has been studied empirically, and the evidence is consistent with our results. Our model predicts that targets facing more asymmetric bidders should be sold using more biased procedures—the bias in a procedure requires deal protection devices. Bidder asymmetry is more likely to be significant if there is more scope for informational problems, for example, if firms are regarded as opaque and outsiders find it harder to value a target than the target's managers or direct competitors. Thus, deal protection devices should be used more frequently in the sale of opaque targets. That seems to be the case. Coff (2003) finds that the incidence of lock-up options is higher for R&D–intensive firms, and Bates and Lemmon (2003) find that termination fees are more common in technology and pharmaceutical industries. These are firms whose assets are harder to value for outsiders than fixed assets, so they offer more scope for bidder asymmetry. Officer (2003) reports similar findings, and additionally that conglomerates have the highest incidence of termination fees. That is consistent with our results, since conglomerates are usually regarded as opaque. And using a target's market-to-book ratio as a proxy for the presence of growth opportunities, Bates and Lemmon (2003) confirm that larger growth opportunities make termination fees more likely. Again, this is consistent with our results, since growth opportunities are harder to value than existing assets.

As we argued above, MBOs are probably the best example for a situation in which one bidder has superior information. Our model predicts that if an MBO team competes with less well-informed outside bidders, the likelihood of an exclusive deal should be higher, and the use of deal protection devices should be heavier. Unfortunately, there are no empirical studies that report any related findings. It would be useful to take this prediction to the data, since it contradicts the prediction of a model based on managerial entrenchment: a strong and independent board should agree to deal protection devices with any type of bidder, but only in the presence of bidder asymmetry; in contrast, a weak board that is controlled by the manager should grant deal protection only to the MBO team.

A second key variable for the optimal procedure is α, which measures the relative size of private value and common value components in the bidders' valuations.

Proposition 4. *An increase in α increases the likelihood of a Stage I sale to bidder* 1.

Proof. See the Appendix. ■

The intuition behind this result is that the target's focus is on extracting a high price from the better-informed bidder, while using a possible exclusive deal with the less well-informed bidder as a threat to extract that high payment. As α increases, both bidders put more weight on their own signals, so both become better informed, but the effect on bidder 1 is stronger.[23] It therefore becomes costlier to bias the selling procedure against the strong bidder, since more value is destroyed by letting bidder 2 win even if bidder 1's signal is high. Consequently, the cut-off $z_1(\bar{t})$ decreases. With pure common values (if $\alpha = \frac{1}{2}$), the problem does not arise, since both bidders always value the target equally. This explains why in the case $\alpha = \frac{1}{2}$ there is no need for a "carrot," that is, no threshold $z_1(\bar{t})$ such that, if $s_1 \geq z_1(\bar{t})$, then bidder 1 wins with certainty.

Of course, this discussion only applies if the bidders are asymmetric—if they are symmetric, then the optimal selling procedure is always a standard auction. For example, it applies to contested MBO bids, in which a management team competes with a competitor of the target, and both have a significant private value component: the competitor may expect to realize synergies due to overlapping products and distribution channels, while the management team may enjoy control rents or fear losing their jobs. The more important these private value components, the more likely it should become that a takeover deal is concluded with the management team in Stage I.

4.3 Bid Premia and Target Shareholder Gains

An important question in the context of takeovers is how much target shareholders benefit. An additional assumption on the distribution of the signals allows us to derive results about the expected transaction price. Specifically, we assume that f is uniform with support [1,2] [we choose a lower bound of 1 to satisfy the assumption that $\underline{t}H(\underline{t}) \geq 1$]. This simplifies the analysis, since we can derive closed form solutions for all variables of interest.

In our setup, bidder 2 is a weak bidder from an ex ante perspective, because she is less well-informed. (This should not be confused with weakness from an ex post perspective, meaning that her signal realization was low, and therefore her valuation is lower.) A weakening of the weak bidder (a decrease in δ) reduces the degree of competition between the bidders, to which the target responds by designing a more biased selling procedure (see Proposition 3). It has the following effects.

Proposition 5. *If f is uniform with support* [1,2], *then a decrease in δ, that is an increase in bidder asymmetry, results in*:

1. *An increased expected transaction price;*
2. *An increased price paid by the winner ($\bar{b}_1$, $\underline{b}_2$, $b_1(s_1)$ and $b_2(s_2)$ increase)*;

[23] If the change in α did not change the *relative* informativeness of the signals, then the allocation rule would not change. That can be verified by rewriting Inequality (7) as an equation, which is correct for any s_2 if $s_1 = z_1(s_2)$, and simplifying; the variable φ can be eliminated, which implies that a change in φ leaves z_1 unchanged.

3. *A reduced likelihood of bidder* 1 *winning; and*
4. *A reduction in the total value created in the sale*.

Proof. See the Appendix. ■

The target benefits from a reduction in δ, since a weaker bidder 2 is a more effective tool when it comes to extracting a higher payment from the strong bidder. It becomes less costly for the target to execute the threat not to sell to bidder 1 (if her bid is low), because bidder 2 trusts her signal less, so her value estimate moves closer to the expected value independent of her own signal (but conditional on $s_1 < z_1(\underline{t})$). In other words, the less informative her signal, the smaller the rent that bidder 2 expects to earn. Knowing that she will leave less money on the table when dealing with bidder 2, the target can extract a higher payment from bidder 1 (this is analogous to an auctioneer posting a higher reserve price). Thus, if the informativeness of the less well-informed bidder's signal decreases, the target can expect a higher transaction price.[24]

However, the expected transaction price is increased by biasing the allocation more, which means that the winner is less likely to be the bidder with the higher valuation (recall that marginal revenue determines the winner). This explains the last result in Proposition 5, that total value creation is reduced if δ decreases. We will come back to this result in Section 4.4.

Empirical studies have shown that the use of deal protection devices that favor one bidder does not necessarily harm target shareholders [see, e.g., Jennings and Mazeo (1993), Comment and Schwert (1995), Bates and Lemmon (2003), Officer (2003), or Peck (2002)]. Even in the case of MBO offers, where bidder asymmetry should be most pronounced, Kaplan (1989) finds significant premia and argues that management cannot use informational advantages to purchase the target at a lower price [see also Lee (1992)]. This is consistent with targets optimally biasing their selling procedure in response to bidder asymmetry and using deal protection devices to cement their commitment to their selling procedure.

Proposition 6. *If f is uniform with support* [1,2], *then an increase in α leads to a higher expected transaction price and to increased value creation. Bidder* 1*'s chances of winning increase*.

Proof. See the Appendix. ■

As before (see Proposition 5), assuming that f is uniform allows us to analyze the equilibrium outcome in more detail. An increase in α benefits

[24] Povel and Singh (2004) obtain the same result for a pure common values model.

the shareholders, since the expected transaction price increases. However, this is not achieved by biasing the selling procedure more, since total value creation increases, too.[25] Instead, an increase in α strengthens competition between the bidders: the relevance of their signals for their valuation increases, and that makes both bidders better informed. Reducing the bias in the selling procedure is optimal. Competition between the bidders is fiercer, so it is easier to extract value, and a reduced bias generates more value which can then be extracted by the target.

This result has implications for empirical work. When estimating bid premia or announcement effects, it is important to control for the type of bidders that participated in the contest, since our model predicts a higher expected transaction price if bidders are trade buyers, and a lower expected transaction price if they are financial buyers.

4.4 Implications for Posttakeover Performance

A large number of studies have analyzed a target's posttakeover performance, testing (for example) whether targets are more efficient after a change in control. The empirical evidence on long-term performance after takeovers seems inconclusive [cf. Andrade, Mitchell, and Stafford (2001)]. Our results can shed new light on these issues.

One of our key insights is that the optimal procedure is biased and may lead to a distorted allocation; the target is sold to the bidder with the highest *marginal revenue*, not necessarily the bidder with the highest *valuation*. A bidder's valuation may be lower because less synergies can be realized, or because a bidder is less experienced at running the target. In other words, less value is created (on average) after a takeover, which should be reflected in a poorer posttakeover performance of the target, or at least below its true potential.

Our model predicts (cf. Proposition 5) that the wider the asymmetry between the bidders, the more biased the allocation rule, and the wider the gap between realized and potential productivity and profitability. So all else equal, we should expect a lower posttakeover performance if, say, one of the bidders was the management team, than if all bidders were symmetrically informed. Similarly, all else equal, we should expect an opaque diversified conglomerate to perform less well than a focused one-division firm after a takeover.

Our model also predicts (cf. Proposition 6) that, if α increases, value creation becomes relatively more relevant than value extraction, and the allocation rule becomes less biased. Consequently, our model predicts that the target's posttakeover performance should be more clearly below its potential after a contest between financial buyers, compared with an otherwise identical contest between trade buyers.

[25] It can be shown that the effect on the transaction price is larger than that on value creation.

These predictions are relevant when interpreting a takeover target's pretakeover and posttakeover performance. Some authors have argued that improved posttakeover performance is evidence of agency problems (inefficient management) that were resolved through the takeover.[26] Our results show that changes in the target's performance are also affected by bidder asymmetry, which produces a more biased selling procedure and therefore reduces the target's expected performance after a takeover. Similarly, it is necessary to control for bidder types when estimating how the target's performance changes, since it tends to be lower if bidders can be characterized as financial buyers.

5. Extensions

5.1 When Setting a Reserve Price Is Optimal

In practice, bids may be invited for a target, and yet there is no takeover in the end. The reason for this can be that the bids were so low that the target prefers to remain independent (or to retain a division that was for sale). Put differently, the bidders' valuations may not be much higher than the seller's. Under our assumption that $\underline{t}H(\underline{t}) \geq 1$, the bidders' valuations are always significantly higher than the seller's, so the seller always wants to conclude a sale. In other words, the seller never finds it optimal to post reserve prices.[27] We now relax this assumption and assume that $z_1(\underline{t})H(z_1(\underline{t})) \geq 1$, and that $\underline{t} \geq 0$ [this is less restrictive, since $z_1(\underline{t}) > \underline{t}$ and H is monotonically increasing]. Under these assumptions, we show that the target may find it optimal to abort the sale (post a reserve price) if the bids seem low. We first develop the intuition why she may do so in the case of a Stage I sale to bidder 2; then we describe under what circumstances the sale will be aborted.

Consider the target's situation if bidder 1 declined both offers in Stage I of the sequential procedure, and the target should be sold to bidder 2, in an exclusive deal. Intuitively, the target may want to set a reserve price in this deal. More precisely, she may want to make a take-it-or-leave-it offer that bidder 2 will accept if her signal s_2 is sufficiently high, but reject if the signal is sufficiently low. Losing the sale in the low-s_2 cases is costly, but this cost may be outweighed by the increase in the price that bidder 2 pays if her signal is sufficiently high. The sequential procedure requires that the target is sold at a price $\underline{b}_2$, which bidder 2 would accept with any signal $s_2 \geq \underline{t}$. Suppose the target increases the price by a small $\varepsilon > 0$. Bidder 2 would now decline the offer if $s_2 \leq \underline{t} + \frac{\varepsilon}{\delta\varphi\alpha}$ and accept it

[26] For an overview, see Agrawal and Jaffe (2003).

[27] More precisely, we have assumed that $\underline{t}H(\underline{t}) \geq 1$ and that the seller values the target at zero. The latter is a normalization. Suppose the seller's valuation is $v_0 > 0$, and that bidder i's valuation is $\alpha t_i + (1-\alpha)t_j + v_0$. Strategically, if v_0 is common knowledge, the seller's problem is equivalent to that in our model.

otherwise. Doing so is beneficial for the target if the incremental expected payment

$$\left(1 - F\left(\underline{t} + \tfrac{\varepsilon}{\delta\varphi\alpha}\right)\right) \cdot \varepsilon - F\left(\underline{t} + \tfrac{\varepsilon}{\delta\varphi\alpha}\right) \cdot \underline{b}_2$$

is positive, which is the case if $\underline{b}_2$ is not too large and the product $\delta\varphi\alpha$ is not too small.

If we assume that $\underline{t}H(\underline{t}) \geq 1$, the incremental expected payment is negative for any ε, so it is not beneficial for the target to threaten bidder 2 with a reserve price in Stage I. The derivative of the incremental expected payment with respect to ε is:

$$\begin{aligned} 1 - F\left(\underline{t} + \frac{\varepsilon}{\delta\varphi\alpha}\right) &- f\left(\underline{t} + \frac{\varepsilon}{\delta\varphi\alpha}\right)\frac{\underline{b}_2 + \varepsilon}{\delta\varphi\alpha} \\ &= \left(1 - F\left(\underline{t} + \frac{\varepsilon}{\delta\varphi\alpha}\right)\right)\left(1 - H\left(\underline{t} + \frac{\varepsilon}{\delta\varphi\alpha}\right)\frac{\underline{b}_2 + \varepsilon}{\delta\varphi\alpha}\right) \\ &< \left(1 - F\left(\underline{t} + \frac{\varepsilon}{\delta\varphi\alpha}\right)\right)(1 - H(\underline{t})\underline{t}), \end{aligned}$$

since $\underline{b}_2 \geq \underline{t}$ and $\delta\varphi\alpha \leq 1$. If $\underline{t}H(\underline{t}) \geq 1$, the above is negative.

If we relax the assumption and allow for lower values of $\underline{t}$, then the target may want to post a reserve price for bidder 2 in Stage I. Of course, if it is optimal to post a reserve price, then that should be part of the optimal selling procedure. Also, given that our model allows for both private and common value components, we should generally expect the optimal reserve price policy to be more complex than simply posting a fixed value. In fact, it is optimal to post bidder-specific reserve prices, which depend on the realization of both signals.

To determine the reserve prices, we need to describe the signal pairs for which a bidder's marginal revenue [see Equations (4)–(5)] is equal to zero. This implicitly defines two threshold functions λ_1 (for bidder 1) and μ_1 (for bidder 2) by $MR_1(\lambda_1(s_2), s_2) = 0$ and $MR_2(\mu_1(s_2), s_2) = 0$. Using this notation, we can rewrite the optimal allocation rule from Equation (9) (see Lemma 1):

$$p_1(s_1, s_2) = \begin{cases} 1 & \text{if} \quad s_1 \geq z_1(s_2) \quad \text{and} \quad s_1 \geq \lambda_1(s_2), \\ 0 & \text{otherwise} \end{cases}$$

$$p_2(s_1, s_2) = \begin{cases} 1 & \text{if} \quad s_1 < z_1(s_2) \quad \text{and} \quad s_1 \geq \mu_1(s_2), \\ 0 & \text{otherwise} \end{cases}$$

Implicit differentiation of Equations (4)–(5) shows that both λ_1 and μ_1 have negative slope,

$$\frac{\partial\lambda_1(s_2)}{\partial s_2} = -\frac{\delta\varphi(1-\alpha)}{\varphi\alpha}\frac{(H(s_1))^2}{(H(s_1))^2 + H'(s_1)},$$
$$\frac{\partial\mu_1(s_2)}{\partial s_2} = -\frac{\delta\varphi\alpha}{\varphi(1-\alpha)}\frac{(H(s_2))^2 + H'(s_2)}{(H(s_2))^2}.$$

A comparison of these terms shows that the constant-marginal revenue curves for bidder 2 are steeper than those of bidder 1.

Under the assumptions that $z_1(\underline{t})H(z_1(\underline{t})) \geq 1$ and $\underline{t} \geq 0$, we can show that bidder 1's marginal revenue is positive whenever $s_1 \geq z_1(s_2)$ [more precisely, $s_1 \geq z_1(s_2)$ implies $s_1 \geq \lambda_1(s_2)$]. In other words, the target will not abort a sale procedure that would otherwise be won by bidder 1. The lowest signal pair realization with which bidder 1 can win is $[z_1(\underline{t}), \underline{t}]$. Bidder 1's marginal revenue is nonnegative if

$$E[t] + \varphi\alpha(z_1(\underline{t}) - E[t]) + \delta\varphi(1-\alpha)(\underline{t} - E[t]) - \varphi\alpha\frac{1}{H(z_1(\underline{t}))} \geq 0.$$

We assume that $z_1(\underline{t}) \cdot H(z_1(\underline{t})) \geq 1$, so it is sufficient to show that the following is satisfied:

$$E[t] + \varphi\alpha(z_1(\underline{t}) - E[t]) + \delta\varphi(1-\alpha)(\underline{t} - E[t]) - \varphi\alpha z_1(\underline{t}) \geq 0$$
$$\iff \quad E[t](1 - \varphi\alpha - \delta\varphi(1-\alpha)) + \delta\varphi(1-\alpha)\underline{t} \geq 0,$$

which is always satisfied, since $\underline{t} \geq 0$ and $\alpha, \delta, \varphi \in [0, 1]$. Since the slope of λ_1 is negative and that of z_1 is positive, this implies that, if the target aborts the sale, it happens only if bidder 1 would not have won anyway—either in Stage I, during exclusive talks with bidder 2, or in Stage II, after competitive bids were submitted such that $b_1 < \hat{z}_1(b_2)$.

In sum, the possible optimality of reserve prices complicates the optimal selling procedure, but the main result remains valid. The optimal procedure makes it easier for bidder 1 to win when her willingness to pay is high and harder if her willingness to pay is low. The complications arise if both signals are low, which affects mostly bidder 2's chances of winning.

5.2 Misrepresentation of the Quality of Information

We now analyze whether the target can trust the bidders in detailing how reliable their information is, how experienced they are in evaluating it, etc. Not surprisingly, we find that this is not the case; bidders may claim to be either better or less well informed than they really are, depending on the circumstances. Given that this often reduces the expected transaction price, the target (or its investment bank) should do its own research to estimate how well informed each bidder really is or may be.

First, consider the possibility that bidder 1 may convince the target and bidder 2 that she is not better informed than bidder 2, and that instead both bidders observe their true factor t_i with probability $\delta\varphi$ [cf. Equation (1)]. It follows immediately that the optimal selling procedure is a symmetric auction, for example a first-price auction. Bidder 2 will bid according to the equilibrium strategies for this symmetric setup, which can be derived from Lemma 3 by setting $z_1(s_2) = s_2$. We need to specify a functional form for F to derive bidder 1's best response; we assume (like we did in Section 4) that the signals are distributed uniformly on [1,2]. Bidder 1's bidding strategy can be derived by taking that of bidder 2 as given and choosing the optimal b_1, given a signal s_1. This yields the "bidder 1-misrepresentation" bidding strategies:

$$
\begin{aligned}
b_1^{\mathrm{mis}_1}(s_1) &= \frac{3}{2} + \frac{1}{2}\varphi\left(s_1 - 2 + \frac{1-\delta}{2}\right) \\
b_2^{\mathrm{mis}_1}(s_2) &= \frac{3}{2} + \frac{1}{2}\delta\varphi(s_2 - 2).
\end{aligned}
$$

It is straightforward to calculate the bidders' rents and the expected transaction price (bidder 1 wins if $s_1 \geq \delta s_2 + (1-\delta)\frac{3}{2}$). A comparison with the corresponding values for the sequential procedure shows who benefits or suffers. Bidder 1 always benefits from the misrepresentation, which is not surprising, given that the main goal of the sequential procedure is to extract value from bidder 1. In contrast, bidder 2 suffers. She bids as if bidder 1 was not better informed and, therefore, suffers the full cost of the "winner's curse." The target, finally, may either benefit or suffer. She suffers because she does not extract as much value as would be possible when using the sequential procedure; however, because bidder 2 tends to overbid, the target also benefits, and the net effect depends on the parameters.

An alternative possibility is that bidder 2 may claim to be better informed than she really is. Suppose she convinces the target and her rival that both bidders' probability of observing t_i is φ. Bidder 2 can then take her rival's symmetric bidding strategy as given, and we obtain these bidding strategies:

$$
\begin{aligned}
b_1^{\mathrm{mis}_2}(s_1) &= \frac{3}{2} + \frac{1}{2}\varphi(s_1 - 2) \\
b_2^{\mathrm{mis}_2}(s_2) &= \frac{3}{2} + \frac{1}{2}\delta\varphi\left(s_2 - 2 - \frac{1-\delta}{2\delta}\right).
\end{aligned}
$$

Bidder 1 is more cautious when bidding than she should be; she overcompensates for the winner's curse, because she believes her rival to be better informed than she really is. As before, we can calculate the bidders' rents and the expected transaction price. Bidder 1 benefits from bidder 2's misrepresentation, since the symmetric procedure cannot extract as much value from her as the sequential procedure. The target suffers, for two reasons: first, because a standard auction is suboptimal, and second, because bidder 1 bids less aggressively (over-compensating for the winner's curse), and that allows bidder 2 to bid less aggressively, too. Bidder 2, finally, may either benefit or suffer, depending on the parameters. She benefits from making bidder 1 overly conservative, but she also benefits from participating in the sequential procedure (which uses bidder 2 as a tool to extract value from bidder 1, focusing less on extracting value from bidder 2).

It would be interesting to analyze the problem of unobservable signal quality in a model in which bidders have to acquire their signals at a cost. In other words, bidders decide on the probability of observing the true signal (e.g., with probability φ or $\delta\varphi$). Some bidders can be assumed to have lower costs of generating useful information (say, a close competitor), or to be endowed with better information from the start (say, a CEO competing with outside bidders). Even if the bidders anticipate that the optimal selling procedure will exploit any information asymmetry, there should remain benefits to being better informed. A target's problem then is to elicit information about two variables from each bidder: how informative the signal is, and what signal was realized. Solving this problem would be interesting, but it is beyond the scope of this article.

6. Conclusion

We have analyzed how a takeover contest should optimally be designed in the presence of bidders that are not equally well informed. We have derived the general properties of optimal selling procedures and the details of a sequential procedure that is optimal. This sequential procedure encourages bidders to compete by treating them in an asymmetric fashion.

At first sight, it might seem that the target should increase bidder competition by handicapping the better informed bidder, since a less well-informed bidder is a weak competitor to the better-informed bidder. However, the optimal selling procedure is more resourceful; it actively uses bidder asymmetry to better play off the bidders against each other, by offering exclusive deals to one bidder while ignoring any bids from another bidder. Specifically, the possibility of an exclusive deal encourages the better-informed bidder to reveal a high willingness to pay (and then pay a high price), and the threat of an exclusive deal with the other bidder discourages the announcement of a low willingness to pay.

Our results also show why shareholder value maximizing target boards should in practice make frequent use of deal protection devices. The sequential selling procedure requires commitment to its rules, and deal protection devices help the target cement this commitment. Evaluating the costs and benefits of deal protections devices then requires that they are evaluated within the context of an entire takeover contest. The use of deal protection devices should be upheld in court if a target's board can show that they were agreed upon as part of an optimally designed selling procedure. While target shareholders may benefit by opportunistically accepting late bids that are submitted after a winner has been declared, this possibility undermines the effectiveness of the sequential selling procedure, thereby harming the shareholders of future takeover targets.

Appendix: Proofs

Proof of Lemma 2

Since Ψ is continuous and strictly increasing, z_1 is continuous and strictly increasing, too. If $\alpha > \frac{1}{2}$, the function Ψ as defined in Equation (6) attains a negative value for $s_2 = \underline{t}$ and a positive value for $s_2 = \bar{t}$. So there must be a signal $\sigma \in (\underline{t}, \bar{t})$ such that $\Psi(\sigma) = 0$. This implies that $z_1(\sigma) = \sigma$, that is, z_1 has a fixed point in $(s_1, s_2) = (\sigma, \sigma)$. This fixed point is defined implicitly by

$$\Psi(\sigma) = 0 \iff \sigma = E[t] + \frac{\alpha}{2\alpha - 1}\frac{1}{H(\sigma)}.$$

This implies that $\sigma > E[t]$. Since $\delta\Psi(\underline{t}) < 0$, $\delta \leq 1$, and Ψ is increasing, it follows that $z_1(\underline{t}) \in (\underline{t}, \sigma)$. Similarly, since $\delta\Psi(\bar{t}) > 0$, it follows that $z_1(\bar{t}) \in (\sigma, \bar{t})$. If $\alpha = \frac{1}{2}$, the only change in our arguments is that the first term in the definition of Ψ vanishes [cf. Equation (6)], and we have $\Psi(\sigma) = 0 \iff \sigma = \bar{t}$, i.e. $\sigma = z_1(\bar{t}) = \bar{t}$. ■

Proof of Lemma 3

Define

$$\widehat{b}_1(s_1) \equiv \begin{cases} \overline{b}_1 & \text{in Stage I} \\ b_1(s_1) & \text{in Stage II} \end{cases}$$

$$\widehat{b}_2(s_2) \equiv \begin{cases} \underline{b}_2 & \text{in Stage I} \\ b_2(s_2) & \text{in Stage II} \end{cases}$$

$$V_i(s_i) \equiv \int_{\underline{t}}^{\bar{t}} (v_i(s_1, s_2) - \widehat{b}_i(s_i))p_i(s_1, s_2)f(s_j)ds_j, \quad i = 1, 2,$$

$$U_1(s_1, \widehat{s}_1) \equiv \int_{\underline{t}}^{\bar{t}} (v_1(s_1, s_2) - \widehat{b}_1(\widehat{s}_1))p_1(\widehat{s}_1, s_2)f(s_2)ds_2,$$

$$U_2(s_2, \widehat{s}_2) \equiv \int_{\underline{t}}^{\bar{t}} (v_2(s_1, s_2) - \widehat{b}_2(\widehat{s}_2))p_2(s_1, \widehat{s}_2)f(s_1)ds_1,$$

$$Q_i(s_i) \equiv \int_{\underline{t}}^{\bar{t}} p_i(s_1, s_2)f(s_j)ds_j \qquad i = 1, 2.$$

$V_i(s_i)$ is bidder i's expected net payoff, given a realized signal s_i. $U_i(s_i, \widehat{s}_i)$ is her expected payoff if the realized signal is s_i, but she uses the strategy for the case in which the signal realization is $\widehat{s}_i$. And $Q_i(s_i)$ is bidder i's probability of winning, given a realized signal s_i.

In Step 1 of the proof, we show that incentive compatibility follows from these sufficient conditions:

$$\frac{\partial V_1(s_1)}{\partial s_1} = \varphi\alpha Q_1(s_1), \quad \frac{\partial V_2(s_2)}{\partial s_2} = \delta\varphi\alpha Q_2(s_2), \quad Q_1^{'}(s_1) \geq 0 \quad \text{and} \quad Q_2^{'}(s_2) \geq 0.$$

Then we show (in Step 2) that the strategies in Equations (10)–(14) satisfy these sufficient conditions and therefore are equilibrium strategies.

Step 1. Consider the case of bidder 1 first. Rewrite $U_1(s_1, \widehat{s}_1)$ as

$$U_1(s_1, \widehat{s}_1) = V_1(\widehat{s}_1) + \varphi\alpha(s_1 - \widehat{s}_1)Q_1(\widehat{s}_1). \tag{A1}$$

We need to show that $V_1(s_1) \geq U_1(s_1, \widehat{s}_1) \forall s_1, \widehat{s}_1$. From $\frac{\partial V_1(s_1)}{\partial s_1} = \varphi\alpha Q_1(s_1)$ we get

$$V_1(s_1) = V_1(\widehat{s}_1) + \int_{\widehat{s}_1}^{s_1} \frac{\partial V_1(s)}{\partial s} ds = V_1(\widehat{s}_1) + \int_{\widehat{s}_1}^{s_1} \varphi\alpha Q_1(s) ds. \tag{A2}$$

Substituting for $V_1(\widehat{s}_1)$ from Equation (A1) in Equation (A2),

$$V_1(s_1) = U_1(s_1, \widehat{s}_1) - \varphi\alpha(s_1 - \widehat{s}_1)Q_1(\widehat{s}_1) + \int_{\widehat{s}_1}^{s_1} \varphi\alpha Q_1(s) ds. \tag{A3}$$

If $s_1 > \widehat{s}_1$ then since Q is weakly increasing, we can substitute for the lower bound on $Q_1(s_1)$ to obtain the following inequality:

$$\begin{aligned} V_1(s_1) &\geq U_1(s_1, \widehat{s}_1) - \varphi\alpha(s_1 - \widehat{s}_1)Q_1(\widehat{s}_1) + \int_{\widehat{s}_1}^{s_1} \varphi\alpha Q_1(\widehat{s}_1) ds \\ &= U_1(s, \widehat{s}_1) - \varphi\alpha(s_1 - \widehat{s}_1)Q_1(\widehat{s}_1) + \varphi\alpha Q_1(\widehat{s}_1)(s_1 - \widehat{s}_1) \\ &= U_1(s_1, \widehat{s}_1). \end{aligned}$$

The argument for $s_1 \leq \widehat{s}_1$ is similar.

The analysis for bidder 2 is analogous and therefore omitted.

Step 2. The functions b_1 and b_2 are increasing, and since by construction $\widehat{z}_1$ implements Equation (9), we have

$$Q_1(s_1) = \begin{cases} 1 & \text{if} \quad s_1 > z_1(\bar{\mathrm{t}}) \\ F(z_2(s_1)) & \text{if} \quad z_1(\underline{t}) < s_1 \leq z_1(\bar{t}) \\ 0 & \text{if} \quad s_1 \leq z_1(\underline{t}) \end{cases} \quad \text{and} \quad Q_2(s_2) = F(z_1(s_2)).$$

Notice that $Q_1'(s_1) \geq 0$ and $Q_2'(s_2) \geq 0$. If $s_1 > z_1(\bar{t})$, then $V_1(s_1) = \int_{\underline{t}}^{\bar{t}} v_1(s_1, s_2) f(s_2) ds_2 - \overline{b}_1$ and

$$V_1'(s_1) = \int_{\underline{t}}^{\bar{t}} \frac{\partial}{\partial s_1} v_1(s_1, s_2) f(s_2) ds_2 = \alpha\varphi = \alpha\varphi Q_1(s_1).$$

If $z_1(\underline{t}) < s_1 \leq z_1(\bar{t})$, then $V_1(s_1) = \int_{\underline{t}}^{z_2(s_1)} [v_1(s_1, s_2) - b_1(s_1)] f(s_2) ds_2$ and

$$\begin{aligned} V_1'(s_1) = &\int_{\underline{t}}^{z_2(s_1)} \left[\frac{\partial}{\partial s_1} v_1(s_1, s_2) - \frac{\partial}{\partial s_1} b_1(s_1)\right] f(s_2) ds_2 \\ &+ z_2'(s_1)[v_1(s_1, z_2(s_1)) - b_1(s_1)] f(z_2(s_1)) \\ = &\left[\alpha\varphi - \frac{\partial}{\partial s_1} b_1(s_1)\right] F(z_2(s_1)) + z_2'(s_1)[v_1(s_1, z_2(s_1)) - b_1(s_1)] f(z_2(s_1)). \end{aligned}$$

Since

$$\frac{\partial}{\partial s_1} b_1(s_1) = [v_1(s_1, z_2(s_1)) - b_1(s_1)] \frac{f(z_2(s_1)) z_2'(s_1)}{F(z_2(s_1))},$$

we have $V_1'(s_1) = \alpha\varphi F(z_2(s_1)) = \alpha\varphi Q_1(s_1)$. If $s_1 \le z_1(\underline{t})$, then bidder 1 cannot win, and therefore $V_1'(s_1) = 0 = \alpha\varphi Q_1(s_1)$.

The analysis for bidder 2 is analogous and therefore omitted. This completes Step 2.

We have, thus, shown that bidder i has no incentive to deviate from $\widehat{b}_i(s_i)$ to $\widehat{b}_i(\widehat{s}_i) \in [\widehat{b}_i(\underline{t}), \widehat{b}_i(\overline{t})]$. For either bidder, there is clearly no benefit to deviating to bids $\widehat{b}_i' < \widehat{b}_i(\underline{t})$ or $\widehat{b}_i'' > \widehat{b}_i(\overline{t})$, since $\widehat{b}_i'$ is equivalent to $b_i(\underline{t})$ (the bidder loses for sure), and $\widehat{b}_i''$ cannot increase the chances of winning above 1, while possibly leading to higher payments. Thus, the bidders will not deviate from the strategies in Equations (10)–(14). ■

Proof of Proposition 2

First, we show that bidder 2 may want to top up bidder 1's price if bidder 1 closes an exclusive deal in Stage I. Bidder 1 is expected to pay a price

$$\begin{aligned}
\overline{b}_1 &= v_1(z_1(\overline{t}), E[t]) - \varphi\alpha \int_{z_1(\underline{t})}^{z_1(\overline{t})} F(z_2(s))ds \\
&< E[t] + \varphi\alpha(z_1(\overline{t}) - E[t]) \\
&< E[t] + \varphi(1-\alpha)(E[s_1|s_1 > z_1(\overline{t})] - E[t]) + \varphi\alpha(\overline{t} - E[t]) \\
&= E_{s_1 \in [z_1(\overline{t}), \overline{t}]}[v_2(s_1, \overline{t})].
\end{aligned}$$

That is, for sufficiently high signals s_2, bidder 2's valuation is higher than $\overline{b}_1$.

Similarly, consider the case in which bidder 2 closed an exclusive deal in Stage I. She is expected to pay

$$\begin{aligned}
\underline{b}_2 &= E[t] + \varphi(1-\alpha)(E[s_1|s_1 < z_1(\underline{t})] - E[t]) + \delta\varphi\alpha(\underline{t} - E[t]) \\
&< E[t] + \varphi(1-\alpha)(z_1(\underline{t}) - E[t]) \\
&\le E[t] + \varphi\alpha(z_1(\underline{t}) - E[t]) \\
&= v_1(z_1(\underline{t}), E[t]).
\end{aligned}$$

That is, for sufficiently high signals s_1, bidder 1's valuation is higher than $\underline{b}_2$. ■

Proof of Proposition 3

From Lemma 2, the function z_1 has a fixed point in σ, which does not depend on δ or φ, since they are not arguments of Ψ, [cf. Equation (6)]. Elements of z_1 are characterized by $\Psi(s_1) = \delta\Psi(s_2)$. Suppose δ decreases by a small $\varepsilon > 0$. This affects the right-hand side of $\Psi(s_1) = \delta\Psi(s_2)$, which becomes less positive (if $s_2 > \sigma$) or less negative (if $s_2 < \sigma$). In order to remain on the z_1 curve, for a given s_2 we must select a signal s_1 such that $\Psi(s_1) = (\delta - \varepsilon)\Psi(s_2)$. If $s_2 = \sigma$, no change to s_1 is needed. If $s_2 < \sigma$, then $\Psi(s_2) < 0$ and $\Psi(z_1(s_2)) < 0$, too, requiring an increase in s_1. If $s_2 > \sigma$, then $\Psi(s_2) > 0$ and $\Psi(z_1(s_2)) > 0$, too, requiring a decrease in s_1. ■

Proof of Proposition 4

By definition, Inequality (7) is binding for any s_2 if $s_1 = z_1(s_2)$. Rearranging yields

$$z_1(s_2) + \tfrac{\alpha}{2\alpha-1}\left(\tfrac{\delta}{H(s_2)} - \tfrac{1}{H(z_1(s_2))}\right) - [(\delta(s_2 - E[t])) + E[t]] \equiv 0.$$

After implicit differentiation,

$$\left.\frac{\partial z_1(s_2)}{\partial \alpha}\right|_{s_2=\overline{t}} = -\left.\frac{-\frac{1}{(2\alpha-1)^2}\left(\frac{\delta}{H(s_2)} - \frac{1}{H(z_1(s_2))}\right)}{1 + \frac{\alpha}{2\alpha-1}\frac{H'(z_1(s_2))}{H^2(z_1(s_2))}}\right|_{s_2=\overline{t}} \propto \frac{\delta}{H(\overline{t})} - \frac{1}{H(z_1(\overline{t}))},$$

which is (weakly) negative, because $\delta < 1$, H is increasing, and $z_1(\bar{t}) \leq \bar{t}$. ■

Proof of Proposition 5

The expected transaction price is [cf. Myerson (1981); Bulow and Roberts (1989); Bulow and Klemperer (1996)]

$$R \equiv \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{\bar{t}} \left[\left(v_1(s_1, s_2) - \frac{\varphi\alpha}{H(s_1)} \right) p_1(s_1, s_2) + \left(v_2(s_1, s_2) - \frac{\delta\varphi\alpha}{H(s_2)} \right) p_2(s_1, s_2) \right] f(s_1) ds_1 f(s_2) ds_2.$$

With signals distributed uniformly on [1,2], we have

$$\begin{aligned} z_1(s_2) &= \delta s_2 + (1-\delta)\frac{10\alpha - 3}{2(3\alpha - 1)}, \\ z_2(s_1) &= \frac{1}{\delta} s_1 - \frac{1-\delta}{\delta} \frac{10\alpha - 3}{2(3\alpha - 1)}, \\ \bar{b}_1 &= \frac{3}{2} + \frac{\varphi(1-\delta)}{2(3\alpha - 1)}\alpha^2, \\ \underline{b}_2 &= \frac{3}{2} + \frac{\varphi(1-\delta)}{2(3\alpha - 1)}\alpha^2 - \frac{\varphi(1+\delta)}{4}, \end{aligned}$$

$$\begin{aligned} b_1(s_1) &= \frac{3}{2} + \frac{\varphi(1-\delta)}{2(3\alpha-1)}\alpha^2 - \frac{\varphi(1+\delta)}{4} - \frac{\varphi(1-\delta)}{4(3\alpha-1)}\alpha + \frac{1}{2}\varphi(s_1 - 1), \\ b_2(s_2) &= \frac{3}{2} - \frac{\varphi(1-\delta)}{2(3\alpha-1)}\alpha^2 + \frac{\varphi(1-\delta)}{2(3\alpha-1)}\alpha - \delta\varphi + \frac{1}{2}\delta\varphi s_2, \\ R &= \int_1^2 \left(\int_1^{z_1(1)} \underline{b}_2 ds_1 + \int_{z_1(1)}^{z_1(s_2)} b_2(s_2) ds_1 + \int_{z_1(s_2)}^{z_1(2)} b_1(s_1) ds_1 + \int_{z_1(2)}^{2} \bar{b}_1 ds_1 \right) ds_2. \end{aligned}$$

Part (a) follows by substituting z_1 and taking derivatives (and recalling that $\delta < 1$ and $\alpha \geq \frac{1}{2}$). Similarly, part (b) follows by taking derivatives of $\bar{b}_1, \underline{b}_2, b_1(s_1)$ and $b_2(s_2)$. Bidder 1's probability of winning is

$$\int_{\underline{t}}^{\bar{t}} (1 - F(z_1(s_2))) f(s_2) ds_2 = \frac{(2\alpha - 1) + \delta\alpha}{2(3\alpha - 1)}.$$

This is increasing in δ, which proves part (c). Finally, the total value created is

$$\int_1^2 \left(\int_1^{z_1(1)} v_2(s_1, s_2) ds_1 + \int_{z_1(1)}^{z_1(s_2)} v_2(s_1, s_2) ds_1 + \int_{z_1(s_2)}^{z_1(2)} v_1(s_1, s_2) ds_1 + \int_{z_1(2)}^{2} v_1(s_1, s_2) ds_1 \right) ds_2.$$

Substituting z_1 and taking derivatives proves part (d). ■

Proof of Proposition 6

Analogous to the proof of Proposition 5. ■

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