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Using bidder asymmetry to increase seller revenue

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Abstract

We construct the optimal selling mechanism in a pure common value environment with two bidders that are not equally well informed. With an optimal mechanism, the seller benefits from bidder asymmetry: her expected revenue increases if the bidder asymmetry increases.

Keywords: Auctions; Common value; Asymmetric bidders

JEL classification: D44; D82

1. Introduction

Studies of auctions in asymmetric environments (see e.g. Maskin and Riley, 2000) focus on the properties of 'standard' auction types. We go beyond these studies by analyzing the properties of an *optimal* selling mechanism. In particular, we analyze how bidder asymmetry affects the seller's expected revenue, if the seller can freely design the selling procedure (instead of using a specific auction type for exogenous reasons).

We analyze a simple common value environment with two bidders. A key variable in our analysis is the degree of bidder asymmetry: both bidders have some private information about the unknown value of the asset, but one bidder's estimate is more precise. Unlike earlier studies, we do not restrict our attention to the extreme case in which one of the bidders is either perfectly informed, or completely uninformed (our model allows for this special case, and also for symmetric bidders). We show that the seller benefits from bidder asymmetry, if she uses an optimal mechanism.

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2. The model

A seller owns an indivisible asset that can be sold to one of two bidders, $i, j \in \{1, 2\}$. All players are risk-neutral. Both bidders value the asset equally, but the value is unknown to them. The value of the asset comprises of two components, t_i, $i \in \{1, 2\}$, drawn independently from the same density function f, with support $[\underline{t}, \bar{t}]$, c.d.f. F and hazard rate $H(t_i) = f(t_i)/(1 - F(t_i))$. The full information value of the asset is $v(t_1, t_2) = \psi_1 t_1 + \psi_2 t_2$, a weighted sum of the two components. We assume that $\psi_1 \in [1/2, 1)$ and $\psi_2 = 1 - \psi_1$. Each bidder observes only one of the components. Specifically, bidder 1 observes t_1 and bidder 2 observes t_2. Assuming that $\psi_1 < 1$ ensures that both signals are informative.

We call bidder 1 the 'strong bidder' and bidder 2 the 'weak bidder', since bidder 1's signal is more informative in the following sense: $\mathrm{Var}(v|t_1) = \psi_2^2 < \psi_1^2 = \mathrm{Var}(v|t_2)$. The assumptions that the weights ψ_1 and ψ_2 add up to one and that the signals t_i are i.i.d. ensure that the expected value of the asset does not depend on ψ_1 and ψ_2, allowing us to examine the effect of bidder asymmetry on the seller's expected revenue, while keeping the ex-ante expected value constant (if $\psi_1 + \psi_2 \neq 1$, the results are similar; details are available from the authors).

We assume that the seller's valuation of the asset is zero, and that her only goal is to maximize expected revenue. We assume that the lower bound $\underline{t}$ of the signals' support is sufficiently high, such that imposing a reserve price will turn out to be sub-optimal; a sufficient condition is that $\underline{t}H(\underline{t}) \geq \psi_1$. We also assume that the hazard rate H is increasing.

3. The optimal mechanism

From the revelation principle, we can restrict attention to direct mechanisms. Denote by $p_i(t_1, t_2)$ the probability with which the optimal mechanism allocates the asset to bidder i, given reported signal realizations t_1 and t_2, and let $x_i(t_1, t_2)$ be the corresponding payment from bidder i to the seller. Let $Q_i(t_i)$ be bidder i's probability of winning, if his signal is t_i; his expected payoff, conditional on signal t_i and announcement $\hat{t}_i$, is

$$U_i(\hat{t}_i|t_i) \equiv \int_{\underline{t}}^{\bar{t}} (v(t_i, t_j)p_i(\hat{t}_i, t_j) - x_i(\hat{t}_i, t_j))f(t_j)\mathrm{d}t_j,$$

and his truthtelling payoff is $V_i(t_i) \equiv U_i(t_i|t_i)$. The optimal mechanism solves the following problem:

$$\max_{x_1, x_2 \in \mathbb{R}, p_1, p_2 \in [0,1]} \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{\bar{t}} (x_1(t_1, t_2) + x_2(t_1, t_2))f(t_1)\mathrm{d}t_1 f(t_2)\mathrm{d}t_2 \tag{1}$$

$$\text{s.t.} \qquad V_i(t_i) \geq 0 \qquad \forall t_i, \ i = 1, 2 \tag{2}$$

$$V_i(t_i) \geq U_i(\hat{t}_i|t_i) \qquad \forall \hat{t}_i, \ \forall t_i, \ i = 1, 2 \tag{3}$$

and such that the probabilities $p_i(t_1, t_2)$ satisfy standard conditions. (1) is the seller's expected revenue, (2) is bidder i's participation constraint, (3) is his truthtelling constraint, and the last two conditions are the feasibility constraints. This can be transformed into a more tractable problem.

Lemma 1. *The truthtelling constraint (3) is satisfied iff* $\frac{\partial V_i(t_i)}{\partial t_i} = \psi_i Q_i(t_i)$ *and* $\frac{\partial Q_i(t_i)}{\partial t_i} \geq 0$.

Proof 1. The proof is adapted from Myerson (1981) and therefore omitted. □

Lemma 1 allows us to substitute (3) by the requirements that

$$V_i(t_i) = V_i(\underline{t}) + \psi_i \int_{\underline{t}}^{t_i} Q_i(s_i)\mathrm{d}s_i \qquad \text{for } i = 1,2 \tag{4}$$

and $Q_i'(t_i) \geq 0$ for $i = 1, 2$; and (2) by the requirement that $V_i(\underline{t}) \geq 0$ for $i = 1, 2$. Substituting for $x_i(t_1, t_2)$ using (4) allows us to rewrite (1) as

$$\max_{p_i, V_i(\underline{t})} \sum_{i=1,2} -V_i(\underline{t}) + \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{\bar{t}} \left[v(t_1, t_2) - \frac{\psi_i}{H(t_i)} \right] p_i(t_1, t_2) f(t_1)\mathrm{d}t_1 f(t_2)\mathrm{d}t_2. \tag{5}$$

Lemma 2. *The optimal mechanism sets* $V_1(\underline{t}) = V_2(\underline{t}) = 0$ *and* $p_1(t_1, t_2) = 1 - p_2(t_1, t_2) = 1$ *if and only if* $\frac{\psi_1}{H(t_1)} \leq \frac{\psi_2}{H(t_2)}$ *(and zero otherwise).*

Proof 2. Clearly, $V_i(\underline{t}) > 0$ is sub-optimal, and setting $V_i(\underline{t}) < 0$ violates (2). Since $\underline{t}H(\underline{t}) \geq \psi_1$ and $H'(t_i) > 0$, the term in square brackets in (5) is positive for all (t_1, t_2). Thus, it is sub-optimal to have $p_1(t_1, t_2) + p_2(t_1, t_2) < 1$ (no reserve price). $H'(t_i) > 0$ also implies $Q_i'(t_i) \geq 0$, satisfying $Q_i'(t_i) \geq 0$ for $i = 1, 2$. The optimal allocation rule is found by comparing the term in square brackets in (5) for a given (t_1, t_2): set $p_1(t_1, t_2) = 1$ iff $\left(v(t_1, t_2) - \frac{\psi_1}{H(t_1)}\right) \geq \left(v(t_1, t_2) - \frac{\psi_2}{H(t_2)}\right)$. □

The optimal mechanism is biased against the strong bidder, since he wins the asset only if his signal is sufficiently higher than the weak bidder's. Consider the case in which both bidders received the same signal, t. Under the optimal allocation rule described in Lemma 2, bidder 2 wins with certainty in this case: the hazard rate is the same for both bidders, and by assumption, $\psi_1 > \psi_2$. Thus, a necessary condition for bidder 1 to win is that his signal is strictly higher than that of bidder 2.

This bias is an intuitive result, which has a counterpart in the analysis of asymmetric private value auctions. For sufficiently low signal reports, the strong bidder's chances of winning are nil: if $\frac{\psi_1}{H(t_1)} \geq \frac{\psi_2}{H(\underline{t})}$ then $p_1(t_1, t_2) = 0$. Nevertheless, the optimal mechanism does not categorically exclude the strong bidder from bidding; it just ignores low announcements, encouraging the strong bidder to reveal high signals (and then pay high transfers).

Biasing the mechanism against the strong bidder increases competition between the bidders, by forcing the strong bidder to submit high bids if he wants to win. One would expect this bias to somewhat mitigate the adverse effect that increased bidder asymmetry has on the seller's expected revenue. In fact, the optimal mechanism achieves more than that:

Theorem 3. *The seller's expected revenue is strictly increasing in* ψ_1.

Proof 3. Consider the optimal mechanism for a given ψ_1. The seller's expected revenue is given by

$$E[v(t_1, t_2)] - \sum_{i=1,2} \int_{\underline{t}}^{\bar{t}} V_i(t_i) f(t_i)\mathrm{d}t_i = E[v(t_1, t_2)] - \sum_{i=1,2} \psi_i \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{t_i} Q_i(t)\mathrm{d}t f(t_i)\mathrm{d}t_i.$$

For ε satisfying $0<\varepsilon<\psi_2$, this is strictly less than

$$E[v(t_1,t_2)] - \sum_{i=1,2}\psi_i \int_{\underline{t}}^{\bar{t}}\int_{\underline{t}}^{t_i} Q_i(t)\mathrm{d}tf(t_i)\mathrm{d}t_i + \varepsilon\int_{\underline{t}}^{\bar{t}}\int_{\underline{t}}^{t}(Q_2(s)-Q_1(s))\mathrm{d}sf(t)\mathrm{d}t, \tag{6}$$

since for a given signal t, the strong bidder's probability of winning is smaller than the weak bidder's,

$$Q_1(t) = F\left(H^{-1}\left(\frac{\psi_2}{\psi_1}H(t)\right)\right) < F\left(H^{-1}\left(\frac{\psi_1}{\psi_2}H(t)\right)\right) = Q_2(t) \qquad \forall t.$$

(This follows directly from $\psi_1>\psi_2$, $H'(\cdot)>0$ and $F'(\cdot)>0$.) We can rewrite (6) as

$$E[v(t_1,t_2)] - (\psi_1+\varepsilon)\int_{\underline{t}}^{\bar{t}}\int_{\underline{t}}^{t_1} Q_1(s_1)\mathrm{d}s_1f(t_1)\mathrm{d}t_1 - (\psi_2-\varepsilon)\int_{\underline{t}}^{\bar{t}}\int_{\underline{t}}^{t_2} Q_2(s_2)\mathrm{d}s_2f(t_2)\mathrm{d}t_2.$$

This term describes the seller's expected revenue if the true parameter is $\psi_1+\varepsilon$, but she uses the allocation rule that would be optimal if the true parameter was $\psi_1<\psi_1+\varepsilon$ (while maintaining incentive compatibility: the transfers $x_i(t_1, t_2)$ are implicitly determined by (4)). Thus, expected revenue can be increased if ψ_1 increases, without changing the allocation rule. By switching to the optimal allocation rule, the seller may additionally increase expected revenue. □

The seller uses bidder asymmetry to her advantage in a way that resembles price discrimination. The higher is ψ_1, the higher the asset's value if the strong bidder's signal is high. With high strong bidder signals, the seller can extract most of the bidders' rents by selling to the strong bidder and requiring a high transfer. For incentive reasons, the mechanism must threaten not to sell to the strong bidder if he reveals a low signal. This is the same threat that makes reserve prices effective, but the difference is that instead of, say, destroying the asset, the seller can instead sell it to the weak bidder. The weak bidder's willingness to pay is reduced if he wins the asset only because the strong bidder revealed a high signal; but this reduction in expected transfers is outweighed by the increased rent extraction from the strong bidder. Notice that bidder asymmetry is necessary for this type of bidder discrimination to be optimal: if the bidders' signals are equally informative, it is optimal to treat bidders symmetrically.

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