Internet Appendix for

Competition for Talent under Performance Manipulation

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A Proofs and Derivations

Contents

A.1 Cost of Misreporting Depends on $(r-y)$ Instead of $(r-q)$

The cost of misreporting could be a function of $(r-y)^2$ instead of $(r-q)^2$. One reason is that y is observable (eventually), while q is not. Such a cost would be uncertain, since the realization of y is uncertain. We could model the cost of misreporting as $\frac{\kappa_1}{2}(r-E[y])^2 + \frac{\kappa_2}{2}E\left[(r-y)^2\right]$ (the first term can easily be omitted), which can be rewritten as

$$
\begin{aligned}
&\frac{\kappa_1}{2}(r-q)^2 + \frac{\kappa_2}{2}\int_{-\infty}^{\infty}(r-q-\varepsilon)^2 f(\varepsilon)\,d\varepsilon \\
&= \frac{\kappa_1}{2}(r-q)^2 + \frac{\kappa_2}{2}\left(\int_{-\infty}^{\infty}(r-q)^2 f(\varepsilon)\,d\varepsilon - \int_{-\infty}^{\infty}(2(r-q)\varepsilon) f(\varepsilon)\,d\varepsilon + \int_{-\infty}^{\infty}\varepsilon^2 f(\varepsilon)\,d\varepsilon\right) \\
&= \frac{\kappa_1}{2}(r-q)^2 + \frac{\kappa_2}{2}\left((r-q)^2 - 2(r-q)E[\varepsilon] + \int_{-\infty}^{\infty}\varepsilon^2 f(\varepsilon)\,d\right) \\
&= \frac{\kappa_1}{2}(r-q)^2 + \frac{\kappa_2}{2}\left((r-q)^2 - 0 + \sigma_\varepsilon^2\right) \\
&= \frac{\kappa_1+\kappa_2}{2}(r-q)^2 + \frac{\kappa_2}{2}\sigma_\varepsilon^2.
\end{aligned}
$$

Now consider a compensation contract $\widehat{w} = (\widehat{\alpha}, \widehat{\beta}, \widehat{\delta})$ such that $\widehat{\alpha} = \alpha + \frac{\kappa_2}{2}\sigma^2$; $\widehat{\beta} = \beta + \xi$; $\widehat{\delta} = \delta - \xi$; and $\kappa_1 + \kappa_2 = c$. The CEO's expected payoff would be

$$
\begin{aligned}
&E\left[\widehat{\alpha} + \widehat{\beta}r + \widehat{\delta}y - \xi(r-y)\right] - \frac{\rho}{2}Var\left(\widehat{\alpha} + \widehat{\beta}r + \widehat{\delta}y - \xi(r-y)\right) - \frac{g}{2}L^2 - \frac{\kappa_1}{2}(r-E[y])^2 - \frac{\kappa_2}{2}E\left[(r-y)^2\right] \\
&= E\left[\widehat{\alpha} + \widehat{\beta}r + \widehat{\delta}y - \xi(r-y)\right] - \frac{\rho}{2}Var\left(\widehat{\alpha} + \widehat{\beta}r + \widehat{\delta}y - \xi(r-y)\right) - \frac{g}{2}L^2 - \frac{\kappa_1+\kappa_2}{2}(r-q)^2 - \frac{\kappa_2}{2}\sigma^2 \\
&= E[\widehat{\alpha} + \beta r + \delta y] - \frac{\rho}{2}Var(\widehat{\alpha} + \beta r + \delta y) - \frac{g}{2}L^2 - \frac{\kappa_1+\kappa_2}{2}(r-q)^2 - \frac{\kappa_2}{2}\sigma^2 \\
&= E\left[\widehat{\alpha} - \frac{\kappa_2}{2}\sigma^2 + \beta r + \delta y\right] - \frac{\rho}{2}Var(\widehat{\alpha} + \beta r + \delta y) - \frac{g}{2}L^2 - \frac{\kappa_1+\kappa_2}{2}(r-q)^2 \\
&= \widehat{\alpha} - \frac{\kappa_2}{2}\sigma^2 + \beta r + \delta \cdot E[y] - \frac{\rho}{2}\delta^2\sigma^2 - \frac{g}{2}L^2 - \frac{\kappa_1+\kappa_2}{2}(r-q)^2 \\
&= \alpha + \beta r + \delta E[y] - \frac{\rho}{2}\sigma^2\delta - \frac{g}{2}L^2 - \frac{c}{2}(r-q)^2,
\end{aligned}
$$

which is the payoff described in equation (4). Thus, our assumption that the cost of misreporting depends on $(r-q)$ is equivalent to assuming that it depends on $(r-y)$.

A.2 Derivation of the Incentive Constraints, (10) and (11)

The incentive constraints are

$$U(\tau_h, w_h) \geq U(\tau_h, w_\ell) \tag{A1}$$

$$U(\tau_\ell, w_\ell) \geq U(\tau_\ell, w_h). \tag{A2}$$

Using (5),

$$U(\tau_h, w_h) \geq \alpha_\ell + \frac{\beta_\ell^2}{2c} - \frac{\rho\sigma^2}{2}\delta_\ell^2 + \frac{(\beta_\ell + \delta_\ell)^2}{2g}\tau_h^2 \tag{A3}$$

$$U(\tau_\ell, w_\ell) \geq \alpha_h + \frac{\beta_h^2}{2c} - \frac{\rho\sigma^2}{2}\delta_h^2 + \frac{(\beta_h + \delta_h)^2}{2g}\tau_\ell^2. \tag{A4}$$

Using (8), (9), and (5),

$$\begin{aligned} U(\tau_h, w_h) \geq & U(\tau_\ell, w_\ell) - \left(\alpha_\ell + \frac{\beta_\ell^2}{2c} - \frac{\rho\sigma^2}{2}\delta_\ell^2 + \frac{(\beta_\ell + \delta_\ell)^2}{2g}\tau_\ell^2\right) \\ & + \left(\alpha_\ell + \frac{\beta_\ell^2}{2c} - \frac{\rho\sigma^2}{2}\delta_\ell^2 + \frac{(\beta_\ell + \delta_\ell)^2}{2g}\tau_h^2\right) \end{aligned} \tag{A5}$$

$$\begin{aligned} U(\tau_\ell, w_\ell) \geq & U(\tau_h, w_h) - \left(\alpha_h + \frac{\beta_h^2}{2c} - \frac{\rho\sigma^2}{2}\delta_h^2 + \frac{(\beta_h + \delta_h)^2}{2g}\tau_h^2\right) \\ & + \left(\alpha_h + \frac{\beta_h^2}{2c} - \frac{\rho\sigma^2}{2}\delta_h^2 + \frac{(\beta_h + \delta_h)^2}{2g}\tau_\ell^2\right). \end{aligned} \tag{A6}$$

Simplify, to obtain (10) and (11).

A.3 Proof of Lemma 1

The benevolent planner's objective function is (3). The first-order conditions are

$$\frac{\partial}{\partial \beta_i} S(\tau_i, w) = -\frac{1}{cg}\left(g\beta_i - c\tau_i^2 + c\beta_i\tau_i^2 + c\delta_i\tau_i^2\right) = 0$$

$$\frac{\partial}{\partial \delta_i} S(\tau_i, w) = -\frac{1}{g}\left(\beta_i\tau_i^2 - \tau_i^2 + \delta_i\tau_i^2 + g\sigma^2\delta_i\rho\right) = 0$$

Combine the two first-order conditions and solve for β and δ, to obtain β_i^* and δ_i^*.

The lowest-rent implementation sets α_ℓ such that the low-talent participation constraint (9) is binding, which can be solved for α_ℓ^*,

$$\alpha_\ell^* = \left(\frac{\rho\sigma^2}{2} - \frac{\left(c\sigma^2\rho\right)^2}{2c} - \frac{\left(c\sigma^2\rho+1\right)^2\tau_\ell^2}{2g}\right)(\delta_\ell^*)^2. \tag{A7}$$

With $U(\tau_\ell, w_\ell^*) = 0$, the high-talent incentive constraint (10) implies that the high-talent participation constraint is not binding, so it can be ignored. The incentive constraints (10) and (11) can be rewritten as (replacing $U(\tau_\ell, w_\ell^*) = 0$)

$$\begin{aligned} U(\tau_h, w_h^*) &\geq \frac{\left(c\sigma^2\rho+1\right)^2 \cdot (\delta_\ell^*)^2}{2g}\left(\tau_h^2 - \tau_\ell^2\right) \\ 0 &\geq U(\tau_h, w_h^*) - \frac{\left(c\sigma^2\rho+1\right)^2 \cdot (\delta_h^*)^2}{2g}\left(\tau_h^2 - \tau_\ell^2\right). \end{aligned}$$

Substitute $U(\tau_h, w_h^*)$ using (5), and isolate α_h,

$$\begin{aligned} \alpha_h^* &\geq \frac{\left(c\sigma^2\rho+1\right)^2 \cdot (\delta_\ell^*)^2}{2g}\left(\tau_h^2 - \tau_\ell^2\right) - \left(\frac{\left(c\rho\sigma^2\right)^2}{2c}(\delta_h^*)^2 - \frac{\rho\sigma^2}{2}(\delta_h^*)^2 + \frac{\left(c\rho\sigma^2+1\right)^2}{2g}(\delta_h^*)^2\tau_h^2\right) \\ \alpha_h^* &\leq \frac{\left(c\sigma^2\rho+1\right)^2 \cdot (\delta_h^*)^2}{2g}\left(\tau_h^2 - \tau_\ell^2\right) - \left(\frac{\left(c\rho\sigma^2\right)^2}{2c}(\delta_h^*)^2 - \frac{\rho\sigma^2}{2}(\delta_h^*)^2 + \frac{\left(c\rho\sigma^2+1\right)^2}{2g}(\delta_h^*)^2\tau_h^2\right) \end{aligned}$$

The lowest value of α_h that satisfies the conditions is

$$\alpha_h^* = \frac{\left(c\sigma^2\rho+1\right)^2\left(\tau_h^2 - \tau_\ell^2\right)}{2g}(\delta_\ell^*)^2 - \left(\frac{\left(c\sigma^2\rho\right)^2}{2c} - \frac{\rho\sigma^2}{2} + \frac{\left(c\sigma^2\rho+1\right)^2\tau_h^2}{2g}\right)(\delta_h^*)^2. \tag{A8}$$

The high-talent incentive constraint is binding, while the low-talent incentive constraint is slack. The surplus generated by a low-talent CEO is (using (3))

$$\begin{aligned} S(\tau_\ell, w_\ell^*) &= \frac{\tau_\ell^2}{g}\left(c\sigma^2\rho+1\right)\delta_\ell^* - \frac{\tau_\ell^2}{2g}\left(c\sigma^2\rho+1\right)^2 \cdot (\delta_\ell^*)^2 - \frac{\rho\sigma^2}{2}(\delta_\ell^*)^2 - \frac{\left(c\sigma^2\rho\right)^2}{2c}(\delta_\ell^*)^2 \\ &= \frac{\tau_\ell^2}{g}\left(c\sigma^2\rho+1\right)\delta_\ell^* - \frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2 + g\sigma^2\rho}{\tau_\ell^2}\frac{\tau_\ell^2}{2g}\left(c\rho\sigma^2+1\right)\cdot(\delta_\ell^*)^2 \end{aligned} \tag{A9}$$

The large fraction equals $\frac{1}{\delta_i^*}$; replace, and simplify, to obtain

$$S(\tau_\ell, w_\ell^*) = \frac{\tau_\ell^2}{2g}\left(c\rho\sigma^2+1\right)\delta_\ell^*. \tag{A10}$$

The surplus is thus strictly positive if a low-talent CEO is hired. Since with $\alpha_\ell = \alpha_\ell^*$ a low-talent CEO's expected net payoff is zero, this implies that the firm's profit is strictly positive if it hires a low-talent CEO. Similarly, a high-talent CEO's surplus under the contract w_h^* is

$$S(\tau_h, w_h^*) = \frac{\tau_h^2}{2g}\left(c\rho\sigma^2 + 1\right)\delta_h^*. \tag{A11}$$

Since $\delta_h^* > \delta_\ell^*$ and $\tau_h > \tau_\ell$, this implies that $S(\tau_h, w_h^*) > S(\tau_\ell, w_\ell^*)$, so a benevolent planner wants both CEO types to be active.

A high-talent CEO's expected payoff is (using $\beta_i^* = c\sigma^2\rho \cdot \delta_i^*$, (5), (12) and (A8))

$$\begin{aligned}
U(\tau_h, w_h^*) =& \frac{\left(c\sigma^2\rho + 1\right)^2\left(\tau_h^2 - \tau_\ell^2\right)}{2g}\left(\delta_\ell^*\right)^2 - \left(\frac{\left(c\sigma^2\rho\right)^2}{2c} - \frac{\rho\sigma^2}{2} + \frac{\left(c\sigma^2\rho + 1\right)^2}{2g}\tau_h^2\right)\left(\delta_h^*\right)^2 \\
&+ \left(\frac{\left(c\rho\sigma^2\right)^2}{2c} - \frac{\rho\sigma^2}{2} + \frac{\left(c\rho\sigma^2 + 1\right)^2}{2g}\tau_h^2\right)\left(\delta_h^*\right)^2 \\
=& \frac{\left(c\sigma^2\rho + 1\right)^2\left(\tau_h^2 - \tau_\ell^2\right)}{2g}\left(\delta_\ell^*\right)^2
\end{aligned}$$

A high-talent CEO's expected payoff is smaller than the surplus she generates (cf. (A11)), and thus the firm's profit is positive, if

$$\begin{aligned}
\frac{\tau_h^2}{2g}\left(c\rho\sigma^2 + 1\right)\delta_h^* &\geq \frac{\left(c\sigma^2\rho + 1\right)^2\left(\tau_h^2 - \tau_\ell^2\right)}{2g}\left(\delta_\ell^*\right)^2 \\
\frac{1}{c\sigma^2\rho + 1}\frac{\tau_h^2}{\tau_h^2 - \tau_\ell^2}\delta_h^* &\geq \left(\delta_\ell^*\right)^2.
\end{aligned}$$

That is satisfied, since

$$\frac{1}{c\sigma^2\rho + 1}\frac{\tau_h^2}{\tau_h^2 - \tau_\ell^2}\delta_h^* > \frac{\tau_h^2}{\left(c\sigma^2\rho + 1\right)\tau_h^2 + g\sigma^2\rho}\delta_h^* = \left(\delta_h^*\right)^2 > \left(\delta_\ell^*\right)^2.$$

So the firm's profit is positive if it hires a high-talent CEO.

There exists a continuum of contracts that implement the efficient outcome. They are found by increasing α_ℓ and α_h such that the ICs remain satisfied. The upper bound to the set of efficient contracts is where a low-talent CEO extracts all the surplus she generates. That is the case if her expected payoff (cf. (5)),

$$U(\tau_\ell, w_\ell^{**}) = \alpha_\ell^{**} + \frac{\left(c\rho\sigma^2\right)^2}{2c}\left(\delta_\ell^*\right)^2 - \frac{\rho\sigma^2}{2}\left(\delta_\ell^*\right)^2 + \frac{\left(c\rho\sigma^2+1\right)^2}{2g}\left(\delta_\ell^*\right)^2\tau_\ell^2,$$

equals the surplus she generates (using $S(\tau_\ell, w_\ell^*)$ as described in (A10)),

$$S(\tau_\ell, w_\ell^*) = \frac{\tau_\ell^2}{2g}\left(c\rho\sigma^2+1\right)\delta_\ell^*, \tag{A12}$$

and solving for α_ℓ^{**} yields

$$\alpha_\ell^{**} = \frac{\tau_\ell^2}{2g}\left(c\rho\sigma^2+1\right)\delta_\ell^* - \left(\frac{\left(c\rho\sigma^2\right)^2}{2c}\left(\delta_\ell^*\right)^2 - \frac{\rho\sigma^2}{2}\left(\delta_\ell^*\right)^2 + \frac{\left(c\rho\sigma^2+1\right)^2}{2g}\left(\delta_\ell^*\right)^2\tau_\ell^2\right)$$

Expand the first term, using (12),

$$\alpha_\ell^{**} = \frac{\tau_\ell^2}{2g}\left(c\rho\sigma^2+1\right)\delta_\ell^*\frac{\delta_\ell^*}{\left(\frac{\tau_\ell^2}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}\right)} - \left(\frac{\left(c\rho\sigma^2\right)^2}{2c}\left(\delta_\ell^*\right)^2 - \frac{\rho\sigma^2}{2}\left(\delta_\ell^*\right)^2 + \frac{\left(c\rho\sigma^2+1\right)^2}{2g}\left(\delta_\ell^*\right)^2\tau_\ell^2\right)$$

and simplify to obtain

$$\alpha_\ell^{**} = \rho\sigma^2\cdot\left(\delta_\ell^*\right)^2. \tag{A13}$$

(The firm's profit then equals zero.) Incentive compatibility is maintained if α_h is increased, such that $\alpha_h^{**} \geq \alpha_h^* + (\alpha_\ell^{**} - \alpha_\ell^*)$ without violating the low-talent incentive constraint, (11). The highest feasible α_h^{**} is thus defined by $\alpha_\ell = \alpha_\ell^{**}$ and a binding incentive constraint for a low-talent CEO,

$$U(\tau_\ell, w_\ell^{**}) = U(\tau_h, w_h^{**}) - \frac{\left(c\sigma^2\rho+1\right)^2}{2g}\cdot\left(\delta_h^*\right)^2\left(\tau_h^2-\tau_\ell^2\right)$$

(where $w_i^{**} = (\alpha_i^{**}, \beta_i^*, \delta_i^*)$). Replace $U(\tau_\ell, w_\ell^{**}) = S(\tau_\ell, w_\ell^{**})$,

$$S(\tau_\ell, w_\ell^{**}) = U(\tau_h, w_h^{**}) - \frac{\left(c\sigma^2\rho+1\right)^2}{2g}\cdot\left(\delta_h^*\right)^2\left(\tau_h^2-\tau_\ell^2\right),$$

and replace both $U(\tau_h, w_h^{**})$ and $S(\tau_\ell, w_\ell^{**})$, to obtain

$$\frac{\tau_\ell^2}{2g}\left(c\rho\sigma^2+1\right)\delta_\ell^* = \alpha_h^{**} + \frac{\left(c\rho\sigma^2\right)^2}{2c}\left(\delta_h^*\right)^2 - \frac{\rho\sigma^2}{2}\left(\delta_h^*\right)^2 + \frac{\left(c\rho\sigma^2+1\right)^2}{2g}\left(\delta_h^*\right)^2\tau_h^2 - \frac{\left(c\sigma^2\rho+1\right)^2}{2g}\cdot\left(\delta_h^*\right)^2\left(\tau_h^2-\tau_\ell^2\right).$$

Solve for α_h^{**}, to obtain

$$\alpha_h^{**} = \frac{\tau_\ell^2}{2g}\left(c\rho\sigma^2+1\right)\delta_\ell^* - \left(\frac{\left(c\rho\sigma^2\right)^2}{2c} - \frac{\rho\sigma^2}{2} + \frac{\left(c\rho\sigma^2+1\right)^2\tau_\ell^2}{2g}\right)\left(\delta_h^*\right)^2. \tag{A14}$$

A high-talent CEO's expected payoff is smaller than the surplus she generates if

$$\begin{aligned}
S(\tau_h, w_h^{**}) &> U(\tau_h, w_h^{**}) \\
\frac{\tau_h^2}{2g}\left(c\rho\sigma^2+1\right)\delta_h^* &> \alpha_h^{**} + \frac{\left(c\rho\sigma^2\right)^2}{2c}\left(\delta_h^*\right)^2 - \frac{\rho\sigma^2}{2}\left(\delta_h^*\right)^2 + \frac{\left(c\rho\sigma^2+1\right)^2}{2g}\left(\delta_h^*\right)^2\tau_h^2
\end{aligned}$$

$$\begin{aligned}
\frac{\tau_h^2}{2g}\left(c\rho\sigma^2+1\right)\delta_h^* >& \frac{\tau_\ell^2}{2g}\left(c\rho\sigma^2+1\right)\delta_\ell^* - \left(\frac{\left(c\rho\sigma^2\right)^2}{2c} - \frac{\rho\sigma^2}{2} + \frac{\left(c\rho\sigma^2+1\right)^2}{2g}\tau_\ell^2\right)\left(\delta_h^*\right)^2 \\
&+ \frac{\left(c\rho\sigma^2\right)^2}{2c}\left(\delta_h^*\right)^2 - \frac{\rho\sigma^2}{2}\left(\delta_h^*\right)^2 + \frac{\left(c\rho\sigma^2+1\right)^2}{2g}\left(\delta_h^*\right)^2\tau_h^2
\end{aligned}$$

$$\frac{\tau_h^2}{2g}\left(c\rho\sigma^2+1\right)\delta_h^* > \frac{\tau_\ell^2}{2g}\left(c\rho\sigma^2+1\right)\delta_\ell^* + \left(\frac{\left(c\rho\sigma^2+1\right)^2\left(\tau_h^2-\tau_\ell^2\right)}{2g}\right)\left(\delta_h^*\right)^2$$

$$\tau_h^2\delta_h^* > \tau_\ell^2\delta_\ell^* + \left(c\rho\sigma^2+1\right)\left(\tau_h^2-\tau_\ell^2\right)\left(\delta_h^*\right)^2$$

$$\tau_h^2\frac{\tau_h^2}{\left(c\sigma^2\rho+1\right)\tau_h^2+g\sigma^2\rho} > \tau_\ell^2\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho} + \left(c\rho\sigma^2+1\right)\left(\tau_h^2-\tau_\ell^2\right)\left(\frac{\tau_h^2}{\left(c\sigma^2\rho+1\right)\tau_h^2+g\sigma^2\rho}\right)^2$$

$$\tau_h^2\frac{\tau_h^2}{\left(c\sigma^2\rho+1\right)\tau_h^2+g\sigma^2\rho} - \tau_\ell^2\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho} - \left(c\rho\sigma^2+1\right)\left(\tau_h^2-\tau_\ell^2\right)\left(\frac{\tau_h^2}{\left(c\sigma^2\rho+1\right)\tau_h^2+g\sigma^2\rho}\right)^2 > 0$$

$$g\sigma^2\rho\left(\tau_h^2-\tau_\ell^2\right)\frac{2\left(c\sigma^2\rho+1\right)\tau_h^2\tau_\ell^2+\left(\tau_h^2+\tau_\ell^2\right)g\sigma^2\rho}{\left(\left(c\sigma^2\rho+1\right)\tau_h^2+g\sigma^2\rho\right)^2\left(\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho\right)} > 0$$

That is satisfied, so the firm's profit is positive if it hires a high-talent CEO.

A.4 Properties of Efficient Contracts

$$\frac{\partial}{\partial c}\delta_i^* = \frac{\partial}{\partial c}\frac{\tau_i^2}{\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho} = -\frac{\tau_i^4\sigma^2\rho}{\left(\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho\right)^2} < 0$$

$$\frac{\partial}{\partial c}\beta_i^* = \frac{\partial}{\partial c}c\sigma^2\rho\frac{\tau_i^2}{\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho} = \frac{\sigma^2\rho\tau_i^2\left(\tau_i^2+g\sigma^2\rho\right)}{\left(\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho\right)^2} > 0$$

$$\frac{\partial}{\partial\sigma^2\rho}\delta_i^* = \frac{\partial}{\partial\sigma^2\rho}\frac{\tau_i^2}{\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho} = -\frac{\tau_i^2\left(c\tau_i^2+g\right)}{\left(\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho\right)^2} < 0$$

$$\frac{\partial}{\partial\sigma^2\rho}\beta_i^* = \frac{\partial}{\partial\sigma^2\rho}c\sigma^2\rho\frac{\tau_i^2}{\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho} = \frac{c\tau_i^4}{\left(\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho\right)^2} > 0$$

$$\frac{\partial}{\partial\tau_i^2}\delta_i^* = \frac{\partial}{\partial\tau_i^2}\frac{\tau_i^2}{\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho} = \frac{g\sigma^2\rho}{\left(\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho\right)^2} > 0$$

$$\frac{\partial}{\partial\tau_i^2}\beta_i^* = \frac{\partial}{\partial\tau_i^2}c\sigma^2\rho\frac{\tau_i^2}{\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho} = \frac{c\sigma^4\rho^2 g}{\left(\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho\right)^2} > 0$$

$$\frac{\partial}{\partial c}L(\tau_i,w_i^*) = \frac{\partial}{\partial c}\left(c\sigma^2\rho+1\right)\frac{1}{g}\tau_i\frac{\tau_i^2}{\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho} = \frac{\sigma^4\rho^2\tau_i^3}{\left(\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho\right)^2}$$

$$\frac{\partial}{\partial\sigma^2\rho}L(\tau_i,w_i^*) = \frac{\partial}{\partial\sigma^2\rho}\left(c\sigma^2\rho+1\right)\frac{1}{g}\tau_i\frac{\tau_i^2}{\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho} = -\frac{\tau_i^3}{\left(\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho\right)^2}$$

$$\frac{\partial}{\partial\tau_i}L(\tau_i,w_i^*) = \frac{\partial}{\partial\tau_i}\left(c\sigma^2\rho+1\right)\frac{1}{g}\tau_i\frac{\tau_i^2}{\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho} = \frac{\left(c\sigma^2\rho+1\right)\tau_i^2}{g}\frac{\left(c\sigma^2\rho+1\right)\tau_i^2+3g\sigma^2\rho}{\left(\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho\right)^2}$$

$$\lim_{c\to0}\beta_i^* = \lim_{c\to0}c\sigma^2\rho\cdot\frac{\tau_i^2}{\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho} = 0$$

$$\lim_{c\to0}\delta_i^* = \lim_{c\to0}\frac{\tau_i^2}{\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho} = \frac{\tau_i^2}{\tau_i^2+g\sigma^2\rho}$$

$$\lim_{c\to0}L_i^*(\tau_i,w_i^*) = \lim_{c\to0}\left(c\sigma^2\rho+1\right)\frac{1}{g}\tau_i\frac{\tau_i^2}{\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho} = \frac{1}{g}\frac{\tau_i^3}{\tau_i^2+g\sigma^2\rho}$$

$$\lim_{c\to\infty}\beta_i^* = \lim_{c\to\infty} c\sigma^2\rho\cdot\frac{\tau_i^2}{\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho} = 1$$
$$\lim_{c\to\infty}\delta_i^* = \lim_{c\to\infty}\frac{\tau_i^2}{\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho} = 0$$
$$\lim_{c\to\infty}L(\tau_i,w_i^*) = \lim_{c\to\infty}\left(c\sigma^2\rho+1\right)\frac{1}{g}\tau_i\frac{\tau_i^2}{\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho} = \frac{1}{g}\tau_i$$

$$\lim_{\sigma^2\rho\to 0}\beta_i^* = \lim_{\sigma^2\rho\to 0} c\sigma^2\rho\cdot\frac{\tau_i^2}{\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho} = 0$$
$$\lim_{\sigma^2\rho\to 0}\delta_i^* = \lim_{\sigma^2\rho\to 0}\frac{\tau_i^2}{\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho} = 1$$
$$\lim_{\sigma^2\rho\to 0}L(\tau_i,w_i^*) = \lim_{\sigma^2\rho\to 0}\left(c\sigma^2\rho+1\right)\frac{1}{g}\tau_i\frac{\tau_i^2}{\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho} = \frac{1}{g}\tau_i$$

$$\lim_{\sigma^2\rho\to\infty}\beta_i^* = \lim_{\sigma^2\rho\to\infty} c\sigma^2\rho\cdot\frac{\tau_i^2}{\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho} = \frac{c\tau_i^2}{c\tau_i^2+g}$$
$$\lim_{\sigma^2\rho\to\infty}\delta_i^* = \lim_{\sigma^2\rho\to\infty}\frac{\tau_i^2}{\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho} = 0$$
$$\lim_{\sigma^2\rho\to\infty}L(\tau_i,w_i^*) = \lim_{\sigma^2\rho\to\infty}\left(c\sigma^2\rho+1\right)\frac{1}{g}\tau_i\frac{\tau_i^2}{\left(c\sigma^2\rho+1\right)\tau_i^2+g\sigma^2\rho} = \frac{1}{g}\frac{c\tau_i^3}{c\tau_i^2+g}$$

A.5 Additive Effort and Talent

Assume that $y = KL + k\tau_i + \varepsilon$, i.e., effort and talent have an additive effect on performance instead of having a multiplicative effect. The CEO's indirect utility function is then

$$u = \alpha + \beta r + \delta\left(KL + k\tau_i\right) - \frac{\rho\sigma^2}{2}\delta^2 - \frac{g}{2}L^2 - \frac{c}{2}\left(r - \left(KL + k\tau_i\right)\right)^2.$$

To find the optimal choices of L and r, given a contract w, solve

$$0 = \frac{\partial}{\partial L}\left(\alpha + \beta r + \delta\left(KL + k\tau_i\right) - \frac{\rho\sigma^2}{2}\delta^2 - \frac{g}{2}L^2 - \frac{c}{2}\left(r - \left(KL + k\tau_i\right)\right)^2\right)$$
$$0 = \frac{\partial}{\partial r}\left(\alpha + \beta r + \delta\left(KL + k\tau_i\right) - \frac{\rho\sigma^2}{2}\delta^2 - \frac{g}{2}L^2 - \frac{c}{2}\left(r - \left(KL + k\tau_i\right)\right)^2\right)$$

$$0 = \delta K - gL + cKr - cK^2L - cKk\tau_i$$
$$0 = \beta - cr + cKL + ck\tau_i$$

to obtain

$$L(\tau_i, w) = K\frac{\beta + \delta}{g}$$
$$r(\tau_i, w) = K^2\frac{\beta + \delta}{g} + \frac{\beta}{c} + k\tau_i.$$

The optimal level of effort $L(\tau_i, w)$ — given a contract w — would thus be independent of talent, and the optimal level of misreporting

$$\begin{aligned} r(\tau_i, w) - (KL(\tau_i, w) + k\tau_i) &= K^2\frac{\beta + \delta}{g} + \frac{\beta}{c} + k\tau_i - \left(K^2\frac{\beta + \delta}{g} + k\tau_i\right) \\ &= \frac{\beta}{c} \end{aligned}$$

would also be independent of talent.

A benevolent planner would maximize

$$\max_{\beta,\delta} KL + k\tau_i - \frac{\rho\sigma^2}{2}\delta^2 - \frac{g}{2}L^2 - \frac{c}{2}\left(r - KL - k\tau_i\right)^2$$

Substituting $L(\tau_i, w)$ and $r(\tau_i, w)$, the benevolent planner maximizes

$$\begin{aligned} &\max_{\beta,\delta} K \cdot K\frac{\beta + \delta}{g} + k\tau_i - \frac{\rho\sigma^2}{2}\delta^2 - \frac{g}{2}\left(K\frac{\beta + \delta}{g}\right)^2 \\ &- \frac{c}{2}\left(K^2\frac{\beta + \delta}{g} + \frac{\beta}{c} + k\tau_i - K \cdot K\frac{\beta + \delta}{g} - k\tau_i\right)^2 \end{aligned}$$

The FOCs are

$$0 = \frac{\partial}{\partial \beta} \left(\begin{array}{c} K \cdot K\frac{\beta+\delta}{g} + k\tau_i - \frac{\rho\sigma^2}{2}\delta^2 - \frac{g}{2}\left(K\frac{\beta+\delta}{g}\right)^2 \\ -\frac{c}{2}\left(K^2\frac{\beta+\delta}{g} + \frac{\beta}{c} + k\tau_i - K \cdot K\frac{\beta+\delta}{g} - k\tau_i\right)^2 \end{array} \right)$$

$$0 = \frac{\partial}{\partial \delta} \left(\begin{array}{c} K \cdot K\frac{\beta+\delta}{g} + k\tau_i - \frac{\rho\sigma^2}{2}\delta^2 - \frac{g}{2}\left(K\frac{\beta+\delta}{g}\right)^2 \\ -\frac{c}{2}\left(K^2\frac{\beta+\delta}{g} + \frac{\beta}{c} + k\tau_i - K \cdot K\frac{\beta+\delta}{g} - k\tau_i\right)^2 \end{array} \right)$$

The benevolent planner's optimal contract sets $\beta^* = c\sigma^2\rho \cdot \delta^*$ and

$$\delta^* = \frac{K^2}{(c\sigma^2\rho + 1)\, K^2 + g\sigma^2\rho}.$$

The efficient contract creates incentives that do not depend on the talent level. In equilibrium, irrespective of the talent level,

$$L(\tau_i, w^*) = K\frac{c\sigma^2\rho + 1}{g} \frac{K^2}{(c\sigma^2\rho + 1)\, K^2 + g\sigma^2\rho}.$$

The expected compensation of the two talent types differ only because the more talented CEO's reported performance is higher. The extent of misreporting is identical, so the separating incentive constraints are satisfied if a pooling contract is offered.

A.6 Proof of Lemma 2

The firm's objective is to maximize

$$E\Pi = p_h \left(S(\tau_h, w_h) - U(\tau_h, w_h)\right) + (1 - p_h) \left(S(\tau_\ell, w_\ell) - U(\tau_\ell, w_\ell)\right) \tag{A15}$$

subject to the incentive constraints (cf. (10) and (11)),

$$U(\tau_h, w_h) \geq U(\tau_\ell, w_\ell) + \frac{(\beta_\ell + \delta_\ell)^2}{2g} \left(\tau_h^2 - \tau_\ell^2\right) \tag{A16}$$

$$U(\tau_\ell, w_\ell) \geq U(\tau_h, w_h) - \frac{(\beta_h + \delta_h)^2}{2g} \left(\tau_h^2 - \tau_\ell^2\right) \tag{A17}$$

and the binding participation constraint for a low-talent CEO (cf. (9))

$$U(\tau_\ell, w_\ell) = 0.$$

From (A16), the high-talent participation constraint is satisfied if the low-talent participation constraint is satisfied. We show below that the firm wants to employ both types of CEO, so the low-talent participation constraint is satisfied, and that the high-talent participation constraint is not binding since $\beta_\ell + \delta_\ell > 0$.

The high-talent incentive constraint (A16) must be binding: if it was not, then the firm could profitably reduce $U(\tau_h, w_h)$ by reducing α_h, without violating any of the constraints. The IC thus can be rewritten as

$$U(\tau_h, w_h) = 0 + \frac{(\beta_\ell + \delta_\ell)^2}{2g} \left(\tau_h^2 - \tau_\ell^2\right).$$

We can temporarily ignore the low-talent incentive constraint (A17) and verify later that it is satisfied.

The program can thus be simplified,

$$\max_{\beta_h, \delta_h, \beta_\ell, \delta_\ell} E\Pi = p_h \left(S(\tau_h, w_h) - \frac{(\beta_\ell + \delta_\ell)^2}{2g} \left(\tau_h^2 - \tau_\ell^2\right) \right) + (1 - p_h)\, S(\tau_\ell, w_\ell)$$

Substitute $S(\tau_h, w_h)$ and $S(\tau_\ell, w_\ell)$ using (3),

$$\begin{aligned} \max_{\beta_h, \delta_h, \beta_\ell, \delta_\ell} E\Pi = & p_h \left(\left(\frac{\tau_h^2}{g} (\beta_h + \delta_h) - \frac{\rho\sigma^2}{2} \delta_h^2 - \frac{\tau_h^2}{2g} (\beta_h + \delta_h)^2 - \frac{\beta_h^2}{2c} \right) - \frac{(\beta_\ell + \delta_\ell)^2}{2g} \left(\tau_h^2 - \tau_\ell^2\right) \right) \\ & + (1 - p_h) \left(\frac{\tau_\ell^2}{g} (\beta_\ell + \delta_\ell) - \frac{\rho\sigma^2}{2} \delta_\ell^2 - \frac{\tau_\ell^2}{2g} (\beta_\ell + \delta_\ell)^2 - \frac{\beta_\ell^2}{2c} \right). \end{aligned} \tag{A18}$$

The first-order conditions (with reference to δ_h, β_h, δ_ℓ, and β_ℓ) are

$$0 = p_h \left(\frac{\tau_h^2}{g} - \rho\sigma^2\delta_h - \frac{\tau_h^2}{g}(\beta_h + \delta_h) \right) \tag{A19}$$

$$0 = p_h \left(\frac{\tau_h^2}{g} - \frac{\tau_h^2}{g}(\beta_h + \delta_h) - \frac{\beta_h}{c} \right) \tag{A20}$$

$$0 = -p_h \frac{(\beta_\ell + \delta_\ell)}{g}\left(\tau_h^2 - \tau_\ell^2\right) + (1 - p_h)\left(\frac{\tau_\ell^2}{g} - \rho\sigma^2\delta_\ell - \frac{\tau_\ell^2}{g}(\beta_\ell + \delta_\ell) \right) \tag{A21}$$

$$0 = -p_h \frac{(\beta_\ell + \delta_\ell)}{g}\left(\tau_h^2 - \tau_\ell^2\right) + (1 - p_h)\left(\frac{\tau_\ell^2}{g} - \frac{\tau_\ell^2}{g}(\beta_\ell + \delta_\ell) - \frac{\beta_\ell}{c} \right). \tag{A22}$$

Combining the first two FOCs yields

$$\beta_h^{sf} = c\sigma^2\rho \cdot \delta_h^{sf} \tag{A23}$$

and $\delta_h^{sf} = \delta_h^*$. (Note: the contract offered to a high-talent CEO is efficient.) Similarly, combining the third and fourth FOC yields

$$\beta_\ell^{sf} = c\sigma^2\rho \cdot \delta_\ell^{sf}. \tag{A24}$$

and δ_ℓ^{sf} as described in (13). The equilibrium value of α_ℓ^{sf} is found by rewriting the low-talent participation constraint,

$$U(\tau_\ell, w_\ell^{sf}) = \alpha_\ell^{sf} + \frac{(c\rho\sigma^2)^2}{2c}\left(\delta_\ell^{sf}\right)^2 - \frac{\rho\sigma^2}{2}\left(\delta_\ell^{sf}\right)^2 + \frac{(c\rho\sigma^2 + 1)^2}{2g}\left(\delta_\ell^{sf}\right)^2\tau_\ell^2 = 0,$$

and solving it for α_ℓ^{sf}, which yields

$$\alpha_\ell^{sf} = \left(\frac{\rho\sigma^2}{2} - \frac{(c\rho\sigma^2)^2}{2c} - \frac{(c\rho\sigma^2 + 1)^2\tau_\ell^2}{2g} \right)\left(\delta_\ell^{sf}\right)^2. \tag{A25}$$

The value of α_h^{sf} is found by rewriting the binding high-talent incentive constraint,

$$U(\tau_h, w_h) = \frac{(c\rho\sigma^2 + 1)^2}{2g}\left(\delta_h^{sf}\right)^2\left(\tau_h^2 - \tau_\ell^2\right)$$

$$\alpha_h^{sf} + \frac{(c\rho\sigma^2)^2}{2c}\left(\delta_h^{sf}\right)^2 - \frac{\rho\sigma^2}{2}\left(\delta_h^{sf}\right)^2 + \frac{(c\rho\sigma^2 + 1)^2}{2g}\left(\delta_h^{sf}\right)^2\tau_h^2 = \frac{(c\rho\sigma^2 + 1)^2}{2g}\left(\delta_h^{sf}\right)^2\left(\tau_h^2 - \tau_\ell^2\right),$$

and solving it for α_h^{sf}, which yields

$$\alpha_h^{sf} = \left(\frac{\rho\sigma^2}{2} - \frac{\left(c\rho\sigma^2\right)^2}{2c} - \frac{\left(c\rho\sigma^2+1\right)^2 \tau_\ell^2}{2g} \right) \left(\delta_h^{sf}\right)^2 . \tag{A26}$$

We now verify that the low-talent incentive constraint is slack:

$$0 > U(\tau_h, w_h) - \frac{\left(c\rho\sigma^2+1\right)^2}{2g} \left(\delta_h^{sf}\right)^2 \left(\tau_h^2 - \tau_\ell^2\right).$$

The high-talent incentive constraint is binding,

$$U(\tau_h, w_h) = 0 + \frac{\left(c\rho\sigma^2+1\right)^2}{2g} \left(\delta_\ell^{sf}\right)^2 \left(\tau_h^2 - \tau_\ell^2\right). \tag{A27}$$

The low-talent incentive constraint can thus be rewritten as

$$0 > \frac{\left(c\rho\sigma^2+1\right)^2}{2g} \left(\delta_\ell^{sf}\right)^2 \left(\tau_h^2 - \tau_\ell^2\right) - \frac{\left(c\rho\sigma^2+1\right)^2}{2g} \left(\delta_h^{sf}\right)^2 \left(\tau_h^2 - \tau_\ell^2\right).$$

The result follows if $\delta_h^{sf} > \delta_\ell^{sf}$. That is the case, since $\delta_h^{sf} = \delta_h^*$, which is strictly larger than δ_ℓ^* (cf. (12)), and that in turn is strictly larger than δ_ℓ^{sf}.

Next, we show that the firm's profit is positive with either of the CEO types. Using (7), (A25) and (A24),

$$\begin{aligned}
\Pi(\tau_\ell, w_\ell^{sf}) &= \frac{\tau_\ell^2}{g} \left(c\rho\sigma^2+1\right) \delta_\ell^{sf} - \alpha_\ell^{sf} - \frac{\tau_\ell^2}{g} \left(c\rho\sigma^2+1\right)^2 \left(\delta_\ell^{sf}\right)^2 - \frac{\left(c\rho\sigma^2\right)^2}{c} \left(\delta_\ell^{sf}\right)^2 \\
&= \frac{\tau_\ell^2}{g} \left(c\rho\sigma^2+1\right) \delta_\ell^{sf} - \left(\delta_\ell^{sf}\right)^2 \left(c\rho\sigma^2+1\right) \frac{\left(c\rho\sigma^2+1\right) \tau_\ell^2 + \rho\sigma^2 g}{2g}.
\end{aligned}$$

That is positive if

$$2 \frac{\tau_\ell^2}{\left(c\rho\sigma^2+1\right) \tau_\ell^2 + \rho\sigma^2 g} > \delta_\ell^{sf},$$

which is the case, since the fraction on the left-hand side equals δ_ℓ^*, and $\delta_\ell^* > \delta_\ell^{sf}$. Similarly, using

(7), (A26) and (A23),

$$\begin{aligned}\Pi(\tau_h, w_h^{sf}) &= \frac{\tau_h^2}{g}\left(c\rho\sigma^2+1\right)\delta_h^{sf} - \alpha_h^{sf} - \frac{\tau_h^2}{g}\left(c\rho\sigma^2+1\right)^2\left(\delta_h^{sf}\right)^2 - \frac{\left(c\rho\sigma^2\right)^2}{c}\left(\delta_h^{sf}\right)^2 \\ &= \frac{\tau_h^2}{g}\left(c\rho\sigma^2+1\right)\delta_h^{sf} - \left(c\rho\sigma^2+1\right)\frac{\left(2\tau_h^2-\tau_\ell^2\right)\left(c\rho\sigma^2+1\right)+\rho\sigma^2 g}{2g}\left(\delta_h^{sf}\right)^2.\end{aligned}$$

That is positive if

$$\frac{\tau_h^2}{\left(2\tau_h^2-\tau_\ell^2\right)\left(c\rho\sigma^2+1\right)+\rho\sigma^2 g} > \frac{\delta_h^{sf}}{2}.$$

Replace δ_h^{sf},

$$\frac{\tau_h^2}{\left(2\tau_h^2-\tau_\ell^2\right)\left(c\rho\sigma^2+1\right)+\rho\sigma^2 g} > \frac{\tau_h^2}{2\tau_h^2\left(c\sigma^2\rho+1\right)+2g\sigma^2\rho}.$$

That is satisfied, and the firm's profit is strictly positive if $\tau_\ell > 0$.

For completeness, we now show that the firm always wants both types of CEO to accept a contract. Specifically, the firm could offer contracts that are incentive compatible but so unattractive that a low-talent CEO rejects and a high-talent CEO accepts, but her expected payoff is zero. Thus, the firm would be able to extract the entire surplus if the CEO has high talent, but it would lose the entire surplus if the CEO has low talent. That is not beneficial (and the firm prefers to hire both types) if $(1-p_h)\,S_\ell^{sf} \geq p_h U_h^{sf}$. From (A27) and (A23),

$$U(\tau_h, w_h^{sf}) = \frac{\left(c\sigma^2\rho+1\right)^2}{2g}\left(\delta_\ell^{sf}\right)^2\left(\tau_h^2-\tau_\ell^2\right) \tag{A28}$$

and from (3) and (A24),

$$S(\tau_\ell, w_\ell^{sf}) = \frac{\tau_\ell^2}{g}\left(c\sigma^2\rho+1\right)\delta_\ell^{sf} - \frac{\rho\sigma^2}{2}\left(\delta_\ell^{sf}\right)^2 - \frac{\tau_\ell^2}{2g}\left(c\sigma^2\rho+1\right)^2\left(\delta_\ell^{sf}\right)^2 - \frac{\left(c\sigma^2\rho\right)^2}{2c}\left(\delta_\ell^{sf}\right)^2. \tag{A29}$$

We want to show that

$$(1-p_h)\,S(\tau_\ell, w_\ell^{sf}) - p_h U(\tau_h, w_h^{sf}) \geq 0.$$

Replace $S(\tau_\ell, w_\ell^{sf})$ and $U(\tau_h, w_h^{sf})$ using (A28) and (A29), to obtain

$$(1-p_h)\,\delta_\ell^{sf}\left(\frac{\tau_\ell^2}{g}\left(c\sigma^2\rho+1\right)-\frac{\rho\sigma^2}{2}\delta_\ell^{sf}-\frac{\tau_\ell^2}{2g}\left(c\sigma^2\rho+1\right)^2\delta_\ell^{sf}-\frac{\left(c\sigma^2\rho\right)^2}{2c}\delta_\ell^{sf}\right)$$
$$-p_h\frac{\left(c\sigma^2\rho+1\right)^2}{2g}\left(\tau_h^2-\tau_\ell^2\right)\delta_\ell^{sf}\geq 0$$

$$\frac{\tau_\ell^2}{g}\left(1-p_h\right)\left(c\sigma^2\rho+1\right)\delta_\ell^{sf}\left(1-\frac{1}{2}\left(c\sigma^2\rho+1\right)\delta_\ell^{sf}-\frac{g}{\tau_\ell^2}\frac{\sigma^2\rho}{2}\delta_\ell^{sf}-\frac{p_h}{1-p_h}\frac{c\sigma^2\rho+1}{2}\left(\frac{\tau_h^2-\tau_\ell^2}{\tau_\ell^2}\right)\delta_\ell^{sf}\right)\geq 0$$

$$\frac{\tau_\ell^2}{g}\left(1-p_h\right)\left(c\sigma^2\rho+1\right)\delta_\ell^{sf}\left(1-\frac{1}{2}\frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho+\frac{p_h}{1-p_h}\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)}{\tau_\ell^2}\delta_\ell^{sf}\right)\geq 0$$

The fraction inside the parenthesis is equal to $\frac{1}{\delta_\ell^{sf}}$, so the requirement is

$$\frac{\tau_\ell^2}{2g}\left(1-p_h\right)\left(c\sigma^2\rho+1\right)\cdot\delta_\ell^{sf}\geq 0,$$

which is satisfied.

A.7 Properties of Equilibrium Contracts in the Single-Firm Setup

The comparative statics for β_h^{sf} and δ_h^{sf} are omitted since they can be found above (recall that $\beta_h^{sf}=\beta_h^*$ and $\delta_h^{sf}=\delta_h^*$).

$$\begin{aligned}\frac{\partial}{\partial c}\delta_\ell^{sf}&=\frac{\partial}{\partial c}\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho+\frac{p_h}{1-p_h}\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)}\\&=-\sigma^2\rho\left(1+\frac{p_h}{1-p_h}\frac{\tau_h^2-\tau_\ell^2}{\tau_\ell^2}\right)\left(\frac{\tau_\ell^2}{\left(\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho+\frac{p_h}{1-p_h}\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)\right)^2}\right)^2<0\end{aligned}$$

$$\begin{aligned}\frac{\partial}{\partial c}\beta_\ell^{sf}&=\frac{\partial}{\partial c}\frac{c\sigma^2\rho\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho+\frac{p_h}{1-p_h}\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)}\\&=\sigma^2\rho\left(1+\frac{g\rho\sigma^2}{\tau_\ell^2}+\frac{p_h}{1-p_h}\frac{\tau_h^2-\tau_\ell^2}{\tau_\ell^2}\right)\left(\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho+\frac{p_h}{1-p_h}\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)}\right)^2>0\end{aligned}$$

$$\frac{\partial}{\partial \sigma^2 \rho} \delta_\ell^{sf} = \frac{\partial}{\partial \sigma^2 \rho} \frac{\tau_\ell^2}{\left(c\sigma^2\rho + 1\right)\tau_\ell^2 + g\sigma^2\rho + \frac{p_h}{1-p_h}\left(c\sigma^2\rho + 1\right)\left(\tau_h^2 - \tau_\ell^2\right)}$$
$$= -\left(c + \frac{g}{\tau_\ell^2} + c\frac{p_h}{1-p_h}\frac{\tau_h^2 - \tau_\ell^2}{\tau_\ell^2}\right)\left(\frac{\tau_\ell^2}{\left(c\sigma^2\rho + 1\right)\tau_\ell^2 + g\sigma^2\rho + \frac{p_h}{1-p_h}\left(c\sigma^2\rho + 1\right)\left(\tau_h^2 - \tau_\ell^2\right)}\right)^2 < 0$$

$$\frac{\partial}{\partial \sigma^2 \rho} \beta_\ell^{sf} = \frac{\partial}{\partial \sigma^2 \rho} \frac{c\sigma^2\rho\tau_\ell^2}{\left(c\sigma^2\rho + 1\right)\tau_\ell^2 + g\sigma^2\rho + \frac{p_h}{1-p_h}\left(c\sigma^2\rho + 1\right)\left(\tau_h^2 - \tau_\ell^2\right)}$$
$$= c\left(1 + \frac{p_h}{1-p_h}\frac{\tau_h^2 - \tau_\ell^2}{\tau_\ell^2}\right)\left(\frac{\tau_\ell^2}{\left(c\sigma^2\rho + 1\right)\tau_\ell^2 + g\sigma^2\rho + \frac{p_h}{1-p_h}\left(c\sigma^2\rho + 1\right)\left(\tau_h^2 - \tau_\ell^2\right)}\right)^2 > 0$$

$$\frac{\partial}{\partial \tau_\ell^2} \delta_\ell^{sf} = \frac{\partial}{\partial \tau_\ell^2} \frac{\tau_\ell^2}{\left(c\sigma^2\rho + 1\right)\tau_\ell^2 + g\sigma^2\rho + \frac{p_h}{1-p_h}\left(c\sigma^2\rho + 1\right)\left(\tau_h^2 - \tau_\ell^2\right)}$$
$$= \frac{\frac{p_h}{1-p_h}\left(c\sigma^2\rho + 1\right)\tau_h^2 + g\sigma^2\rho}{\left(\left(c\sigma^2\rho + 1\right)\tau_\ell^2 + g\sigma^2\rho + \frac{p_h}{1-p_h}\left(c\sigma^2\rho + 1\right)\left(\tau_h^2 - \tau_\ell^2\right)\right)^2} > 0$$

$$\frac{\partial}{\partial \tau_\ell^2} \beta_\ell^{sf} = c\sigma^2\rho \frac{\partial}{\partial \tau_\ell^2} \delta_\ell^{sf} = c\sigma^2\rho \frac{\frac{p_h}{1-p_h}\left(c\sigma^2\rho + 1\right)\tau_h^2 + g\sigma^2\rho}{\left(\left(c\sigma^2\rho + 1\right)\tau_\ell^2 + g\sigma^2\rho + \frac{p_h}{1-p_h}\left(c\sigma^2\rho + 1\right)\left(\tau_h^2 - \tau_\ell^2\right)\right)^2} > 0$$

$$\frac{\partial}{\partial \tau_h^2} \delta_\ell^{sf} = \frac{\partial}{\partial \tau_h^2} \frac{\tau_\ell^2}{\left(c\sigma^2\rho + 1\right)\tau_\ell^2 + g\sigma^2\rho + \frac{p_h}{1-p_h}\left(c\sigma^2\rho + 1\right)\left(\tau_h^2 - \tau_\ell^2\right)}$$
$$= -\frac{\frac{p_h}{1-p_h}\left(c\sigma^2\rho + 1\right)\tau_\ell^2}{\left(\left(c\sigma^2\rho + 1\right)\tau_\ell^2 + g\sigma^2\rho + \frac{p_h}{1-p_h}\left(c\sigma^2\rho + 1\right)\left(\tau_h^2 - \tau_\ell^2\right)\right)^2} < 0$$

$$\frac{\partial}{\partial \tau_h^2} \beta_\ell^{sf} = c\sigma^2\rho \frac{\partial}{\partial \tau_h^2} \delta_\ell^{sf} = -c\sigma^2\rho \frac{\frac{p_h}{1-p_h}\left(c\sigma^2\rho + 1\right)\tau_\ell^2}{\left(\left(c\sigma^2\rho + 1\right)\tau_\ell^2 + g\sigma^2\rho + \frac{p_h}{1-p_h}\left(c\sigma^2\rho + 1\right)\left(\tau_h^2 - \tau_\ell^2\right)\right)^2} < 0$$

$$\frac{\partial}{\partial p_h} \delta_\ell^{sf} = \frac{\partial}{\partial p_h} \frac{\tau_\ell^2}{\left(c\sigma^2\rho + 1\right)\tau_\ell^2 + g\sigma^2\rho + \frac{p_h}{1-p_h}\left(c\sigma^2\rho + 1\right)\left(\tau_h^2 - \tau_\ell^2\right)}$$
$$= -\frac{\left(c\sigma^2\rho + 1\right)\left(\tau_h^2 - \tau_\ell^2\right)\tau_\ell^2}{\left(1 - p_h\right)^2\left(\left(c\sigma^2\rho + 1\right)\tau_\ell^2 + g\sigma^2\rho + \frac{p_h}{1-p_h}\left(c\sigma^2\rho + 1\right)\left(\tau_h^2 - \tau_\ell^2\right)\right)^2} < 0$$

$$\frac{\partial}{\partial\sigma^2\rho}L(\tau_\ell,w_\ell^{sf}) = \frac{\partial}{\partial\sigma^2\rho}\frac{1}{g}\tau_\ell\left(c\sigma^2\rho+1\right)\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho+\frac{p_h}{1-p_h}\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)}$$
$$= -\frac{\tau_\ell^3}{\left(\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho+\frac{p_h}{1-p_h}\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)\right)^2} < 0$$

$$\frac{\partial}{\partial c}L(\tau_\ell,w_\ell^{sf}) = \frac{\partial}{\partial c}\frac{1}{g}\tau_\ell\left(c\sigma^2\rho+1\right)\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho+\frac{p_h}{1-p_h}\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)}$$
$$= \frac{\sigma^2\rho^2\tau_\ell^3}{\left(\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho+\frac{p_h}{1-p_h}\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)\right)^2} > 0$$

$$\frac{\partial}{\partial c}L(\tau_\ell,w_\ell^{*}) = \frac{\partial}{\partial c}\frac{1}{g}\tau_\ell\left(c\sigma^2\rho+1\right)\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho} = \frac{\sigma^2\rho^2\tau_\ell^3}{\left(\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho\right)^2} > 0$$

$$\lim_{c\to\infty}\beta_\ell^{sf} = \lim_{c\to\infty}\frac{c\sigma^2\rho\cdot\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho+\frac{p_h}{1-p_h}\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)} = \frac{\tau_\ell^2}{\tau_\ell^2+\frac{p_h}{1-p_h}\left(\tau_h^2-\tau_\ell^2\right)} < 1 = \lim_{c\to\infty}\beta_\ell^*$$
$$\lim_{c\to\infty}\delta_\ell^{sf} = \lim_{c\to\infty}\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho+\frac{p_h}{1-p_h}\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)} = 0 = \lim_{c\to\infty}\delta_\ell^*$$

$$\lim_{c\to 0}\beta_\ell^{sf} = \lim_{c\to 0}c\sigma^2\rho\cdot\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho+\frac{p_h}{1-p_h}\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)} = 0$$
$$\lim_{c\to 0}\delta_\ell^{sf} = \lim_{c\to 0}\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho+\frac{p_h}{1-p_h}\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)} = \frac{\tau_\ell^2}{\tau_\ell^2+g\sigma^2\rho+\frac{p_h}{1-p_h}\left(\tau_h^2-\tau_\ell^2\right)}$$

$$\lim_{\sigma^2\rho\to 0}\beta_\ell^{sf} = \lim_{\sigma^2\rho\to 0}c\sigma^2\rho\cdot\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho+\frac{p_h}{1-p_h}\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)} = 0 = \lim_{\sigma^2\rho\to 0}\beta_\ell^*$$
$$\lim_{\sigma^2\rho\to 0}\delta_\ell^{sf} = \lim_{\sigma^2\rho\to 0}\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho+\frac{p_h}{1-p_h}\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)} = \frac{\tau_\ell^2}{\tau_\ell^2+\frac{p_h}{1-p_h}\left(\tau_h^2-\tau_\ell^2\right)} < 1 = \lim_{\sigma^2\rho\to 0}\delta_\ell^*$$

$$
\begin{aligned}
\lim_{\sigma^2\rho\to\infty}\beta_\ell^{sf} &= \lim_{\sigma^2\rho\to\infty}\frac{c\sigma^2\rho\cdot\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho+\frac{p_h}{1-p_h}\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)} = \frac{c\tau_\ell^2}{c\tau_\ell^2+g+\frac{p_h}{1-p_h}c\left(\tau_h^2-\tau_\ell^2\right)}\\
\lim_{\sigma^2\rho\to\infty}\delta_\ell^{sf} &= \lim_{\sigma^2\rho\to\infty}\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho+\frac{p_h}{1-p_h}\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)} = 0
\end{aligned}
$$

A.8 Profit Positive if High-talent CEO Accepts Low-talent Contract w_ℓ^{**}

It is always possible to poach a low-talent CEO by offering her the efficient contract $w_\ell^{**} = (\alpha_\ell^{**}, \beta_\ell^*, \delta_\ell^*)$. Offering that contract lets the firm break even if it hires a low-talent CEO, and if a high-talent CEO accepts it, the firm actually benefits: Using (7), we have $\Pi(\tau_h, w_\ell^{**}) > \Pi(\tau_\ell, w_\ell^{**})$ if

$$
\begin{aligned}
&\frac{\tau_h^2}{g}\left(c\sigma^2\rho+1\right)\delta_\ell^* - \alpha_\ell^{**} - \frac{\tau_h^2}{g}\left(c\sigma^2\rho+1\right)^2(\delta_\ell^*)^2 - \frac{\left(c\sigma^2\rho\right)^2}{c}(\delta_\ell^*)^2\\
>&\frac{\tau_\ell^2}{g}\left(c\sigma^2\rho+1\right)\delta_\ell^* - \alpha_\ell^{**} - \frac{\tau_\ell^2}{g}\left(c\sigma^2\rho+1\right)^2(\delta_\ell^*)^2 - \frac{\left(c\sigma^2\rho\right)^2}{c}(\delta_\ell^*)^2,
\end{aligned}
$$

or

$$
\tau_h^2\frac{c\sigma^2\rho+1}{g}\delta_\ell^*\left(1-\left(c\sigma^2\rho+1\right)\delta_\ell^*\right) > \tau_\ell^2\frac{c\sigma^2\rho+1}{g}\delta_\ell^*\left(1-\left(c\sigma^2\rho+1\right)\delta_\ell^*\right).
$$

That is satisfied, since $\tau_h^2 > \tau_\ell^2$ and the term in parentheses is positive (from (12), we have $\left(c\sigma^2\rho+1\right)\delta_i^* < 1$).

A.9 Proof of Proposition 3

The firms' equilibrium contract is defined by the program (14)-(16). A high-talent CEO's decisions must be distorted, since as shown above, the set of efficient contracts is incentive compatible but leads to positive profits for the firm. Clearly, one of the incentive constraints (15) and (16) must be binding: If not, the distortion in a high-talent CEO's decisions could be mitigated, increasing the surplus she generates. As before, we ignore one of the incentive constraints and assume that the other is binding. However, as we will show, the binding constraint is the low-talent incentive constraint (16). (The low-talent incentive constraint is slack in the equilibrium contract in the single-firm setup.)

With a binding low-talent incentive constraint (16), and ignoring the high-talent incentive constraint (15), we can rewrite the firms' optimization program as a Lagrangian:

$$\max_{\beta_h,\delta_h,\lambda} \mathcal{L} = S(\tau_h, w_h) - \lambda \left(S(\tau_\ell, w_\ell^{**}) + \frac{(\beta_h+\delta_h)^2}{2g}\left(\tau_h^2 - \tau_\ell^2\right) - S(\tau_h, w_h) \right)$$

(The equilibrium value of α_h can be found later, by setting a high-talent CEO's equilibrium expected payoff equal to the surplus she generates.) Replacing $S(\tau_h, w_h^c)$ using (3) and $S(\tau_\ell, w_\ell^{**})$ (which equals $S(\tau_\ell, w_\ell^{*})$) using (A10) and (12), the firm's optimization program is

$$\begin{aligned}\max_{\beta_h,\delta_h,\lambda} \mathcal{L} =& \frac{\tau_h^2}{g}(\beta_h+\delta_h) - \frac{\rho\sigma^2}{2}\delta_h^2 - \frac{\tau_h^2}{2g}(\beta_h+\delta_h)^2 - \frac{\beta_h^2}{2c} \\ & - \lambda \left(\frac{\tau_\ell^2}{2g}\left(c\rho\sigma^2+1\right)\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2 + g\sigma^2\rho} + \frac{(\beta_h+\delta_h)^2}{2g}\left(\tau_h^2-\tau_\ell^2\right)\right) \\ & + \lambda \left(\frac{\tau_h^2}{g}(\beta_h+\delta_h) - \frac{\rho\sigma^2}{2}\delta_h^2 - \frac{\tau_h^2}{2g}(\beta_h+\delta_h)^2 - \frac{\beta_h^2}{2c} \right)\end{aligned}$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial \beta_h} = 0 = \frac{\tau_h^2}{g} - \frac{\tau_h^2}{g}(\beta_h+\delta_h) - \frac{\beta_h}{c} - \lambda\frac{\beta_h+\delta_h}{g}\left(\tau_h^2-\tau_\ell^2\right) + \lambda\left(\frac{\tau_h^2}{g} - \frac{\tau_h^2}{g}(\beta_h+\delta_h) - \frac{\beta_h}{c}\right) \tag{A30}$$

$$\frac{\partial \mathcal{L}}{\partial \delta_h} = 0 = \frac{\tau_h^2}{g} - \rho\sigma^2\delta_h - \frac{\tau_h^2}{g}(\beta_h+\delta_h) - \lambda\frac{\beta_h+\delta_h}{g}\left(\tau_h^2-\tau_\ell^2\right) + \lambda\left(\frac{\tau_h^2}{g} - \rho\sigma^2\delta_h - \frac{\tau_h^2}{g}(\beta_h+\delta_h)\right) \tag{A31}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \lambda} = 0 =& -\frac{\tau_\ell^2}{2g}\left(c\rho\sigma^2+1\right)\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2 + g\sigma^2\rho} - \frac{(\beta_h+\delta_h)^2}{2g}\left(\tau_h^2-\tau_\ell^2\right) \\ & + \frac{\tau_h^2}{g}(\beta_h+\delta_h) - \frac{\rho\sigma^2}{2}\delta_h^2 - \frac{\tau_h^2}{2g}(\beta_h+\delta_h)^2 - \frac{\beta_h^2}{2c}\end{aligned} \tag{A32}$$

Rewrite (A30) and (A31) as

$$\lambda = \frac{\frac{\tau_h^2}{g} - \frac{\tau_h^2}{g}(\beta_h+\delta_h) - \frac{\beta_h}{c}}{\frac{\beta_h+\delta_h}{g}\left(\tau_h^2-\tau_\ell^2\right) - \frac{\tau_h^2}{g} + \frac{\tau_h^2}{g}(\beta_h+\delta_h) + \frac{\beta_h}{c}} \tag{A33}$$

$$\lambda = \frac{\frac{\tau_h^2}{g} - \rho\sigma^2\delta_h - \frac{\tau_h^2}{g}(\beta_h+\delta_h)}{\frac{\beta_h+\delta_h}{g}\left(\tau_h^2-\tau_\ell^2\right) - \frac{\tau_h^2}{g} + \rho\sigma^2\delta_h + \frac{\tau_h^2}{g}(\beta_h+\delta_h)}. \tag{A34}$$

Combining the two equations yields

$$\frac{\frac{\tau_h^2}{g}-\frac{\tau_h^2}{g}\left(\beta_h+\delta_h\right)-\frac{\beta_h}{c}}{\frac{\beta_h+\delta_h}{g}\left(\tau_h^2-\tau_\ell^2\right)-\frac{\tau_h^2}{g}+\frac{\tau_h^2}{g}\left(\beta_h+\delta_h\right)+\frac{\beta_h}{c}}=\frac{\frac{\tau_h^2}{g}-\rho\sigma^2\delta_h-\frac{\tau_h^2}{g}\left(\beta_h+\delta_h\right)}{\frac{\beta_h+\delta_h}{g}\left(\tau_h^2-\tau_\ell^2\right)-\frac{\tau_h^2}{g}+\rho\sigma^2\delta_h+\frac{\tau_h^2}{g}\left(\beta_h+\delta_h\right)},$$

which can be simplified to yield

$$\beta_h^c=c\sigma^2\rho\cdot\delta_h^c. \tag{A35}$$

Substitute that in (A32),

$$\begin{aligned}0=&-\frac{\tau_\ell^2}{2g}\left(c\rho\sigma^2+1\right)\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}-\frac{\left(c\sigma^2\rho+1\right)^2}{2g}\delta_h^2\left(\tau_h^2-\tau_\ell^2\right)\\&+\frac{\tau_h^2}{g}\left(c\sigma^2\rho+1\right)\delta_h-\frac{\rho\sigma^2}{2}\delta_h^2-\frac{\tau_h^2}{2g}\left(c\sigma^2\rho+1\right)^2\delta_h^2-\frac{\left(c\sigma^2\rho\right)^2}{2c}\delta_h^2,\end{aligned} \tag{A36}$$

and rearrange,

$$0=\frac{1}{2g}\left(c\rho\sigma^2+1\right)\left(-\left(\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)+\tau_h^2\left(c\sigma^2\rho+1\right)+g\sigma^2\rho\right)\cdot\delta_h^2+2\tau_h^2\cdot\delta_h-\frac{\tau_\ell^4}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}\right),$$

to obtain

$$\delta_h=\frac{\tau_h^2\pm\sqrt{\frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}}}{\left(c\sigma^2\rho+1\right)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}.$$

The solution is found by considering the limit case as the disutility from bearing risk goes to zero (because either ρ or σ go to zero): In that case, long-term compensation based on realized performance becomes a highly effective incentive tool, and the optimal contract should set $\delta_h=1$ (which is equal to the efficient level δ_h^* if $\sigma^2\rho=0$, cf. (12)). That is the case if the square root is added, not if it is subtracted,

$$\begin{aligned}\frac{\tau_h^2+\sqrt{\frac{\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2}{\tau_\ell^2}}}{2\tau_h^2-\tau_\ell^2}&=\frac{\tau_h^2+\left(\tau_h^2-\tau_\ell^2\right)}{2\tau_h^2-\tau_\ell^2}=1\\\frac{\tau_h^2-\sqrt{\frac{\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2}{\tau_\ell^2}}}{2\tau_h^2-\tau_\ell^2}&=\frac{\tau_h^2-\left(\tau_h^2-\tau_\ell^2\right)}{2\tau_h^2-\tau_\ell^2}=\frac{\tau_\ell^2}{2\tau_h^2-\tau_\ell^2}<1.\end{aligned}$$

So the optimal value of δ_h is δ_h^c as defined in (17).

We can now determine α_h^c, using the requirement that $U(\tau_h, w_h^c) = S(\tau_h, w_h^c)$. Using (5) and (3),

$$\alpha_h^c + \frac{\left(\beta_h^c\right)^2}{2c} - \frac{\rho\sigma^2}{2}\left(\delta_h^c\right)^2 + \frac{\left(\beta_h^c + \delta_h^c\right)^2}{2g}\tau_h^2 = \frac{\tau_h^2}{g}\left(\beta_h^c + \delta_h^c\right) - \frac{\rho\sigma^2}{2}\left(\delta_h^c\right)^2 - \frac{\tau_h^2}{2g}\left(\beta_h^c + \delta_h^c\right)^2 - \frac{\left(\beta_h^c\right)^2}{2c}.$$

Replace $\beta_h^c = c\sigma^2\rho \cdot \delta_h^c$, and rearrange, to obtain

$$\alpha_h^c = \frac{\tau_h^2}{g}\left(c\sigma^2\rho + 1\right)\delta_h^c - \frac{\tau_h^2}{g}\left(c\sigma^2\rho + 1\right)^2\left(\delta_h^c\right)^2 - \frac{\left(c\sigma^2\rho\right)^2}{c}\left(\delta_h^c\right)^2. \tag{A37}$$

The equilibrium value of α_ℓ equals

$$\alpha_\ell^c = \alpha_\ell^{**} = \sigma^2\rho \cdot \left(\delta_\ell^*\right)^2, \tag{A38}$$

the value of α_ℓ that lets a low-talent CEO fully extract the efficient surplus it generates under an efficient contract.

We now verify that the high-talent incentive constraint (15) is satisfied:

$$S(\tau_h, w_h^c) \geq S(\tau_\ell, w_\ell^{**}) + \frac{\left(\beta_\ell^* + \delta_\ell^*\right)^2}{2g}\left(\tau_h^2 - \tau_\ell^2\right).$$

Recall that the low-talent incentive constraint (16) is binding, i.e.,

$$S(\tau_h, w_h^c) = S(\tau_\ell, w_\ell^{**}) + \frac{\left(\beta_h^c + \delta_h^c\right)^2}{2g}\left(\tau_h^2 - \tau_\ell^2\right).$$

So the requirement is that

$$\frac{\left(\beta_h + \delta_h\right)^2}{2g}\left(\tau_h^2 - \tau_\ell^2\right) \geq \frac{\left(\beta_\ell^* + \delta_\ell^*\right)^2}{2g}\left(\tau_h^2 - \tau_\ell^2\right).$$

Since $\beta_h^c = c\sigma^2\rho \cdot \delta_h^c$ and $\beta_\ell^* = c\sigma^2\rho \cdot \delta_\ell^*$, the requirement is that $\delta_h^c \geq \delta_\ell^*$. We show in the proof of Corollary 4 (below) that $\delta_h^c \geq \delta_h^*$. Since $\delta_h^* > \delta_\ell^*$ (cf. (12)), this implies that $\delta_h^c \geq \delta_\ell^*$.

The low-talent participation constraint is satisfied, since by construction, $U(\tau_\ell, w_\ell^c) = S(\tau_\ell, w_\ell^c) = S(\tau_\ell, w_\ell^*) > 0$, and the high-talent participation constraint is then also satisfied, since $\beta_\ell^c + \delta_\ell^c > 0$.

The remainder of this proof shows that there are no profitable deviation strategies for the firms. A deviation contract is one of the following: (1) a separating contract that a low-talent CEO accepts and a high-talent CEO rejects; (2) a separating contract that a high-talent CEO accepts and a low-talent CEO rejects; or (3) a pooling contract that both types accept. (A profitable deviation to separating contracts that both types accept is not feasible, since by construction, the postulated equilibrium separating contract is the optimal separating contract.)

The first deviation cannot be profitable: Under the equilibrium contract, a low-talent CEO generates the highest possible surplus and fully extracts it. Any departures from that contract are either rejected, or they lead to losses for the firm.

The second deviation consists of a contract that a high-talent CEO accepts but a low-talent CEO rejects. Say, the deviation contract offered to a high-talent CEO is $w_d = (\alpha_d, \beta_d, \delta_d)$. A high-talent CEO prefers the deviation contract w_d to the separating contract w_h^c if $U(\tau_h, w_d) > U(\tau_h, w_h^c)$, i.e., if (cf. (5))

$$\alpha_d + \frac{\beta_d^2}{2c} - \frac{\rho\sigma^2}{2}\delta_d^2 + \frac{(\beta_d + \delta_d)^2}{2g}\tau_h^2 > \alpha_h^c + \frac{(\beta_h^c)^2}{2c} - \frac{\rho\sigma^2}{2}(\delta_h^c)^2 + \frac{(\beta_h^c + \delta_h^c)^2}{2g}\tau_h^2.$$

A low-talent CEO will not accept the deviation contract w_d targeted at a high-talent CEO if $U(\tau_\ell, w_\ell^c) \geq U(\tau_\ell, w_d)$. Since her incentive constraint (16) is binding, that holds if $U(\tau_\ell, w_h^c) \geq U(\tau_\ell, w_d)$, i.e., if

$$\alpha_d + \frac{\beta_d^2}{2c} - \frac{\rho\sigma^2}{2}\delta_d^2 + \frac{(\beta_d + \delta_d)^2}{2g}\tau_\ell^2 \leq \alpha_h^c + \frac{(\beta_h^c)^2}{2c} - \frac{\rho\sigma^2}{2}(\delta_h^c)^2 + \frac{(\beta_h^c + \delta_h^c)^2}{2g}\tau_\ell^2$$

Combine the two inequalities:

$$\begin{aligned}
&\alpha_d + \frac{\beta_d^2}{2c} - \frac{\rho\sigma^2}{2}\delta_d^2 + \frac{(\beta_d + \delta_d)^2}{2g}\tau_h^2 - \alpha_h^c - \frac{(\beta_h^c)^2}{2c} + \frac{\rho\sigma^2}{2}(\delta_h^c)^2 - \frac{(\beta_h^c + \delta_h^c)^2}{2g}\tau_h^2 \\
&- \alpha_d - \frac{\beta_d^2}{2c} + \frac{\rho\sigma^2}{2}\delta_d^2 - \frac{(\beta_d + \delta_d)^2}{2g}\tau_\ell^2 + \alpha_h^c + \frac{(\beta_h^c)^2}{2c} - \frac{\rho\sigma^2}{2}(\delta_h^c)^2 + \frac{(\beta_h^c + \delta_h^c)^2}{2g}\tau_\ell^2 > 0
\end{aligned}$$

$$\frac{(\beta_d + \delta_d)^2}{2g}\tau_h^2 - \frac{(\beta_h^c + \delta_h^c)^2}{2g}\tau_h^2 - \frac{(\beta_d + \delta_d)^2}{2g}\tau_\ell^2 + \frac{(\beta_h^c + \delta_h^c)^2}{2g}\tau_\ell^2 > 0$$

$$\left(\frac{(\beta_d + \delta_d)^2}{2g} - \frac{(\beta_h^c + \delta_h^c)^2}{2g}\right)(\tau_h^2 - \tau_\ell^2) > 0$$

That requires $(\beta_d + \delta_d) > (\beta_h^c + \delta_h^c)$, i.e., a distortion of a high-talent CEO's incentives worse than under the separating contract. (We show in the proof of Corollary 4 (below) that $\delta_h^c \geq \delta_h^*$, which implies $\beta_h^c > \beta_h^*$.) But such a contract reduces the surplus that a high-talent CEO generates, and thus the rent she can extract. It would not be accepted, unless it violates the firm's break-even constraint, which would make the deviation unattractive to the firm.

The third possible deviation is to a pooling contract that both CEO types accept. Such a deviation pooling contract is feasible if p_h is sufficiently large, which leads to the equilibrium existence condition $p_h \leq p_o$. The upper bound p_o is the value of p_h at which the firm breaks even, and a high-talent CEO is indifferent between the equilibrium contract w_h^c and the deviation pooling contract w_{pool}^d. Such a deviation pooling contract w_{pool}^d must maximize a high-talent CEO's expected payoff, such that she prefers this contract to the separating equilibrium contract, such that a low-talent CEO also prefers the deviation pooling contract, and such that the firm breaks even on average

The reason for focusing on maximizing a high-talent CEO's payoff under the deviation pooling contract is that a low-talent CEO prefers a pooling deviation contract w_{pool}^d to her separating equilibrium contract w_ℓ^c (i.e., $U(\tau_\ell, w_{pool}^d) > U(\tau_\ell, w_\ell^c)$) if a high-talent CEO weakly prefers w_{pool}^d to *her* equilibrium contract w_h^c (i.e., $U(\tau_h, w_{pool}^d) \geq U(\tau_h, w_h^c)$): Using (5),

$$U(\tau_\ell, w_{pool}^d) = U(\tau_h, w_{pool}^d) - \frac{\left(\beta_{pool}^d + \delta_{pool}^d\right)^2}{2g} \left(\tau_h^2 - \tau_\ell^2\right).$$

Since $U(\tau_h, w_{pool}^d) \geq U(\tau_h, w_h^c)$, we have

$$U(\tau_\ell, w_{pool}^d) \geq U(\tau_h, w_h^c) - \frac{\left(\beta_{pool}^d + \delta_{pool}^d\right)^2}{2g} \left(\tau_h^2 - \tau_\ell^2\right),$$

and using (5), substitute $U(\tau_h, w_h^c)$ to obtain

$$U(\tau_\ell, w_{pool}^d) \geq U(\tau_\ell, w_h^c) + \frac{\left(\beta_h^c + \delta_h^c\right)^2}{2g} \left(\tau_h^2 - \tau_\ell^2\right) - \frac{\left(\beta_{pool}^d + \delta_{pool}^d\right)^2}{2g} \left(\tau_h^2 - \tau_\ell^2\right).$$

By construction, a low-talent CEO is indifferent between the equilibrium separating contracts: $U(\tau_\ell, w_\ell^c) = U(\tau_\ell, w_h^c)$. Thus,

$$U(\tau_\ell, w_{pool}^d) \geq U(\tau_\ell, w_\ell^c) + \frac{\left(\tau_h^2 - \tau_\ell^2\right)}{2g} \left(\left(\beta_h^c + \delta_h^c\right)^2 - \left(\beta_{pool}^d + \delta_{pool}^d\right)^2\right).$$

A deviation pooling contract is feasible only if it reduces the distortion in a high-talent CEO's incentives: Otherwise, the overall surplus would be reduced (under the separating contract, a low-talent CEO's incentives are efficient), and since both types of CEO prefer the pooling contract, the zero-profit condition must be violated. So we have $\beta_{pool}^d + \delta_{pool}^d < \beta_h^c + \delta_h^c$, and thus $U(\tau_\ell, w_{pool}^d) >$

$U(\tau_\ell, w_\ell^c)$ whenever $U(\tau_h, w_{pool}^d) \geq U(\tau_h, w_h^c)$.

When constructing a pooling deviation, the firm's problem is

$$\max_{\beta,\delta} U(\tau_h, w)$$
$$\text{s.th.} \quad E_i\left[\Pi(\tau_i, w)\right] = 0$$

Using (6), the zero-profit constraint can be rewritten as

$$E_i\left[\tau_i L(\tau_i, w)\right] - \alpha - \beta \cdot E_i\left[r(\tau_i, w)\right] - \delta \cdot E_i\left[\tau_i L(\tau_i, w)\right] = 0,$$

and using (1) and (2), it can be rewritten as

$$\begin{aligned} E_i\left[\tau_i \frac{1}{g}\tau_i\left(\beta+\delta\right)\right] - \alpha - \beta \cdot E_i\left[\frac{1}{g}\tau_i^2\left(\beta+\delta\right) + \frac{\beta}{c}\right] - \delta \cdot E_i\left[\tau_i \frac{1}{g}\tau_i\left(\beta+\delta\right)\right] &= 0 \\ \frac{1}{g}\left(\beta+\delta\right) \cdot E_i\left[\tau_i^2\right] - \alpha - \frac{1}{g}\beta\left(\beta+\delta\right) \cdot E_i\left[\tau_i^2\right] - \frac{\beta^2}{c} - \frac{1}{g}\delta\left(\beta+\delta\right) \cdot E_i\left[\tau_i^2\right] &= 0 \\ \left(\frac{1}{g}\left(\beta+\delta\right)\left(1-\beta-\delta\right)\right) \cdot E_i\left[\tau_i^2\right] - \alpha - \frac{\beta^2}{c} &= 0. \end{aligned}$$

Using (5), replace $U(\tau_h, w)$ and rewrite the firm's program as

$$\max_{\beta,\delta} U(\tau_i, w) = \alpha + \frac{\beta^2}{2c} - \frac{\rho\sigma^2}{2}\delta^2 + \frac{\left(\beta+\delta\right)^2}{2g}\tau_h^2$$
$$\text{s.th.} \quad \alpha = \left(\frac{1}{g}\left(\beta+\delta\right)\left(1-\beta-\delta\right)\right) \cdot E_i\left[\tau_i^2\right] - \frac{\beta^2}{c}$$

Replace α in the objective function using the constraint,

$$\max_{\beta,\delta} U(\tau_i, w) = \left(\frac{1}{g}\left(\beta+\delta\right)\left(1-\beta-\delta\right)\right) \cdot E_i\left[\tau_i^2\right] - \frac{\beta^2}{c} + \frac{\beta^2}{2c} - \frac{\rho\sigma^2}{2}\delta^2 + \frac{\left(\beta+\delta\right)^2}{2g}\tau_h^2 \tag{A39}$$

and the first-order conditions are

$$0 = \left(\frac{1}{g}\left(1-2\beta-2\delta\right)\right) \cdot E_i\left[\tau_i^2\right] - \frac{\beta}{c} + \frac{\left(\beta+\delta\right)}{g}\tau_h^2$$
$$0 = \left(\frac{1}{g}\left(1-2\beta-2\delta\right)\right) \cdot E_i\left[\tau_i^2\right] - \rho\sigma^2\delta + \frac{\left(\beta+\delta\right)}{g}\tau_h^2$$

Combine them to obtain

$$\beta^d_{pool} = c\rho\sigma^2 \cdot \delta^d_{pool} \tag{A40}$$

$$\delta^d_{pool} = \frac{E_i\left[\tau_i^2\right]}{2\left(c\rho\sigma^2+1\right)\cdot E_i\left[\tau_i^2\right] - \left(c\rho\sigma^2+1\right)\tau_h^2 + g\sigma^2\rho} \tag{A41}$$

Note that this requires that p_h is sufficiently large,

$$p_h > 1 - \frac{\left(c\rho\sigma^2+1\right)\tau_h^2 + g\sigma^2\rho}{2\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)}.$$

Otherwise, both β^d_{pool} and δ^d_{pool} would be negative, discouraging effort, so the contract w^d_{pool} could not be an improvement.

Using the zero-profit constraint in the firm's program, find α,

$$\alpha^d_{pool} = \frac{1}{g}\left(c\rho\sigma^2+1\right)\cdot E_i\left[\tau_i^2\right]\cdot\delta^d_{pool} - \frac{1}{g}\left(c\rho\sigma^2+1\right)^2\cdot E_i\left[\tau_i^2\right]\cdot\left(\delta^d_{pool}\right)^2 - c\left(\rho\sigma^2\right)^2\left(\delta^d_{pool}\right)^2.$$

Expand the first summand by $\frac{\delta^d_{pool}}{\delta^d_{pool}}$ and replace δ^d_{pool} in that fraction's denominator,

$$\begin{aligned}
\alpha^d_{pool} =& \frac{1}{g}\left(c\rho\sigma^2+1\right)\cdot E_i\left[\tau_i^2\right]\cdot\tfrac{\left(2\left(c\rho\sigma^2+1\right)\right)\cdot E_i\left[\tau_i^2\right]-\left(c\rho\sigma^2+1\right)\tau_h^2+g\rho\sigma^2}{E_i\left[\tau_i^2\right]}\left(\delta^d_{pool}\right)^2 \\
&-\frac{1}{g}\left(c\rho\sigma^2+1\right)^2\cdot E_i\left[\tau_i^2\right]\cdot\left(\delta^d_{pool}\right)^2 - c\left(\rho\sigma^2\right)^2\left(\delta^d_{pool}\right)^2 \\
=& \frac{2\left(c\rho\sigma^2+1\right)^2\cdot E_i\left[\tau_i^2\right]-\left(c\rho\sigma^2+1\right)^2\tau_h^2+\left(c\rho\sigma^2+1\right)g\rho\sigma^2}{g}\cdot\left(\delta^d_{pool}\right)^2 \\
&-\frac{1}{g}\left(c\rho\sigma^2+1\right)^2\cdot E_i\left[\tau_i^2\right]\cdot\left(\delta^d_{pool}\right)^2 - c\left(\rho\sigma^2\right)^2\left(\delta^d_{pool}\right)^2 \\
=& \frac{1}{g}2\left(c\rho\sigma^2+1\right)^2\cdot E_i\left[\tau_i^2\right]\cdot\left(\delta^d_{pool}\right)^2 - \frac{1}{g}\left(c\rho\sigma^2+1\right)^2\tau_h^2\cdot\left(\delta^d_{pool}\right)^2 \\
&+\left(c\rho\sigma^2+1\right)\rho\sigma^2\cdot\left(\delta^d_{pool}\right)^2 - \frac{1}{g}\left(c\rho\sigma^2+1\right)^2\cdot E_i\left[\tau_i^2\right]\cdot\left(\delta^d_{pool}\right)^2 - c\left(\rho\sigma^2\right)^2\left(\delta^d_{pool}\right)^2 \\
=& \frac{1}{g}\left(c\rho\sigma^2+1\right)^2\cdot E_i\left[\tau_i^2\right]\cdot\left(\delta^d_{pool}\right)^2 - \frac{1}{g}\left(c\rho\sigma^2+1\right)^2\tau_h^2\cdot\left(\delta^d_{pool}\right)^2 \\
&+\left(c\rho\sigma^2+1\right)\rho\sigma^2\cdot\left(\delta^d_{pool}\right)^2 - c\left(\rho\sigma^2\right)^2\left(\delta^d_{pool}\right)^2,
\end{aligned}$$

to obtain

$$\alpha^d_{pool} = \frac{1}{g}\left(c\rho\sigma^2+1\right)^2 \cdot E_i\left[\tau_i^2\right] \cdot \left(\delta^d_{pool}\right)^2 - \frac{1}{g}\left(c\rho\sigma^2+1\right)^2 \tau_h^2 \cdot \left(\delta^d_{pool}\right)^2 + \sigma^2\rho\left(\delta^d_{pool}\right)^2.$$

Calculate $U(\tau_h, w^d_{pool})$ using (5), replacing $\beta^d_{pool} = c\rho\sigma^2 \cdot \delta^d_{pool}$:

$$\begin{aligned}
U(\tau_h, w^d_{pool}) =& \alpha^d_{pool} + \frac{\left(c\rho\sigma^2\right)^2}{2c}\left(\delta^d_{pool}\right)^2 - \frac{\rho\sigma^2}{2}\left(\delta^d_{pool}\right)^2 + \frac{\left(c\rho\sigma^2+1\right)^2}{2g}\tau_h^2\left(\delta^d_{pool}\right)^2 && \text{(A42)}\\
=& \frac{1}{g}\left(c\rho\sigma^2+1\right)^2 \cdot E_i\left[\tau_i^2\right] \cdot \left(\delta^d_{pool}\right)^2 - \frac{1}{g}\left(c\rho\sigma^2+1\right)^2 \tau_h^2 \cdot \left(\delta^d_{pool}\right)^2 + \sigma^2\rho\left(\delta^d_{pool}\right)^2\\
&+ c\frac{\left(\rho\sigma^2\right)^2}{2}\left(\delta^d_{pool}\right)^2 - \frac{\rho\sigma^2}{2}\left(\delta^d_{pool}\right)^2 + \frac{\left(c\rho\sigma^2+1\right)^2}{2g}\tau_h^2\left(\delta^d_{pool}\right)^2 && \text{(A43)}
\end{aligned}$$

$$\begin{aligned}
=& \frac{1}{g}\left(c\rho\sigma^2+1\right)^2 \cdot E_i\left[\tau_i^2\right] \cdot \left(\delta^d_{pool}\right)^2 - \frac{1}{2g}\left(c\rho\sigma^2+1\right)^2 \tau_h^2 \cdot \left(\delta^d_{pool}\right)^2\\
&+ \frac{\rho\sigma^2}{2}\left(\delta^d_{pool}\right)^2 + c\frac{\left(\rho\sigma^2\right)^2}{2}\left(\delta^d_{pool}\right)^2 && \text{(A44)}\\
=& \left(\frac{1}{g}\left(c\rho\sigma^2+1\right)^2 \cdot E_i\left[\tau_i^2\right] - \frac{1}{2g}\left(c\sigma^2\rho+1\right)\left(\left(c\sigma^2\rho+1\right)\tau_h^2 - g\sigma^2\rho\right)\right)\left(\delta^d_{pool}\right)^2. && \text{(A45)}
\end{aligned}$$

When p_h is so high that such a deviation contract is feasible only at the margin, the payoff $U(\tau_h, w^d_{pool})$ is equal to $U(\tau_h, w^c_h)$, which in turn equals $S(\tau_h, w^c_h)$. Using (3) and $\beta^c_h = c\rho\sigma^2 \cdot \delta^c_h$,

$$\begin{aligned}
S(\tau_h, w^c_h) &= \frac{\tau_h^2}{g}(\beta+\delta) - \frac{\rho\sigma^2}{2}\delta^2 - \frac{\tau_h^2}{2g}(\beta+\delta)^2 - \frac{\beta^2}{2c} && \text{(A46)}\\
&= \frac{\tau_h^2}{g}\left(c\sigma^2\rho+1\right)\delta^c_h - \frac{\rho\sigma^2}{2}(\delta^c_h)^2 - \frac{\tau_h^2}{2g}\left(\left(c\sigma^2\rho+1\right)\right)^2(\delta^c_h)^2 - \frac{\left(c\sigma^2\rho\right)^2}{2c}(\delta^c_h)^2 && \text{(A47)}\\
&= \frac{\tau_h^2}{g}\left(c\sigma^2\rho+1\right)\delta^c_h - \frac{1}{2g}\left(c\sigma^2\rho+1\right)\left(\left(c\sigma^2\rho+1\right)\tau_h^2 + g\sigma^2\rho\right)(\delta^c_h)^2. && \text{(A48)}
\end{aligned}$$

So we have $U(\tau_h, w^d_{pool}) = U(\tau_h, w^c_h)$ if

$$\begin{aligned}
&\left(\frac{1}{g}\left(c\rho\sigma^2+1\right)^2 \cdot E_i\left[\tau_i^2\right] - \frac{1}{2g}\left(c\sigma^2\rho+1\right)\left(\left(c\sigma^2\rho+1\right)\tau_h^2 - g\sigma^2\rho\right)\right)\left(\delta^d_{pool}\right)^2\\
&= \frac{\tau_h^2}{g}\left(c\sigma^2\rho+1\right)\delta^c_h - \frac{1}{2g}\left(c\sigma^2\rho+1\right)\left(\left(c\sigma^2\rho+1\right)\tau_h^2 + g\sigma^2\rho\right)(\delta^c_h)^2. && \text{(A49)}
\end{aligned}$$

The right-hand side does not depend on p_h. The left-hand side is increasing in p_h if it is increasing in $E_i\left[\tau_i^2\right]$, since $\frac{\partial}{\partial p_h}E_i\left[\tau_i^2\right] = \frac{\partial}{\partial p_h}\left(p_h\tau_h^2+(1-p_h)\,\tau_\ell^2\right) = \left(\tau_h^2-\tau_\ell^2\right)$ and since p_h enters δ^d_{pool} only through $E_i\left[\tau_i^2\right]$. Using (A39) to describe $U(\tau_h, w^d_{pool})$,

$$U(\tau_h, w^d_{pool}) = \left(\frac{1}{g}\left(\beta+\delta\right)\left(1-\beta-\delta\right)\right)\cdot E_i\left[\tau_i^2\right] - \frac{\beta^2}{c} + \frac{\beta^2}{2c} - \frac{\rho\sigma^2}{2}\delta^2 + \frac{(\beta+\delta)^2}{2g}\tau_h^2,$$

and applying the envelope theorem (cf. the two first-order conditions), we find

$$\begin{aligned}\frac{d}{dE_i\left[\tau_i^2\right]}U(\tau_h, w^d_{pool}) =& \frac{1}{g}\left(\beta^d_{pool}+\delta^d_{pool}\right)\left(1-\beta^d_{pool}-\delta^d_{pool}\right)\\ &+\frac{\partial U(\tau_h, w^d_{pool})}{\partial\beta^d_{pool}}\frac{\partial\beta^d_{pool}}{\partial E_i\left[\tau_i^2\right]}+\frac{\partial U(\tau_h, w^d_{pool})}{\partial\delta^d_{pool}}\frac{\partial\delta^d_{pool}}{\partial E_i\left[\tau_i^2\right]}\\ =&\left(\delta^d_{pool}+\beta^d_{pool}\right)\left(1-\beta^d_{pool}-\delta^d_{pool}\right),\end{aligned} \tag{A50}$$

which is positive if $\delta^d_{pool}+\beta^d_{pool}<1$. That must be the case if the firm is to break even: If $\delta^d_{pool}+\beta^d_{pool}>1$, then a high-talent CEO's incentives would more distorted than under the separating contract w_h^c, since under that contract we have $\beta_h^c+\delta_h^c<1$:

$$\begin{aligned}\beta_h^c+\delta_h^c &= \frac{\left(c\sigma^2\rho+1\right)\tau_h^2+\left(c\sigma^2\rho+1\right)\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}{\left(c\sigma^2\rho+1\right)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}\\ &= \frac{\left(c\sigma^2\rho+1\right)\tau_h^2+\left(c\sigma^2\rho+1\right)\sqrt{\left(\tau_h^2-\tau_\ell^2+\frac{g\sigma^2\rho}{(c\sigma^2\rho+1)}\right)^2-\frac{(g\sigma^2\rho)^2\left((c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho\right)}{(c\sigma^2\rho+1)^2\left((c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho\right)}}}{\left(c\sigma^2\rho+1\right)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}\\ &< \frac{\left(c\sigma^2\rho+1\right)\tau_h^2+\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2+\frac{g\sigma^2\rho}{(c\sigma^2\rho+1)}\right)}{\left(c\sigma^2\rho+1\right)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}\\ &= 1.\end{aligned}$$

In addition, a low-talent CEO's incentives would be distorted instead of being efficient, since $\delta^d_{pool}>\delta^*_\ell$:

$$\frac{p_h\tau_h^2+(1-p_h)\,\tau_\ell^2}{2\left(c\rho\sigma^2+1\right)\left(p_h\tau_h^2+(1-p_h)\,\tau_\ell^2\right)-\left(c\rho\sigma^2+1\right)\tau_h^2+g\sigma^2\rho} > \frac{\tau_\ell^2}{\left(c\rho\sigma^2+1\right)\tau_\ell^2+g\sigma^2\rho}$$

$$\left(p_h \tau_h^2 + (1-p_h)\,\tau_\ell^2\right)\left(c\rho\sigma^2+1\right)\tau_\ell^2 + \left(p_h \tau_h^2 + (1-p_h)\,\tau_\ell^2\right) g\sigma^2\rho$$
$$> 2\left(p_h \tau_h^2 + (1-p_h)\,\tau_\ell^2\right)\left(c\rho\sigma^2+1\right)\tau_\ell^2 - \left(c\rho\sigma^2+1\right)\tau_h^2\tau_\ell^2 + g\sigma^2\rho\tau_\ell^2$$

$$\left(p_h \tau_h^2 + (1-p_h)\,\tau_\ell^2 - \tau_\ell^2\right) g\sigma^2\rho$$
$$> \left(2\left(p_h \tau_h^2 + (1-p_h)\,\tau_\ell^2\right) - \tau_h^2 - \left(p_h \tau_h^2 + (1-p_h)\,\tau_\ell^2\right)\right)\left(c\rho\sigma^2+1\right)\tau_\ell^2$$

$$p_h\left(\tau_h^2 - \tau_\ell^2\right) g\sigma^2\rho > -\left(1-p_h\right)\left(c\rho\sigma^2+1\right)\left(\tau_h^2-\tau_\ell^2\right)\tau_\ell^2,$$

which is satisfied.

So the overall surplus would be reduced if $\delta^d_{pool} + \beta^d_{pool} > 1$, and both types of CEO would prefer the pooling contract, which means that the zero-profit condition must be violated. Hence, we must have $\delta^d_{pool} + \beta^d_{pool} < 1$ for a pooling deviation to be feasible. Therefore, we must have $\frac{\partial}{\partial p_h} U(\tau_h, w^d_{pool}) > 0$.

We can now solve (A49) for p_h, which defines an upper bound p_o such that if $p_h < p_o$, there exists no profitable pooling deviation contract. Rearrange and simplify (A49),

$$\frac{1}{g}\left(c\rho\sigma^2+1\right)\left(\left(c\rho\sigma^2+1\right)\cdot E_i\left[\tau_i^2\right] - \frac{1}{2}\left(\left(c\rho\sigma^2+1\right)\tau_h^2 - g\sigma^2\rho\right)\right)\left(\delta^d_{pool}\right)^2$$
$$= \frac{1}{g}\left(c\rho\sigma^2+1\right)\left(\tau_h^2\cdot\delta_h^c - \frac{1}{2}\left(\left(c\rho\sigma^2+1\right)\tau_h^2 + g\sigma^2\rho\right)\left(\delta_h^c\right)^2\right) \tag{A51}$$

$$\left(\left(c\rho\sigma^2+1\right)\cdot E_i\left[\tau_i^2\right] - \frac{1}{2}\left(\left(c\rho\sigma^2+1\right)\tau_h^2 - g\sigma^2\rho\right)\right)\left(\delta^d_{pool}\right)^2$$
$$= \left(\tau_h^2\cdot\delta_h^c - \frac{1}{2}\left(\left(c\rho\sigma^2+1\right)\tau_h^2 + g\sigma^2\rho\right)\left(\delta_h^c\right)^2\right) \tag{A52}$$

Substitute δ^d_{pool} and simplify,

$$\frac{1}{2}\left(2\left(c\rho\sigma^2+1\right)\cdot E_i\left[\tau_i^2\right] - \left(c\rho\sigma^2+1\right)\tau_h^2 + g\sigma^2\rho\right)\left(\frac{E_i\left[\tau_i^2\right]}{2\left(c\rho\sigma^2+1\right)\cdot E_i\left[\tau_i^2\right] - \left(c\rho\sigma^2+1\right)\tau_h^2 + g\sigma^2\rho}\right)^2$$
$$= \left(\tau_h^2\cdot\delta_h^c - \frac{1}{2}\left(\left(c\rho\sigma^2+1\right)\tau_h^2 + g\sigma^2\rho\right)\left(\delta_h^c\right)^2\right) \tag{A53}$$

$$\frac{1}{2}\frac{\left(E_i\left[\tau_i^2\right]\right)^2}{2\left(c\rho\sigma^2+1\right)\cdot E_i\left[\tau_i^2\right] - \left(c\rho\sigma^2+1\right)\tau_h^2 + g\sigma^2\rho} = \left(\tau_h^2\cdot\delta_h^c - \frac{1}{2}\left(\left(c\rho\sigma^2+1\right)\tau_h^2 + g\sigma^2\rho\right)\left(\delta_h^c\right)^2\right)$$

$$
\begin{aligned}
0 = &\left(E_i\left[\tau_i^2\right]\right)^2 \\
&- 2\left(2\tau_h^2 \cdot \delta_h^c - \left(\left(c\rho\sigma^2+1\right)\tau_h^2 + g\sigma^2\rho\right)\left(\delta_h^c\right)^2\right)\left(c\rho\sigma^2+1\right)\cdot E_i\left[\tau_i^2\right] \\
&+ \left(\left(c\rho\sigma^2+1\right)\tau_h^2 - g\sigma^2\rho\right)\left(2\tau_h^2\cdot\delta_h^c - \left(\left(c\rho\sigma^2+1\right)\tau_h^2 + g\sigma^2\rho\right)\left(\delta_h^c\right)^2\right)
\end{aligned}
$$

Solve this quadratic equation in $E_i\left[\tau_i^2\right]$, to obtain the cut-off value[1]

$$
\left(E_i\left[\tau_i^2\right]\right)_{\max} = \xi(\delta_h^c)\left(c\rho\sigma^2+1\right) + \sqrt{\left(\left(\xi(\delta_h^c)\left(c\rho\sigma^2+1\right)\right)^2 - \left(\left(c\rho\sigma^2+1\right)\tau_h^2 - g\sigma^2\rho\right)\xi(\delta_h^c)\right)},
$$

where

$$
\xi(\delta_h^c) \equiv \left(2\tau_h^2\cdot\delta_h^c - \left(\left(c\rho\sigma^2+1\right)\tau_h^2 + g\sigma^2\rho\right)\left(\delta_h^c\right)^2\right).
$$

Since by definition, $\left(E_i\left[\tau_i^2\right]\right)_{\max} = \left(p_o\tau_h^2 + (1-p_o)\,\tau_\ell^2\right)$, the corresponding upper bound p_o is then

$$
p_o = \frac{\left(E_i\left[\tau_i^2\right]\right)_{\max}}{\tau_h^2 - \tau_\ell^2} - \frac{\tau_\ell^2}{\tau_h^2 - \tau_\ell^2},
$$

which after substituting $\left(E_i\left[\tau_i^2\right]\right)_{\max}$ yields

$$
p_o = \frac{1}{\tau_h^2-\tau_\ell^2}\xi(\delta_h^c)\left(c\rho\sigma^2+1\right) - \frac{\tau_\ell^2}{\tau_h^2-\tau_\ell^2} + \frac{\sqrt{\left(\left(\xi(\delta_h^c)\left(c\rho\sigma^2+1\right)\right)^2 - \left(\left(c\rho\sigma^2+1\right)\tau_h^2 - g\sigma^2\rho\right)\xi(\delta_h^c)\right)}}{\tau_h^2-\tau_\ell^2}.
$$

Substitute $\left(\left(c\rho\sigma^2+1\right)\tau_h^2 + g\sigma^2\rho\right) = \frac{\tau_h^2}{\delta_h^*}$ and $\xi(\delta_h^c)$, to obtain

$$
p_o = \frac{\left(c\rho\sigma^2+1\right)\tau_h^2\left(2\delta_h^c - \frac{\left(\delta_h^c\right)^2}{\delta_h^*}\right) - \tau_\ell^2}{\tau_h^2-\tau_\ell^2} + \frac{\sqrt{\left(\left(c\rho\sigma^2+1\right)\tau_h^2\left(2\delta_h^c - \frac{\left(\delta_h^c\right)^2}{\delta_h^*}\right)\right)^2 - \left(\left(c\rho\sigma^2+1\right)\tau_h^2 - g\sigma^2\rho\right)\tau_h^2\left(2\delta_h^c - \frac{\left(\delta_h^c\right)^2}{\delta_h^*}\right)}}{\tau_h^2-\tau_\ell^2}. \quad \text{(A54)}
$$

[1] The equation has a second solution, in which the square root is subtracted. However, at that value, $U(\tau_h, w_{pool}^d)$ is decreasing in p_h, which is inconsistent with the contract being feasible. As shown above (see the text around equation (A50)), $U(\tau_h, w_{pool}^d)$ is increasing in p_h in the range where pooling deviation contracts are feasible. As is easily confirmed, with very high values of p_0, δ_{pool}^d is close to δ_h^*; in contrast, the distortion in δ_h^c does not depend on p_h. So for very high values of p_h, we must have $U(\tau_h, w_{pool}^d) > U(\tau_h, w_h^c)$, and the solution in which the square root is added is the relevant cut-off for $E_i\left[\tau_i^2\right]$.

A.10 Properties of the Existence Condition

An equilibrium separating contract exists if $p_h \leq p_o$, where p_o is defined in (A54).

For low values of τ_ℓ, p_o is positive but smaller than one. We have:

$$\lim_{\tau_\ell \to 0} \frac{\tau_h^2 + \sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}{\left(c\sigma^2\rho+1\right)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} = \frac{2\tau_h^2}{\left(c\sigma^2\rho+1\right)2\tau_h^2+g\sigma^2\rho}$$

$$\lim_{\tau_\ell \to 0}\left(2\delta_h^c - \frac{\left(\delta_h^c\right)^2}{\delta_h^*}\right) = \left(2\frac{2\tau_h^2}{(c\sigma^2\rho+1)2\tau_h^2+g\sigma^2\rho} - \frac{\left(\frac{2\tau_h^2}{(c\sigma^2\rho+1)2\tau_h^2+g\sigma^2\rho}\right)^2}{\frac{\tau_h^2}{(c\sigma^2\rho+1)\tau_h^2+g\sigma^2\rho}}\right) = \left(c\sigma^2\rho+1\right)\left(\frac{2\tau_h^2}{(c\sigma^2\rho+1)2\tau_h^2+g\sigma^2\rho}\right)^2$$

So

$$\begin{aligned}
\lim_{\tau_\ell \to 0} p_o =& \frac{\left(c\rho\sigma^2+1\right)^2\tau_h^2\left(\frac{2\tau_h^2}{(c\sigma^2\rho+1)2\tau_h^2+g\sigma^2\rho}\right)^2}{\tau_h^2} \\
&+ \frac{\sqrt{\left(\left(c\rho\sigma^2+1\right)^2\tau_h^2\left(\frac{2\tau_h^2}{(c\sigma^2\rho+1)2\tau_h^2+g\sigma^2\rho}\right)^2\right)^2 - \left(\left(c\rho\sigma^2+1\right)\tau_h^2 - g\sigma^2\rho\right)\tau_h^2\left(c\sigma^2\rho+1\right)\left(\frac{2\tau_h^2}{(c\sigma^2\rho+1)2\tau_h^2+g\sigma^2\rho}\right)^2}}{\tau_h^2} \\
=& \frac{\left(2\left(c\sigma^2\rho+1\right)\tau_h^2\right)^2 + \sqrt{\left(4\left(c\sigma^2\rho+1\right)\tau_h^2 g\sigma^2\rho + \left(g\sigma^2\rho\right)^2\right)^2 - \left(g\sigma^2\rho\right)^2}}{\left(2\left(c\sigma^2\rho+1\right)\tau_h^2+g\sigma^2\rho\right)^2} \\
<& \frac{\left(2\left(c\sigma^2\rho+1\right)\tau_h^2\right)^2 + \sqrt{\left(4\left(c\sigma^2\rho+1\right)\tau_h^2 g\sigma^2\rho + \left(g\sigma^2\rho\right)^2\right)^2}}{\left(2\left(c\sigma^2\rho+1\right)\tau_h^2+g\sigma^2\rho\right)^2} \\
=& 1.
\end{aligned}$$

So $\lim_{\tau_\ell \to 0} p_o \in (0,1)$. For increasing values of τ_ℓ, p_o decreases, and in the limit as $\tau_\ell = \tau_h$, we have $p_o = 0$, i.e., the separating equilibrium does not exist. It is easily confirmed that (A49) is satisfied in the limit as $\tau_\ell \to \tau_h$: Rearrange (A49), to define $\Delta U = U(\tau_h, w_{pool}^d) - U(\tau_h, w_h^c)$,

$$\begin{aligned}
\Delta U =& \left(\frac{1}{g}\left(c\rho\sigma^2+1\right)^2\left(p_h\tau_h^2+(1-p_h)\tau_\ell^2\right) - \frac{1}{2g}\left(c\sigma^2\rho+1\right)\left(\left(c\sigma^2\rho+1\right)\tau_h^2 - g\sigma^2\rho\right)\right)\left(\delta_{pool}^d\right)^2 \\
&- \frac{\tau_h^2}{g}\left(c\sigma^2\rho+1\right)\delta_h^c + \frac{1}{2g}\left(c\sigma^2\rho+1\right)\left(\left(c\sigma^2\rho+1\right)\tau_h^2+g\sigma^2\rho\right)\left(\delta_h^c\right)^2. \qquad \text{(A55)}
\end{aligned}$$

We have $\lim_{\tau_\ell \to \tau_h} \delta^d_{pool} = \lim_{\tau_\ell \to \tau_h} \delta^c_h = \delta^*_h$, so

$$
\begin{aligned}
\lim_{\tau_\ell \to \tau_h} \Delta U = & \left(\frac{1}{g} \left(c\rho\sigma^2 + 1 \right)^2 \tau_h^2 - \frac{1}{2g} \left(c\sigma^2\rho + 1 \right) \left(\left(c\sigma^2\rho + 1 \right) \tau_h^2 - g\sigma^2\rho \right) \right) \left(\delta^*_h \right)^2 \\
& - \frac{\tau_h^2}{g} \left(c\sigma^2\rho + 1 \right) \cdot \delta^*_h + \frac{1}{2g} \left(c\sigma^2\rho + 1 \right) \left(\left(c\sigma^2\rho + 1 \right) \tau_h^2 + g\sigma^2\rho \right) \left(\delta^*_h \right)^2 \\
= & \frac{1}{2g} \left(c\sigma^2\rho + 1 \right) \left(2 \left(c\rho\sigma^2 + 1 \right) \tau_h^2 - \left(c\sigma^2\rho + 1 \right) \tau_h^2 + g\sigma^2\rho \right) \left(\delta^*_h \right)^2 \\
& - \frac{1}{2g} \left(c\sigma^2\rho + 1 \right) \left(\left(c\sigma^2\rho + 1 \right) \tau_h^2 + g\sigma^2\rho \right) \left(\delta^*_h \right)^2 \\
= & 0.
\end{aligned}
$$

Also, the slope of ΔU in τ_ℓ^2 is negative in the limit as $\tau_\ell \to \tau_h$:

$$
\begin{aligned}
\frac{\partial}{\partial \tau_\ell^2} \Delta U = & \left(1 - p_h \right) \frac{1}{g} \left(c\rho\sigma^2 + 1 \right)^2 \left(\delta^d_{pool} \right)^2 \\
& + \frac{\left(c\rho\sigma^2 + 1 \right) \left(2 \left(c\rho\sigma^2 + 1 \right) \left(p_h \tau_h^2 + \left(1 - p_h \right) \tau_\ell^2 \right) - \left(c\sigma^2\rho + 1 \right) \tau_h^2 + g\sigma^2\rho \right) \left(2\delta^d_{pool} \left(\frac{\partial}{\partial \tau_\ell^2} \delta^d_{pool} \right) \right)}{2g} \\
& - \frac{\left(c\sigma^2\rho + 1 \right) \tau_h^2 \left(\frac{\partial}{\partial \tau_\ell^2} \delta^c_h \right)}{g} + \frac{\left(c\sigma^2\rho + 1 \right) \left(\left(c\sigma^2\rho + 1 \right) \tau_h^2 + g\sigma^2\rho \right) \left(2\delta^c_h \left(\frac{\partial}{\partial \tau_\ell^2} \delta^c_h \right) \right)}{2g},
\end{aligned}
\tag{A56}
$$

and taking limits,

$$
\lim_{\tau_\ell \to \tau_h} \frac{\partial}{\partial \tau_\ell^2} \Delta U = -p_h \frac{\sigma^2\rho \left(c\sigma^2\rho + 1 \right) \tau_h^2}{\left(\left(c\sigma^2\rho + 1 \right) \tau_h^2 + g\sigma^2\rho \right)^2} < 0.
$$

(Note that the slope is zero if $p_0 = 0$, so $\lim_{\tau_\ell \to \tau_h} p_o = 0$.) That is, the existence condition is the most restrictive when the adverse selection problem is almost insignificant; and it becomes less restrictive as the adverse selection problem becomes more relevant (i.e., the talent levels become more different).

A.11 Properties of Equilibrium Contracts under Competition for Talent

The comparative statics for β^c_ℓ and δ^c_ℓ are omitted since they can be found above (recall that $\beta^c_\ell = \beta^*_\ell$ and $\delta^c_\ell = \delta^*_\ell$).

$\frac{\partial}{\partial c}\delta_h^c < 0$:

$$
\begin{aligned}
\frac{\partial}{\partial c}\delta_h^c &= \frac{\partial}{\partial c}\frac{\tau_h^2+\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} \\
&= \frac{\frac{-2\frac{\left(\tau_\ell^2\right)^2\left(\sigma^2\rho\right)^2 g\left(\tau_h^2-\tau_\ell^2\right)}{\left((c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho\right)^2}}{2\cdot\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}} - \frac{\tau_h^2+\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}\sigma^2\rho\left(2\tau_h^2-\tau_\ell^2\right)}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} \\
&= -\frac{\frac{\left(\tau_\ell^2\right)^2\left(\sigma^2\rho\right)^2 g\frac{\left(\tau_h^2-\tau_\ell^2\right)}{\left((c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho\right)^2}}{\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}} + \delta_h^c\cdot\sigma^2\rho\left(2\tau_h^2-\tau_\ell^2\right)}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} \\
&< 0.
\end{aligned}
$$

$\frac{\partial}{\partial c}\beta_h^c > 0$:

$$
\begin{aligned}
\frac{\partial}{\partial c}\beta_h^c &= \frac{\partial}{\partial c}c\sigma^2\rho\frac{\tau_h^2+\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} \\
&= \sigma^2\rho\delta_h^c + c\sigma^2\rho\frac{\partial}{\partial c}\delta_h^c \\
&= \sigma^2\rho\delta_h^c - c\sigma^2\rho\frac{\frac{\left(\tau_\ell^2\right)^2\left(\sigma^2\rho\right)^2 g\frac{\left(\tau_h^2-\tau_\ell^2\right)}{\left((c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho\right)^2}}{\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} - c\sigma^2\rho\frac{\sigma^2\rho\left(2\tau_h^2-\tau_\ell^2\right)}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}\delta_h^c \\
&= \sigma^2\rho\frac{2\tau_h^2-\tau_\ell^2+g\sigma^2\rho}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}\delta_h^c - c\sigma^2\rho\frac{\frac{\left(\tau_\ell^2\right)^2\left(\sigma^2\rho\right)^2 g\frac{\left(\tau_h^2-\tau_\ell^2\right)}{\left((c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho\right)^2}}{\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}.
\end{aligned}
$$

That's positive if

$$\sigma^2\rho\frac{2\tau_h^2-\tau_\ell^2+g\sigma^2\rho}{\left(c\sigma^2\rho+1\right)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}\delta_h^c > c\sigma^2\rho\frac{\frac{\left(\tau_\ell^2\right)^2\left(\sigma^2\rho\right)^2 g\frac{\left(\tau_h^2-\tau_\ell^2\right)}{\left(\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho\right)^2}}{\sqrt{\frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}}}}{\left(c\sigma^2\rho+1\right)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}$$

$$\left(2\tau_h^2-\tau_\ell^2+g\sigma^2\rho\right)\delta_h^c > c\frac{\left(\sigma^2\rho\right)^2 g\left(\tau_h^2-\tau_\ell^2\right)\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}\cdot\delta_\ell^*}{\sqrt{\frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}}}.$$

Since $\delta_h^c > \delta_\ell^*$, a sufficient condition is

$$\left(2\tau_h^2-\tau_\ell^2+g\sigma^2\rho\right) > c\frac{\left(\sigma^2\rho\right)^2 g\left(\tau_h^2-\tau_\ell^2\right)\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}}{\sqrt{\frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}}}$$

Since

$$\sqrt{\frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}} = \left(\tau_h^2-\tau_\ell^2\right)\sqrt{\frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho\frac{\tau_h^2+\tau_\ell^2}{\tau_h^2-\tau_\ell^2}}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}}, \tag{A57}$$

the sufficient condition is

$$\left(2\tau_h^2-\tau_\ell^2+g\sigma^2\rho\right) > c\frac{\left(\sigma^2\rho\right)^2 g\left(\tau_h^2-\tau_\ell^2\right)\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}}{\left(\tau_h^2-\tau_\ell^2\right)\sqrt{\frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho\frac{\tau_h^2+\tau_\ell^2}{\tau_h^2-\tau_\ell^2}}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}}}$$

$$\left(2\tau_h^2-\tau_\ell^2+g\sigma^2\rho\right) > c\left(\sigma^2\rho\right)^2 g\frac{\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}}{\sqrt{\frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho\frac{\tau_h^2+\tau_\ell^2}{\tau_h^2-\tau_\ell^2}}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}}}.$$

Since the square root is larger than one, a sufficient condition is

$$\left(2\tau_h^2-\tau_\ell^2+g\sigma^2\rho\right) > c\left(\sigma^2\rho\right)^2 g\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}$$

$$\left(2\tau_h^2-\tau_\ell^2+g\sigma^2\rho\right)\left(\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho\right)-c\left(\sigma^2\rho\right)^2 g\tau_\ell^2 > 0$$

$$\tau_\ell^2\left(2\tau_h^2-\tau_\ell^2\right)\left(c\sigma^2\rho+1\right)+2\tau_h^2 g\sigma^2\rho+\sigma^4\rho^2g^2>0,$$

which is satisfied.

$\frac{\partial}{\partial c}L(\tau_h, w_h^c) > 0$**:**

$$\begin{aligned}
\frac{\partial}{\partial c}L(\tau_h, w_h^c) \propto & \frac{\partial}{\partial c}\delta_h^c+\frac{\partial}{\partial c}\beta_h^c \\
= & -\frac{\frac{\left(\tau_\ell^2\right)^2\left(\sigma^2\rho\right)^2 g\frac{\left(\tau_h^2-\tau_\ell^2\right)}{\left(\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho\right)^2}}{\sqrt{\frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}}}}{\left(c\sigma^2\rho+1\right)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}-\frac{\sigma^2\rho\left(2\tau_h^2-\tau_\ell^2\right)}{\left(c\sigma^2\rho+1\right)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}\delta_h^c \\
& +\sigma^2\rho\frac{2\tau_h^2-\tau_\ell^2+g\sigma^2\rho}{\left(c\sigma^2\rho+1\right)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}\delta_h^c-c\sigma^2\rho\frac{\frac{\left(\tau_\ell^2\right)^2\left(\sigma^2\rho\right)^2 g\frac{\left(\tau_h^2-\tau_\ell^2\right)}{\left(\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho\right)^2}}{\sqrt{\frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}}}}{\left(c\sigma^2\rho+1\right)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} \\
= & -\left(c\sigma^2\rho+1\right)\frac{\frac{\left(\tau_\ell^2\right)^2\left(\sigma^2\rho\right)^2 g\frac{\left(\tau_h^2-\tau_\ell^2\right)}{\left(\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho\right)^2}}{\sqrt{\frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}}}}{\left(c\sigma^2\rho+1\right)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}+\frac{\sigma^4\rho^2 g}{\left(c\sigma^2\rho+1\right)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}\delta_h^c.
\end{aligned}$$

That's positive if

$$\left(c\sigma^2\rho+1\right)\frac{\left(\tau_h^2-\tau_\ell^2\right)\frac{\left(\tau_\ell^2\right)^2}{\left(\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho\right)^2}}{\sqrt{\frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}}}<\delta_h^c$$

$$\frac{\left(\tau_h^2-\tau_\ell^2\right)\frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}}{\sqrt{\frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}}}<\frac{\delta_h^c}{\delta_\ell^*}$$

The square root in the fraction on the left-hand side is larger than $\left(\tau_h^2-\tau_\ell^2\right)$ (see (A57)), so the condition is satisfied if

$$\frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}<\frac{\delta_h^c}{\delta_\ell^*}.$$

We show in the proof of Corollary 4 (below) that $\delta_h^c \geq \delta_h^*$. Since $\delta_h^* > \delta_\ell^*$ (cf. (12)), this implies that $\delta_h^c \geq \delta_\ell^*$, so the right-hand side is larger than one. The left-hand side is smaller than one, so the condition is satisfied.

$\frac{\partial}{\partial\sigma^2\rho}\delta_h^c < 0$**:**

$$\frac{\partial}{\partial\sigma^2\rho}\delta_h^c = \frac{\partial}{\partial\sigma^2\rho}\frac{\tau_h^2 + \sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}$$

$$= \frac{\frac{g\left(\tau_h^2-\tau_\ell^2\right)\frac{\tau_\ell^4}{\left((c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho\right)^2}}{\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} - \frac{\delta_h^c}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}\left(c\left(2\tau_h^2-\tau_\ell^2\right)+g\right)$$

$$= \frac{\frac{\left(\tau_h^2-\tau_\ell^2\right)g\frac{\tau_\ell^2}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}{\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}\delta_\ell^* - \frac{\left(c\left(2\tau_h^2-\tau_\ell^2\right)+g\right)}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}\delta_h^c$$

That's negative if

$$\frac{\left(\tau_h^2-\tau_\ell^2\right)g\frac{\tau_\ell^2}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}{\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}\delta_\ell^* < \left(c\left(2\tau_h^2-\tau_\ell^2\right)+g\right)\delta_h^c.$$

We show in the proof of Corollary 4 (below) that $\delta_h^c \geq \delta_h^*$. Since $\delta_h^* > \delta_\ell^*$ (cf. (12)), this implies that $\delta_h^c \geq \delta_\ell^*$. Since the square root in the denominator on the left-hand side is larger than $\left(\tau_h^2-\tau_\ell^2\right)$ (see (A57)), a sufficient condition is

$$g\frac{\tau_\ell^2}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho} < c\left(2\tau_h^2-\tau_\ell^2\right)+g.$$

That is satisfied, since the fraction is smaller than one.

$\frac{\partial}{\partial\sigma^2\rho}\beta_h^c > 0$**:**

$$
\begin{aligned}
\frac{\partial}{\partial\sigma^2\rho}\beta_h^c &= \frac{\partial}{\partial\sigma^2\rho}c\sigma^2\rho\frac{\tau_h^2+\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} \\
&= c\delta_h^c + c\sigma^2\rho\left(\frac{\frac{g\tau_\ell^4\frac{\left(\tau_h^2-\tau_\ell^2\right)}{\left((c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho\right)^2}}{\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} - \frac{\delta_h^c\left(c\left(2\tau_h^2-\tau_\ell^2\right)+g\right)}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}\right) \\
&= \frac{\left((c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho\right)c-c\sigma^2\rho\left(c\left(2\tau_h^2-\tau_\ell^2\right)+g\right)}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}\delta_h^c + \frac{\frac{c\sigma^2\rho g\tau_\ell^4\frac{\left(\tau_h^2-\tau_\ell^2\right)}{\left((c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho\right)^2}}{\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} \\
&= \frac{c\left(2\tau_h^2-\tau_\ell^2\right)}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}\delta_h^c + c\sigma^2\rho\frac{\frac{g\tau_\ell^4\frac{\left(\tau_h^2-\tau_\ell^2\right)}{\left((c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho\right)^2}}{\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} \\
&> 0.
\end{aligned}
$$

$\frac{\partial}{\partial\sigma^2\rho}L(\tau_h, w_h^c) < 0$:

$$\frac{\partial}{\partial\sigma^2\rho}L(\tau_h, w_h^c) \propto \frac{\partial}{\partial\sigma^2\rho}\left(\delta_h^c + \beta_h^c\right)$$

$$= \frac{\frac{g\tau_\ell^4 \frac{\left(\tau_h^2-\tau_\ell^2\right)}{\left((c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho\right)^2}}{\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} - \frac{\left(c\left(2\tau_h^2-\tau_\ell^2\right)+g\right)}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}\delta_h^c$$

$$+ \frac{c\left(2\tau_h^2-\tau_\ell^2\right)}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}\delta_h^c + c\sigma^2\rho\frac{\frac{g\tau_\ell^4 \frac{\left(\tau_h^2-\tau_\ell^2\right)}{\left((c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho\right)^2}}{\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}$$

$$= \left(c\sigma^2\rho+1\right)\frac{\frac{g\tau_\ell^4 \frac{\left(\tau_h^2-\tau_\ell^2\right)}{\left((c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho\right)^2}}{\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} - \frac{g}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}\delta_h^c.$$

That's negative if

$$\frac{\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)\frac{\tau_\ell^4}{\left((c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho\right)^2}}{\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}} < \delta_h^c$$

$$\frac{\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)\frac{\tau_\ell^2}{\left((c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho\right)}}{\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}} < \frac{\delta_h^c}{\delta_\ell^*}.$$

We show in the proof of Corollary 4 (below) that $\delta_h^c \geq \delta_h^*$. Since $\delta_h^* > \delta_\ell^*$ (cf. (12)), this implies that $\delta_h^c \geq \delta_\ell^*$. So the right-hand side is larger than one. The fraction on the left-hand side is larger than $\left(\tau_h^2 - \tau_\ell^2\right)$ (see (A57)), so a sufficient condition is

$$\frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2}{\left(\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho\right)} < 1,$$

which is satisfied.

Limits:

$$\lim_{c\to\infty}\delta_h^c = \lim_{c\to\infty}\frac{\tau_h^2+\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} = \lim_{c\to\infty}\frac{\tau_h^2+\sqrt{\frac{\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2}{\tau_\ell^2}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)} = 0$$

$$\lim_{c\to\infty}\beta_h^c = \lim_{c\to\infty}c\sigma^2\rho\frac{\tau_h^2+\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} = \lim_{c\to\infty}c\sigma^2\rho\frac{\tau_h^2+\sqrt{\frac{\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2}{\tau_\ell^2}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)} = 1$$

$$\lim_{c\to0}\delta_h^c = \lim_{c\to0}\frac{\tau_h^2+\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} = \frac{\tau_h^2+\sqrt{\frac{\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{\tau_\ell^2+g\sigma^2\rho}}}{\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} > 0$$

$$\begin{aligned}\lim_{c\to0}\beta_h^c &= \lim_{c\to0}c\sigma^2\rho\frac{\tau_h^2+\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}\\ &= \lim_{c\to0}c\sigma^2\rho\frac{\tau_h^2+\sqrt{\frac{\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{\tau_\ell^2+g\sigma^2\rho}}}{\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} = 0\end{aligned}$$

$$\lim_{\sigma^2\rho\to0}\delta_h^c = \lim_{\sigma^2\rho\to0}\frac{\tau_h^2+\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} = \frac{\tau_h^2+\sqrt{\frac{\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2}{\tau_\ell^2}}}{2\tau_h^2-\tau_\ell^2} = 1$$

$$\lim_{\sigma^2\rho\to0}\beta_h^c = \lim_{\sigma^2\rho\to0}c\sigma^2\rho\cdot\frac{\tau_h^2+\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} = 0$$

$$\lim_{\sigma^2\rho\to\infty}\delta_h^c = \lim_{\sigma^2\rho\to\infty}\frac{\tau_h^2+\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} = 0$$

$$\lim_{\sigma^2\rho\to\infty}\beta_h^c = \lim_{\sigma^2\rho\to\infty}c\sigma^2\rho\cdot\frac{\tau_h^2+\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}$$
$$= c\frac{\tau_h^2+\sqrt{\frac{\left(\tau_h^2-\tau_\ell^2\right)\left(c\left(\tau_h^2-\tau_\ell^2\right)\tau_\ell^2+g\left(\tau_h^2+\tau_\ell^2\right)\right)}{c\tau_\ell^2+g}}}{c\left(2\tau_h^2-\tau_\ell^2\right)+g} > 0$$

Misreporting:

$$\begin{aligned}
&\lim_{c\to\infty} r(\tau_h, w_h^c) - q(\tau_h, w_h^c)\\
&= \lim_{c\to\infty}\frac{\beta_h^c}{c}\\
&= \lim_{c\to\infty}\sigma^2\rho\frac{\tau_h^2+\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}\\
&= \lim_{c\to\infty}\sigma^2\rho\frac{\tau_h^2+\sqrt{\frac{\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2}{\tau_\ell^2}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)}\\
&= 0.
\end{aligned}$$

A.12 Proof of Corollary 4

In all setups, we have $\beta = c\sigma^2\rho\cdot\delta$. Using (1), (2), and $q = \tau_i L$, we immediately obtain that L, r and q are increasing in δ. From (17) and (12), we have $\delta_h^c > \delta_h^*$ if

$$\frac{\tau_h^2+\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} > \frac{\tau_h^2}{(c\sigma^2\rho+1)\,\tau_h^2+g\sigma^2\rho},$$

or, equivalently,

$$\frac{\tau_h^2 + \left(\tau_h^2 - \tau_\ell^2\right)\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2 + g\sigma^2\rho\frac{\tau_h^2+\tau_\ell^2}{\tau_h^2-\tau_\ell^2}}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}{\left(c\sigma^2\rho+1\right)\left(2\tau_h^2 - \tau_\ell^2\right) + g\sigma^2\rho} > \frac{\tau_h^2}{\left(c\sigma^2\rho+1\right)\tau_h^2 + g\sigma^2\rho}.$$

Since the square root is larger than one, we have

$$\delta_h^c > \frac{2\tau_h^2 - \tau_\ell^2}{\left(c\sigma^2\rho+1\right)\left(2\tau_h^2 - \tau_\ell^2\right) + g\sigma^2\rho}.$$

The fraction on the right-hand side is larger than δ_h^*, since $\tau_h > \tau_\ell$. Thus, we have $\delta_h^c > \delta_h^*$. That in turn implies that $\delta_h^c \geq \delta_\ell^*$ since $\delta_h^* > \delta_\ell^*$ (see (12)). Comparing (13) and (12) shows that and we have $\delta_\ell^{sf} < \delta_\ell^*$.

In all setups, we have $\beta = c\sigma^2\rho \cdot \delta$, so the incentive power is increasing in δ. As just shown, $\delta_h^c > \delta_h^* = \delta_h^{sf}$ and $\delta_\ell^{sf} < \delta_\ell^* = \delta_\ell^c$, so $\varphi_h^c > \varphi_h^* = \varphi_h^{sf}$ and $\varphi_\ell^{sf} < \varphi_\ell^* = \varphi_\ell^c$.

A.13 Proof of Corollary 5

Using $q = \tau_i L$, (2) and (1),

$$r\left(\tau_i, w\right) - q\left(\tau_i, w\right) = \frac{1}{g}\tau_i^2\left(\beta + \delta\right) + \frac{\beta}{c} - \frac{1}{g}\tau_i^2\left(\beta+\delta\right) = \frac{\beta}{c}.$$

In all setups, the equilibrium contract sets $\beta = c\sigma^2\rho\delta$. The result then follows from $\delta_\ell^{sf} < \delta_\ell^* = \delta_\ell^c$ and $\delta_h^{sf} = \delta_h^* < \delta_h^c$.

A.14 Derivation of Equation (18)

The surplus under competition is

$$E_i\left[S(\tau_i, w_i^c)\right] = p_h S(\tau_h, w_h^c) + (1 - p_h) S(\tau_\ell, w_\ell^*).$$

Replace $S(\tau_h, w_h^c)$ and $S(\tau_\ell, w_\ell^*)$ using (3); replace $\beta_\ell^* = c\sigma^2\rho \cdot \delta_\ell^*$; and replace δ_ℓ^*, β_h^c and δ_h^c using (12), (A35) and (17), to obtain

$$E_i\left[S(\tau_i, w_i^c)\right] = p_h\left(c\sigma^2\rho + 1\right)\left(\frac{1}{g}\tau_h^2 - \frac{1}{2g}\tau_h^2\frac{\left(c\sigma^2\rho+1\right)\tau_h^2 + g\sigma^2\rho}{\tau_h^2}\delta_h^c\right)\delta_h^c + (1-p_h)\left(c\rho\sigma^2+1\right)\frac{\tau_\ell^2}{2g}\delta_\ell^*.$$

Replace the fraction inside the parentheses by $\frac{1}{\delta_h^*}$ (cf. (12)), to obtain

$$E_i\left[S(\tau_i, w_i^c)\right] = p_h\left(c\sigma^2\rho + 1\right)\frac{\tau_h^2}{2g}\frac{1}{\delta_h^*}\left(2\delta_h^*\delta_h^c - (\delta_h^c)^2\right) + (1-p_h)\left(c\rho\sigma^2+1\right)\frac{\tau_\ell^2}{2g}\delta_\ell^*. \tag{A58}$$

As is easily verified, this surplus is decreasing in δ_h^c: The more severe the distortion in a high-talent CEO's effort, the lower the surplus.

The surplus in the single-firm setup is

$$E_i\left[S(\tau_i, w_i^{sf})\right] = p_h S(\tau_h, w_h^*) + (1-p_h)S(\tau_\ell, w_\ell^{sf}).$$

Replace $S(\tau_h, w_h^*)$ and $S(\tau_\ell, w_\ell^{sf})$ using (3); replace $\beta_h^* = c\sigma^2\rho \cdot \delta_h^*$; and replace δ_h^*, β_ℓ^{sf} and δ_ℓ^{sf} using (12), (A24) and (13), to obtain

$$E_i\left[S(\tau_i, w_i^{sf})\right] = p_h\left(c\rho\sigma^2+1\right)\frac{\tau_h^2}{2g}\delta_h^* + (1-p_h)\left(c\sigma^2\rho+1\right)\left(\frac{1}{g}\tau_\ell^2 - \frac{1}{2g}\tau_\ell^2\frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2 + g\sigma^2\rho}{\tau_\ell^2}\delta_\ell^{sf}\right)\delta_\ell^{sf}.$$

Replace the fraction inside the parentheses by $\frac{1}{\delta_\ell^*}$(cf. (12)), to obtain

$$E_i\left[S(\tau_i, w_i^{sf})\right] = p_h\left(c\rho\sigma^2+1\right)\frac{\tau_h^2}{2g}\delta_h^* + (1-p_h)\left(c\sigma^2\rho+1\right)\frac{\tau_\ell^2}{2g}\frac{1}{\delta_\ell^*}\left(2\delta_\ell^*\delta_\ell^{sf} - \left(\delta_\ell^{sf}\right)^2\right). \tag{A59}$$

Since $\delta_\ell^{sf} < \delta_\ell^*$, $E_i\left[S(\tau_i, w_i^{sf})\right]$ increases in δ_ℓ^{sf}; that is, if the distortion is stronger (δ_ℓ^{sf} is lower), surplus is reduced.

Define the change in surplus caused by introducing competition for talent,

$$\begin{aligned}\Delta S =& E_i\left[S(\tau_i, w_i^c)\right] - E_i\left[S(\tau_i, w_i^{sf})\right]\\ =& p_h\left(c\sigma^2\rho+1\right)\frac{\tau_h^2}{2g}\frac{1}{\delta_h^*}\left(2\delta_h^*\delta_h^c - (\delta_h^c)^2 - (\delta_h^*)^2\right)\\ &+ (1-p_h)\left(c\rho\sigma^2+1\right)\frac{\tau_\ell^2}{2g}\frac{1}{\delta_\ell^*}\left((\delta_\ell^*)^2 - 2\delta_\ell^*\delta_\ell^{sf} + \left(\delta_\ell^{sf}\right)^2\right).\end{aligned}$$

Rearrange to obtain (18).

A.15 Proof of Proposition 6

The formal results we prove are the following:

(i) (a) $\lim_{c\to\infty}\Delta S > 0$*; (b)* $\lim_{\rho\to 0}\Delta S > 0$*; (c)* $\lim_{\sigma\to 0}\Delta S > 0$*.*

(ii) (a) $\Delta S < 0$ *if* $\sigma^2\rho$ *is large and c small. (b)* $\lim_{\tau_\ell\to 0}\Delta S < 0$*; (c)* $\lim_{\tau_\ell\to\tau_h}\Delta S = 0$ *and* $\lim_{\tau_\ell\to\tau_h}\frac{\partial}{\partial\tau_\ell}\Delta S > 0$*; (d)* $\lim_{p_h\to 1}\Delta S < 0$*; (e)* $\lim_{p_h\to 0}\Delta S = 0$ *and* $\lim_{p_h\to 0}\frac{\partial}{\partial p_h}\Delta S < 0$*.*

A.15.1 Proof of Proposition 6*(i)(a)*, $\lim_{c\to\infty}\Delta S > 0$.

As is easily verified, $\lim_{c\to\infty}\delta_h^{sf} = \lim_{c\to\infty}\delta_\ell^{sf} = \lim_{c\to\infty}\delta_h^{c} = \lim_{c\to\infty}\delta_\ell^{c} = 0$. The contracts provide incentives to exert effort exclusively through β, the compensation based on the reported performance:

$$\lim_{c\to\infty}\beta_h^{sf} = \lim_{c\to\infty}\beta_h^{*} = \lim_{c\to\infty}\frac{c\sigma^2\rho\tau_h^2}{\left(c\sigma^2\rho+1\right)\tau_h^2+g\sigma^2\rho} = 1$$

$$\lim_{c\to\infty}\beta_\ell^{sf} = \lim_{c\to\infty}\frac{c\sigma^2\rho\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho+\frac{p_h}{1-p_h}\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)} = \frac{\tau_\ell^2}{\tau_\ell^2+\frac{p_h}{1-p_h}\left(\tau_h^2-\tau_\ell^2\right)} < 1$$

$$\lim_{c\to\infty}\beta_h^{c} = \lim_{c\to\infty}c\sigma^2\rho\frac{\tau_h^2+\sqrt{\frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}}}{\left(c\sigma^2\rho+1\right)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} = 1$$

$$\lim_{c\to\infty}\beta_\ell^{c} = \lim_{c\to\infty}\beta_\ell^{*} = \lim_{c\to\infty}\frac{c\sigma^2\rho\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho} = 1$$

Thus, a distortion remains in the single-firm setup, so in the limit as manipulation becomes infinitely costly, the competitive setup is more efficient than the single-firm setup. Equilibrium existence is not an issue: Since $\lim_{c\to\infty}\delta_h^c = 0$ and $\lim_{c\to\infty}c\rho\sigma^2\delta_h^c = 1$, we have $\lim_{c\to\infty}\left(c\rho\sigma^2+1\right)\delta_h^c = 1$. Therefore,

$$\begin{aligned}
&\lim_{c\to\infty}\left(\left(2\tau_h^2\cdot\delta_h^c-\left(\left(c\rho\sigma^2+1\right)\tau_h^2+g\sigma^2\rho\right)\left(\delta_h^c\right)^2\right)\left(c\rho\sigma^2+1\right)\right)\\
&= 2\tau_h^2\cdot\lim_{c\to\infty}\left(c\rho\sigma^2+1\right)\delta_h^c-\tau_h^2\cdot\lim_{c\to\infty}\left(c\rho\sigma^2+1\right)^2\left(\delta_h^c\right)^2-g\sigma^2\rho\cdot\lim_{c\to\infty}\left(c\rho\sigma^2+1\right)\left(\delta_h^c\right)^2\\
&= 2\tau_h^2-\tau_h^2-0,
\end{aligned}$$

and thus

$$\lim_{c\to\infty} p_o = \frac{1}{\tau_h^2-\tau_\ell^2}\cdot\left(2\tau_h^2-\tau_h^2\right) - \frac{\tau_\ell^2}{\tau_h^2-\tau_\ell^2} + \frac{1}{\tau_h^2-\tau_\ell^2}\sqrt{\left(\left(2\tau_h^2-\tau_h^2\right)^2 - \tau_h^2\left(2\tau_h^2-\tau_h^2\right)+0\right)},$$

which equals 1.

A.15.2 Proof of Proposition 6*(i)(b,c)*, $\lim_{\sigma\to 0}\Delta S > 0$ and $\lim_{\rho\to 0}\Delta S > 0$.

Consider the incentives in the limit as $\sigma \to 0$ (the results are identical for the limit $\rho \to 0$),

$$\begin{aligned}
\lim_{\sigma\to 0}\delta_h^{sf} &= \lim_{\sigma\to 0}\delta_h^{*} = \lim_{\sigma\to 0}\frac{\tau_h^2}{\left(c\sigma^2\rho+1\right)\tau_h^2+g\sigma^2\rho} = 1\\
\lim_{\sigma\to 0}\delta_\ell^{sf} &= \lim_{\sigma\to 0}\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho+\frac{p_h}{1-p_h}\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)} = \frac{\tau_\ell^2}{\tau_\ell^2+\frac{p_h}{1-p_h}\left(\tau_h^2-\tau_\ell^2\right)} < 1\\
\lim_{\sigma\to 0}\delta_h^{c} &= \lim_{\sigma\to 0}\frac{\tau_h^2+\sqrt{\frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}}}{\left(c\sigma^2\rho+1\right)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho} = 1\\
\lim_{\sigma\to 0}\delta_\ell^{c} &= \lim_{\sigma\to 0}\delta_\ell^{*} = \lim_{\sigma\to 0}\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho} = 1
\end{aligned}$$

A low-talent CEO's decisions in the single-firm setup remain distorted, while they are efficient in the competitive setup; so the surplus produced under the single-firm setup is smaller. The separating equilibrium exists, because $\lim_{\sigma\to 0} p_o = 1$: Since $\lim_{\sigma\to 0}\delta_h^c = 1$, we have

$$\begin{aligned}
\lim_{\sigma\to 0} p_o = \frac{1}{\tau_h^2-\tau_\ell^2}&\left(\left(2\tau_h^2-\left(\tau_h^2+0\right)\right)\cdot 1-\tau_\ell^2+\sqrt{\left(\left(2\tau_h^2-\left(\tau_h^2+0\right)\right)^2-\left(\tau_h^2-0\right)\left(2\tau_h^2-\left(\tau_h^2+0\right)\right)\right)}\right)\\
&= 1.
\end{aligned}$$

A.15.3 Proof of Proposition 6*(ii)(a)*, $\Delta S < 0$ if $\sigma^2\rho$ is large and c small

Since the effects of σ and ρ are similar, we focus on changes in z, where $z = \sigma^2\rho$. Define (simplifying the notation slightly)

$$\Delta S\left(p_h\right) \equiv \left(1-p_h\right)\left(S_\ell^{*}-S_\ell^{sf}\right) - p_h\left(S_h^{*}-S_h^{c}\right).$$

If $p_h > 0$, and the surpluses S_h^* and S_h^c are positive and finite, that is negative if

$$\frac{(1-p_h)\left(S_\ell^* - S_\ell^{sf}\right)}{p_h\left(S_h^* - S_h^c\right)} < 1.$$

We first consider the case when $c = 0$. Observe that (using (18))

$$\begin{aligned}
&\lim_{c\to 0}\lim_{z\to\infty}\frac{S_\ell^* - S_\ell^{sf}}{S_h^* - S_h^c} \\
&= \lim_{c\to 0}\lim_{z\to\infty}\frac{\frac{\tau_\ell^2}{2g}\frac{1}{\left(\frac{\tau_\ell^2}{(cz+1)\tau_\ell^2+gz}\right)}\left(\frac{\tau_\ell^2}{(cz+1)\tau_\ell^2+gz} - \frac{\tau_\ell^2}{(cz+1)\tau_\ell^2+gz+\frac{p_h}{1-p_h}(cz+1)\left(\tau_h^2-\tau_\ell^2\right)}\right)^2}{\frac{\tau_h^2}{2g}\frac{1}{\left(\frac{\tau_h^2}{(cz+1)\tau_h^2+gz}\right)}\left(\frac{\tau_h^2+\sqrt{\frac{(cz+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+gz\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(cz+1)\tau_\ell^2+gz}}}{(cz+1)\left(2\tau_h^2-\tau_\ell^2\right)+gz} - \frac{\tau_h^2}{(cz+1)\tau_h^2+gz}\right)^2} = 0.
\end{aligned} \tag{A60}$$

Also, from (A63),

$$\Delta S(0) = 0,$$

and since $\delta_h^c > \delta_h^*$ (cf. the proof of Corollary 4),

$$\Delta S(1) = -\left(c\sigma^2\rho + 1\right)\frac{\tau_h^2}{2g}\frac{1}{\delta_h^*}\left(\delta_h^c - \delta_h^*\right)^2 < 0.$$

Hence for any $p_h \in [0,1]$, we can find $z(p_h)$ such that if $z \geq z(p_h)$ then

$$\Delta S(p_h) = (1-p_h)\left(S_\ell^* - S_\ell^{sf}\right) - p_h\left(S_h^* - S_h^c\right) \leq 0.$$

From equation (A60) we know that for any arbitrary number $\varepsilon \in (0,1)$ we can make $\frac{(1-p_h)\left(S_\ell^*-S_\ell^{sf}\right)}{p_h\left(S_h^*-S_h^c\right)} \leq \varepsilon$ if we let z be sufficiently large (say if $z \geq z_\varepsilon(p_h)$ for some number $z_\varepsilon(p_h)$). This means that when $z \geq z_\varepsilon(p_h)$,

$$\begin{aligned}
\Delta S(p_h) &= (1-p_h)\left(S_\ell^* - S_\ell^{sf}\right) - p_h\left(S_h^* - S_h^c\right) \\
&\leq \varepsilon p_h\left(S_h^* - S_h^c\right) - p_h\left(S_h^* - S_h^c\right) \quad \left(\text{since } (1-p_h)\left(S_\ell^* - S_\ell^{sf}\right) \leq \varepsilon\cdot p_h\left(S_h^* - S_h^c\right)\right) \\
&= (\varepsilon - 1)\, p_h\left(S_h^* - S_h^c\right) \\
&\leq 0.
\end{aligned}$$

Define

$$\underline{z}\left(p_h\right) = \inf\left\{x | z \geq x \Rightarrow \Delta S\left(p_h\right) \leq 0\right\}.$$

There exists $\overline{z}$, defined as

$$\overline{z} \equiv \max_{x \in [0,1]} \underline{z}\left(x\right).$$

Now we can state the result: If $z \geq \overline{z}$, then $\Delta S\left(p_h\right) \leq 0$ for all $p_h \in [0,1]$. Now, fix any $z \geq \overline{z}$. Since $p_o\left(z\right) \in (0,1)$, if $p_h \leq p_o\left(z\right)$ then the competitive equilibrium exists and it's weakly less efficient than the single firm equilibrium.

By continuity, the same result holds for small $c > 0$. The results $\Delta S\left(0\right) = 0$ and $\Delta S\left(1\right) < 0$ hold generally, for any c. Next,

$$\lim_{z\to\infty} \frac{S_\ell^* - S_\ell^{sf}}{S_h^* - S_h^c} = \lim_{z\to\infty} \frac{\frac{\tau_\ell^2}{2g} \frac{1}{\left(\frac{\tau_\ell^2}{(cz+1)\tau_\ell^2+gz}\right)} \left(\frac{\tau_\ell^2}{(cz+1)\tau_\ell^2+gz} - \frac{\tau_\ell^2}{(cz+1)\tau_\ell^2+gz+\frac{p_h}{1-p_h}(cz+1)\left(\tau_h^2-\tau_\ell^2\right)}\right)^2}{\frac{\tau_h^2}{2g} \frac{1}{\left(\frac{\tau_h^2}{(cz+1)\tau_h^2+gz}\right)} \left(\frac{\tau_h^2+\sqrt{\frac{(cz+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+gz\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(cz+1)\tau_\ell^2+gz}}}{(cz+1)\left(2\tau_h^2-\tau_\ell^2\right)+gz} - \frac{\tau_h^2}{(cz+1)\tau_h^2+gz}\right)^2}$$

$$= c^2 \frac{p_h^2\tau_\ell^4\left(\tau_h^2-\tau_\ell^2\right)^2\left(c\tau_h^2+g\right)\left(c\left(2\tau_h^2-\tau_\ell^2\right)+g\right)^2}{\left((1-p_h)\left(c\tau_\ell^2+g\right)+p_h c\left(\tau_h^2-\tau_\ell^2\right)\right)^2\left(c\tau_\ell^2+g\right)\left(c\tau_h^2\left(\tau_h^2-\tau_\ell^2\right)-\left(c\tau_h^2+g\right)\sqrt{\left(\tau_h^2-\tau_\ell^2\right)\frac{c\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)+g\left(\tau_h^2+\tau_\ell^2\right)}{c\tau_\ell^2+g}}\right)^2},$$

which is strictly positive if $c > 0$. However, for small c, the term is small:

$$\begin{aligned}
&\lim_{c\to 0} c^2 \frac{p_h^2\tau_\ell^4\left(\tau_h^2-\tau_\ell^2\right)^2\left(c\tau_h^2+g\right)\left(c\left(2\tau_h^2-\tau_\ell^2\right)+g\right)^2}{\left((1-p_h)\left(c\tau_\ell^2+g\right)+p_h c\left(\tau_h^2-\tau_\ell^2\right)\right)^2\left(c\tau_\ell^2+g\right)\left(c\tau_h^2\left(\tau_h^2-\tau_\ell^2\right)-\left(c\tau_h^2+g\right)\sqrt{\left(\tau_h^2-\tau_\ell^2\right)\frac{c\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)+g\left(\tau_h^2+\tau_\ell^2\right)}{c\tau_\ell^2+g}}\right)^2} \\
&= \lim_{c\to 0} c^2 \cdot \frac{p_h^2\tau_\ell^4\left(\tau_h^2-\tau_\ell^2\right)^2\left(0+g\right)\left(0+g\right)^2}{\left(\left(1-p_h\right)\left(0+g\right)+0\right)^2\left(0+g\right)\left(0-\left(0+g\right)\sqrt{\left(\tau_h^2-\tau_\ell^2\right)\frac{0+g\left(\tau_h^2+\tau_\ell^2\right)}{0+g}}\right)^2} \\
&= \lim_{c\to 0} c^2 \cdot \frac{p_h^2\tau_\ell^4\left(\tau_h^2-\tau_\ell^2\right)}{g^2\left(1-p_h\right)^2\left(\tau_h^2+\tau_\ell^2\right)} \\
&= 0.
\end{aligned}$$

We can thus repeat the steps to obtain the result.

A.15.4 Proof of Proposition 6*(ii)(b)*, $\lim_{\tau_\ell \to 0} \Delta S < 0$.

Using (18),

$$\lim_{\tau_\ell \to 0} \Delta S = -p_h \left(c\sigma^2\rho + 1\right) \frac{\tau_h^2}{2g} \frac{1}{\delta_h^*} \cdot \lim_{\tau_\ell \to 0} \left(\delta_h^c - \delta_h^*\right)^2 + (1 - p_h) \left(c\rho\sigma^2 + 1\right) \lim_{\tau_\ell \to 0} \frac{\tau_\ell^2}{2g} \frac{1}{\delta_\ell^*} \left(\delta_\ell^* - \delta_\ell^{sf}\right)^2$$

We have

$$\begin{aligned}
\lim_{\tau_\ell \to 0} \delta_h^{sf} &= \lim_{\tau_\ell \to 0} \delta_h^* = \frac{\tau_h^2}{\left(c\sigma^2\rho + 1\right) \tau_h^2 + g\sigma^2\rho} = \delta_h^* \\
\lim_{\tau_\ell \to 0} \delta_\ell^{sf} &= \lim_{\tau_\ell \to 0} \frac{\tau_\ell^2}{\left(c\sigma^2\rho + 1\right) \tau_\ell^2 + g\sigma^2\rho + \frac{p_h}{1-p_h} \left(c\sigma^2\rho + 1\right) \left(\tau_h^2 - \tau_\ell^2\right)} = 0 \\
\lim_{\tau_\ell \to 0} \delta_h^c &= \lim_{\tau_\ell \to 0} \frac{\tau_h^2 + \sqrt{\frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2\left(\tau_h^2 - \tau_\ell^2\right)^2 + g\sigma^2\rho\left(\left(\tau_h^2\right)^2 - \left(\tau_\ell^2\right)^2\right)}{\left(c\sigma^2\rho+1\right)\tau_\ell^2 + g\sigma^2\rho}}}{\left(c\sigma^2\rho + 1\right) \left(2\tau_h^2 - \tau_\ell^2\right) + g\sigma^2\rho} = \frac{2\tau_h^2}{2\left(c\sigma^2\rho + 1\right) \tau_h^2 + g\sigma^2\rho} \\
\lim_{\tau_\ell \to 0} \delta_\ell^c &= \lim_{\tau_\ell \to 0} \delta_\ell^* = \lim_{\tau_\ell \to 0} \frac{\tau_\ell^2}{\left(c\sigma^2\rho + 1\right) \tau_\ell^2 + g\sigma^2\rho} = 0,
\end{aligned}$$

so

$$\begin{aligned}
\lim_{\tau_\ell \to 0} \Delta S &= -p_h \left(c\sigma^2\rho + 1\right) \frac{\tau_h^2}{2g} \frac{1}{\delta_h^*} \left(\frac{2\tau_h^2}{2\left(c\sigma^2\rho + 1\right) \tau_h^2 + g\sigma^2\rho} - \frac{\tau_h^2}{\left(c\sigma^2\rho + 1\right) \tau_h^2 + g\sigma^2\rho}\right)^2 + 0 \\
&= -\frac{1}{2} p_h \frac{\left(c\sigma^2\rho + 1\right) \left(\sigma^2\rho\tau_h^2\right)^2 g}{\left(\left(c\sigma^2\rho + 1\right) \tau_h^2 + g\sigma^2\rho\right) \left(2\left(c\sigma^2\rho + 1\right) \tau_h^2 + g\sigma^2\rho\right)^2},
\end{aligned}$$

which is negative.

A.15.5 Proof of Proposition 6*(ii)(c)*, $\lim_{\tau_\ell \to \tau_h} \Delta S = 0$ and $\lim_{\tau_\ell \to \tau_h} \frac{\partial}{\partial \tau_\ell} \Delta S > 0$.

As is easily verified, $\lim_{\tau_\ell \to \tau_h} \delta_h^c = \lim_{\tau_\ell \to \tau_h} \delta_\ell^{sf} = \delta_h^*$. The incentives are the same and they are efficient, since with identical talent there is no benefit from distorting the incentives. So the surplus is the same in both setups, and thus $\Delta S = 0$. The derivative of ΔS with respect to τ_ℓ^2 is

$$\begin{aligned}\frac{\partial}{\partial \tau_\ell^2}\Delta S = &- p_h \left(c\sigma^2\rho + 1\right) \frac{\tau_h^2}{2g}\frac{1}{\delta_h^*}\left(2\left(\delta_h^c - \delta_h^*\right)\frac{\partial}{\partial \tau_\ell^2}\delta_h^c\right)\\ &+ (1-p_h)\left(c\rho\sigma^2 + 1\right)\frac{1}{2g}\left(\left(\delta_\ell^* - \delta_\ell^{sf}\right)^2 + \tau_\ell^2 \frac{1}{2\cdot\delta_\ell^*}\left(\frac{\partial}{\partial \tau_\ell^2}\delta_\ell^*\right)\left(\delta_\ell^* - \delta_\ell^{sf}\right)^2\right)\\ &+ (1-p_h)\left(c\rho\sigma^2 + 1\right)\frac{1}{2g}\left(\tau_\ell^2\frac{1}{\delta_\ell^*}2\left(\delta_\ell^* - \delta_\ell^{sf}\right)\left(\frac{\partial}{\partial \tau_\ell^2}\delta_\ell^* - \frac{\partial}{\partial \tau_\ell^2}\delta_\ell^{sf}\right)\right).\end{aligned} \tag{A61}$$

The limit as $\tau_\ell \to \tau_h$ is

$$\begin{aligned}\lim_{\tau_\ell\to\tau_h}\frac{\partial}{\partial \tau_\ell^2}\Delta S = &- p_h\left(c\sigma^2\rho + 1\right)\frac{\tau_h^2}{2g}\frac{1}{\delta_h^*}\cdot\lim_{\tau_\ell\to\tau_h}\left(2\left(\delta_h^c - \delta_h^*\right)\frac{\partial}{\partial \tau_\ell^2}\delta_h^c\right)\\ &+ (1-p_h)\left(c\rho\sigma^2 + 1\right)\frac{1}{2g}\\ &\times\lim_{\tau_\ell\to\tau_h}\left(\left(\delta_\ell^* - \delta_\ell^{sf}\right)^2 + \tau_\ell^2\frac{1}{2\cdot\delta_\ell^*}\left(\frac{\partial}{\partial \tau_\ell^2}\delta_\ell^*\right)\left(\delta_\ell^* - \delta_\ell^{sf}\right)^2 + \tau_\ell^2\frac{1}{\delta_\ell^*}2\left(\delta_\ell^* - \delta_\ell^{sf}\right)\left(\frac{\partial}{\partial \tau_\ell^2}\delta_\ell^* - \frac{\partial}{\partial \tau_\ell^2}\delta_\ell^{sf}\right)\right)\end{aligned}$$

The second term vanishes in the limit as $\tau_\ell \to \tau_h$, since $\lim_{\tau_\ell\to\tau_h}\delta_\ell^{sf} = \lim_{\tau_\ell\to\tau_h}\delta_\ell^* = \delta_h^*$,

$$\begin{aligned}\lim_{\tau_\ell\to\tau_h}\delta_\ell^{sf} &= \lim_{\tau_\ell\to\tau_h}\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2 + g\sigma^2\rho + \frac{p}{1-p}\left(c\sigma^2\rho+1\right)\left(\tau_h^2 - \tau_\ell^2\right)} = \frac{\tau_h^2}{c\sigma^2\rho\tau_h^2 + \tau_h^2 + g\sigma^2\rho}\\ \lim_{\tau_\ell\to\tau_h}\delta_\ell^* &= \lim_{\tau_\ell\to\tau_h}\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2 + g\sigma^2\rho} = \frac{\tau_h^2}{c\sigma^2\rho\tau_h^2 + \tau_h^2 + g\sigma^2\rho},\end{aligned}$$

and since the limits of the corresponding derivatives are positive and finite,

$$\begin{aligned}\lim_{\tau_\ell\to\tau_h}\frac{\partial}{\partial \tau_\ell^2}\delta_\ell^{sf} &= \lim_{\tau_\ell\to\tau_h}\frac{\partial}{\partial \tau_\ell^2}\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2 + g\sigma^2\rho + \frac{p}{1-p}\left(c\sigma^2\rho+1\right)\left(\tau_h^2 - \tau_\ell^2\right)}\\ &= \lim_{\tau_\ell\to\tau_h}\frac{g\sigma^2\rho + \frac{p}{1-p}\left(c\sigma^2\rho+1\right)\tau_h^2}{\left(\left(c\sigma^2\rho+1\right)\tau_\ell^2 + g\sigma^2\rho + \frac{p}{1-p}\left(c\sigma^2\rho+1\right)\left(\tau_h^2 - \tau_\ell^2\right)\right)^2}\\ &= \frac{\frac{p}{1-p}\left(c\sigma^2\rho+1\right)\tau_h^2 + g\sigma^2\rho}{\left(\left(c\sigma^2\rho+1\right)\tau_h^2 + g\sigma^2\rho\right)^2}\end{aligned}$$

$$\lim_{\tau_\ell\to\tau_h}\frac{\partial}{\partial \tau_\ell^2}\delta_\ell^* = \lim_{\tau_\ell\to\tau_h}\frac{\partial}{\partial \tau_\ell^2}\frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2 + g\sigma^2\rho} = \lim_{\tau_\ell\to\tau_h}\frac{g\sigma^2\rho}{\left(\left(c\sigma^2\rho+1\right)\tau_\ell^2 + g\sigma^2\rho\right)^2} = \frac{g\sigma^2\rho}{\left(\left(c\sigma^2\rho+1\right)\tau_h^2 + g\sigma^2\rho\right)^2}.$$

So

$$\lim_{\tau_\ell\to\tau_h}\frac{\partial}{\partial\tau_\ell^2}\Delta S=-p_h\frac{\left(c\sigma^2\rho+1\right)\tau_h^2}{2g}\frac{1}{\delta_h^*}\cdot\lim_{\tau_\ell\to\tau_h}\left(2\left(\delta_h^c-\delta_h^*\right)\frac{\partial}{\partial\tau_\ell^2}\delta_h^c\right)$$

$$=-p_h\frac{\left(c\sigma^2\rho+1\right)\tau_h^2}{2g}\frac{1}{\delta_h^*}\cdot\lim_{\tau_\ell\to\tau_h}\left(\begin{array}{c}2\left(\frac{\tau_h^2+\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}-\frac{\tau_h^2}{(c\sigma^2\rho+1)\tau_h^2+g\sigma^2\rho}\right)\\ \times\left(\frac{\partial}{\partial\tau_\ell^2}\frac{\tau_h^2+\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}\right)\end{array}\right)$$

$$=-p_h\frac{\left(c\sigma^2\rho+1\right)\tau_h^2}{2g}\frac{1}{\delta_h^*}\cdot\lim_{\tau_\ell\to\tau_h}\left(\begin{array}{c}2\left(\frac{\tau_h^2+\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}{(c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho}-\frac{\tau_h^2}{(c\sigma^2\rho+1)\tau_h^2+g\sigma^2\rho}\right)\\ \times\frac{\left(\begin{array}{c}\frac{-\tau_\ell^2\frac{\left(c\sigma^2\rho+1\right)^2\left(\tau_h^2-\tau_\ell^2\right)\tau_\ell^2+\left(c\sigma^2\rho+1\right)\left(2\tau_h^2-\tau_\ell^2\right)\rho\sigma^2g+\left(g\sigma^2\rho\right)^2}{\left((c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho\right)^2}\left((c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho\right)}{\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}\\ +\left(\tau_h^2+\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}\right)\left(c\sigma^2\rho+1\right)\end{array}\right)}{\left((c\sigma^2\rho+1)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho\right)^2}\end{array}\right)$$

$$=-p_h\left(c\sigma^2\rho+1\right)\frac{\left(c\sigma^2\rho+1\right)\tau_h^2+g\sigma^2\rho}{2g}\cdot\frac{2}{\left((c\sigma^2\rho+1)\tau_h^2+g\sigma^2\rho\right)^3}$$

$$\times\lim_{\tau_\ell\to\tau_h}\left(\begin{array}{c}\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}\\ \times\left(\begin{array}{c}\frac{-\tau_\ell^2\frac{\left(c\sigma^2\rho+1\right)^2\left(\tau_h^2-\tau_\ell^2\right)\tau_\ell^2+\left(c\sigma^2\rho+1\right)\left(2\tau_h^2-\tau_\ell^2\right)\rho\sigma^2g+\left(g\sigma^2\rho\right)^2}{\left((c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho\right)^2}}{\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}}\left(\left(c\sigma^2\rho+1\right)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho\right)\\ +\left(\tau_h^2+\sqrt{\frac{(c\sigma^2\rho+1)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{(c\sigma^2\rho+1)\tau_\ell^2+g\sigma^2\rho}}\right)\left(c\sigma^2\rho+1\right)\end{array}\right)\end{array}\right)$$

$$= - p_h \frac{1}{g} \frac{c\sigma^2\rho + 1}{\left(\left(c\sigma^2\rho+1\right)\tau_h^2 + g\sigma^2\rho\right)^2}$$

$$\times \lim_{\tau_\ell \to \tau_h} \left(\begin{array}{c} -\tau_\ell^2 \frac{\left(c\sigma^2\rho+1\right)^2\left(\tau_h^2-\tau_\ell^2\right)\tau_\ell^2+\left(c\sigma^2\rho+1\right)\left(2\tau_h^2-\tau_\ell^2\right)\rho\sigma^2 g+\left(g\sigma^2\rho\right)^2}{\left(\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho\right)^2} \left(\left(c\sigma^2\rho+1\right)\left(2\tau_h^2-\tau_\ell^2\right)+g\sigma^2\rho\right) \\ +\sqrt{\frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}} \left(\tau_h^2+\sqrt{\frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2+g\sigma^2\rho\left(\left(\tau_h^2\right)^2-\left(\tau_\ell^2\right)^2\right)}{\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho}}\right)\left(c\sigma^2\rho+1\right) \end{array} \right)$$

$$= -p_h \frac{1}{g} \frac{c\sigma^2\rho + 1}{\left(\left(c\sigma^2\rho+1\right)\tau_h^2 + g\sigma^2\rho\right)^2} \left(\begin{array}{c} -\tau_h^2 \frac{\left(c\sigma^2\rho+1\right)^2\cdot 0\cdot\tau_h^2+\left(c\sigma^2\rho+1\right)\tau_h^2\rho\sigma^2 g+\left(g\sigma^2\rho\right)^2}{\left(\left(c\sigma^2\rho+1\right)\tau_h^2+g\sigma^2\rho\right)^2}\left(\left(c\sigma^2\rho+1\right)\tau_h^2+g\sigma^2\rho\right) \\ +\sqrt{\frac{\left(c\sigma^2\rho+1\right)\tau_h^2\cdot 0+g\sigma^2\rho\cdot 0}{\left(c\sigma^2\rho+1\right)\tau_h^2+g\sigma^2\rho}}\left(\tau_h^2+\sqrt{\frac{\left(c\sigma^2\rho+1\right)\tau_h^2\cdot 0+g\sigma^2\rho\cdot 0}{\left(c\sigma^2\rho+1\right)\tau_h^2+g\sigma^2\rho}}\right)\left(c\sigma^2\rho+1\right) \end{array} \right)$$

$$= -p_h \frac{c\sigma^2\rho+1}{g} \left(\frac{1}{\left(\left(c\sigma^2\rho+1\right)\tau_h^2+g\sigma^2\rho\right)^2} \left(-\tau_h^2\rho\sigma^2 g \cdot 1 + 0\right) \right).$$

So

$$\lim_{\tau_\ell \to \tau_h} \frac{\partial}{\partial \tau_\ell^2} \Delta S = p_h \frac{\left(c\sigma^2\rho+1\right)\tau_h^2\rho\sigma^2}{\left(\left(c\sigma^2\rho+1\right)\tau_h^2+g\sigma^2\rho\right)^2},$$

which is positive.

A.15.6 Proof of Proposition 6*(ii)(d)*, $\lim_{p_h \to 1} \Delta S < 0$.

$$\begin{aligned} \lim_{p_h \to 1} \delta_\ell^{sf} &= \lim_{p_h \to 1} \frac{\tau_\ell^2}{\left(c\sigma^2\rho+1\right)\tau_\ell^2 + g\sigma^2\rho + \frac{p_h}{1-p_h}\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)} \\ &= \lim_{p_h \to 1} \frac{\left(1-p_h\right)\tau_\ell^2}{\left(1-p_h\right)\left(\left(c\sigma^2\rho+1\right)\tau_\ell^2+g\sigma^2\rho\right)+p_h\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)} \\ &= \lim_{p_h \to 1} \frac{0}{0+\left(c\sigma^2\rho+1\right)\left(\tau_h^2-\tau_\ell^2\right)} \\ &= 0. \end{aligned} \tag{A62}$$

The distortion in the single-firm setup becomes extreme, but the expected value of the lost surplus goes to zero as p_h grows. In contrast, since δ_h^c does not depend on p_h, the distortion remains unchanged in the competitive setup, and the expected value of the lost surplus grows with p_h. Thus, $\lim_{p_h \to 1} \frac{\partial}{\partial p_h} \Delta S < 0$.

In taking this limit, p_h may cross the threshold p_o, above which a separating equilibrium does not exist. It is straightforward to find numerical examples for which $\Delta S < 0$ and either $p_h > p_o$ or $p_h < p_o$. For example, if $\tau_\ell = 1$, $\tau_h = 2$, $c = 1$, $\sigma = 1$, $\rho = 1$, and $g = 1$ then $\Delta S < 0$ implies

$p_h > p_o$. But $\Delta S < 0$ and $p_h < p_o$ for $p_h \in (0.76946, 0.96707)$ if we change τ_ℓ to $\tau_\ell = \frac{1}{2}$ (we then have $p_o = 0.96707$).

A.15.7 Proof of Proposition 6*(ii)(e)*, $\lim_{p_h \to 0} \Delta S = 0$ and $\lim_{p_h \to 0} \frac{\partial}{\partial p_h} \Delta S < 0$.

Using (18),

$$\lim_{p_h \to 0} \Delta S = \lim_{p_h \to 0} -p_h \left(c\sigma^2\rho + 1\right) \frac{\tau_h^2}{2g} \frac{1}{\delta_h^*} \left(\delta_h^c - \delta_h^*\right)^2 + \lim_{p_h \to 0} (1 - p_h) \left(c\rho\sigma^2 + 1\right) \frac{\tau_\ell^2}{2g} \frac{1}{\delta_\ell^*} \left(\delta_\ell^* - \delta_\ell^{sf}\right)^2.$$

Changes in p_h have no effect on δ_h^c, δ_h^*, or δ_ℓ^*, while

$$\lim_{p_h \to 0} \delta_\ell^{sf} = \lim_{p_h \to 0} \frac{\tau_\ell^2}{\left(c\sigma^2\rho + 1\right) \tau_\ell^2 + g\sigma^2\rho + \frac{p_h}{1-p_h} \left(c\sigma^2\rho + 1\right) \left(\tau_h^2 - \tau_\ell^2\right)} = \frac{\tau_\ell^2}{\left(c\sigma^2\rho + 1\right) \tau_\ell^2 + g\sigma^2\rho} = \delta_\ell^*.$$

So

$$\begin{aligned} \lim_{p_h \to 0} \Delta S &= -0 \cdot \left(c\sigma^2\rho + 1\right) \frac{\tau_h^2}{2g} \frac{1}{\delta_h^*} \left(\delta_h^c - \delta_h^*\right)^2 + (1 - 0) \left(c\rho\sigma^2 + 1\right) \frac{\tau_\ell^2}{2g} \frac{1}{\delta_\ell^*} \left(\delta_\ell^* - \delta_\ell^*\right)^2 \\ &= 0. \end{aligned} \tag{A63}$$

Using (18),

$$\begin{aligned} \frac{\partial}{\partial p_h} \Delta S = &- \left(c\sigma^2\rho + 1\right) \frac{\tau_h^2}{2g} \frac{1}{\delta_h^*} \left(\delta_h^c - \delta_h^*\right)^2 - \left(c\rho\sigma^2 + 1\right) \frac{\tau_\ell^2}{2g} \frac{1}{\delta_\ell^*} \left(\delta_\ell^* - \delta_\ell^{sf}\right)^2 \\ &+ (1 - p_h) \left(c\rho\sigma^2 + 1\right) \frac{\tau_\ell^2}{2g} \frac{1}{\delta_\ell^*} \cdot 2 \left(\delta_\ell^* - \delta_\ell^{sf}\right) \cdot (-1) \cdot \frac{\partial}{\partial p_h} \delta_\ell^{sf}. \end{aligned}$$

We have $= \frac{\partial}{\partial p_h} \delta_h^c = \frac{\partial}{\partial p_h} \delta_h^* = \frac{\partial}{\partial p_h} \delta_\ell^* = 0$, and

$$\begin{aligned} \lim_{p_h \to 0} \frac{\partial}{\partial p_h} \delta_\ell^{sf} &= \lim_{p_h \to 0} \frac{\partial}{\partial p_h} \frac{\tau_\ell^2}{\left(c\sigma^2\rho + 1\right) \tau_\ell^2 + g\sigma^2\rho + \frac{p_h}{1-p_h} \left(c\sigma^2\rho + 1\right) \left(\tau_h^2 - \tau_\ell^2\right)} \\ &= \lim_{p_h \to 0} - \frac{\tau_\ell^2 \frac{1}{(1-p_h)^2} \left(c\sigma^2\rho + 1\right) \left(\tau_h^2 - \tau_\ell^2\right)}{\left(\left(c\sigma^2\rho + 1\right) \tau_\ell^2 + g\sigma^2\rho + \frac{p_h}{1-p_h} \left(c\sigma^2\rho + 1\right) \left(\tau_h^2 - \tau_\ell^2\right)\right)^2} \\ &= - \frac{\left(c\sigma^2\rho + 1\right) \tau_\ell^2 \left(\tau_h^2 - \tau_\ell^2\right)}{\left(\left(c\sigma^2\rho + 1\right) \tau_\ell^2 + g\sigma^2\rho\right)^2}. \end{aligned}$$

So

$$\begin{aligned}
\lim_{p_h \to 0} \frac{\partial}{\partial p_h} \Delta S =& - \left(c\sigma^2\rho + 1\right) \frac{\tau_h^2}{2g} \frac{1}{\delta_h^*} \left(\delta_h^c - \delta_h^*\right)^2 - \left(c\rho\sigma^2 + 1\right) \frac{\tau_\ell^2}{2g} \frac{1}{\delta_\ell^*} \cdot \lim_{p_h \to 0} \left(\delta_\ell^* - \delta_\ell^{sf}\right)^2 \\
& + \left(c\rho\sigma^2 + 1\right) \frac{\tau_\ell^2}{2g} \frac{1}{\delta_\ell^*} \cdot 2 \cdot \lim_{p_h \to 0} \left(\delta_\ell^* - \delta_\ell^{sf}\right) \cdot \frac{\left(c\sigma^2\rho + 1\right) \tau_\ell^2 \left(\tau_h^2 - \tau_\ell^2\right)}{\left(\left(c\sigma^2\rho + 1\right) \tau_\ell^2 + g\sigma^2\rho\right)^2} \\
=& - \left(c\sigma^2\rho + 1\right) \frac{\tau_h^2}{2g} \frac{1}{\delta_h^*} \left(\delta_h^c - \delta_h^*\right)^2 \\
<& 0.
\end{aligned}$$

A.16 Proof of Proposition 7

In the single-firm setup, the high-talent incentive constraint is binding, while in the competitive case, the low-talent incentive constraint is binding. Thus, using (10) and (11),

$$\begin{aligned}
\Delta_u^{sf} &= \frac{\left(\beta_\ell^{sf} + \delta_\ell^{sf}\right)^2}{2g} \left(\tau_h^2 - \tau_\ell^2\right) \\
\Delta_u^c &= \frac{\left(\beta_h^c + \delta_h^c\right)^2}{2g} \left(\tau_h^2 - \tau_\ell^2\right).
\end{aligned}$$

Since we have

$$\beta_h^c + \delta_h^c > \beta_h^* + \delta_h^* > \beta_\ell^* + \delta_\ell^* > \beta_\ell^{sf} + \delta_\ell^{sf},$$

it follows that $\Delta_u^c - \Delta_u^{sf} > 0$.

Compensation inequality may increase or decrease, depending on the parameters. Figure 1 plots $\Delta_w^c - \Delta_w^{sf}$ for different values of p_h (on the horizontal axis) and $c \in \{\frac{1}{2}, 1, 2, 4, 8\}$), $\sigma = 1$, $\rho = 1$, $\tau_\ell = 1$, $\tau_h = 2$, and $g = 1$. For these examples, $\Delta_w^c - \Delta_w^{sf}$ is positive if c is sufficiently small, and negative if p_h and c are sufficiently large. However, this effect does not hold for all parameter constellations. If $\sigma = 1$, $\rho = 1$, $\tau_\ell = 1$, and $\tau_h = 2$, the derivative of $\Delta_w^c - \Delta_w^{sf}$ with respect to c,

$$
\begin{aligned}
\frac{\partial}{\partial c}\left(\Delta_w^c - \Delta_w^{sf}\right) = & \frac{\rho\sigma^2}{g}\left(\Delta_w^c - \Delta_w^{sf}\right) \\
& - \frac{\left(c\sigma^2\rho + 1\right)\tau_h^2}{g} \frac{\left(\tau_\ell^2\right)^2\left(\sigma^2\rho\right)^2 g \frac{\left(\tau_h^2 - \tau_\ell^2\right)}{\left(\left(c\sigma^2\rho+1\right)\tau_\ell^2 + g\sigma^2\rho\right)^2}}{\left(\left(c\sigma^2\rho + 1\right)\left(2\tau_h^2 - \tau_\ell^2\right) + g\sigma^2\rho\right)\sqrt{\frac{\left(c\sigma^2\rho+1\right)\tau_\ell^2\left(\tau_h^2-\tau_\ell^2\right)^2 + g\sigma^2\rho\left(\left(\tau_h^2\right)^2 - \left(\tau_\ell^2\right)^2\right)}{\left(c\sigma^2\rho+1\right)\tau_\ell^2 + g\sigma^2\rho}}} \\
& - \frac{c\sigma^2\rho + 1}{g}\sigma^2\rho\frac{\tau_h^2\left(2\tau_h^2 - \tau_\ell^2\right)}{\left(c\sigma^2\rho + 1\right)\left(2\tau_h^2 - \tau_\ell^2\right) + g\sigma^2\rho} \cdot \delta_h^c \\
& + \frac{c\sigma^2\rho + 1}{g}\tau_\ell^2\sigma^2\rho\left(\delta_\ell^*\right)^2 \\
& + \frac{c\sigma^2\rho + 1}{2g}\rho\sigma^2\left(\left(2\tau_h^2 - \tau_\ell^2\right)\left(c\rho\sigma^2 + 1\right) + \frac{\tau_\ell^2}{\tau_h^2}\rho\sigma^2 g\right)\left(\delta_h^*\right)^3 \\
& - \frac{c\sigma^2\rho + 1}{2g}\sigma^2\rho\left(\left(c\rho\sigma^2 + 1\right)\tau_\ell^2 + \rho\sigma^2 g + \frac{p_h}{1-p_h}\frac{\tau_h^2 - \tau_\ell^2}{\tau_\ell^2}\left(\left(c\rho\sigma^2 + 1\right)\tau_\ell^2 + 2\rho\sigma^2 g\right)\right)\left(\delta_\ell^*\right)^3,
\end{aligned}
$$

is negative (as expected) if g is below 3 (for any p_h), but it is positive if $g > 6$ (again assuming $\sigma = 1$, $\rho = 1$, $\tau_\ell = 1$, and $\tau_h = 2$).