Efficient Estimation of General Equilibrium Models Using Asset Returns and Industry Production Data*

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This Version: February 23, 2021

Abstract

Consumers choose consumption bundles and hold equity ownership in firms that produce consumption goods. Thus, consumer preferences are reflected in firms' production decisions and their security returns. Using this insight, we test moment restrictions from a multi-sector general equilibrium model using production data from manufacturing industries along with market- and industry-level returns. Our approach leads to significant improvement in identification of discount rate and risk aversion relative to consumption CAPM estimation. We also obtain reliable estimates of consumers' elasticity of substitution among products. Restrictions from industry equilibrium and industry IVs are critical for identification but returns-based instruments are also informative.

Keywords: Consumer preferences, Efficient estimation, General equilibrium, Moment conditions, Instrumental variables

*We thank Franklin Allen, Tim Bresnahan, Tom George, Nils Gottfries, Kris Jacobs, Ravi Jagannathan, Mamoud Medhat, Elisa Pazaj, Valerie Ramey, Enrique Schroth, and Vijay Yerramilli for helpful comments or discussions.

1 Introduction

Economic agents make consumption and financial investment choices contemporaneously. The extant asset pricing literature builds on this insight and utilizes time-series data on stock returns and aggregated consumption to estimate structural parameters that describe consumer preferences—in particular, the subjective discount factor and risk aversion (Hansen and Singleton (1982, 1983) and onwards). At the microeconomic level, however, agents choose consumption bundles (or baskets) of various types of consumption goods produced by firms in different sectors—as considered in standard general equilibrium theory (e.g., Arrow and Hahn, 1971). Indeed, estimation of structural parameters describing consumers' choice among product varieties is the focus of a large literature in international trade and applied industrial organization.[1] Meanwhile, equity value maximization by firms implies that the production decisions of public firms reflect the preferences of their equity owners.[2] Hence, in a realistic multi-sector context, parameters describing consumer preferences should be reflected not only in aggregate consumption and asset returns but also in *industry-specific* product and asset markets related information. Somewhat surprisingly, this estimation approach does not appear to have been implemented in the literature.

In this paper, we employ a new approach to estimation of parameters describing consumer preferences by using conditional moment restrictions from a multi-sector general equilibrium model with production and securities trading. Our empirical tests utilize asset returns at the market level as well as returns and production data at the industry level. We find that adding restrictions from industry production equilibrium paths and using strong instrument variables (IVs) offered by highly autocorrelated industry-level production inputs substantially enhance identification of structural parameters describing consumer preferences.

[1] Estimation of demand parameters related to product variety when preferences are assumed to have the constant elasticity of substitution (CES) form (Spence,1976; Dixit and Stiglitz, 1977) is undertaken, for example, by Feenstra (1994), Broad and Weinstein (2006), and Redding and Weinstein (2020)). Another literature undertakes nonparameteric identification in differentiated products markets (e.g., Berry, Levinsohn and Pakes 1995, 2004).

[2] This applies to general equilibrium production models with complete markets (Cochrane, 1991; Jermann, 1998) or with incomplete markets (Horvath, 2000). More generally, even with agency problems due to separation of ownership and control (Berle and Means, 1932; Jensen and Meckling, 1976), the preferences of equity owners are not *irrelevant* for managers.

We assume a competitive market structure with Cobb-Douglas constant returns to scale (CRS) technology (consistent with the competitive assumption) that is subject to Markov industry productivity shocks. The output in the aggregate sector also follows a Markov process. Thus, the economy is subject to both aggregate and sectoral productivity shocks, which is consistent with the real business cycle literature (Long and Plosser, 1987; Foerster et al., 2011). Firms in both sectors are widely-held and the representative consumer simultaneously chooses the consumption basket and equity investment in both sectors, as well as in a risk free asset. Industry firms choose investment and materials input to maximize the present discounted expected value of real dividends, where the discount rate is the representative consumer's marginal utility of consumption. We then derive our equilibrium restrictions on the representative consumer's consumption-investment policies and firms' capital investment-input choice policies from a stationary competitive general equilibrium path with simultaneous clearing of product and asset markets.

To generate explicit testable restrictions, we parameterize consumer preferences to exhibit time-additive constant relative risk aversion (CRRA) defined over a consumption basket with constant elasticity of substitution (CES). We are thus able to benchmark our estimates with asset pricing literature that uses stock market returns (e.g., Hansen and Singleton, 1982, 1983; Stock and Wright, 2000) and the applied microeconomics literature that uses CES preferences (Feenstra, 1994; Broda and Weinstein, 2006; Redding and Weinstein, 2020).

For empirical tests of the model, we need industry data on capital, investment, materials input, and industry productivity. We take these data from the NBER-CES database of U.S. manufacturing industries; the latest data available are for 1958-2011 (annual). Using sectoral output data from the Bureau of Economic Affairs (BEA), we form measures of sectoral output consistent with our theoretical framework. For the financial returns, we use value-weighted market equity returns; equity returns on the value-weighted industries portfolio; and the risk free rate. Consistent with the literature, we use per capita nondurables consumption as the measure of consumption. We estimate the model using generalized method of moments (GMM) with heteroskedasticity- and autocorrelation-consistent (HAC) inference.

An important aspect of industry production data is the high levels of own and cross-autocorrelations in the endogenous variables and industry productivity shocks. This is consistent with the literature that highlights the relatively high short run predictability of capital investment (Eberly, Rebelo and Vincent, 2012). Thus, in contrast to the well known weak IV problem with asset returns and consumption growth (Stock and Wright, 2000), using lagged industry production inputs as IVs allows stronger correlation with the optimality conditions related to firms' investment and input choices and, hence, a potential for stronger identification of the structural parameters of interest.

Indeed, to evaluate the incremental contribution of industry production-based Euler conditions and IVs to identification (or estimation efficiency), we first estimate the intertemporal consumption capital asset pricing (CCAPM) version of our model, where we use only the subset of moment conditions that characterize consumer equilibrium in the aggregate stock and risk free bond markets with only lagged market returns and consumption growth as IVs. This estimation exercise focuses, of course, on the estimation of the discount factor and risk aversion. We generally find insignificant and widely dispersed point estimates, consistent with weak identification noted in the literature (Hansen and Singleton, 1983; Stock and Wright, 2000). The point estimates of the discount rates are close to zero in all specifications (and even negative in some specifications) and the RRA coefficient is statistically significant only for IVs with three or four year lags; but these estimates (19.9 and 28.3, respectively) are significantly higher than the upper end of risk aversion considered reasonable in the literature (Mehra and Prescott, 1985; Bansal and Yaron, 2004).[3] Adding the Euler condition for asset market equilibrium in the industry equity portfolio and using lagged industry returns as IVs improves estimation efficiency: We obtain statistically significant point estimates for risk aversion with shorter—that is, one and two year—lags. However, the discount rate is still not identified and the range of risk aversion estimates, namely, 15.7–33.3, is wide and significantly higher than the reasonable range.[4]

Our general equilibrium (GE) model provides four Euler conditions along the general equilibrium path: The aforementioned Euler conditions for capital investment and materials

[3] The estimated values are, however, consistent with the implied RRA from aggregate returns data for CRRA preferences (e.g., Lettau and Uhlig, 2002).

[4] The J statistics are not significant in all these tests, and hence do not provide evidence against model specification. This is consistent with earlier results in the literature using monthly value-weighted market returns (Hansen and Singleton, 1983).

input at the industry level, and the Euler conditions for equity risk free at the aggregate and industry levels. However, the literature cautions against adding moment conditions (especially for a fixed sample length) because increased estimation efficiency comes at the cost of increasing estimator bias (Han and Phillips, 2006; Newey and Windmeijer, 2009). We, therefore, estimate the GE model with three Euler conditions: the two industry production equilibrium conditions and the condition for asset market equilibrium for the industry equity portfolio. For parsimony of instrumentation, and exploiting the high cross-autocorrelation between market and industry portfolio stock returns, we use as IV lagged market returns, consumption growth, industry capital investments, materials input, and productivity. Because the industry equilibrium optimality conditions involve parameters describing consumer preferences toward product variety, we estimate four structural parameters: the discount rate, risk aversion, the intratemporal elasticity of substitution (ES) among products, and the utility weights of manufactured products and the "rest."

The GE estimation yields strong identification of all parameters. The point estimates of the four parameters are statistically highly significant for all moment conditions (that is, choice of IVs); are not widely dispersed across different specifications (or IVs); and are economically appealing. Furthermore, the test statistics do not reject the null hypothesis of the joint validity of the optimality conditions and the IVs. The RRA coefficient is reliably estimated as 8.5 for IVs with one year lag and is thus placed in the range considered reasonable in the literature. Across the various specifications of IVs with up to four year lagged returns, the range of estimates for relative risk aversion is 8.5–12.6. Moreover, in contrast to estimation using moment conditions with only asset returns, the estimation of restrictions from the industry production and asset returns provides reliable estimates of the discount factor in the range 0.78–0.89, with the one year lag point estimate being at the lower end of this range.

Meanwhile, reliable estimates of the elasticity of substitution (ES) between the manufacturing sector and the "rest " have a range from 1.9 to 6.8. These estimates significantly exceed 1, which is the requirement of the theoretical model. In addition, the point estimates are broadly consistent with the estimates reported by papers that use import data (Feenstra, 1994; Broda and Weinstein, 2006) as well studies that use consumer level purchases (Redding and Weinstein, 2020). The estimates of the utility weights of manufactured products are in a tight range between 0.32 and 0.39, with the 95% confidence intervals being located strictly between 0 and 1 for all specifications.

In sum, our empirical results support the view that using conditional moment conditions based on Euler conditions from a general equilibrium model and IVs—with both industry level production and asset returns, as well as market returns—leads to strong identification of the whole profile of structural parameters describing consumer preferences with respect to intertemporal decision making (subjective discount factor and relative risk aversion) and with respect to intratemporal product variety choice (ES and utility weights). This view is based on the uniform statistically high significance of the point estimates, their relatively low dispersion, and economic appeal.

It is useful to examine the relative contributions of the intertemporal and capital investment and intratemporal materials input Euler conditions, as well as instrumentation by production and returns, to identification in the GE model. We adopt two approaches to address this issue, both of which generate complementary results. We analyze parameter identification by using various combinations of the production and asset market Euler conditions, as well as using only industry production as instruments. We find that both the industry Euler conditions are required to enhance identification; that is, identification deteriorates sharply if we "eliminate" either of the industry production Euler conditions. Moreover, while the role of industry level production IVs is critical, the asset return IVs also improve identification. We also examine the sensitivity of the moment conditions to variations in the parameters and find that Euler equations that are most sensitive to such variations have a greater impact on identification (Du, 2011). Our analysis indicates that the intertemporal investment Euler condition contributes most significantly to the identification of the subjective discount factor and risk aversion, whereas intratemporal materials input condition plays a greater role in the identification of parameters related to preferences over product variety.

To our knowledge, this is the first study to use economically motivated—through a multisector general equilibrium model with production and security trading—conditional moment restrictions and exploit high serial correlation in production data at the industry level to estimate simultaneously parameters describing consumer preferences from both asset and product markets. The results indicate that jointly using information in securities trading—in both the market and (manufacturing) industry portfolios—and the production decisions of public firms enhances identification of structural parameters in general equilibrium models.

Related Literature Our paper connects the literature that estimates relative risk aversion and subjective discount rates with time-additive CRRA expected utility (e.g., Hansen and Singleton, 1982, 1983; Stock and Wright, 2000) with the literature that estimates parameters that describe consumer preferences for product variety in the CES setting (e.g., Feenstra, 1994; Broda and Weinstein, 2006; Redding and Weinstein, 2020). Our contribution here is to show that a multisector general equilibrium model with production and securities trading can be used to generate conditional moment restrictions that allow joint estimation of structural parameters using asset returns and industry level production data. Furthermore, the Euler conditions from the industry equilibrium and informed IVs based on industry production inputs significantly enhance identification.

There is a large macrofinance literature on production-based general equilibrium asset pricing in single consumption good settings (e.g., Cochrane, 1991; Jermann, 1998) that uses macroeconomic business cycle data to examine time series properties of stock returns. Cochrane (1996) uses an investment-based asset pricing model to generate investment-related risk factors and examines their ability to explain variations in stock returns in the cross-section and over time. However, this literature does not use Euler conditions, as well as production and returns data at the industry level, to undertake estimation of structural parameters relating to consumer preferences in a multi-good environment.

Because our study adds moment conditions—in the form of new Euler conditions and IVs—to the canonical asset pricing tests that use GMM, it is related to the econometrics literature that considers the number and choice of moment conditions in GMM estimation because of the conflicting effects of adding moment conditions on efficiency and consistency (Han and Phillips, 2006). One strand of this literature examines the validity of moment conditions (Andrews, 1999). Another strand presumes validity, but addresses the choice of IVs (Donald and Newey, 2001; Newey and Windmeijer, 2009; Donald, Imbens and Newey, 2009). By testing the model with subsets of conditions (Eichenbaum, Hansen and Singleton, 1988), we show that a parsimonious construction of IVs and use of Euler conditions can facilitate identification.

Finally, our study is related to the literature on estimation efficiency of asset pricing models. Using a linear factor model, Fama and French (1997) highlight the large standard errors in estimating industry and firm level equity risk premia because of uncertainty on factor risk premiums and on risk loadings. Jagannathan and Wang (2002) compare the estimation efficiency of linear factor models versus the stochastic discount factor (SDF) method using GMM. Liu, Whited and Zhang (2009) derive q-theory implications for cross-sectional stock returns and test the model using levered investment returns. When matching the average return of testing portfolios they find that the q-theory model outperforms traditional models, including the CCAPM. Our contribution is to use restrictions derived from a production-based general equilibrium asset pricing model that distinguishes between aggregate and industry-level returns and exploit higher time-series correlations of industry production and investment data to improve the estimation efficiency of structural parameters, compared in particular to the CCAPM.

In the remaining paper, section 2 describes and characterizes the multi-sector general equilibrium model. Section 3 describes the data. Section 4 undertakes estimation with only moment restrictions from asset markets. Section 5 estimates the general equilibrium model. Section 6 analyzes the roles of different Euler conditions and IVs in identification, and section 7 concludes.

2 A Structural Multi-Sector General Equilibrium Model

We develop a two-sector dynamic production-based asset pricing model. The objective is to develop additional moment conditions from an equilibrium model that allow use of industry data that can help identification of structural model parameters.

2.1 Production

There are two sectors in the economy, specializing in the production of non-storable goods x and y. We will identify these as sectors x and y, respectively, and use capital letters to denote their outputs. For simplicity, output in sector x is modeled as an exogenous stochastic process $\{X_t\}_{t=0}^{\infty}$ that is sold competitively. This good x can be either consumed or used to facilitate production in the other sector; it also serves as the numeraire, and its price (p^x) is normalized to unity each period. That is, sector x sells X_t at unit price each period.

The second sector produces y and is composed of a continuum of identical competitive firms. Without loss of generality, the number of firms is normalized to unity and for notational convenience the exposition proceeds in terms of the optimization problem of the representative firm.

Sector y produces output Y_t through a technology that stochastically converts its beginning-of-the-period capital (K_t) and materials input (H_t) chosen during the period, using the production function

$$F(K_t, H_t, \lambda_t) = \lambda_t K_t^{\alpha} H_t^{\psi}. \tag{1}$$

Here, λ_t represents the stochastically evolving industry-wide productivity level and $\alpha > 0, \psi > 0$ are the output elasticities of capital and materials input, respectively. Consistent with the competitive industry structure assumption, we will assume that the technology exhibits constant returns to scale, that is, $\alpha + \psi = 1$.

Capital stock K_t evolves according to

$$K_{t+1} = (1 - \delta)K_t + I_t, K_{i0} = \bar{K}_{i0}, \tag{2}$$

where δ is the per-period depreciation rate and I_t is the investment at t. Investment also involves strictly convex adjustment costs so that the total investment cost function is

$$Z(I, K_t) = I + 0.5\upsilon \left(\frac{I_t}{K_t}\right)^2 K_t. \tag{3}$$

All firms in the model are unlevered and publicly owned, with their equity being traded in frictionless security markets. The number of shares outstanding in the two sectors at the beginning of t is denoted by Q_t^x and Q_t^y.Because of the "Lucas tree" structure of sector x, we fix the number of outstanding shares in this sector to unity, without loss of generality, that is, $Q_t^x \equiv 1$. Dividends per share at t are denoted by $a_t^j, j = x, y$. Because the cash flow of sector x at t is X_t, its sectoral dividend payout is $D_t^x = \int_0^1 a_t^x(i)di = X_t$. Meanwhile, the dividends from sector y are

$$D_t^y = p_t^y Y_t - H_t - Z(I_t, K_t). \tag{4}$$

Dividends can be negative, financed by equity issuance.[5] Per share dividends in sector y at t are determined by $Q_t^y a_t^y = D_t^y$.

2.2 Consumers

There is a continuum of identical consumers in the economy; the number of consumers is normalized to unity, without loss of generality. The representative consumer-investor (CI) maximizes the expected discounted time-additive utility of random consumption streams of the two goods subject to period-by-period budget constraints. In addition to investing in the stocks issued by firms, the CI has access in every period to a (one-period) risk free security (f) that pays a unit of the numeraire good next period. The mass of risk free securities is also fixed at unity. The profile of securities outstanding at t is thus $\mathbf{Q}_t = (1, Q_t^y, 1)$.

Thus, in each period t, the representative consumer chooses the consumption vector $\mathbf{c}_t = (c_t^x, c_t^y)$ taking as given product prices $\mathbf{p}_t = (1, p_t^y)$. The portfolio of asset holdings at the beginning of the period is $\mathbf{q}_t = (q_t^x, q_t^y, q_t^f)$. Along with consumption, the CI simultaneously chooses the new asset holdings $\mathbf{q}_{t+1}$, taking as given the corresponding share prices $\mathbf{s}_t = (s_t^x, s_t^y, s_t^f)$. For simplicity, there is no other endowment or labor income. Hence, the CI is subject to a wealth constraint determined by the dividend payouts per security of $\mathbf{a}_t = (a_t^x, a_t^y, 1)$.

More precisely, let W_t be the wealth net of new asset purchases during the period—that is, the the disposable income available for consumption. Then, the representative CI's

[5] In the absence of taxes and transactions costs, negative dividends are equivalent to the market value of new equity share issuance.

optimization problem is

$$\max \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma} - 1}{1-\gamma} \right], \gamma \geq 0, \beta < 1, \tag{5}$$

$$\text{s.t., } \mathbf{p}_t \cdot \mathbf{c}_t \leq \mathbf{q}_t \cdot (\mathbf{a}_t + \mathbf{s}_t) - \mathbf{q}_{t+1} \cdot \mathbf{s}_t \equiv W_t,\ \mathbf{c}_t \geq 0. \tag{6}$$

In (5), γ determines the representative CI's degree of risk aversion; β is the subjective discount factor; and $C_t \equiv C(\mathbf{c}_t)$ is an aggregated consumption index with constant elasticity of substitution (ES) between the consumption of the two goods:

$$C(\mathbf{c}_t) = \left[(1-\phi)(c_t^x)^{(\sigma-1)/\sigma} + \phi(c_t^y)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}. \tag{7}$$

Here, $\sigma > 1$ is the ES and $0 < \phi < 1$ is a pre-specified consumption weight for good y. Because preferences are strictly increasing, the budget constraint (6) will be binding in any optimum and hence W_t also represents the total consumption expenditure at t.

The optimal consumption demand functions derived from the optimization problem (5)-(6) are multiplicatively separable in W_t and $\mathbf{p}_t$ (see the Appendix)

$$c_t^{j*}(\mathbf{p}_t, W_t) = \frac{W_t}{P_t} \left[\frac{P_t \phi^j}{p_t^j} \right]^\sigma,\ j = x, y, \tag{8}$$

where $p_t^x = 1$, $\phi^x \equiv (1-\phi), \phi^y \equiv \phi$, and $P_t \equiv P(\mathbf{p}_t)$ is the aggregate price index

$$P(\mathbf{p}_t) = \left[(1-\phi)^\sigma + (\phi)^\sigma (p_t^y)^{1-\sigma} \right]^{1/(1-\sigma)}. \tag{9}$$

Note that, at the optimum, the aggregate real consumption $C_t^* = \frac{W_t}{P_t}$, that is, the real income.

2.3 Asset Markets

In the usual fashion, the pricing kernel (or the SDF) for future equity payoffs is defined in terms of the intertemporal marginal rate of substitution of real consumption (IMRS). Noting that $C_t = \frac{W_t}{P_t}$, the SDF (or pricing kernel) is given by $M_{t+1} \equiv \beta \left(\frac{W_{t+1}}{W_t} \right)^{-\gamma} \left(\frac{P_{t+1}}{P_t} \right)^{\gamma-1}$. In terms of the gross returns $R_{t+1}^j = (a_{t+!}^j + s_{t+1}^j)/s_t^j$, $j = x, y$ and $R_{t+1}^f = 1/s_t^f$, the asset market equilibrium condition can be written in the standard form (see the Appendix) as

$$\mathbf{1} = \mathbb{E}_t \left[M_{t+1} \mathbf{R}_{t+1} \right], \tag{10}$$

where $\mathbf{1}$ is the unit column vector and $\mathbf{R}_t = (R_t^x, R_t^y, R_t^f)'$.

2.4 Equilibrium

Given the stationary (or time-invariant) consumer preferences and production technology, and Markov structure of the exogenous stochastic variables, the pay-off relevant state at the beginning of each t can be written $\Gamma_t = (K_t, X_t, \lambda_t)$. Then, along the equilibrium path, the representative CI chooses consumption and asset demand vectors $(\mathbf{c}_t^*, \mathbf{q}_{t+1}^*)$ to solve the constrained optimization problem (5)-(6) such that the product and asset price vectors $(\mathbf{p}_t^*, \mathbf{S}_t^*)$ clear both the asset and product markets, that is,

$$\begin{aligned} \mathbf{q}_{t+1}^* &= \mathbf{Q}_{t+1}, & (11) \\ c_t^{x*}(\mathbf{p}_t^*, W_t^*) &= X_t, & (12) \\ c_t^{y*}(1, p_t^{y*}, W_t^*) &= Y_t - H_t^* - Z(I_t^*, K_t). & (13) \end{aligned}$$

In general, there will not exist complete contingent markets in this model; hence, the discount rate is given by the representative consumer's marginal utility of real consumption (Horvath, 2000). Thus, for every state Γ_τ, $\tau \geq 0$, the representative firm in sector y chooses investment and materials input to maximize the conditional expected present value of real dividends, which is recursively computed as

$$\max_{I_\tau, H_\tau} \mathbb{E}_\tau \left[\sum_{t=\tau}^{\infty} \beta^{t-\tau} \left(\frac{W_t}{P_t} \right)^{-\gamma} \left(\frac{p_t^y Y_t - H_t - Z(I_t, K_t)}{P_t} \right) \mid \Gamma_\tau \right], \text{ s.t., (1)–(3).} \tag{14}$$

Using the Bellman representation of (14), it follows that along the equilibrium path firm value can be recursively computed as

$$V_t^*(\Gamma_t) = \left(\frac{W_t}{P_t} \right)^{-\gamma} \left(\frac{1}{P_t} \right) \left[p_t^{y*}(F(K_t, H_t^*, \lambda_t) - (H_t^* + Z(I_t^*, K_t)) \right] + \beta \mathbb{E}_t \left[V_{t+1}^*(\Gamma_{t+1}^*) \right], \tag{15}$$

where $\Gamma^*_{t+1} = (K^*_{t+1}, X_{t+1}, \theta_{t+1})$ and $K^*_{t+1} = (1-\delta)K_t + I^*_t$.

We now characterize the industry equilibrium with the specification of product price, investment, and materials input paths.

2.5 Industry Equilibrium

Along the equilibrium path, the firm takes the product price as given and equates it to marginal cost of materials input. Note that the marginal cost of materials input is the inverse of their marginal product. For notational ease, we will write $\eta \equiv \phi/(1-\phi)$, the partial derivatives of the investment cost function as $Z_I(I,K) \equiv 1+\upsilon(I/K)$ and $Z_K(I,K) \equiv -0.5\upsilon(I/K)^2$.

Proposition 1 *Along an equilibrium path, for any* Γ_t*,*

$$p^{y*}_t = \left[\frac{X_t - (Z(I^*_t, K_t) + H^*_t)}{Y_t}\right]^{1/\sigma} \eta = [F_H(K_t, H^*_t, \lambda_t)]^{-1}, \tag{16}$$

$$\begin{aligned} Z_I(I^*_t, K_t) &= \mathbb{E}_t[M_{t+1}\{p^{y*}_{t+1} F_K(K^*_{t+1}, H^*_{t+1}, \lambda_{t+1}) - Z_K(I^*_{t+1}, K^*_{t+1}) + \\ &\qquad (1-\delta) Z_I(I^*_{t+1}, K^*_{t+1})\}] \,. \end{aligned} \tag{17}$$

And the asset market equilibrium satisfies Equation (10).

Equation (16) reflects the competitive equilibrium pricing condition where prices clear markets in both sectors and industry price equals the marginal cost. In a general equilibrium, the relative price of y (in terms of the numeraire), p^y, should be decreasing with the supply of y relative to that of x. And the sensitivity of p^y to this relative supply should be increasing (in algebraic terms) with the ES. Furthermore, ceteris paribus, p^y should be positively related to the weight of good y in the consumer's utility function, ϕ. These properties are satisfied by the equilibrium price function. Meanwhile, standard cost minimization implies that the marginal cost is the inverse of the marginal productivity of materials input. Finally, Equation (17) is the Euler condition with respect to investment that trades of current marginal cost of investment—represented by the left-hand side—with the discounted expected marginal value of current investment (or the right hand size).

2.6 Equilibrium Restrictions

The equilibrium investment and product prices given by Eqs. (16)-(17) determine the time-path of firms' capital stocks K_t^* (through the law of motion (2)) and dividends,

$$D_t^{y*} = p_t^{y*} F(K_t, H_t^*, \lambda_t) - (H_t^* + Z(I_t^*, K_t)). \tag{18}$$

These dividends, along with X_t and the unit payout from the riskless security, then determine the disposable income of the representative consumer W_t^* from (6). And, given p_t^{y*}, the aggregate price index P_t^* is determined by Equation (9). These quantities then determine the optimal consumption vector (c_t^{x*}, c_t^{y*}) and the aggregate consumption index C_t^*, according to Equations (8)-(9) and Equation (7), respectively. Finally, with the knowledge of the equilibrium investment and product pricing rules and, conditional on the state Γ_t, the representative CI forms expectations on the pricing kernel $M_{t+1}^* = \beta \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma}$, which determines the equilibrium returns according to Equation (10).

In sum, the model at hand imposes four equilibrium restrictions: the product market conditions Equation (16)-(17) and two asset markets conditions that are derived from Equation (10) in the form of equity risk premia, namely,

$$\mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(R_{t+1}^j - R_{t+1}^f \right) \right] = 0, j = x, y. \tag{19}$$

Note that these restrictions are in the form of moment conditions that we will exploit in the structural estimation that we now describe.

3 Structural Estimation of the Model

There are two objectives of our empirical analysis. The first is to estimate the "deep" parameters of the model related to consumer preferences, namely, $\theta = (\beta, \gamma, \sigma, \phi)$. While estimation of the discount rate and risk aversion parameters (β, γ) are the focus of the asset pricing literature, (σ, ϕ) control the consumer-investors' preferences over product varieties and their estimation is a focus of a long-standing literature in applied microeconomics and trade mentioned above. The second objective is to test the validity of the economic restrictions imposed by Equations (16)-(17) through tests of overidentifying restrictions using GMM.

3.1 Data and Empirical Measures

For empirical estimates tests of the model, we need industry data on capital, investment, materials input, sales, and productivity. We take these data from the NBER-CES manufacturing database. The latest data available are for 1958-2011 (annually). We map the NBER-CES manufacturing database to the 17 Fama-French industries using the mapping of 1997 NAIC codes to four-digit 1987 Standard Industry Classification (SIC) codes. We are able to map 13 industries from the 17 Fama-French industries to the NAIC codes in the NBER-CES manufacturing database.

Consistent with our theoretical framework, we measure the output (X) of the "aggregate" sector (x) as the difference between the *aggregate* output of all sectors obtained from the US Bureau of Economic Affairs (BEA) and the output of the manufacturing industry, which proxies for the industry output (Y).[6] For these quantities, the data also provide information about the relevant price deflator in 1997 dollars, which we utilize for computing values in real terms. Although data on the price index P are not readily available, we exploit the fact that, along the equilibrium path, the consumption basket $C_t^* = \frac{W_t}{P_t}$ is a sufficient statistic for computing the SDF. Consistent with the literature (e.g., Hansen and Singleton, 1982), we use the real U.S. annualized per capita nondurables consumption (ND),[7] obtained from Federal Reserve Bank of St. Louis (converted to 1997-dollars).

We compute average productivity across industries as the weighted average of the productivity for each industry, where the weights are computed using the proportion of output of a given industry relative to the total output of all industries in our sample. We use change in total factor productivity for each industry from NBER-CES database to compute

[6] To be consistent with the definition of sector x in our model,we use the aggregate output in all sectors rather than the Gross Domestic Product (GDP).

[7] Hansen and Singleton (1982) also use per capita real consumption nondurables and services, but they report small standard errors for risk aversion estimates with nondurables.

the weighted average, which is transformed into a level variable after computing the average change in productivity.

To compute the financial variables of the model, we use annual CRSP value-weighted returns and the annual risk free rate (denoted R_t^f) obtained from Kenneth French's website (the inflation data to derive the real rate is obtained from the US Bureau of Labor Statistics); we apply the consumption price deflator to adjust the returns data. We use four-digit SIC codes to compute the portfolio returns. Following the standard procedure in the literature, we compute the value-weighted index monthly returns of all firms in all manufacturing industries in the NBER-CES database (denoted R_t^y). In a similar fashion, we obtain the financial variables for the aggregate sector (x)—or the "market"—using the annual CRSP value-weighted index returns (denoted R_t^m) as the proxy.

3.2 Time Series Characteristics

Because lagged endogenous variables are widely used as instruments in the literature, the sample own and cross-autocorrelations of endogenous variables in the moment conditions of our model are of particular interest. Table 1 presents these correlations for the asset market conditions for one and two year lags. Consistent with the literature, there is relatively low own and cross-autocorrelations (for annualized observations) in consumption growth (C_t^g) and market ERP ($\tilde{R}_t^m$) in our sample. Not surprisingly, there is high contemporaneous correlation between industry and market returns. But we also find that the own and cross-autocorrelation of manufacturing industry ERP ($\tilde{R}_t^y$) are essentially commensurate with those observed for market returns. For example, the correlation between current and one-period lagged market return is -0.11, while the corresponding correlation is about -0.19 for the industry returns. Furthermore, the cross-autocorrelation between lagged industry returns and current consumption growth is not significantly different than the corresponding correlations between market returns and consumption growth. In sum, utilizing industry returns in IV estimation would add information but not necessarily resolve the weak IV problem in structural estimation of asset pricing models.

In contrast to Table 1, Table 2 shows very high own and cross-autocorrelations in the industry-level capital investment and other production related variables, namely, the productivity shocks and materials input. First, industry productivity (λ_t) is highly serially correlated and also has high cross-autocorrelations with industry investment (I_t) and materials input (H_t). In a similar fashion, I_t and H_t each have high own and cross-autocorrelations. The high serial correlation in investment is also noted elsewhere in the literature (e.g., Eberly, Rebelo and Vincent, 2012). Thus, as we mentioned already, there is a potential here that industry investment and other production related variables can be utilized as strong IVs in empirical estimation, to which we now turn.

3.3 Other Parameterization

The "deep" model parameters θ are to be estimated. But we need calibrations for the other parameters of the model. We calibrate the production function elasticities of capital and materials (α, ψ) by estimating the production function specified in Equation (1) using GMM. We utilize the orthogonality restrictions given by

$$\Lambda_t(Y_t - \lambda_t K_t^{\alpha} H_t^{\psi}) = 0, \tag{20}$$

where Λ_t is the IV vector that uses one-year lagged inputs $(\lambda_{t-1}, K_{t-1}, H_{t-1})$ as instruments and using heteroskedasticity-robust standard errors. This yields the estimates $\hat{\alpha} = 0.3$ (*p-value* $= 0.037$) and $\hat{\psi} = 0.7$ (*p-value* $= 0.000$).[8]

Turning to the depreciation rate δ, as pointed out by the literature (e.g., Oliner, 1989), depreciation rates in manufacturing generally have been rising because of increased use of computer equipment and software that have higher depreciation rates compared with machinery and structures (Gomme and Rupert, 2007). We use an annual depreciation rate of 25%, which is consistent with the calibration in macroeconomic models with capital investment (e.g., Jermann and Quadrini, 2012). Meanwhile, there is a wide variation in the literature regarding estimates of the capital adjustment cost parameter υ. In particular, uti-

[8] Hansen's (1982) J (χ^2) statistic with two degrees of freedom is 8.74 with a *p-value* of 0.014. Hence, and not surprisingly for the Cobb-Douglas specification, the validity of the overidentifying restrictions in (20) is reliably rejected.

lizing US plant level data, Cooper and Haltiwanger (2006) find υ of around 10% for a strictly convex adjustment cost function. However, Hall (2004) presents evidence against significant capital adjustment costs. Hence, we use $\upsilon = 0.01$ for our tests.

4 GMM Estimation with Asset Markets Equilibrium Conditions

We now present the results of GMM estimation of the model. We begin by estimating the consumption CAPM (CCAPM) version of our model based on the aggregate consumption and market data. This sets up a useful benchmark and facilitates comparison with the received literature. We then estimate the moment restrictions using both the product and asset market equilibrium conditions. Because of the autocorrelation in the data—for both returns and the production variables—seen in Table 1, we use heteroskedasticity- and autocorrelation-consistent (HAC) inference.

4.1 Consumption CAPM Estimation

We follow the standard approach and estimate (β, γ) by using the equilibrium asset return equation (10) and setting up moment conditions in terms of the market (or aggregate) equity risk premium, $(R^m_{t+1} - R^f_{t+1})$. Noting that in equilibrium, $\Lambda_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$, we use—in the usual way—the orthogonality conditions,[9]

$$\mathbb{E}_t\left[\mathbf{A}^m_t\left\{\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(R^m_{t+1} - R^f_{t+1}\right)\right\}\right] = 0. \tag{21}$$

Here, $\mathbf{A}^m_t$ is a vector of IVs that—in the usual fashion—use only market and aggregate data. Similar to Hansen and Singleton (1982) and many others, we use lagged values of the market and risk free returns, as well as lagged consumption growth as IVs. Letting $\tilde{R}^m_t \equiv R^m_t - R^f_t$, that is, the market risk premium at t, and $C^g_t = \frac{C_t}{C_{t-1}}$, that is, the consumption growth at t,

[9] Untabulated results show that in our sample the moment restriction (21) generally performs better—or at least no worse than—the pair of restrictions $\mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R^m_{t+1}\right] = 1$ and $\mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R^f_{t+1}\right] = 1$.

we use four lagged values of $(\tilde{R}_t^m, C_t^g)$ as IVs, namely, $A_{-it}^m = (\tilde{R}_{t-i}^m, C_{t-i}^g), i = 1, 2, 3, 4$. Given our annual data, a maximum of four year lag appears appropriate in striking the balance between the serial correlation present in the data and preserving sample size. In addition to the lagged covariates, we use an IV that is a nonlinear function of $(\tilde{R}_{t-1}^m, C_{t-1}^g)$ to allow for nonlinear effects:

$$A_{nl,t}^m \equiv \left(\tilde{R}_{t-1}^m, C_{t-1}^g, \tilde{R}_{t-1}^m \times C_{t-1}^g, (\tilde{R}_{t-1}^m)^2, (C_{t-1}^g)^2\right). \tag{22}$$

Table 3 shows the estimation results, as well as tests of over-identifying restrictions through the chi-square statistic $\chi^2(DF)$. The point estimates are not significant for short-lag linear IVs—that is, for one and two year lags—or for the nonlinear IV. For longer-lag IVs—that is, for three and four years—we find significant point estimates (at conventional levels) for the risk aversion parameter $\hat{\gamma}$; this is especially the case when we use four-year lagged market returns and consumption growth, where the estimates are highly significant. The estimated risk aversion ranges from 5.8 to 28.3, which is a rather wide dispersion. The upper end of estimated range is significantly higher than 10, which is considered to be toward the maximum of the reasonable range of risk aversion in the macrofinance literature (Mehra and Prescott, 1985; Bansal and Yaron, 2004). However, the estimates are consistent with the risk aversion implied by imposing CCAPM on market returns; for example, Lettau and Uhlig (2002) point out that the aggregate Sharpe ratios observed in the post-War U.S. data are consistent with risk aversion of around 27. In addition, these estimates are consistent with the 90% S-sets range (with annual data) computed by Stock and Wright (2000) to account for weak IVs. Meanwhile, the estimates of the subjective discount factor (β) are economically unappealing for all IV specifications because they are either statistically insignificant or infinitesimally negative.

The weak identification of the parameters here is consistent with the weak empirical performance of the CCAPM reported in the literature (Hansen and Singleton, 1983). We note, however, that the very high p-values of the J-statistics for all specifications indicate that the validity of the overidentifying conditions is not rejected at very high levels of confidence. But although the conventional J statistics do not indicate model mispecification, the dispersed and insignificant estimates are consistent with weak IVs (Stock and Wright, 2000).

4.2 Estimation with Aggregate and Industry Asset Markets Conditions

It facilitates intuition on the role of industry-level moment conditions to first restrict attention to the asset markets conditions (see Equation (10)) and derive the attendant orthogonality conditions from the market and industry equity risk premia denoted by $\tilde{R}_t^m$ and $\tilde{R}_t^y$, respectively. Thus, we now estimate (β, γ) by utilizing the orthogonality conditions:

$$\mathbb{E}_t \left[\mathbf{A}_t^{my} \left\{ \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \tilde{R}_{t+1}^j \right\} \right] = 0, \; j = m, y. \tag{23}$$

A natural extension of the IVs used earlier is to use lagged values of the market and industry risk premia, along with the consumption growth rates. That is, $A_{-it}^{my} = (\tilde{R}_{t-i}^m, \tilde{R}_{t-i}^y, C_{t-i}^g), i = 1, 2, 3, 4$. We also extend the nonlinear IV from the market estimation in Equation (21) to include lagged industry ERP, that is, $A_{nl,t}^{my} = A_{nl,t}^m \cup \tilde{R}_{t-1}^y$.

The results are displayed in Table 4. There is marked improvement in identification of risk aversion compared to Table 3 (where we use only market data). The point estimates are statistically significant for the IVs with two and four year lags and for the nonlinear IV. The estimated range widens relative to Table 3—here the range is from 2.8 to 33.3. However, the estimates of the subjective discount factor $\hat{\beta}$ are all statistically insignificant and sometimes negative. As in the CCAPM estimation above, the J-statistic does not reject model specification and instrument validity at high levels of confidence. In sum, using returns data on the portfolio of manufacturing industries by adding the equilibrium industry equity risk premium moment condition somewhat improves the significance of the point-estimates of risk aversion. Nevertheless, the wide dispersion in the estimates is notable and, therefore, still indicative of weak identification.

5 General Equilibrium Estimation

We now use orthogonality conditions given by the equilibrium path in both product and asset markets—that is, the general equilibrium path—to estimate the full vector of unknown parameters $\theta = (\beta, \gamma, \sigma, \phi)$.

Note that in our model the general equilibrium path offers four moment conditions, namely, the two asset market conditions in Equation (23) as well as the optimality conditions for investment (Equation (17)) and materials input (Equation (16)). But, as we mentioned before, there are well known pitfalls in adding moment conditions with fixed sample size, especially if they include weak moment conditions since increased estimation efficiency comes at the cost of increasing estimator bias (Han and Phillips, 2006; Newey and Windmeijer, 2009). Furthermore, a large number of moment conditions raises the likelihood of misspecification bias through utilization of possible invalid restrictions (Andrews, 1999).

Consequently, our test design for GE estimation uses the two product market moment conditions and the industry equity risk premium (ERP) moment conditions. To set these out concisely, we define $F^*_{K,t+1} \equiv F_K(K^*_{t+1}, H^*_{t+1}, \lambda_{t+1})$, $Z^*_t \equiv Z(I^*_t, K^*_t)$, $Z^*_{K,t+1} \equiv Z_K(I^*_{t+1}, K^*_{t+1})$, $Z^*_{I,t+1} \equiv Z_I(I^*_{t+1}, K^*_{t+1})$. Then the system of orthogonality conditions we use are:

$$\mathbb{E}_t \left[\mathbf{A}^m_t \left\{ \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \tilde{R}^y_{t+1} \right\} \right] = 0, \tag{24}$$

$$B^y_{-1t} \left\{ \left(\frac{X_t - (Z^*_t + H^*_t)}{Y_t} \right)^{1/\sigma} \eta F_H(K_t, H^*_t, \lambda_t) - 1 \right\} = 0, \tag{25}$$

$$B^y_{-1t} \left\{ p^{y*}_t Z^*_{I,t} - \mathbb{E}_t \left[M_{t+1} [p^{y*}_{t+1} F^*_{K,t+1} - Z^*_{K,t+1} + (1-\delta) Z^*_{I,t+1} \right] \right\} = 0. \tag{26}$$

In light of the high correlation between market and industry returns, for the asset market moment condition we use lagged market equity premium and consumption growth (for $t-1$ through $t-4$), as well as the nonlinear IV with market and consumption growth terms that is used in the CCAPM estimation. That is, $\mathbf{A}^m_t = (A^m_{-it} = (\tilde{R}^m_{t-i}, C^g_{t-i}), i = 1, 2, 3, 4, A^m_{nl,t})$. For parsimony, we use one-year lagged industry production inputs as IVs, that is, $B^y_{-1t} = (I_{t-1}, H_{t-1}, \lambda_{t-1})$. Untabulated analysis indicates that the results do not materially change if we replace the industry ERP moment condition with the market moment condition (see

Equation (21)). For notational ease, we will represent the "general equilibrium" IVs as $G_{-it} = (A^m_{-it}, B^y_{-1t})$, $i = 1, 2, 3, 4$, and $G_{nl,t} = (A^m_{nl,t}, B^y_{-1t})$.

The estimation results are displayed in Table 5. In striking contrast to the previous estimation results, the point estimates of the entire unknown parameter vector $\theta = (\beta, \gamma, \sigma, \phi)$ are statistically significant for each specification and the J-statistics reliably do not reject the null of the joint validity of the moment conditions and the IVs. The estimates for the subjective discount factor (β) are around 0.78 for IVs with one year lag on market returns (that is, A^m_{-1t} and $A^m_{nl,t}$) and range from 0.82 for the IV with two year lag market returns to 0.89 for the IV with the longest (four year) lag. The 95% confidence interval across the different specifications is between 0.72 and 1.00. The corresponding interval for one and two year lagged IVs is 0.72 to 0.94.[10]

Meanwhile, the estimates of risk aversion in Table 5 are between 8.5-12.6, which is a much narrower range than is encountered in Tables 3 and 4. As mentioned above, the macrofinance literature views RRA coefficients of upto 10 as reasonable (Mehra and Prescott, 1985; Bansal and Yaron, 2004). The point estimates with one and two year lagged IVs are therefore consistent with this range. The 95% confidence interval across all specifications is between 6.7 and 16 and the corresponding interval for one and two year lagged IVs is between 6.3 and 14.1. Comparing the statistically significant risk aversion estimates in Table 4 with Table 5, it appears that industry-level product and asset market moment conditions reduce the relative risk aversion required to explain the aggregate returns and consumption data. Overall, we find that using industry-level production moment conditions and strong IVs with lagged endogenous production related variables significantly improves the identification of both the subjective discount factor and risk aversion with a CRRA parameterization.

Turning to the parameters relating to the CES product variety preferences, the estimates of the elasticity of substitution (σ) between the manufacturing sector and the "rest" have a range from 1.9 to 6.8. These estimates significantly exceed 1, which is the requirement of the

[10] As a comparison, using annual stock and bond returns data from 1871-1993 and using consumption of nondurables and services, with one year lagged IVs, Stock and Wright (2000) estimate the equilibrium asset returns restriction (Eq. (10)) and report point estimates between 0.9 and 0.97. The estimates of the discount factor using monthly returns data in CCAPM models with power expected utility are typically higher (e.g., Hansen and Singleton, 1983; Stock and Wright, 2000).

theoretical model. Feenstra (1994) estimates σ at around 6 for two consumer manufactured goods (athletic shoes and knit shirts) and around 4.2 for steel sheets. Broda and Weinstein (2006) use import data on product varieties from 1972-1988 and 1990-2001 and report σ estimates of a number of manufactured products–at the three digit SITC (Standard International Trade Classification) aggregation level–that range from 1.2 to 6.7. Hottman and Monarch (2018) use import data from 1998 to 2014 and report the median cross-sectoral σ (at the four digit North American Industry Classification System (NAICS) level) of 4.1. And Redding and Weinstein (2020) utilize scan data from consumer sales across a wide variety of products from 2004-2014 and report a median ES estimate of 6.5. In sum, our estimates of σ between the manufacturing sectors and the "rest" are not economically unreasonable. Furthermore, the estimated utility weights, or the taste parameter (ϕ) for manufactured products is also estimated in a tight range to be between 0.32 and 0.39. We note that the the 95% confidence intervals for these estimates lie strictly between 0 and 1, so the estimates are economically sensible.

In sum, the results in Table 5 support the view that using the general equilibrium system of moment conditions and IVs—with both industry production and asset returns and market returns—lead to strong identification of the entire parameter set $\theta = (\beta, \gamma, \sigma, \phi)$. This view is based on the uniform statistical significance of the point estimates for the entire parameter vector in Table 5, their relatively low dispersion (compared with Tables 3 and 4 for (β, γ)), and economic appeal. Furthermore, the J test for the joint validity of overidentifying restrictions and IVs reliably does not reject any of the specifications, even with a parsimonious choice of IVs.

6 Euler Conditions, IVs and Estimation Efficiency

In undertaking the general equilibrium estimation in the previous section, we have simultaneously introduced new moment conditions—namely, the equilibrium Euler conditions for investment and materials input from Proposition 1—as well as new IVs—namely, lagged investment, inputs, and industry productivity shocks—to the Euler condition for equilibrium equity returns (Tables 3 and 4). The consequent improved identification or estimation efficiency (as discussed in the previous section) could, therefore, arise from the use of one or both of the economically motivated Euler conditions and/or the use of strong industry production based IVs (as discussed in section 3.2). Hence, it is useful to examine the relative role of the intertemporal Euler condition for investment, the intratemporal materials input Euler condition, and the industry-based IVs in the improved estimation efficiency. We adopt two approaches to examine this issue.

First, we analyze parameter identification by using various combinations of the production and asset market Euler conditions, as well as using only production IVs. Since our estimation uses IVs with various lags and none of them are rejected by test statistics (Table 5), we use the IV with one year lag, consistent with the literature (e.g., Stock and Wright, 2000). This analysis is presented in Table 6. In the first row, we use only the investment Euler condition and the asset returns moment conditions from section 4.2 using the industry IV B^{y}_{-1t} (defined in the previous section) and one-year lagged market and industry returns (A^{my}_{-1t}) as IVs. That is, we use the model

$$\begin{aligned} B^{y}_{-1t}\left\{p^{y*}_t Z^*_{I,t} - \mathbb{E}_t\left[M_{t+1}[p^{y*}_{t+1}F^*_{K,t+1} - Z^*_{K,t+1} + (1-\delta)Z^*_{I,t+1}]\right\}\right. &= 0, \qquad (27)\\ \mathbb{E}_t\left[A^{my}_{-1t}\left\{\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\tilde{R}^j_{t+1}\right\}\right] &= 0,\ j = m, y. \qquad (28)\end{aligned}$$

(For notational ease, we put $G^{my}_{-1t} = A^{my}_{-1t} \cup B^{y}_{-1t}$.) As indicated in Table 6, we are unable to attain convergence of the GMM estimator (even with different starting points), and hence some *p-values* of point estimates are not available. In any case, the point estimates are not economically sensible.

In the second row of Table 6, we replace the investment Euler condition (27) with the materials input Euler condition

$$B^{y}_{-1t}\left\{\left(\frac{X_t - (Z^*_t + H^*_t)}{Y_t}\right)^{1/\sigma}\eta F_H(K_t, H^*_t, \lambda_t) - 1\right\} = 0, \qquad (29)$$

and continue to use (28). This model only provides statistically significant estimates of the risk aversion, which is in the range estimated in Table 5. However, the other point estimates are not significant and the estimates of the discount rate are unappealing (given Table 5). Furthermore, the J statistic rejects the validity of the moment conditions with high levels of confidence.

The last two rows of the Table show the estimates when we use only the industry production Euler conditions (27)-(28). The third row uses only one year lagged endogenous variables and exogenous industry productivity as IVs. The last row uses G_{-1t}^{my}, that is, both lagged production and returns data as IVs. As seen in the third row, using both the industry production Euler conditions and the industry IVs dramatically improves estimation efficiency relative to models that use only one of the Euler conditions. All the point estimates are statistically significant and in the range of estimates from the full model (Table 5), In addition, the J-statistic test does not reliably indicate model misspecification. Nevertheless, the results in the last row indicate that using industry and equity returns enhances estimation efficiency. Specifically, the statistical significance of all estimates improve, especially the estimate of the ES σ. Moreover, the *p-value* of the J statistic is significantly higher, that is, the data provide stronger support for this specification reltive to the specification in the third row.

Our second approach is to examine the sensitivity of the estimated moment conditions to variations in the four parameters. Specifically, for each estimated parameter in $\theta = (\beta, \gamma, \sigma, \phi)$, we first multiply the parameter by 0.5, holding everything else same. Then we compute all the 11 moments (accounting for the instruments G_{-1}^{my}) and scale it by the moment values using the estimated parameters. We repeat this for the case where we multiply the estimated parameter by 1.5. We then graph the three points for each moment/parameter. The Euler equations that are more sensitive to variations in parameter values can then be viewed to have greater contributions to identification (e.g., Du, 2011). Figures 1 and 2 present this analysis. (Equation numbers in the graph represent Euler conditions in Equations (26), (25), and (24)). Figure 1 indicates that the intertemporal investment Euler condition contributes most significantly to the identification of the subjective discount factor and risk aversion. Figure 2 shows that both the investment and intratemporal materials input conditions contribute significantly to the identification of the ES and the utility weights, with the materials condition playing a greater role.

We conclude from Table 6 and Figures 1-2, as well as the previous analysis (Table 3 and 4), that utilizing both the intertemporal and intratemporal industry production Euler conditions as well as the strong industry-based IVs are required for enhancing estimation precision of the parameter vector of interest θ. However, adding lagged industry and market returns as IVs improves estimation efficiency.

7 Conclusion

Consumers generally choose consumption bundles or baskets of various types of consumption goods produced by firms in different sectors; they also hold equity ownership in these firms. Hence, consumer preferences should be reflected in not only security trading, as has been highlighted by the vast asset pricing literature, but also in the shareholder value-maximizing production decision of firms. Based on this observation, we employ a new approach to estimation of parameters describing consumer preferences relating to intertemporal decision making and intratemporal product variety choice by using conditional moment restrictions from a multi-sector general equilibrium model with production and securities trading. To benchmark our results to the literature, preferences are parameterized with time additive power utility defined over constant elasticity of substitution (CES) consumption baskets. Our empirical tests utilize U.S. stock and risk free returns at the market level as well as equity returns and production data for the U.S. manufacturing industry.

A notable aspect of industry level data on capital investments, materials input, and industry productivity is their high level of autocorrelation, which makes them potentially strong instruments (IVs) because of high correlation with Euler conditions for investment and materials input. Generalized method of moments (GMM) estimation indeed shows that using the industry Euler conditions and IVs facilitates identification. The point estimates of the representative consumer's subjective discount factor, relative risk aversion, elasticity of substitution, and sectoral utility weights are uniformly statistically high significant; have relatively low dispersion across different IVs; and are economically appealing based on the values considered reasonable and/or estimated with alternative datasets and methodologies in the literature. Tests of subsets of moment conditions of the model indicate that Euler conditions from the industry production equilibrium and industry IVs are critical for identification, but instruments based on market and industry returns are also informative and enhance identification.

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Table 1. Matrix of Autocorrelation Coefficients: Asset Markets Variables

This table uses annual data from 1958 to 2011. It presents own and cross-autocorrelations of yearly growth rates of U.S. per capita real (in 1997 dollars) nondurables consumption (C^g); the annualized market equity risk premium (ERP) $\tilde{R}^m = R^m - R^f$, where R^m is the value-weighted CRSP return and R^f is the annual risk free rate; and $\tilde{R}^y$ is the annualized ERP from value-weighted index monthly returns of all firms in all manufacturing industries in the NBER-CES database.

Variables	C^g_t	C^g_{t-1}	C^g_{t-2}	$\tilde{R}^m_t$	$\tilde{R}^m_{t-1}$	$\tilde{R}^m_{t-2}$	$\tilde{R}^y_t$	$\tilde{R}^y_{t-1}$	$\tilde{R}^y_{t-2}$
C^g_t	1.000								
C^g_{t-1}	0.567	1.000							
C^g_{t-2}	0.259	0.575	1.000						
$\tilde{R}^m_t$	-0.254	-0.160	-0.028	1.000					
$\tilde{R}^m_{t-1}$	0.196	-0.253	-0.164	-0.111	1.000				
$\tilde{R}^m_{t-2}$	-0.066	-0.156	-0.270	-0.260	-0.095	1.000			
$\tilde{R}^y_t$	|-0.320	-0.170	-0.006	0.972	-0.207	-0.274	1.000		
$\tilde{R}^y_{t-1}$	0.177	-0.321	-0.179	-0.106	0.972	-0.174	-0.193	1.000	
$\tilde{R}^y_{t-2}$	-0.082	0.138	-0.335	-0.243	-0.088	0.974	-0.263	-0.157	1.000

Table 2. Matrix of Autocorrelation Coefficients: Industry Production Variables

This table uses the NBER-CES database of 473 manufacturing (annual) from 1958 to 2011. It presents own and cross-autocorrelations of annual investment (I), materials input (H), and average productivity (λ).

Variables	λ_t	λ_{t-1}	λ_{t-2}	I_t	I_{t-1}	I_{t-2}	H_t	H_{t-1}	H_{t-2}
λ_t	1.000								
λ_{t-1}	0.931	1.000							
λ_{t-2}	0.867	0.939	1.000						
I_t	0.884	0.929	0.909	1.000					
I_{t-1}	0.801	0.897	0.930	0.950	1.000				
I_{t-2}	0.795	0.830	0.90	0.895	0.955	1.000			
H_t	0.881	0.895	0.882	0.943	0.930	0.925	1.000		
H_{t-1}	0.839	0.893	0.896	0.923	0.945	0.934	0.985	1.000	
H_{t-2}	0.820	0.852	0.895	0.883	0.926	0.949	0.965	0.986	1.000

Table 3. Estimation of CCAPM Using Market Returns

This table presents the point estimates, standard errors, and J statistics from two step GMM estimation (with heteroskedasticity- and autocorrelation-consistent inference) of the subjective discount rate ($\hat{\beta}$) and the constant relative risk aversion coefficient ($\hat{\gamma}$) from the moment restrictions

$$\mathbb{E}_t\left[\mathbf{A}_t^m\left\{\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\tilde{R}_{t+1}^m\right\}\right]=0, i=1,2,3,4,$$

The vector of IVs is $\mathbf{A}_t^m=(A_{-it}^m\equiv(\tilde{R}_{t-i}^m,C_{t-i}^g), i=1,2,3,4,\ A_{nl,t}^m)$, where $A_{nl,t}^m$ is the nonlinear IV, $A_{nl,t}^m=\left(\tilde{R}_{t-1}^m,C_{t-1}^g,\tilde{R}_{t-1}^m\times C_{t-1}^g,(\tilde{R}_{t-1}^m)^2,(C_{t-1}^g)^2\right)$. $\tilde{R}^m$ and C^g are defined in Table 1. The sample period is 1958-2011 (annual) and the data are described in the text. The p-value of J statistics are calculated with Chi-square distribution with degrees of freedom DF. Statistical significance at 10%, 5%, and 1% levels are denoted by *,** , and ***, respectively.

IV	$\hat{\gamma}$	SE($\hat{\gamma}$)	$\hat{\beta}$	SE($\hat{\beta}$)	J	DF	p-Value
A_{-1}^m	5.83	7.06	1.58e-06	1.37e-06	9.9e-06	1	1.00
A_{-2}^m	17.64	12.05	-1.05e-05	8.31e-06	1.1e-05	1	0.99
A_{-3}^m	19.92*	12.49	-7.71e-10	6.43-10	2.3e-05	1	1.00
A_{-4}^m	28.29***	8.13	-7.68e-11	6.22e-11	3.8e-07	1	1.00
A_{nl}^m	4.50	4.85	-1.29e-06	1.05e-06	0.15	4	1.00

Table 4. Estimation of Asset Market Restrictions Using Market and Industry Returns

This table presents the point estimates, standard errors, and J statistics from two step GMM estimation (with heteroskedasticity- and autocorrelation-consistent inference) of the subjective discount rate ($\hat{\beta}$) and the constant relative risk aversion coefficient ($\hat{\gamma}$) from the moment restrictions

$$\mathbb{E}_t\left[\mathbf{A}_t^{my}\left\{\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\tilde{R}_{t+1}^{j}\right\}\right]=0,\ j=m,y,$$

The vector IVs is $A_{-it}^{my}=((\tilde{R}_{t-i}^{m},\tilde{R}_{t-i}^{y},C_{t-i}^{g}),i=1,2,3,4,A_{nl,t}^{my})$, $A_{nl,t}^{my}=A_{nl,t}^{m}\cup\tilde{R}_{t-1}^{y}$. The sample period and is 1958-2011 and the variables are defined in Tables 1 and 3. The p-value of J statistics are calculated with Chi-square distribution with degrees of freedom DF. Statistical significance at 10%, 5%, and 1% levels are denoted by *,** , and ***, respectively.

IV	$\hat{\gamma}$	SE($\hat{\gamma}$)	$\hat{\beta}$	SE($\hat{\beta}$)	J	DF	p-Value
A_{-1}^{my}	2.38	1.81	-1.10e-07	8.34e-08	3.50e-05	5	1.00
A_{-2}^{my}	32.57***	5.02	-2.58e-11	1.93e-11	1.60e-09	3	1.00
A_{-3}^{my}	3.17	3.32	-1.80e11	1.42e-11	2.90e-09	4	1.00
A_{-4}^{my}	15.37***	2.35	2.89e-09	9.22e-10	23.9e-09	6	1.00
A_{nl}^{my}	33.33***	1.73	-1.40e-11	1.04e-11	2.90e-09	5	1.00

Table 5. General Equilibrium Estimation

This table presents the point estimates, standard errors, and J statistics from two step GMM estimation (with heteroskedasticity- and autocorrelation-consistent inference) of subjective discount rate ($\hat{\beta}$), constant relative risk aversion coefficient ($\hat{\gamma}$), intratemporal elasticity of substitution ($\hat{\sigma}$) and the utility weight for the manufacturing industry ($\hat{\phi}$) from the system of moment restrictions based on the Euler conditions for capital investment and materials input, as well as the equity and riskfree asset markets considered in Table 4. These moment conditions are

$$\begin{aligned}
\mathbb{E}_t\left[\mathbf{A}_t^m\left\{\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\tilde{R}_{t+1}^y\right\}\right] &= 0,\\
B_{-1t}^y\left\{\left(\frac{X_t-(Z_t+H_t)}{Y_t}\right)^{1/\sigma}\eta F_H(K_t,H_t,\lambda_t)-1\right\} &= 0,\\
B_{-1t}^y\left\{p_t^y Z_{I,t}-\mathbb{E}_t\left[M_{t+1}[p_{t+1}^y F_{K,t+1}-Z_{K,t+1}+(1-\delta)Z_{I,t+1}\right]\right\} &= 0,
\end{aligned}$$

where λ and H are defined in Table 2. K is the capital stock and $Z(I,K)$ are the total investment costs, and $Y=F(K,H,\lambda)$ is the output from production function F. And F_H, F_K, Z_K, and Z_I are the associated partial derivatives. X is the output of the rest of the economy, and $p^y = \left(\frac{X-(Z+H)}{Y}\right)^{1/\sigma}\eta$, where $\eta \equiv \frac{\phi}{1-\phi}$, is the equilibrium price of manufacturing output. A_t^m is defined in Table 3, $B_{-1t}^y=(I_{t-1},H_{t-1},\lambda_{t-1})$ is the vector of industry production IVs, and we put $G_{-it} = (A_{-it}^m, B_{-1t}^y)$, $i=1,2,3,4$, $G_{nl,t}=(A_{nl,t}^m, B_{-1t}^y)$. The sample period is 1958-2011 (annual). The p-value of J statistics are calculated with Chi-square distribution with degrees of freedom DF. Statistical significance at 10%, 5%, and 1% levels are denoted by *,** , and ***, respectively.

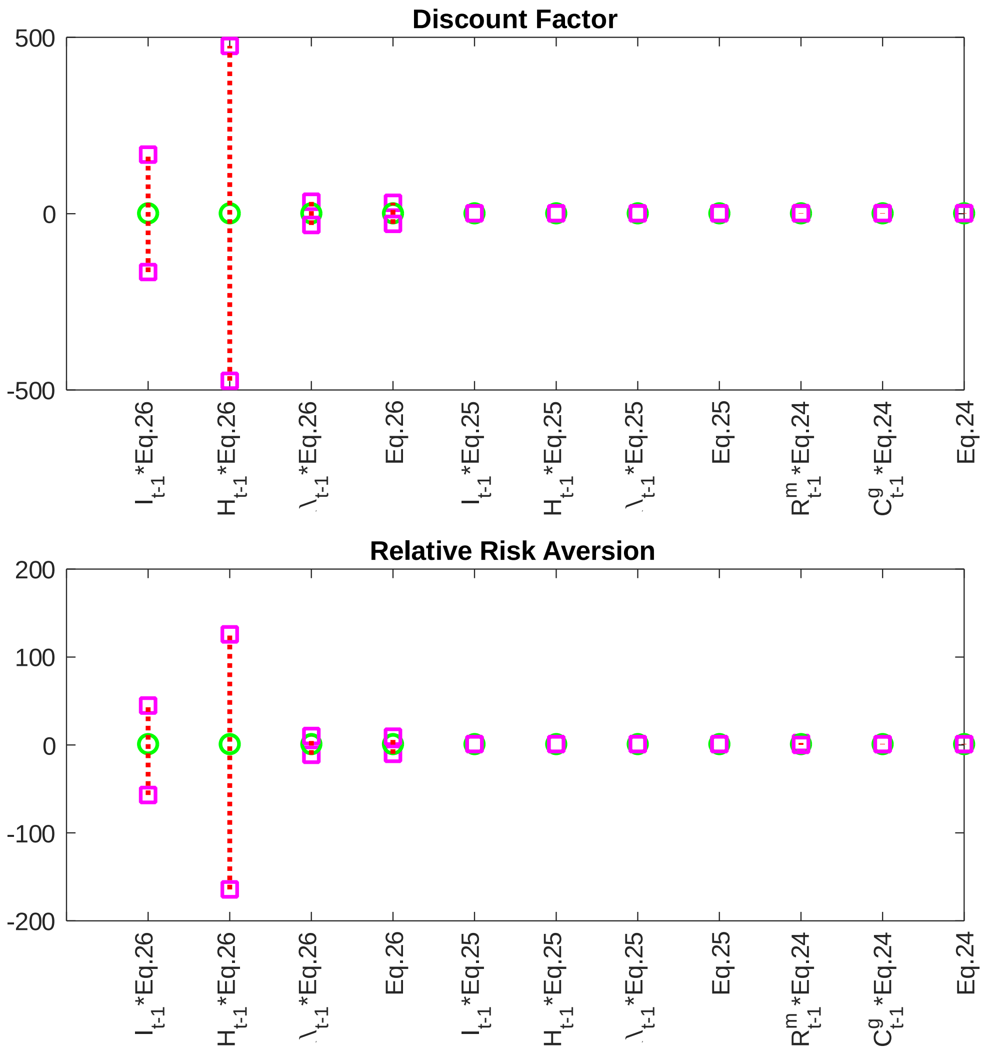
IV	$\hat{\gamma}$	SE($\hat{\gamma}$)	$\hat{\beta}$	SE($\hat{\beta}$)	$\hat{\sigma}$	SE($\hat{\sigma}$)	$\hat{\phi}$	SE($\hat{\phi}$)	χ^2	DF	p-Value
G_{-1}	8.549***	0.959	0.779***	0.026	6.768***	1.147	|0.387***	0.111	1.797	7	0.970
G_{-2}	10.235***	1.989	0.827***	0.055	4.604***	1.429	0.375***	0.030	1.750	7	0.972
G_{-3}	12.160***	1.027	0.882***	0.037	2.761***	0.410	0.351***	0.0205	1.697	7	0.975
G_{-4}	12.569***	1.749	0.893***	0.053	1.885***	0.390	0.323***	0.040	1.681	7	0.975
G_{nl}	8.530***	0.891	0.778***	0.024	6.723***	1.067	0.387***	0.108	1.815	9	0.994

Table 6. Role of Moment Conditions

This table presents the point estimates, standard errors, and J statistics from two step GMM estimation (with heteroskedasticity- and autocorrelation-consistent inference) of subjective discount rate ($\hat{\beta}$), constant relative risk aversion coefficient ($\hat{\gamma}$), intratemporal elasticity of substitution ($\hat{\sigma}$) and the utility weight for the manufacturing industry ($\hat{\phi}$) from subsets of moment conditions used in Table 5. The sample period is 1958-2011 (annual) and the data are described in the text. The p-value of J statistics are calculated with Chi-square distribution with degrees of freedom DF. Statistical significance at 10%, 5%, and 1% levels are denoted by *,**, and ***, respectively.

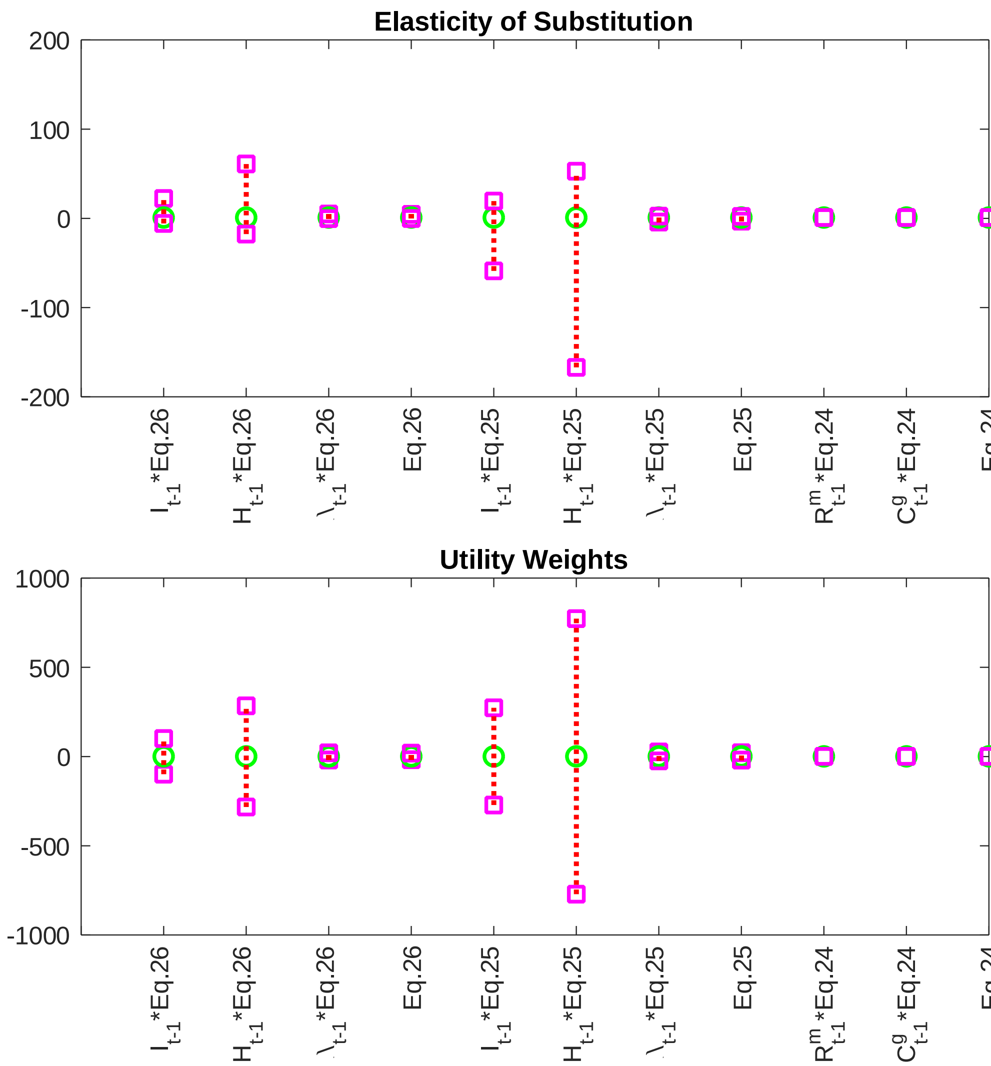
Euler Equations	IV	$\hat{\gamma}$	SE($\hat{\gamma}$)	$\hat{\beta}$	SE($\hat{\beta}$)	$\hat{\sigma}$	SE($\hat{\sigma}$)	$\hat{\phi}$	SE($\hat{\phi}$)	χ^2	DF	p-Value
Investment, Asset Markets	G_{-1}^{my}	7.864		0.476	0.420	-10.000		|0.270	5.566	0.027	10	0.85
Inputs, Asset Markets	G_{-1}^{my}	12.196***	4.212	0.001	0.001	2.517	5.526	0.346	0.355	20189.9***	8	0.000
Investment, Materials	B_{-1}^{y}	8.230***	2.424	|0.770***	0.062	|7.262**	3.698	0.388***	0.033	1.765	4	0.779
Investment, Materials	G_{-1}^{my}	8.378***	2.238	|0.775***	0.0576	|6.792***	2.612	0.387***	0.027	1.887	8	0.984

Figure 1: Sensitivity of Various Moments to Discount Factor (β) and Relative Risk Aversion (γ)



Notes to Figure: This graph shows the effects of variations in the discount factor (β) and risk aversion (γ) on the moment conditions Eqs. (24)-(26), represented on the x-axis. I_{t-1}*Eq26 is the product of lagged investments I_{t-1} and Eq. (26) and similarly for the other items on the x-axis. In all cases, we vary each parameter from 0.5 times to 1.5 times the estimated parameter value, holding all other parameters at their estimated levels. The red line with the pink box shows the change in the moment scaled by the moment evaluated at the estimated parameter values. The green circle represents the moment at the estimated parameter values.

Figure 2: Sensitivity of Various Moments to Elasticity of Substitution (σ) and Utility Weights (ϕ)



Notes to Figure: This graph shows the effects of variations in the elasticity of substitution (σ) and utility weights (ϕ) on the moment conditions Eqs. (24)-(26), represented on the x-axis. I_{t-1}*Eq26 is the product of lagged investments I_{t-1} and Eq. (26) and similarly for the other items on the x-axis. In all cases, we vary each parameter from 0.5 times to 1.5 times the estimated parameter value, holding all other parameters at their estimated levels. The red line with the pink box shows the change in the moment scaled by the moment evaluated at the estimated parameter values. The green circle represents the moment at the estimated parameter values.

Appendix
Derivations and Proofs

Derivation of Optimal Consumption and Portfolio Policies

Since the objective function is strictly increasing and concave, the optimal consumption and portfolio policies can be characterized through a two-step process, where optimal consumption c_t is determined as a function of available consumption expenditure W_t, and the optimal portfolio is then determined taking as given the optimal consumption policy. Using the dynamic programming principle (DP), at any t, the representative consumers optimization problem (5)-(6 can be written as

$$\max_{\mathbf{c}_t, \mathbf{q}_{t+1}} E_t \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{C_\tau^{1-\gamma} - 1}{1-\gamma} \right] + \xi_t \left[W_t - \mathbf{p}_t \cdot \mathbf{c}_t \right]. \tag{A1.1}$$

Here, ξ_t is the Lagrange multiplier for the budget constraint (6). Since preferences are strictly increasing, the budget constraint is binding and $\xi_t > 0$. Next, using the definition of aggregate consumption (7), the first order optimality conditions for $c_t^j, j = x, y$, can be written

$$(C_t)^{\frac{1-\gamma\sigma}{\sigma}} (c_t^j)^{-\frac{1}{\sigma}} \phi^j = \xi_t p_t^j, \tag{A1.2}$$

where $p_t^x = 1$, $\phi^x \equiv (1-\phi), \phi^y \equiv \phi$. It follows from (A1.2) that

$$p_t^j c_t^j = \xi_t^{-\sigma} (p_t^j)^{1-\sigma} (C_t)^{-(1-\gamma\sigma)} (\phi^j)^\sigma \tag{A1.3}$$

Then recognizing that $W_t = p_t \cdot c_t$, and using (A1.3), and the definition of the aggregate price index P_t (see (9)) allows one to solve for the Lagrange multiplier as

$$\xi_t = \left(\frac{W_t}{P_t} \right)^{-\frac{1}{\sigma}} P_t^{-1} (C_t)^{\frac{1-\gamma\sigma}{\sigma}}. \tag{A1.4}$$

Substituting this in (A1.2) and rearranging terms then gives the optimal consumption functions given in (8).

Next, for any $\tau \geq t$, let $U_\tau \equiv \beta^{\tau-t} \frac{C_\tau^{*1-\gamma} - 1}{1-\gamma}$ denote the indirect period utility function with the optimal consumption functions given in (8). The envelope theorem then yields $\xi_\tau = \frac{\partial U_\tau}{\partial W_\tau}$. Using the fact that $W_\tau = q_\tau \cdot (D_\tau + S_\tau) - q_{\tau+1} \cdot S_\tau$ then yields the optimality conditions for q_{t+1}

$$\xi_t \mathbf{S}_t = \mathbb{E}_t \left[\beta \chi_{t+1} (\mathbf{D}_{t+1} + \mathbf{S}_{t+1}) \right]. \tag{A1.5}$$

But using $C_t^* = \frac{W_t}{P_t}$ and substituting in (A1.4) gives $\xi_t = (C_t^*)^{-\gamma} P_t^{-1}$. Since this holds for any τ, inserting in (A1.5) and dividing through by $\mathbf{S}_t$ yields Equation (10). ∎

Proof of Proposition 1: Substituting the optimal consumption functions (8) in the market clearing conditions (11)-(13) in a symmetric equilibrium yield

$$\begin{aligned} \frac{W_t}{P_t} \left[P_t (1-\phi) \right]^\sigma &= X_t - [Z(I_{it}, K_{it}) + H_{it}] && \text{(A2.1)} \\ \frac{W_t}{P_t} \left[\frac{P_t \phi}{p_t^y} \right]^\sigma &= Y_t && \text{(A2.2)} \end{aligned}$$

Dividing (A2.1) by (A2.2) and rearranging terms yields $p_t^y = \left(\frac{W_t}{Y_t} \right)^{1/\sigma} \eta_t$ in a symmetric equilibrium. Since competitive firms equate marginal costs with any given price, in equilibrium the marginal cost $[F_H(K_t, H_t, \lambda_t)]^{-1}$ is equated with the price as given in (16). Next, using the Bellman-representation (15), along any competitive equilibrium path, at any t, conditional on Γ_t, the optimization problem for the typical competitive firm is

$$V_t(\Gamma_t) = \max_{I, H_t \geq 0} \left(\frac{W_t}{P_t} \right)^{-\gamma} \left(\frac{p_t^y Y_t - H_t - Z(I_t, K_t)}{P_t} \right) + \beta \mathbb{E}_t \left[V_{t+1}(\Gamma_{t+1}) \right], \text{ s.t., (1)–(2).} \tag{A2.3}$$

Then, subject to (1)–(2), the optimal (interior) investment input path satisfies

$$\begin{aligned} 0 &= -\frac{\partial V_t(\Gamma_t)}{\partial K_t} + \left(\frac{W_t}{P_t} \right)^{-\gamma} \left(\frac{1}{P_t} \right) \frac{\partial D_t^y}{\partial K_t} + \beta (1-\delta) \frac{\partial \mathbb{E}_t \left[V_t(\Gamma_{t+1}) \right]}{\partial K_{t+1}}, && \text{(A2.4)} \\ 0 &= \left(\frac{W_t}{P_t} \right)^{-\gamma} \left(\frac{1}{P_t} \right) \frac{\partial D_t^y}{\partial I_t} + \beta \frac{\partial \mathbb{E}_t \left[V_{t+1}(\Gamma_{t+1}) \right]}{\partial I_t}. && \text{(A2.5)} \end{aligned}$$

Now, $D_t^y = p_t^y Y_t - H_t - Z(I_t, K_t)$. Hence, $\frac{\partial D_t^y}{\partial I_t} = -Z_I(I_t, K_t)$. Furthermore, $\frac{\partial K_{t+1}}{\partial I_t} = 1$ and thus $\frac{\partial \mathbb{E}_t[V_{t+1}(\Gamma_{t+1})]}{\partial I_t} = \frac{\partial \mathbb{E}_t[V_{t+1}(\Gamma_{t+1})]}{\partial K_{t+1}}$. Recalling that the SDF is $M_{t+1} \equiv \beta \left(\frac{W_{t+1}}{W_t} \right)^{-\gamma} \left(\frac{P_{t+1}}{P_t} \right)^{\gamma-1}$, (A2.4) and (A2.5) then together imply that the Euler condition characterizing the equilibrium investment path is

$$-\frac{\partial D_t^y}{\partial I_t} = \mathbb{E}_t \left[M_{t+1} \left(\frac{\partial D_{t+1}^y}{\partial K_{t+1}} - (1-\delta) \frac{\partial D_{t+1}^y}{\partial I_{t+1}} \right), \right] \tag{A2.6}$$

where in (A2.6) we have used iterated expectations and recursively substituted the optimality condition for I_{t+1}. Now, using the envelope theorem (that sets the indirect effects of ∂K_{t+1} on the optimally chosen I_{t+1} and H_{t+1} to zero), in a symmetric competitive (price-taking) equilibrium with $Y_{it+1} = Y_{t+1}$, we have

$$\frac{\partial D_{t+1}^y}{\partial K_{t+1}} = p_{t+1}^y F_K(K_{t+1}, H_{t+1}, \lambda_{t+1}) - Z_K(I_{t+1}, K_{t+1}). \tag{A2.7}$$

(A2.6)-(A2.7) and $\frac{\partial D_{t+1}^y}{\partial I_{t+1}} = -Z_I(I_{t+1}, K_{t+1})$ then together characterize the equilibrium path for investment in a symmetric competitive equilibrium viz.,

$$Z_I(I_t, K_t) = E_t \left[M_{t+1} \left(\{ p_{t+1}^y F_K(K_{t+1}, H_{t+1}, \lambda_{t+1}) - Z_K(I_{t+1}, K_{t+1}) \} + (1-\delta) Z_I(I_{t+1}, K_{t+1}) \right) \right]. \tag{A2.8}$$

■