Asset Valuation with known cash flows

– Annuities and Perpetuities
– care loan, saving for retirement, mortgage
Simple Perpetuity

A perpetuity is a stream of cash flows each of the amount of “CF” dollars, that are received at the end of each period forever.

• Note:
  – Cash flows are the same over time
  – There is no cash flow today (i.e. you receive the first cash flow one period from now)
Simple Perpetuity
Valuing a perpetuity

The PV of a perpetuity is,

\[ PV = \frac{CF}{1+r} + \frac{CF}{(1+r)^2} + \frac{CF}{(1+r)^3} + \ldots \]

\[ = \sum_{i=1}^{\infty} \frac{CF}{(1+r)^i} = \frac{CF}{r} \]
Example: You will receive $100 forever beginning the next year. The annual interest rate is 10%. Find PV.

\[ \text{PV} = \frac{100}{0.1} = \$1,000 \]

Check:
If we invest $1,000 then we should be able to “replicate” the stream of cash flows generated by the perpetuity. That is by investing $1,000 today we should receive a payment of $100 each year forever.

This is how we can do this:
Simple Annuity

An annuity is a stream of cash flows each of the amount of “CF” dollars, that are received at the end of each period for the duration of “n” periods.

Note:

- Cash flows are the same over time
- There is no cash flow today (i.e. you receive the first cash flow one period from now)
Simple five year Annuity

![Chart showing cash flows for a five-year annuity with payments of $5 at each of the six time periods.]
Simple annuity formula

The PV of an annuity for n years is,

\[
PV = \frac{CF}{1+r} + \frac{CF}{(1+r)^2} + \frac{CF}{(1+r)^3} + \ldots + \frac{CF}{(1+r)^n}
\]

\[
= \sum_{i=1}^{n} \frac{CF}{(1+r)^i} = \frac{CF}{r} \left(1 - \frac{1}{(1+r)^n}\right)
\]
Example: Find the present value of an annuity that pays $500 for the duration of 7 years (beginning at the end of the first year). The annual interest rate is 5%.
“n” year annuity versus perpetuity when \( r = 10\% \)
Growing perpetuity

A growing perpetuity is a stream of cash flows that grows over time with growth rate \( g \) where cash flows are received at the end of each period forever.

- **Note:**
  - Cash flows grow over time with rate \( g \)
  - There is no cash flow today (i.e. you receive the first cash flow one period from now)
Growing perpetuity with growth rate of 8%
Growing perpetuity formula

- The first cash flow “CF” is received at the end of the first period and is growing at rate “g” afterwards

In particular, cash flows look like:

<table>
<thead>
<tr>
<th>t=0</th>
<th>t=1</th>
<th>t=2</th>
<th>t=3</th>
<th>......</th>
<th>t=n</th>
<th>......</th>
</tr>
</thead>
<tbody>
<tr>
<td>CF</td>
<td>CF(1+g)</td>
<td>CF(1+g)^2</td>
<td>......</td>
<td>CF(1+g)^n-1</td>
<td>......</td>
<td>......</td>
</tr>
</tbody>
</table>

\[ PV = \frac{CF}{1+r} + \frac{CF(1+g)}{(1+r)^2} + \frac{CF(1+g)^2}{(1+r)^3} + \ldots \]

\[ = \sum_{i=1}^{\infty} \frac{CF(1+g)^{i-1}}{(1+r)^i} = \frac{CF}{r - g} \]
Growing perpetuity with growth rate “g” and interest rate r=10%
Growing Annuity

A growing annuity is a stream of cash flows that grows over time with growth rate “g” where cash flows are received at the end of each period for the duration of “n” years.

• Note:
  – Cash flows grow over time with rate “g”
  – There is no cash flow today (i.e. you receive the first cash flow one period from now)
Five year growing Annuity with growth rate of 8%
Growing annuity formula

The PV of a growing annuity for n years is,

\[
PV = \frac{CF}{1+r} + \frac{CF(1+g)}{(1+r)^2} + \frac{CF(1+g)^2}{(1+r)^3} + \cdots + \frac{CF(1+g)^{n-1}}{(1+r)^n}
\]

\[
= \sum_{i=1}^{n} \frac{CF(1+g)^{i-1}}{(1+r)^i} = \frac{CF}{r-g} \left(1 - \frac{(1+g)^n}{(1+r)^n}\right)
\]
Growing annuity with growth rate “g” and interest rate r=10%
Growing annuity formula for \( r = g \)

\[
PV = \frac{CF}{1+r} + \frac{CF(1+g)}{(1+r)^2} + \frac{CF(1+g)^2}{(1+r)^3} + \ldots + \frac{CF(1+g)^{n-1}}{(1+r)^n}
\]

\[
= \frac{CF}{1+r} + \frac{CF}{(1+r)} + \frac{CF}{(1+r)} + \ldots + \frac{CF}{(1+r)}
\]

\[
= \frac{n \cdot CF}{1+r}
\]
• **Example 1**: if you save $1,000 each year for 35 years, how much will you have in your bank account after 35 years if the interest rate is 10%?

• How much would you need to save each year in order to accumulate $300,000 after 35 years?
• What is the present value of your installments if you save $1,000 each year for (a) 35 years and (b) forever?

• What is the present value of your installments if the interest rate changes to 9%?

• What is the future value of your installments if the interest rate changes to 9%?
Example 2: You want to rent an apartment in Houston for one year. The landlord is not willing to reduce the monthly rent of $1,000 but offers the first month for no charge. You can also stay in your old apartment and pay rent of $915 (at the beginning of each month). What should you do? Assume an interest rate of 1% per month.

PV(current rent payments) =

PV(alternative rent payments) =

Would your choice be the same if you got the last month free?
• **Example 3:** You need a parking space for the period of two years. You can either buy a parking space for $10,000 and then sell it in two years for $10,500, or rent a parking space for the period of 2 years. The monthly rent is currently $75 and is expected to rise by 0.5% each month (starting from the next). What should you do? Assume an interest rate of 1% per month.

\[
\text{PV(buy parking space)} =
\]

\[
\text{PV(rent parking space)} =
\]
Example 4: You have just earned a Federal tax return and are thinking to donate $2,000 to the Museum of Contemporary Art in Houston. In return Museum offers free annual membership ($100 per year paid at the beginning of the year) forever or a growing perpetuity of $70 with growth rate of 3% per year (the first payment of $70 is in one year). What should you do? Assume an interest rate of 7% per year.

PV(free membership offer) =

PV(growing perpetuity) =
Example 5: 30 years ago, André François Raffray agreed to pay the 90 year old Jeanne Calment 2,500 francs ($500) per month until she dies. In return he will receive her apartment when she dies. The apartment is worth $184,000. Suppose the monthly interest rate is 1%. Assuming M. Raffray thought this was a good deal, how long did he think Jeanne Calment would live?

Mr. Raffray will break even if Jeanne Clament lives less than $n$ additional months.

This implies that .
Example 6: An insurance agent offers you the following contract: you pay $5,000 per year (end) for the next 15 years and in return you will receive $7,000 a year (end) for the following 15 years. Suppose interest rates are 9%. Should you buy this contract?
Example cont’d: suppose that the insurance agent sweetens the deal and says that the payments that you receive will grow at 3% per year. Would you take the contract now?