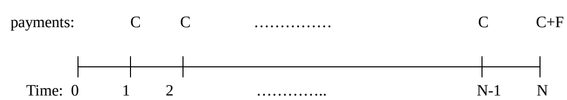
Bond Valuation

- The Structure of *fixed income securities*
- Price & *yield to maturity* (ytm)
- *Term structure* of interest rates
- Treasury STRIPS
- No-arbitrage pricing of coupon bonds

Structure of fixed income securities

- A Fixed Income Security promises to pay fixed **coupon** payments at a prespecified dates and a fixed **principal** amount (the face value) at the maturity date.
- When there are no coupon payments then the bond is called a **zero coupon bond** or a **pure discount bond**.

Payments of a "N" year bond with annual coupon C and face value F



Coupon Bonds

- The coupon payments on coupon bonds are typically stated as a percentage of the principal (or face value) paid per year.
- If coupon payments are made n times per year, then the coupon amount is (c x F)/n, where c is the coupon rate and F is the face value.

Understanding the terms: U.S. Treasury Notes and Bonds are typically issued with face value of $10,000, and pay semi-annual coupons. Assume a 30 year coupon rate of 7.5%. What payments do you receive from buying this bond?

You receive a coupon payment of (0.075 x $10,000)/2 = $375 twice a year and on the maturity date you will receive the coupon of $375 plus the principal of $10,000.

The U.S. government issues bonds

Default free bonds issued by the government:

Treasury bills have an initial maturity less than one year and are all discount bonds

Treasure notes have initial maturities between one and ten years and pay coupons

Treasury bonds have initial maturities longer than ten years and pay coupons

Default free bonds issued by government sponsored agencies

Fannie Mae: Federal National Mortgage Association

Ginnie Mae: Government National Mortgage Association

Freddie Mac: Federal Home Loan Mortgage Corporation

Federal Home Loan Bank

Price & Yield to Maturity

The **Yield to Maturity (ytm)** is the (constant) discount rate which equates the discounted cash flows to the price.

YTM on a n year Zero Coupon Bond with face value F and price P

$$P = \frac{F}{(1+y)^n}$$

- **Example**: Suppose that a 5 year discount bond with face value \$100 is selling for \$95. What is the yield to maturity on this bond?

$$95 = \frac{100}{(1+y)^5} \Rightarrow \left(\frac{100}{95}\right)^{0.2} = 1+y \Rightarrow y = 1.03\%$$

Calculating the YTM of a Coupon Bond

Example: A 7% coupon bond that pays annually, has 2 years to maturity, face value of \$100 and price of \$98.22. Find the bond's ytm.

$$98.22 = \frac{7}{1+y} + \frac{107}{(1+y)^2} \Rightarrow y = 8\%$$

- What if the price was \$100?

$$100 = \frac{7}{1+y} + \frac{107}{(1+y)^2} \Rightarrow y = 7\%$$

- What if the coupon rate was 10% (price still \$98.22)?

$$98.22 = \frac{10}{1+y} + \frac{110}{(1+y)^2} \Rightarrow y = 11\%$$

The Yield Curve/Term Structure of Interest Rates

- So fare we have assumed that interest rates are constant across time
- In fact, interest rates vary considerably depending on how long you are going to borrow/lend:
  - Long term interest rates reflect investors' expectations of future real interest rates and inflation in coming years
  - If either inflation or the real interest rate are expected to change in the future, then long term rates will differ from short term rates.

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Treasury STRIPS & the Yield Curve

- U.S. Treasury **STRIPS** are the "stripped" coupons from T-Bills and Bonds . The prices of these STRIPS are reported as price per \$100 of face value.
- Denote the price of a "n" year STRIP bond by B_n
- The ytm of a "n" year STRIP bond, y_n, is

$$B_n = \frac{100}{(1+y_n)^n} \Rightarrow y_n = \left(\frac{100}{B_n}\right)^{\frac{1}{n}} - 1$$

Example: The prices of a 1,2 and 3 year STRIPS are \$96, \$91 and \$85. Find the yields to maturity of these bonds?

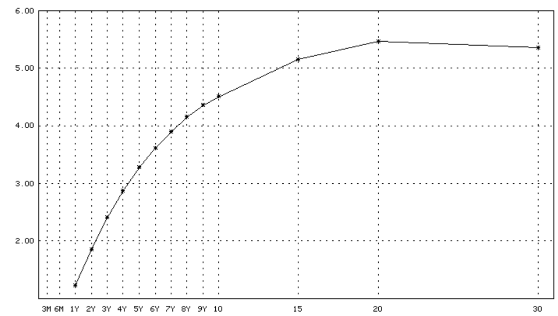
$$y_1 = \frac{100}{96} - 1 = 4.175\%,$$

$$y_2 = \left(\frac{100}{91}\right)^{\frac{1}{2}} - 1 = 4.83\%, \quad y_3 = \left(\frac{100}{85}\right)^{\frac{1}{3}} - 1 = 5.57\%$$

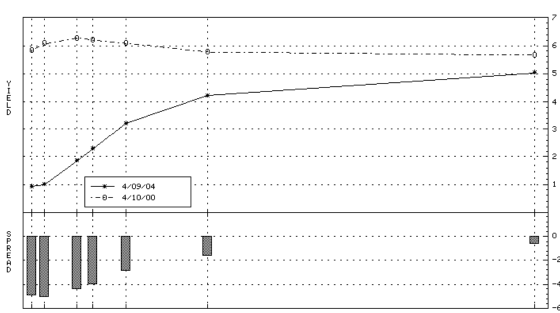
Prices – U.S. Government Strips (2004)

	DESCRIPTION	PRICE	SRC	UPDATE	YIELD	HEDGED YIELD
3MO	1)					
6MO	2)					
1YR	3) S 0 02/15/05	B 98.9728	BGN	16:00	1.2280	1.2280
2YR	4) S 0 02/15/06	B 96.6712	BGN	16:00	1.8450	1.8450
3YR	5) S 0 02/15/07	B 93.4467	BGN	16:00	2.3980	2.3980
4YR	6) S 0 02/15/08	B 89.6602	BGN	16:00	2.8600	2.8600
5YR	7) S 0 02/15/09	B 85.4620	BGN	16:00	3.2700	3.2700
6YR	8) S 0 02/15/10	B 81.0597	BGN	16:00	3.6260	3.6260
7YR	9) S 0 02/15/11	B 76.8086	BGN	16:00	3.8930	3.8930
8YR	10) S 0 02/15/12	B 72.4463	BGN	16:00	4.1520	4.1520
9YR	11) S 0 02/15/13	B 68.2884	BGN	16:00	4.3600	4.3600
10YR	12) S 0 02/15/14	B 64.4367	BGN	16:00	4.5150	4.5150
15YR	13) S 0 02/15/18	B 49.4847	BGN	16:00	5.1470	5.1470
20YR	14) S 0 02/15/23	B 36.1386	BGN	16:00	5.4750	5.4750
30YR	15) S 0 02/15/31	B 24.2190	BGN	16:00	5.3530	5.3530

Yield Curve – U.S. Government Strips (2004)



Change in Yield Curve – U.S. Government Strips (2000 vs. 2004)



Using market bond prices to discount cash flows

- Corporations use bond prices when evaluating a stream of certain cash flows.

Example: Suppose that you are considering an investment in a project that generates the following cash flows with perfect certainty:

Date	1	2	3	4
Cash flow	$50	$100	$100	$100

You also know the discount bond (STRIPS) prices

Date	1	2	3	4
Price B_i	$98	$95	$92	$88

What is the PV of the cash flows from the project?

There are two ways to calculate the PV:

Method I: use the discount bond (STRIPS) prices directly …

The PV of $1 received in "n" years is $B_n/100$.

$$PV = 50\times\frac{98}{100} + 100\times\frac{95}{100} + 100\times\frac{92}{100} + 100\times\frac{88}{100} = 324$$

Method II: use yields to maturity …

Date	1	2	3	4
y_i	2.04%	2.60%	2.82%	3.25%

$$PV = \frac{50}{1.0204} + \frac{100}{(1.0260)^2} + \frac{100}{(1.0282)^3} + \frac{100}{(1.0325)^4} = 324$$

No-Arbitrage Pricing of Coupon Bonds

- We can value coupon bonds using the prices of STRIPS

Example: Suppose that you are given the option to purchase a 5 year coupon bond with annual coupon rate 10% and face value of $1,000. Also you have the following Treasury STRIPS data:

Years to Maturity	1	2	3	4	5
STRIPS Price	$98	$95	$92	$89	$85

What <u>must</u> the price of the bond be?

$$PV = 100\times\frac{98}{100} + 100\times\frac{95}{100} + 100\times\frac{92}{100} + 100\times\frac{89}{100}$$
$$+100\times\frac{85}{100} + 1000\times\frac{85}{100} = \$1{,}309$$

Pricing Coupon Bonds with STRIPS prices

- In general the price of a bond is,

$$\text{Bond price} = \sum_{i=1}^{n} C_i \times \left(\frac{B_i}{100}\right)$$

Where C_i is the i'th payment (including coupons and principal) and B_i is the "time matched" STRIPS price.

Term Structure Implied by Coupon Bonds

- We have showed how to price coupon bonds using the prices of STRIPS.
- STRIPS are priced using the prices of coupon bonds.

Example: Consider the following coupon bond prices:

	Years to maturity	Face value	Coupon Rate	Current price
Bond A	1	$1,000	5%	$997.5
Bond B	2	$1,000	8%	$1,048

What are the prices and ytm's of the one and two year discount bonds?

Method I:

- The one year ytm, $997.5 = \frac{1000+50}{1+y_1} \Rightarrow y_1 = 5.26\%$

$$\Rightarrow B_1 = \frac{100}{1+y_1} = \$95$$

- The two year ytm,

$$1048 = \frac{80}{1+y_1} + \frac{1000+80}{(1+y_2)^2} \Rightarrow y_2 = 5.4\% \Rightarrow B_2 = \$90$$

Method II:

- The price of a one year STRIP, $997.5 = 1050 \times \frac{B_1}{100} \Rightarrow B_1 = 95$

- The price of a two year STRIP,

$$1048 = 80 \times \frac{B_1}{100} + 1080 \times \frac{B_2}{100} \Rightarrow B_2 = 90$$

Synthetic Discount Bonds

*How could you **synthetically** construct 1 and 2 discount bonds (with face value $100) from the two previous coupon bonds?*

Synthetic 1 year zero coupon bond:

Bond A pays $1050 at time 1. So buy 100/1050=0.09523 units of bond A.

the price is 0.09523 x 997.5 = **$95**

Synthetic 2 year zero coupon bond:

Bond B pays $1080 at time 2. So buy 100/1080 = 0.09259 units of bond B. But then you also get a coupon in the amount $80 x 0.09259 = $7.407.

To eliminate this sell 7.407/1050 = 0.00705 units of bond A.

Total cost is 0.09259 x 1048 – 0.00705 x 997.5 = **$90**