Inflation

– *real* and *nominal* cash flows.

– *real* and *nominal* interest rates.

Taxes

– *After-tax* cash flows

– *After-tax* interest rate.

Purchasing Power & Inflation

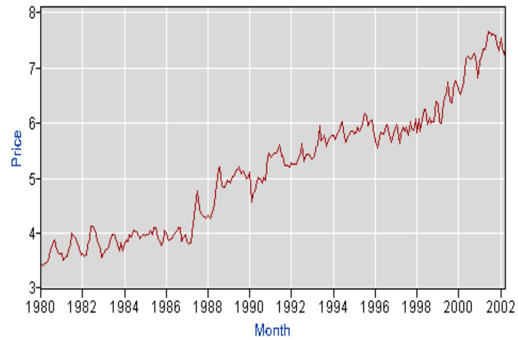
What is "Purchasing Power"?

- The amount of goods we can buy with one unite of currency represents the "Purchasing Power" of that currency
- Inflation is defined as the rate of increase in prices. If prices increase by 5% per year then we say that the inflation rate is 5% per year.
- Inflation is often measured by the Consumer Price Index (CPI). The CPI measures the rate of increase of the price of a bundle of goods that a typical consumer would purchase in a year.

Inflation

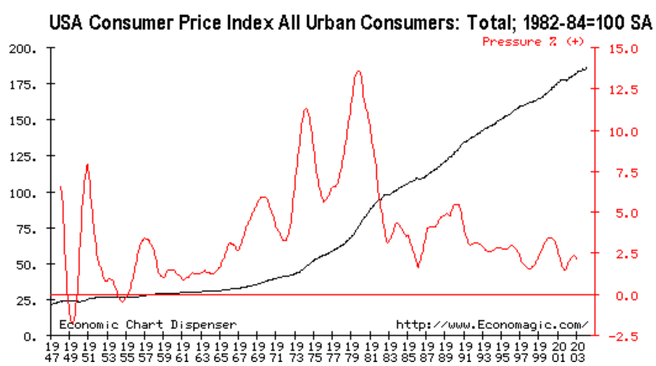
"Purchasing Power" can change dramatically over time…

U.S. city average
Item: Steak, T-Bone, USDA Choice, bone-in, per lb. (453.6 gm)



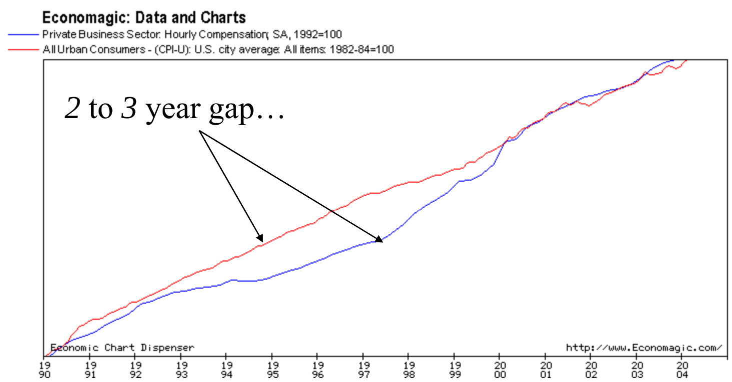
Consumer Price Index

The "Consumer Price Index" measures the price of a "basket of goods"



Wages & Inflation

Is your salary catching up with prices?



Real and Nominal Cash Flows

- The nominal cash flow is the dollar amount received
- The real cash flow equals the purchasing power and depends on the inflation rate, *i*.

$$CF_{real} = \frac{CF_{nominal}}{(1+i)^n}$$

- n = time receiving CF

Example: You are offered $100 in one year. The expected inflation rate this year is 3% and your borrowing and lending interest rate is 4%. What is the real value of this payment?

In one year, the real value of $100 will be

$100/(1.03) = $97.09

What is the present value of this payment?

Discounting Cash Flows

When faced with inflation, there are **two** methods for calculating present value. They always give the **same** result:

1. Discount nominal cash flows with nominal interest rates
2. Discount real cash flows with real interest rates

Nominal vs. Real Interest Rates

The interest rate we have been using, which is called the **nominal interest rate**, tells us how to convert dollars today to dollars in the future

- For example, if the nominal rate is r=10%, one dollar today is worth 1.1 dollars in one year from now

The **real interest rate** tells us how much more "stuff" you will be able to buy next year if you save one dollar today.

- If the nominal rate is *r=10%* per year and the inflation rate is 5% per year, then if we save for an extra year, we will be able to buy 1.1/1.05 = 1.048 times as much, or 4.8% more stuff.

Nominal vs. Real Interest Rates

- r_r = annual real interest rate
- r_n = annual nominal interest rate
- i = annual inflation rate

$$1 + r_r = \frac{1 + r_n}{1 + i}$$

Example: Suppose that Mr. Bond is planning to buy a new 25" SONY HD-TV one year from now. How much money should Mr. Bond put aside in a saving account today in order to have enough money to purchase the TV one year from now? The inflation rate is 3% and the interest rate Mr. Bond receives on his saving account is 4%. Currently such a TV costs $350.

<u>Using nominal interest rate:</u>

- One year from now the price of the TV will be $350(1.03)=$360.5
- You need to save $360.5/1.04**=$346.6** today.

<u>Using real interest rate:</u>

- The real value of the TV one year from now will be $350
- The real interest rate is 1.04/1.03-1=0.978%
- You need to save $350/1.00978**=$346.6** today.

Example: You are forecasting next year's revenues based on this year's sales of $1M. You expect a 2% real growth in your sales (units). Further you expect inflation to be 4%. Suppose that the appropriate nominal interest rate is 5%. What is the present value of next year's sales revenue?

Next year's nominal sales forecast is $1M(1.02)(1.04) = $1.0608M
This implies a PV of 1.0608/1.05 = $1.0103M

Next year's real sales forecast IS $1M(1.02) = $1.02M
This implies a PV of $1.02M/(1.05/1.04) = $1.0103M

Example: Mr. Bond is currently 40 years old (time t=0) and will work for another 25 years while retiring at age 65 (time t=25). His employer is offering him the following retirement plan: an annuity of fixed cash flows each equal to 80% of his last annual salary for the rest of his life (the first payment is paid at the age of 66, time t=26). The expected inflation rate for this period is 3% and the nominal interest rate is 10%. Mr. Bond's annual salary for the previous year, received at time t=0, is $100,000 and is expected to increase by 2% each year.

1. What is Mr. Bond's last annual salary (at age 65)?

$$100{,}000 \times (1.02)^{25} = \$164{,}061$$

2. What is Mr. Bonds real salary at age 65?

$$\frac{\text{nominal salary}}{\text{inflation}} = \frac{\$164{,}061}{(1.03)^{25}} = \$78{,}356$$

3. What is Mr. Bond's annual pension payment?

$$80\% \times 164{,}061 = \$131{,}249$$

4. What is Mr. Bond's real annual pension payment at ages 66, 75?

$$age\ 66: \frac{131{,}249}{(1.03)^{26}} = \$60{,}859, \quad age\ 75: \frac{131{,}249}{(1.03)^{35}} = \$46{,}644$$

5. What is Mr. Bond's real annual pension payment at age 85?

$$\frac{131{,}249}{(1.03)^{45}} = \$34{,}707$$

6. Assume that Mr. Bond will live for another 45 years (until the age of 85). What is the fund's current pension liability for Mr. Bond, i.e. what is the PV of the pension benefits.

$$\frac{1}{(1.1)^{25}} \times \frac{131{,}249}{0.1}\left(1 - \frac{1}{(1.1)^{20}}\right) = \$103{,}131$$

7. Assume that Mr. Bond will live for another 45 years (until the age of 85). Instead of a fixed **nominal** payment Mr. Bond wants a fixed **real** payment. Holding the value of the pension benefits Mr. Bond receives, what fixed real pension payment could he get?

$$\$103{,}131 = \frac{1}{\left(1.1/1.03\right)^{25}} \times \frac{\$X}{\left(1.1/1.03 - 1\right)}\left(1 - \frac{1}{\left(1.1/1.03\right)^{20}}\right) \Rightarrow X = \$49{,}580$$

8. What is Mr. Bond's nominal annual pension payments at ages 66, 75, and 85?

$$age\ 66: \$49{,}580(1.03)^{26} = \$106{,}924$$

$$age\ 75: \$49{,}580(1.03)^{35} = \$139{,}511$$

$$age\ 85: \$49{,}580(1.03)^{45} = \$187{,}491$$

Dealing with tax

- All businesses pay taxes and should take tax effects into consideration. This implies that, in calculating the project value for a business you should:
  - Use **after-tax cash flows**
  - Use the **after-tax interest rate**

- After-tax interest rates take into account the fact that interest received is income, and interest paid is an expense.
  - Suppose that the market rate of interest is r = 5%, and you pay a marginal tax rate of t = 35%.
  - The after-tax interest rate = r(1-t) = 5%(1-0.35) = 3.25%

Treating taxes

- r = before tax interest rate
- τ = marginal tax rate
- n = time of payment
- $CF_{\text{after tax}}$ =after tax cash-flow

$$PV = \frac{CF_{\text{after tax}}}{(1+r(1-\tau))^n}$$

Example: You win the lottery which either:

- Pays you income of $120,000 at the end of each year for 20 years, or
- Pays a lump sum of $1M now

The before tax interest rate is 10%. Either way, your winnings are taxable and your tax rate is 30%. Which should you choose?

After tax interest rate = 10% (1-0.3) = 7%

After tax annuity payment = $120,000(0.7) = $84,000

PV (annuity 20 years, interest 7%, payment $84,000) = $0.889M

PV (after tax lump sum) = $1M (1-0.3) = $0.7M

CPI: Houston-Galveston-Brazoria, TX

All Urban Consumers CPI U.S. city average