Quoting interest rates

– Compounded annual percentage rate (APR)
– Effective annual yield (EAY)

Mortgages

– Payments/Principal and interest
– Refinancing

## Quoting interest rates

*"the CD offers a 6% A.P.R. compounded quarterly"*

- Okay…so what is the annual interest rate?
  - A.P.R. – annual percentage rate
  - quarterly interest rate is 6% / 4 = 1.5%
  - annual interest rate (or effective annual yield – EAY) is $(1.015)^4 = 6.14\%$

## Quoting interest rates

- Let $r$= A.P.R. and $k$ = *# of compounding intervals*.
- EAY is given by, $1+EAY = (1+r/k)^k$
  - *Always make sure that you get EAY close to r*
- Example: A.P.R. = 6%

| k | 1+EAY |
|---|---|
| 1 | 1.06 |
| 2 | $(1+0.06/2)^2 = 1.0609$ |
| 4 | $(1+0.06/4)^4 = 1.0614$ |
| 12 | $(1+0.06/12)^{12} = 1.0617$ |
| 365 | $(1+0.06/365)^{365} = 1.06183$ |
| 8,760 | $(1+0.06/8760)^{8760} = 1.061836$ |

## Continuous compounding

- What if interest is compounded very frequently or continuously?

$$1+\mathrm{EAY} = \lim_{k\to\infty}\left(1+\frac{r}{k}\right)^k = e^r \quad (e \approx 2.7)$$

**Example**: Assume 6% A.P.R. compounded continuously, what is the EAY?

$$1+\mathrm{EAY} = e^{0.06} = 1.061837$$

**Car loan (example)**: You want to buy a \$20,000 car but you have only enough for the down payment of \$4,000. You can borrow from a bank at 9% APR compounded monthly or from "Houston Dealers" who offers financing at 6% APR for a 30 month loan but then you must forego a \$1,000 discount. What should you do?

- Bank: pay a high interest rate on a smaller loan
- Dealer: pay a lower interest rate on a larger loan

monthly payment **bank**: $\$15{,}000 = \frac{X}{0.09/12}\left(1 - \frac{1}{(1+0.09/12)^{30}}\right) \Rightarrow X = \$560$

monthly payment **dealer**: $\$16{,}000 = \frac{X}{0.06/12}\left(1 - \frac{1}{(1+0.06/12)^{30}}\right) \Rightarrow X = \$576$

- Given your credit history, the bank changes your rate to 12%. What should you do?

monthly payment **bank**: $\$15{,}000 = \frac{X}{0.12/12}\left(1 - \frac{1}{(1+0.12/12)^{30}}\right) \Rightarrow X = \$581$

**Example (continued)**: Suppose now that you had \$10,000 to pay on the car (that is \$6,000 above the required down payment). You can save your money in the bank and earn 7% APR. Should you save your extra money or use it to pay for the car (consider the 12% APR case for this part)?

(1) Borrow from bank @ 12% and use the remaining \$6,000 to pay for the car
(2) Borrow from bank @ 12% and invest the remaining \$6,000 @ 7%
(3) Borrow from dealer @ 6% and use the remaining \$6,000 to pay for the car
(4) Borrow from dealer @ 6% and invest the remaining \$6,000 @ 7%

We should consider two alternatives,

(1) Borrow from bank @ 12% and use the money to pay for the car

$$\$20{,}000 - \$1{,}000 - \$10{,}000 = \$9{,}000 = \frac{X}{0.12/12}\left(1 - \frac{1}{(1+0.12/12)^{30}}\right) \Rightarrow X = \$349$$

(4) Borrow from dealer @ 6% and save the money in the bank @ 7%

$$\$6{,}000 = \frac{X}{0.07/12}\left(1 - \frac{1}{(1+0.07/12)^{30}}\right) \Rightarrow X = \$218 \Rightarrow 576 - 218 = \$358$$

# Calculating Mortgage Payments

**Example**: You have managed to save \$50,000 and are buying a house for \$250,000. You are offered a 9% APR (compounded monthly) 30 year mortgage. What is your monthly payment?

monthly payment: $$\$200{,}000 = \frac{X}{0.09/12}\left(1 - \frac{1}{(1+0.09/12)^{30\times 12}}\right) \Rightarrow X = \$1{,}609$$

- How long would it take you to pay back the mortgage if you are willing to cut back on your monthly expenses and increase the monthly payment by \$200?

$$200{,}000 = \frac{\$1{,}809}{0.09/12}\left(1 - \frac{1}{(1+0.09/12)^{n}}\right) \Rightarrow \frac{1}{(1+0.09/12)^{n}} = 0.1708$$

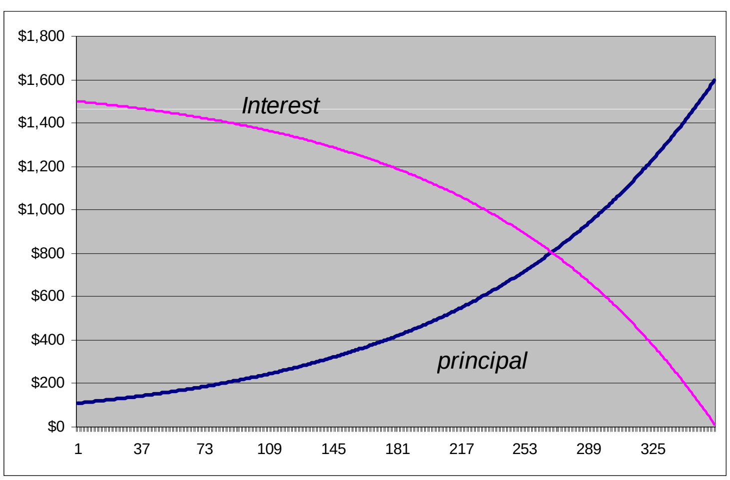
$$-n\ln(1.0075) = \ln(0.1708) \Rightarrow n = 236.5 \qquad \Rightarrow 19 \text{ years and } 8 \text{ months}$$

# Calculating "interest" and "principal".

**Example (continued)**

| time | Loan outstanding (beginning month) | Payment (end month) | Interest (end month) | Principal (end month) |
|---|---|---|---|---|
| 1 | 200,000 | 1,609 | 200,000(0.09/12)=1,500 | 109 |
| 2 | 200,000-109 = 199,891 | 1,609 | 199,891(0.09/12)=1,499 | 110 |
| 3 | 199,781 | 1,609 | 199,781(0.09/12)=1,498 | 111 |
| … | … | … | … | … |
| 360 | 1,597 | 1,609 | 12 | 1,597 |

# Composition of mortgage payments



## Loan Outstanding

**Example**: Consider the previous 9% APR (compounded monthly) 30 year mortgage for $200,000. What is the remaining principal (loan outstanding) on your mortgage after 10 and 20 years?

Outstanding loan:

$$\frac{\$1{,}609}{0.09/12}\left(1-\frac{1}{(1+0.09/12)^{20\times 12}}\right)=\$178{,}832$$

$$\frac{\$1{,}609}{0.09/12}\left(1-\frac{1}{(1+0.09/12)^{10\times 12}}\right)=\$127{,}017$$

- Suppose that after ten years the interest rate on mortgages drops to 8% APR. (1) What is the loan outstanding at that time and (2) what is the PV of the remaining mortgage payments after 10 years.

$$\frac{\$1{,}609}{0.08/12}\left(1-\frac{1}{(1+0.08/12)^{20\times 12}}\right)=\$192{,}363$$

## Refinancing

- How does it work?
  - You are allowed to close the mortgage by paying the **outstanding principal** at any point in time (this is called "*the option to refinance*"). But, you must pay refinancing fees when singing the new mortgage.
  - The **outstanding principal** is determined by the mortgage contract and is usually specified to be the present value of remaining payments under the mortgage interest rate (this is not the market interest rate at the time of refinancing).

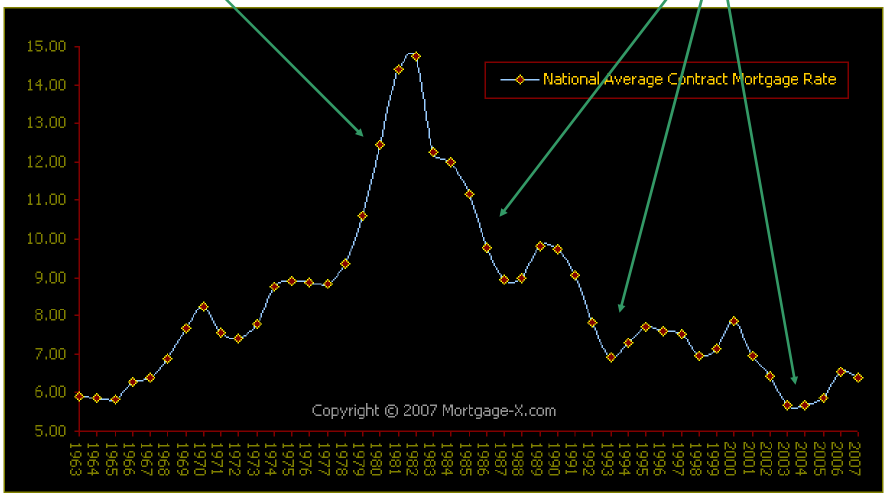
*Why should I ever refinance?*

*When should I refinance?*

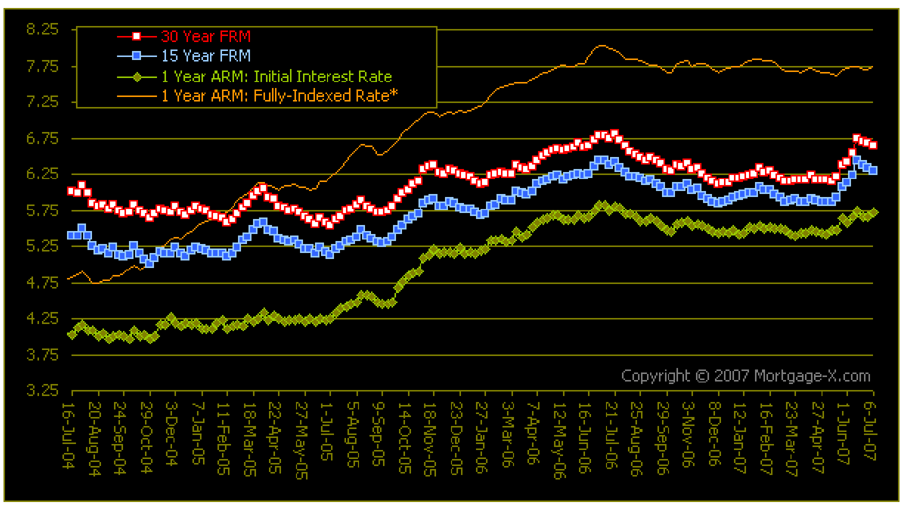
# Mortgage rates in the U.S. 1963-2007

Say you signed your mortgage in 1980

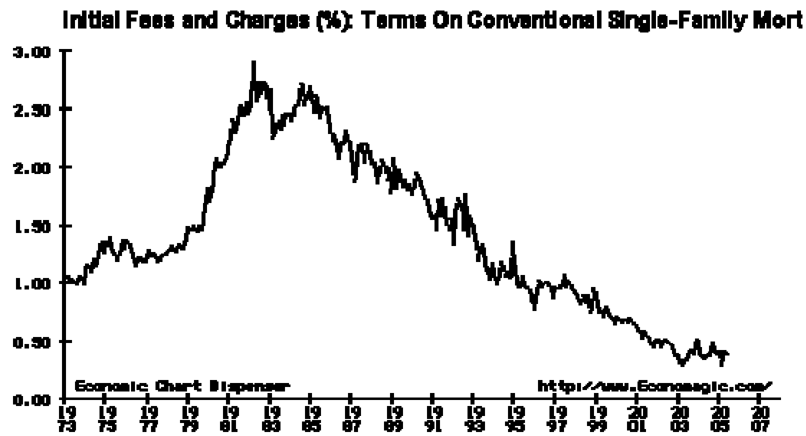
Potential refinancing points




# FRM & ARM in the U.S. 2004-2007



How costly is it to refinance?




**<u>Mortgage example (continued)</u>**: After 5 years of payments, mortgage interest rates dropped from 9% to 8.75% for a 30 year term. The refinancing fee is $2,000. Should you refinance [assume that you will stay in the house for the next 30 years and that interest rates will not change for the next 30 years]?

*In order to refinance we must*

- Take a new mortgage for 30 years and pay $2,000 in fees
- Pay the outstanding loan on the old mortgage

*When should we refinance?*

- If the fees we pay plus the outstanding loan is smaller than the present value of the remaining payments under the old mortgage then we should refinance.

- loan outstanding after 5 years (300 remaining payments)

$$\frac{1{,}609}{0.09/12}\left(1-\frac{1}{(1+0.09/12)^{30\times 12\ -\ 5\times 12}}\right)\Rightarrow \$191{,}731$$

- PV of payments of old mortgage,

$$\frac{1{,}609}{0.0875/12}\left(1-\frac{1}{(1+0.0875/12)^{30\times 12\ -\ 5\times 12}}\right)\Rightarrow \$195{,}708$$

- By refinancing you gain

$$\$195{,}708-\$191{,}731-\$2{,}000=\$1{,}977$$

**<u>Mortgage example (continued)</u>**: Assume that you are planning to move and sell your house in 5 years. Should you still refinance?

Why does this matter?

- By selling the house we stop paying the (otherwise) “high” mortgage payments in five years from now (by closing the mortgage at that time).
- By refinancing now, we save on paying the “high” mortgage payments for the next five years.
- In previous case, by refinancing we were avoiding paying the “high” mortgage payments for the next thirty years.

- Thus, it is not clear that we should refinance.

- loan outstanding in 5 years from now (year 10, 240 remaining payments)

$$\frac{1{,}609}{0.09/12}\left(1-\frac{1}{(1+0.09/12)^{30\times12\text{ - }10\times12}}\right)\Rightarrow \$178{,}832$$

- PV of payments of old mortgage,

$$\frac{1{,}609}{0.0875/12}\left(1-\frac{1}{(1+0.0875/12)^{5\times12}}\right)+\frac{178{,}832}{(1+0.0875/12)^{5\times12}}\Rightarrow$$

$$77{,}966+115{,}646=\$193{,}612$$

- By refinancing you gain

$$\$193{,}612-\$191{,}731-\$2{,}000=-\$119$$