Risk and Return

- Interpretation of the MVE portfolio
- "BETA" of securities
- Capital Asset Pricing Model

From Last Time…

- Investors require high returns for risky stocks
- It is optimal for investors to divide their investment between the **risk free** asset and the **MVE** portfolio
- Investors seeking safer investments should invest a larger fraction of their wealth in the risk free asset.

If all investors act this way then, what is the MVE portfolio?

It must be the portfolio of *all* risky securities in the economy

(1) Stocks

(2) Risky bonds

(3) Real-estate

(4) Human capital ….

The Market Portfolio

- The proxy for the MVE portfolio is the **Market Portfolio**
- The Market Portfolio is the portfolio of all risky securities held in the proportion to their market value. The return on the market portfolio is given by:

$$\tilde{r}_m = \sum_{i=1}^{N} v_i \tilde{r}_i$$

Where the weights v_i are given by,

$$v_i = \frac{\text{total dollar value of security i}}{\text{total dollar value of all risky securities}}$$

The Market Portfolio

Example: Consider an economy consisting of 4 risky assets:

Asset	A	B	C	D
Value	\$100K	\$200K	\$150K	\$50K

This implies the following weights of the market portfolio:

$$v_1 = \frac{\$100K}{\$500K} = 0.2 \qquad v_3 = \frac{\$150K}{\$500K} = 0.3$$

$$v_2 = \frac{\$200K}{\$500K} = 0.4 \qquad v_4 = \frac{\$50K}{\$500K} = 0.1$$

As of 2003, the combined market capitalization of the S&P500 Index stocks was \$7.82 Trillion. Wal-Mart (whose market cap is \$216 Billion) had a weight of,

$$v_{\text{Wal-Mart}} = \frac{\$216B}{\$7820B} = 2.76\%$$

The Market Portfolio

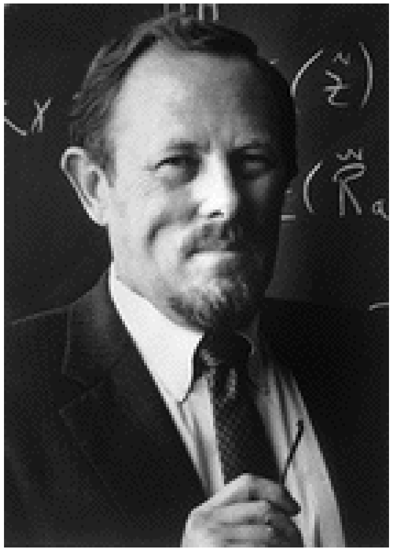
What risk is in the market portfolio?

- The firm specific (**idiosyncratic**) risk is diversified away but the portfolio still has macro economic (**systematic**) risk

- *What expected return should we require from a risky asset that is not correlated with the return on the market portfolio – that is, a security that has only idiosyncratic risk?*

- *What expected return should we require from a risky asset that has a correlation of 1 with the return on the market portfolio?*

- *Should stocks that are highly correlated with the market portfolio earn higher or lower returns?*



William F. Sharpe extended the ideas of Harry M. Markowitz to provide a relation between risk and return based on systematic risk (1964). The model he developed is the “Capital Asset Pricing Model (CAPM). He won the 1990 Nobel Prize in Economics for his work.

Relation Between Individual Stocks and the Market Portfolio

Terminology

- The return on the market portfolio in period "t" – $\tilde{r}_{m,t}$

- The return on firm i in period "t" – $\tilde{r}_{i,t}$

- Specific (idiosyncratic) risk of firm i – $\tilde{\varepsilon}_{i,t}$ $[E(\tilde{\varepsilon}_{i,t}) = 0]$

- We measure systematic risk of asset "i" by $BETA_i$, β_i

Estimating BETA

$BETA_i$ satisfies the following regression equation:

$$\underbrace{\tilde{r}_{i,t} - r_{f,t}}_{\text{excess return on firm } i} = \alpha_i + \beta_i \underbrace{\left(\tilde{r}_{m,t} - r_{f,t}\right)}_{\text{excess return on market portfolio}} + \underbrace{\tilde{\varepsilon}_{i,t}}_{\text{firm specific risk}}$$

excess return on firm i *excess return on market portfolio* *firm specific risk*

- Theoretically $BETA_i$ is given by $$\beta_i = \frac{\operatorname{cov}\left(\tilde{r}_m, \tilde{r}_i\right)}{V\left(\tilde{r}_m\right)} = \frac{\rho_{i,m}\sigma_i}{\sigma_m}$$

- We **estimate** $BETA_i$ by running the above regression while using past data on *market* returns, *firm i* returns and *risk free* rates.

Estimating BETA of General Motors (GM)

Month	GM Return	Market Return	Monthly T-Bill Rate	Excess GM Return	Excess Market Return
January	6.06	7.89	0.65	5.41	7.24
February	−2.86	1.51	0.58	−3.44	0.93
March	−8.18	0.23	0.62	−8.79	−0.38
April	−7.36	−0.29	0.72	−8.08	−1.01
May	7.76	5.58	0.66	7.10	4.92
June	0.52	1.73	0.55	−0.03	1.18
July	−1.74	−0.21	0.62	−2.36	−0.83
August	−3.00	−0.36	0.55	−3.55	−0.91
September	−0.56	−3.58	0.60	−1.16	−4.18
October	−0.37	4.62	0.65	−1.02	3.97
November	6.93	6.85	0.61	6.32	6.25
December	3.08	4.55	0.65	2.43	3.90
Mean	0.02	2.38	0.62	−0.60	1.75
Std Dev	4.97	3.33	0.05	4.97	3.32

Regression Results $r_{GM} - r_f = \alpha + \beta(r_M - r_f)$

	α	β
Estimated coefficient	−2.590	1.1357
Standard error of estimate	(1.547)	(0.309)

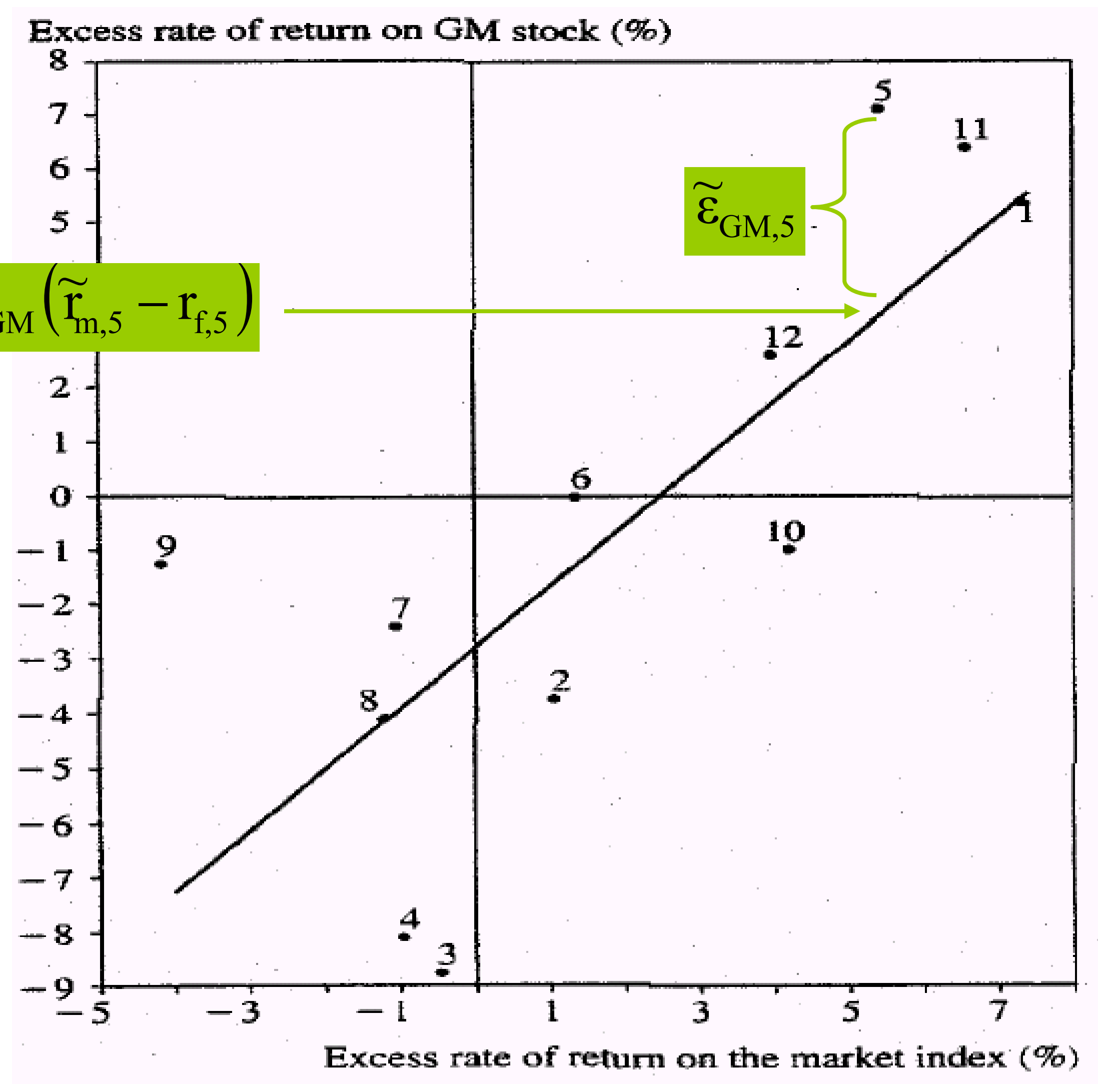
Variance of residuals = 12.601

Standard deviation of residuals = 3.550

R-SQR = 0.575

Estimation of the Theoretical Regression Line

$$\tilde{r}_{GM,5} - r_{f,5} = \alpha_i + \beta_{GM}\left(\tilde{r}_{m,5} - r_{f,5}\right)$$



$BETA_{GM}$ is estimated to be 1.136. This means that when the market moves by 1%, GM's return on average moves by 1.136%.

BETA of a Portfolio

Consider a portfolio of N stocks with returns $r_1, r_2, \ldots, r_N$ and weights $w_1, w_2, \ldots, w_N$.

- *The covariance with the market portfolio is,*

$$\operatorname{cov}(\tilde{r}_m, \tilde{r}_p) = \operatorname{cov}\left(\tilde{r}_m, \sum_{i=1}^{N} w_i \times \tilde{r}_i\right)$$

$$= w_1 \times \operatorname{cov}(\tilde{r}_m, \tilde{r}_1) + w_2 \times \operatorname{cov}(\tilde{r}_m, \tilde{r}_2) + \ldots + w_N \times \operatorname{cov}(\tilde{r}_m, \tilde{r}_N)$$

- *The portfolio's BETA is,*

$$\beta_p = \frac{\operatorname{cov}(\tilde{r}_m, \tilde{r}_p)}{V(\tilde{r}_m)} = w_1 \times \frac{\operatorname{cov}(\tilde{r}_m, \tilde{r}_1)}{V(\tilde{r}_m)} + \ldots + w_N \times \frac{\operatorname{cov}(\tilde{r}_m, \tilde{r}_N)}{V(\tilde{r}_m)}$$

$$= w_1 \times \beta_1 + \ldots + w_N \times \beta_N$$

Example: Suppose that the expected annual return on Microsoft's stock is 23% with standard deviation of 21.25%, the expected return on the S&P500 Index (the market) is 15% with standard deviation of 10%, and the correlation between the return on Microsoft and the S&P500 Index is 0.8.

- What is Microsoft's BETA?

$$\beta_{\text{Microsoft}} = \frac{\operatorname{cov}(\tilde{r}_m, \tilde{r}_{\text{Microsoft}})}{V(\tilde{r}_m)} = \frac{\operatorname{cov}(\tilde{r}_m, \tilde{r}_{\text{Microsoft}})}{0.10^2}$$

The covariance is,

$$\operatorname{cov}(\tilde{r}_m, \tilde{r}_{\text{Microsoft}}) = \rho_{\text{Microsoft,m}} \sigma_{\text{Microsoft}} \sigma_m = 0.8 \times 0.2125 \times 0.10 = 0.017$$

Microsoft's BETA is then, $\beta_{\text{Microsoft}} = \dfrac{0.017}{0.10^2} = 1.7$

Example (continued): Now suppose that you put half of your wealth into Microsoft and half into GM. The expected annual return on GM's stock is 16%, and the covariance between the return on GM and the S&P500 is 0.011.

- What is GM's BETA?

$$\beta_{GM} = \frac{\text{cov}(\tilde{r}_m, \tilde{r}_{GM})}{V(\tilde{r}_m)} = \frac{0.011}{0.10^2} = 1.1$$

- What is the portfolio's BETA?

$$\beta_p = w_{Microsoft} \times \beta_{Microsoft} + w_{GM} \times \beta_{GM} = 0.5 \times 1.7 + 0.5 \times 1.1 = 1.4$$

- What is the portfolio's expected return?

$$E(r_p) = w_{Microsoft} \times E(r_{Microsoft}) + w_{GM} \times E(r_{GM}) = 0.5 \times 23\% + 0.5 \times 16\% = 19.5\%$$

Capital Asset Pricing Model (CAPM)

- *What is the expected rate of return on a stock with* $\beta = 0$?

$$E(r_i) = r_f + 0 \times \left(E(r_m) - r_f\right) = r_f$$

- *What is the expected rate of return on a stock with* $\beta = 1$?

$$E(r_i) = r_f + 1 \times \left(E(r_m) - r_f\right) = E(r_m)$$

- *What is the expected rate of return on a portfolio that is made up of 50% of the risk free asset and 50% of a stock with* $\beta = 1$?

$$E(r_i) = r_f + 0.5 \times \left(E(r_m) - r_f\right) = 0.5 \times E(r_m) + 0.5 \times r_f$$

- *In general according to the CAPM,*

$$E(r_i) = r_f + \beta_i \times \left(E(r_m) - r_f\right)$$

Capital Asset Pricing Model (CAPM)

$$E(r_i) = r_f + \beta_i \times \left(E(r_m) - r_f\right)$$

- Only market risk matters in determining the required rate of return. Diversifiable (idiosyncratic) risk does not affect the required rate of return.
- Assets with high BETA's (not necessary high variance) yield high expected returns.

What is the excess return on a stock with a negative BETA?

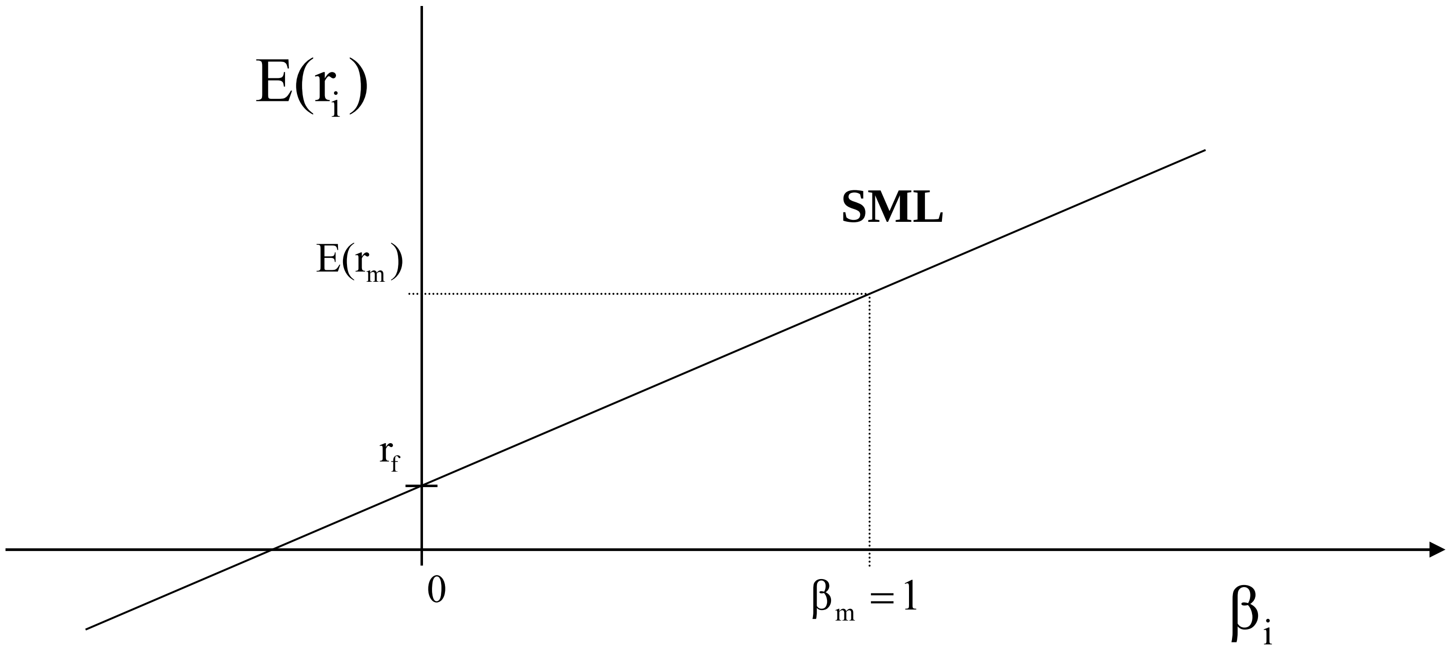
Could a stock have a negative BETA?

Stocks with negative BETA exist (e.g. Blockbuster's BETA is –0.276). Such stocks earn expected return below the risk free rate. Such stocks provide relatively high income during down turns in the economy and can be viewed as insurance.

Security Market Line (SML).

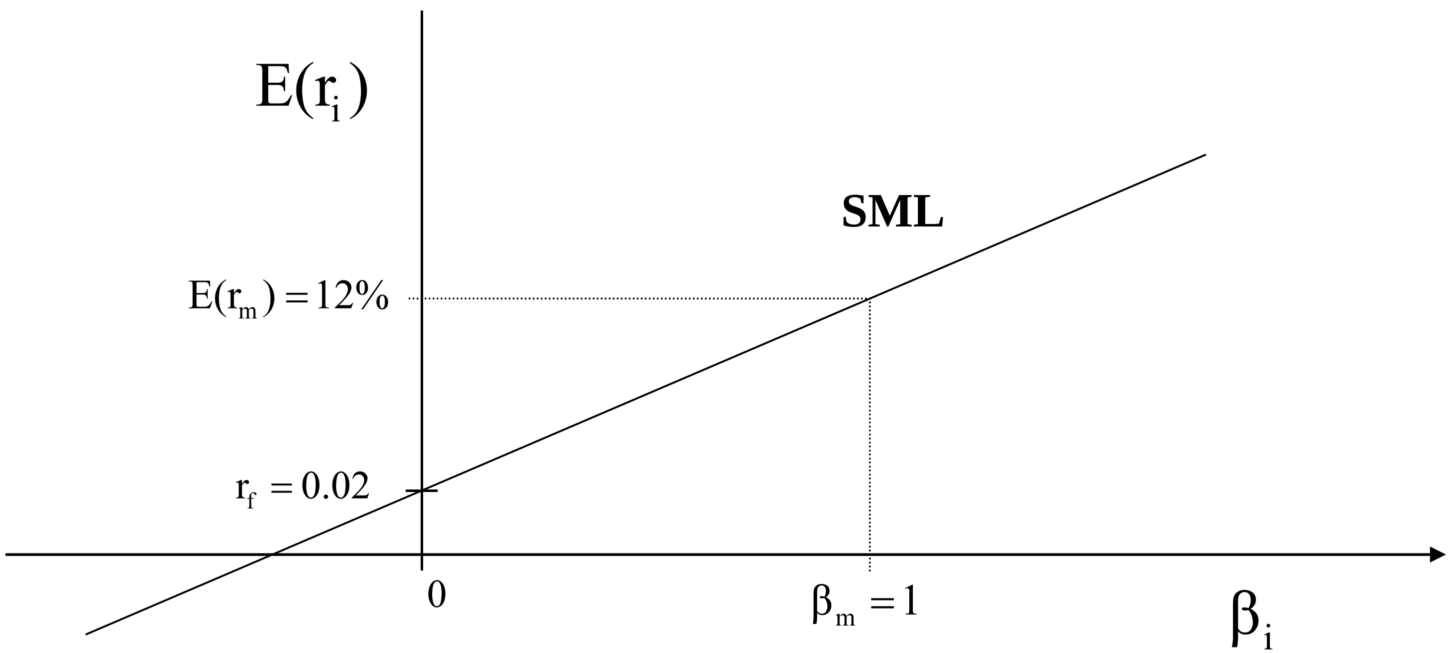
The Security Market Line represents the relation between the expected return (on the "y" axis) and beta (on the "x" axis) - implied by the CAPM

$$E(r_i) = r_f + \beta_i \left(E(r_m) - r_f \right)$$



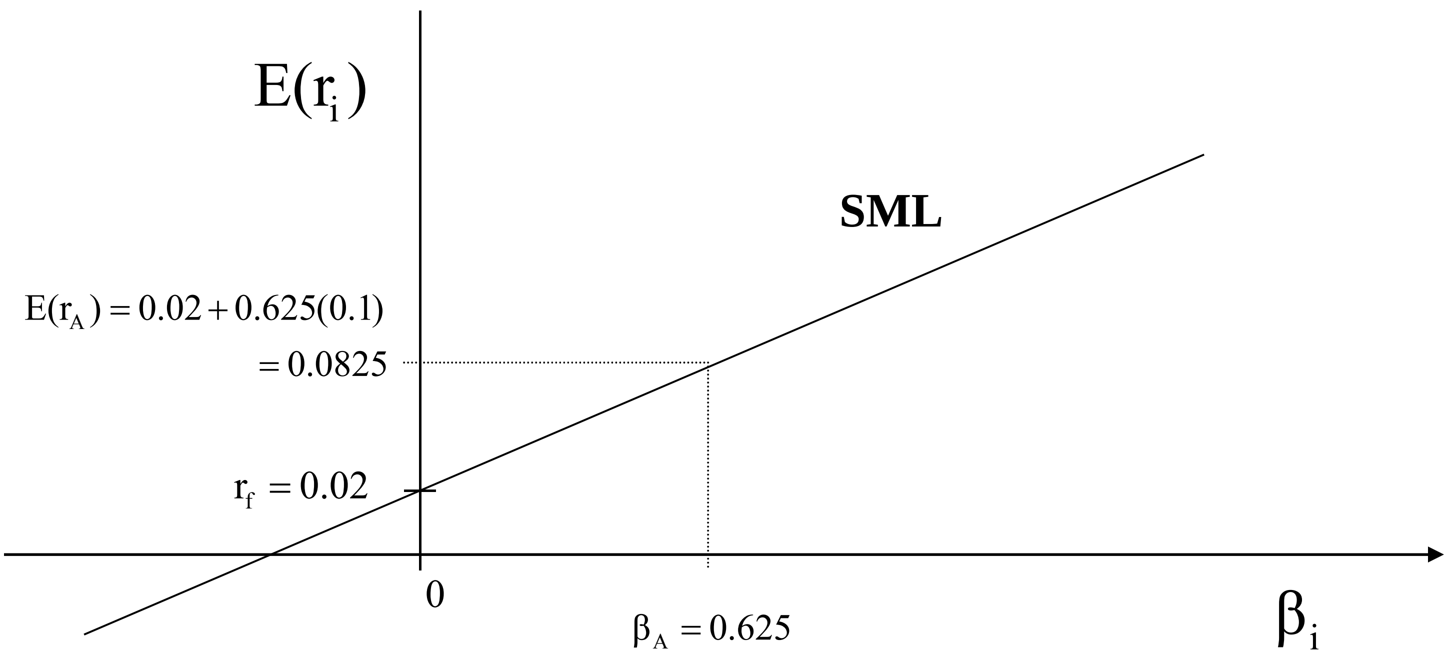
Example: The expected return on the S&P500 Index is 12%. The risk free rate is 2%. Plot the relationship between the expected return on a security and its BETA – i.e. the Security Market Line.

According to the CAPM: $E(r_i) = 0.02 + \beta_i \times (0.10)$



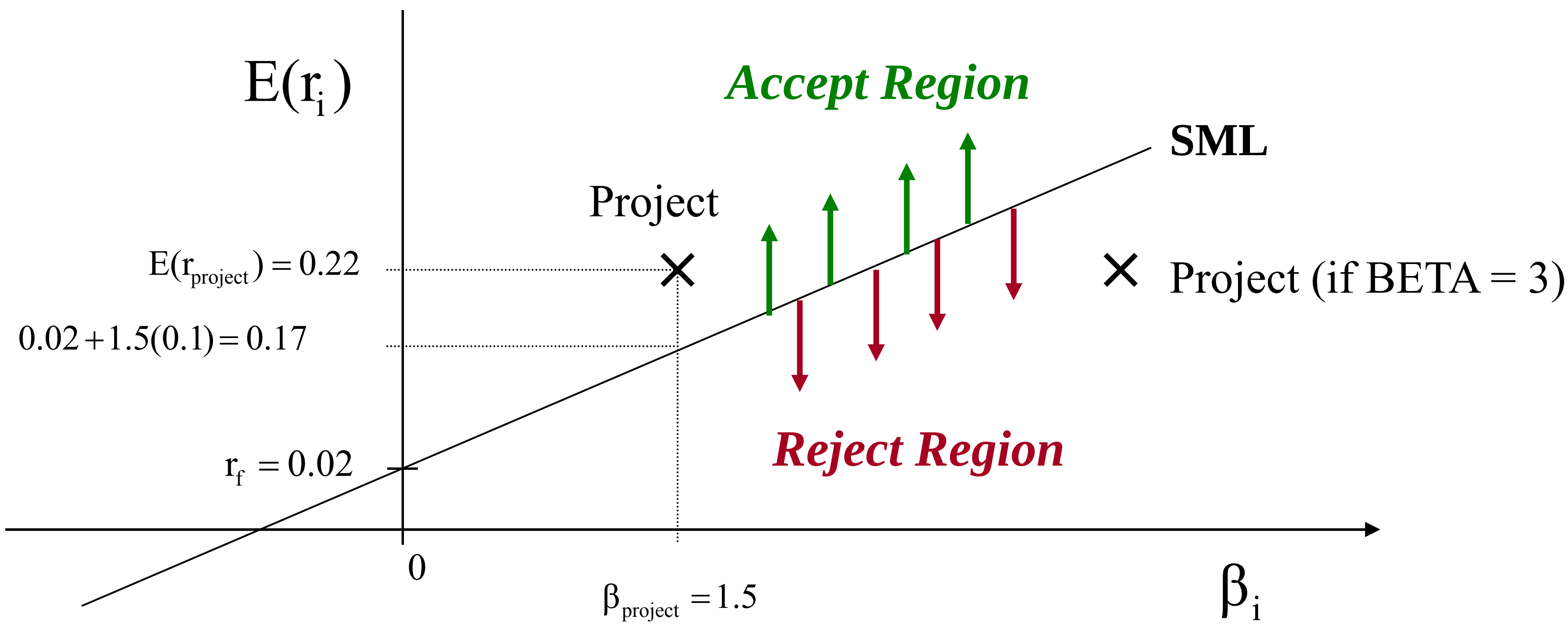
Example (continued): The standard deviation on the S&P500 Index is 20%. The return on security A has a correlation of 50% with the return on the S&P500 Index, with standard deviation 25%. Using the Security Market Line, what is the expected return on security A?

The BETA of security A is: $$\beta_A = \frac{\rho_{A,m}\sigma_A}{\sigma_m} = \frac{0.5 \times 0.25}{0.20} = 0.625$$



Using the CAPM to evaluate projects

Example (continued): You can invest in a project that will yield expected return of 22%. The project's return is correlated with the return on the S&P500 Index and has a BETA of 1.5. Place the project graphically below. Should you invest in the project? What if the project had a BETA of 3?



Using the CAPM to calculate stock prices

Example: Consider stock A with the following characteristics:

Price in one year – P_1	*Probability*
$95	50%
$110	30%
$130	20%

We also know that: $\beta_A = 0.75,\ E(r_m) = 20\%$ and $r_f = 5\%$

- What is the required rate of return on an investment in stock A?

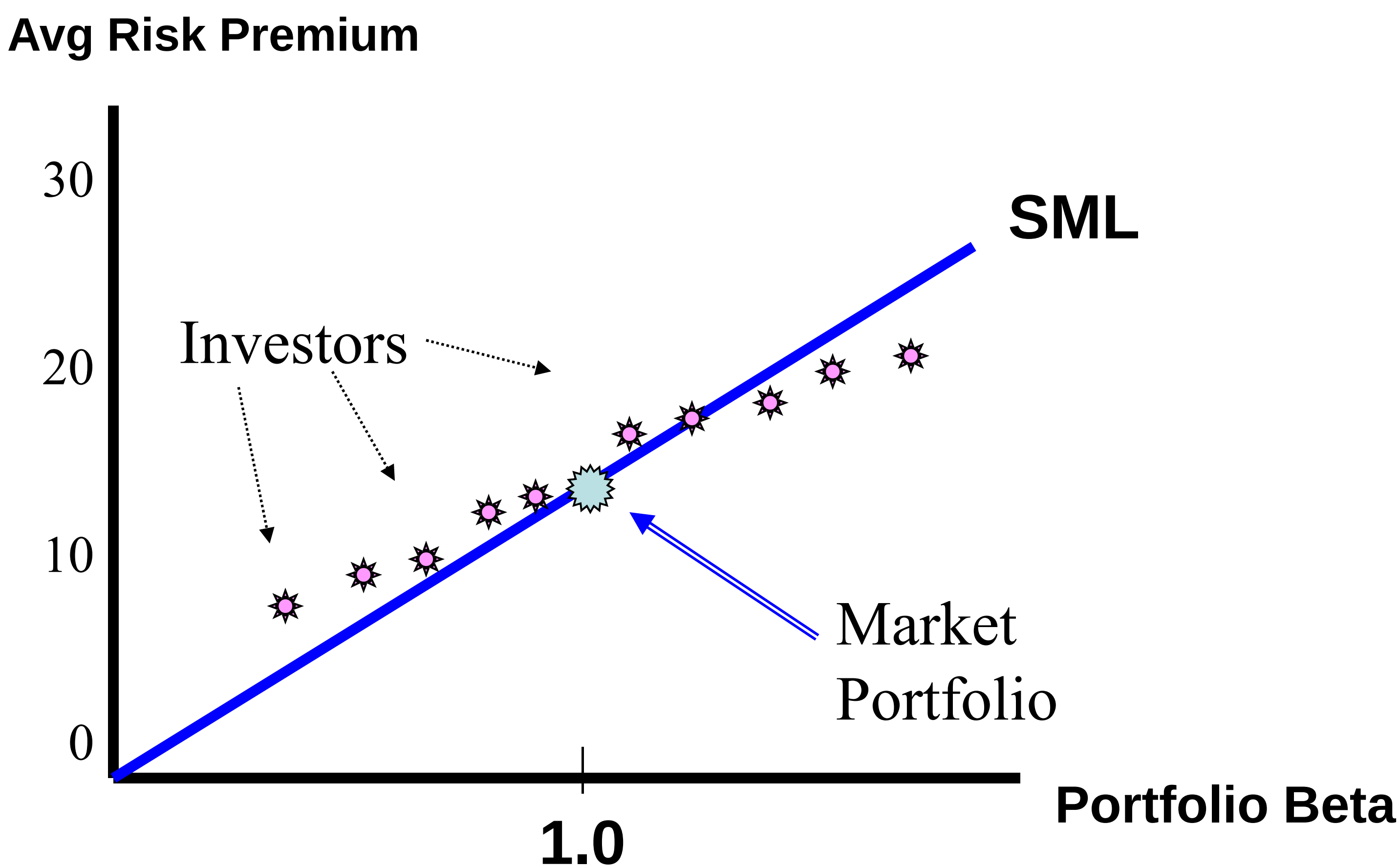
$$E(r_A) = 0.05 + 0.75 \times (0.20 - 0.05) = 16.25$$

- According to the CAPM, what is the current price of stock A?

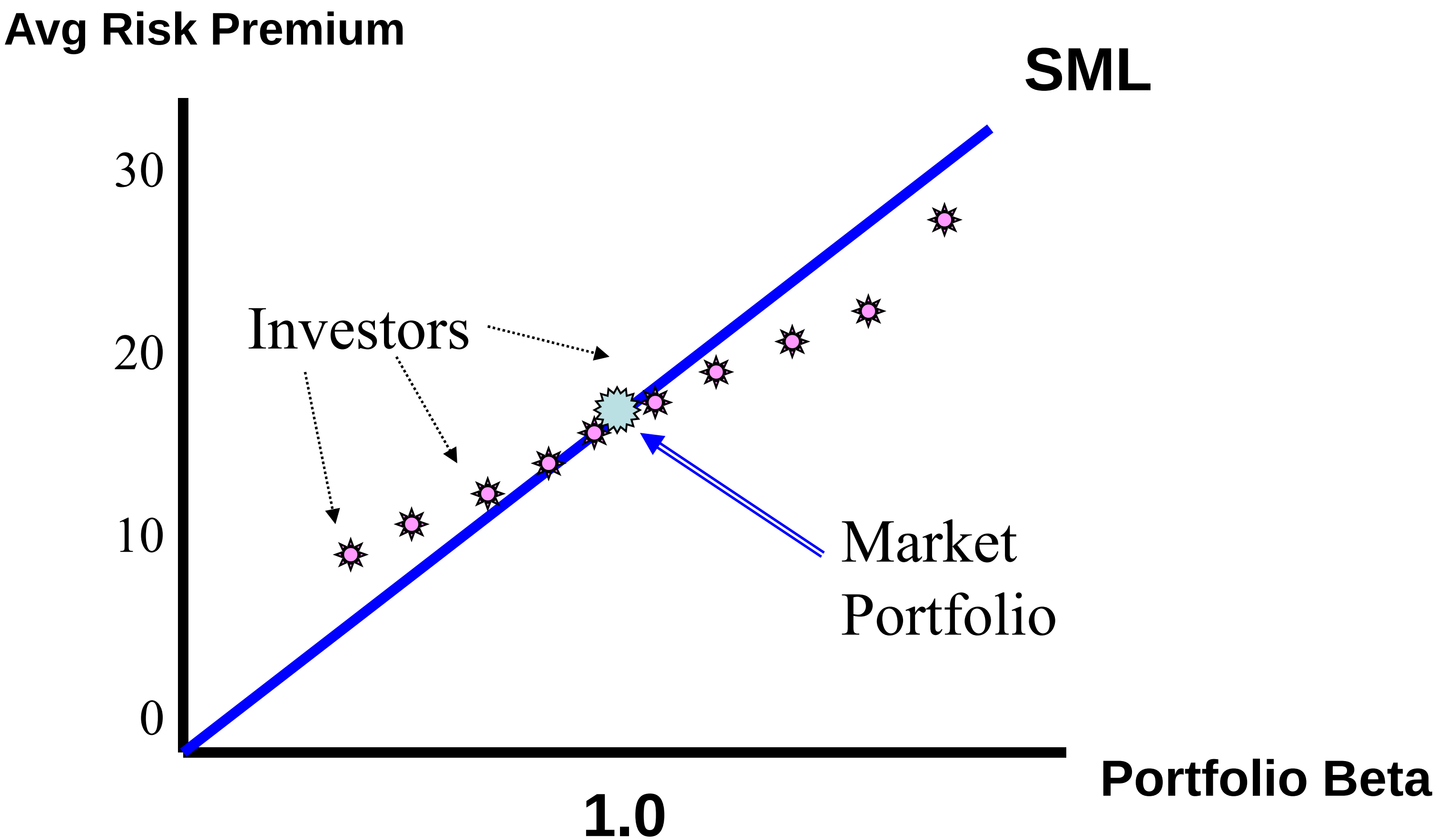
$$P_0 = \frac{E(P_1)}{1 + E(r_A)} = \frac{0.5 \times 95 + 0.3 \times 110 + 0.2 \times 130}{1.1625} = \$91.61$$

Does the CAPM work?

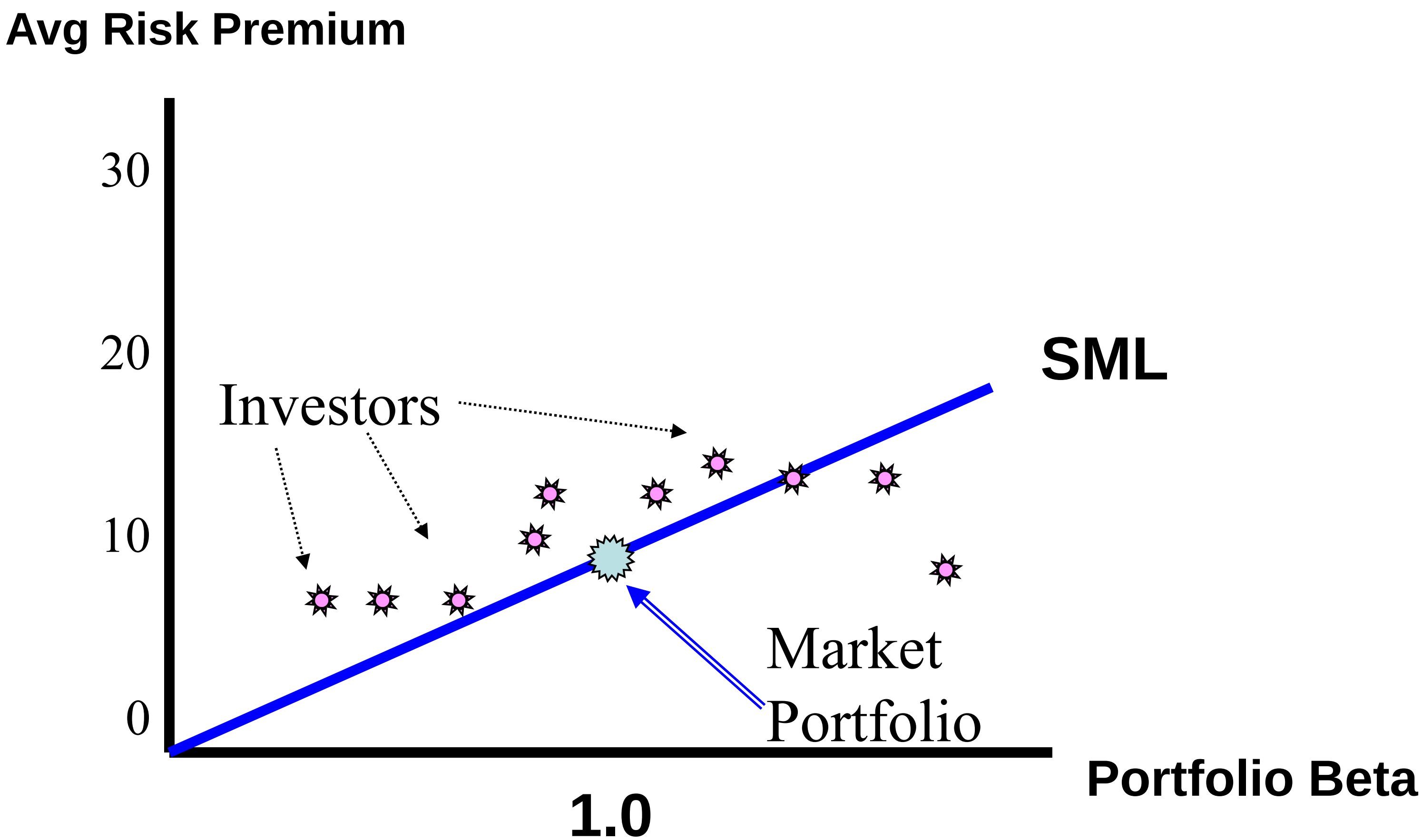
Beta vs. Average Risk Premium
1931-2002



Beta vs. Average Risk Premium
1931-1965

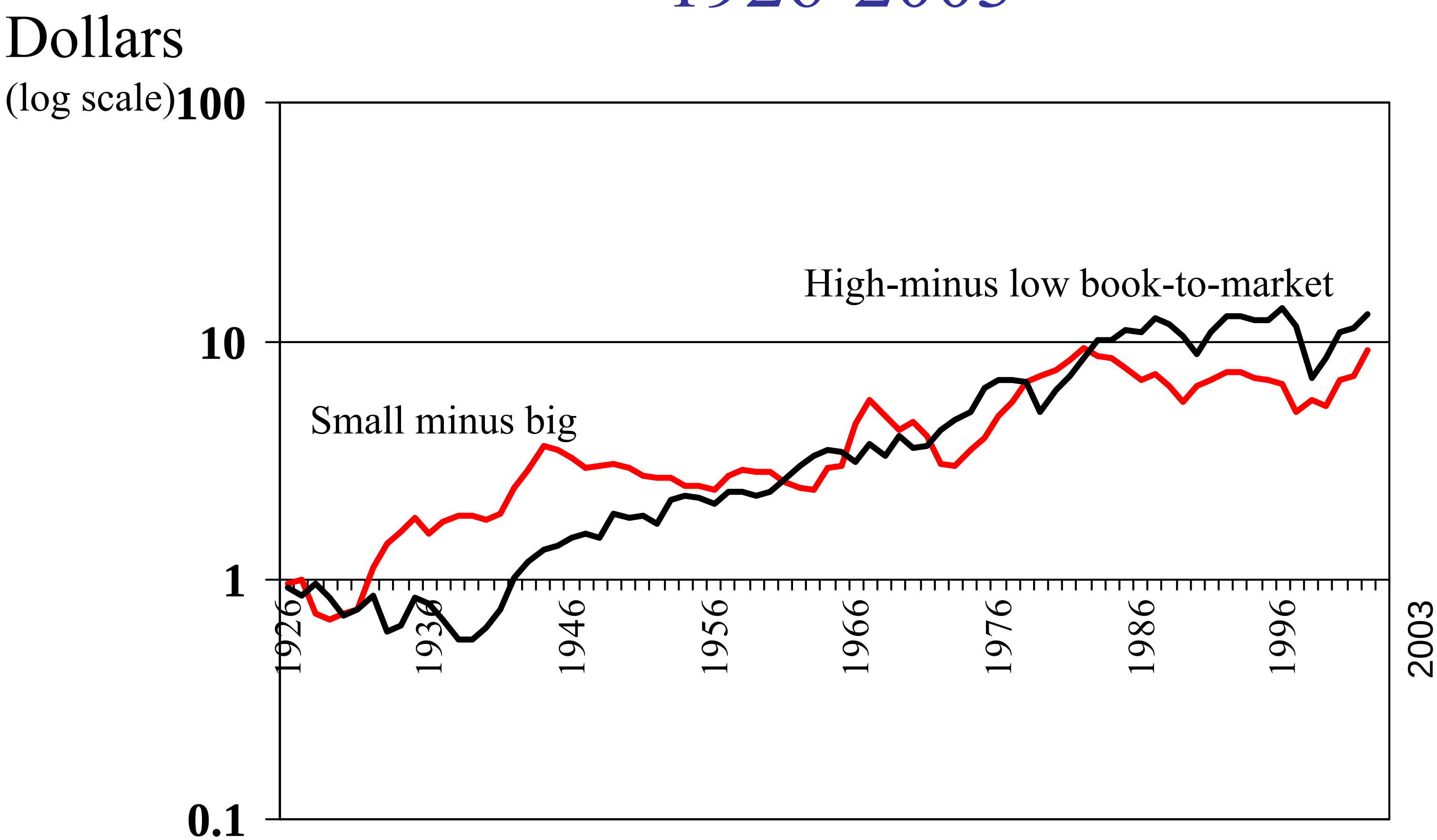


Beta vs. Average Risk Premium
1966-2002



Other factors that drive expected returns

Return vs. Book-to-Market 1926-2003



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