- The correct discount rate for a cash flow is the expected return available in the market on other investments of comparable risk and term.
- If the interest on an investment is taxed at rate τ, or if the interest on a loan is tax deductible, then the effective after-tax interest rate is

$$r(1 - \tau) \tag{5.8}$$

5.5 The Opportunity Cost of Capital

- The opportunity cost of capital is the best available expected return offered in the market on an investment of comparable risk and term.
- The opportunity cost of capital provides the benchmark against which the cash flows of a new investment should be evaluated.

Key Terms

adjustable rate mortgages (ARMs) *p. 148*
after-tax interest rate *p. 157*
amortizing loan *p. 146*
annual percentage rate (APR) *p. 144*
continuous compounding *p. 144*
(opportunity) cost of capital *p. 159*
effective annual rate (EAR) *p. 142*
federal funds rate *p. 152*
mid-year convention *p. 168*
nominal interest rate *p. 148*
real interest rate *p. 148*
simple interest *p. 144*
term structure *p. 150*
yield curve *p. 150*

Further Reading

For an interesting account of the history of interest rates over the past four millennia, see S. Homer and R. Sylla, *A History of Interest Rates* (John Wiley & Sons, 2005).

For a deeper understanding of interest rates, how they behave with changing market conditions, and how risk can be managed, see J. C. Van Horne, *Financial Market Rates and Flows* (Prentice Hall, 2000).

For further insights into the relationship between interest rates, inflation, and economic growth, see a macroeconomics text such as A. Abel, B. Bernanke, and D. Croushore, *Macroeconomics* (Prentice Hall, 2010).

For further analysis of the yield curve and how it is measured and modeled, see M. Choudhry, *Analyzing and Interpreting the Yield Curve* (John Wiley & Sons, 2004).

Problems

All problems are available in MyFinanceLab. *An asterisk (*) indicates problems with a higher level of difficulty.*

Interest Rate Quotes and Adjustments

1. Your bank is offering you an account that will pay 20% interest in total for a two-year deposit. Determine the equivalent discount rate for a period length of
   a. Six months. b. One year. c. One month.
2. Which do you prefer: a bank account that pays 5% per year (EAR) for three years or
   a. An account that pays 2½% every six months for three years?
   b. An account that pays 7½% every 18 months for three years?
   c. An account that pays ½% per month for three years?

3. Many academic institutions offer a sabbatical policy. Every seven years a professor is given a year free of teaching and other administrative responsibilities at full pay. For a professor earning \$70,000 per year who works for a total of 42 years, what is the present value of the amount she will earn while on sabbatical if the interest rate is 6% (EAR)?

4. You have found three investment choices for a one-year deposit: 10% APR compounded monthly, 10% APR compounded annually, and 9% APR compounded daily. Compute the EAR for each investment choice. (Assume that there are 365 days in the year.)

5. You are considering moving your money to a new bank offering a one-year CD that pays an 8% APR with monthly compounding. Your current bank's manager offers to match the rate you have been offered. The account at your current bank would pay interest every six months. How much interest will you need to earn every six months to match the CD?

6. Your bank account pays interest with an EAR of 5%. What is the APR quote for this account based on semiannual compounding? What is the APR with monthly compounding?

7. Suppose the interest rate is 8% APR with monthly compounding. What is the present value of an annuity that pays \$100 every six months for five years?

8. You can earn \$50 in interest on a \$1000 deposit for eight months. If the EAR is the same regardless of the length of the investment, determine how much interest you will earn on a \$1000 deposit for
   a. 6 months. b. 1 year. c. $1\frac{1}{2}$ years.

9. Suppose you invest \$100 in a bank account, and five years later it has grown to \$134.39.
   a. What APR did you receive, if the interest was compounded semiannually?
   b. What APR did you receive if the interest was compounded monthly?

10. Your son has been accepted into college. This college guarantees that your son's tuition will not increase for the four years he attends college. The first \$10,000 tuition payment is due in six months. After that, the same payment is due every six months until you have made a total of eight payments. The college offers a bank account that allows you to withdraw money every six months and has a fixed APR of 4% (semiannual) guaranteed to remain the same over the next four years. How much money must you deposit today if you intend to make no further deposits and would like to make all the tuition payments from this account, leaving the account empty when the last payment is made?

11. You make monthly payments on your mortgage. It has a quoted APR of 5% (monthly compounding). What percentage of the outstanding principal do you pay in interest each month?

Application: Discount Rates and Loans

12. Capital One is advertising a 60-month, 5.99% APR motorcycle loan. If you need to borrow \$8000 to purchase your dream Harley Davidson, what will your monthly payment be?

13. Oppenheimer Bank is offering a 30-year mortgage with an EAR of $5\frac{3}{8}\%$ If you plan to borrow \$150,000, what will your monthly payment be?

14. You have decided to refinance your mortgage. You plan to borrow whatever is outstanding on your current mortgage. The current monthly payment is \$2356 and you have made every payment on time. The original term of the mortgage was 30 years, and the mortgage is exactly four years and eight months old. You have just made your monthly payment. The mortgage interest rate is $6\frac{3}{8}\%$ (APR). How much do you owe on the mortgage today?

15. You have just sold your house for \$1,000,000 in cash. Your mortgage was originally a 30-year mortgage with monthly payments and an initial balance of \$800,000. The mortgage is currently exactly $18\frac{1}{2}$ years old, and you have just made a payment. If the interest rate on the mortgage is 5.25% (APR), how much cash will you have from the sale once you pay off the mortgage?

16. You have just purchased a home and taken out a $500,000 mortgage. The mortgage has a 30-year term with monthly payments and an APR of 6%.

a. How much will you pay in interest, and how much will you pay in principal, during the first year?

b. How much will you pay in interest, and how much will you pay in principal, during the 20th year (i.e., between 19 and 20 years from now)?

17. Your mortgage has 25 years left, and has an APR of 7.625% with monthly payments of $1449.

a. What is the outstanding balance?

b. Suppose you cannot make the mortgage payment and you are in danger of losing your house to foreclosure. The bank has offered to renegotiate your loan. The bank expects to get $150,000 for the house if it forecloses. They will lower your payment as long as they will receive at least this amount (in present value terms). If current 25-year mortgage interest rates have dropped to 5% (APR), what is the lowest monthly payment you could make for the remaining life of your loan that would be attractive to the bank?

***18.** You have an outstanding student loan with required payments of $500 per month for the next four years. The interest rate on the loan is 9% APR (monthly). You are considering making an extra payment of $100 today (that is, you will pay an extra $100 that you are not required to pay). If you are required to continue to make payments of $500 per month until the loan is paid off, what is the amount of your final payment? What effective rate of return (expressed as an APR with monthly compounding) have you earned on the $100?

***19.** Consider again the setting of Problem 18. Now that you realize your best investment is to prepay your student loan, you decide to prepay as much as you can each month. Looking at your budget, you can afford to pay an extra $250 per month in addition to your required monthly payments of $500, or $750 in total each month. How long will it take you to pay off the loan?

***20.** Oppenheimer Bank is offering a 30-year mortgage with an APR of 5.25%. With this mortgage your monthly payments would be $2000 per month. In addition, Oppenheimer Bank offers you the following deal: Instead of making the monthly payment of $2000 every month, you can make half the payment every two weeks (so that you will make $52/2 = 26$ payments per year). With this plan, how long will it take to pay off the mortgage of $150,000 if the EAR of the loan is unchanged?

***21.** Your friend tells you he has a very simple trick for shortening the time it takes to repay your mortgage by one-third: Use your holiday bonus to make an extra payment on January 1 of each year (that is, pay your monthly payment due on that day twice). If you take out your mortgage on July 1, so your first monthly payment is due August 1, and you make an extra payment every January 1, how long will it take to pay off the mortgage? Assume that the mortgage has an original term of 30 years and an APR of 12%.

22. You need a new car and the dealer has offered you a price of $20,000, with the following payment options: (a) pay cash and receive a $2000 rebate, or (b) pay a $5000 down payment and finance the rest with a 0% APR loan over 30 months. But having just quit your job and started an MBA program, you are in debt and you expect to be in debt for at least the next $2\frac{1}{2}$ years. You plan to use credit cards to pay your expenses; luckily you have one with a low (fixed) rate of 15% APR (monthly). Which payment option is best for you?

23. The mortgage on your house is five years old. It required monthly payments of $1402, had an original term of 30 years, and had an interest rate of 10% (APR). In the intervening five years, interest rates have fallen and so you have decided to refinance—that is, you will roll over the outstanding balance into a new mortgage. The new mortgage has a 30-year term, requires monthly payments, and has an interest rate of $6\frac{5}{8}\%$ (APR).

a. What monthly repayments will be required with the new loan?

b. If you still want to pay off the mortgage in 25 years, what monthly payment should you make after you refinance?

c. Suppose you are willing to continue making monthly payments of $1402. How long will it take you to pay off the mortgage after refinancing?
d. Suppose you are willing to continue making monthly payments of $1402, and want to pay off the mortgage in 25 years. How much additional cash can you borrow today as part of the refinancing?

24. You have credit card debt of $25,000 that has an APR (monthly compounding) of 15%. Each month you pay the minimum monthly payment only. You are required to pay only the outstanding interest. You have received an offer in the mail for an otherwise identical credit card with an APR of 12%. After considering all your alternatives, you decide to switch cards, roll over the outstanding balance on the old card into the new card, and borrow additional money as well. How much can you borrow today on the new card without changing the minimum monthly payment you will be required to pay?

The Determinants of Interest Rates

25. In 1975, interest rates were 7.85% and the rate of inflation was 12.3% in the United States. What was the real interest rate in 1975? How would the purchasing power of your savings have changed over the year?

26. If the rate of inflation is 5%, what nominal interest rate is necessary for you to earn a 3% real interest rate on your investment?

27. Can the nominal interest rate available to an investor be significantly negative? (*Hint*: Consider the interest rate earned from saving cash "under the mattress.") Can the real interest rate be negative? Explain.

28. Consider a project that requires an initial investment of $100,000 and will produce a single cash flow of $150,000 in five years.
a. What is the NPV of this project if the five-year interest rate is 5% (EAR)?
b. What is the NPV of this project if the five-year interest rate is 10% (EAR)?
c. What is the highest five-year interest rate such that this project is still profitable?

29. Suppose the term structure of risk-free interest rates is as shown below:

Term	1 year	2 years	3 years	5 years	7 years	10 years	20 years
Rate (EAR, %)	1.99	2.41	2.74	3.32	3.76	4.13	4.93

a. Calculate the present value of an investment that pays $1000 in two years and $2000 in five years for certain.
b. Calculate the present value of receiving $500 per year, with certainty, at the end of the next five years. To find the rates for the missing years in the table, linearly interpolate between the years for which you do know the rates. (For example, the rate in year 4 would be the average of the rate in year 3 and year 5.)
*c. Calculate the present value of receiving $2300 per year, with certainty, for the next 20 years. Infer rates for the missing years using linear interpolation. (*Hint*: Use a spreadsheet.)

30. Using the term structure in Problem 29, what is the present value of an investment that pays $100 at the end of each of years 1, 2, and 3? If you wanted to value this investment correctly using the annuity formula, which discount rate should you use?

31. What is the shape of the yield curve given the term structure in Problem 29? What expectations are investors likely to have about future interest rates?

32. Suppose the current one-year interest rate is 6%. One year from now, you believe the economy will start to slow and the one-year interest rate will fall to 5%. In two years, you expect the economy to be in the midst of a recession, causing the Federal Reserve to cut interest rates drastically and the one-year interest rate to fall to 2%. The one-year interest rate will then rise to

3% the following year, and continue to rise by 1% per year until it returns to 6%, where it will remain from then on.

a. If you were certain regarding these future interest rate changes, what two-year interest rate would be consistent with these expectations?
b. What current term structure of interest rates, for terms of 1 to 10 years, would be consistent with these expectations?
c. Plot the yield curve in this case. How does the one-year interest rate compare to the 10-year interest rate?

Risk and Taxes

33. Figure 5.4 shows that CBS's five-year borrowing rate is 1.3% and JPMorgan Chases' is 2.6%. Which would you prefer? $500 from CBS paid today or a promise that the firm will pay you $550 in five years? Which would you choose if JPMorgan Chase offered you the same alternative?

34. Your best taxable investment opportunity has an EAR of 4%. Your best tax-free investment opportunity has an EAR of 3%. If your tax rate is 30%, which opportunity provides the higher after-tax interest rate?

35. Your uncle Fred just purchased a new boat. He brags to you about the low 7% interest rate (APR, monthly compounding) he obtained from the dealer. The rate is even lower than the rate he could have obtained on his home equity loan (8% APR, monthly compounding). If his tax rate is 25% and the interest on the home equity loan is tax deductible, which loan is truly cheaper?

36. You are enrolling in an MBA program. To pay your tuition, you can either take out a standard student loan (so the interest payments are not tax deductible) with an EAR of 5½% or you can use a tax-deductible home equity loan with an APR (monthly) of 6%. You anticipate being in a very low tax bracket, so your tax rate will be only 15%. Which loan should you use?

37. Your best friend consults you for investment advice. You learn that his tax rate is 35%, and he has the following current investments and debts:

- A car loan with an outstanding balance of $5000 and a 4.8% APR (monthly compounding)
- Credit cards with an outstanding balance of $10,000 and a 14.9% APR (monthly compounding)
- A regular savings account with a $30,000 balance, paying a 5.50% EAR
- A money market savings account with a $100,000 balance, paying a 5.25% APR (daily compounding)
- A tax-deductible home equity loan with an outstanding balance of $25,000 and a 5.0% APR (monthly compounding)

a. Which savings account pays a higher after-tax interest rate?
b. Should your friend use his savings to pay off any of his outstanding debts? Explain.

38. Suppose you have outstanding debt with an 8% interest rate that can be repaid any time, and the interest rate on U.S. Treasuries is only 5%. You plan to repay your debt using any cash that you don't invest elsewhere. Until your debt is repaid, what cost of capital should you use when evaluating a new risk-free investment opportunity? Why?

The Opportunity Cost of Capital

39. In the summer of 2008, at Heathrow Airport in London, Bestofthebest (BB), a private company, offered a lottery to win a Ferrari or 90,000 British pounds, equivalent at the time to about $180,000. Both the Ferrari and the money, in 100-pound notes, were on display. If the U.K. interest rate was 5% per year, and the dollar interest rate was 2% per year (EARs), how much did it cost the company in dollars each month to keep the cash on display? That is, what was the opportunity cost of keeping it on display rather than in a bank account? (Ignore taxes.)

40. Your firm is considering the purchase of a new office phone system. You can either pay \$32,000 now, or \$1000 per month for 36 months.

a. Suppose your firm currently borrows at a rate of 6% per year (APR with monthly compounding). Which payment plan is more attractive?

b. Suppose your firm currently borrows at a rate of 18% per year (APR with monthly compounding). Which payment plan would be more attractive in this case?

41. After reading the Novy-Marx and Rauh article (see the Common Mistake Box on page 159), you decide to compute the total obligation of the state you live in. After some research you determine that your state's promised pension payments amount to \$10 billion annually, and you expect this obligation to grow at 2% per year. You determine that the riskiness of this obligation is the same as the riskiness of the state's debt. Based on the pricing of that debt you determine that the correct discount rate for the fund's liabilities is 3% per annum. Currently, based on actuarial calculations using 8% as the discount rate, the plan is neither over- nor underfunded—the value of the liabilities exactly matches the value of the assets. What is the extent of the true unfunded liability?

CHAPTER 5 APPENDIX

Continuous Rates and Cash Flows

NOTATION

e	2.71828...
ln	natural logarithm
r_{cc}	continuously compounded discount rate
g_{cc}	continuously compounded growth rate
$\overline{C}_1$	total cash flows received in first year

In this appendix, we consider how to discount cash flows when interest is paid, or cash flows are received, on a continuous basis.

Discount Rates for a Continuously Compounded APR

Some investments compound more frequently than daily. As we move from daily to hourly ($k = 24 \times 365$) to compounding every second ($k = 60 \times 60 \times 24 \times 365$), we approach the limit of continuous compounding, in which we compound every instant ($k = \infty$). Eq. 5.3 cannot be used to compute the discount rate from an APR quote based on continuous compounding. In this case, the discount rate for a period length of one year—that is, the EAR—is given by Eq. 5A.1:

The EAR for a Continuously Compounded APR

$$(1 + EAR) = e^{APR} \tag{5A.1}$$

where the mathematical constant[12] $e = 2.71828 \ldots$. Once you know the EAR, you can compute the discount rate for any compounding period length using Eq. 5.1.

Alternatively, if we know the EAR and want to find the corresponding continuously compounded APR, we can invert Eq. 5A.1 by taking the natural logarithm (ln) of both sides:[13]

The Continuously Compounded APR for an EAR

$$APR = \ln(1 + EAR) \tag{5A.2}$$

Continuously compounded rates are not often used in practice. Sometimes, banks offer them as a marketing gimmick, but there is little actual difference between daily and continuous compounding. For example, with a 6% APR, daily compounding provides an EAR of $(1 + 0.06/365)^{365} - 1 = 6.18313\%$, whereas with continuous compounding the EAR is $e^{0.06} - 1 = 6.18365\%$.

Continuously Arriving Cash Flows

How can we compute the present value of an investment whose cash flows arrive continuously? For example, consider the cash flows of an online book retailer. Suppose the firm forecasts cash flows of $10 million per year. The $10 million will be received throughout each year, not at year-end; that is, the $10 million is paid *continuously* throughout the year.

We can compute the present value of cash flows that arrive continuously using a version of the growing perpetuity formula. If cash flows arrive, starting immediately, at an initial

[12] The constant e raised to a power is also written as the function *exp*. That is, $e^{APR} = exp(APR)$. This function is built into most spreadsheets and calculators.

[13] Recall that $\ln(e^x) = x$

rate of \$$C$ per year, and if the cash flows grow at rate g per year, then given a discount rate (expressed as an EAR) of r per year, the present value of the cash flows is

Present Value of a Continuously Growing Perpetuity[14]

$$PV = \frac{C}{r_{cc} - g_{cc}} \tag{5A.3}$$

where $r_{cc} = \ln(1 + r)$ and $g_{cc} = \ln(1 + g)$ are the discount and growth rates expressed as continuously compounded APRs, respectively.

There is another, approximate method for dealing with continuously arriving cash flows. Let $\overline{C}_1$ be the total cash flows that arrive during the first year. Because the cash flows arrive throughout the year, we can think of them arriving "on average" in the middle of the year. In that case, we should discount the cash flows by ½ year less:

$$\frac{C}{r_{cc} - g_{cc}} \approx \frac{\overline{C}_1}{r - g} \times (1 + r)^{1/2} \tag{5A.4}$$

In practice, the approximation in Eq. 5A.4 works quite well. More generally, it implies that when cash flows arrive continuously, we can compute present values reasonably accurately by following a "**mid-year convention**" in which we pretend that all of the cash flows for the year arrive in the middle of the year.

EXAMPLE 5A.1 Valuing Projects with Continuous Cash Flows

Problem

Your firm is considering buying an oil rig. The rig will initially produce oil at a rate of 30 million barrels per year. You have a long-term contract that allows you to sell the oil at a profit of \$1.25 per barrel. If the rate of oil production from the rig declines by 3% over the year and the discount rate is 10% per year (EAR), how much would you be willing to pay for the rig?

Solution

According to the estimates, the rig will generate profits at an initial rate of (30 million barrels per year) × (\$1.25/barrel) = \$37.5 million per year. The 10% discount rate is equivalent to a continuously compounded APR of $r_{cc} = \ln(1 + 0.10) = 9.531\%$; similarly, the growth rate has an APR of $g_{cc} = \ln(1 - 0.03) = -3.046\%$. From Eq. 5A.3, the present value of the profits from the rig is

$$PV(\text{profits}) = 37.5/(r_{cc} - g_{cc}) = 37.5/(0.09531 + 0.03046) = \$298.16 \text{ million}$$

Alternatively, we can closely approximate the present value as follows. The initial profit rate of the rig is \$37.5 million per year. By the end of the year, the profit rate will have declined by 3% to $37.5 \times (1 - 0.03) = \$36.375$ million per year. Therefore, the average profit rate during the year is approximately $(37.5 + 36.375)/2 = \$36.938$ million. Valuing the cash flows as though they occur at the middle of each year, we have

$$\begin{aligned} PV(\text{profits}) &= [36.938/(r - g)] \times (1 + r)^{1/2} \\ &= [36.938/(0.10 + 0.03)] \times (1.10)^{1/2} = \$298.01 \text{ million} \end{aligned}$$

Note that both methods produce very similar results.

[14]Given the perpetuity formula, we can value an annuity as the difference between two perpetuities.