Information production, dilution costs, and optimal security design

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Abstract

We investigate the problem of a firm wishing to finance a project by issuing securities under asymmetric information. We find that, when outside investors can produce (noisy) information on the firm’s quality, the degree of information asymmetry resulting in equilibrium is endogenous and depends on the information sensitivity of the security issued. Thus, in contrast to the prediction of the pecking order theory (see, e.g. Myers and Majluf, J. Financial Econom. 13 (1984) 187-221) a security with low sensitivity to private information, such as debt, does not always dominate one with high information sensitivity, such as equity. A firm’s preference for equity rather than debt depends on the costs of information production, the precision of the information-production technology, and the extent of the information asymmetry. We also study the optimal security design problem and find that, depending on the cost and precision of the

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information-production technology, risky debt or a composite security with a convex payoff emerges as optimal securities. © 2001 Elsevier Science S.A. All rights reserved.

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1. Introduction

The problem of raising funds by selling securities under asymmetric information is one of the main themes of contemporary corporate finance. In a typical situation, a firm with limited resources and private information on its investment opportunities would like to raise cash by issuing new equity, debt, or both. Alternatively, funds may be raised by securitizing an asset or a pool of assets, or by (partial) divestiture of a division. Under these circumstances, a firm of superior quality will find that the price at which it can sell its securities is less than the value attributed by its insiders, given their favorable private information. For these firms, this difference represents a dilution cost that is due to the informational asymmetry between insiders and outside investors. The standard intuition of the pecking order theory (see, e.g., Myers and Majluf, 1984; Myers, 1984) is that, in these cases, firms of above-average quality should use less information-sensitive securities, that is securities with a lower discrepancy between their market value and the value attributed by the insiders, given their information.

We show that the predictions of the pecking order theory depend crucially on the assumption that the extent of informational asymmetry between a firm’s insiders and outside investors is fixed and remains so when the firm issues different securities. In Myers and Majluf, outside investors cannot produce (additional) information on the value of the issuing firm at any cost and, thus, the market for information acquisition is essentially closed. Once outside investors are allowed to produce (noisy) information, the extent of the informational asymmetry between the firm and outside investors becomes endogenous and depends on the degree of information sensitivity of the security used. By issuing a more information-sensitive security, a firm may promote information production by outside investors and thus reduce the extent of information asymmetry.

In our model, firms are endowed with a project requiring a certain investment. The value of the project depends on the firm’s quality, which is private information to its insiders. For simplicity, we assume that firms may be either of “good” or of “bad” quality. The firm has the choice of issuing either a security with a high sensitivity to private information or one with a low
sensitivity. Securities are sold in capital markets populated by two types of investors: uninformed investors, who exert an exogenous and inelastic demand for firms’ securities, and other investors who can potentially become informed. This second group of investors, whom we call “specialized investors”, has access to a costly information-production technology and may produce information on the quality of the firm, thus becoming informed. Market clearing is guaranteed by a group of competitive market makers who set prices after observing the total demand for the securities issued by the firm.

In the presence of asymmetric information, the price at which a firm can sell its securities in the marketplace will depend on the market makers’ posterior belief in the quality of the issuing firm, after observation of total demand. If overall demand from informed and uninformed investors is sufficiently low, the perceived quality of the issuing firm may decrease to the point that it may not be able to raise the desired funds (or it may wish, at these depressed securities prices, to withdraw the issue). By offering a more information-sensitive security such as equity, a firm of good quality encourages information production by specialized investors and promotes informed trading. This greater volume of informed trading increases expected demand and induces higher issuing prices, thus reducing the chance that the issue will fail. We find that in this case a firm is more likely to issue equity when investors’ cost of becoming informed is lower, and the precision of the signal observed by investors choosing to become informed is greater. If the cost of becoming informed is sufficiently large, however, our model predicts in accordance with the pecking order theory that firms should use securities that are less information-sensitive, such as risky debt. We also find that the likelihood that a firm will issue equity increases with the value of the project relative to the amount of external funds raised and with the extent of the informational asymmetry between its insiders and outside investors.

We develop our analysis in two steps. We first examine the case in which the domain of securities available to the firm is artificially restricted to equity and risky debt. This case allows the predictions of the pecking order theory to be addressed directly.\footnote{Gale (1992) discusses reasons that firms may wish to restrict their choice of financing instruments to a standard set of securities such as equity and debt.} We then extend our analysis by characterizing the solution to the more general problem of optimally designing the security issued by the firm. The main results of our paper extend easily to this more general setting. Specifically, when the cost of becoming informed is sufficiently high, the solution to this optimal security design problem is risky debt; that is, a security with the same structure as the one in the basic case. Conversely, if the information acquisition costs are sufficiently low, the firm will optimally choose a security with a convex payoff; that is, a composite security consisting of
equity and call options which, in some circumstances, may be interpreted as a traditional warrant.

Several papers are now part of the security design literature. In an important paper, DeMarzo and Duffie (1999) examine the problem faced by a firm raising funds under asymmetric information. At the moment the firm offers its securities, the insiders’ private information creates an illiquidity loss that has to be traded off against the costs of retention. Their paper then examines the insider’s “ex-ante” problem (i.e., before becoming informed) of optimally designing the securities to be offered so as to resolve efficiently the illiquidity that arises ex-post. In a similar spirit, Narayanan (1988) considers the optimal financing choice for a firm issuing securities under asymmetric information. The main findings of these papers support the implication of the pecking order theory that firms of superior quality should minimize dilution costs by first issuing securities with a lower information sensitivity.

Nachman and Noe (1994) examine a security design problem similar to the one of DeMarzo and Duffie, but at the interim stage (in the sense of Holmstrom and Myerson, 1983), that is after the insiders become informed about the firm’s quality (its type). Their paper establishes the characteristics of the probability distribution of the project’s future cash flow necessary to ensure that a firm’s insiders prefer debt over equity. The authors find that the predictions of the pecking order theory hold if, and only if, a certain (conditional) stochastic dominance condition is satisfied.2

The main difference between these papers and ours is that these works do not consider the effect of information production by investors. We show that, when the choice of information acquisition is explicitly recognized, a firm’s preference for a security with high, or low, information sensitivity is not unambiguously determined by the probability distribution of future cash flow. Instead, given a probability distribution of future cash flow, we find that the desirable information sensitivity of a security depends on variables such as the cost and precision of the information-production technology.

In a related seminal paper, Boot and Thakor (1993) examine the problem faced by a firm wishing to sell an asset under asymmetric information. That paper shows that the seller’s revenue maximizing strategy is to split the claims on the cash flow from the asset into an information-sensitive security that promotes informed trading and a second claim that is less information-sensitive.

Our paper differs from theirs on several important dimensions. First, in our paper, firms do not sell the project in its entirety but retain a residual claim on their assets. This is an important difference because it makes the cost of

2 Other papers in this strand of literature include Brennan and Kraus (1987), Constantinides and Grundy (1990), Rahi (1996) and Ravid and Spiegel (1997). An extensive survey of this, and related, literature may be found in Daniel and Titman (1995).
dilution a concern for the issuing firm. In Boot and Thakor’s model, sellers offer the entirety of their assets for sale, thus eliminating the effect of dilution. An implication is that, in their model, it is always optimal to split the cash flow in safe debt and an information-sensitive security. In our model, the decision to issue a security with high or low information sensitivity depends on some critical parameters. This difference is important because it allows us to make predictions on when a certain security will be optimal. (An additional implication is that, without endogenous information production, in Boot and Thakor a firm is indifferent on the structure of the securities issued. In our model, a security with low information sensitivity will always be optimal, for precisely the same reasons as the one discussed in Myers and Majluf.)

Second, in our model the firm is required to raise a fixed amount of funds to undertake the investment project. Because the price of the securities offered by the firm depends on the realized demand under an adverse realization of the uninformed investors’ demand, the firm may not be able (or willing) to issue any security, in which case the issue is not successful. In Boot and Thakor’s model, the firm always sells the asset, accepting all funds that it is able to raise.

Third, Boot and Thakor focus on the problem of splitting the cash flow from the asset into two securities, one more information-sensitive than the other. We consider the problem of choosing a marginal security that is more (in the case of equity) or less (in the case of risky debt) information-sensitive. In our setting, this choice is important because it affects the extent of dilution costs suffered by the firm. Our model may be easily reinterpreted as one in which firms have already issued all possible risk-free securities (as predicted by Boot and Thakor), and must now decide on the information sensitivity of a residual security to be issued. Hence, our model complements theirs.

Finally, we explicitly characterize the solution to the optimal security design problem, an issue not addressed in Boot and Thakor.

Other related papers in the security design literature are Allen and Gale (1988, 1994) and Madan and Soubra (1991). These papers examine the optimal security design problem from the viewpoint of providing optimal risk-sharing opportunities to investors. We disregard risk-sharing problems by assuming universal risk neutrality. We also do not consider arguments for optimal security design that are based on corporate control considerations, such as those examined in Zender (1991) and Kalay and Zender (1997).

Our paper is organized as follows. Section 2 presents the basic model. For simplicity, we assume that information-production technology perfectly reveals the insiders’ private information to outside investors producing information. Section 3 defines the equilibrium for our model. Section 4 characterizes the equilibrium on the securities markets in the debt and equity case, and it examines the choice of financing for the firm. Section 5 examines the impact of
the characteristics of the project on the financing choices. Section 6 shows that the main results of our paper also hold when outside investors produce noisy information. Section 7 solves the optimal security design problem. Section 8 contains the empirical predictions of the model. Section 9 concludes. All proofs are in Appendix A.

2. The model

We consider a one-period economy with two dates, \( t = 0, 1 \). This economy has three type of agents: firms endowed with investment projects, investors, and market makers. All agents are assumed to be risk-neutral, and the riskless interest rate is normalized to zero.

2.1. The agents in the economy

2.1.1. The issuing firms

At the beginning of the period, \( t = 0 \), an all-equity firm is endowed with an investment project, which is the firm’s only asset (the model’s sequence of events is displayed in Fig. 1). To undertake the project this firm has to make a fixed initial investment \( I \) at date \( t = 0 \).\(^3\) At date \( t = 1 \), the value of the project, \( \hat{V} \), is realized. The value of project at \( t = 1 \) depends on the quality of the firm, which is private information to its insiders. A firm’s (or, equivalently, a project’s) quality may be one of two types: It may be good, denoted by \( f = G \), or bad, denoted by \( f = B \). The two-type specification of our model eliminates the possibility of partially pooling equilibria, avoiding the type of equilibria discussed in Noe (1988). If the quality of the firm is good, the value of the project at \( t = 1 \) is \( V^G \); if the quality of the firm is bad, the value of the project is \( V^B \). Firm’s outsiders do not know the true quality of the firm, but they have a common prior belief \( \theta \) that the firm is of good quality and a belief \( 1 - \theta \) that the firm is of bad quality. The value of initial investment \( I \) is such that only the project of a good firm is profitable; that is, \( 0 < V^B < I < V^G \).\(^4\) Also, the prior probability that the firm has a good project, \( \theta \), is such that \( E(\hat{V}) = \theta V^G + (1 - \theta) V^B > I \). Thus, the project is expected to be profitable when evaluated on the basis of outsiders’ prior beliefs. Finally, if a project is implemented, a firm’s manager will earn a non-contractible private benefit. This implies that managers of bad firms are willing to undertake the project if they can obtain financing.

\(^3\) Alternatively, it may be assumed that \( I \) represents the incremental investment that is necessary to continue an existing project.

\(^4\) Cooney and Kalay (1993) also consider the case in which firms may be endowed with a project with negative Net Present Value (NPV).
At $t = 0$,

- Firms decide which security to issue: $S = E, D$.
- Specialized investors choose whether to pay a cost $c$ and become informed or to remain uninformed.
- Investors who choose to become informed observe a signal.
- Informed and uninformed traders submit orders.
- Market makers observe total order flow and set prices.
- Securities are issued and trade occurs.
- Project is undertaken.

At $t = 1$,

Value of the project is realized.

Fig. 1. Sequence of events.

The firm has no cash and at $t = 0$ must raise all the funds necessary to invest in the project by issuing claims sold in a market for firms’ securities. We initially assume that firms can raise funds by issuing only one type of security, and we restrict this choice to be either a security with high information sensitivity, equity, or one with low information sensitivity, debt.

The decision on the amount of funds, $I$, to raise, and whether to secure them by issuing debt or equity, is made by a firm at the beginning of the period, $t = 0$, when the securities markets are open. It is easy to see that, to minimize dilution costs, a firm of good quality will always prefer to raise only the minimum amount of funds, $I$, necessary to invest in the project. Therefore, a firm of good quality will issue only an amount of debt or equity whose market value is equal to the required investment $I$. Firms of bad quality will not be able to sell securities having a market value greater than $I$ without revealing their type to uninformed investors and market makers. Because projects of

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5 We assume that the firm has already decided to raise these funds by selling securities in a public offering instead of in a private placement.

6 For simplicity, we assume that the firm (or the project) has no outstanding equity that is already publicly trading. Therefore, if the firm chooses to issue equity, strictly speaking it will happen in the context of an initial public offering (IPO). Our basic setting, however, would arise in several other contexts, such as asset securitizations, equity carve-outs, and divestitures.
firms of bad quality are not profitable, this type of firm will not be able to raise any amount of funds other than $I$.

Under these assumptions, an issuing firm will announce at $t = 0$ the intention to raise an amount of funds equal to the fixed investment $I$ and whether it will raise these funds by issuing equity or debt. The face value of the debt, $D$ (if it chooses to issue debt), or the fraction of equity, $\gamma$ (if it chooses to issue equity), that the firm must sell to outside investors will be determined in equilibrium by the market makers after observation of total demand.

2.1.2. Uninformed and specialized investors

The securities markets have two types of investors: uninformed and specialized investors. Uninformed investors are traders without privileged information on the issuing firm. This class of investors is assumed to exert a stochastic and inelastic demand for the securities issued by the firm. Their total dollar demand, $u$, is assumed to be exogenous and distributed on $(-\infty, \infty)$ with the probability density function $g(u)$.\footnote{The main results of our paper will hold also when uninformed and specialized traders are restricted to non-negative trades; that is, when short sales are prohibited. Because adding a no-short sale constraint would only increase the complexity of the analysis, without adding any commensurate economic insight, we maintain this assumption for expositional simplicity.}

Specialized investors are atomistic traders who have access to an information-production technology. This technology enables them to acquire information on the quality of the issuing firm and thus to become informed. Each specialized investor is endowed with $1 + c$ dollars. Before deciding whether to invest in the securities sold by the firm, specialized investors must decide whether to pay an information producing cost $c$ and to become informed, or to remain uninformed.\footnote{Without loss of generality, we may assume that specialized investors who chose to remain uninformed contribute to form the random demand $u$ of the general pool of uninformed investors.} We assume that if a specialized investor decides to become informed, he or she will learn the true quality of the firm. Based on the observation of the quality of the firm, the informed investor must then decide whether to buy the security offered by the firm and to invest the remaining dollar of his or her wealth, or to sell this security short. We assume that an informed investor may sell short the securities only up to one dollar’s worth, which will then be invested in the risk-free asset.\footnote{Our results will remain unchanged if we assume that informed investors may sell short up to $k \geq 0$ dollars’ worth of securities.} The measure of investors who decide to become informed is denoted by $x \in (0, T)$, and it will be determined endogenously.\footnote{We will assume that $T$ is sufficiently large that it will never be binding in equilibrium.}

The decision of whether to pay the cost $c$ and become informed, or to remain uninformed is made by specialized investors at the beginning of the period,
after the firm has announced its intention to raise funds for a total value $I$ and whether it wishes to raise these funds by issuing debt or equity. Specialized investors who become informed will then condition their trades to the (private) observation of the quality of the firm. Because informed investors are atomistic players, we assume that they behave as price takers. Given the mechanism for the formation of security prices described below, informed investors will buy one dollar’s worth of securities when they learn that the quality of the issuing firm is good, and they will sell short one dollar’s worth of securities when they learn that the quality of the issuing firm is bad. Thus, if a measure $n$ of specialized investors decides to become informed, the total dollar demand for the security from these informed investors, denoted by $x_I$, will be equal to $x_I = n$ if the issuing firm is good and it will be equal to $x_I = -n$ if the issuing firm is bad.

2.1.3. Market makers

There is a large number of competitive market makers. Market makers observe only the net total aggregate demand of the uninformed and informed investors, $x = u + x_I$. After observing total demand, $x$, market makers set the price for the security that the firm has chosen to issue to be equal to its expected value, conditional on the observed demand. Because, at that price, there may be an excess demand or excess supply of securities, market makers must also take a long or a short position in the market so as to balance supply and demand and clear the market.\(^\text{11}\)

2.2. The securities market

We assume that informed and uninformed investors submit their orders simultaneously to the market makers. After observing the total net demand, market makers set the issuing price of the security issued by the firm at a level equal to its conditional expected value. Given these prices, the issuing firm must then offer a face value of debt, $D(x)$, if it has chosen to issue debt, or sell a fraction of equity, $γ(x)$, if it has chosen to issue equity, that is necessary (and sufficient) to raise the desired amount of funds, $I$. Finally, given the supply of the firm’s securities and the total realized demand, the market makers will take a position in the market so as to clear the market.

The face value of debt, $D(x)$, or the fraction of equity, $γ(x)$, that the firm must sell to outside investors to raise the required funds $I$ will depend on the market makers’ posterior probability that the firm is of good quality, conditional on the observed demand $x$. Because, in equilibrium, firms of bad quality

\(^{11}\)This assumption avoids the necessity that, if the demand for the securities exceeds the supply, the issuing firm is obliged to ration investors. The IPO literature reveals that rationing between informed and uninformed investors may lead to underpricing of the securities offered for sale (see, e.g., Rock, 1986).
quality will pool with good ones, market makers update their prior probability that the firm is of good quality by using Bayes’ rule and obtain that the posterior probability $\hat{\theta}$ is given by

$$
\hat{\theta}(x, z^M) = \frac{\theta g(x - z^M)}{\theta g(x - z^M) + (1 - \theta)g(x + z^M)},
$$

(2.1)

where $z^M$ is the market makers’ beliefs about the amount of informed trading by specialized investors that occurs in equilibrium. We make the following assumption on the probability density function of the uninformed demand.

**Assumption 1.** Noise trading is distributed on $(-\infty, \infty)$ with a unimodal continuously differentiable probability density function $g(u)$, which satisfies the following conditions:

(i) $g(u) > 0$, $u \in (-\infty, \infty)$,

(ii) $\lim_{u \to \pm\infty} \frac{g(u + a)}{g(u)} = \lim_{u \to -\infty} \frac{g(u)}{g(u + a)} = 0$, $\nabla a > 0$,

(iii) $\frac{\partial^2 \ln(g(u))}{\partial u^2} < 0$, $u \in (-\infty, \infty)$.

(2.2)

(2.3)

(2.4)

It may be verified that the Normal and Student distributions, for example, satisfy the conditions of Assumption 1.

**Lemma 1.** Under the conditions of Assumption 1, the posterior probability that the firm is of good quality, $\hat{\theta}(x, z^M)$, is an increasing function of the observed level of demand $x$ for all $z^M > 0$; also

$$
\lim_{x \to \infty} \hat{\theta}(x, z^M) = 1 \quad \text{and} \quad \lim_{x \to -\infty} \hat{\theta}(x, z^M) = 0.
$$

(2.5)

Furthermore, $\hat{\theta}(x, 0) = \theta$, for all $x$.

Lemma 1 establishes that if the amount of informed trading expected by market makers is strictly positive ($z^M > 0$), then the posterior probability $\hat{\theta}$ has the desirable property that it is an increasing function of total realized demand $x$. Furthermore, when total demand $x$ is sufficiently large, the probability that the quality of the firm is good is close to 1. Conversely, when the demand is sufficiently low, the probability that the firm is of good quality is close to 0. This implies that there is a critical value of the total demand, $x^c$, such that for all $x < x^c$ the conditional expected value of the firm’s project is less than the required investment $I$, and thus the net present value of the project is negative. This critical value of the demand, $x^c(z^M)$, is implicitly defined by

$$
\hat{\theta}(x^c, z^M)V^G + [1 - \hat{\theta}(x^c, z^M)]V^B = I.
$$

(2.6)
If market makers observe a demand equal to \( x = x^c \), the firm must sell the full expected value of the project to outside investors. In this case, the firm will be required to set \( D(x^c, x^M) = V^G \) if it has decided to issue debt and \( \gamma(x^c, x^M) = 1 \) if it has decided to issue equity. For all \( x < x^c(x^M) \), the expected value of the securities the firm can issue is always less than the required investment \( I \), and the firm will never be able to raise the desired funds. In this case, the issue fails (or is withdrawn by the firm), and the investment project is not undertaken.

Consider a firm of good quality. In this case, the informed investors’ demand will be \( x_I = \alpha \) and the total demand will be \( x = u + \alpha \). Define \( u^G \equiv x^c - \alpha \) as the realization of the uninformed investors’ demand that will result in the critical demand \( x^c \), given the amount of informed trading occurring in the case of a firm of good quality. Again, if the uninformed investors’ demand is below this critical value \( u^G \), a firm of good quality will not be able (or willing) to raise the required funds and the issue will fail. Because, in equilibrium, market makers have rational beliefs, set \( x^M = \alpha \). Substituting the value of the critical demand \( x^c = u^G + \alpha \) into Eq. (2.6) and using Eq. (2.1) for \( \theta(u^G + \alpha, \alpha) \), we obtain that \( u^G \) must satisfy

\[
\frac{g(u^G)}{g(u^G + 2\alpha)} = \frac{1 - \theta I - V^B}{\theta V^G - I}.
\]  

Eq. (2.7) implicitly defines a function \( u^G(\alpha) \) relating the critical cutoff point of the uninformed demand, \( u^G \), to the amount of informed trading when the quality of the firm is good and informed investors’ demand for its securities is positive. 12 The function \( u^G(\alpha) \) is displayed in Fig. 2.

Consider a firm of bad quality. Following a similar procedure, when the quality of the firm is bad, informed investors’ demand will be \( x_I = -\alpha \) and total demand will be \( x = u - \alpha \). Define \( u^B \equiv x^c + \alpha \) as the realization of the uninformed investors’ demand that will result in the critical demand \( x^c \), given the amount of informed trading occurring in the case of a firm of bad quality. In this case, Eq. (2.7) becomes

\[
\frac{g(u^B - 2\alpha)}{g(u^B)} = \frac{1 - \theta I - V^B}{\theta V^G - I}.
\]  

Eq. (2.8) again implicitly defines a function \( u^B(\alpha) \), which relates the cutoff value of the uninformed demand, \( u^B \), to the amount of informed trading when the quality of the issuing firm is bad and the informed investors’ demand for its securities is negative (that is when they sell short the securities offered by the issuing firm).

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12 Note that condition (2.7) requires that at the cutoff point of the uninformed demand, \( u^G \), the likelihood ratio \( \ell(u, z) = g(u)/g(u + 2z) \) is equal to a certain critical level \( \ell^c \) given by \( \ell^c \equiv (1 - \theta) \times (I - V^B)/\theta (V^G - I) \).
Lemma 2. Under the conditions of Assumption 1, the function $u^G(z)$ has a unique maximum at $z > 0$, which solves

$$\frac{g(\hat{u} - 2z)}{g(\hat{u})} = \frac{1 - \theta I - V^B}{\theta V^G - I}.$$  \hspace{1cm} (2.9)

where $\hat{u} = \arg\max u$. Furthermore, $u^B(z) = u^G(z) + 2z$ and is an increasing function of $z$, for all $z > 0$.

Consider a firm of good quality. Lemma 2 establishes that for this type of firm the critical level of the uninformed demand, $u^G$, is an inverted u-shaped function of the amount of informed demand $z$. When informed trading $z$ is very low, demand is not very informative. In this case, the terms at which a
firm will be able to sell its securities on the market will be relatively insensitive to realized demand $x$. Because the unconditional expected value of the project is more than the funds that firms need to raise in the market ($E(V) > I$), in this case the project will be financed for most realizations of the demand $x$. Hence, the issue will fail only when the realized uninformed demand $u$ and, therefore, total demand $x$ are extremely low. On the other extreme, when informed trading $z$ is very high, total demand $x$ is very likely to be very high as well. In this case, the issue will fail only if the realization of the uninformed demand $u$ will be sufficiently low as to generate a total demand $x$ below the critical level $x^c$. Hence, when the amount of informed trading is large, firms of the good type will be able again to issue securities successfully for most realizations of uninformed demand $u$. For intermediate levels of informed demand $z$, total demand $x$ is instead rather informative of the issuing firm’s quality. In this case, even a moderate realization of the uninformed demand $u$ may generate a total demand $x$ which is below the critical level $x^c$, triggering a failure of the issue. In this situation, this type of firm will be able to issue securities successfully only if the realization of uninformed demand $u$ is sufficiently large. These remarks lead to the inverted u-shaped function displayed in Fig. 2.

Consider a firm of bad quality. For this type of firm, an increase of the level of informed trading decreases total demand and increases monotonically the minimum amount of uninformed demand, $u_B$, which is necessary to meet the critical demand $x^c$. Hence, for a firm of the bad quality, the critical level $u_B = u^G(z) + 2z$ is always an increasing function of the level of informed trading $z$.

3. Definition of equilibrium

We use the notion of noisy rational expectations Nash-equilibrium, as in Grossman and Stiglitz (1980).

Definition. An equilibrium of our model is given by (i) the amount of informed trading by specialized investors, $z^*_S$, when the firm issues equity, $S = E$, and when it issues debt, $S = D$; (ii) the market makers’ beliefs about the amount of informed trading, $z^M_S$, when the firm issues equity, $S = E$, and when it issues debt, $S = D$; (iii) the fraction of equity, $\gamma(x_E, z^M_E)$, if the firm issues equity, or the face value of the debt, $D(x_D, z^M_D)$, if it issues risky debt, that a firm must issue to raise the required funds $I$, and (iv) a choice of the form of financing for the issuing firm, $S = E, D$, with the properties that

1. ex-ante expected profits of each informed investor are zero;
2. market makers’ beliefs are rational: $z^*_S = z^*_S$, for $S = E, D$;
3. markets clear; and
4. firms issue the security, equity or risky debt, that maximizes the ex-ante wealth of their initial shareholders.
4. The choice of financing

We first characterize the equilibrium of our model for a given choice by the firm to issue either equity or debt. Then we consider the choice of financing by the issuing firm. We will focus on those equilibria that maximize the ex-ante value of the initial shareholders of the firm of good type.\(^{13}\)

4.1. Equity financing

If the firm chooses to issue equity, it will offer for sale a certain proportion \(\gamma\) of its equity to outside investors. The ex-ante payoff to the outside equity holders is then given by \(\gamma V^G\), if the firm is good, and by \(\gamma V^B\), if the firm is bad. The fraction of equity, \(\gamma\), that the firm is required to sell in equilibrium to outside investors is a function of the realization of total demand, \(x_E\), and of the amount of informed trading, \(\varphi_E^M\), that market makers believe to occur in the equity market in equilibrium. The fraction of equity \(\gamma(x_E, \varphi_E^M)\) is determined by the competitive market makers so that the expected value of the equity sold by the firm, conditional on the observed demand, is equal to the amount of funds that the firm wishes to raise, \(I\). That is

\[
\gamma(x_E, \varphi_E^M) [\hat{\theta}(x_E, \varphi_E^M) V^G + (1 - \hat{\theta}(x_E, \varphi_E^M)) V^B] = I, \tag{4.1}
\]

where \(\hat{\theta}(x_E, \varphi_E^M)\) is given by Eq. (2.1).

The amount of informed trading, \(\varphi_E\), that will occur in the equity market will depend on the profits that specialized investors expect to make by producing information. Given that market makers expect an amount of trade from informed investors equal to \(\varphi_E^M\), the profits that each specialized investor expects ex-ante to earn from becoming an informed investor are given by

\[
\pi_E(\varphi_E, \varphi_E^M) \equiv \theta \int_{u^G(\varphi_E^M) - \varphi_E + \varphi_E^M}^{\infty} \frac{\gamma(u + \varphi_E, \varphi_E^M) V^G - I}{I} g(u) \, du

+ (1 - \theta) \int_{u^B(\varphi_E^M) + \varphi_E - \varphi_E^M}^{\infty} \frac{I - \gamma(u - \varphi_E, \varphi_E^M) V^B}{I} g(u) \, du - c, \tag{4.2}
\]

where \(u^G(\varphi_E^M)\) and \(u^B(\varphi_E^M)\) are defined in Eq. (2.7) and Eq. (2.8), respectively.\(^{14}\)

\(^{13}\)This choice may be justified on the basis of the notion of perfect sequentiality of Grossman and Perry (1986).

\(^{14}\)Note that if \(\varphi_E \neq \varphi_E^M\), the critical level of the uninformed demand below which the issue fails, \(u^c\), is given by \(u^c = x^c - \varphi_E\). From the definition of \(u^G(\varphi_E^M)\), for a firm of good quality this is equal to \(u^c = u^G(\varphi_E^M) - \varphi_E + \varphi_E^M\). By a similar procedure, one can obtain that the critical value of uninformed demand for a firm of bad quality is given by \(u^c = u^B(\varphi_E^M) + \varphi_E - \varphi_E^M\).
Because in equilibrium the amount of informed trading expected by the market makers, \( a^M_E \), is equal to the actual amount of informed trading \( a^E \), the measure of specialized investors that in equilibrium will choose to become informed investors, \( a^*_E \), is given by the value at which expected profits from investing in information acquisition are zero; that is, when

\[
\pi_E(a^*_E, a^E) = \theta \int_{\gamma^E(a^*_E)}^{\infty} \frac{\gamma(u + a^*_E, a^E) G - I}{I} g(u) \, du \\
+ (1 - \theta) \int_{\gamma^E(a^*_E)}^{\infty} \frac{I - \gamma(u - a^*_E, a^E) V^B}{I} g(u) \, du - c = 0. \tag{4.3}
\]

In what follows, it will be helpful to use the fact that the first and the second terms in Eq. (4.3) are equal: \(^{15}\)

\[
\begin{align*}
\theta \int_{\gamma^E(a^*_E)}^{\infty} &\frac{\gamma(u + a^*_E, a^E) G - I}{I} g(u) \, du = (1 - \theta) \\
\times &\int_{\gamma^E(a^*_E)}^{\infty} \frac{I - \gamma(u - a^*_E, a^E) V^B}{I} g(u) \, du.
\end{align*} \tag{4.4}
\]

Thus, ex-ante expected profits of informed investors from buying equity when the firm is found to be of good quality are equal to the expected profits from selling the firm’s equity short when the firm is found to be of bad quality. This and the zero-profit condition, Eq. (4.3), together require that

\[
2\theta \int_{\gamma^E(a^*_E)}^{\infty} [\gamma(u + a^*_E, a^E) G - I] g(u) \, du - cI = 0. \tag{4.5}
\]

4.2. Debt financing

Instead of issuing equity, a firm may choose to raise funds by issuing debt. The face value of debt, \( D \), maturing at \( t = 1 \) that a firm must issue to raise the desired funds \( I \) will depend on the realization of the total demand \( x_D \) observed by the market makers and on their beliefs about the equilibrium amount of informed trading, \( x^M_D \); that is, \( D = D(x_D, x^M_D) \). Because \( V^B < I \), a firm must set \( D > I > V^B \), and debt will be necessarily risky. If the firm is of a good quality, the issuing firm will be solvent at \( t = 1 \), and bondholders will receive the face value \( D \) of their bonds. If the firm is of a bad quality, it will default at \( t = 1 \) and its bondholders will seize the assets, receiving a payoff \( V^B \). As in the case of equity, market makers will price the bonds issued by the firm to be equal to their expected value, conditional on the realization of observed demand. Hence, a firm will be required to issue debt with a face value \( D \), such that its

\(^{15}\)This may be easily verified by using the fair pricing condition, Eq. (4.1), and by changing the variables of integration in the first integral of Eq. (4.3).
expected value, conditional on the realization of the demand \( x_D \), is equal to the amount of funds \( I \) to be raised. Thus,

\[
\hat{\theta}(x_D, x_D^M)D(x_D, x_D^M) + (1 - \hat{\theta}(x_D, x_D^M))V^B = I,
\]

where \( \hat{\theta}(x_D, x_D^M) \) is given by Eq. (2.1). If a firm issues debt, the expected profits that each specialized investor expects to earn from becoming informed (per dollar invested), given that market makers expect a measure, \( x_D^M \), of specialized investors to become informed, are given by

\[
\pi_D(x_D, x_D^M) \equiv \theta \int_{u^D(x_D^M) - y_D^M}^{\infty} \frac{D(u + x_D, x_D^M) - I}{I} g(u) \, du - \frac{I - V^B}{I} g(u) \, du - c.
\]

In a rational expectation equilibrium, the amount of informed trading expected by the market makers, \( x_D^* \), is equal to its equilibrium value, \( x_D^* \). Hence, the measure \( x_D^* \) of specialized investors who in equilibrium choose to become informed is given by the value at which expected profits from becoming informed are zero, that is \( \pi_D(x_D^*, x_D^*) = 0 \). As in the equity case, the profits that a specialized investor expects to make by buying the firm’s debt, if its observed quality is good, or by short selling it, if the firm’s observed quality is bad, are again equal:

\[
\theta \int_{u^D(x_D^*)}^{\infty} \frac{D(u + x_D^*, x_D^*) - I}{I} g(u) \, du = (1 - \theta) \int_{u^D(x_D^*)}^{\infty} \frac{I - V^B}{I} g(u) \, du = 0.
\]

Hence, the zero-profit condition requires that

\[
2\theta \int_{u^D(x_D^*)}^{\infty} \frac{I - V^B}{I} g(u) \, du = cI = 0.
\]

4.3. Equilibrium in the securities markets

Given the security issued by the firm, specialized investors will choose to become informed as long as they expect to earn positive profit from information acquisition. The total amount of informed trading will then depend on the degree of information sensitivity of the security issued by the firm and on the cost of information production, \( c \). In particular, if information-production costs are not too large, in equilibrium a positive amount of informed trading will result in both the equity and debt markets.

Proposition 1. A \( \bar{c} > 0 \) exists with the property that, for a given choice of debt or equity issued by firms, there is a positive equilibrium amount of informed trading: \( x_E^*(c) > 0 \), and \( x_D^*(c) > 0 \), for all \( c < \bar{c} \).
Comparing the equilibrium amount of informed trading when the firm issues equity with the one prevailing when the firm issues debt, reveals that the amount of informed trading is greater in the equity case, as established in the following proposition.

**Proposition 2.** The equilibrium amount of informed trading is greater if the firm issues equity than if it issues debt: \( a^*_E(c) > a^*_D(c) \), for all \( 0 < c < c^* \). Furthermore, the equilibrium amount of informed trading is a decreasing function of the information-production costs \( c \).

Because equity is a junior claim and is more information-sensitive than debt, it will generate greater expected profits from information acquisition. Hence, all else being equal, the use of an equity claim will generate a greater amount of informed trading and in equilibrium will induce a greater number of specialized investors to become informed: \( a^*_E(c) > a^*_D(c) \). Finally, lower information-production costs will produce greater profits from informed trading and will induce a larger measure of specialized investors to become informed as well. Therefore, for both debt and equity, the equilibrium amount of informed trading, \( a^*_S(c) \), is a decreasing function of the information-production cost \( c \).

### 4.4. The choice of financing

We can now examine a firm’s choice of financing, debt versus equity. We derive the value of the equity of the original shareholders when the firm finances the investment project by issuing equity, \( W_E \), and when the firm issues debt, \( W_D \). Because firms of bad quality will always wish to pool with firms of good quality (otherwise they would be revealed as bad, and they would not be able to raise any funds), we will focus only on the payoff to the shareholders of firms of good quality.

If a firm of good quality decides to issue equity, the payoff to its original shareholders, \( W^G_E \), is given by

\[
W^G_E(a^*_E) = \int_{u^G(a^*_E)}^{\infty} [1 - \gamma(u + a^*_E, a^*_E)] V^G g(u) \, du \\
= (V^G - I) \int_{u^G(a^*_E)}^{\infty} g(u) \, du \\
- \int_{u^G(a^*_E)}^{\infty} [\gamma(u + a^*_E, a^*_E) V^G - I] g(u) \, du.
\]

Inspection of Eq. (4.10) reveals that the payoff to the original shareholders from issuing equity may be decomposed into two terms. The first term is the value of the project minus its cost; that is, its NPV multiplied by the probability that the equity issue is successful (the firm is able to raise the desired funds and
implement the project). The second term is an adverse selection component, which represents the cost of dilution that a good firm expects to incur when selling at a market value \( I \) equity that the firm’s insiders value at \( \gamma(u + z_E^*, z_E^*)V^G \), given their private information.

From Eq. (4.5), the value of equity of the original shareholders is

\[
W^G_E = (V^G - I) \int_{\tilde{u}^G(z_E^*)}^{\infty} g(u) \, du - \frac{cI}{2\theta}
\]  

(4.11)

The value of the equity of the original shareholders of a firm of good quality, when the firm raises the desired funds \( I \) by issuing debt, \( W^G_D \), is given by

\[
W^G_D(z_D^*) = \int_{\tilde{u}^G(z_D^*)}^{\infty} [V^G - D(u + z_D^*, z_D^*)]g(u) \, du
\]

\[
= (V^G - I) \int_{\tilde{u}^G(z_D^*)}^{\infty} g(u) \, du - \int_{\tilde{u}^G(z_D^*)}^{\infty} [D(u + z_D^*, z_D^*) - I]g(u) \, du.
\]

(4.12)

Similar to the equity case, the value of the original shareholders may be decomposed into two terms. The first term is equal to the net profit from taking the project multiplied by the probability that the firm will be successful in raising the desired funds in the debt market. The second term is again equal to the dilution cost that the firm expects to sustain in the debt market by selling at market value \( I \) an amount of debt valued by the firm’s insiders at its face value \( D(u + z_D^*, z_D^*) \).

From Eq. (4.9), the value of equity of the original shareholders when the firm issues debt, \( W^G_D \), may be rewritten as

\[
W^G_D(z_D^*) = (V^G - I) \int_{\tilde{u}^G(z_D^*)}^{\infty} g(u) \, du - \frac{cI}{2\theta}
\]

(4.13)

Comparing Eq. (4.11) and Eq. (4.13), the dilution costs that the firm expects to sustain in the debt and equity markets are the same and are equal to \( cI/2\theta \). Securities with high information sensitivity will also generate larger informed trading profits and thus induce a greater measure of investors to become informed. Given the information sensitivity of the security issued by the firm, specialized investors will then choose to become informed until expected profits, per dollar of trade, are equal in both markets. This implies that the dilution costs that the firm expects to sustain are the same for both debt and equity, and will not depend on their degree of information sensitivity.

Because expected dilution costs are the same for both debt and equity, the issuing firm will then choose to use the security that maximizes the probability that the issue will succeed. In particular, a firm of good quality will prefer equity to debt if \( u^G(z_E^*) < u^G(z_D^*) \), and it will prefer debt to equity if \( u^G(z_E^*) > u^G(z_D^*) \). A firm of bad quality will always mimic the choice made by...
a firm of good quality (otherwise it would reveal its type and be unable to raise any funds). Because, from Proposition 2, the equilibrium amount of informed trading depends on information-production costs, the optimal choice of financing will also depend on the size of the information-production costs $c$, as it is established in the following proposition.

Proposition 3. There exists a critical value of the information-production cost, $c^*$, such that equity is preferred to debt for all $c \in (0, c^*)$ and debt is preferred to equity for all $c \in (c^*, \infty)$.

If information-production costs are sufficiently large, in equilibrium there will be little investment in information production by specialized investors and a correspondingly low amount of informed trading. At these depressed levels of informed trading, issuing equity, not debt, has the effect of raising the minimum threshold of uninformed demand, $u^G$, needed for the security issue to succeed. Hence, a firm of good quality increases its chances of raising the required funds if it discourages information production by issuing debt. At the opposite extreme, if information-production costs are sufficiently low, the effect of issuing equity is to raise informed trading and, by making security prices more informative, decrease the minimum level of uninformed demand that is necessary for a successful issue. Hence, in this case, firms of good quality will prefer to stimulate information production by issuing equity rather than debt.

Proposition 3 implies that, if information-production costs are sufficiently large, firms will prefer, in accordance with the pecking order theory, to issue debt; that is, a security with lower information sensitivity. If, on the contrary, the costs of information production are sufficiently low, our model predicts that the firm should instead issue a more information-sensitive security such as equity. This implies that a firm’s preference for debt over equity depends on the properties of the information-production technology, such as the cost of information production, and that such preference is not uniquely determined by the stochastic properties of the probability distribution over future cash flow.\(^{16}\)

5. Project characteristics and financing choices

In this section we examine how the characteristics of the investment project, such as its size, value, and degree of informational asymmetry, affect the choice

\(^{16}\)This result contrasts with the findings of Nachman and Noe (1994), which were that the characteristics of the probability distribution of future cash flow determine a firm’s preference for debt over equity. Our model shows that, given the probability distribution of future cash-flow, this choice depends on the properties of the information-production technology as well.
of financing. This analysis allows us in Section 8 to draw some empirical predictions of our model. For simplicity, we consider the case in which uninformed trading is distributed normally, with mean $\mu$ and variance $\sigma^2$. In this case, by substituting the normal density function into Eq. (2.7), we obtain that the explicit form of the functions $u^G(a)$ and $u^B(a)$ is given by

$$u^G(a) = \mu - \frac{K}{2} a, \quad u^B(a) = \mu + \frac{K}{2} a,$$

where $K \equiv \sigma^2 \ln(\theta/(1-\theta)(V^G - I)/(I - V^B))/2$. Further, in Appendix A we show that the critical value, $c^*$, is independent of the mean, $\mu$, and the standard deviation, $\sigma$, of the uninformed investors density function. This implies that in the case of normally distributed uninformed trading, our main results are independent from changes in the mean and variance of the uninformed investors’ demand distribution. As such, we assume that the demand from uninformed investors is distributed according to a standard normal distribution.

We consider the effect on the critical value $c^*$ resulting from to an increase of the size of the project, but without changing its value (that is, its NPV). We assume that the value of the cash flows at $t = 1$ is given by

$$\tilde{V} = \begin{cases} 
V^G + I \text{ with probability } \theta, \\
V^B + I \text{ with probability } 1 - \theta.
\end{cases}$$

An increase in the size of the project, $I$, will increase the amount of required investment but leave its net present value unchanged.

**Proposition 4.** For a given net present value of the project to be financed, the critical value $c^*$ is a decreasing function of its size: $\partial c^*/\partial I < 0$.

The effect of increasing the size of the project while holding its net present value constant is to decrease the reward for information production, per dollar invested. Because equity is more information-sensitive than debt, the adverse impact on the incentives for information production is proportionally more severe for equity. Thus, firms of good quality will prefer equity over debt only at relatively lower information-production costs, decreasing the critical level $c^*$.

We next consider the effect of a variation of the extent of information asymmetry on the financing choices. We assume that projects are parametrized by a variable $\eta$ that affects the variance of their possible full information value, but leaves unchanged expected value and NPV. We parametrize the projects as

$$\tilde{V} = \begin{cases} 
V^G + \eta/\theta \text{ with probability } \theta, \\
V^B - \eta/(1-\theta) \text{ with probability } 1 - \theta,
\end{cases}$$

where $K \equiv \sigma^2 \ln(\theta/(1-\theta)(V^G - I)/(I - V^B))/2$. Further, in Appendix A we show that the critical value, $c^*$, is independent of the mean, $\mu$, and the standard deviation, $\sigma$, of the uninformed investors density function. This implies that in the case of normally distributed uninformed trading, our main results are independent from changes in the mean and variance of the uninformed investors’ demand distribution. As such, we assume that the demand from uninformed investors is distributed according to a standard normal distribution.
where $\eta$ is common knowledge to all market participants. The extent of informational asymmetry between the issuing firm and investors increases with the parameter $\eta$.

Proposition 5. Let $\theta \geq \theta_0$ (as $\theta_0$ is defined in Appendix A). Then, the critical value $c^*$ is an increasing function of the degree of informational asymmetry with outside investors: $\partial c^*/\partial \eta > 0$.

By issuing equity instead of debt, firms that are characterized by a greater informational asymmetry between insiders and outside investors stimulate more informed trading by specialized investors. The greater amount of informed trading results in a lower threshold of the uninformed demand, $u^G$. This increases the probability that the issue will be successful and makes financing the project by issuing equity more desirable. Hence, firms of good quality are willing to issue equity even at higher information-production costs, thus raising the critical level, $c^*$.

6. The case of noisy information production

In this section we extend our main findings to the case in which specialized investors choosing to become informed do not perfectly observe the firm’s quality. In particular, we assume that after paying the cost $c$, a specialized investor will observe a signal $e \in \{g, b\}$ on the firm’s quality. The signal is noisy and has precision $\delta$:

$$\Pr(e = g|f = G) = \delta,$$

$$\Pr(e = b|f = B) = \delta.$$ (6.1)

Also, specialized investors that choose to become informed may receive the wrong signal with a positive probability $1 - \delta$. This probability is the same for firms of good and bad quality. We also assume that the signals received by different informed investors are independent. Hence, if the number of informed investors is $a$, a measure $\delta a$ will receive the correct signal, while a measure $(1 - \delta)a$ will receive the wrong signal.

When an informed investor receives a signal $e = g$, his or her posterior probability that the firm is good, $\theta'$, is given by Bayes’ rule as

$$\theta'(g) = \frac{\delta \theta}{\delta \theta + (1 - \delta)(1 - \theta)} > \delta.$$ (6.3)

In equilibrium, informed investors who receive a good signal, $e = g$, will buy one dollar’s worth of the firm’s security, and those who receive a bad signal,
\( e = b \), will sell short one dollar’s worth. If the number of investors choosing to become informed in equilibrium is \( x \), then the total net demand from informed investors is determined as follows. For a type \( G \) firm, a measure \( \delta x \) of specialized traders will buy the security issued by the firm, and a measure \( (1 - \delta) x \) will sell them short. Thus, the total demand from informed investors is equal to

\[
x_I(f = G) = \delta x - (1 - \delta) x = (2\delta - 1)x.
\] (6.4)

Similarly, for a type \( B \) firm,

\[
x_I(f = B) = -\delta x + (1 - \delta) x = -(2\delta - 1)x.
\] (6.5)

Let us denote

\[
x' = (2\delta - 1)x.
\] (6.6)

Consider the case in which a firm issues equity. Ex-ante expected profits of the informed investors are given by

\[
\pi_E(x', x') = \theta \int_{u''(x')}^{\infty} \left[ \frac{\gamma(u + x', x')V^G - I}{I} - (1 - \delta) \frac{\gamma(u + x', x')V^G - I}{I} \right] g(u) \, du
\]
\[
\times g(u) \, du - (1 - \delta) \int_{u''(x')}^{\infty} \frac{1 - \gamma(u - x', x')V^B}{I} \int_{u''(x')}^{\infty} \frac{g(u)}{I} \, du - c.
\] (6.7)

Using Eq. (6.7), the zero-profit condition becomes

\[
\pi_E(x'_E, x'_E) = \theta \int_{u''(x'_E)}^{\infty} \frac{\gamma(u' + x'_E, x'_E)V^G - I}{I} g(u) \, du
\]
\[
+ (1 - \theta) \int_{u''(x'_E)}^{\infty} \frac{1 - \gamma(u - x'_E, x'_E)V^B}{I} \int_{u''(x'_E)}^{\infty} \frac{g(u)}{I} \, du - \frac{c}{(2\delta - 1)} = 0.
\] (6.8)

Note that the zero-profit condition Eq. (6.8) is equivalent to the zero-profit condition Eq. (4.3) derived for the case in which agents receive perfect signals, but with the higher information acquisition cost \( c' = c/(2\delta - 1) \). Using the results from the previous sections, we obtain that

\[
W^G_E = (V^G - I) \int_{u''(x'_E)}^{\infty} g(u) \, du - \frac{cI}{2\theta(2\delta - 1)}.
\] (6.9)

Following a similar procedure for the debt case, we have

\[
W^D_E = (V^G - I) \int_{u''(x'_D)}^{\infty} g(u) \, du - \frac{cI}{2\theta(2\delta - 1)}.
\] (6.10)
By comparing Eq. (6.9) and Eq. (6.10) with the corresponding expressions Eq. (4.11) and Eq. (4.13), we obtain that the model with noisy independent signals is equivalent to the model with perfect signals, but with higher information-production cost \( c' = c/(2\delta - 1) \). We can then prove the following proposition.

**Proposition 6.** The critical value \( c^* \) increases with the precision of the information-production technology: \( \partial c^*/\partial \delta > 0 \).

As the information-production technology becomes more precise, expected profits from informed trading increase and more specialized investors choose to become informed. Because the increase in informed demand will be stronger if the firm issues equity instead of debt, an increase in the precision of the information-production technology will make firms prefer equity over debt for greater information-production costs \( c \).

7. Optimal security design

In this section we extend the previous analysis by explicitly addressing the question of the optimal design of the security issued by the firm. We model this optimal security design problem as the optimal choice of a security that pays at \( t = 1 \) a fraction \( \gamma_1 \in [0, 1] \) of the firm, if the firm turns out to be good, and a fraction \( \gamma_0 \in [0, 1] \), if the firm turns out to be bad. Thus, this security has the structure of a contingent claim whose value depends on the realized value of the underlying assets of the firm. The fractions \( \gamma_0 \) and \( \gamma_1 \) that the firm must sell to outside investors are determined endogenously after realization of the security’s total market demand, \( x \). These fractions are determined by the condition that the total market value of the securities issued by the firm is equal to the amount of funds, \( I \), that it wishes to raise. Therefore, in our framework, the design of a security is the choice of a pair of functions \( \{\gamma_0(x), \gamma_1(x)\} \) that specify, for each realization of total demand \( x \), the fractions of the firm’s value that insiders must promise to pay (in each state) to outside investors to raise the desired funds \( I \). Note that the securities considered in the previous sections can be described as particular cases of this general security. Specifically, for all \( x \geq x^c \), risky debt is obtained by setting \( \gamma_0(x) = 1 \) and \( \gamma_1(x) = D(x)/V^G \), while equity is obtained by setting \( \gamma_0(x) = \gamma_1(x) = \gamma(x) \).

The sequence of events is now the same as in the basic model. The firm announces first the structure of the security that it intends to issue; that is, the pair \( \{\gamma_0(x), \gamma_1(x)\} \). After this announcement, specialized investors choose whether to become informed or not. We assume that investors who choose to become informed receive, after paying the cost \( c \), a perfect signal revealing the type of the issuing firm. Investors who receive a good signal will buy one
dollar’s worth of the security, and investors who receive a bad signal will sell short one dollar’s worth. After observing total demand $x$, market makers price the security again using the posterior probability Eq. (2.1). Thus, the fractions $\gamma_0$ and $\gamma_1$ that the firm must promise to outside investors are determined by the condition that the expected value of the contingent claim (conditional on the realized demand $x$ and on the amount of informed trading believed to occur in equilibrium, $x^M$) is equal to $I$. Hence

$$\gamma_1(x, x^M) \hat{\theta}(x, x^M) V^G + \gamma_0(x, x^M)(1 - \hat{\theta}(x, x^M))V^B = I. \tag{7.1}$$

As in the basic model, Eq. (7.1) and Lemma 1 together imply a critical level of total demand, $x^c$, for which the firm must sell the full expected value of the project to raise the desired funds $I$. This critical level $x^c$ for which market makers require that $\gamma_0(x^c) = \gamma_1(x^c) = 1$ is implicitly defined by Eq. (2.6). If realized demand is below this critical level $x^c$, the firm will not be able to raise the desired funds $I$. Thus, for a firm of good quality, the critical level of total demand, $x^c$, is reached when uninformed demand is equal to $u^G(a) = x^c/C_0a$ and is implicitly defined by Eq. (2.7). Similarly, for a firm of bad quality the critical level of total demand $x^c$ is reached when uninformed demand is equal to $u^B = x^c + a$ and is implicitly defined by Eq. (2.8). Note that the critical levels of uninformed demand, $u^G$ and $u^B$, do not directly depend on the structure of the security issued by the firm (that is the pair $\{\gamma_0(x), \gamma_1(x)\}$), but only indirectly, through the equilibrium amount of informed trading, $a$.

Consider the specialized investors’ decision to become informed. The zero-profit condition requires that, given a security structure $\{\gamma_0, \gamma_1\}$, the equilibrium amount of informed trading, $a^*$, is determined by

$$\pi(x^*, a^*) = \theta \int_{\hat{u}^G(a^*)}^{\infty} \frac{\gamma_1(u + x^*, a^*) V^G - I}{I} g(u) \, du$$

$$+ (1 - \theta) \int_{\hat{u}^B(a^*)}^{\infty} \frac{I - \gamma_0(u - x^*, a^*) V^B}{I} g(u) \, du - c = 2\theta \int_{\hat{u}^G(a^*)}^{\infty} \frac{\gamma_1(u + x^*, a^*) V^G - I}{I} g(u) \, du - c = 0. \tag{7.2}$$

Inspection of Eq. (7.2) reveals that, given the structure of the security, the equilibrium amount of informed trading is a decreasing function of the information-production cost, $c$. By using the zero-profit condition Eq. (7.2), the ex-ante expected value of shareholders’ wealth for a firm of good type can be expressed as

$$W^G(x^*) = \int_{\hat{u}^G(x^*)}^{\infty} [1 - \gamma_1(u + x^*, a^*)] V^G g(u) \, du$$

$$= (V^G - I) \int_{\hat{u}^G(x^*)}^{\infty} g(u) \, du - \frac{cI}{2\theta}. \tag{7.3}$$
From Eq. (7.3), it can be seen that ex-ante shareholders’ wealth is maximized by designing a security \( \{\gamma_0(x), \gamma_1(x)\} \) that maximizes the probability of the issue’s success, by minimizing \( u^G \). These considerations lead to the following proposition characterizing the optimal security design problem.

**Proposition 7.** There is a critical value of the information production cost, \( \hat{c} > 0 \), such that

Case (i): if \( c \geq \hat{c} \), the optimal security issued by the firm is given by

\[
\begin{align*}
\gamma_0^*(x) &= 1, \\
\gamma_1^*(x) &= \frac{I - (1 - \hat{\theta}(x))V^B}{\hat{\theta}(x)V^G} \geq \frac{I}{V^G^*},
\end{align*}
\]

for all \( x \geq x^c \);

Case (ii): if \( c < \hat{c} \), there is a \( \hat{x} > x^c \) (where \( \hat{x} \) is implicitly defined by \( \hat{\theta}(\hat{x}) = I/V^G \)) such that the optimal security issued by the firm is given by

if \( x > \hat{x} \)

\[
\begin{align*}
\gamma_0^*(x) &= 0, \\
\gamma_1^*(x) &= \frac{I}{\hat{\theta}(x)V^G} \geq \frac{I}{V^G^*},
\end{align*}
\]

and if \( x^c \leq x \leq \hat{x} \)

\[
\begin{align*}
\gamma_0^*(x) &= \frac{I - \hat{\theta}(x)V^G}{(1 - \hat{\theta}(x))V^B}, \\
\gamma_1^*(x) &= 1.
\end{align*}
\]

In Case (i), the solution to the optimal security design involves a claim with the same payoff as a risky debt contract. In Case (ii), the solution to the optimal security design problem is a security with a convex payoff; that is, a composite security consisting of a combination of equity and call options.

Proposition 7 extends Proposition 3 to the case in which the firm optimally designs the structure of the security that it chooses to issue. It shows that when the cost of information production, \( c \), is sufficiently large, the optimal security design problem requires that the firm issue a security with low information sensitivity. Furthermore, the optimal security has the structure of risky debt, and it is the very security we have analyzed in the basic case. Conversely, if the cost of information production, \( c \), is sufficiently low, then the optimal security design problem requires that the firm issues a security with high information sensitivity. The exact structure of the optimal security is determined in equilibrium by the amount of realized total demand \( x \). If demand is sufficiently large, \( x > \hat{x} \), then the firm will sell only an option-like instrument; that is, a
security with the highest degree of information sensitivity. In particular, if
\( \theta(x) \geq 1/(V^G - V^B) \), this security may be directly interpreted as a standard
warrant.\(^{18}\) If, instead, realized total demand is low, \( x \leq \hat{x} \), then the firm will be
obliged to sell a fraction of equity as well.

Finally, the results of this section extend immediately to the case of noisy
information production presented in Section 6. Following an argument similar
to the one developed in that section, we have

\[
W^G(\hat{x}^*) = (V^G - I) \int_{u^G(\hat{x}^*)}^{\infty} g(u) \, du - \frac{c' I}{2\delta},
\]

where \( \hat{x}' = (2\delta - 1)x \) and \( c' = c/(2\delta - 1) \). Thus, we have the following
proposition.

**Proposition 8.** The critical value \( \hat{c} \) increases with the precision of the information-
production technology: \( \partial \hat{c} / \partial \delta > 0 \).

8. Extensions and empirical implications

Our model could easily be extended to include the case where firms,
in addition to an investment project, have assets in place as well. The
main difference is that in such a case firms of different quality may have a
different critical threshold level of total demand, \( x^* \), below which they
will withdraw the issue from the market. This difference of threshold levels
reflects the different evaluations of the status-quo in absence of a security issue;
that is, the value of original shareholders’ wealth given by the assets in place.
The main intuitions of our paper extend easily to this case. Firms of good
quality may prefer to promote information production by issuing a security
with high information sensitivity, rather than to use one with low information
sensitivity.

Our model may also be extended to other situations such as asset
securitization, divestitures, and equity carve-outs. In the case of asset
securitization, a firm pools a set of assets and issues securities backed by it.
An important security design problem is the determination of the information
sensitivity of the securities sold to outside investors. The main implication of
our paper is that if the selling firm maintains a residual equity position in the
pool, it may prefer to issue a security with high information sensitivity. (This
possibility may arise when the issuing firm must monitor the pooled assets. In

\(^{18}\) This may be seen as follows. Denote by \( \beta \) the fraction of equity obtained by exercising
the warrants, and by \( EX \) the (total) exercise price. Then, the desired security is obtained by setting
\( \beta(x) = \gamma(x)V^G/(V^G - V^B) \), and \( EX = \beta V^B/(1 - \beta) \). We would like to thank our referee for
pointing this out to us.
these cases, by maintaining an equity position the issuing firm preserves its incentives to monitor.) Furthermore, our analysis shows that the optimal security may be a combination of equity and derivatives. A similar result holds if the issuing firm has a reservation price below which it prefers to withdraw the offering. By using information sensitive securities, the issuing firm will increase demand and thus the probability of a successful issue.

Our model extends as well to situations in which a firm must finance a project in one of its divisions. A possibility in these cases is to finance this expenditure by issuing debt secured by the assets of the division or by selling equity in the division in a carve-out or a partial divestiture. (Another possibility, which has become popular recently, is the use of tracking stocks.) Our model would again predict that in some circumstances, issuing an information-sensitive security such as equity dominates issuing one with low information sensitivity.

While the pecking order theory has been successful at predicting some of the empirical regularities of the stock issuing process, empirical tests of this theory as a predictor of a firm’s capital structure have produced overall mixed results (see MacKie-Mason, 1990; Opler and Titman, 1996; and Shyam-Sunder and Myers, 1999, among others). Our basic model may help shed some further light on the determinants of a firm’s security design and capital structure choice.19

Implication 1: Firms with relatively large growth opportunities are more likely to be equity financed. Firms endowed with substantial growth opportunities tend to have projects with greater net present value per unit of investment. Our model predicts that for this class of firms, the amount of information production and informed trading generated by an equity issue may be sufficiently large to make equity attractive even at relatively high information-production costs. Hence, all else being equal, they are more likely to prefer equity over debt financing.

Implication 2: Younger firms are more likely to be equity financed, while older ones are more likely to be debt financed. Younger firms tend to be characterized by a higher degree of informational asymmetry and high growth opportunities. Older and more established firms tend to have a lower degree of informational asymmetry with outside investors and fewer growth opportunities. Our model implies that, all else being equal, younger firms prefer equity issues, while older firms prefer debt financing. This observation may help to explain why, when making a public security offering, firms prefer first to issue public equity in an initial public offering (IPO) and only later publicly traded debt.

These predictions are consistent with the more traditional explanations, based on moral hazard and agency costs of debt arguments, as to why young

19We would like to emphasize that these implications are derived from a simplified model that explicitly omits other relevant components of the capital structure and security design problem such as taxes and agency costs (for an extensive survey of this literature, see Harris and Raviv, 1991).
growth firms prefer equity over debt financing. In these models, growth options provide a bad collateral for debt, because debt financing may induce firms to give up positive NPV projects at future dates (see, for instance, the under investment problem of Myers, 1977). The traditional view is that these firms face a fundamental trade-off between the use of debt and equity. Specifically, adverse selection considerations would lead firms to prefer debt financing, while moral hazard considerations would induce them to prefer equity financing. Our model instead suggests that for these young firms adverse selection motives as well may lead to a preference for equity financing. Hence, our model may help to explain why young growth firms are predominantly equity financed.

Implication 3: Mature firms with low market-to-book ratio are more likely to be debt financed. Firms characterized by a low market-to-book ratio are those in which the amount of required invested capital is large relative to market value. For these firms the ratio of the investment projects’ net present value to invested capital is lower. Our model predicts that for this class of firm the amount of information production and informed trading may be relatively low, making equity issues less attractive. Therefore, for this type of firm, our model predicts that issuers tend to prefer debt to equity financing, and these firms prefer (secured or unsecured) debt to seasoned equity, divestitures, or carve-outs.

9. Conclusion

We have investigated the problem faced by a firm wishing to raise funds to implement an investment project by issuing securities in a financial market characterized by asymmetric information. We have shown that, contrary to the intuition behind the pecking order theory, when investors may produce information on the quality of the underlying firm, insiders may prefer to issue a more information-sensitive security such as equity, rather than a less information-sensitive one such as risky debt. The use of an information-sensitive security increases the amount of informed trading in the market, making security prices more informative about the value of the underlying firm. We have also shown that a firm is more likely to issue equity if the cost of producing information is lower, and the information acquisition process is more precise. Finally, we have characterized the solution to the optimal security design problem and have shown that the optimal security with low information sensitivity is risky debt, while the one with high information sensitivity is a composite security consisting, in some cases, of a combination of equity and warrants.
Appendix A

Proof of Lemma 1. Differentiating Eq. (2.1) with respect to \( x \), we have:

\[
\frac{\partial \hat{\theta}(x, z^M)}{\partial x} = \frac{\theta(1-\theta)[g'(x - z^M)/g(x - z^M) - g'(x + z^M)/g(x + z^M)]}{g(x - z^M)g(x + z^M)(\theta g(x - z^M) + (1-\theta)g(x + z^M)^2)}.
\]  

(A.1)

The term in the square brackets in the numerator can be rewritten as

\[
\frac{\partial \ln(g(x - z^M))}{\partial x} - \frac{\partial \ln(g(x + z^M))}{\partial x}.
\]

Because, from Assumption 1, \( \partial \ln(g(x))/\partial x \) is a decreasing function, we have \( \frac{\partial \hat{\theta}(x, z^M)}{\partial x} > 0 \). Properties in Eq. (2.5) are obtained by taking the appropriate limits. \( \square \)

Proof of Lemma 2. The function \( u^G(z) \) is defined implicitly Eq. (2.7). By implicit function differentiation,

\[
\frac{du^G(z)}{d\hat{u}} = \frac{2g'(u + 2\hat{z})/g(u + 2\hat{z})}{[g'(u)/g(u) - g'(u + 2\hat{z})/g(u + 2\hat{z})]}.
\]  

(A.2)

The term in the denominator is positive from Assumption 1. Hence, the sign of \( \frac{du^G(z)}{d\hat{u}} \) is the same as the sign of \( g'(u + 2\hat{z})/g(u + 2\hat{z}) \). Because, by Assumption 1, the density function \( g(u) \) is unimodal, define \( \hat{u} \equiv \arg \max g(u) \) and note that \( g'(u) > 0 \) for all \( u \in (-\infty, \hat{u}) \), \( g'(u) < 0 \) for all \( u \in (\hat{u}, \infty) \), and \( g'(u) = 0 \) for \( u = \hat{u} \). Define \( \hat{\hat{z}} > 0 \) as the unique solution to \( g(\hat{u})/g(\hat{u} - 2\hat{z}) = \theta/(1-\theta)(V^G - I)/(I - V^B) \). To see that such \( \hat{\hat{z}} > 0 \) exists and is unique, note that \( \theta/(1-\theta)(V^G - I)/(I - V^B) > 1 \) and that \( g(\hat{u})/g(\hat{u} - 2\hat{z}) \) is a monotonically increasing function of \( \hat{z} \), ranging from \( 1 \) to \( \infty \) as \( \hat{z} \) goes from \( 0 \) to \( \infty \). Then, \( u^G(\hat{z}) = \hat{u} - 2\hat{z} \), and the derivative of \( u^G(z) \) at \( \hat{z} \) is zero:

\[
\left. \frac{du^G(z)}{d\hat{u}} \right|_{\hat{z}} = \frac{2g'(u^G(\hat{z}) + 2\hat{z})/g(u^G(\hat{z}) + 2\hat{z})}{[g'(u^G(\hat{z}))/g(u^G(\hat{z})) - g'(u^G(\hat{z}) + 2\hat{z})/g(u^G(\hat{z}) + 2\hat{z})]} = 0.
\]  

(A.3)

Consider some \( \hat{z} > 0 \). For \( \hat{z} = \hat{u} - 2\hat{\hat{z}} \),

\[
\frac{g(\hat{u} + 2\hat{z})}{g(\hat{u})} = \frac{g(\hat{u})}{g(\hat{u} - 2\hat{z})} = \frac{\theta}{1-\theta} \frac{V^G - I}{I - V^B},
\]

because \( \hat{z} = \hat{u} - 2\hat{\hat{z}} < \hat{u} - 2\hat{z} \) gives \( g(\hat{u}) < g(\hat{u} - 2\hat{z}) \). Furthermore

\[
\frac{\partial (g(u + 2\hat{z})/g(u))}{\partial u} = \frac{1}{g(u)g(u + 2\hat{z})} \left( \frac{g'(u + 2\hat{z})}{g(u + 2\hat{z})} - \frac{g'(u)}{g(u)} \right) < 0.
\]  

(A.4)
From Eq. (A.4) and (A.5), $u^G(z')$ has to be greater than $u^0$. Hence
\[ u^G(z') + 2z' > u^0 + 2z' = \tilde{u}, \] (A.6)
and
\[
\frac{du^G(z)}{dz} \bigg|_{z=z'} = \frac{2g'(u^G(z') + 2z') \left[ g(u^G(z')) - g'(u^G(z') + 2z') \right]}{g'(u^G(z'))} < 0,
\] (A.7)
which implies that $u^G(z)$ has a negative slope for all $z > \tilde{z}$. Repeating this procedure, $u^G(z)$ has a positive slope for all $z < \tilde{z}$. To prove that $u^B(z)$ is increasing in $z$, we take the full differential of Eq. (2.8) to obtain
\[
\frac{du^B(z)}{dz} = \frac{2g(u - 2z) / g(u - 2z)}{g'(u - 2z) / g(u - 2) - g'(u) / g(u)}.
\] (A.8)
The term in the denominator is positive, so the sign of $du^B(z)/dz$ depends on the sign of $g'(u - 2z)$. From Eq. (2.8), $g(u) > g(u - 2z)$. Hence, $u - 2z < \tilde{u}$ and $g'(u - 2z) > 0$. 

\textbf{Proof of Proposition 1.} Define $c_E$ as the information-production cost $c$ with the property that expected profits from informed trading in the equity markets are zero when $z_E = 0$, that is $\pi_E(0, 0) = 0$. Similarly, define $c_D$ as the information-production cost $c$ at which $\pi_D(0, 0) = 0$. Let $\tilde{c} = \min\{c_E; c_D\}$. When $z = \infty$, $\pi_E = \pi_D = -c < 0$; that is, information is fully revealed to the market makers and prices are equal to the true value of the securities. Hence, by continuity, there exist a $z_D > 0$ and $z_E > 0$ such that $\pi_E(z_E^*, z_D^*) = \pi_D(z_D^*, z_E^*) = 0$. Because expected profit in the debt market, $\pi_D(z_D^*, z_E^*)$, is a monotonically decreasing function of $z_D$, $z_E^*$ is unique. This may be verified by substituting Eq. (4.8) into Eq. (4.9) to obtain
\[
\pi_D(z_D^*, z_E^*) = 2(1 - \theta) \int_{u^B(z_D^*)}^{\infty} \frac{I - V^B}{I} g(u) du - c.
\] (A.9)
Taking the derivative with respect to $z_D^*$, $d\pi_D(z_D^*, z_E^*) / dz_D^* < 0$. In the case of equity there may be multiple $z_E$ solving $\pi_E(z_E^*, z_D^*) = 0$. In this case, we will choose the largest solution $z_E^*$, for which the amount of informed trading is the greatest. 

\textbf{Proof of Proposition 2.} Rewriting Eq. (4.3) using Eq. (4.4) for the equity case, and rewriting Eq. (4.9) using Eq. (4.8) for the debt case,
\[
\pi_E(z_E^*, z_D^*) = 2(1 - \theta) \int_{u^B(z_E^*)}^{\infty} \frac{I - \gamma(u - z_E^*, z_E^*)}{I} g(u) du - c = 0,
\] (A.10)
\[ \pi_D(\alpha_D^*, \alpha_E^*) = 2(1 - \theta) \int_{u^G(\alpha_E^*)}^{\infty} \frac{I - V^B}{I} g(u) \, du - c = 0, \quad (A.11) \]

where \( g(u - \alpha_E^*, \alpha_E^*) \leq 1 \) by definition. Comparing Eq. (A.10) and Eq. (A.11), \( \pi_D(\alpha_E^*, \alpha_E^*) < 0 \). Because, from the proof of Proposition 1, \( \pi_D(\alpha, \alpha) \) is decreasing in \( \alpha \), \( \alpha_E > \alpha_D \). The profits of informed investors are decreasing in \( \alpha \) for both debt and equity. It then follows that the equilibrium amount of informed trading is decreasing in \( \alpha \). \( \square \)

**Proof of Proposition 3.** From Eq. (A.10) and Eq. (A.11), \( d\alpha_E^*/dc < 0 \) and \( d\alpha_D^*/dc < 0 \). Also, \( \lim_{c \to 0} \alpha_E^* = \infty \), \( \lim_{c \to 0} \alpha_D^* = \infty \), and \( \alpha_E^* > \alpha_D^* \). When the cost of information acquisition increases, the amount of informed trading goes to zero in both debt and equity markets. Hence, given that \( u^G(\alpha) \) is an inverted u-shaped function, \( u^G(\alpha_E^*) = u^G(\alpha_D^*) \) for some \( c^* > 0 \). For the levels of information acquisition costs below \( c^* \), \( u^G(\alpha_E^*) < u^G(\alpha_D^*) \), and, therefore, equity is preferred to debt. For levels of \( c \) above \( c^* \), \( u^G(\alpha_E^*) > u^G(\alpha_D^*) \), and debt is preferred to equity. \( \square \)

In the case of normal distributions, at the threshold level \( c^* \), the equilibrium values for \( \alpha \) satisfy \( \alpha_E^* \alpha_D^* = K \). (For simplicity we will omit asterisks when denoting the equilibrium amount of informed trading \( \alpha \).) To see this, consider the function ~\( u^G(\alpha) = -K/\alpha - \alpha \). The following relationship holds for both \( \alpha_E^* \) and \( \alpha_D^* \): \( -K/\alpha - \alpha = u_0 \) where \( u_0 \) is some constant. For \( \alpha > 0 \) there exist two solutions to the preceding equation, given by \( \alpha = (\pm u_0 \pm \sqrt{u_0^2 - 4K})/2 \). Hence,\(^20\)

\[
\alpha_E^* - \alpha_D^* = \sqrt{u_0^2 - 4K} = \sqrt{\left(\frac{K}{\alpha} - \alpha\right)^2 - 4K} = \sqrt{\left(\frac{K}{\alpha} - \alpha\right)^2} \quad \text{or} \quad \frac{K}{\alpha} - \alpha = \frac{K}{\alpha} + \alpha.
\]

Substituting \( \alpha_D^* \) and noting that \( |K/\alpha - \alpha_D^*| = K/\alpha_D^* - \alpha_D^* \),

\[
\alpha_E^* - \alpha_D^* = \frac{K}{\alpha_D^*} - \alpha_D^*, \quad (A.13)
\]

from which it follows that \( \alpha_E^* \alpha_D^* = K \). To see that the critical value \( c^* \) is independent from the mean and the standard deviation of the uninformed investors density function, it suffices to make two changes of variables: \( u' = u - \mu \), and then \( u'' = u'/\sigma \). Then it is easy to see that \( \alpha_E^* \alpha_D^* = K \) continues to hold for all \( \mu \) and \( \sigma \). Let \( N_{0,1}(u) \) be the density function of a standard normal distribution.

\(^{20}\)At \( c^* \), \( K/\alpha_D^* - \alpha_D^* = -(K/\alpha_E^* - \alpha_E^*) \), so it does not matter which \( \alpha \) we use in the right-hand side of the Eq. (A.12).
Proof of Proposition 4. Note that

$$K = \frac{1}{2} \ln \left( \frac{\theta v^G + I - I}{1 - \theta (I - v^B - I)} \right) = \frac{1}{2} \ln \left( \frac{\theta v^G}{1 - \theta v^B} \right)$$

is independent of $I$. If $dc^*/dI = 0$, then by taking the differential of $\pi_D(\alpha_D) = K$ with respect to $\alpha_E$, $\alpha_D$, and $I$, and by rearranging terms,

$$\varepsilon_{x_D,I} + \varepsilon_{x_E,I} = 0,$$

(A.14)

where $\varepsilon_{x,I}$ is the elasticity of the amount of informed trading with respect to $I$. Consequently, if $\varepsilon_{x_D,I} + \varepsilon_{x_E,I} > 0$, then $dc^*/dI > 0$; and vice versa, if $\varepsilon_{x_D,I} + \varepsilon_{x_E,I} < 0$, then $dc^*/dI < 0$. The zero-profit condition in the debt case is given by

$$\pi_D(\alpha_D, \alpha_D) = 2(1 - \theta) \int_{-K/\alpha_D + \alpha_D}^{\infty} \frac{-v^B}{I} N_{0,1}(u) du - c = 0. \quad (A.15)$$

Taking the differential of Eq. (A.15) with respect to $\alpha_D$ and $I$ we have

$$A_1 dI + A_2 d\alpha_D = 0,$$

(A.16)

where

$$A_1 = \int_{-K/\alpha_D + \alpha_D}^{\infty} \frac{v^B}{I^2 N_{0,1}(u)} du 0,$$

$$A_2 = \left( \frac{K}{\alpha_D} + \alpha_D \right) \frac{v^B}{I \alpha_D} N_{0,1} \left( \frac{-K}{\alpha_D} + \alpha_D \right).$$

Both $A_1$ and $A_2$ are positive. Hence, from Eq. (A.16),

$$\varepsilon_{x_D,I} < 0.$$  

(A.17)

The zero-profit condition in the equity case is given by

$$\pi_E(\alpha_E, \alpha_E) = 2(1 - \theta) \int_{-K/\alpha_E + \alpha_E}^{\infty} \frac{\theta (v^G - v^B) N_{0,1}(u - 2\alpha_E)}{\theta (v^G + I) N_{0,1}(u - 2\alpha_E) + (1 - \theta)(v^B + I) N_{0,1}(u)} N_{0,1}(u) du - c = 0. \quad (A.18)$$
Taking the differential of Eq. (A.18) with respect to \( a \) and \( I \),

\[
B_1 \, dI + B_2 \, d\alpha_E = 0,
\]

(A.19)

where

\[
B_1 \equiv \frac{\partial \pi_E(\alpha_E, \alpha_E)}{\partial I}, \quad B_2 \equiv \frac{d \pi_E(\alpha_E, \alpha_E)}{d \alpha_E}.
\]

One can verify that \( B_1 = 0 \). From the proof of Proposition 1, at \( \alpha_E^* \) it is \( B_2 = 0 \). Hence,

\[
e_{\alpha_E, I} < 0,
\]

(A.20)

and the desired result follows. \( \square \)

We need the following lemma for proving Proposition 5.

**Lemma A.1.** There exists \( \theta_0 \) such that at \( c^* \),

\[
D_3 \equiv \int_{-\infty}^{\infty} \frac{\partial}{\partial \alpha_E} \left( \frac{I - \gamma(u - \alpha_E, \alpha_E) V^B}{I} N_{0,1}(u) \right) du < 0
\]

(A.21)

for all \( \theta > \theta_0 \).

**Proof.** Substituting for \( \gamma(u - \alpha_E, \alpha_E) \) from Eq. (4.1), Eq. (A.21) can be rewritten as

\[
D_3 = \int_{-K/\alpha_E + \alpha_E}^{\infty} \frac{\partial}{\partial \alpha_E} \left( \frac{\theta(V^G - V^B) N_{0,1}(u - 2\alpha_E)}{\theta V^G N_{0,1}(u - 2\alpha_E) + (1 - \theta) V^B N_{0,1}(u) N_{0,1}(u)} \right) du
\]

\[
= \theta(V^G - V^B) \int_{-K/\alpha_E + \alpha_E}^{\infty} \frac{\partial}{\partial \alpha_E} \left( \frac{N_{0,1}(u - 2\alpha_E)}{\theta V^G N_{0,1}(u - 2\alpha_E) + (1 - \theta) V^B N_{0,1}(u) N_{0,1}(u)} \right) du
\]

\[
= (1 - \theta) \theta V^B (V^G - V^B)
\]

(A.22)

\[
\times \int_{-K/\alpha_E - \alpha_E}^{\infty} \left( \frac{-N_{0,1}^2(u)}{[\theta V^G N_{0,1}(u)/(N_{0,1}(u + 2\alpha_E)) + (1 - \theta) V^B]^2} \right) du.
\]
One can see that

\[ \text{sign } D_3 = \text{sign } \int_{-\infty}^{\infty} \left( \frac{-N'_{0,1}(u)}{[\theta V^G N_{0,1}(u)/(N_{0,1}(u + 2z_E)) + (1 - \theta)V^B]^2} \right) \, du. \]

The function \([\theta V^G N_{0,1}(u)/(N_{0,1}(u + 2z_E)) + (1 - \theta)V^B]^2\) is increasing in \(u\), and the function \(-N'_{0,1}(u)\) is symmetric around zero.

It follows that

\[ \int_{-\infty}^{\infty} \left( \frac{-N'_{0,1}(u)}{[\theta V^G N_{0,1}(u)/(N_{0,1}(u + 2z_E)) + (1 - \theta)V^B]^2} \right) \, du < 0. \]

Hence, there exists \(u^* < 0\) such that

\[ \int_{u^*}^{\infty} \left( \frac{-N'_{0,1}(u)}{[\theta V^G N_{0,1}(u)/(N_{0,1}(u + 2z_E)) + (1 - \theta)V^B]^2} \right) \, du = 0. \]  \(\text{(A.23)}\)

Keeping \(z_E\) fixed, the function \(u^*(\theta)\) is bounded for \(\theta \in [\theta_{\text{min}}, 1]\), where \(\theta_{\text{min}}\) is a threshold prior probability. Define \(U^*(z_E) \equiv \min_{\theta \in [\theta_{\text{min}}, 1]} u^*(\theta)\). If \(-K/\alpha_E - \alpha_E < U^*\), then \(D_3 < 0\). Consider

\[ \int_{U^*}^{\infty} \frac{\partial}{\partial z_E} \left( \frac{-N'_{0,1}(u)}{[\theta V^G N_{0,1}(u)/(N_{0,1}(u + 2z_E)) + (1 - \theta)V^B]^2} \right) \, du \]

\[ = \int_{U^*}^{\infty} \left( \frac{N'_{0,1}(u)}{[\theta V^G N_{0,1}(u)/(N_{0,1}(u + 2z_E)) + (1 - \theta)V^B]^2} \right) \]

\[ \times \left( \frac{\partial(\theta V^G N_{0,1}(u)/(N_{0,1}(u + 2z_E)))/(\partial z_E)}{[\theta V^G N_{0,1}(u)/(N_{0,1}(u + 2z_E)) + (1 - \theta)V^B]} \right) \, du. \]

\(\text{(A.24)}\)

The second bracketed term is increasing in \(u\). Hence, the whole expression is negative. It follows that

\[ \int_{U^*}^{\infty} \left( \frac{N'_{0,1}(u)}{[\theta V^G N_{0,1}(u)/(N_{0,1}(u + 2z_E)) + (1 - \theta)V^B]^2} \right) \, du \leq 0 \]

for all \(\alpha \geq \alpha_E\) and \(\frac{\partial U^*}{\partial z_E} \geq 0\).

At \(c^*\): \(\alpha_E > \sqrt{K(\theta)} > \sqrt{K(\theta_{\text{min}})}\), and \(-K/\alpha_E - \alpha_E < -2\sqrt{K(\theta)}\), where \(K \times (\theta) \equiv \frac{1}{2} \ln(\theta/(1 - \theta)(V^G - I)/(I - V^B))\). Define \(\theta_0\) such that \(-2\sqrt{K(\theta_0)} = U^* \times (\sqrt{K(\theta_{\text{min}})})\). Then for all \(\theta > \theta_0\), \(D_3 < 0\). \(\Box\)
Proof of Proposition 5. Following a procedure similar to the one adopted in the proof of Proposition 4, if $\frac{dc^*}{d\eta} = 0$, and taking differential of $x_D x_D = K$ with respect to $x_D$, $x_E$, and $\eta$:

$$
\varepsilon_{x_D,\eta} + \varepsilon_{x_E,\eta} = -\frac{\eta}{2K} \frac{E[\tilde{V}] - I}{(\theta(V^G - I) + \eta)((1 - \theta)(I - V^B) + \eta)}.
$$

(A.25)

Hence, if

$$
\varepsilon_{x_D,\eta} + \varepsilon_{x_E,\eta} > -\frac{\eta}{2K} \frac{E[\tilde{V}] - I}{(\theta(V^G - I) + \eta)((1 - \theta)(I - V^B) + \eta)},
$$

then $\frac{dc^*}{d\eta} > 0$, and if

$$
\varepsilon_{x_D,\eta} + \varepsilon_{x_E,\eta} < -\frac{\eta}{2K} \frac{E[\tilde{V}] - I}{(\theta(V^G - I) + \eta)((1 - \theta)(I - V^B) + \eta)},
$$

then $\frac{dc^*}{d\eta} < 0$.

Taking the differential of the zero-profit condition in the debt case with respect to $x_D$ and $\eta$,

$$
C_1 \eta - C_2 \left( \frac{dx_D}{x_D} \left( \frac{K}{x_D} + x_D \right) + \frac{1}{2x_D} \frac{E[\tilde{V}] - I}{(\theta(V^G - I) + \eta)((1 - \theta)(I - V^B) + \eta)} \eta \frac{dx_D}{\eta} \right) = 0,
$$

(A.26)

where

$$
C_1 = 2 \int_{-K/2x_D + x_D}^{\infty} \frac{\eta}{I} N_{0.1}(u) \, du,
$$

$$
C_2 = 2(1 - \theta) I - V^B + \eta/(1 - \theta) \frac{1}{I} N_{0.1} \left( -\frac{K}{x_D} + x_D \right).
$$

It follows that

$$
\varepsilon_{x_D,\eta} = -C_2 \frac{1}{2x_D} \frac{E[\tilde{V}] - I}{((\theta(V^G - I) + \eta)((1 - \theta)(I - V^B) + \eta))^{\eta}} + C_1 \eta \left( \frac{K}{x_D} + x_D \right) C_2.
$$

(A.27)

Combining the above with $C_1 > 0$, and $C_2 > 0$,

$$
\varepsilon_{x_D,\eta} > \frac{1}{2x_D} \frac{E[\tilde{V}] - I}{((\theta(V^G - I) + \eta)((1 - \theta)(I - V^B) + \eta))^{\eta}} \left( \frac{K}{x_D} + x_D \right).
$$

(A.28)
Taking the zero-profit condition in the equity case,
\[
2(1 - \theta) \int_{-K/\alpha_E + \alpha_E}^{\infty} \frac{\theta(V^G - V^B + \eta/(1 - \theta))N_{0,1}(u - 2\alpha_E)}{\theta(V^G + \eta/(1 - \theta))N_{0,1}(u - 2\alpha_E) + (1 - \theta)(V^B - \eta/(1 - \theta))N_{0,1}(u)} \times N_{0,1}(u) \, du - c = 0,
\]
and differentiating it with respect to \( \alpha_E \) and \( \eta \),
\[
D_1 \frac{d\eta}{\eta} - D_2 \left( \frac{d\alpha_E}{\alpha_E} \left( \frac{K}{\alpha_E} + \alpha_E \right) + \frac{1}{2\alpha_E} \left( \frac{\text{E}[\tilde{V}] - I}{\theta(V^G - I) + \eta((1 - \theta)(I - V^B) + \eta)} \frac{d\eta}{\eta} \right) \right) + D_3 \frac{d\alpha_E}{\alpha_E} = 0,
\]
(A.29)

where
\[
D_1 \equiv \eta \int_{-K/\alpha_E + \alpha_E}^{\infty} \frac{\partial}{\partial \eta} \frac{\theta(V^G - V^B + \eta/(1 - \theta))N_{0,1}(u - 2\alpha_E)}{\theta(V^G + \eta/(1 - \theta))N_{0,1}(u - 2\alpha_E) + (1 - \theta)(V^B - \eta/(1 - \theta))N_{0,1}(u)} \times N_{0,1}(u) \, du,
\]
\[
D_2 \equiv \frac{\theta(V^G - V^B + \eta/(1 - \theta))N_{0,1}(-K/\alpha_E - \alpha_E)}{\theta(V^G + \eta/(1 - \theta))N_{0,1}(-K/\alpha_E - \alpha_E) + (1 - \theta)(V^B - \eta/(1 - \theta))N_{0,1}(-K/\alpha_E + \alpha_E)} \times N_{0,1} \left( \frac{K}{\alpha_E} + \alpha_E \right),
\]
\[
D_3 \equiv \alpha_E \int_{-K/\alpha_E + \alpha_E}^{\infty} \frac{\partial}{\partial \alpha_E} \frac{\theta(V^G - V^B + \eta/(1 - \theta))N_{0,1}(u - 2\alpha_E)}{\theta(V^G + \eta/(1 - \theta))N_{0,1}(u - 2\alpha_E) + (1 - \theta)(V^B - \eta/(1 - \theta))N_{0,1}(u)} \times N_{0,1}(u) \, du.
\]

It is easy to check that \( D_1 > 0, D_2 > 0, \) and \( D_3 \leq 0 \) from Lemma A.1. We can rewrite Eq. (A.29) as
\[
\epsilon_{\alpha_E,\eta} = -D_2 \frac{1}{2\alpha_E} \left( \frac{\text{E}[\tilde{V}] - I}{\theta(V^G - I) + \eta((1 - \theta)(I - V^B) + \eta)} \frac{d\eta}{\eta} \right) + D_1 \frac{K}{\alpha_E + \alpha_E} D_2 - D_3.
\]
(A.30)
and
\[ e_{x_E,\eta} > \frac{1}{2e_F (\theta(V^G - I) + \eta)((1 - \theta)(I - V^B) + \eta))^\eta} \]
\[ \left( \frac{K}{e_E} + e_E \right)^\eta \] (A.31)

From Eq. (A.28) and Eq. (A.30),
\[ e_{x_E,\eta} + e_{x_D,\eta} > \left(1 - \frac{1}{2e_E (\theta(V^G - I) + \eta)((1 - \theta)(I - V^B) + \eta)^\eta} \left( \frac{K}{e_E} + e_E \right) \right) \]
\[ + \frac{1}{2e_D (\theta(V^G - I) + \eta)((1 - \theta)(I - V^B) + \eta)^\eta} \left( \frac{K}{e_D} + e_D \right) \]
\[ = \frac{1}{2 (\theta(V^G - I) + \eta)((1 - \theta)(I - V^B) + \eta)^\eta} \]
\[ \times \left( \frac{1}{\frac{K}{e_E} + e_E} + \frac{1}{\frac{K}{e_D} + e_D} \right) \] (A.32)

Noting that \( e_D = K/e_E \), it follows
\[ e_{x_E,\eta} + e_{x_D,\eta} > \frac{1}{2 (\theta(V^G - I) + \eta)((1 - \theta)(I - V^B) + \eta)^\eta} \frac{e_{\tilde{V}} - I}{K} \] (A.33)

concluding the proof. \( \Box \)

**Proof of Proposition 6.** If at the threshold level \( c^* \) we marginally increase the precision of the signal \( \delta \), then the effective cost \( c^*/(2\delta - 1) \) will decrease and equity will be preferred to debt. To reach the threshold level again, \( c^* \) must increase. Hence, \( dc^*/d\delta > 0 \). \( \Box \)

**Proof of Proposition 7.** Let \( e_{\text{min}} \) and \( e_{\text{max}} \), respectively, be the minimal and the maximal amount of information production that can be achieved by the firm by issuing different securities. Following a procedure similar to the one adopted in the proof of Proposition 3, monotonicity of \( e_{\text{min}} \) and \( e_{\text{max}} \) in the information-production costs, \( c \), implies that there is a critical value of the information-production cost, \( \hat{c} > 0 \), with the property that if \( c < \hat{c} \), the optimal security issued by the firm maximizes the
equilibrium amount of information production, and if \( c > \hat{c} \), the optimal security minimizes the equilibrium amount of information production. To complete the proof, we have to show that the security defined by Eq. (7.4) minimizes the equilibrium amount of informed trading and the security defined by Eq. (7.5) maximizes the equilibrium amount of informed trading.

Consider case (i). Rewrite the zero-profit condition Eq. (7.2) as

\[
2 \left( \frac{1}{c} \right) \int_{u^B(x)}^\infty \frac{I - \gamma_0(u - x^*, x^*) V^B I}{g(u)} \, du - c = 0. \tag{A.34}
\]

This equation determines the equilibrium amount of informed trading \( x^* \). The function \( u^B(x) \) is monotonic in \( x \), so to minimize \( x^* \), \( \gamma_0 \) and \( \gamma_1 \) must minimize the lower limit of the integral in Eq. (A.34). Because the integrand is non-negative, minimization of the lower limit of the integral will be obtained by setting \( \gamma_0 \) to be equal to 1 for each \( u \). Then, \( \gamma_1 \) is determined by the fair pricing condition Eq. (7.1).

The proof proceeds by contradiction. Let \( x^*_D \) be the equilibrium amount of informed trading for the security defined by Eq. (7.4). Assume that there exists a different security \( S \), \( \gamma_{0S}(x, x^*_S) \) and \( \gamma_{1S}(x, x^*_S) \), such that \( x^*_S \leq x^*_D \). From the monotonicity of \( u^B(x) \),

\[
u^B(x^*_S) \leq u^B(x^*_D). \tag{A.35}
\]

The zero-profit conditions for these securities can be written as

\[
\pi(x^*_S) = 2(1 - \theta) \int_{u^B(x^*_S)}^\infty \frac{I - V^B I}{g(u)} \, du - c = 0, \tag{A.36}
\]

\[
\pi(x^*_D) = 2(1 - \theta) \int_{u^B(x^*_D)}^\infty \frac{I - \gamma_{0S}(u - x^*_S, x^*_S) V^B I}{g(u)} \, du - c = 0. \tag{A.37}
\]

Substituting \( u^B(x^*_D) \) as a lower limit of integration in Eq. (A.37) and using Eq. (A.35) we have

\[
2(1 - \theta) \int_{u^B(x^*_D)}^\infty \frac{I - \gamma_{0S}(u - x^*_S, x^*_S) V^B I}{g(u)} \, du - c \leq 0. \tag{A.38}
\]

Subtracting Eq. (A.36) from Eq. (A.38) and rearranging the terms,

\[
\int_{u^B(x^*_D)}^\infty (1 - \gamma_{0S}(u - x^*_S, x^*_S)) g(u) \, du \leq 0. \tag{A.39}
\]

Because the security \( S \) is different from the one defined by Eq. (7.4), given the continuity of \( \gamma_{0S} \), there must exist an interval \([u', u''] \subset [u^B(x^*_D), \infty)\), such that \( \gamma_{0S}(u - x^*_S, x^*_S) < 1, \forall u \in [u', u''] \). This implies that the integrand in Eq. (A.39) is non-negative for all \( u \) and strictly positive on a non-zero interval. Hence, the integral has to be strictly positive, contradicting Eq. (A.39). So the security
defined by Eq. (7.4) minimizes the equilibrium amount of information production.

Consider case (ii). Following a procedure similar to case (i), the zero-profit condition Eq. (A.34) implies that to maximize the equilibrium amount of informed trading, the issuer has to set $\gamma_0$ to the lowest possible value for every realization of $u$. If the realization of $u$ (or $x$) is high enough the issuer can raise all the necessary funds by promising to pay only if the firm is good and by setting accordingly $\gamma_0(x) = 0$; $\gamma_1$ is determined by the fair pricing condition Eq. (7.1): $\gamma_1(x) = I/\hat{\theta}(x)V^G$. For low realizations of $u$, the issuer cannot raise the necessary amount $I$ and keep $\gamma_0 = 0$, even by setting $\gamma_1 = 1$. Hence, the optimal security must have $\gamma_1 = 1$, and $\gamma_0$ is determined by the fair pricing condition Eq. (7.1): $\gamma_0 = (I - \hat{\theta}(x)V^G)/((1 - \hat{\theta}(x))V^B)$. The value of $\hat{\gamma}$ is defined implicitly through $I/\hat{\theta}(\hat{x})V^G = 1$. The remainder of the proof is completely analogous to the proof of case (i). □

References