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Why Do Hot Dogs Come in Packs of 10 and Buns in 8s or 12s? A Demand-Side Investigation*

I. Introduction

In a recent front-page article in the *Wall Street Journal* titled "Why Do Hot Dogs Come in Packs of 10 and Buns in 8s or 12s?" the author concluded from his interview with several marketing executives, "The dark secrets of packaging aren't always easy to unwrap. While the issue of how much product a company should put in a box, bottle or tube is about as basic as any in business, it also can be fraught with complexities" (Koten 1984). Another puzzling issue is why the "large economy-size" can of some products, such as tuna fish, is often more expensive per ounce than the small can (as reported in Widrick [1979a, 1979b] and Cude and Walker [1984a, 1984b]).

The objective of this paper is to present a theory that will give insight into the determination of package sizes and prices and that will capture some important stylized facts about package pricing. Recent empirical research has shown that package sizes and unit prices (price per pound, per ounce, etc.) vary significantly across brands and stores. Moreover, it has been shown that the incidence of quantity premiums (higher unit prices for larger sizes) was surprisingly high. Table 1 illustrates this.

* We would like to thank the referee for help in improving the style and content of this paper.

(*Journal of Business*, 1987, vol. 60, no. 4)

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0021-9398/87/6004-0006\$01.50

This paper presents a theory that yields insight into the determination of package prices and sizes. Consumer heterogeneity, consumption rates, storage costs, and transaction costs (cost of making trips to the store) explains differences in package size and unit prices. In the model, fully informed consumers and a monopolist seller pursue optimizing behavior. The seller chooses package sizes and prices to maximize profits, and the consumers select package sizes that maximize their utilities. We show that consumer heterogeneity may induce the seller to offer more than one size and that larger sizes would be sold either at unit price discounts or unit price premiums. Welfare implications and empirical tests of the theory are presented.

TABLE 1 **Package Size and Unit Prices**

Unit Price Variation	% of 472 Brands	Size Ratios (Large/Small)		% Discount or Premium	
		Average	SD	Average	SD
Discount	91.5	2.28	1.12	15.8	11.8
Uniform	1.3	2.66	1.03	0	0
Premium	7.2	1.84	.39	11.3	12.7

Table 1 was constructed using 1984 data from a North Carolina supermarket and includes brand sizes and their prices. The table illustrates package size and unit price variations across the 472 brands that came in two sizes. As can be seen, 91.5% of the brands were sold at a quantity discount and 7.2% at a quantity premium. The average unit price discount was 15.8%, and the average premium was 11.3%. The average ratio of larger size to smaller size was 2.28 for brands sold at quantity discounts and 1.84 for brands sold at quantity premiums. The standard deviations given in the third and fifth columns of the table show, however, that package size and unit price variations are large. Only six of the 472 brands had uniform unit prices.

What could explain package size and unit price variation? Why are larger sizes sold sometimes at unit price discounts and sometimes at premiums? Our model studies fully informed consumers and a monopolist seller as they optimize. The package sizes and unit prices that maximize the seller's profits must account for consumers' selections of the most attractive offer. Consumer heterogeneity in consumption rates, storage costs, and transaction costs results in a variety of package sizes at unit prices that may reflect quantity discounts or quantity premiums.

The model presented here has similarities to the product line models of Mussa and Rosen (1978), Spence (1980), Dolan (1983), Maskin and Riley (1984), Moorthy (1984), and Oren, Smith, and Wilson (1984). In these models, a monopolist plans product and price offers in order to sort consumers in the most profitable way (extracting as much consumer surplus as possible), given that consumers are free to self-select the offer they most prefer. The underlying family of consumer demand curves is described by a single parameter that represents higher marginal utilities. (This and other assumptions of the above models are criticized in Maskin and Riley's [1984] concluding remarks and in Shugan [1984].) Self-selection induces the monopolist to plan inefficient offers to consumers with low marginal utilities, since efficient offers (which would be offered if market separation was possible) would induce consumers with high marginal utilities to select offers not designed for them. This result is also obtained in our model, but with fewer restrictive assumptions on consumers' preferences. We derive

consumer willingness-to-pay functions using three parameters that represent consumption rates, storage costs, and transaction costs of making a trip to the store. This formulation enables us to derive richer results and testable hypotheses on the relation between the optimal offers and these parameters. We show that the same heterogeneity that induces the seller to offer more than one package size will also lead to unit price discounts or premiums.

Stigler (1963), Adams and Yellen (1976), and Schmalensee (1982) have shown that bundling units of the same good (or different goods) into packages can be an effective device for sorting consumers into groups between which a monopolist can profitably price discriminate. Offering multiple sizes of the same good constitutes a mixed-bundling strategy. In their model, however, mixed bundling always results in quantity discounts. Why?

Adams and Yellen in effect assume that the monopolist is constrained to offer larger package sizes that are exact integer multiples of smaller sizes. In this case it is clear that premiums cannot prevail since consumers can always compose the larger size from the small packages (Narasimhan [1984] also claims that quantity discounts prevail, making essentially the same assumption). Adams and Yellen do not show how the optimal bundles are determined. How many units of each product should be combined in each package in order to maximize profit? In our model, the seller may design package sizes so that combining two small packages yields a size significantly larger than one large package. In this case, the seller may be able to charge a higher unit price for the large package.

It is possible that when small packages are produced in larger volume they will have lower unit costs because of economies of scale. (Other authors emphasize savings in production and selling costs as the source of quantity discounts [see Dolan 1980; and Eppen and Lieberman 1984].) Even if the larger package is more costly to produce per unit, pricing it at a premium invites consumers to compose the large size from multiple small packages. Our model assumes away cost differences in order to focus on demand-side explanations for package size and unit price variations. This also helps us address the problem of multiple purchases of small packages.

An alternative explanation for quantity premiums that is based on imperfect information of consumers is provided by Salop (1977). In his theory, sellers occasionally offer larger sizes at higher unit prices so consumers must compare sizes and package prices to find the best buy. If some consumers do not become informed by comparing sizes and prices (because of high opportunity costs of time), a monopolist can earn higher profits by charging different unit prices for different sizes and exploiting the ill-informed customers. This model does not explain why some products are consistently priced at a premium, apparently

fooling consumers over long periods of time. Our model is the demand-side alternative to this approach.

II. The Model

Our model is kept simple to reduce the mathematical complexity of the analysis without destroying the message. All agents are assumed to be fully rational. We set aside time preference issues by assuming that agents have a discount factor of one.

On the supply side, a monopolist sells a product to heterogeneous customers. The product is sold in differently sized and priced packages with the objective of maximizing the profit per time period. Since our main interest is in the effect of demand heterogeneity on the relation between package sizes and prices, it will be assumed that all the seller's costs are zero. Bulk breaking packages and repacking for the purpose of arbitrage is assumed to be too costly to be economical.

On the demand side, a minimum degree of heterogeneity is specified—only two types of customers. A fraction, α_1 , are type 1 customers, and α_2 are type 2 customers ($\alpha_1 + \alpha_2 = 1$), where population units are normalized to one. A crucial assumption of this paper is that a customer of type i consumes the product at a constant rate x_i per period or not at all. Consumption rates are price sensitive only in the sense that high prices might force the consumer to drop the product from the consumption plan entirely. For many grocery items, this might be reasonable (see Granger and Billson 1972; and Eppen and Liebermann 1984).

The consumers operate in an environment described by the classical economic order quantity model (EOQ). Inventory costs per period are a multiple h_i of the average stock of product, which equals half the package size since the consumption rate is constant. The transaction costs of going to the store are a constant multiple t_i of the number of packages consumed per period, x_i/Q_i , where Q_i is the package size. According to the EOQ model, the costs per period are

$$C(Q_i, i) = t_i x_i / Q_i + h_i Q_i / 2. \quad (1)$$

The second argument in $C(\cdot)$ represents the parameters of consumer i , a notational convention used in other functions below.

Let $U(x_i, i)$ be the utility per period derived by type i customers. Their willingness to pay per period is the utility less the storage/transaction cost per period,

$$V(Q_i, i) = U(x_i, i) - C(Q_i, i). \quad (2)$$

We assume that type 2 consumers place a higher value per unit of consumption on any given package size than do consumers of type 1; that is,

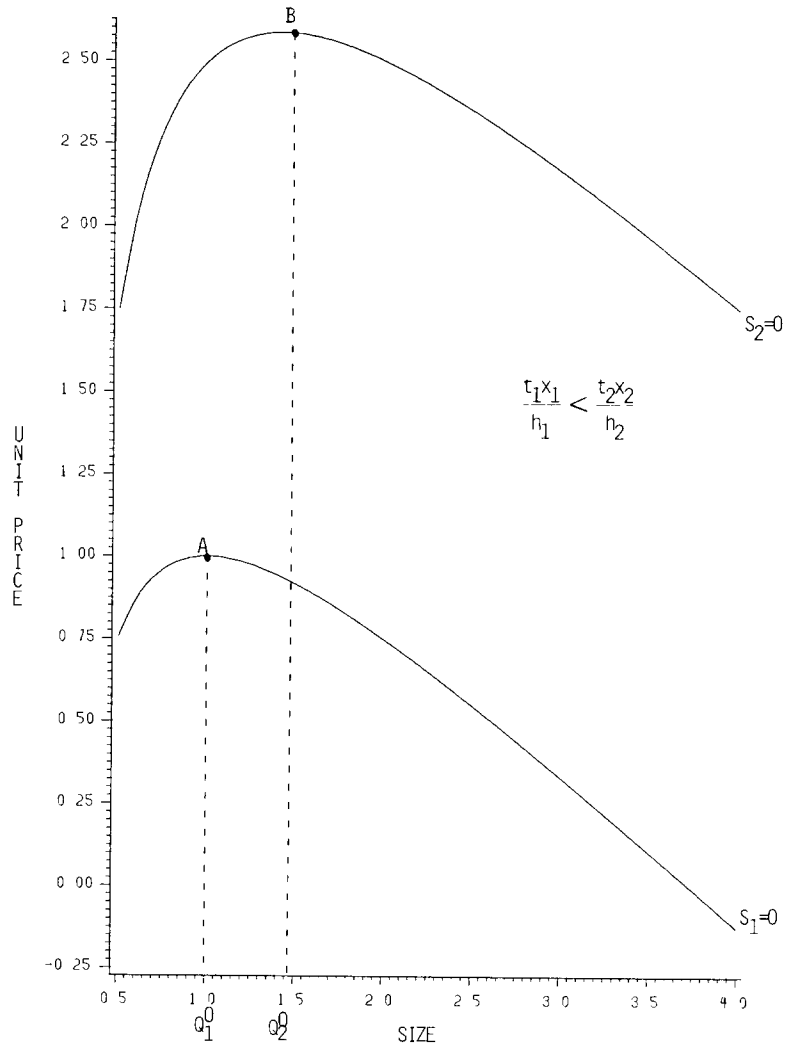


FIG. 1

$$V(Q, 1)/x_1 < V(Q, 2)/x_2, \quad (3)$$

for all $Q \geq 0$. Type 1 consumers are called low-value customers and type 2 consumers are referred to as high-value customers. Assumption (3) is weaker than assumptions in the existing literature, where it is common to add the restriction that one group has uniformly lower marginal valuations. Figures 1 and 2 illustrate possible relations between the consumers' willingness-to-pay curves when inequality (3) holds.

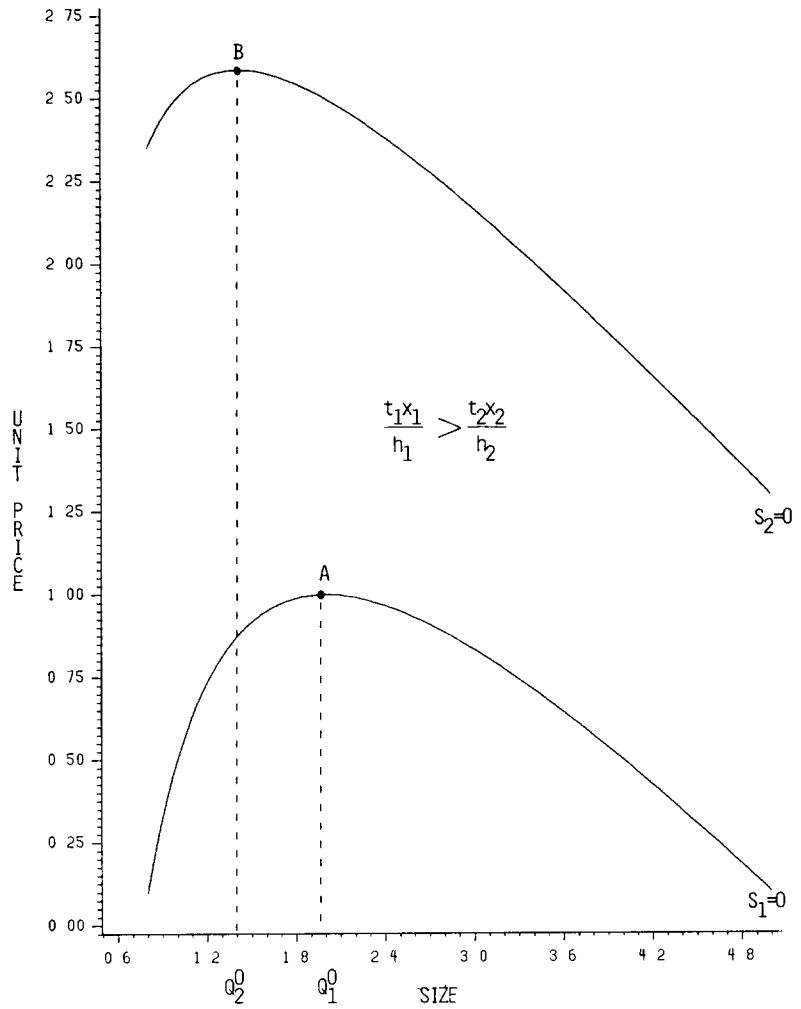


FIG. 2

III. Equilibrium Package Sizes and Unit Prices

Since there are only two groups of customers and customers of the same group are identical, it is clear that no more than two package sizes can prevail in the market. Let P_i be the package price of the container sold to type i customers; therefore, P_i/Q_i is the unit price, and the average expenditure per period by the customer is $x_i P_i/Q_i$. The consumer surplus per period obtained by consumer i when buying a package of size Q_i equals the willingness to pay less the expenditure per period:

$$S_i = V(Q_i, i) - x_i P_i/Q_i. \quad (4)$$

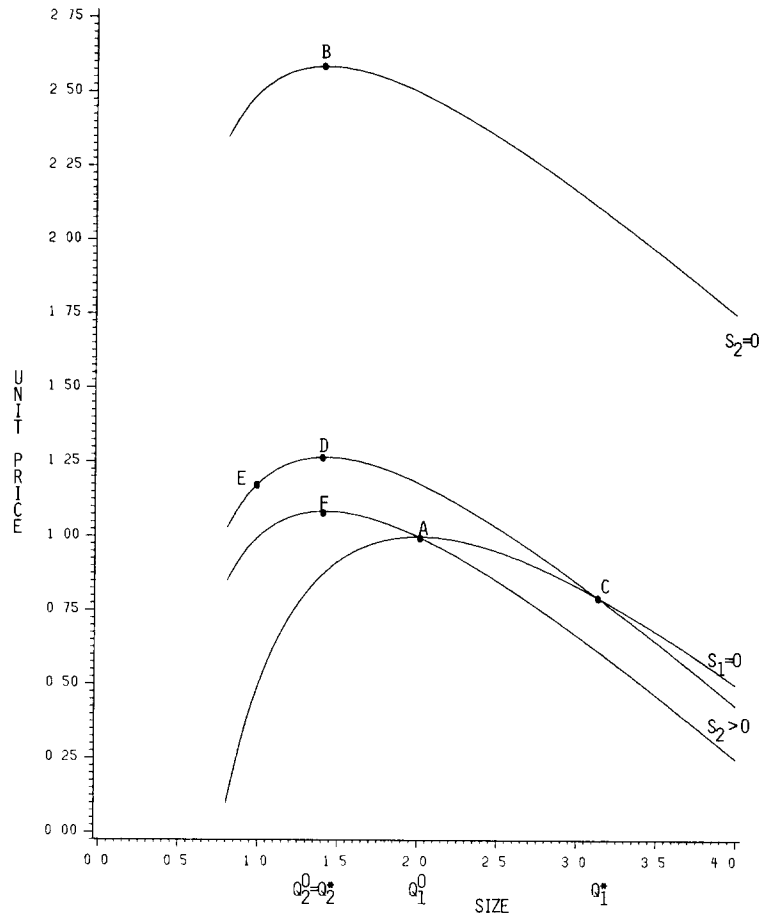


FIG. 3

In Figure 3, the curve labeled $S_2 = 0$ represents trade-offs of unit price and size that leave type 2 consumers with zero consumer surplus. A parallel downward shift of this curve results in an indifference curve that gives constant positive consumer surplus.

In equilibrium, consumers choose package sizes the seller designates for them. Therefore, package size and price offered to customer i must satisfy the following inequalities:

$$V(Q_i, i) - x_i P_i / Q_i \geq 0, \quad (5)$$

$$V(Q_i, i) - x_i P_i / Q_i \geq V(sQ_k, i) - x_i P_k / Q_k, \quad (6)$$

for all $i, k = 1, 2$, and $s = 1, 2, 3, \dots$

The weak inequality (5) states that type i consumers will purchase the packaged good only when they obtain nonnegative surplus per

period. When (5) does not hold, consumers i are worse off buying the package Q_i than spending all their money on other goods, so they will not buy the package. The weak inequality (6) states that type i consumers prefer buying size Q_i at unit price P_i/Q_i to buying size Q_k or any multiple of this size at unit price P_k/Q_k . Inequality (5) will be referred to as the “surplus” condition, and inequalities (6) will be called the “self-selection” conditions.

When any of the above constraints are binding, consumers are indifferent between two courses of action, and it is assumed that they always take the choice designed specifically for them. Consumers’ choices must be incorporated into the design and pricing of the packages by the seller. Similarly, when the seller finds two potential offers are equally profitable, it is assumed he will make the offer that is most attractive to the consumers.

When selling size Q_i at price P_i , the seller’s profit per period from the consumer is $x_i P_i/Q_i$, so the total profit per period is

$$\pi = \alpha_1 x_1 P_1/Q_1 + \alpha_2 x_2 P_2/Q_2. \quad (7)$$

The equilibrium sizes and unit prices are nonnegative values Q_1^* , Q_2^* , P_1^*/Q_1^* , and P_2^*/Q_2^* that maximize profit subject to surplus and self-selection constraints.

To derive equilibrium sizes and unit prices, it will be assumed the seller can if necessary limit the number of packages purchased each trip to one per customer. This will allow us to set $s = 1$ in the self-selection constraints. It will be shown, however, that, for a large range of parameter configurations, such a limit will not be necessary because in equilibrium the self-selection inequalities will hold for $s = 2, 3, 4, \dots$.

Three observations will facilitate derivation of the equilibrium. First, in equilibrium either a surplus constraint or a self-selection constraint is binding for each consumer type. If neither constraint is binding for a given consumer type, the seller could increase profit by slightly increasing the unit price of the package designated for these consumers without violating the surplus or self-selection constraints. The higher unit price will make this package even less attractive to the other consumer type, so they will not select it.

Second, in equilibrium a surplus constraint must be binding for at least one consumer type. If a surplus constraint is not binding for either type, the seller could increase unit prices for both packages by equal small amounts without violating the self-selection constraints, and this extraction of slightly more of the consumers’ surplus increases profits.

Third, assumption (3) implies that the seller will not be able to extract all the surplus of the high-value customers. If the seller makes the high-value customers an offer $(Q_2^*, P_2^*/Q_2^*)$ that leaves them with zero surplus while making the low-value customers an offer $(Q_1^*, P_1^*/Q_1^*)$ that

gives them nonnegative surplus, the high-value customers would self-select the offer designated for the low-value customers since from (3)

$$V(Q_1^*, 2)/x_2 - P_1^*/Q_1^* > V(Q_1^*, 1)/x_1 - P_1^*/Q_1^* \geq 0, \quad (8)$$

and this implies that type 2 consumers get positive surplus from the package designed for type 1 consumers.

These three observations imply that when deriving equilibrium only one case need be considered: the surplus constraint binds the low-value consumers (type 1), and the self-selection constraint binds the high-value consumers (type 2).

Before analyzing the case with self-selection it will be useful to consider a situation in which the monopolist can sell to each consumer type in a separate market, so self-selection is excluded. The most profitable way to extract consumer's surplus is to offer in each market the package size that minimizes their transaction and storage cost $C(Q_i, i)$. By setting package sizes at these efficient levels, unit prices set to extract the entire consumer surplus are the highest.

The package size that minimizes storage and transaction costs (1) is

$$Q_i^0 = (2t_i x_i / h_i)^{1/2}. \quad (9)$$

In Q_i^0 the ratio t_i/h_i represents consumers' cost trade-offs of trips to the store versus storage, and x_i reflects the speed of consumption. Therefore, the crucial value that determines *efficient* size is $t_i x_i / h_i$.

The unit prices that extract the surplus in this case are easily derived from the surplus condition and the efficient size (9):

$$P_i^0/Q_i^0 = U(x_i, i)/x_i - (2t_i h_i / x_i)^{1/2}. \quad (10)$$

In Figures 1 and 2 the efficient quantities and unit prices are points *A* and *B*, which maximize unit prices on the zero surplus indifference curves for type 1 and 2, respectively.

Now consider the relevant case in which the monopolist extracts the entire surplus of the low-value customers and leaves the high-value customer indifferent between the two packages. The profit-maximizing package sizes are

$$Q_1^* = \left[\frac{(\alpha \mathbf{x} + 1)\mathbf{t} - 1}{(\alpha \mathbf{x} + 1)\mathbf{h}/\mathbf{x} - 1} \right]^{1/2} Q_2^*, \quad (11)$$

and

$$Q_2^* = (2t_2 x_2 / h_2)^{1/2}, \quad (12)$$

where boldface variables refer to relative values $\alpha = \alpha_1/\alpha_2$, $\mathbf{t} = t_1/t_2$, $\mathbf{h} = h_1/h_2$, and $\mathbf{x} = x_1/x_2$ (for a derivation, see App. sec. 1).

Equilibrium unit prices are determined by substituting Q_1^* , Q_2^* , into the zero surplus and self-selection constraints (5) and (6) and solving.

Rearranging the self-selection constraint, the equilibrium unit price difference (for a derivation, see App. sec. 2) equals

$$P_2^*/Q_2^* - P_1^*/Q_1^* = t_2(Q_1^* - Q_2^*)^2/(Q_2^{*2}Q_1^*). \quad (13)$$

The above situation presumes that two different packages will be offered. Under what conditions would the seller be better off ignoring consumer heterogeneity by offering just one size? As demonstrated in Section C of the Appendix, necessary and sufficient conditions for the two-package strategy to be optimal are that

$$tx/h \neq 1, \quad (14)$$

and

$$\alpha x > \frac{V(Q_1^*, 2)/x_2}{V(Q_1^*, 1)/x_1} - 1. \quad (15)$$

The left-hand side of the inequality measures the relative number of low-value consumers and their relative consumption rates. The right-hand side of this inequality measures how much the high-value consumers appreciate Q_1^* relative to the low-value consumers. The smaller this magnitude is relative to the left-hand side, the less benefit the seller obtains from concentrating exclusively on high-value customers, ignoring low-value customers. When (14) and (15) are satisfied, two distinct packages will be offered at two different prices.

IV. Determinants of Unit Price and Package Size Variation

We are now ready to answer the following questions. (a) Who buys the larger package? (b) Who is offered the efficient size? (c) What determines size and unit price variation? The answers to these questions are embodied in the following inequalities.

When comparing (9) with (11) and (12), it follows directly that

$$Q_1^* \geq Q_1^0 \geq Q_2^0 = Q_2^* \quad (16)$$

if and only if $tx/h \geq 1$. Three important results are combined in (16).

RESULT A. The larger package will be sold to the consumer with the larger crucial value, $t_i x_i/h_i$.

RESULT B. The size designated for the high-value consumer group equals its efficient size, but the size given to the low-value group in general differs from its efficient size.

RESULT C. Package size differences are larger in a situation in which consumers are allowed to self-select than in a situation in which the seller can prevent self-selection and sell to each consumer group in a separate market.

The rationale for result A is clear. Consumers with high consumption rates, high transaction costs, and low storage costs desire large sizes,

and, hence, it is profitable for the seller to offer the larger package to these customers.

To understand result B, recall that Q_1^* and Q_2^* were derived under the assumption that the consumer group for which the surplus constraint is binding is group 1 and the group for which the self-selection constraint is binding is group 2. Comparing (11) and (12) with (9) shows that $Q_2^* = Q_2^0$, but Q_1^* is not necessarily equal to the efficient size Q_1^0 . Why?

Taking them in order, suppose that Q_2^* was not equal to the efficient size Q_2^0 ; that is, in figure 3 the offer to type 2 is at point E rather than at point D . It will be shown that Q_2^* cannot be the profit-maximizing package. Adjust Q_2^* toward Q_2^0 . This makes the package size more attractive to type 2 consumers, and the seller could correspondingly increase the unit price without violating self-selection (slide point E along the indifference curve toward D). Since type 1 consumers get negative surplus from the package designed for type 2 consumers, the above adjustment can be made small enough so that type 1 consumers still would not prefer package 2. The postulated adjustment increases the unit price of type 2 without violating any of the constraints, and profits are increased; therefore, Q_2^* is profit maximizing only if it equals the efficient size.

Next, why does Q_1^* not equal the efficient size Q_1^0 ? In figure 3, suppose the offers were at points A and F , so the high-value consumers are indifferent between the offers, and the low-value consumer gets zero surplus. If the offer to the low-value customer is adjusted to the right along the indifference curve, infinitesimal reduction in unit price is necessary because A is at a point at which the slope of the indifference curve is zero. This movement distinctly decreases the attractiveness of this offer from the perspective of the high-value consumers, so the unit price P_2^*/Q_2^* can be distinctly increased from point F to point D without violating the self-selection constraint. The optimal offers are C and D in figure 3.

To gain intuition about result C, recall that, if the seller could prevent self-selection and sell to each consumer group separately, he would offer the efficient sizes. Inequality (16) implies that the difference between Q_1^* and Q_2^* must be larger than the difference between Q_1^0 and Q_2^0 . Why? From the previous discussion it is optimal to offer the size offered the low-value consumers away from the sizes Q_1^0 and Q_2^0 (slide offer A along the indifference curve toward C), only an infinitesimal reduction in the unit price P_1^*/Q_1^* is needed to give the low-value consumers zero surplus, but a significant increase in the unit price P_2^*/Q_2^* can be made without violating self-selection conditions. A movement of offer A along the indifference curve away from C will decrease unit prices for both customers, reducing profits. A similar argument can be given to show that $Q_1^* < Q_1^0$ when $tx/h < 1$.

Inequalities (16) relate to the package sizes. What is the relation between unit prices?

$$P_1^*/Q_1^* \leq P_1^0/Q_1^0 \leq P_2^*/Q_2^* \leq P_2^0/Q_2^0. \quad (17)$$

Two important results are combined in (17).

RESULT D. The unit price paid by the low-value consumers is less than the unit price paid by the high-value consumer group when either market separation or self-selection occurs.

RESULT E. Each consumer group pays a lower unit price when self-selection is possible than when markets are completely separated.

The rationale for result D is as follows. When market separation occurs, the high-value customers have more consumer surplus available to be extracted, and they pay a higher unit price. When self-selection occurs, recall that high-value customers are indifferent between the offers. As in figure 3, the package designated for them is the efficient size, and it therefore must have a higher unit price if they are to view it as equivalent to a package of inefficient size.

Intuition for result E is based on the fact that consumers are not obligated to buy a package. Under market separation, low-value customers get a package that equals their efficient size, yet their entire surplus is extracted. Under self-selection their surplus is still extracted, but they do not get their efficient size (result B above). Without a unit price reduction they would refuse to buy the package. High-value consumers are offered their efficient size in both cases. If the unit price for the low-value customer is reduced when self-selection occurs, the unit price for the high-value customer must also be reduced to prevent self-selection of the package designed for low-value customers.

All the above is based on the assumption that the seller can restrict purchases to one per customer, so only $s = 1$ was considered in the self-selection constraint. When the seller plans to offer quantity premiums, he must consider the possibility that consumers may avoid the premium by multipackage purchases. Under what configurations of the parameters can the seller assume that multiple purchases will not occur? Returning to the general self-selection constraint, high-value consumers are happier with one of the packages designed for them than they are with s packages designed for low-value consumers when

$$P_2^*/Q_2^* - P_1^*/Q_1^* \leq [C(sQ_1^*, 2) - C(Q_2^*, 2)]/x_2. \quad (18)$$

A sufficient condition for this to hold for all $s \geq 2$ is

$$Q_2^*/Q_1^* \leq \sqrt{2} \approx 1.41. \quad (19)$$

For a derivation, see App. sec. 4. The ratio of the two package sizes is easily found from equations (11) and (12), and for a substantial range of values of t , x , h , and α , inequality (19) will hold. Intuitively, (19) implies that, when quantity premiums are considered, a relative small-

TABLE 2 Quantity Discounts and Premiums

Consumer Type	Necessary and Sufficient Condition	Discount or Premium
Low value	$tx/h \geq 1$	Discount
High value	$tx/h \leq 1$	Premiums

size ratio will discourage multipackage purchases (two small packages yield a size too large).

V. Quantity Discounts and Premiums

It was shown in table 1 that prepackaged supermarket products are sold at either quantity discounts or premiums, an observation made by other studies. When should the seller offer quantity discounts, and when should he offer quantity premiums (higher unit price for a larger quantity)? Also, who buys at a discount, and who buys at a premium? The following results are obtained from (16) and (17), noting that the inequalities are strict for $Q_1^* \neq Q_2^*$.

RESULT F. When the larger package is designed for the low-value consumer group as in figure 2, quantity discounts are offered. If the larger package is designated for the high-value consumer group as in figure 1, quantity premiums are offered.

Table 2 gives necessary and sufficient conditions for quantity discounts and quantity premiums (these terms include the boundary case of identical unit prices). The table suggests the following important implications for cases in which selling two sizes is most profitable for the seller.

RESULT G. When customers differ only in their storage costs, quantity *premiums* will prevail.¹

RESULT H. When customers differ only in their transaction costs, quantity *discounts* will prevail.²

The implications make sense. Considering result G, if customers differ only in storage cost, customers with lower storage costs (e.g.,

1. More generally, when one of the customer groups has the highest storage and transaction costs but consumption rates and utility functions are the same, if there is greater heterogeneity in storage costs than in transaction costs, the larger package should be designated for customers with lower storage costs and should be offered at a higher unit price.

2. More generally, when one of the customer groups has the highest storage and transaction costs but consumption rates and utility functions are the same, if there is greater heterogeneity in transaction costs than in storage costs, the larger package should be designated for customers with the higher transaction costs and should be offered at a lower unit price.

TABLE 3 Examples of Discounts and Premiums

Quantity Discounts	Quantity Premiums
Consumer parameters:	
$x_i = 1, U(x_i, i) = 4, \alpha_i = 0.5,$ $t_1 = 1.5, t_2 = 1.0, h_1 = h_2 = 1.0$	$x_i = 1, U(x_i, i) = 4, \alpha_i = 0.5,$ $t_1 = t_2 = 1.0, h_1 = 1.5, h_2 = 1.0$
Solution values:	
$Q_1^* = 2.00, P_1^*/Q_1^* = 2.25$ $Q_2^* = 1.41, P_2^*/Q_2^* = 2.34$	$Q_1^* = 1.00, P_1^*/Q_1^* = 2.25$ $Q_2^* = 1.41, P_2^*/Q_2^* = 2.34$

customers with large garages) value the product more and also value the larger size more (i.e., there is positive correlation between willingness to pay and desire for large sizes). See the situation described in figure 1. Since these customers value the product more, they can be charged a higher unit price, and quantity premiums occur.

Considering result H, if customers differ only in transaction costs, customers with higher transaction costs (e.g., customers with high transportation costs) value the product less but value the larger size more (i.e., there is negative correlation between willingness to pay and desire for large sizes). See the situation described in figure 2. Since these customers value the product less, they will be charged a lower unit price, and quantity discounts occur.

Table 3 gives two numerical examples. In the first, consumers differ only in their transaction costs, and, hence, quantity discounts prevail. In the second example, consumers differ only in their storage costs, and, hence, quantity premiums prevail. It is easy to check that these offers are more profitable than selling only one package and that no consumer will buy multiple packages.

VI. Welfare Implications of Packaging and Unit Price Discrimination

This model involves both package size and unit price discrimination. Some consumers are not offered the size package that keeps their storage and transaction costs at a minimum, and they are charged different unit prices for the same good. Any payment for goods produced at zero cost is a transfer, so a social loss occurs either when the unit price offered to customers, who place a positive value on the package, is too high so that no exchange takes place or when the size of the package does not minimize consumers' storage and transaction costs.

RESULT I. The equilibrium offers are inefficient.

If the seller finds it profitable to price discriminate, result B shows that the low-value consumers always get an inefficient size: $Q_1^* \neq Q_1^0$. This result is common to all models with quantity-dependent prices and self-selection (Katz 1983; Maskin and Riley 1984).

planner, however, might not be very useful because of high costs associated with correcting the situation. A regulation preventing unit price differentiation might be enforced at a lower cost. Would such a regulation enhance efficiency?

RESULT J. A regulation that prevents unit price discrimination may or may not induce an increase in social welfare.

This result is consistent with results of other models of price discrimination such as those of Robinson (1933), Schmalensee (1981), Chiang and Spatt (1982), Varian (1985), and Gerstner and Holthausen (1986).

To show that result J holds, first consider the offers that the monopolist makes when price discrimination is prohibited. A regulation forcing identical unit prices would result in a unit price equal to either P_1^0/Q_1^0 or P_2^0/Q_2^0 . Any unit price below these would give both customers positive consumer surplus, and prices could be increased by the seller. Any price between these extremes forces the low-value consumers out of the market, and then it is profitable to raise the price to the high-value consumers as much as possible. When both customers are served, the most profitable offers are $(Q_1^0, P_1^0/Q_1^0)$ and $(Q_2^0, P_1^0/Q_1^0)$, and the resulting profit is $(\alpha_1 x_1 + \alpha_2 x_2)P_1^0/Q_1^0$. (The tiebreaking assumption of Sec. III implies that the efficient size is offered to the high-value customers even though the seller earns equal profits with other sizes.) This pair of offers is efficient because each customer gets the size that minimizes his transaction/storage costs. If the product is sold only to high-value customers, the profit-maximizing offer is $(Q_2^0, P_2^0/Q_2^0)$, and the profit is $\alpha_2 x_2 P_2^0/Q_2^0$. This offer is inefficient because type 1 customers are not sold any package even though they have a positive value for the product.

When will price discrimination be inefficient relative to uniform pricing? If it is more profitable to sell to all customers at a uniform price than to sell just to the high-value customers, price discrimination is detrimental. In this case, each group would get its efficient size if equal unit prices were required, whereas, under price discrimination, low-value consumers are offered an inefficient size.

When will price discrimination be efficient relative to uniform pricing? Price discrimination is advantageous if it is more profitable at a uniform price to sell to just high-value customers than to all customers. Under both pricing methods, high-value customers get their efficient package size. Under uniform pricing low-value customers do not consume the good at all. This is worse than buying a package that is slightly different than their efficient size, as is the case with price discrimination. Table 4 specifies necessary and sufficient conditions for the seller's choice under the uniform unit price regulation to be a social improvement (for a derivation, see App. sec. 5).

To verify that both situations are possible, consider the examples in table 5 and corresponding figures 4 and 5. In figure 4, if the unit price

TABLE 4 Efficiency of Uniform Price Regulation

A. Necessary and Sufficient Condition for Price Discrimination to Occur	
$tx/h \neq 1, \quad \alpha x > \frac{V(Q_1^*, 2)/x_2}{V(Q_1^*, 1)/x_1} - 1$	
B. Necessary and Sufficient Conditions for Improvement in Social Welfare by Eliminating Price Discrimination	
Improvement in welfare:	
$\alpha x > \frac{V(Q_2^0, 2)/x_2}{V(Q_1^0, 1)/x_1} - 1$	
Reduction in welfare:	
$\alpha x < \frac{V(Q_2^0, 2)/x_2}{V(Q_1^0, 1)/x_1} - 1$	

were P_1^0/Q_1^0 , the package sizes would be efficient at points *A* and *G*. The profit would equal $2P_1^0/Q_1^0$; but this is less than P_2^0/Q_2^0 (the profit from selling only to the high-value customers), so the profit-maximizing choice is to offer only point *B*, $(Q_2^0, P_2^0/Q_2^0)$. The low-value customers will be forced out of the market even though their willingness to pay exceeds the cost of providing the good. This loss is greater than the inefficiency created by price discrimination.

In figure 5 the valuation of the low-value consumer is higher than in

TABLE 5 Examples of Welfare Reduction and Improvement from Uniform Unit Price Regulation

Welfare Reduction	Welfare Improvement
Consumer parameters:	
$x_1 = 1, \quad U(x_1, 1) = 3.0, \quad U(x_2, 2) = 3.44$ $t_1 = 2.0, \quad t_2 = 1.0, \quad h_1 = h_2 = 1.0,$ $\alpha_1 = .5$	$x_1 = 1, \quad U(x_1, 1) = 3.5, \quad U(x_2, 2) = 4.0$ $t_1 = 2.0, \quad t_2 = 1.0, \quad h_1 = h_2 = 1.0,$ $\alpha_1 = .5$
Solution values with price discrimination:	
$Q_1^* = 3.16, \quad P_1^*/Q_1^* = 0.79, \quad V(Q_1^*, 1) = 0.79$ $Q_2^* = 1.41, \quad P_2^*/Q_2^* = 1.27, \quad V(Q_2^*, 2) = 2.03$ $S_1 = .0, \quad S_2 = .76, \quad \pi = 2.06$ Total welfare: $S_1 + S_2 + \pi = 2.82$	$Q_1^* = 3.16, \quad P_1^*/Q_1^* = 1.29, \quad V(Q_1^*, 1) = 1.29$ $Q_2^* = 1.41, \quad P_2^*/Q_2^* = 1.77, \quad V(Q_2^*, 2) = 2.59$ $S_1 = .0, \quad S_2 = .82, \quad \pi = 3.06$ $S_1 + S_2 + \pi = 3.88$
Solution values with uniform unit price regulation:	
Q_1^0 not offered $Q_2^0 = 1.41, \quad P_2^0/Q_2^0 = 2.03, \quad V(Q_2^0, 2) = 2.03$ $S_1 = .0, \quad S_2 = .0, \quad \pi = 2.03$ Total welfare: $S_1 + S_2 + \pi = 2.03$	$Q_1^0 = 2.00, \quad P_1^0/Q_1^0 = 1.50, \quad V(Q_1^0, 1) = 1.50$ $Q_2^0 = 1.41, \quad P_2^0/Q_2^0 = 1.50, \quad V(Q_2^0, 2) = 2.59$ $S_1 = .0, \quad S_2 = 1.09, \quad \pi = 3.0$ $S_1 + S_2 + \pi = 4.09$

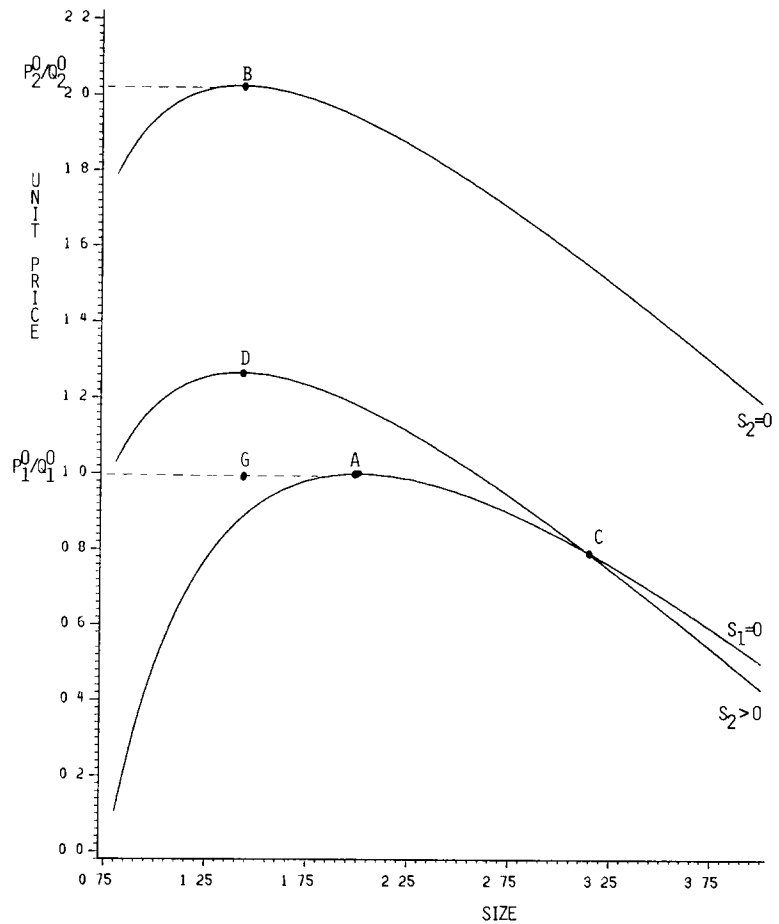


FIG. 4

figure 4, so the profit-maximizing offers are at points A and G, $(Q_1^0, P_1^0/Q_1^0)$ and $(Q_2^0, P_1^0/Q_1^0)$. Both customers are given their efficient-sized package, and this is a social improvement on the price discrimination equilibrium.

VII. Comparative Statics

Keeping results A–E in mind, consider the response of the relative package size $Q^* = Q_1^*/Q_2^*$ and the unit price difference to changes in the four relative parameters t , h , x , and α . Table 6 summarizes the signed response to parameter changes for the case in which self-selection occurs (see App. Sec. F for derivation). Care must be taken in interpreting these results because finite changes in parameters may

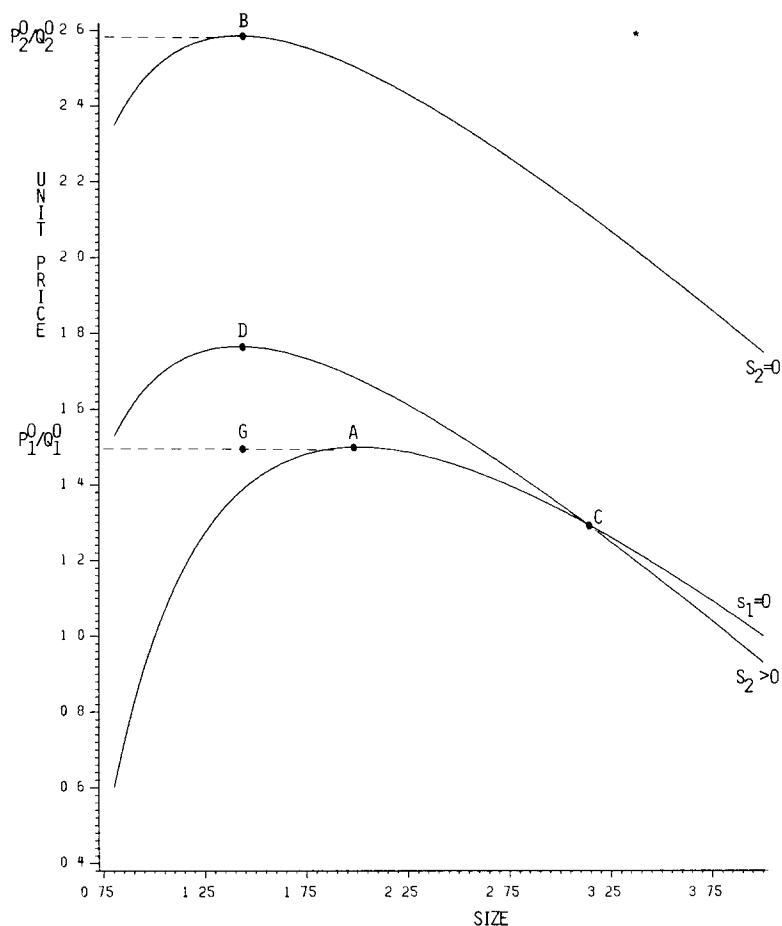


FIG. 5

move the solution from a region in which two packages are offered to one in which only one package is available.

The intuition for these responses is based on the following reasoning. If the seller could prevent self-selection and sell to each consumer group in a different market, the ratio between the sizes sold to the different customers would be $Q^0 = Q_1^0/Q_2^0 = (tx/h)^{1/2}$. As tx/h diverges from one, greater consumer heterogeneity leads to a larger degree of package-size dispersion in the markets. The first three columns of table 6 show that the same logic applies when self-selection is possible. To see this, assume that the ratio tx/h is less than one, which implies through (16) that $Q^* < 1$. An increase in either t or x or a decrease in h would contribute to an increase in the crucial ratio tx/h , bringing it

TABLE 6 Package Size and Unit Price Responses

Variable	Parameter			
	t	x	h	α
Q^*	+	+	-	-
$P_2^*/Q_2^* - P_1^*/Q_1^*$	$\text{sign}(tx/h - 1)^*$	$\text{sign}(tx/h - 1)^*$	$-\text{sign}(tx/h - 1)^*$	$-\text{sign}(Q^* - 1)$

* True only for changes in consumer 1's parameters.

closer to one. This represents a decrease in the "relevant" consumer heterogeneity, and Q^* increases to diminish the package size dispersion, as shown in the first row in table 6. The second row shows, as expected, that a smaller unit price dispersion is created. When the ratio tx/h exceeds one, increases in the parameters t , x , and $1/h$ represent greater consumer heterogeneity, and the difference in unit prices will increase to reflect this. If tx/h is less than one, the changes represent less consumer heterogeneity, and the unit price difference will decrease.

To understand the response to an increase in the relative population proportion α , first consider the upper entry in the last column of table 6. As α increases, selling to type 1 customers is relatively more attractive for the seller, and intuition suggests that the size offered to these customers should approach the efficient size Q_1^0 . Expression (16) implies that, when Q_1^* gets closer to Q_1^0 , it also gets closer to Q_2^* . Therefore, size variation is smaller when α increases, as shown in the upper entry.

Consider now the lower entry of the fourth column. As shown by the upper entry, Q_1^* converges to Q_2^* when α increases. This reduction in size difference makes Q_1^* more attractive to type 2 consumers, and to prevent them from self-selecting Q_1^* , the unit price of Q_2^* must be decreased relative to the price of Q_1^* , as shown by the lower entry.

Two implications are clear from table 6.

RESULT K. Greater size variation implies larger unit price variation, a hypothesis tested in Section VIII.

RESULT L. If consumers differ only in their storage costs, then the seller should respond to larger differences in storage costs by offering larger quantity premiums. If customers differ only in their transaction costs, then the seller should respond to larger differences in transaction costs by offering larger quantity discounts.

When there are only storage cost differences, premiums are offered. If $h > 1$, then an increase in h is an increase in consumer heterogeneity, and larger premiums are justified. If $h < 1$, then an increase in h is a reduction in heterogeneity, and the premiums should be reduced. Similar reasoning applies to changes in transaction costs.

TABLE 7 Regression of Percentage Size Increases on Percentage Unit Price Changes

	472 Brands %	Elasticity of Size Increase	<i>t</i> -Value	<i>R</i> ²
Discount	91.5	.084	21.9	.53
Premium	7.2	.107	4.14	.34

VIII. Empirical Results

To test some of the model's implications, consider first the hypothesis implied by table 6 and expression (13) that larger size variation implies larger unit price variation. We used the North Carolina supermarket data discussed in the introduction and regressed the table 1 percentage size increases separately on unit price discount percentages and unit price premium percentages. The intercept of the regression was suppressed since zero size variation obviously results in zero unit price variation. Table 7 summarizes the regression results. The positive and significant *t*-values and the high *R*² indicate that larger size variation does imply larger unit price variation.

Next, consider expression (19), the sufficient condition that excludes multipackage purchases. As mentioned above, the condition becomes relevant when quantity premiums prevail. Therefore, if the condition is empirically valid, we should observe that it holds more frequently for cases in which premiums occur than for cases in which discounts occur. Table 8 gives the percentage of products for which the size ratio was less than the crucial value 1.41 in each case. Since expression (19) is just a sufficient condition, crucial values of 1.50 and 1.60 are also given as approximations. The *t*-statistics for the differences in proportions show that the proportions for premiums are significantly larger than for discounts at the 10% level. Therefore, it is unlikely that such observed differences in proportions would occur if the true proportions were identical.

Finally, to test some of the model's assumptions, we conducted a survey of 263 respondents (mostly students and secretaries). The sur-

TABLE 8 Percentage of Products with Size Ratio Less than the Crucial Value

	Crucial Value		
	1.41	1.50	1.60
Discount (%)	3.4	16.0	20.6
Premium (%)	8.8	23.5	38.2
<i>t</i> -ratio	1.56*	1.14	2.39**
<i>p</i> -value	.06	.13	.01

* Significant at 10% confidence.

** Significant at 5% confidence.

vey data were used to determine the importance of storage and transaction cost trade-offs (described by the EOQ model) to consumers when making decisions about package sizes and to observe heterogeneity in consumer storage and transaction costs. The survey was also used to test our explanation for quantity premiums, which, unlike the Salop (1977) theory, relies on perfect information. We asked survey respondents to answer the following questions.

When you do your shopping and have to choose among several sizes offered of a given brand:

- (1) How often do you compare package sizes and unit prices (price per ounce, pound, etc.) to help you choose the best buys?
☐ Never ☐ Sometimes ☐ Often
- (2) How important is it to you that the size would be large enough so you would not have to go to the store too often?
☐ Never important ☐ Sometimes important
☐ Often important
- (3) How important is it to you that the size would be convenient for storage?
☐ Never important ☐ Sometimes important
☐ Often important
- (4) Have you ever bought a larger size even when you noticed that this size had a higher unit price (price per ounce, pound, etc.)?
☐ Never ☐ Sometimes ☐ Often

The results, summarized in the first three rows of table 9, indicate that storage and transaction costs trade-offs are important to consumers in making package size decisions. The results also reveal consumer heterogeneity in storage and transaction costs. Finally, the first row shows that 61.2% of the respondents often compare package sizes and unit prices to help determine best buys, and the fourth row shows that 45.2% sometimes buy a larger size even when there is a unit price premium. This result is consistent with our claim that quantity premium can prevail even with perfect information.

TABLE 9 **Frequencies of Survey Responses**

Question	Never	Sometimes	Often
1	4 (1.5)	98 (37.3)	161 (61.2)
2	12 (4.6)	157 (59.7)	94 (35.7)
3	69 (26.2)	127 (48.3)	67 (25.5)
4	144 (54.7)	112 (42.6)	7 (2.6)

Note.—236 respondents. Numbers in parentheses are percentages.

IX. Issues for Future Investigation

Consider now some of the model's limitations and implications for future research. We assumed a monopolist sells a single product to customers who consume the product at constant rates per period and make trips to the store each time they run out of the product. We also assumed that the seller can limit to one the number of packages he sells to a customer each visit to the store and that bulk breaking and repackaging for the purpose of arbitrage is too costly. Finally, we assumed that the seller's production and selling costs are zero.

The structure of the model does not consider the following.

a) Consumers most often buy more than one product in each store and might visit more than one store on each shopping trip. Stores and shopping centers carry a variety of merchandise and brands to help consumers economize on their trips to the market, and when going to a store for one product, customers might "piggyback"; that is, they might purchase other needed products at the same time. As a result, some authors (e.g., Jeuland and Narasimhan 1985) assume that consumers have zero transaction costs.

There are products, however, such as fresh milk, bread and buns, and certain meats, that seem to play a major role in determining shopping schedules. For these types of items, transaction cost is a major consideration. Moreover, an interpretation of t_i different than transaction cost might be given: stock-out cost. This is the opportunity lost when the inventory at home is used up prior to the next shopping trip and the consumption rate is temporarily forced to zero. There is an inverse relation between package size and these stock-out costs, so they lead qualitatively to results similar to those obtained under the EOQ model.

b) Significant unit price variation might affect consumption rates. This is not considered in our model of fixed per-period consumption rates, although it can be argued that this is the limiting case of a more general model. It ought to be easy to parameterize the price elasticity of consumption rates and to conclude that continuity implies there is a range of behavior that incorporates price sensitivity yet exhibits conclusions similar to those found here.

c) While we have shown that for a range of parameters consumers will buy a single package, outside this range the consumers may be tempted to buy a few small packages instead of the large size. If the seller cannot impose a one-per-customer restriction, multiple purchases must be taken into account.

d) Quantity discounts can open up the possibility of profitable arbitrage by a repackager. Not only might firms buy in bulk and repackage, but consumer coalitions might also share large packages to take advantage of quantity discounts.

e) Considerations of production and selling costs might dominate demand-related considerations of consumer heterogeneity in decisions about package sizes and unit prices.

f) Competition would limit the seller's ability to extract surplus from consumers through packaging.

Even given these limitations, the model developed here has some attractive features. Most obviously, the model allows explanation of some important stylized facts of product packaging and pricing in a reasonably simple fashion. The model can be a good approximation for product markets that are less sensitive to the limitations. For product markets more sensitive to the assumptions, the model can serve as a first approximation. The major parameters of the model are potentially observable, and the conclusions are sharp, providing some clear testable hypotheses. Future research should focus on relaxing the above limitations.

X. Conclusions

Standard models of supply and demand pretend that goods can be purchased in infinitely divisible amounts. In fact, many goods are pre-packed in containers that come in only a few sizes. Empirical research has shown that package sizes and unit prices vary significantly across brands and stores. This paper has analyzed the source of variation among package sizes and unit prices using diversity in buyers' consumption rates, storage costs, and transaction costs as the principal determinants rather than buyers' misperceptions of unit prices or differences in production or packaging costs.

In our paper, package size variety is not created to fool customers but to allow them to trade off storage costs for shopping costs, and prices are set to induce self-selection by the appropriate consumers. By offering several package sizes at different unit prices, customers are automatically sorted into market segments. This allows the seller to extract more consumer surplus and to earn higher profits. When only transaction costs differ (e.g., when only the opportunity cost of consumers' time varies), quantity discounts are most profitable. Quantity premiums will prevail when only storage cost differences (such as availability of refrigeration) induce variations in package sizes. Our empirical results are consistent with the model's implications.

Appendix

1. *Derivation of optimal package size.* Rearranging (6) to solve for the revenue from a sale to consumer 2, we get

$$P_2 x_2 / Q_2 = P_1 x_2 / Q_1 + t_2 x_2 (1/Q_1 - 1/Q_2) + h_2 (Q_1 - Q_2) / 2. \quad (A1)$$

Substituting $(P_1x_1/Q_1)/x$ for the first term on the right-hand side of (A1), the total profit can be written

$$\pi = (P_1x_1/Q_1)(\alpha_1 + \alpha_2/x) + \alpha_2t_2x_2(1/Q_1 - 1/Q_2) + \alpha_2h_2(Q_1 - Q_2)/2. \quad (\text{A2})$$

From (5), the revenue from sales to a consumer of type 1 equals

$$P_1x_1/Q_1 = U(x_1, 1) - t_1x_1/Q_1 - h_1Q_1/2. \quad (\text{A3})$$

Substituting (A3) into (A2) so that profits are expressed entirely in terms of quantities and then setting the derivatives with respect to Q_1 and Q_2 equal to zero, the first-order conditions are

$$(t_1x_1/Q_1^2 - h_1/2)(\alpha_1 + \alpha_2/x) - \alpha_2t_2x_2/Q_1^2 + \alpha_2h_2/2 = 0, \quad (\text{A4})$$

and

$$\alpha_2t_2x_2/Q_2^2 - \alpha_2h_2/2 = 0. \quad (\text{A5})$$

Solving for Q_1 and Q_2 gives equations (11) and (12).

Second-order conditions will be satisfied if the parameters x , t , h are all positive and $(\alpha x + 1)t > 1$.

2. *Derivation of unit price difference.* Divide both sides of (A1) by x_2 and rearrange so that

$$P_2/Q_2 - P_1/Q_1 = t_2(1/Q_1 - 1/Q_2) + h_2(Q_1 - Q_2)/(2x_2). \quad (\text{A6})$$

From the formula (12) for the optimal value of Q_2^* , it is easy to see that $2x_2 = h_2Q_2^{*2}/t_2$. Substituting this into the second term on the right-hand side of (A6) and simplifying gives (13).

3. *Conditions for two different packages to be offered.* When the only offer made is $(Q_1^0, P_1^0/Q_1^0)$, both consumers will accept the offer since by assumption (3) high-value consumers like any offer more than low-value consumers and by construction low-value consumers get zero surplus from this offer. Think of this as two identical offers, one to type 1 and one to type 2 consumers. Since these offers satisfy the surplus and self-selection constraints, the two different offers $(Q_1^*, P_1^*/Q_1^*)$ and $(Q_2^*, P_2^*/Q_2^*)$ must produce by definition at least as much profit and strictly more profit except in the case in which $tx/h = 1$, when from (11) $Q_1^* = Q_2^*$. On the other hand, if the seller ignores the low-value consumers and sells only to high-value consumers, the most profitable offer is $(Q_2^0, P_2^0/Q_2^0)$. The two offers, $(Q_1^*, P_1^*/Q_1^*)$ and $(Q_2^*, P_2^*/Q_2^*)$, generate higher profit than the offer just to high-value customers when

$$\alpha_1x_1P_1^*/Q_1^* + \alpha_2x_2P_2^*/Q_2^* > \alpha_2x_2P_2^0/Q_2^0. \quad (\text{A7})$$

A necessary and sufficient condition for the two package strategy to be optimal can be derived as follows. From the surplus and self-selection constraints it must be that

$$P_2^0/Q_2^0 = V(Q_2^0, 2)/x_2, \quad (\text{A8})$$

$$P_1^*/Q_1^* = V(Q_1^*, 1)/x_2, \quad (\text{A9})$$

and

$$V(Q_2^*, 2) - x_2P_2^*/Q_2 = V(Q_1^*, 2) - x_2P_1^*/Q_1^*. \quad (\text{A10})$$

Substituting (A9) into (A10) and rearranging gives

$$P_2^*/Q_2^* = V(Q_2^*, 2)/x_2 + V(Q_1^*, 1)/x_1 - V(Q_1^*, 2)/x_2. \quad (A11)$$

Comparing (9) and (12) it is clear that $Q_2^* = Q_2^0$, so (A8) can be substituted into (A11) to get

$$P_2^*/Q_2^* = P_2^0/Q_2^0 + V(Q_1^*, 1)/x_1 - V(Q_1^*, 2)/x_2. \quad (A12)$$

When inequality (A7) is divided by $\alpha_2 x_2$ and rearranged, it becomes

$$\alpha x > (P_2^0/Q_2^0 - P_2^*/Q_2^*)/(P_1^*/Q_1^*). \quad (A13)$$

Substituting (A9) and (A12) into (A13) gives the desired inequality (15).

4. *Conditions for single package purchases.* From the definition of transaction and storage costs (1) and the formula for efficient package sizes (9), we can write

$$C(sQ, i) = h_i s Q / 2 [(Q_i^0 / s Q)^2 + 1]. \quad (A14)$$

From the self-selection condition (6) at the optimum we know that

$$P_2^*/Q_2^* - P_1^*/Q_1^* = \{h_2 Q_1^* / 2 [(Q_2^0 / Q_1^*)^2 + 1] - h_2 Q_2^* / x_2\}. \quad (A15)$$

Substituting (A14) and (A15) into (18) and cross-multiplying by x_2/h_2 gives

$$Q_1^* / 2 [(Q_2^0 / Q_1^*)^2 + 1] - Q_2^* \leq s Q_1^* / 2 [(Q_2^0 / s Q_1^*)^2 + 1] - Q_2^*. \quad (A16)$$

After algebraic rearrangement, (A16) becomes

$$(Q_2^0 / Q_1^*)^2 \leq s, \quad (A17)$$

for all $s = 2, 3, 4$. Condition (19) follows immediately from this and $Q_2^* = Q_2^0$.

We ought to remark that when the low-value customer is offered an inefficient package too small to minimize transaction/storage cost, the low-value customer might buy two of these packages. Condition (19) also rules this out. By (3), if the high-value customer is worse off buying two packages, the low-value customer is even less happy with such a purchase.

5. *Condition for relative efficiency of price discrimination.* The relative efficiency of price discrimination depends directly on whether the seller will respond to a requirement of equal unit prices by selling to both consumer types or just to the high-value customers. It is more profitable to sell to both when

$$(\alpha_1 x_1 + \alpha_2 x_2) P_1^0 / Q_1^0 > \alpha_2 x_2 P_2^0 / Q_2^0. \quad (A18)$$

Divide both sides of this by $\alpha_2 x_2$ and rearrange to get

$$\alpha x > P_2^0 / Q_2^0 / (P_1^0 / Q_1^0) - 1. \quad (A19)$$

From the zero surplus condition $P_i^0 / Q_i^0 = V(Q_i^0, i) / x_i$, substituting this into (A19) results in the inequality of table 4.

6. *Responses of relative package sizes and unit prices.* From equations (11) and (12), it is easy to see that Q^* is increasing with t and x and decreasing with h . This gives the first three entries in the first row of table 6. To obtain the last entry, we differentiate (11) with respect to α to get

$$dQ^* / d\alpha = (h - tx)(1/2Q^*) / [(\alpha x + 1)h/x - 1]^2. \quad (A20)$$

From this it is clear that

$$\text{sign}(dQ^*/d\alpha) = -\text{sign}(tx/h - 1). \quad (A21)$$

From (A21) and (16), it is clear that, when α increases, Q_1^* approaches Q_1^0 and also Q_2^* .

The response of the difference in unit prices is best seen by rearranging (13) using the formula for the optimal package sizes (11) and (12) to get the following:

$$P_2^*/Q_2^* - P_1^*/Q_1^* = [(Q^* - 1)^2/Q^*] [(t_2h_2/2x_2)^{1/2}]. \quad (A22)$$

If we hold t_2 , x_2 , and h_2 constant, the only effect t_1 , x_1 , and h_1 can have is through the effect tx/h on $(Q^* - 1)^2/Q^*$. This is increasing in t_1x_1/h_1 if and only if $Q^* > 1$. Note that the effect of changes in the parameters of consumer 2 are ambiguous because of the second term of (A22). These parameters influence both package sizes.

The effect of α on the unit price difference is established by noting that (16) and (A21) imply that Q increases with larger α if and only if $Q < 1$. From (22), it is clear that the unit price difference becomes less with increases in Q if and only if $Q < 1$. Combining these gives the desired result.

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