Asset Pricing Implications and Estimation of a Multi-Sector General Equilibrium Model

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Abstract

Constructing a multi-sector production based asset pricing model, we analyze industry-level equity risk premium (ERP) and utilize general equilibrium moment restrictions to estimate salient parameters of recursive constant elasticity of substitution (CES) consumer preferences. Industry ERP is determined by exposure to volatilities of aggregate income and (CES) price index and their covariation, as well as the covariation of industry productivity and input prices with the aggregate factors. With reasonable risk aversion and low intertemporal elasticity of substitution (IES), the model can explain the observed ERP for (manufacturing) consumer durables and consumer non-durables sectors. The results also highlight the effects of sectoral demand price elasticity on the risk premium. General equilibrium restrictions and strong IVs from industry production data yield robust and reasonable estimates of risk aversion, IES and sectoral elasticity of substitution (ES). Industry restrictions and IVs are critical for estimation efficiency, but asset market restrictions and IVs enhance identification.

Keywords: Asset pricing, multi-sector, industry equity premium, efficient estimation, moment conditions, instrumental variables

1 Introduction

Asset pricing at the *industry* (or sectoral) level—in particular, the industry equity risk premium (ERP)—is of long-standing interest in the literature (e.g., Fama and French 1997).[1] Furthermore, a large literature estimates risk aversion and the intertemporal elasticity of substitution in consumption (IES) (e.g., Hansen and Singleton 1982, 1983, Epstein and Zin 1991) using data on aggregate (or market level) asset returns. At the microeconomic level, however, agents choose consumption bundles (or baskets) of various types of consumption goods produced by firms in different sectors. Received microeconomic theory addresses consumption baskets in general equilibrium (e.g., Arrow and Hahn 1971), and in more parameterized form through constant elasticity of substitution (CES) preferences (e.g., Dixit and Stiglitz 1977). But equity value maximization by firms implies that the production decisions of public firms reflect the preferences of their equity owners. Hence, in a realistic multi-sector context, parameters describing consumer preferences should be reflected not only in aggregate consumption and asset returns but also in industry-specific product and asset markets related information,[2] potentially enhancing empirical identification. In this paper, we construct a multi-sector general equilibrium model with production and securities trading to study industry-level ERP, and employ a new approach to estimation of parameters governing consumer preferences by using conditional moment restrictions from the joint clearing of product and asset markets.

We theoretically and empirically analyze an infinite-horizon multi-sectoral model with multiple competitive sectors, each producing a different basic good. Firms in each industry are homogenous and use an industry-specific production technology that is subject to random sectoral productivity shocks and stochastically converts capital and materials inputs to output. Firms are unlevered and widely-held. The representative consumer is endowed with Epstein and Zin's (1989) recursive preferences and every period simultaneously chooses the consumption basket and portfolio investment in stocks and a risk free asset. Firms combine intermediate goods to form their composite investment and materials input every period to maximize the present discounted expected value of real dividends.

We then derive restrictions on the representative consumer's consumption-portfolio investment and

[1] Throughout, we will use industy-level (or sectoral) ERP to mean the ERP of a representative firm in the industry.

[2] This applies to general equilibrium production models with complete markets (Cochrane 1991, Jermann 1998) or with incomplete markets (Horvath 2000). More generally, even with agency problems due to separation of ownership and control (Berle and Means 1932, Jensen and Meckling 1976), the preferences of equity owners are not irrelevant for managers.

firms' capital investment and input policies, along a dynamic competitive general equilibrium path with simultaneous clearing of product and asset markets. While our multi-sector model considers consumer goods as well as intermediate goods sectors, for expositional parsimony we focus on consumer goods industries to facilitate comparisons with the received asset pricing and applied economics literatures. We utilize these restrictions on product and equity markets in two ways.

First, we quantitatively analyze industry-level ERP through (log-linear) analytic approximation of the equilibrium path. In doing so, we derive a linear factor model where the risk factors and factor loadings are endogenously generated: Industry ERP is determined by exposure of firms' equity payoffs to volatilities of aggregate income and price index and their covariation, as well as the covariation of industry-level productivity shocks and input prices with aggregate shocks. We analyze the ERP separately for the consumer durables and non-durables manufacturing sectors because the role of durability is emphasized in the literature (Yogo 2008, Gomes, Kogan and Yogo 2009).

Second, we use these restrictions to form moment conditions in generalized method of moments (GMM) estimation of salient parameters related to consumer preferences—namely, risk aversion, intertemporal elasticity of substitution (IES) and the *intratemporal* elasticity of substitution (ES)—in the manufactured consumer goods sector.[3] While estimates of risk aversion and IES are of substantial interest in the asset pricing literature, estimation of sectoral ES is the focus of a long-standing literature in economics (e.g., Feenstra 1994, Broda and Weinstein 2006, Redding and Weinstein 2017).

For these analyses, we need industry data on capital, investment, materials inputs, and industry productivity. We obtain data on consumer goods manufacturing industries from the NBER-CES database of U.S. manufacturing industries; the latest data available are for 1958-2016 (annual). We use standard sources for National Income Product Accounting (NIPA) aggregate income and consumption data, and for financial returns data.

As is well known from the macrofinance literature (Mehra and Prescott 1985 and onwards), it is challenging to reconcile the observed *aggregate* ERP with single-good, Markov equilibrium asset pricing models under plausible values of risk aversion—that is, the equity premium puzzle (EPP). For our analysis of sectoral ERP, we calibrate the aggregate moments of the model with NIPA data and

[3] In a similar spirit, Liu, Whited and Zhang (2009) use restrictions on expected stock returns and expected investment returns to estimate adjustment costs and capital share (under constant returns to scale) on the production side.

parameterize the subjective discount factor, depreciation, and adjustment costs consistent with the literature. The sectoral productivities and input costs are calibrated to match the sample mean output, materials inputs and capitals stocks in the model's steady state. Meanwhile, the literature generally considers risk aversion up to 10 as reasonable (e.g., Mehra and Prescott 1985). We use risk aversion levels of 5 (the risk aversion estimate from our sample), 7.5 and 10—since the latter are used in literature (e.g., Bansal and Yaron 2004). Similarly, there is no consensus in the literature regarding the IES—with estimates ranging from 0 to 2 (see Best et al. 2020). Similarly, there are a range of estimates in the literature for the sectoral ES (Feenstra 1994, Broda and Weinstein). We therefore use GMM estimates from our sample that cluster around 0.2 for IES and 2 for the ES. These estimates are consistent with estimated ranges in the received literature. We calibrate the industry taste parameter (in the CES preferences) based on the percentage of income allocated to consumer manufactured goods in highest income decile countries (Duarte and Reusticca 2016, Duarte 2018).

The model generates sizeable unconditional ERP for both consumer durables and consumer non-durables sectors with reasonable levels of risk aversion and low levels of IES. Specifically, it matches the sample mean annual ERP in consumer durables sector of 6.71% for risk aversion between 7.5 and 10. In the consumer non-durables sector, as well, the model matches the mean annual ERP of 10.05% with risk aversion between 7.5 and 10.[4] As a benchmark, the model-generated ERP of the aggregate asset that pays the equilibrium consumption basket each period is very low compared to the sample mean of the market ERP: Thus, the EPP continues to hold in our framework for an aggregate asset with passive (consumption basket) payoff.

The relatively high *industry* ERP generated by our model—in comparison with the benchmark aggregate asset—arises from four sources: (1) greater sensitivity of firms' equity payoffs to aggregate risk factors—namely, aggregate income and aggregate (CES) price index—because of endogenous production and capital investment; (2) effects of the interaction between the aggregate risk factors; (3) contributions to the risk premium by the covariation between industry-level and aggregate risk factors; and, (4) effects of sectoral ES (or, equivalently, sectoral demand price elasticity) on the risk premium.

To explicate, we recall that consumer optimum with CES preferences implies that $C_t = \frac{W_t}{P_t}$, where

[4] Bansal and Yaron (2004) and Croce (2014) generate relatively high values of aggregate ERP with reasonable values of risk aversion in models with long-run risk. But they require the IES to exceed 1.

C_t is the aggregate consumption basket, W_t is the aggregate income and P_t is the (CES) aggregate price index.[5] But the aggregate consumption basket is not a sufficient statistic for income and the price index for *industry* product demand. Specifically, for an industry j with output Y_t^j, the inverse demand function $\omega_t^j(Y_t^j)$ is proportional to $(W_t/P_t)^{1/\sigma}\,P_t\left(Y_t^j\right)^{-1/\sigma}$, where σ is the elasticity of substitution (ES). Hence, the industry demand curve shifts outward with exogenous increases in aggregate income *or* the price index—that is, C_t does not subsume the effects of the aggregate risk factors on industry fundamentals. Consequently, industry firms respond optimally to variations in these aggregate risk factors. Consistent with the data, our model generates production and investment that are positively related to the aggregate risk factors. In particular, equity payoffs (dividends plus capital gains) are also positively related to the aggregate factors. Hence, the risk premium (SDF) is affected by volatilities of aggregate income and price index, as well as their *covariation*. For the standard reasons, the loading on aggregate income volatility is positive. Conversely, the loading on aggregate price index volatility is negative since index shocks are positively related to the stochastic discount factor (SDF) and equity payoffs (due to the industry demand effect). In addition, because of the cyclical behavior of equity payoffs, the positive covariation of W_t and P_t in the data adds to the industry risk premium.

The risk premium is also affected by the covariation between *industry-level* production related factors—namely, sectoral productivity shocks and intermediate goods prices—with aggregate income and price index. The covariation between sectoral *input* prices and the aggregate price index has a positive risk premium because higher production and investment costs tend to coincide with higher P_t, when shareholders' utility cost of lower profits is higher. Conversely, the loading on the covariation of input prices and aggregate income is negative. Meanwhile, the covariation between sectoral productivity and aggregate income amplifies the procyclicality of equity payoffs and hence the risk premium, whereas the loading on the covariation of aggregate price index and sectoral productivity is negative because higher productivity raises equity payouts in high marginal utility regimes.

Furthermore, in our multi-sector model, the industry-level ERP depends not only on the discount factor, risk aversion and IES—as in the canonical model—but also on the sectoral ES and taste parame-

[5] Consistent with the CES model, we derive the CES price index as $P_t = W_t/C_t$, using NIPA income and consumption data. Hence, by construction, the variance and covariance of log changes in aggregate income and price index utilized in our analysis imply the (annual) volatility of log per capita consumption growth in the data. In sum, our results are not driven by "large" values of implied consumption growth volatility.

ters. In particular, ES and ERP are negatively related (in the region around our structural estimates). Quantitatively, the ERP can be quite high for reasonable risk aversion levels and low IES over a range of ES values; conversely, the ERP can be low even with relatively high risk aversion for relatively high ES. More generally, because the sectoral ES measures the demand price elasticity of the good, the sectoral ERP is significantly influenced by the price elasticity of the industry good.

There is a large literature that modifies the canonical aggregate asset pricing model to help resolve the EPP. One strand of this literature argues that consumption volatility measured from NIPA may not correctly represent the actual consumption risk faced by investors. For example, Ait-Sahalia et al. (2004) argue that NIPA consumption weights focus on basic consumption goods. By incorporating luxury good consumption through nonhomothetic preferences, they are able to explain observed equity premium at relatively low levels of risk aversion: These results are consistent with our analysis because differentiated goods have lower price elasticity compared to basic goods or commodities (Berry, Levinsohn and Pakes, 1995; Broda and Weinstein, 2006), and as mentioned above, we find a negative relation of ERP and elasticity. Meanwhile, Parker and Juillard (2005), Savov (2011) and Kroencke (2017) point out other sources of mis-measurement of consumption risk. Another strand of the literature emphasizes the role of long run risks in exchange and production economies (Bansal and Yaron 2004, Croce 2014). Our analysis focuses on the ERP at the industry-level in multi-sector production economies with investment and finds that the EPP need not hold because of sectoral risk factors. Our results also indicate that the *composition* of the aggregate consumption basket—in terms of the weighted average demand price elasticity—matters in the measurement of consumption risk.

Our estimation analysis uses GMM with heteroskedasticity- and autocorrelation-consistent (HAC) inference. An important aspect of industry production data is the high levels of own and cross-autocorrelations in the endogenous variables (investment and material inputs). This is consistent with the literature that highlights the high short run predictability of capital investment (Eberly, Rebelo and Vincent 2012). Thus, in contrast to the well known weak IV problem with asset returns and consumption growth (Stock and Wright 2000), using lagged industry production inputs as IVs allows stronger correlation with the optimality conditions related to firms' investment and input choices and, hence, a potential for stronger identification of the structural parameters. We find that adding restrictions from industry production equilibrium paths and using strong instrument variables (IVs) offered by highly autocorrelated industry variables substantially enhance identification of structural parameters describing consumer preferences, relative to benchmark estimations using equity returns data alone.

The GE estimation yields strong identification of the three parameters (risk eversion, IES and ES): The point estimates are statistically highly significant for all moment conditions (that is, choice of IVs); are not widely dispersed across different specifications (or IVs); and are economically appealing. The risk aversion estimate is clustered around 5, well within the range of risk aversion values considered reasonable in the literature (Mehra and Prescott 1985). The IES estimates cluster around 0.2 and the tests reject the null of zero IES at probability values below 0.01. As we mentioned above, the literature reports a wide range of estimates for IES. A long-standing literature finds IES in the low range using aggregate data (Hall 1988, Campbell 1999). More recently, using a novel structural estimation approach, based on "mortgage notches" in the UK, Best et al. (2020) also report estimates of IES around 0.1, while a meta-analysis of IES estimates in the literature finds estimates clustered between 0.3-0.4 (Havránek 2015). However, another strand of the literature reports estimates of IES in excess of 1 (see Bansal and Yaron 2004). Meanwhile, our estimates of the intratemporal elasticity of substitution (ES) for manufactured consumption goods cluster around 2. These estimates significantly exceed 1, which is the requirement of the theoretical model. In addition, the point estimates are consistent with the estimates in the literature for manufactured goods using import data (e.g., Broda and Weinstein 2006).

It is useful to examine the contributions of the intertemporal capital investment and intratemporal materials input Euler conditions and production data to identification in the GE model. We analyze parameter identification by using various combinations of the production and asset market Euler conditions, as well as using only industry production as instruments. We find that identification of the three parameters (risk aversion, IES and ES) deteriorates if we "eliminate" either of the industry production Euler conditions. Consistent with intuition, our analysis indicates that the intertemporal investment Euler condition contributes most significantly to the identification of risk aversion and IES, whereas both the intratemporal materials input condition and investment Euler conditions contribute to the identification of ES. However, while the role of industry level production IVs is critical, the asset return IVs also improve identification.

To our knowledge, this is the first study to utilize a multi-sector general equilibrium production-based asset pricing model to analyze industry-level ERP and estimate consumer preferences related parameters.[6] This realistic setting can generate relatively high industry ERP with reasonable consumer preferences and production technology parameterization. In particular, we quantitatively highlight the role of intratemporal ES in the determination of equilibrium industry ERP. Furthermore, the multi-sector general equilibrium setting allows us to use production-based conditional moment restrictions and exploit high serial correlation in production data at the industry level in structural estimation. We find that jointly using information in securities trading and the production decisions of public firms enhances identification of structural parameters.

2 A Multi-Sector General Equilibrium Model

The economy consists of J competitive sectors (or industries), each producing a different good. Sectors are partitioned into J_c sectors that produce (final) consumption goods and sectors that produce two types of intermediate goods: J_h sectors that produce material inputs for production and J_k sectors that produce capital inputs.

2.1 Consumption and Portfolio Investment

There is a continuum of identical consumer-investors (CI) in the economy; the number of CIs is normalized to unity, without loss of generality. The (representative) CI's income each period comprises of dividend payouts from firms, net changes in the value of her/his stock portfolio, and income from a riskless security. At each t, the CI chooses the consumption vector $\mathbf{c}_t = (c_{1t}, ...c_{Jc,t})$, taking as given the vector of consumption good prices $\mathbf{p}_t^c = (1, ..., p_t^{J_c})$; that is, the first consumption good serves as the numeraire and its price is normalized to unity each period. The CI also has access every period to a (one-period) risk-free security (f) that pays a unit of the numeraire good next period. The mass of risk-free securities is fixed at unity. The CI's asset holdings at the beginning of the period are denoted by the $(J+1)-$dimensional vector $\mathbf{q}_t = (.., q_{nt}^j, .., q_t^f)$. Along with consumption, the CI simultaneously chooses her/his new asset holdings $\mathbf{q}_{t+1}$, taking as given the corresponding (ex-dividend) asset prices $\mathbf{s}_t$. The dividend payouts per share are denoted by $\mathbf{d}_t$ (with $d_t^f \equiv 1$).

[6] There is a long-standing literature on multi-sector models in production economies (e.g., Samuelson and Solow 1956, Johansen 1960) that studies computable general equilibrium models (e.g., Shoven and Whalley 1992) or aggregate output volatility (e.g., Horvath 2000).

The representative CI has Epstein and Zin (1989) preferences over intertemporal streams of consumption bundles $\{C_t\}_{t=1}^{\infty}$ that are expressed in recursive form as

$$\mathcal{U}_t = \left[(1-\alpha)C_t^{1-\eta} + \alpha\mathbb{E}_t\left[\mathcal{U}_{t+1}^{1-\gamma}\right]^{\frac{1-\eta}{1-\gamma}}\right]^{\frac{1}{1-\eta}}, \tag{1}$$

when $\gamma \neq 1$ and $\eta \neq 1$. In (1), $C_t \equiv C(\mathbf{c}_t)$ is an aggregated consumption index with constant elasticity of substitution (CES) among consumption goods, that is,

$$C(\mathbf{c}_t) = \left[\sum_{j=1}^{J_c} \phi^j (c_t^j)^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}, \tag{2}$$

where $\sigma > 1$ is the ES and $0 < \phi^j < 1$ are the utility weights; α controls the subjective rate of impatience; γ determines the degree of risk aversion; and η^{-1} measures the intertemporal elasticity of substitution (IES) over consumption baskets.

For model simplicity and tractability, the CI is assumed to have only dividend income.[7] Hence the CI's budget constraint is given by

$$\mathbf{p}_t^c \cdot \mathbf{c}_t \leq \mathbf{q}_t \cdot (\mathbf{d}_t + \mathbf{s}_t) - \mathbf{q}_{t+1} \cdot \mathbf{s}_t \equiv W_t. \tag{3}$$

Because preferences are strictly increasing, the budget constraint (3) will be binding in any optimum and hence W_t also represents the total consumption expenditure at t. In the standard fashion for Dixit and Stiglitz (1997) preferences, intratemporal optimization yields the consumption demand functions (see the Online appendix)

$$c_t^j(\mathbf{p}_t^c, W_t) = \frac{W_t}{P_t}\left[\frac{P_t\phi^j}{p_t^j}\right]^{\sigma}, \; j = 1, ..J, \tag{4}$$

[7] Our purpose in this study is to develop a multi-sector model for quantitative analysis of industry ERP and demonstrate the value of using general equilibrium conditions for the estimation of consumer preference related parameters. Considering endogenous labor income, for example, with leisure as a component of consumer preferences and labor as a factor of production significantly complicates the analysis and empirical implementation of the model. We discuss briefly the implications of non-dividend sources of income on the ERP in Section 4.3.

where $P_t \equiv P(\mathbf{p}_t^c)$ is the aggregate consumer price index

$$P(\mathbf{p}_t^c) = \left[\sum_{j=1}^{J_c} (\phi^j)^{\sigma} (p_t^j)^{1-\sigma} \right]^{1/(1-\sigma)}. \tag{5}$$

At the optimum, the aggregate real consumption $C_t \equiv C(\mathbf{c}_t) = \frac{W_t}{P_t}$, which is the real income.

Meanwhile, the CI's portfolio optimization equates the real current security price to expected present value of real equity payoffs next period. The real SDF (or pricing kernel) is the intertemporal marginal rate of substitution of real consumption (IMRS). Letting $\Lambda_t \equiv \frac{\partial \mathcal{U}_t}{\partial \mathcal{C}_t} = (1-\alpha) C_t^{-\eta} \mathcal{U}_t^{\eta}$ denote the marginal valuation at t, the SDF for the one-period investment horizon is $\Lambda_{t,t+1} \equiv \frac{\Lambda_{t+1}}{\Lambda_t}$. Using $C_t = \frac{W_t}{P_t}$ and following Epstein and Zin (1989), the real SDF can be written

$$\frac{\Lambda_{t+1}}{\Lambda_t} = \alpha^{\theta} \left(\frac{G_{t+1}^W}{G_{t+1}^P} \right)^{-\eta\theta} R_{C,t+1}^{\theta-1}, \theta \equiv \frac{1-\gamma}{1-\eta}, \tag{6}$$

where G_{t+1}^W and G_{t+1}^P are the gross growth rates in aggregate income and the price index between t and $t+1$, respectively, and $R_{C,t+1}$ is the gross one-period (real) return on an asset that pays aggregate consumption as its dividend.[8] Hence, asset prices satisfy

$$\frac{\mathbf{s}_t}{P_t} = \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\frac{\mathbf{d}_{t+1} + \mathbf{s}_{t+1}}{P_{t+1}} \right), \right] \tag{7}$$

which can be expressed in more conventional (or nominal terms) by defining the one-period ahead *nominal* SDF $M_{t,t+1} \equiv P_t \left(\frac{\Lambda_{t,t+1}}{P_{t+1}} \right)$. Since the one-period return between t and $t+1$ in each sector is $R_{t+1}^j = \left(\frac{d_{t+1}^j + s_{t+1}^j}{s_t^j} \right)$, and the return on the one-period nominally riskless bond is $R_{t+1}^f = (1/s_t^f)$, Equation (7) can be written as the restriction

$$\mathbf{1} = \mathbb{E}_t \left[M_{t,t+1} \mathbf{R}_{t+1} \right], \tag{8}$$

where $\mathbf{1}$ and $\mathbf{R}_{t+1}$ are the unit and gross nominal returns vectors, respectively.

[8] More precisely, $\frac{\Lambda_{t+1}}{\Lambda_t} = \left(\frac{G_{t+1}^W}{G_{t+1}^P} \right)^{\eta} \left(\frac{U_{t+1}}{\mathbb{E}_t [U_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{\eta-\gamma}$, which can be shown to equal (6).

2.2 Production and Dividends

There are a continuum of identical firms of unit mass in each sector. It is therefore convenient to exposit the model at the sectoral level. Each sector j utilizes a production technology that stochastically converts capital stock at the beginning of t (K_t^j) and materials input (H_t^j) chosen during the period to output (Y_t^j) through the production function:

$$F^j(K_t^j, H_t^j, A_t^j) = A_t^j (K_t^j)^{\psi_K^j} (H_t^j)^{\psi_H^j}, \tag{9}$$

where A_t^j represents the stochastically evolving industry-wide (sectoral) productivity level. $\psi_K^j > 0, \psi_H^j > 0$ are the output elasticities of capital and inputs, respectively, with the restriction of non-increasing returns to scale, that is, $\psi_K^j + \psi_H^j \leq 1$. The sectoral productivity shocks follow a first order log-autoregressive stochastic process with possibly contemporaneously correlated innovations. Writing $a_t^j = \ln(A_t^j)$, the productivity stochastic process is specified as:

$$a_t^j = \rho_a^j a_{t-1}^j + \varepsilon_{at}^j, \tag{10}$$

where ε_{at}^j is a normal mean zero variable with a stationary (or time-invariant) variance-covariance matrix $\mathbb{E}[\boldsymbol{\varepsilon}_a \boldsymbol{\varepsilon}_a'] = \Phi_a$ (for $\boldsymbol{\varepsilon}_a = (\varepsilon_a^1, ..., \varepsilon_a^J)$). Capital stock K_t^j evolves according to

$$K_{t+1}^j = (1 - \delta^j) K_t^j + I_t^j, \tag{11}$$

where δ^j is the per-period depreciation rate and I_t^j is the investment at t.

Similar to the literature (Kiyotaki 1988, Horvath 2000), firms in each sector combine intermediate goods to form a composite material input and investment good using the sector-specific CES functions:

$$H_t^j = \left[\sum_{n=J_c+1}^{J_h} \varphi_{nj}^h (H_{nt}^j)^{(\zeta_j^h - 1)/\zeta_j^h} \right]^{\zeta_j^h/(\zeta_j^h - 1)}; I_t^j = \left[\sum_{n=J_h+1}^{J} \varphi_{nj}^k (I_{nt}^j)^{(\zeta_j^k - 1)/\zeta_j^k} \right]^{(\zeta_j^k - 1)/\zeta_j^k}, \tag{12}$$

where H_{nt}^j is the material intermediate good purchased by sector j from sector $n = J_c + 1, ..., J_h$; φ_{nj}^h is the sector-specific weight of this good and $\zeta_j^h \geq 1$ is the ES among material intermediate goods in sector j. Analogously, one interprets I^j_{nt}, φ^k_{nj} and ζ^k_j for investment intermediate goods. The costs of material and investment intermediate goods are $\Upsilon^j_{ht} = \sum_{J_c+1}^{J_h} p^n_t H^j_{nt}$ and $\Upsilon^j_{kt} = \sum_{J_h+1}^{J} p^n_t I^j_{nt}$, respectively.

Sectors choose intermediate goods in a two-step process. In the first step, H^j_t and I^j_t are determined; in the second stage, conditional on (H^j_t, I^j_t), the individual intermediate goods H^j_{nt} and I^j_{nt} are chosen to minimize the intermediate cost expenditures Υ^j_{ht} and Υ^j_{kt}. This process yields the demand function for intermediate goods as (see the Online appendix):

$$H^j_{nt} = (\varphi^h_{nj})^{\zeta^h_j} \left[\frac{p^n_t}{X^j_t}\right]^{-\zeta^h_j} H^j_t; I^j_{nt} = (\varphi^k_{nj})^{\zeta^k_j} \left[\frac{p^n_t}{Z^j_t}\right]^{-\zeta^k_j} I^j_t, \tag{13}$$

where $X^j_t = \left[\sum_{n=J_c+1}^{J_h} (\varphi^h_{nj})^{\zeta^h_j} (p^n_t)^{1-\zeta^h_j}\right]^{1/(1-\zeta^h_j)}$ and $Z^j_t = \left[\sum_{n=J_h+1}^{J} (\varphi^k_{nj})^{\zeta^k_j} (p^n_t)^{1-\zeta^k_j}\right]^{1/(1-\zeta^k_j)}$ are the materials and investment intermediate goods price indices for sector j. It can be shown that the effective composite material input demand H^j_t is such that $X^j_t H^j_t = \Upsilon^j_{ht}$, and similarly $Z^j_t I^j_t = \Upsilon^j_{kt}$. Apart from the costs of investment goods, firms are subject to convex capital adjustment costs so that the total investment cost function is

$$O^j(I^j_t, K^j_t) = Z^j_t I^j_t + 0.5\upsilon^j \left(\frac{I^j_t}{K^j_t}\right)^2 K^j_t, \tag{14}$$

where υ^j is the sector-specific capacity adjustment cost parameter. The process for determination of (H^j_t, I^j_t) will be specified below in the characterization of equilibrium.

All firms in the model are unlevered and publicly owned, with their equity being traded in frictionless security markets. The number of shares outstanding in each sector at the beginning of t is denoted by Q^j_t. Cash flows are paid out as dividends. Let $\Psi^j_t \equiv p^j_t Y^j_t$ denote revenues or sales. Then payouts from sector j at t are

$$D^j_t = \Psi^j_t - X^j_t H^j_t - O^j(I^j_t, K^j_t). \tag{15}$$

Dividends can be negative, financed by equity issuance, that is $Q^j_{t+1} = \max[Q^j_t, Q^j_t - \frac{D^j_t}{s^j_t}]$.[9] Per share dividends in sector j at t are determined by $Q^j_t d^j_t = D^j_t$.

[9] In the absence of taxes and transactions costs, negative dividends are equivalent to the market value of new equity share issuance.

2.3 Equilibrium

The state vector for firms in sector j at the beginning of t is $\Omega_t^j = (W_t, P_t, A_t^j, X_t^j, Z_t^j, K_t^j, Q_t^j)$; the first four elements of this vector are taken as exogenous by firms, while K_t^j and q_t^j are dynamically endogenous for $t \geq 1$ (K_0^j is pre-specified and $q_0^j = 1$). In general, there will not exist complete contingent markets in this model; hence, the discount rate is given by the representative consumer's marginal utility of real consumption (Brock 1982; Horvath 1998). At every t, conditional on Ω_t^j, the representative firm in each sector is instructed by shareholders to choose $\{H_{t+\tau}^j, I_{t+\tau}^j\}_{\tau=0}^{\infty}$ to maximize the conditional present value of real dividends given by[10]

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \Lambda_{t,t+\tau} \left(\frac{D_{t+\tau}^j}{P_{t+\tau}} \right) \right], \tag{16}$$

subject to the production and capital accumulation constraints specified above.

In equilibrium, firms follow optimal investment and input choice strategies taking as given the prices for the industry good p_t^j, while consumers follow optimal consumption and portfolio policies represented by (4) and (7). The model is closed by the requirement that product and asset markets clear. Thus, $c_t^j(\mathbf{p}_t^c, W_t) = Y_t^j, j = 1, ...J$; $\sum_{j=1}^{J} H_{nt}^j = Y_t^n, n = 1, ..., J$; and $q_{t+1}^j = Q_{t+1}^j$, that is, the asset market clearing conditions (7) (or (8)) hold. Note that in equilibrium the share holdings do not affect the income W_t of the CI, since these are determined by $\sum_{j=1}^{J} D_t^j$. Hence, we normalize Q_{t+1}^j to unity at every t without loss of generality (see, e.g., Horvath 1998). With some abuse of notation, we will use the sectoral state vector $\Omega_t^j = (W_t, P_t, A_t^j, X_t^j, Z_t^j, K_t^j)$ for further analysis.

It is convenient, from the viewpoint of our empirical analysis, to express firms' objective functions in nominal terms. Defining the nominal SDF for future payoffs at t as $M_{t,t+\tau} \equiv P_t \left(\frac{\Lambda_{t,t+\tau}}{P_{t+\tau}} \right), \tau = 0, 1, ..,$ the objective function in (16) can be re-expressed as $\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau} D_{t+\tau}^j \right]$. Since $\Lambda_{t,t+\tau} \equiv \frac{\Lambda_{t+\tau}}{\Lambda_t}$ by construction, it will follow that $M_{t,t+\tau} = \frac{M_{t+\tau}}{M_t}, \tau = 0, 1, 2...(M_{t,t} = 1)$. Then, using the Bellman representation, we can define the nominal *cum-dividend* value function of the representative firm along the equilibrium path recursively as a function of the state:

$$V_t^j(\Omega_t^j) = \max_{I_t^j, H_t^j \geq 0} D_t^j + \mathbb{E}_t \left[V_{t+1}^j(\Omega_{t+1}^j) \right]. \tag{17}$$

[10] Recall that $\Lambda_{t,t} \equiv \Lambda_t = \frac{\partial \mathcal{U}_t}{\partial \mathcal{C}_t} = \frac{(1-\alpha)}{(1-\eta)} C_t^{-\eta} \mathcal{U}_t^{\eta}$.

The ex-dividend value of the firm will be denoted S_t^j.

2.4 Equilibrium Characterization for Consumer Goods

We will focus on the asset pricing implication of consumer goods industries (sectors $j = 1, ..., J_c$) for reasons of space and expositional parsimony. There are two principal reasons for this focus. First, the existing empirical asset pricing literature mostly focuses on ERP driven by the (direct) consumption of the representative consumer, so it facilitates intuition on the determinants of sectoral ERP —through comparisons of our results with the literature—to focus on consumer goods industries. Second, the demand functions for consumer goods—and, hence, their sales and dividends—involve the intratemporal ES and sectoral taste parameters (see (4)); these parameters have been estimated by a long literature, which provides a useful benchmark for checking the "reasonableness" of our estimates. In contrast, there is sparse literature on the estimation of the sector-specific CES production and investment parameters ζ_j^h and ζ_j^k that drive the demand functions of intermediate goods producers.[11]

We now present the optimality conditions for the representative firm in a consumer goods sector along the equilibrium path. Since the goods markets clear in equilibrium, we can use (4) to solve for the inverted demand functions for consumer goods sectors as

$$\omega_t^j(Y_t^j) = \phi^j \left(W_t P_t^{\sigma-1}\right)^{1/\sigma} \left(Y_t^j\right)^{-1/\sigma}, j = 1, ..., J. \tag{18}$$

As we mentioned at the outset, and as is apparent from (18), the mapping from quantity to product prices *can not* be expressed in terms of the consumption basket alone. Using $C_t = \frac{W_t}{P_t}$, we can rewrite (18) as $\omega_t^j(Y_t^j) = \phi^j (C_t)^{1/\sigma} P_t \left(Y_t^j\right)^{-1/\sigma}$. Hence, the aggregate (CES) price index still affects the inverted demand function. Intuitively, equilibrium prices for goods are ceteris paribus positively related to the aggregate index, holding fixed the consumption basket. This observation will be important when we consider the equity risk premium below. We will represent the equilibrium path quantities by $(p_t^{j*}, H_t^{j*}, I_t^{j*}, D_t^{j*}, S_t^{j*})$.

[11] Conceptually, the extension of the equilibrium conditions to intermediate goods sectors in Proposition 1 below is straightforward, since it requires substituting the consumer goods demand functions with intermediate goods demand functions from (13). However, extending the quantitative and empirical analysis to study the asset pricing implications for intermediate goods producers is an interesting avenue for future research.

Proposition 1 *Along an equilibrium path, for each t and state Ω_t^j, for any sector $j = 1, ..., J_c$, the optimality conditions for (H_t^{j*}, I_t^{j*}) are given, respectively, by*

$$p_t^{j*} F_H(K_t^j, H_t^{j*}, A_t^j) = X_t^j, \tag{19}$$

$$\begin{aligned} Z_t^j + \upsilon^j \left(\frac{I_t^{j*}}{K_t^j}\right) \quad = \quad & \mathbb{E}_t \left[M_{t,t+1} \left\{ p_{t+1}^{j*} F_K(K_{t+1}^{j*}, H_{t+1}^{j*}, A_{t+1}^j) + 0.5\upsilon^j \left(\frac{I_{t+1}^{j*}}{K_{t+1}^j}\right)^2 + \right.\right. \\ & \left.\left. (1-\delta)\left(Z_t^j + \upsilon^j \left(\frac{I_{t+1}^{j*}}{K_{t+1}^{j*}}\right)\right)\right\}\right], \end{aligned} \tag{20}$$

where $K_{t+1}^{j*} = K_t^j(1-\delta) + I_t^{j*}$. The goods market clears in each sector so that

$$p_t^{j*} = \phi^j \left(\frac{W_t}{P_t}\right)^{1/\sigma} P_t \left[F(K_t^j, H_t^{j*}, A_t^j)\right]^{-1/\sigma}. \tag{21}$$

Finally, the equilibrium dividends D_t^{j*} then satisfy (15), while the ex-dividend value S_t^{j*} is given recursively by

$$\mathbb{E}_t \left[M_{t,t+1} \left(\frac{D_{t+1}^{*j} + S_{t+1}^{j*}}{S_t^{j*}} \right) \right] = 1. \tag{22}$$

Equation (19) reflects the optimality condition that equates marginal cost of inputs (X_t^j) to the marginal revenue productivity of inputs at the competitive (or market clearing) price. Equation (20) is the Euler condition showing the trade-off between the current marginal cost of investment—represented by the left-hand side—with its discounted expected marginal value—given by or the right hand size.

3 Quantitative Implications: Sectoral Risk Premia

We will analyze the quantitative implications of the model with respect to equilibrium equity risk premia through log-linear approximations to product and asset market equilibria around a deterministic steady state. Our approach thus combines methods from the asset pricing literature (e.g., Campbell and Shiller 1988) to approximate the SDF and equity risk premia, with the methods from the macrofinance literature (e.g. King, Plosser and Rebelo 1988) to approximate real quantities, which we apply to analyzing industry equilibria (see Proposition 1). The details of all the analytic approximations below, including the industry and asset market equilibria in the steady state, are given in the Online appendix.

3.1 Analytic Approximations

For firms in any sector j, the vector of state variables $\Omega_t^j = (W_t, P_t, A_t^j, X_t^j, Z_t^j, K_t^j)$ includes exogenous stochastic processes, namely, $\tilde{\Omega}_t^j = (W_t, P_t, A_t^j, X_t^j, Z_t^j)$ and an endogenous capital stock process K_t^j. In the standard fashion (King et al. 1988), the solution method involves log-linearizing the equilibrium conditions of Proposition 1 around a deterministic steady state and solving the resultant linear dynamic system. Because the only exogenous shocks to our economy, namely, the sectoral productivity shocks follow a first order log-autoregressive process (see Equation (10)), we will solve the model (with log-linear approximations of industry and asset markets' equilibrium) under the assumption that the exogenous—at the level of the industry—stochastic state variables $\tilde{\Omega}_t^j$ follow a log-autoregressive system with multivariate normal i.i.d correlated shocks. The model's solution will then give, at any t, the sector capital stock K_{t+1}^j, product prices p_t^j, the log of dividends d_t^j, as well as the log of the SDF m_t and log of equity prices s_t^j as linear combinations of the industry's state variables Ω_t^j, consistent with the assumption that state variable follow a first order autoregressive system.

For the exogenous stochastic state variables, we establish the notation: $w_t \equiv \log(W_t), \pi_t \equiv \log(P_t)$, $x_t^j = \log(X_t^j)$, $a_t^j = \log(A_t^j)$ (as in (10)), $\boldsymbol{\mu}_t^j \equiv (w_t, \pi_t, a_t^j, x_t^j, z_t^j)'$, $\boldsymbol{\rho} = (\rho_w, \rho_\pi, \rho_a^j, \rho_x^j, \rho_z^j)$, $0 \leq \boldsymbol{\rho} \leq 1$, and $\boldsymbol{\varepsilon}_t^j = (\varepsilon_{wt,} \varepsilon_{\pi t,} \varepsilon_{at,}^j, \varepsilon_{xt,}^j \varepsilon_{zt}^j)'$. The log-state vector $\boldsymbol{\mu}_t^j$ follows the recursive law of motion[12]

$$\boldsymbol{\mu}_{t+1}^j = \boldsymbol{\rho}' \boldsymbol{\mu}_t^j + \boldsymbol{\varepsilon}_{t+1}^j. \tag{23}$$

As noted earlier, $\boldsymbol{\mu}_t^j$ includes two aggregate quantities—w_t and π_t—that are common across industries—and three industry-specific quantities: a_t^j, x_t^j and z_t^j. Correspondingly, the shock vector $\boldsymbol{\varepsilon}_t^j$ is also composed of aggregate and industry-specific shocks. In particular, $\boldsymbol{\varepsilon}_t^j$ are mean zero variables with the time-invariant variance-covariance matrix $\Phi^j = \left[\Phi_{ij}^j\right]$, where $Cov(\varepsilon_{wt,} \varepsilon_{wt,}) = \Phi_w^2, Cov(\varepsilon_{\pi t,} \varepsilon_{\pi t,}) = \Phi_\pi^2$ and $Cov(\varepsilon_{wt,} \varepsilon_{\pi t,}) = \Phi_{w\pi}$ are common across j, while $Cov(\varepsilon_{wt,} \varepsilon_{at}^j) = \Phi_{aw}^j$ etc. are specific to the

[12] The specification in Equation (23) may be considered as the special case of a general vector autoregression system with one lag (VAR(1)) and a matrix of autocorrelation coefficients with only nonzero diagonal terms.

industry.[13] We first derive the log of the SDF using the standard approach and then turn to the derivation of equilibrium law of motion for the real variables.

Now from (6), the log of the real pricing kernel $\lambda_{t+1} \equiv \log(\Lambda_{t,t+1})$ is

$$\lambda_{t+1} = \theta \log \alpha - \eta\theta \left[g_{w,t+1} - g_{\pi,t+1}\right] + (\theta - 1) r_{c,t+1}, \tag{24}$$

where $g_{w,t+1}$ and $g_{\pi,t+1}$ are the log growth rates of income and aggregate price, respectively, and $r_{c,t+1} \equiv \log(R_{C,t+1})$. Using the Campbell and Shiller (1988) log-linearization approach, we can write

$$r_{c,t+1} = f_0 + f_1 z_{c,t+1} - z_{ct} + g_{w,t+1} - g_{\pi,t+1}, \tag{25}$$

where z_{ct} is the log price-consumption ratio. Here f_0 and f_1 are approximating constants that depend on the unconditional mean of z_{ct}, say, z_c. Indeed, $f_0 = \log(1 + \exp(z_c)) - f_1 z_c$ and $f_1 = \frac{\exp(z_c)}{1+\exp(z_c)}$. We take z_{ct} to be a linear function of the logs of the aggregate state variables W_t and P_t, namely, $z_{ct} = \kappa_{c0} + \kappa_{cw} w_t + \kappa_{c\pi}\pi_t$. We show in the Online appendix that

$$\kappa_{cw} = \frac{(\rho_w - 1)(\eta - 1)}{f_1(\rho_w - 1) - 1}, \kappa_{c\pi} = \frac{(\rho_\pi - 1)[\theta(1-\eta) + 1]}{\theta(f_1(\rho_\pi - 1) - 1)}, \kappa_{c0} = \frac{\log \alpha + f_0}{1 - f_1}. \tag{26}$$

Since $g_{w,t+1} = (\rho_w - 1)w_t + \varepsilon^w_{t+1}$ and $g_{\pi,t+1} = (\rho_\pi - 1)\pi_t + \varepsilon^\pi_{t+1}$, substitution of (26) and (25) in (24) allows one to write (see the Online appendix) the log of the nominal SDF $m_{t+1} = \lambda_{t+1} + \pi_t - \pi_{t+1}$ as

$$m_{t+1} = \theta \log \alpha + B_w w_t + (B_\pi + (1 - \rho_\pi))\pi_t + b_w \varepsilon_{w,t+1} + b_\pi \varepsilon_{\pi,t+1}, \tag{27}$$

where the coefficients are

$$\begin{aligned}
B_w &\equiv (\rho_w - 1)\left[\theta(1-\eta) - 1\right] + (\theta - 1)\kappa_{cw}[f_1\rho_w - 1], \\
B_\pi &\equiv (\rho_\pi - 1)\left[\theta(\eta - 1) + 1\right] + (\theta - 1)\kappa_{c\pi}[f_1\rho_\pi - 1], \\
b_w &\equiv -\eta\theta + (\theta - 1)[f_1\kappa_{cw} + 1]; b_\pi \equiv \eta\theta - (\theta - 1)[1 - f_1\kappa_{c\pi}].
\end{aligned} \tag{28}$$

[13] We will derive the analytic approximations to the equilibrium path for general linear first-order autoregressive (that is, AR(1)) processes, allowing for both Gaussian and non-Gaussian innovations (e.g., Sim 1990) to the state variables.

We now turn to the approximation of equilibrium real quantities, which will provide us with the equilibrium sectoral dividend polices and, in turn, stock prices and the industry ERP. We will denote steady state quantities by "bars" and define by $\hat{L}_t = \log(L_t) - \log(\bar{L})$ the log deviation of any variable L_t from its steady state value. Using (21), we start by writing the optimality conditions (19)-(20) as

$$
\begin{aligned}
X_t^j &= \left(\frac{\psi_H^j}{H_t^j}\right)\left[\phi^j (W_t)^{1/\sigma}\left(P_t A_t^j (K_t^j)^{\psi_K^j}\right)^{\frac{\sigma-1}{\sigma}}\right], && (29)\\
Z_t^j + \upsilon^j\left(\frac{I_t^j}{K_t^j}\right) &= \mathbb{E}_t\left[M_{t,t+1}\left\{\psi_K^j \frac{\Psi_{t+1}^j}{K_{t+1}^j} + \left(\frac{\upsilon^j}{2}\right)\left(\frac{I_{t+1}^j}{K_{t+1}^j}\right)^2 + \right.\right.\\
&\left.\left. +(1-\delta^j)\left(Z_{t+1}^j + \upsilon^j\left(\frac{I_{t+1}^j}{K_{t+1}^j}\right)\right)\right\}\right]. && (30)
\end{aligned}
$$

Log-linearization of (29)-(30) around the steady state gives (see the Online appendix)

$$
\begin{aligned}
\sigma \bar{X}^j \hat{X}_t^j &= \left(\frac{\psi_H^j \bar{\Psi}^j}{\bar{H}^j}\right)\left[\hat{W}_t + (\sigma-1)\left\{\hat{P}_t + \hat{A}_t^j + \psi_K^j \hat{K}_t^j\right\} - \nu_H^j \hat{H}_t^j\right], && (31)\\
\upsilon^j \delta^j (\hat{I}_t^j - \hat{K}_t^j) &= \alpha \mathbb{E}_t\left[\hat{M}_{t,t+1} u^{M,j} + \psi_K^j \frac{\bar{\Psi}^j}{\bar{K}^j}(\hat{\Psi}_{t+1}^j - \hat{K}_{t+1}^j) + \right.\\
&\left. \upsilon^j \delta^j\left\{(\hat{I}_{t+1}^j - \hat{K}_{t+1}^j) - (1/\alpha)\right\}\right], && (32)
\end{aligned}
$$

where $u^{M,j} = \left(\psi_K^j \frac{\bar{\Psi}^j}{\bar{K}^j} + (1-\delta^j) + \left(\frac{\upsilon^j \delta^j}{2}\right)(2-\delta^j)\right)$. Writing logs of H_t^j and K_t^j as small letters, from (31) we get the equilibrium materials choice as an affine function of the current state (in logs):

$$
h_t^j = (\nu_H^j)^{-1}\left[\sigma(\log\phi^j + \log\psi_H^j) + w_t + (\sigma-1)(\pi_t + a_t^j + \psi_K^j k_t^j) - \sigma x_t^j\right], \tag{33}
$$

where $\nu_H^j = \psi_H^j + \sigma(1-\psi_H^j)$. Next, using (32), the equilibrium law of motion of capital stock can be shown (see the Online appendix) to be an affine function of the (log) state variables, that is,

$$
k_{t+1}^j = \xi_0^j + \xi_w^j w_t + \xi_\pi^j \pi_t + \xi_a^j a_t^j + \xi_x^j x_t^j + \xi_z^j z_t^j + \xi_k^j k_t^j. \tag{34}
$$

As is apparent from (20) (or 32), the loadings on the aggregate and industry factors combine both elements of consumer preferences and industry production relationships. For instance,

$$
\begin{aligned}
\xi_w^j &= \frac{\left(\frac{\phi^j \psi_K^j \bar{\Psi}^j}{\sigma \bar{K}^j}\right)(1-\rho_w)(1+(\nu_H^j)^{-1}) - u^{M,j} B_w}{\upsilon^j(\rho_w + \xi_k^j) + u_1^{K,j}}, && (35)\\
\xi_a^j &= \frac{\left(\frac{\phi^j \psi_K^j \bar{\Psi}^j}{\sigma \bar{K}^j}\right)(\sigma-1)(1-\rho_a^j)(1+(\nu_H^j)^{-1})}{\upsilon^j(\rho_a^j + \xi_k^j) + u_1^{K,j}}. && (36)
\end{aligned}
$$

Since equilibrium h_t^j and k_{t+1}^j are affine functions of the current state, using (15) we can express equilibrium dividends also as affine functions of the current state:

$$
d_t^j = N_{d,0}^j + N_{d,w}^j w_t + N_{d,\pi}^j \pi_t + N_{d,a}^j a_t^j + N_{d,x}^j x_t^j + N_{d,z}^j z_t^j + N_{d,k}^j k_t^j. \tag{37}
$$

Not surprisingly, the loadings of dividends on aggregate and industry factors inherit the properties of h_t^j and k_{t+1}^j. For example,

$$
\begin{aligned}
N_{d,w}^j &= (\bar{D}^j)^{-1}\left[\frac{\bar{\Psi}^j \phi^j \left(\nu_H^j + \psi_H^j\right)}{\sigma \nu_H^j} - \bar{X}^j \bar{H}^j \left(\frac{\psi_H^j}{\nu_H^j}\right) - \bar{K}^j(1+\upsilon^j\delta^j)\xi_w^j\right], && (38)\\
N_{d,a}^j &= (\bar{D}^j)^{-1}\left[\frac{(\sigma-1)\bar{\Psi}^j \phi^j (\nu_H^j + \psi_H^j(\sigma-1))}{\sigma \nu_H^j} - \bar{K}^j(1+\upsilon^j\delta^j)\xi_a^j\right]. && (39)
\end{aligned}
$$

With the equilibrium dividend function (37) in hand, we then derive the equilibrium equity price by log-linearization of the equilibrium asset pricing condition (22), which gives

$$
\mathbb{E}_t\left[\hat{M}_t^j + (1-\alpha)\hat{D}_{t+1}^j + \alpha \hat{S}_{t+1}^j - \hat{S}_t^j\right] = 0. \tag{40}
$$

Writing (with some abuse of notation) $s_t^j = \log(S_t^j)$, we can derive s_t^j as the affine function:

$$
s_t^j = N_{s,0}^j + N_{s,w}^j w_t + N_{s,\pi}^j \pi_t + N_{s,a}^j a_t^j + N_{s,x}^j x_t^j + N_{s,z}^j z_t^j + N_{s,k}^j k_t^j, \tag{41}
$$

where the loadings in (41) on the state variables are weighted combinations of the loadings of the SDF and dividends (see the Online appendix).

We now utilize the standard reformulation of the Euler (8), namely,

$$\mathbb{E}_t\left[R^j_{t+1} - R^f_{t+1}\right] = -R^f_{t+1} Cov_t\left(M_{t,t+1}, \frac{D^j_{t+1} + S^j_{t+1}}{S^j_t}\right), \tag{42}$$

where Cov_t is the conditional covariance function and R^f_{t+1} is the nominal riskless rate (at t). Log-linear expansion of (43) around the steady state yields the equity risk premium (ERP) equation:

$$\mathbb{E}_t[r^j_{t+1} - r^f_{t+1}] = -\alpha Cov_t\left(m_{t+1} - \mathbb{E}_t[m_{t+1}], \left(\frac{1-\alpha}{\alpha}\right) d^j_{t+1} + s^j_{t+1} - \mathbb{E}_t\left[\left(\frac{1-\alpha}{\alpha}\right) d^j_{t+1} + s^j_{t+1}\right]\right). \tag{43}$$

(43) specifies the standard result that the ERP is positively related to the covariance between the shocks to the log SDF (that is, $m_{t+1} - \mathbb{E}_t[m_{t+1}]$) and shocks to dividends and stock prices. This equation also recognizes that in the steady state (denoted by bars) $\bar{M} = \alpha$ (since $W_{t+1} = W_t = \bar{W}, P_{t+1} = P_t = \bar{P}$, and $\mathcal{U}_t = \mathcal{U}_{t+1}$) and $\bar{S}^j = \frac{\alpha \bar{D}^j}{1-\alpha}$, so that $\frac{\bar{D}^j}{\bar{S}^j} = \left(\frac{1-\alpha}{\alpha}\right)$. But because dividends and stock prices depend on both aggregate and industry-specific states (see (37) and (41)), it follows that the equilibrium ERP should as well. Indeed, straightforward computations yield a linear multi-factor model of the form:

$$\begin{aligned}\mathbb{E}_t[r^j_{t+1} - r^f_{t+1}] \quad = \quad & \beta^j_w Var_t(\varepsilon^w_{t+1}) + \beta^j_\pi Var_t(\varepsilon^\pi_{t+1}) + \beta^j_{w\pi} Cov_t(\varepsilon_{wt,} \varepsilon_{\pi t,}) + \beta^j_{aw} Cov_t(\varepsilon_{wt,} \varepsilon^j_{at}) + \\ & \beta^j_{a\pi} Cov_t(\varepsilon_{\pi t,} \varepsilon^j_{at}) + \beta^j_{xw} Cov_t(\varepsilon_{wt,} \varepsilon^j_{xt}) + \beta^j_{x\pi} Cov_t(\varepsilon_{\pi t,} \varepsilon^j_{xt}) + \\ & \beta^j_{zw} Cov_t(\varepsilon^w_t, \varepsilon^j_{zt}) + \beta^j_{z\pi} Cov_t(\varepsilon^\pi_t, \varepsilon^j_{zt}).\end{aligned} \tag{44}$$

In (44), β^j_w, β^j_w and $\beta^j_{w\pi}$ represent the risk loadings of the representative firm in sector j to aggregate shocks, that is, exposure to income and price index shocks. And $\beta^j_{aw}, \beta^j_{xw}, \beta^j_{x\pi}, \beta^j_{zw}$, and $\beta^j_{z\pi}$ represent the industry's risk sensitivities to industry-specific shocks due to the covariation between shocks to industry productivity and intermediate cost indices with aggregate shocks. The specification of the loadings, in terms of the parameters of the equilibrium (log) dividend and stock price functions—namely, (37) and (41)— and the discount factor α are provided in the Online appendix.

The ERP representation in (44) differs from the received literature in two principal ways. First, unlike single consumption good macrofinance models—both in exchange and production economies—our model allows cross-sectional variation across industries. Second, and distinct from the production-based asset pricing literature, the "real side" of our model specifies the contribution to equilibrium ERP due to the covariation between industry-specific (productivity and materials costs) shocks and aggregate shocks. Note that by using the equilibrium relation $C_t = \frac{W_t}{P_t}$, we can also express the ERP in terms of consumption risk—as is more typical in the asset pricing literature—by eliminating the role of income. But for the reasons given above, the effects of shocks to the aggregate price index cannot be excluded.[14]

But it is important to develop intuition on the relation of the risk loadings to the fundamentals of the model. It is apparent from (42) that the risk premium will depend on the interaction of the SDF and firms' equilibrium response to aggregate and industry shocks. To illustrate, the risk loading on aggregate income shocks above takes the form $\beta_w^j = \alpha b_w \tilde{N}_w^j$, where b_w is the contribution of exposure to income shocks in the SDF (see (27)) and $\tilde{N}_w^j = -[\left(\frac{1-\alpha}{\alpha}\right) N_{d,w}^j + N_{s,w}^j]$ is the value-weighted average of the effects of w_t of dividends and (ex-dividend) equity values, namely,

$$\tilde{N}_w^j = \frac{(1-\alpha)N_{d,w}^j + \alpha(B_w + \xi_w^j(\alpha N_{s,k}^j + (1-\alpha)N_{d,k}^j))}{\alpha(\alpha\rho_w - 1)}. \tag{45}$$

Note that $\tilde{N}_w^j$ incorporates the responsiveness of firms' investment and material input choices to aggregate income shocks in the industry equilibrium: These are represented by ξ_w^j (see (34)) and income-sensitivity parameters in (33). In turn, the optimal investment and material input choices are related to the "deep" parameters of the model, namely, the industry taste weight ϕ^j and the ES σ on the consumer preferences side, and output elasticities ψ_H^j and ψ_K^j and capacity adjustment costs υ^j on the production side. Of course, the equilibrium risk loading also includes the CI's risk aversion (γ) and IES ($1/\eta$), incorporated in B_w. In sum, the general equilibrium model constructed above with multiple industries implicates parameters from the well-known CES consumer preferences (namely, ϕ^j and σ) in the ERP, along with the parameters that are considered in the consumption- and production-based asset pricing models in the literature.

We note that under the maintained assumption of a stationary variance-covariance matrix of the

[14] In this alternative formulation, with some abuse of notation, we write the log-autoregressive consumption basket process as $c_t - \rho_c c_{t-1} + \varepsilon_{t+1}^c$ and express the ERP as: $\mathbb{E}_t[r_{t+1}^j - r_{t+1}^f] = \beta_c^j Var_t(\varepsilon_{t+1}^c) + \beta_\pi^j Var_t(\varepsilon_{t+1}^\pi) + \beta_{ca}^j Cov_t(\varepsilon_{t+1}^c, \varepsilon_{t+1}^a) + \beta_{a\pi}^j Cov_t(\varepsilon_{t+1}^\pi, \varepsilon_{t+1}^a) + \beta_{a\pi}^j Cov_t(\varepsilon_{t+1}^\pi, \varepsilon_{t+1}^a) + \beta_{xw}^j Cov_t(\varepsilon_{t+1}^w, \varepsilon_{t+1}^x) + \beta_{x\pi}^j Cov_t(\varepsilon_{t+1}^\pi, \varepsilon_{t+1}^x)$.

aggregate and industry shocks (see (23)), (44) can also be written as the unconditional ERP

$$\mathbb{E}[r^j_{t+1} - r^f_{t+1}] = \beta^j_w \Phi^2_w + \beta^j_\pi \Phi^2_\pi + \beta^j_{w\pi}\Phi_{w\pi} + \beta^j_{aw}\Phi^j_{aw} + \beta^j_{xw}\Phi^j_{xw} + \beta^j_{zw}\Phi^j_{zw} \quad (46)$$
$$\beta^j_{a\pi}\Phi^j_{a\pi} + \beta^j_{x\pi}\Phi^j_{x\pi} + \beta^j_{z\pi}\Phi^j_{z\pi}.$$

Of course, by allowing non-stationary or heteroskedastic industry productivity shocks, (44) will generate time-varying industry ERP as a linear factor model with aggregate and industry factors. Nevertheless, with appropriate calibration, (46) allows the computation of the unconditional ERP, which is of substantial interest. We now turn to these computations.

3.2 Data and Calibration

For empirical analysis of the model, we need industry data on capital, investment, materials input, sales, and productivity. We take these data from the NBER-CES manufacturing database. The latest data available are for 1958-2018 (annual). However, because industry productivity data are generally available only through 2016, our sample period is 1958-2016. The NBER-CES data are in nominal terms. While deflators for materials costs and investment are provided, the appropriate deflators for output and (especially) capital stock are not apparent. For this reason, we work with the nominal SDF in the Euler condition for investment (Equation 20) and, therefore, also asset returns. Of course, the unconditional ERP, which we will compute, is expressed in real terms.

As the proxies for aggregate income (W) and consumption basket (C), we use per-capita national income and consumption expenditures from the Federal Reserve Bank of St. Louis (FRED). We note that the aggregate price index in the model, P, is not necessarily the CPI, since along the equilibrium path consumption basket $C_t = \frac{W_t}{P_t}$. That is, the appropriate price index is the "ideal" (or CES) price index, which is not generally available (e.g., Redding and Weinstein 2020). We therefore use the implied equilibrium P from the income and consumption data. We use the deflators for the material costs (H^j_t) and investment (I^j_t) as measures of X^j_t and Z^j_t. We compute the covariances between industry factors (A^j_t, X^j_t, Z^j_t) and the aggregate factors (W_t, P_t)—that contribute to the risk premium—from the data using the log-autoregressive specifications in (23).

Table 1 displays the parameterization for the aggregate quantities (which are common across sectors), as well as the sector-specific parameters for all sectors. The (annual) volatility of per capita national income shocks (Φ_w) in our sample period is 2.47% , while the volatility of the shocks to log aggregate price index (Φ_π) is very low, that is, 0.76%; and the covariance $\Phi_{w\pi}$ is even lower (4.8 e-05). Using the relation $\log C_t = \log W_t - \log P_t$, we obtain (annual) volatility of log consumption growth as 2.39%. By construction (since P is forced to equal W/C), this volatility equals (up to rounding) the annual volatility of per-capita consumption in our sample period. The covariances between the industry and aggregate factors tend to be small. Finally, all stochastic processes are highly persistent, that is, have an autocorrelation coefficient exceeding 0.9, with autocorrelation aggregate income and consumer price index being essentially 1.

Consistent with the theoretical focus, we restrict attention to consumer goods industries subsample of the NBER-CES database by mapping 1997 North American Industry Classification System (NAICS) codes to four-digit 1987 Standard Industry Classification (SIC) codes.[15] As we mentioned already, the asset pricing literature highlights the role of durability in expected returns (Yogo 2008, Gomes, Kogan and Yogo 2009). Hence, for our quantitative analysis, we consider separately consumer durable goods industries and consumer non-durable goods industries. For each of these sectors, we take the cross-sectional means of production data across the relevant industry codes.

We now turn to the calibration of parameters relating to consumer preferences, namely, $(\alpha, \gamma, \eta, \sigma, \phi^j)$. Because our production data are at an annual frequency, we use 3% annual discount rate, that is $\alpha = 0.97$, consistent with multi-sector general equilibrium models in the literature (e.g., Horvath 2000). There is no consensus in the literature on the other parameters, however. The estimated of percentage of income spent on manufactured consumer goods in the highest income decile countries (which includes U.S.) range from around 0.3 (Duarte and Restuccia 2016) to 0.39 (Duarte 2018). We set ϕ^j as 0.35 for our manufactured consumer goods sector.

For (γ, η, σ), we utilize the structural estimates from our model using GMM (see Section 5) with our aggregate data and production data for consumer goods industries in our manufacturing industries sample.[16] The risk aversion estimates (γ), shown in Table 5 below, are robust around 5. But we also use risk aversion of 7.5 and 10 for our calculations, which are considered in the literature (e.g.,

[15] The list of consumer goods industry NAICS codes is obtained from Statistics Canada (2021).

[16] This estimation is conducted with $\alpha = 0.97$ and $\phi^j = 0.35$ for consumer goods industries in the manufacturing database.

Bansal and Yaron 2004, Croce 2014). We obtain robust estimates of IES (η^{-1}) of around 0.2. We will discuss this estimate further below in Section 5, but as mentioned before, our estimate is consistent with a strand of the literature on IES estimates. We get reliable estimates of ES (σ) of around 2 for manufactured consumer goods industry. Following Feenstra (1994) and Broda and Weinstein (2006), the interpretation of this sector specific ES is the elasticity of substitution amongst different products in the manufactured consumer goods sector, including between durables and non-durables.[17] The estimated σ reliably exceeds 1, as required for convexity of preferences, and (as we will discuss further below) is within the range estimates of ES reported in the literature.

We compute annual returns for each sector as the value-weighted portfolio returns of all firms in the sector. We first compute the value-weighted portfolio returns for each sector using monthly data and then compute the calendar year returns using the monthly time series. When needed, we apply the consumption price deflator (CPI) to adjust the returns data to real terms. We obtain the market and risk free returns from Kenneth French's website.

Turning to the production side of the model, it is well known that because of varying rates of depreciation for different types of capital (equipment, structures, and intellectual property), estimating depreciation rates is challenging. The literature notes that depreciation rates have been trending upwards because of the increased use of computer equipment and software since this lowers the useful life of capital stock (Oliner 1989). Moreover, the depreciation rates on such equipment have been rising. For example, Gomme and Rupert (2007) note that annual depreciation rates of computer equipment have risen from 15% in 1960-1980 to 40% in 1990s, and give estimates for software depreciation rates of about 50%. Epstein and Denny (1980) estimate the depreciation rate of physical capital (in the first part of our sample-period) to be about 13%. Because of increasing use of computerized and higher technology equipment in manufacturing during our sample period, we use an annual depreciation rate of 20%.[18] There is also a wide variation in the literature regarding estimates of the capital adjustment cost parameter υ. In particular, utilizing US plant level data, Cooper and Haltiwanger (2006) find υ of around

[17] This approach is based on extending the basic CES basket (4) specification to allow for nonsymmetric or good-specific ES (e.g., Broda and Weinstein 2006). Using this more flexible specification does not change the theoretical characterization of the equilibrium and, hence, the sectoral asset pricing implications. For notational parsimony, we continue to use σ to denote the sectoral ES in the manufactured consumer goods sector.

[18] This value is also the mean depreciation rate estimated by a detailed study of Canadian manufacturing data (Gellatly et al 2007).

10% for a strictly convex adjustment cost function, which is the value we utilize in our computations. Table 1 reports the calibration of salient model parameters used in computations.

In the usual fashion, we take the steady state value of per capita income $\bar{W}$ and aggregate (CES) price index $\bar{P}$ as their sample means. By construction of the P_t time-series, the implied steady state consumption-to-income ratio matches the sample mean and the steady state consumption level is also close to its sample mean. In a similar vein, the production elasticities ψ_K^j, ψ_H^j, and the steady state values $\bar{A}^j$, $\bar{X}^j$ and $\bar{Z}^j$ are calibrated so that the model's steady state per capita output, materials inputs, and capital stock match their corresponding sample means.[19] The procedure for the steady state calibration and the steady state values are given in the Online appendix (Section C.1 and Table A.1). Finally, to calibrate the SDF, we set $f_1 = \frac{\exp(z_c)}{1+\exp(z_c)}$ (in (25)) as 0.997, which is consistent with Bansal and Yaron (2004) and Campbell and Shiller (1988).

Prior to considering the asset pricing implications of the calibrated model, we note that in equilibrium firms' demand for material inputs is procyclical (that is, positively related to aggregate income, see (33)). And under the calibration in Table 1, investment is also procyclical (that is, $\xi_w^j > 0$ in (35)). These equilibrium properties are consistent with the data since, in our consumer manufacturing industry sample, log materials inputs and log investment are highly positively correlated with log per capita income. In a similar vein, the model implies a positive relation of materials inputs and investment with the aggregate price index and sectoral productivity, both of which are consistent with the data. In particular, equity payoffs (that is, $D_t^j + S_t^j$) are positively correlated with aggregate income and the aggregate price index.

3.3 Asset Pricing Implications

Table 2 presents the computations for the unconditional equilibrium ERP for consumer durables and consumer non-durable sectors in Panels A and B, respectively. We also provide the mean of sample ERP for each sector in our data as a benchmark. As mentioned before, we analyze the ERP for three

[19] The procedure for calibrating the production elasticities is a follows. We start by estimating, for each sector j (that is, consumer durable and non-durables) (ψ_K^j, ψ_H^j) through the estimation of the production function specified in Equation (9) using GMM. We utilize the orthogonality restrictions given by $\Gamma_t^j(Y_t^j - A_t^j(K_t^j)^{\psi_j}(H_t^j)^{\xi_j}) = 0$, where Γ_t^j is the IV vector that uses two year lagged inputs $(A_{t-1}^j, K_{t-1}^j, H_{t-1}^j)$ as instruments and using heteroskedasticity-robust standard errors and constrain the estimated elasticities to satisfy the restriction of non-increasing returns to scale. The production elasticities are then calibrated along with the other production related parameters to match the sample means of output, materials inputs, and capital stock.

levels of risk aversion: $\gamma = 5,\ 7.5,\ 10$. In each case, we also report the factor loadings (see (46)) to facilitate intuition on the contributions to sectoral ERP.

We consider first the ERP for the consumer durables sector in Panel A. While the annual unconditional ERP with the estimated risk aversion of 5 is lower than that in the data, the model is able to match the observed ERP with risk aversion between 7.5 and 10. Note that the model can match the ERP in the data for reasonable risk aversion levels even when the IES is low. For the consumer non-durables sector, the results in Panel B show that the model matches the sample mean of annual ERP in this sector, 10.05%, with risk aversion between 7.5 and 10.

In the Introduction, we have provided intuition for the signs of the loadings on aggregate risk factors $(\beta^j_w, \beta^j_\pi, \beta^j_{w\pi})$, as well as the loadings on the sectoral risk factors $(\beta^j_{xw}, \beta^j_{zw}, \beta^j_{x\pi}, \beta^j_{z\pi}, \beta^j_{aw}, \beta^j_{a\pi})$. Using census data on U.S. manufacturing plants (1974-2011), Ederhof, Nagar and Rajan (2021) find that materials costs are the largest component of firms' non-investment costs. Furthermore, investment is more sluggish than materials input demand (consistent with our data) and the covariance between investment goods prices and aggregate factors is lower than that of materials goods prices (Table 1). These facts may help explain the substantially larger size of the loadings on materials input price risk compared with investment goods price risk.

To provide a benchmark for the industry ERP in Table 2, we analyze the ERP for the aggregate asset that pays the equilibrium consumption basket $C_t = \frac{W_t}{P_t}$ at every t (see Section B.2.4 of the Online appendix). We find an ERP of 0.57% for a risk aversion of 10 and IES of 0.2 (as in Table 2), in contrast to the (annual) sample mean ERP of 7.33%. The factor loadings for the aggregate asset (when $\gamma = 10, \eta = 5.3$) are $\beta_w = 9.99, \beta_\pi = 10.02, \beta_{w\pi} = -20$. Thus, the equity premium puzzle holds for the asset that *passively* pays the equilibrium consumption basket because of (1) its substantially lower sensitivity to aggregate risk factors compared to the industry loadings (in Table 2) that incorporate endogenous production and investment responses by firms to these factors, (2) the negative loading on the covariation of W_t and P_t,[20] and (3) the absence of industry-level risk factors.

[20]We note that $\beta_\pi > 0$ for the aggregate asset, in contrast to the negative industry loadings for price index shocks at the sectoral level in Table 2. The reason is that higher P_t ceteris paribus raises marginal utility, but also reduces the payoff (W_t/P_t) from the aggregate asset. Hence, the covariance of asset payoffs and the SDF is negative, and consequently the loading on aggregate price index volatility is positive. In contrast, as noted above, equity payoffs in the industry equilibrium are positively correlated with the price index. Meanwhile, $\beta_{x\pi} < 0$ for the aggregate asset because positive shocks to W_t and P_t have opposing effects on equity payoffs (by construction), as opposed to the positive relation of payoffs with *both* aggregate income and price index in the industry equilibrium.

Now it is apparent from the expressions for the factor loadings (see Section 3.1 and the Online appendix) that the effects of risk aversion and IES on the ERP are affected by the (sectoral) intratemporal ES (σ). Indeed, the impact of ES—the focus of a long-standing literature in applied economics—on the ERP is of independent interest. In a similar vein, the effects of risk aversion or IES on the ERP are impacted by the production technology parameters.

To further understand the effects on the ERP by the interaction of the "deep" parameters of the model, Figure 1 displays surface plots of the effects on ERP of bivariate variations in (i) γ and σ, (ii) γ and (the production elasticity for capital) ψ_K, and (iii) η and σ.[21] The top panel in Figure 1 confirms that, for a fixed σ (ES), ERP is positively related to risk aversion. But the ERP is negatively related to the ES. There are clear interaction effects of risk aversion and ES on the ERP; in particular, the positive effects of risk aversion on ERP are greater at lower values ES. Hence, the ERP can be quite large for γ around 10 and σ around 1.5; and conversely the ERP can be relatively small for risk aversion of 10 for ES of 3.

In the CES setting (see (4)), the sectoral ES measures the demand price elasticity of the industry good; hence, the graphical analysis in Figure 1 suggests a negative relation of ERP and price elasticity. Conceptually and empirically, goods with low product differentiation, such as commodities and basic consumption goods, have greater demand price elasticity compared with highly differentiated goods, such as luxury goods (Berry, Levinsohn and Pakes 1995, Broda and Weinstein 2006). Thus, the graphical analysis of Figure 1 implies a negative relation of ERP and the sectoral ES or price elasticity, which appears consistent with the analysis of Ait-Sahalia et al. (2004), mentioned above.

Meanwhile, high capital productivity (ψ_K) ceteris paribus makes investment more procyclical, which ceteris paribus raises the ERP. Hence, as seen in the middle panel of Figure 1, ERP is positively related to ψ_K for a fixed γ, and ERP is large for high ψ_K and γ. Finally, the bottom panel of Figure 1 shows a non-monotone relationship between ERP and IES (η^{-1}), for a fixed σ. But the ERP is large for reasonable values of ES and low values of the IES, which is consistent with our estimation results below (Section 4). Overall, Figure 1 shows that sectoral ERP can be large for ranges of reasonable consumer preferences and production technology parameters.

[21] We plot these figures for intervals around our structural estimates (presented in Section 4) of $\gamma = 5, \eta = 5.3, \sigma = 2$. And we plot only positive values of ERP.

Finally (as we mentioned above), for simplicity we do not incorporate non-dividend income for the CI in the model. An exogenous stochastic non-dividend income component will not significantly affect the results since the analytic approximations above are based on a general first order log-autoregressive process for consumer wealth and we calibrate the model with per capita income in the data. Allowing endogenous labor income with leisure as a component of consumer preferences and labor as a factor of production will not significantly affect the endogenous risk factors, but will affect the factor loadings.

4 Structural Estimation of the Model

In this section, we undertake estimation of three salient parameters governing preferences, namely (γ, η, σ) through GMM using returns and production data from consumer goods sectors in the NBER-CES manufacturing database (our data sources have been described in Section 3.2). We focus on these parameters for estimation parsimony, which enhances (estimation) efficiency.[22] Furthermore, the joint estimation of risk aversion, IES and ES is novel and of intrinsic interest to both finance and economics literatures. Because (γ, η, σ) apply to all sectors, we combine the consumer durables and consumer non-durables industries considered in Section 3 and take their mean values for the production side of estimation; and we use value-weighted portfolio returns for the sectoral equity returns. Consistent with Section 3.2, we set $\alpha = 0.97$ and $\phi^j = 0.35$. Moreover, the production elasticities are re-estimated for the combined consumer goods manufacturing subsector and set as $\psi_H^j = 0.75$ and $\psi_K^j = 0.25$ (to maintain the restriction of CRS, as in the previous Section).

4.1 Basic Moment Conditions

Our general equilibrium model provides two "real side" moment conditions from the Euler conditions (19)-(EulerIC), as well as conditions from the asset market equilibrium (see (8) or (22)). These conditions form the basis of our estimation and are specified below:

$$0 = \mathbb{E}_t \left[\phi^j \left(W_t\right)^{1/\sigma} \left(P_t\right)^{\frac{\sigma-1}{\sigma}} (Y_t^j)^{-\frac{1}{\sigma}} \left(\frac{Y_t^j}{H_t^j} \right) - X_t^j \right], \tag{47}$$

[22] In particular, when estimating the discount factor α, we do not obtain consistent or economically appealing estimates, even when estimating (γ, η, α) with fixed ES. With some specifications, estimates of α exceed 1, similar to the literature (Hansen and Singleton 1983, Epstein and Zin 1991); or, we obtain significantly negative values of the IES, which implies that consumption increases with interest rates.

$$
\begin{aligned}
0 &= -Z_I^j(I_t^j, K_t^j) + \mathbb{E}_t \Big[M_{t,t+1} \Big\{ p_{t+1}^j F_K(K_{t+1}^j, H_{t+1}^j, A_{t+1}^j) - \\
&\quad Z_K^j(I_{t+1}^j, K_{t+1}^j) + (1-\delta) Z_I^j(I_{t+1}^j, K_{t+1}^j) \Big\} \Big], \qquad (48) \\
0 &= \mathbb{E}_t \Big[M_{t,t+1} \tilde{R}_{t+1} \Big], \qquad (49)
\end{aligned}
$$

where $\tilde{R}_{t+1}$ is a vector of excess returns. In particular, we use the aggregate or market ERP, $\tilde{R}_t^{\Sigma} = R_t^{\Sigma} - R_t^f$, the consumer goods manufacturing (CGM) sector ERP, $\tilde{R}_t^j = R_t^j - R_t^f$, and the excess market returns relative to CGM sector returns $\tilde{R}_t^{\Sigma,j} = R_t^{\Sigma} - R_t^j$. In addition, and similar to Epstein and Zin (1991), we use the real aggregate return R_t^{Σ} as a proxy for $R_{C,t}$, that is, the gross return on the asset that pays aggregate consumption as its dividend.[23] In the usual way, we generate overidentifying restrictions through the use of instrumental variables (IVs), which we describe next.

4.2 IVs and Time Series Characteristics

Because lagged endogenous variables are natural IVs, the sample own and cross-autocorrelations of endogenous variables in the moment conditions of our model are of particular interest. Panel A of Table 3 presents these correlations for the asset market conditions for one and two year lags. Consistent with the literature, there is relatively low cross-autocorrelations (for annualized observations) in per capita consumption growth G_t^C and aggregate ERP ($\tilde{R}_t^{\Sigma}$) in our sample. Not surprisingly, there is high contemporaneous correlation (0.75) between CGM sector ERP ($\tilde{R}_t^j$) and aggregate returns. But we also find that the own and cross-autocorrelation of $\tilde{R}_t^j$ are essentially commensurate with those observed for aggregate returns. For example, the correlation between current and one-period lagged aggregate return is -0.08, while the corresponding correlation is about -0.06 for the industry returns. Furthermore, the cross-autocorrelation between lagged industry returns and current consumption growth is not significantly different than the corresponding correlations between market returns and consumption growth. In sum, utilizing industry returns in IV estimation would add information but not necessarily resolve the weak IV problem in structural estimation of asset pricing models.

In contrast to Panel A of Table 3, Panel B shows very high own and cross-autocorrelations in the industry-level industry investment (I_t) and materials input (H_t) in the CGM sector. The high serial

[23] Hence, in our empirical tests, $M_{t,t+1} = P_t \left(\frac{\alpha^{\theta} \left(\frac{C_{t+1}}{C_t} \right)^{-\eta\theta} (R_t^{\Sigma})^{\theta-1}}{P_{t+1}} \right)$.

correlation in capital investment is also noted elsewhere in the literature (e.g., Eberly, Rebelo and Vincent 2012). Thus, as we mentioned already, there is a potential here that industry investment and material inputs can be utilized as strong IVs in empirical estimation, to which we now turn.

4.3 Estimation with Equity Returns

To set up useful benchmarks, we first follow the standard approach and estimate (γ, η) by using the equilibrium asset return equation (49) and setting up moment conditions in terms of the aggregate equity risk premium, that is, $\mathbb{E}_t\left[\Gamma_t^{\Sigma}\left\{M_{t,t+1}\tilde{R}_{t+1}^{\Sigma}\right\}\right] = 0$. Here, $\mathbf{\Gamma}_t^{\Sigma}$ is a vector of IVs that—in the usual fashion—use only market and risk free returns, with aggregate consumption data. Similar to the literature, we use lagged values of the aggregate risk premium as well as lagged consumption growth as IVs. In addition to the lagged covariates, we use an IV that is a nonlinear function of $(\tilde{R}_{t-1}^{\Sigma}, G_{t-1}^{C})$ to allow for nonlinear effects:

$$\Gamma_{nl,t}^{\Sigma} \equiv \left(\tilde{R}_{t-1}^{\Sigma}, G_{t-1}^{C}, \tilde{R}_{t-1}^{\Sigma} \times G_{t-1}^{C}, (\tilde{R}_{t-1}^{\Sigma})^2, (G_{t-1}^{C})^2\right). \tag{50}$$

The first two rows of Table 4 display the estimation results, as well as tests of over-identifying restrictions through the chi-square statistic $\chi^2(DF)$. Based on the SDF for Epstein and Zin (1989) preferences, we estimate $\hat{\theta}$ and $\hat{\eta}$ and deduce the implied $\hat{\gamma}$, which is displayed in parentheses next to $\hat{\theta}$. For various combinations of lagged covariates $(\tilde{R}_{t-\ell}^{\Sigma}, G_{t-\ell}^{C}), \ell = 1, 2, 3, 4$, we either find insignificant estimates or significant estimates with implausibly large values of risk aversion and very low values of IES. We show estimation using a linear IV with lags up to two years and the nonlinear IV. Moreover, the p-values of the J-statistics imply rejection of the overidentifying restrictions. The fragility of the estimates, with respect to both the coefficient values and significance, is indicative of the weak identification noted in the literature (Epstein and Zin 1991, Stock and Wright 2000).[24]

Because we have sectoral—that is, consumer manufactured goods—equity returns, we can undertake estimation using moment conditions in Equation (49) for industry returns, that is, utilize orthogonality conditions of the form $\mathbb{E}_t\left[\Gamma_t^{j}\left\{M_{t,t+1}\tilde{R}_{t+1}^{j}\right\}\right] = 0$. For this estimation, we also construct a nonlinear IV

[24] Ait-Sahalia et al (2004) report that using basic non-durables and service consumption series annual data from NIPA yields very high estimates of risk aversion, which is consistent with our estimates in Table 4. However, when using luxury sales data alone, Ait-Sahalia et al. (2004) find estimates of risk aversion in the 4-10 range.

along the lines of (50). The results are displayed in the third and fourth rows of Table 4. With lagged sectoral ERP and lagged consumption growth as IVs, we get significant estimates, but the estimates of risk aversion remain implausibly high. And using the non-linear sector-based IV yields insignificant estimates. Finally, we can combine aggregate and industry returns by defining $\tilde{R}^{\Sigma,j}_{t+1} \equiv R^{\Sigma}_{t+1} - R^{j}_{t+1}$ and use moment restrictions of the form $\mathbb{E}_t\left[\Gamma^{\Sigma,j}_t \left\{M_{t,t+1}\tilde{R}^{\Sigma,j}_{t+1}\right\}\right] = 0$. For the IVs, here we employ one year lagged market and sector returns. The results, which are barely significant for theta and insignificant for the IES, also imply an implausibly high value of risk aversion.

In sum, estimating parameters of consumer preferences using only moment restrictions from the asset market equilibrium condition—and utilizing both aggregate and consumer goods manufacturing industry returns—is consistent with the equity premium puzzle. That is, explaining the ERP in the data through only variations in aggregate consumption (or, equivalently, variations in aggregate income and the perfect price index) requires implausibly large values of risk aversion. Furthermore, the estimates are indicative of identification fragility due to weak IVs, as has been highlighted in the literature.

4.4 Estimation Using General Equilibrium Restrictions

We now use orthogonality conditions from the equilibrium path in both product and asset markets—namely, Equations (47)-(49)—to estimate the vector of unknown parameters (γ, η, σ). As we mentioned before, there are well known pitfalls in adding moment conditions with fixed sample size, especially if they include weak moment conditions since increased estimation efficiency comes at the cost of increasing estimator bias (Han and Phillips 2006, Newey and Windmeijer 2009). Furthermore, a large number of moment conditions raises the likelihood of mis-specification bias through utilization of possible invalid restrictions (Andrews 1999). Consequently, our test design for GE estimation uses the two product market moment conditions and two of the excess return restrictions discussed above— that is, aggregate or industry ERP or the excess aggregate return over the industry return. However, for robustness we also undertake estimation with all five moment conditions: two from the product market side and three types of excess return restrictions from the asset market side.

Because of the increased number of moment restrictions, we construct the IVs parsimoniously. Hence, we use only lagged returns for the asset market condition, and lagged control variables—$H_{t-\ell}$ and $I_{t-\ell}$—for the corresponding optimality conditions ((47) and (48), respectively). Collectively, we denote these IVs as $\Pi_N, N = 1, 2, 3, 4$, and will specify these when discussing the results.

The results are displayed in Table 5. The first row is based on four moment conditions that include the aggregate and industry ERP restrictions, along with product market restrictions (47) and (48). The IV $\Pi_1 = \tilde{R}^{\Sigma}_{t-\ell}, \tilde{R}^{j}_{t-\ell}, \ell = 1, 2, 3; H_{t-\ell}, I_{t-\ell}, \ell = 1, 2, 3$. The IV Π_2 differs from Π_1 by setting $H_{t-\ell}, \ell = 1$, to examine the implications of asymmetry between short run utilization of material inputs versus the long run effects of investment on capital stock. The IV Π_3 differs from Π_2 by using only two year lags for the market risk premium, that is, $\tilde{R}^{\Sigma}_{t-\ell}$, $\ell = 1, 2$. The specification associated Π_4 substitutes the moment restriction $\mathbb{E}_t\left[\Gamma^{j}_{-1}\left\{M_{t,t+1}(R^{\Sigma}_t - R^{j}_t)\right\}\right] = 0$ for the restriction $\mathbb{E}_t\left[\Gamma^{\Sigma}_{-2}\left\{M_{t,t+1}\tilde{R}^{\Sigma}_t\right\}\right] = 0$ and uses $R^{\Sigma}_{t-1}, R^{j}_{t-1}$, $I_{t-\ell}, \ell = 1, 2, 3$, and $H_{t-\ell}, \ell = 1$ as IVs. Finally, the specification Π_5 uses all five moment restrictions with the (equity return) IVs $\tilde{R}^{\Sigma}_{t-1}, \tilde{R}^{j}_{t-1}, R^{\Sigma}_{t-1}$ and R^{j}_{t-1} and industry IVs in Π_2.

In striking contrast to the previous estimation results (Table 4), the point estimates of the unknown parameter vector (γ, η, σ) in each specification are statistically significant, with plausible values, and quite robust to changes in IVs and moment restrictions. The estimates of risk aversion (rounded to the nearest whole number) in all specifications are centered around 5, which is well within the range of risk aversion values (2-10) considered plausible (or reasonable) by the literature (Mehra and Prescott 1985, Bansal and Yaron 2004). Comparing the statistically significant risk aversion estimates in Table 4 with those in Table 5, it is clear that industry-level product and asset market moment conditions drastically reduce the risk aversion required to explain aggregate returns and consumption data. This is consistent with the ERP constructed in (44) based on aggregate and industry-based risk factors. We note that numerical calculations in Table 2, calibrated with the parameter values (up to rounding) in the first two rows of Table 5, generate an ERP lower than the average industry ERP in the data. This could be due to linear analytic approximations around a fictitious deterministic steady state, whereas the GMM estimation in Table 5 allows nonlinear interactions of the underlying risk factors.

Next, the IES estimates in Table 5 cluster around 0.2, and the empirical tests reject the null of zero IES at p-values significantly below 0.01. As we mentioned earlier, there is no consensus in the literature on the value of IES. With additive utility and using aggregate data, Hall (1988) finds low point estimates of IES that are not significantly different from zero, and sets an upper bound of 0.1. Similarly, Campbell (1999) uses aggregate data and finds low estimates of the IES. More recently, using a novel source of quasi-experimental variation in interest rates (to address the challenge of finding exogenous variations in interest rates) and exploiting "notched" mortgage loan schedules in the UK that imply discrete jumps in mortgage rates at critical loan thresholds, Best et al. (2020) use individual home refinancing data to estimate IES around 0.1. More generally, a large number of studies using micro data report IES estimates below 0.4 (Havránek 2015, Havránek et al. 2015). However, the literature also reports estimates exceeding 1 (Vissing-Jorgensen 2002, Gruber 2013).

Note also that the estimates of $\hat{\theta}$ are reliably positive and strictly less than 1. This implies a negative relation of aggregate returns and the SDF, that is, declines in market returns generate increases in the SDF and hence the risk premium, which is empirically appealing. Indeed, this property is present in empirical parameterizations of Epstein-Zin preferences commonly used in the literature (e.g., Bansal and Yaron 2004, Croce 2014), where the IES exceeds 1 (that is, $\frac{1}{\eta} > 1$) so that θ is negative (since risk aversion is generally taken to be higher than 1). However, the estimates in Table 5 imply a preference for late resolution of uncertainty because $\hat{\gamma}(1/\hat{\eta}) < 1$, which is consistent with other production and investment based asset pricing studies in the literature (e.g., Papanikolau 2011).[25]

Turning to the parameters relating to the CES product variety preferences, the estimates of the elasticity of substitution (σ) for the manufactured consumer goods sector cluster around 2. These estimates significantly exceed 1, which is the requirement of the theoretical model. The estimation of the CES in Dixit and Stiglitz (1977) preferences at the industry level is of long-standing interest in the applied economics literature. Broda and Weinstein (2006) use import data on product varieties from 1972-1988 and 1990-2001 and report median ES estimates for differentiated goods (at the four-digit Standard International Trade Classification (SITC) level) of 2.1 for 1990-2001 (and 2.5 for 19972-1988). Hottman and Monarch (2018) use import data from 1998 to 2014 and report the median ES for tradable consumer goods (at the four digit NAICS level) of 2.75. Thus, our estimates appear reasonably consistent with estimates of sectoral ES in the literature.

In sum, the results in Table 5 support the view that using the general equilibrium system of moment conditions and IVs—with both industry production and asset returns and market returns—lead to strong identification of the parameters related to consumer preferences in a general equilibrium model.

[25] Evidence supporting the assumption of preference for early resolution of uncertainty is mixed. While some studies show that parameterization consistent with preference for early resolution can help match aggregate asset pricing moments (e.g., Bansal and Yaron 2004), cross-sectional studies that examine the relation of risk-premia to investment maturities present confounding evidence: Binsbergen et al. (2012) find that claims to long-maturity dividends carry lower risk premia; and Giglio et al. (2015) and Weber (2018) find negative relation of risk premia and duration of risky cash flows.

This view is based on the uniform statistical significance of the point estimates for the entire parameter vector in Table 5, their low dispersion (compared with Table 4 for (γ, η)), and economic appeal.

5 Euler Conditions, IVs and Estimation Efficiency

The general equilibrium estimation in Table 5 differs from the estimation using only asset returns (Table 4) in two ways: we add industry-based moment conditions and IVs, but also increase the number of estimated parameters from two (γ, η) to three (γ, η, σ). It is useful to disentangle the effects of these two changes in enhancing the identification in Table 5 (relative to Table 4). To this end, we undertake estimation by using the moment conditions and IVs in Π_1, but fixing the value of $\sigma = 2.03$, consistent with the estimate in Table 5. As seen in the first row of Table 6, we find reliable estimates of (γ, η) essentially identical to that in Table 5 (for Π_1). We conclude that the use of industry moment conditions and IVs, rather than the higher number of estimated parameters *per se,* appears to improve identification of risk aversion and IES.

Turning to the role of the moment conditions derived from the industry equilibrium, the improved identification or estimation efficiency in Table 5 could arise from the use of one or both of the Euler conditions. Hence, it is useful to examine the relative roles of the intertemporal Euler condition for investment and the intratemporal materials input Euler condition.

We first exclude one of the product market equilibrium restrictions. We then use only the product market restrictions. To facilitate comparison with Table 5, we maintain the IVs for moment conditions on returns given in Π_1. (We label the specifications with "bars" to reflect constrained estimation.) We first use only the investment Euler condition along with the asset returns moment conditions and IVs in Π_1. We are unable to reliably estimate all three parameters (γ, η, σ). However, using $\sigma = 2.03$ and estimating only (γ, η) generates estimates similar to Table 5, as seen in the second row of Table 6. Comparing these estimates with Table 4, we conclude that adding the intertemporal investment Euler equation significantly enhances identification of risk aversion and IES *conditional* on knowledge of σ. The third row presents estimation results when we include the material optimality condition but exclude the investment Euler condition. In this case, we obtain a reliable estimate of ES, which is consistent with the estimation in Table 5. However, the risk aversion and IES estimates are highly distorted relative to the estimates in Table 5—the former being too high and the latter being too low; and there is also a significant decline in estimation precision of θ. Hence, we conclude that estimation efficiency and/or the economic appeal of estimates appear to significantly deteriorate if either of the two product market conditions are excluded.

The fourth row of Table 6 shows estimates from utilizing only the production and investment equilibrium conditions. The estimates are significant and close to the estimates in Table 5. However, the risk aversion estimate is lower than the robust estimate in Table 5. We conclude that using the general equilibrium moment restrictions from product and asset markets (as in Table 5) together improve identification and estimation efficiency.

In sum, utilizing both the intertemporal and intratemporal industry production Euler conditions as well as the strong industry-based IVs are required for enhancing the precision and economic appeal of consumer preferences related parameters. However, adding lagged industry and market returns as IVs improves estimation efficiency.

6 Conclusion

We develop an infinite-horizon multi-sectoral production-based general equilibrium asset pricing model with competitive industries, where the representative consumer is endowed with recursive, CES preferences on baskets of goods and firms choose material inputs and capital investment. We analyze the determinants of industry-specific equity risk premium (ERP) for consumer goods manufacturing sectors, and utilize equilibrium restrictions on product and asset markets to estimate salient parameters of consumer preferences, namely, risk aversion, and intertemporal and intratemporal elasticities of substitution (IES and ES, respectively). Our empirical tests utilize U.S. stock and risk free returns at the market level as well as equity returns and production data for the U.S. manufacturing industry.

Quantitative analysis of the model using log-linear expansions around the model's steady state generates a linear intertemporal multifactor model for industry-level ERP, where the risk factors are endogenously determined by the variance-covariance matrix of shocks to aggregate income and price index, as well as industry-specific productivity and input price shocks. Numerical analysis with judicious calibration shows that the model generates sizeable ERP with reasonable levels of risk aversion and low levels of IES. In particular, the model matches the observed ERP in the consumer durables sector with risk aversion below 10.

Industry production and investment data are characterized by high levels of own and cross autocorrelation, which makes them potentially strong IVs. GMM estimation indeed shows that using the industry Euler conditions and IVs facilitates identification. The point estimates of the representative consumer's risk aversion, IES and ES are uniformly statistically significant; have relatively low dispersion across different IVs; and are economically appealing, based on the literature. Tests of subsets of moment conditions of the model indicate that Euler conditions from the industry production equilibrium and industry IVs are critical for identification, but instruments based on market and industry returns also enhance identification.

The framework developed in this paper can be extended in several directions in future research. We have focused on the asset pricing implications and estimation of consumer goods sectors. A natural extension of our study is to examine asset pricing in intermediate goods industries. Furthermore, for analytic tractability, we have not considered endogenous labor income in this study; a more realistic model with endogenous labor input in production, with attendant labor income for consumers, would be a natural extension in future research. The production environment can also be enhanced by allowing innovation (or growth) in industry productivity and allowing entry and exit.

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Table 1. Calibration for Numerical Calculations

	Aggregate Parameters				Global Parameters							
Parameter	Φ_w	Φ_π	$\Phi_{w\pi}$	ρ_w	ρ_π	α	υ	δ			η	σ
Value	2.47	0.76	4.8 e-05	0.999	0.999	0.97	0.1	0.2			5.3	2
					Sector-Specific Parameters							
Sector	ψ_K^j	ψ_H^j	Φ_{aw}^j	$\Phi_{a\pi}^j$	Φ_{xw}^j	$\Phi_{x\pi}^j$	Φ_{zw}^j	$\Phi_{z\pi}^j$	ρ_a^j	ρ_x^j	ρ_z^j	ϕ^j
Consumer Durables	0.21	0.79	4.0 e-05	4.2 e-05	3.9 e-04	5.5 e-05	3.0 e-04	-1.2 e-07	0.930	0.966	0.962	0.35
Consumer Non-Durables	0.18	0.82	7.0 e-05	1.7 e-05	6.0 e-04	1.2 e-04	2.9 e-04	-7.8 e-06	0.929	0.970	0.965	0.35

Notes to Table: This table displays the parameterization used for the numerical computations of the model equilibrium presented in Section 3. ρ_w, ρ_π are the estimated autocorrelation coefficients of the first order autoregressive processes of annual log per capita U.S. income ($w_t = \log W_t$) and log price index ($\pi_t = \log\ (W_t/C_t)$, C_t = annual per capita U.S. consumption), that is, $w_t = \rho_w w_{t-1} + \varepsilon_{wt}$ and $\pi_t = \rho_\pi \pi_{t-1} + \varepsilon_{\pi t}$. Φ_w, Φ_π and $\Phi_{w\pi}$ are the volatilities and covariance of the estimated income and price index shocks ε_{wt} and $\varepsilon_{\pi t}$. The intertemporal elasticity of substitution (η^{-1}) and intratemporal product elasticity of substitution are calibrated from the structural estimation of the model (in Table 5). The production and sectoral parameters are calibrated for the U.S. manufacturing sector for 1958-2016 using the NBER-CES (annual) data. The output elasticities of capital and material inputs ψ_K^j, ψ_H^j are based on estimation of the Cobb-Douglas production function in (9), while the autocorrelation coefficients of log productivity shocks and materials and capital input prices ($\rho_a^j, \rho_x^j, \rho_z^j$), as well as the covariances of industry productivity and input shocks with w_t and π_t ($\Phi_{aw}^j, \Phi_{a\pi}^j, \Phi_{xw}^j, \Phi_{x\pi}^j, \Phi_{zw}^j, \Phi_{z\pi}^j$) are estimated from the first order autoregressive processes of log productivity and intermediate goods prices during 1958-2016.

Table 2. Sectoral Equity Risk Premium

	$E[r^j - r^f]$	β^j_w	β^j_π	$\beta^j_{w\pi}$	β^j_{aw}	$\beta^j_{a\pi}$	β^j_{xw}	$\beta^j_{x\pi}$	β^j_{zw}	$\beta^j_{z\pi}$
A: Consumer Durables										
Data	6.71									
Model ($\gamma = 5$, est.)	3.79	85.65	-68.39	19.63	26.10	-20.88	-33.30	26.64	-0.08	0.06
Model ($\gamma = 7.5$)	5.53	127.02	-109.90	20.59	38.70	-33.55	-49.38	42.81	-0.11	0.10
Model ($\gamma = 10$)	7.28	168.39	-151.29	21.69	51.31	-46.19	-65.47	58.93	-0.15	0.14
B: Consumer Non-Durables										
Data	10.05									
Model ($\gamma = 5$, est.)	5.89	135.30	-108.11	31.10	41.73	-33.38	-55.38	44.30	- 0.09	0.07
Model ($\gamma = 7.5$)	8.60	200.65	-173.73	32.67	61.88	-53.64	-82.13	71.20	-0.13	0.11
Model ($\gamma = 10$)	11.32	266.00	-239.162	34.45	82.03	-73.84	-108.88	98.01	-0.17	0.16

Notes to Table: This Table presents the unconditional equity risk premium (ERP) and factor loadings based on the model presented in Section 3 for consumer durables and consumer non-durables sectors and using the calibration of parameters in Table 1. The results are shown for risk aversion levels of $\gamma = 5$ (the estimated value from the model), 7.5, and 10. The sample period is 1958-2016 (annual) and data sources are described in text.

Table 3. Matrix of Autocorrelation Coefficients

Variables	G_t^C	G_{t-1}^C	G_{t-2}^C	$\tilde{R}_t^{\Sigma}$	$\tilde{R}_{t-1}^{\Sigma}$	$\tilde{R}_{t-2}^{\Sigma}$	$\tilde{R}_t^j$	$\tilde{R}_{t-1}^j$	$\tilde{R}_{t-2}^j$
G_t^C	1.000								
G_{t-1}^C	0.910	1.000							
G_{t-2}^C	0.780	0.905	1.000						
$\tilde{R}_t^{\Sigma}$	0.104	-0.072	-0.025	1.000					
$\tilde{R}_{t-1}^{\Sigma}$	0.115	0.121	-0.062	-0.078	1.000				
$\tilde{R}_{t-2}^{\Sigma}$	-0.084	0.100	0.108	-0.228	-0.077	1.000			
$\tilde{R}_t^j$	0.185	-0.152	0.030	0.751	-0.216	-0.415	1.000		
$\tilde{R}_{t-1}^j$	0.217	0.217	0.011	-0.135	0.755	-0.208	-0.057	1.000	
$\tilde{R}_{t-2}^j$	-0.002	0.194	0.191	-0.205	-0.134	0.756	-0.338	-0.034	1.000

B: Industry Production Variables

Variables	I_t	I_{t-1}	I_{t-2}	H_t	H_{t-1}	H_{t-2}
I_t	1.000					
I_{t-1}	0.971	1.000				
I_{t-2}	0.934	0.972	1.000			
H_t	0.972	0.955	0.941	1.000		
H_{t-1}	0.964	0.973	0.984	0.990	1.000	
H_{t-2}	0.943	0.966	0.973	0.979	0.990	1.000

Notes to Table: This table uses annual data from 1958 to 2016. Panel A presents own and cross-autocorrelations of yearly growth rates of U.S. per capita consumption growth G_t^C, annualized aggregate (market) ERP $\tilde{R}_t^{\Sigma} = R_t^{\Sigma} - R_t^f$, where R_t^{Σ} is the value-weighted CRSP return and R_t^f is the annual risk free rate, and annualized ERP $\tilde{R}_t^j$ from value-weighted index monthly returns of all firms in all manufacturing industries in the NBER-CES database. Panel B presents own and cross-autocorrelations of mean annual industry investment (I), materials input (H), and average productivity (A) using the NBER-CES manufacturing database with the sample period 1958-2016.

Table 4. Estimation using Asset Market Data

IV	$\hat{\theta}(\hat{\gamma})$	SE($\hat{\theta}$)	$\hat{\eta}^{-1}$	SE($\hat{\eta}$)	J	DF	p-Value
Γ^{Σ}_{-2}	-0.011 (41)	1.260	0.27e-03	4.232e+04	0.624	3	0.891
Γ^{Σ}_{nl}	2.302 (169)***	0.471	0.013***	27.652	1.414	3	0.702
Γ^{j}_{-2}	2.377 (126)***	0.446	0.019***	9.799	1.607	3	0.658
Γ^{j}_{nl}	-0.800 (47)	0.851	-0.02	66.239	1.933	4	0.748
$\Gamma^{\Sigma,j}_{-1}$	1.995 (56)*	1.204	0.02	59.2690	0.010	1	0.605

Notes to Table: This table presents the point estimates, standard errors, and J statistics from two step GMM estimation (with heteroskedasticity- and autocorrelation-consistent inference) of risk aversion ($\hat{\gamma}$) and the intertemporal elasticity of substitution of consumption ($\hat{\eta}^{-1}$) from moment restrictions derived from the asset market equilibrium condition, using data on aggregate ($\tilde{R}^{\Sigma}_t$) and manufacturing industry ($\tilde{R}^{j}_t$) equity risk premium. The sample period is 1958-2016 (annual) and the data are described in the text. The p-value of J statistics are calculated with Chi-square distribution with degrees of freedom DF. Statistical significance at 10%, 5%, and 1% levels are denoted by *,** , and ***, respectively.

Table 5. Estimation of the General Equilibrium Model

IV	$\hat{\theta}(\hat{\gamma})$	SE($\hat{\theta}$)	$\hat{\eta}^{-1}$	SE($\hat{\eta}$)	$\hat{\sigma}$	SE($\hat{\sigma}$)	χ^2	DF	p-Value
Π_1	0.879 (5)***	0.150	0.19***	0.626	2.03***	0.022	8.155	13	0.833
Π_2	0.865 (5)***	0.185	0.19***	0.992	2.02***	0.023	6.049	11	0.870
Π_3	0.864 (6)***	0.188	0.19***	1.1540	2.02***	0.023	6.048	10	0.811
Π_4	0.861 (5)***	0.190	0.19***	1.059	2.02***	0.030	5.979	10	0.817
Π_5	0.871 (5)***	0.182	0.19***	0.922	2.02***	0.021	6.074	12	0.912

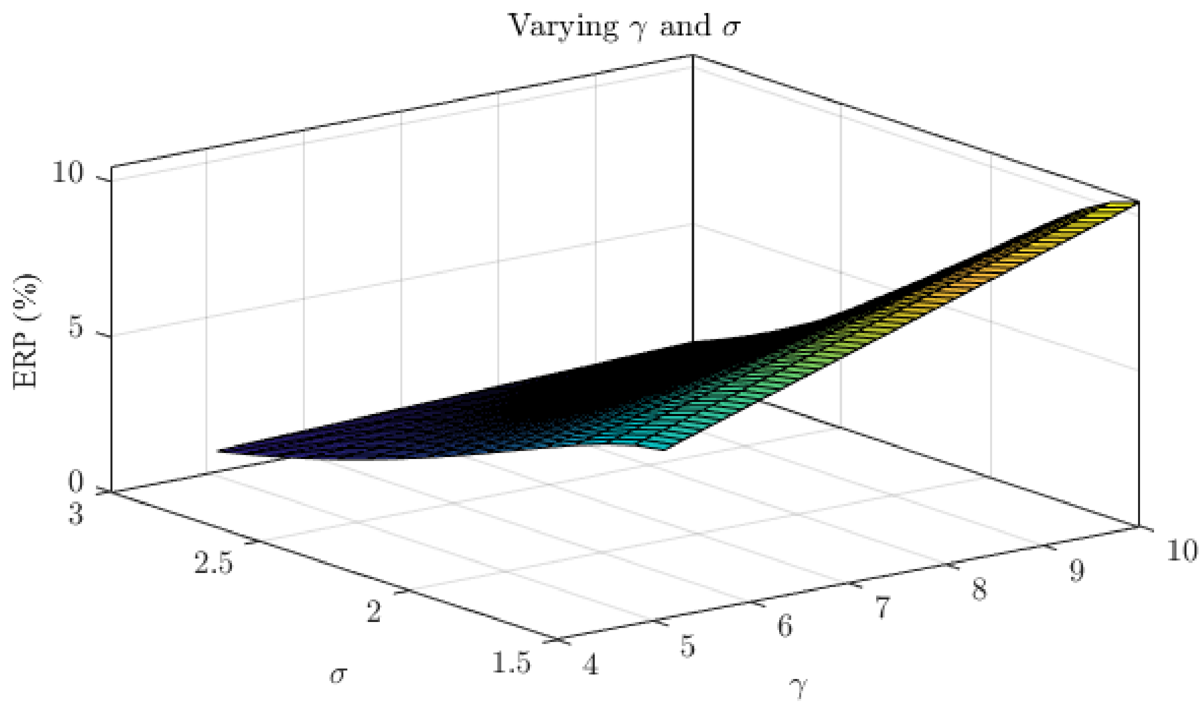
Notes to Table: This table presents the point estimates, standard errors, and J statistics from two step GMM estimation (with heteroskedasticity- and autocorrelation-consistent inference) of risk aversion ($\hat{\gamma}$), the intertemporal elasticity of substitution of consumption ($\hat{\eta}^{-1}$), and the intratemporal elasticity of substitution ($\hat{\sigma}$) from data on U.S. consumer goods manufacturing industries. The sample period is 1958-2016 (annual) and data sources are described in text. The moment restrictions are derived from the Euler conditions for capital investment and materials input, as well as the asset markets considered in Table 4. The other parameters used in the moment conditions are set as $\alpha = 0.97$, $\phi^j = 0.35$, $\psi_H^j = 0.75$ and $\psi_K^j = 0.25$. The sample period is 1958-2016 (annual) and data sources are described in text. The p-value of J statistics are calculated with Chi-square distribution with degrees of freedom DF. Statistical significance at 10%, 5%, and 1% levels are denoted by *,** , and ***, respectively.

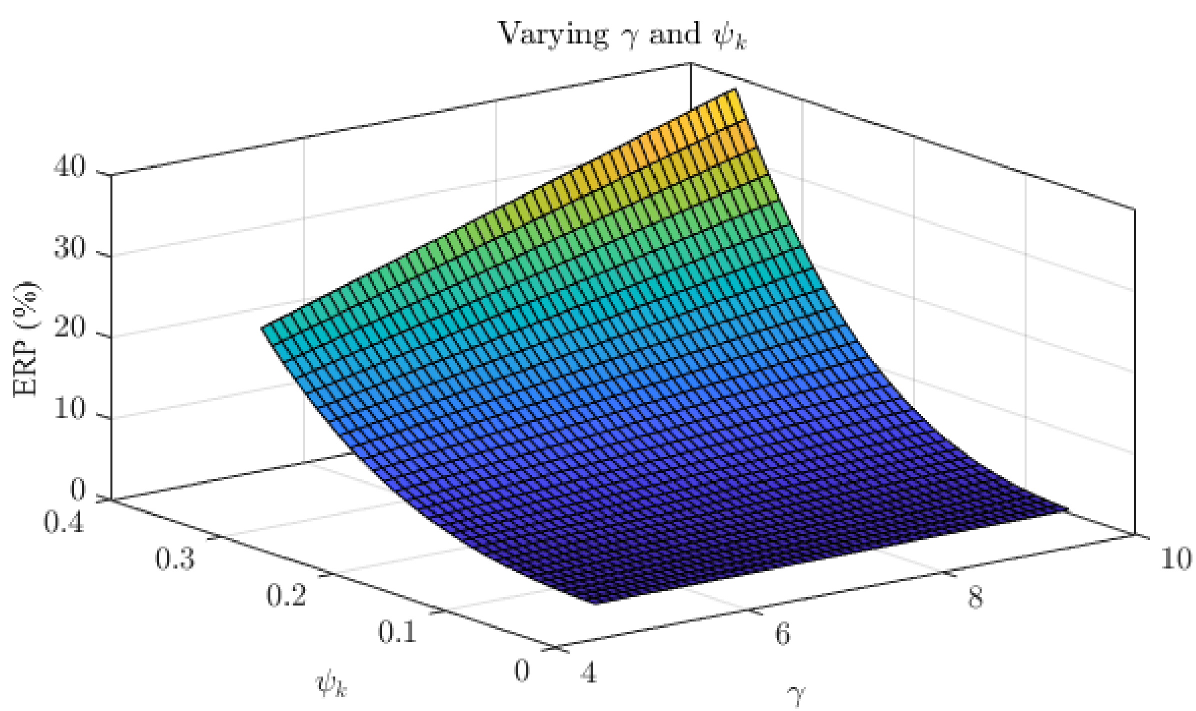
Table 6. Role of Moment Conditions

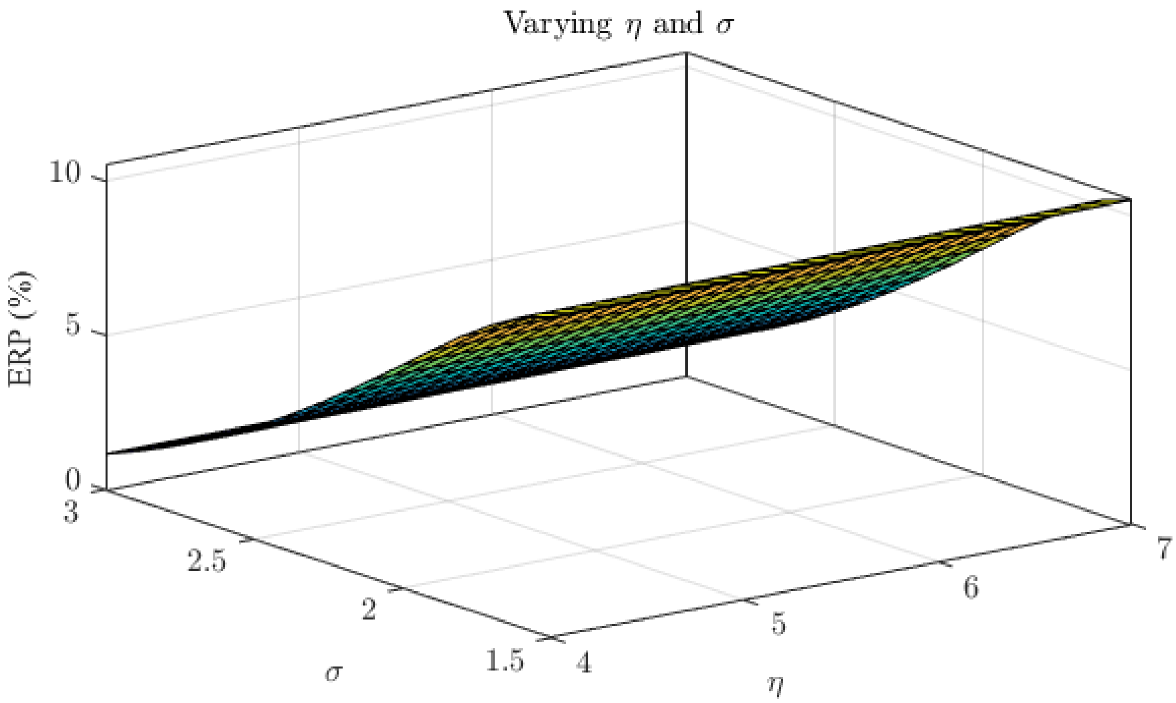
Euler Equations	$\hat{\theta}(\hat{\gamma})$	SE($\hat{\theta}$)	$\hat{\eta}^{-1}$	SE($\hat{\eta}$)	$\hat{\sigma}$	SE($\hat{\sigma}$)	χ^2	DF	p-Value
All, $\sigma = 2.03$	0.888 (5)***	0.129	0.19***	0.515			5.707	14	0.973
Investment, Asset Markets, $\sigma = 2.03$	0.847 (5)***	0.222	0.19***	1.5442			7.284	10	0.698
Material Inputs, Asset Markets	5.817 (836.2)*	3.049	0.01***	29.817	2.04***	0.044	2.892	3	0.409
Investment, Material Inputs	0.887 (4)*	0.146	0.23**	0.529	2.05***	0.043	5.482	5	0.360

Notes to Table: This table presents the point estimates, standard errors, and J statistics from two step GMM estimation (with heteroskedasticity- and autocorrelation-consistent inference) of risk aversion ($\hat{\gamma}$), the intertemporal elasticity of substitution of consumption ($\hat{\eta}^{-1}$), and the intratemporal elasticity of substitution ($\hat{\sigma}$) from data on U.S. consumer goods manufacturing industries. The sample period is 1958-2016 (annual) and data sources are described in text. The combinations of moment restrictions are derived from the Euler conditions for capital investment and materials input, as well as the asset markets considered in Table 4. The other parameters used in the moment conditions are set as $\alpha = 0.97$, $\phi^j = 0.35$, $\psi^j_H = 0.75$ and $\psi^j_K = 0.25$. The p-value of J statistics are calculated with Chi-square distribution with degrees of freedom DF. Statistical significance at 10%, 5%, and 1% levels are denoted by *,** , and ***, respectively.

Figure 1: Consumer Durables Manufacturing Industry Risk Premium and Various Parameters







Notes to Figure: This figure graphically displays, through three-dimensional plots, the relation of equilibrium equity risk premium for the consumer durables manufacturing industry, with various combinations of consumer preference parameters: risk aversion (γ), (inverse of) intertemporal elasticity of substitution (η), intratemporal elasticity of substitution (σ), and the production elasticity of capital (ψ_K). The sample period is 1958-2016 (annual) and data sources are described in text.