Ch 9 Homework Solutions: Waiting Lines

Problem 9-1

With a normally distributed service time, we have:

$$\mu = \frac{60}{1.5} = 40 \text{ per hour}$$

$$\sigma = \frac{.2}{60} = .00333 \text{ hr}$$

$$\sigma^2 = .00001111$$

$$L_q = \frac{(30)^2(.00001111) + (30/40)^2}{2(1-30/40)} = \frac{.01 + .5625}{.5} = 1.145$$

$$L_S = 1.145 + .75 = 1.895$$

$$W_q = 1.895/30 = .06317 \text{ hr} = 3.79 \text{ minutes}$$

$$W_S = .06317 + .025 = .08817 \text{ hr} = 5.29 \text{ minutes}$$

The expense is (.08817)(12) = $1.06 per truck.

Problem 9-2

We use a single-channel model with any service time distribution.

$$\sigma^2 = (2/8)^2 = .0625 \text{ days}$$

$$L_q = \frac{(10)^2(.0625) + (10/11)^2}{2(1-10/11)} = \frac{7.0764}{.181818} = 38.92 \text{ claims}$$

$$L_S = 38.92 + 10/11 = 39.83 \text{ claims}$$

$$W_q = 38.92/10 = 3.89 \text{ days}$$

$$W_S = 3.89 + 1/11 = 3.98 \text{ days average time in system}$$

Problem 9-3

We wish to compute M such that P_M = .05 or less.

Given λ = 20 arrivals per hour and μ = 60/2.5 = 24 customers per hour

We now use a trial and error approach.

M	P_0	P_M
5	.2506	.1007
7	.2172	.0606
8	.2067	.0481

Therefore, the restaurant should allow for 8 spaces including the service area space.

Problem 9-4

Since 6 cars can wait, the maximum number in the system includes 6 plus one auto being serviced: M = 7.

$$P_0 = \frac{1-(10/12)}{1-(10/12)^8} = .2172$$

$P_w = 1 - .2172 = .7828$ or about 78.3% busy working inspections

$P_m = (10/12)^7(.2172) = .061$ or a bit over 6% of the customers lost

Problem 9-5

For one crew working with exponentially distributed service time:

$$L_S = \frac{3}{4-3} = 3 \text{ skidders}$$

Cost ($/hr) = (60)(3) + (5)(8.50) = $222.50 per hour skidder wait plus crew cost.

With the off-loader option, normally distributed:

μ = 7 skidders per hour

$\sigma^2 = (2/60)^2 = .001111$

$$L_q = \frac{(3)^2(.001111) + (3/7)^2}{2(1-3/7)} = \frac{.193671}{1.14286} = .16946 \text{ skidders}$$

$$L_S = .16946 + .42857 = .59803 \text{ skidders}$$

Cost ($/hr) = (60)(.59803) + (2)(5)(8.50) = 120.88 per hour skidder wait plus off-loader cost

With two crews $P_o = .45461$

$$L_S = \frac{(3)(4)(3/4)^2}{(1)!(8-3)^2}(.45461) + \frac{3}{4} = .8727 \text{ skidders}$$

Cost ($/hr) = (.8727)(60) + 14 + (2)(5)(8.50) = $151.36 per hour skidder wait plus two crews

The most effective option is to use the off-loader.

Problem 9-6

In order to ensure the arrivals can be handled, at least 500/60 = 8.3333 players must be provided. So for a first attempt, we try 9 players:

$P_o = .00012$

$$L_S = \frac{(500)(60)(500/60)^9}{(8)!(540-500)^2}(.00012) + \frac{500}{60} = 19.15$$

$$W_S = \frac{19.15}{500} = .0383 \text{ hours/fan} = 2.3 \text{ minutes/fan}$$

Consequently, 9 players are adequate.

Problem 9-7

With 2 agents, the mean length of the queue, L_q, is:

$$L_q = \frac{(30)(20)(30/20)^2}{(1)!(40-30)^2}(.1429) = 1.9292$$

If he expands to 3 agents, where $P_o = .21053$:

$$L_q = \frac{(30(20)(30/20)^3}{(2)!(60-30)^3}(.21053) = .2368$$

This is a reduction in mean line length which is more than 50%. Therefore, three agents are adequate.