Ch 3 Homework Solutions: Inventory

Problem 3-1

$$\text{Number of orders per year } = \sqrt{\frac{(220{,}000)(.18)}{(2)(30)}} = 25.69$$

Problem 3-2

$$\text{Optimal dollars per order } = \sqrt{\frac{(2)(28{,}000)(48)}{.23}} = \$3{,}418.62$$

Problem 3-3

$$\text{Months of supply per order } = 12\sqrt{\frac{(2)(45)}{(96{,}000)(.22)}} = .784 \text{ months}$$

Problem 3-4

$$\sqrt{\frac{(2)(72{,}000)(40)}{(8)^2(.20)}} = 671 \text{ units}$$

Since the containers require 10 ft^2 of space, the required space for 671 units is:

Area = (671)(10) = 6710 ft^2

With the 5000 ft^2 restriction, at most 5000/10 = 500 containers may be ordered.

We now evaluate the costs of the alternatives:

Alternative	Annual Ordering Cost	Annual Carrying Cost	Total Annual Cost
671/order	$\frac{(72{,}000)(40)}{(8)(671)} = 537$	$\frac{(8)(.20)(671)}{2} = 537$	1074
500/order	$\frac{(72{,}000)(40)}{(8)(500)} = 720$	$\frac{(8)(.20)(500)}{2} = 400$	1120

Adding the additional storage area would save $46 per year, and it is not justified economically.

The daily demand is 9000/300 = 30 units/day.

The 500 unit capacity represents 500/30 = 16.67 days' supply

Problem 3-5

Given for last year: N_o = 9.3 orders per year
C = .30 of value per year
P = $46 per order

Solving for the annual dollar value of A:

$$N_o^2 = \frac{AC}{2P}$$

$$(9.3)^2 = \frac{.30A}{(2)(46)}$$

$$A = \frac{(9.3)^2(2)(46)}{.30} = \$26{,}523.60$$

Next year, A = (26,523.60) (1.20) = $31,828.32

$$\text{Number of orders per year} = \sqrt{\frac{(31{,}828.32)(.30)}{(2)(46)}} = 10.19$$

Problem 3-6

Item	A	$\sqrt{A}$	Number of Orders	$N_\$$	$A/N_\$$
A	120,000	346.41	5	24000	5
B	80000	282.84	6	13333	6
C	50000	223.61	6	8333	6
D	24000	154.92	4	6000	4
E	10500	102.47	8	1313	8
F	5200	72.11	6	867	6
G	2400	48.99	7	343	7
H	1100	33.17	8	138	8
I	900	30.00	6	150	6
J	300	17.32	6	50	6
	$\sum\sqrt{A} =$	1,311.84	$\sum N_\$ =$	54,527	$\sum A/N_\$ = 62$

$$X = \frac{\sum\sqrt{A}}{\sum(A/N_\$)} = \frac{1311.84}{62} = 21.1587$$

We now compute the minimum average inventory without increasing the purchasing work load.

Item	$N_\$ = X\sqrt{A}$	Avg. Inventory ($N_\$$ / 2)	Orders/Year (A / $N_\$$)
A	7,329.59	3,664.80	16.37
B	5,984.53	2,992.27	13.37
C	4,731.30	2,365.65	10.57
D	3,277.91	1,638.96	7.32
E	2,168.13	1,084.07	4.84
F	1,525.75	762.88	3.41
G	1,036.56	518.28	2.32
H	701.83	350.92	1.57
I	634.76	317.38	1.42
J	366.47	183.24	0.82
		$13,878.45	62.01

With the old system, where $\sum N_{\$} = \$54{,}527$, the average inventory value was

$$\frac{54{,}527}{2} = \$27{,}263.5$$

The improvement is:

$$\frac{27{,}263.50 - 13{,}878.45}{27{,}263.50} = 49.1\% \text{ (Part a)}$$

Now, we will explore the minimum average inventory possible with a 25% increase in orders:

$$1.25\sum A / N_{\$} = 1.25(62) = 77.5 \text{ orders per year}$$

The new value of X is:

$$X = \frac{E\sqrt{A}}{\sum(A/N_{\$})} = \frac{1311.84}{77.5} = 16.9270$$

This means our new value of $X\sum\sqrt{A}$ is:

$$N_{\$} = (16.9270)(1311.84) = \$22{,}205.52$$

and our new average inventory value is:

$$\frac{N_{\$}}{2} = \frac{22{,}205.52}{2} = \$11{,}102.76 \text{ (Part b)}$$

This represents a reduction of:

$$\frac{13{,}878.45 - 11{,}102.76}{13{,}878.45} = 20\%$$

Now, we explore the question of reducing the number of orders when average inventory is allowed to be increased by 10%. By raising average inventory, which is $N_{\$}$ / 2, by 10%, we in effect are increasing X by 10%. The new value of X is then:

$$X = (21{,}1587)(1.10) = 23.2746$$

The new number of orders per year, which is $\sum A/N_\$$, becomes:

$$\sum A/N_\$ = \frac{\sum\sqrt{A}}{X} = \frac{1311.84}{23.2746} = 56.3636$$

The reduction is $\frac{62 - 56.3636}{62} = 9.1\%$ (Part c)

Problem 3-7

Item	A	$\sqrt{A}$	No. Orders A/N	N$
Leather	2000,000	447.21	4	50,000
v. soles	40,000	200.00	2	20,000
r. soles	30,000	173.21	2	15,000
Lining	30,000	173.21	2	15,000
l. laces	4,000	63.25	2	2,000
c. laces	2,000	44.72	2	1,000
	$\sum\sqrt{A}$	1,101.60	$\sum A/N_\$ = 14$	$\sum N_\$ = 103{,}000$

$$X = \frac{\sum\sqrt{A}}{\sum A/N_\$} = \frac{1101.60}{14} = 78.6857$$

With a constant number of orders, the minimum average inventory becomes:

Item	$N_\$ = X\sqrt{A}$	Avg. Inv. (N$ / 2)	Orders per yr N$
leather	35,189.04	17,594.52	5.68
v. soles	15,737.14	7,868.57	2.54
r. soles	13,629.15	6,814.58	2.18
lining	13,629.15	6,814.58	2.18
l. laces	4,976.87	2,488.44	0.80
c. laces	3,518.83	1,759.42	0.57
		43,340.11	13.95

Compared to the old system, where $\sum N_\$ / 2 = \$51{,}500$, our new average inventory value of $43,340.11 represents a 15.84% improvement.

Problem 3-8

First, we compute the expected total annual stockout costs for the two alternatives:

ROP	Prob. Of Being Out	Number Short	Expected Annual Cost	Total Annual Stockout Cost
250	.04 when 260	10	(10)(.04)(200)(10) = 800	
	.02 when 280	30	(30)(.02)(200)(10) = 1,200	$2,000/year
220	.06 when 240	20	(20)(.06)(200)(10) = 2,400	
	.04 when 260	40	(40)(.04)(200)(10) = 3,200	
	.02 when 280	60	(60)(.02)(200)(10) = 2,400	$8,000/year

The savings in carrying cost when reducing from an ROP of 250 units down to 220 units is:

(30)(20) = $600 per year

Since the stockout cost increase ($6,000) is larger than the carrying cost savings ($600), the ROP should remain at 250.

Problem 3-9

The number of orders per year is:

$$\frac{(4\ units/day)(250\ days)}{100\ units/order} = 10 \text{ orders / year}$$

The average demand in the reorder period is:

(4 units / day) (25 days) = 100 units

The expected annual stockout costs are:

ROP	SS	Prob. Of Being Out	Number Short	Expected Annual Cost	Total Cost
100	0	.20 when 125	25	(25)(.20)(20)(10) = $1,000	
		.15 when 150	50	(50)(.15)(20)(10) = $1,500	
		.10 when 175	75	(75)(.10)(20)(10) = $1,500	$4,000/yr
125	25	.15 when 150	25	(20)(.15)(20)(10) = $750	
		.10 when 175	50	(50)(.10)(20)(10) = $1,000	$1,750/yr
150	50	.10 when 175	25	(25)(.10)(20)(10) - $500	$500/yr
175	75	None			$0/yr

The total annual costs of the safety stock are:

ROP	SS	Expected Stockout	Carrying Cost	Total Annual Cost
100	0	4,000	0	$4,000/yr
125	25	1,750	(25)(5) = 125	$1,875/yr
150	50	500	(50)(5) = 250	$750/yr
175	75	0	(75)(5) = 375	$375/yr

The optimal policy is to set the ROP at 175 units.

Problem 3-10

We first compute the expected annual stockout costs:

ROP	SS	Prob. Of Being Out	Number Short	Expected Annual Cost	Total Cost
30	0	.10 when 32	2	(2)(.10)(100)(10) = 200	
		.05 when 35	5	(5)(.05)(100)(10) = 250	
		.01 when 36	6	(6)(.01)(100)(10) = 60	$510/yr
32	2	.05 when 35	3	(3)(.05)100)(10) = 150	
		.01 when 36	4	(4)(.01)(100)(10) = 40	$190/yr
35	5	.01 when 36	1	(1)(.01)(100)(10) = 10	$10/yr
36	6	none	0		$0/yr

The total annual costs of the safety stock are:

ROP	SS	Expected Stockout	Carrying Cost	Total Annual Cost
30	0	510	0	$510/yr
32	2	190	(2)(18) = 36	$226/yr
35	5	10	(5(18) = 90	$100/yr
36	6	0	(6)(18) = 108	$108/yr

The optimal reorder point should be 35 units. The expected annual stockout cost is $10 per year, or one stockout ($100 bill) in ten years' time.

Problem 3-11

At the current service level of 80 percent, the safety stock represents about .84 standard deviations or (.84)(80) = 67.2 units.

At a proposed service level of 99 percent, we find that we are about 2.33 standard deviations or (2.33)(80) = 186.4 units of stock above the mean. So, to go from 80 to 99 percent service level, we must add 186.4 – 67.2 = 119.2 units of stock, which costs (119.2)(6) = $715.20 per year in additional carrying costs. Therefore, they should not make the change.

Problem 3-12

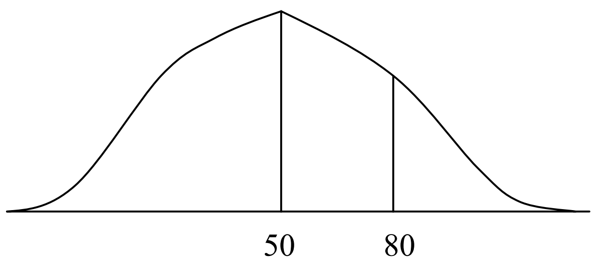
The safety stock of 24 units is to the right of the mean by:

$Z = 24/26 = 0.923$ standard deviations.

The area to the left of 0.923 standard deviations is 0.82199. Therefore, the probability of stocking out in the reorder lead time is 1 – 0.82199, or about 17.8%

Problem 3-13

Currently with a reorder point of 80 games, the service level during the reorder lead time is:



Safety stock = 80 – 50 = 30 units

$$Z = \frac{80 - 50}{25} = 1.2 \text{ standard deviations}$$

The area below 1.2 standard deviations is 0.88493, which corresponds to a service level of 1 – 0.88493 = 0.11507, or about 11.5%.

To yield a service level of 99%, the reorder point must be about 2.33 standard deviations to the right of the mean, and:

Reorder point = 50 + (2.33)(25) = 108 games

The increased holding cost for 108 – 80 = 28 additional units of safety stock is:

Additional cost = (28)(10)(.22) = $61.60 per year

Problem 3-14

Given: U = 200 units
C = $50
P = $30
V = $40

$$N_u = \sqrt{\left(\frac{2UP}{C}\right)\left(\frac{C+V}{V}\right)} = \sqrt{\left(\frac{(2)(200)(30)}{50}\right)\left(\frac{50+40}{40}\right)} = 23 \text{ units in order quantity}$$

$$B = N_u\left(\frac{C}{C+V}\right) = (23)\left(\frac{50}{50+40}\right) = 13 \text{ units on backorder}$$