A NOTE ON FORECAST MODIFICATION BASED UPON RESIDUAL ANALYSIS

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Mabert [4] has presented a scheme to modify the forecasts from linear regression models when the residuals are autocorrelated. In such cases, forecast error patterns sometimes reveal runs of consecutive errors with the same sign. The basis of Mabert's approach is to forecast the residuals themselves with a simple exponential smoothing model. The residual forecasts are then injected as correction factors into the regression forecasts. This approach reduced forecast errors in the application reported in [4]. While Mabert's approach is intuitively appealing, there is an alternative method of forecast adjustment, Cochrane-Orcutt Iterative Least Squares, or COILS [1]. The COILS procedure is easy to use and has some well-established statistical properties that Mabert's approach lacks.

The main effect of autocorrelated residuals is that the estimators of the regression coefficients are inefficient, or have upwardly biased variances. Inefficient forecasts naturally result, with biased t, F, and R^2 statistics. The COILS procedure exploits the autocorrelation information in the residuals to correct these problems. A brief summary of the steps in the procedure for the simplest case, that of first-order autocorrelation (at a lag of one period) is given below:

1. Estimate the sample autocorrelation coefficient, p, in the residuals based on an initial least-squares regression.
2. Transform all data, including the intercept, with the sample p. Letting Y represent the vector of endogenous variables, and X the matrix of exogenous variables, the transformations are

$$Y_t^* = Y_t - pY_{t-1} \tag{1}$$

$$X_t^* = X_t - pX_{t-1} \tag{2}$$

3. Repeat the least-squares regression, using the transformed data, and obtain the new residuals.
4. Estimate a new value of p with the residuals from step 3.
5. Repeat steps 2-4 with the new p. Continue the process until successive estimates of p are arbitrarily close to each other.

This process always converges to a stationary value of p [3]. After convergence, forecasts are generated with the most recent residual value, e_t, and the vector of estimators of the regression coefficients, C, from the last regression in step 3:

$$\hat{Y}_{t+1} = X_{t+1}C + pe_t \tag{3}$$

The COILS forecasts have at least four useful properties. First, the forecasts can be shown to be more efficient, in the asymptotic case, than the ordinary-least-squares forecasts [3]. In small samples, extensive Monte Carlo studies [3] have also shown COILS to be more efficient. Second, the standard statistical tests apply to the regression results from COILS while they do not apply to ordinary regressions, as in Mabert's paper. In particular, we should note that the R^2 and standard error values reported in [4] are biased and do not indicate that a reliable forecasting model has been identified, as Mabert states. Third, statistical confidence intervals can be set around the COILS forecasts with the usual methods. Because of the aforementioned bias and variance problems, confidence intervals cannot be developed for Mabert's forecasts. Fourth, the COILS forecasts are inexpensive to generate and are available as an alternative to ordinary-least-squares regressions in most econometric computer packages (see the Time Series Processor package [2], for example). In my experience, COILS usually converges in three or four iterations, which amounts to perhaps two seconds of additional CPU time compared to ordinary regressions for a time series of 100 observations.

In conclusion, Mabert's approach to the autocorrelation problem in forecasting seems useful only in cases where a COILS program is not available.

REFERENCES

[1] Cochrane, D., and G. H. Orcutt. "Application of Least Squares Regressions to Relationships Containing Auto-correlated Error Terms." *Journal of the American Statistical Association*, Vol. 44, No. 245 (March 1949), pp. 32-61.

[2] Hall, B. H., and L. E. Wyse. *Time Series Processor User's Manual*. Research Triangle Park, N.C.: Triangle Universities Computation Center, 1977.

[3] Johnston, J. *Econometric Methods*. 2nd edition. New York: McGraw-Hill, 1972.

[4] Mabert, V. A. "Forecast Modification Based Upon Residual Analysis: A Case Study of Check Volume Estimation." *Decision Sciences*, Vol. 9, No. 2 (April 1978), pp. 285-296.