Growth, Liquidity Provision, International Reserves, and Sovereign Debt Capacity*

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Abstract

I establish a theoretical framework to address three distinct, but interrelated puzzles in international economics: (1) the occurrence of twin crises, (2) the existence of large amounts of sovereign debt, and (3) the presence of substantial amounts of international reserves. By considering the interaction between growth and banking in a small open economy that is unable to commit to repaying its external sovereign debt, my dynamic stochastic general equilibrium (DSGE) model uses Global Games techniques to study the endogenous relationship between domestic bank runs, sovereign debt capacity, international reserves, growth, and macroeconomic fundamentals. The main findings are as follows. First, when excluded from international credit markets, liquidity demands rise in a country's domestic banking sector, which raise the probability of bank runs and costs of liquidation of long term projects. This creates incentives for repayment and sovereign debt capacity. Second, twin (domestic) banking and (external) sovereign debt crises endogenously emerge within the model. Third, international reserves have "war chest" like properties within the model: they help prevent domestic bank runs, during which incentives for a country to strategically default increase, and can therefore create sovereign debt capacity. Finally, my model can quantitatively generate reasonable amounts of sovereign debt and international reserves in equilibrium.

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1 Introduction

In this paper, I establish a framework to address three distinct, but interrelated puzzles in international economics: (1) the occurrence of twin crises, or joint banking and balance of payments problems (Kaminsky and Reinhart (1999)), (2) the existence of large amounts of (external) sovereign debt, and (3) the presence of substantial amounts of international reserves. Twin (domestic) banking and (external) sovereign debt crises have serious macroeconomic consequences for aggregate quantities such as output and investment, as highlighted by the recent Eurozone crises. These recent crises are just the newest additions to an already long list of twin-crisis episodes.[1] Since twin crisis events are usually preceded by surges in sovereign debt (Reinhart and Rogoff (2009, 2011)), a full understanding of twin crises also requires us to understand why countries are able to borrow so much sovereign debt in the first place.

However, the existence of large amounts of sovereign debt is itself a puzzle. Starting from Eaton and Gersovitz (1981), a well-established view of sovereign default is that it happens for strategic reasons. There is a general lack of international legal enforcement for the repayment of sovereign debt, and countries otherwise lack the ability to commit to honoring their obligations (for more information on the legal issues surrounding sovereign defaults, see Sturzenneger and Zettelmeyer (2006) and Panizza et al. (2009)). In the absence of legal enforcement, other costs of default must be large in magnitude to justify the high levels of external sovereign debt observed in the data. They also need to be pro-cyclical in order to match counter-cyclical sovereign default.

In addition, countries simultaneously hold substantial amounts of sovereign debt and international reserves, even after sovereign defaults.[2] Sovereign debt in the presence of

[1] For the period 1970-2011, the crises database of Laeven and Valencia (2008, 2012) counts a total of 147 banking crises and 66 sovereign defaults episodes; of these, they count 19 episodes of twin (domestic) banking and sovereign debt crises. Reinhart and Rogoff (2009, 2011) also document the prevalence of twin crises in their extended historical sample.

[2] Debt and international reserves levels are plotted for a selection of emerging markets for the year 2003 in Figure 1. During this year, Argentina was in the middle of a sovereign default episode (the default occurred in 2001 and restructuring took place in 2005). Note that Argentina had a reserve to GDP ratio of 10% during this period (the debt levels for Argentina represents defaulted debt). All other countries in the figure were not in default. Furthermore, we see in panel C of Figure 2 that international reserve to output ratios actually

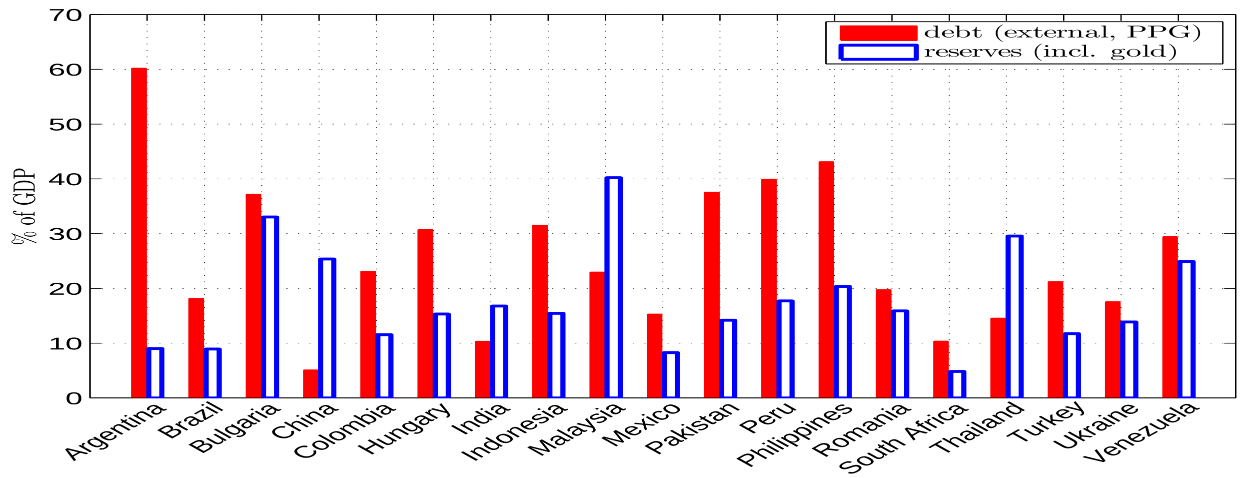

Figure 1: **Sovereign debt and international reserves.** This figure plots the stock of outstanding sovereign debt (solid bar) and international reserves (hollow bar) as a fraction of (annual) GDP for a selection of emerging market economies during 2003. Sovereign debt refers to external public and publicly guaranteed debt while international reserves is inclusive of gold. Note that Argentina was in the middle of its default episode during 2003 (the default occurred in 2001 while restructuring took place in 2005). The data is taken from the World Bank's World Development Indicators (WDI) database.

international reserve accumulation presents an additional challenge. A predominant view is that international reserve accumulation occur due to precautionary motives– to protect the domestic economy against adverse shocks including sudden stops and twin crises.[3] While international reserves stabilize the domestic economy and increase a country's *ability* to repay its sovereign debt, it can paradoxically reduce debt capacity by decreasing its *willingness* to repay because the country can now better self insure during periods of exclusion from international credit markets after a sovereign default.

In this paper I present a theory of twin crises and sovereign default costs in the presence of reserve accumulation that addresses these puzzles. I introduce growth and domestic banking fragility (following Diamond and Dybvig (1983)) into the canonical Eaton and Gersovitz (1981) model of sovereign debt. In my model, maturity transformation by the domestic banking

increase after a sovereign default.

[3]There is also a mercantile view of reserves; this paper focuses on the precautionary view of international reserves (see Aizenman and Lee (2007) for a review of the two perspectives).

sector is vital for driving investment and economic growth. In such a setting, coordination failure can lead to bank runs and costly liquidations, as well as affect ex-ante incentives to invest. These interactions between growth and banking provide the basis for (endogenous) costs of default in my setting.

The threat of domestic bank runs implies a natural role for the government to manage liquidity risk at the country level, and the government's ability to stabilize the domestic economy will be intimately linked to its external balance sheet. For example, the government's ability to act as a lender of last resort crucially depends on the amount of liquidity that it can access on short notice, which provides a motive for international reserve accumulation. Similarly, the amount of sovereign debt on the government's external balance sheet affects its ability to *credibly* provide guarantees to the domestic banking sector. This link between domestic banking fragility and the government's external balance sheet positions will be an important determinant of economic growth and sovereign risk in my model.

Costs of default in my model are motivated by empirical findings of decreased inflows of external private credit after a sovereign default (see Arteta and Hale (2008) and Das et al. (2010)). In addition, sovereign defaults also increase the costs of borrowing for domestic firms (Agca and Celasun (2012)). The key assumption in my model is that losses in foreign credit to private firms place a greater burden on the domestic banking sector while the country is in default, as domestic intermediaries will have to substitute credit in place of lost external private credit. In turn, this can place a greater burden on the government to provide country level liquidity risk management after defaulting, and this is costly. If the government chooses not to increase international reserves after defaulting and maintains the same level of investment and growth, then the heavier burden placed upon domestic intermediaries can result in higher chances of bank runs and costly liquidations. Alternatively, the government can try to manage domestic banking fragility by accumulating more international reserves after defaulting. However, something else must give, be it decreased consumption and/or decreased investment, which can be costly. In Figure 2, I conduct an event study[4] to see

[4]Details for this event study are available in Appendix A.

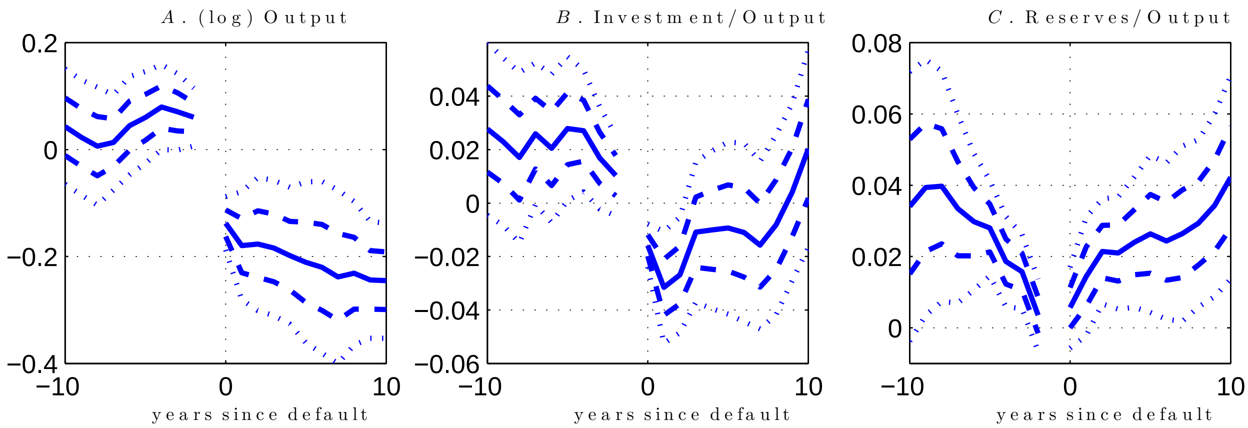

Figure 2: **Output, investment and international reserves around default.** The solid lines in each panel of this figure plot averages responses in (log) output, investment-to-output, and international reserves-to-output around a sovereign default event at time zero. All quantities are relative to their values one year prior to defaulting. The dashed (dotted) lines in each panel plots the one (two) standard error bars. Standard errors are clustered by country and year. Further details are available in Appendix A.

how output, investment and reserves behave around sovereign default episodes. Consistent with the mechanism of my model, we see that international reserve-to-output ratios increase after defaulting (panel C). We see in panel B that the increase in international reserves is accompanied by a drop in investment-to-output ratios. In addition, these are also accompanied by output losses following a sovereign default (panel A).

To summarize, sovereign default and debt capacity are determined as the government compares the costs and consequences of managing a potentially more fragile banking system after defaulting against the costs of debt repayment. These tradeoffs will also vary across the business cycle and co-move with the investment opportunities set. During economic expansions, this "liquidity provision wedge" will be large: hoarding too much international reserves can lead to forgone growth opportunities, which can be especially costly during good times when investment returns are high. In addition, having insufficient reserves and having to inefficiently liquidate projects will be more costly during good times when productivity is high. Therefore, it is more costly for a government to default during good times. The opposite is true during bad times– hoarding more international reserves is less costly if investment returns are low in the first place. As a result, strategic default is more likely to occur during bad times. In this manner, my model generates pro-cyclical costs of default and counter-cyclical sovereign default.

The presence of international reserves in my model closely reflects the Thornton (1802) view– international reserves protect and stabilize the domestic credit markets against domestic and external runs on the economy. Reserves have "war chest" like properties in the model. Even though they may seem idle in equilibrium, they are nevertheless still doing an important job in helping rule out bank runs in the first place. In addition, the presence of international reserves can boost sovereign debt capacity– by making the possibility of bank runs remote, they decrease the possibility of a strategic sovereign default aimed at alleviating domestic bank runs (by defaulting on external parties, the economy will have additional resources with which to pay domestic agents and prevent bank runs). This acts as a countervailing force to the "usual" intuition in the sovereign debt literature whereby reserves kill sovereign debt capacity by making the autarky outcome less painful. This is an important ingredient for generating both debt and reserves in equilibrium.

Finally, it is well known that equilibrium indeterminacy can result in models involving coordination failure and bank runs (Diamond and Dybvig (1983)). To circumvent this, I embed equilibrium selection techniques from the Global Games literature (see Morris and Shin (2003) for a review) into a dynamic stochastic general equilibrium (DSGE) setting. Thus, my model is able to provide unique equilibrium predictions concerning the endogenous relationship between bank runs, sovereign debt capacity, international reserve accumulation, growth, and macroeconomic fundamentals.

1.1 Related literature.

Under fairly general conditions, Bulow and Rogoff (1989) show that if financial exclusion is the only form of punishment for defaulting, then a country could always default and subsequently replicate the original lending relationship through state-contingent savings. This action results in a higher consumption path and so in the absence of other costs of default, sovereign debt capacity will not exist. Furthermore, being able to save through complete markets is not always necessary for this result to hold: Auclert and Rognlie (2014) show that a similar replication argument holds within the incomplete-markets setting of Eaton and Gersovitz (1981). There have been many proposed solutions to the Bulow and Rogoff (1989) puzzle.[5] My assumption of a post-default drop in private credit broadly falls into the "reputation spill over" category (Cole and Kehoe (1998)). My incomplete-markets mechanism for overcoming the replication argument is as follows: there is now an additional liquidity constraint which any potential replicating portfolio must be subject to. When this liquidity constraint is violated, domestic bank runs and costly liquidations will ensue. A drop in private credit after a sovereign default implies that this liquidity constraint will be even more likely to bind after defaulting. As a result, replication arguments that hold in settings with just a single budget constraint might fail to satisfy this additional liquidity constraint and therefore be infeasible. Coordination failure is key for this result to go through in my model, as otherwise such additional liquidity constraints will not be present. The liquidity frictions which give rise to twin crises also generate sovereign debt capacity. In this way, twin crises and sovereign debt capacity come hand in hand in my model– neither can be present without the other.

This paper is related to quantitative models of sovereign debt in the incomplete-markets setting of Eaton and Gersovitz (1981), starting with Aguiar and Gopinath (2006) and Arellano (2008) (see Aguiar and Amador (2014) for a review of this literature). With a few exceptions, much of this literature assumes exogenous costs of default and does not consider the joint accumulation of sovereign debt and international reserves.

Mendoza and Yue (2012) endogenize default costs in terms of decreased trade and output

[5] These include direct sanctions (Bulow and Rogoff (1989)), inability or unwillingness to save after defaulting (Cole and Kehoe (1995), Kletzer and Wright (2000), Wright (2002), Gul and Pesendorfer (2004), and Amador (2012)), reputation spill overs affecting other parts of the domestic economy (Cole and Kehoe (1998)), bubbles (Hellwig and Lorenzoni (2009)), information spill overs (Sandleris (2008)), and non-selective default (Broner et al. (2010), and Gennaioli et al. (2014)).

as a result of lost private trade credit following a sovereign default. Gornemann (2013) extends Mendoza and Yue (2012) to further incorporate endogenous growth.[6] While the underlying frictions in Mendoza and Yue (2012) and Gornemann (2013) are also liquidity related, they assume away the ability for the economy to meet these working capital constraints through international savings, which is a key difference from my setting. In addition, my costs of default are banking related, instead of the trade channel considered in Mendoza and Yue (2012) and Gornemann (2013).

There is a literature that focuses on the role of international reserves in the *absence* of strategic sovereign default. Examples include Aizenman and Lee (2007), Jeanne and Ranciere (2011), and Hur and Kondo (2013). The idea that a main use of international reserves is to stabilize the domestic financial sector traces as far back as Thornton (1802). More recently, Obstfeld et al. (2010) finds evidence in support of this view.

In a setting with strategic sovereign default, but with exogenous costs of default and single period debt, Alfaro and Kanczuk (2009) finds that it is difficult to generate positive levels of international reserves in equilibrium. Their result highlights how international reserves can worsen a country's terms of borrowing by making defaulting less costly and decrease its *willingness* to repay.[7] My model can generate international reserves and debt in equilibrium, even with single period debt and endogenous default costs. A countervailing force is present in my model: international reserves help rule out domestic bank runs, which are times with increased incentives for a sovereign to strategically default, and can thus help improve a sovereign's terms of borrowing.

My paper is also related to a theoretical literature on crises in open economy settings (see Lorenzoni (2014) for a review of this literature). Examples include Krugman (1979), Cole and Kehoe (2000), Caballero and Krishnamurthy (2001), Chang and Velasco (2001), and Holmstrom and Tirole (2002). Within this literature, Chang and Velasco (2001) is

[6] Others sovereign default models with capital accumulation include Gordon and Guerron-Quintana (2013) and Park (2014).

[7] Assuming exogenous default costs, Bianchi et al. (2014) generates debt and international reserves in equilibrium in a model with long term debt.

most similar in spirit to my paper. They also consider how domestic bank runs influence the possibility of twin crises, strategic incentives to default, ex-ante allocations between international reserves and domestic investments, as well as decisions for external creditors to run. The key difference is that their analysis is done in a finite horizon setting in which it is impossible to generate any forms of "reputation" for debt repayment and so they impose exogenous default costs. While their analysis is extremely informative, their theory is still incomplete: as Reinhart and Rogoff (2009, 2011) show, twin crises usually occur after large amounts of sovereign debt have been accumulated, hence a complete theory of twin crises requires endogenizing sovereign debt capacity. My paper is an attempt at building such a "complete" theory.

Many other crises models also stress the link between sovereign default and the domestic banking sector. In particular, a strand of literature explores the implications of non-selective default. In these models, the government cannot selectively default and so sovereign defaults directly hurt domestic agents (including domestic banks) who hold sovereign bonds. Analysis along these lines include Broner et al. (2010), Brutti (2011), Acharya and Rajan (2013), D'Erasmo and Mendoza (2013), Bocola (2014), Gennaioli et al. (2014), and Perez (2014). My model instead assumes selective external sovereign default– the goal is to see how far theories of external sovereign debt capacity can get under the original selective default assumptions of Eaton and Gersovitz (1981) and Bulow and Rogoff (1989).[8]

Finally, my paper also builds on closed-economy analysis of banking and growth. Papers along these lines include Cooper and Ross (1998) and Ennis and Keister (2003, 2006) who investigate how bank runs (modeled after Diamond and Dybvig (1983)) affect growth through both ex-post liquidations as well as by changing ex-ante incentives to invest. From a

[8] Reality lies somewhere in between these two extremes. Selective default can happen in practice (e.g. the government can default and then bail out domestic banks), however obstacles are likely to hinder such behaviour in practice (e.g. Broner et al. (2010) points out one such obstacle). How well a government is able to achieve full selective default in practice is an empirical question which warrants further investigation. Keep in mind that if a sovereign can achieve a sufficiently high level of selective default, then it would be difficult for the government to commit to not selectively defaulting on just its external creditors in the event of an actual sovereign default.

methodological perspective, I embed the Global Games analysis of the closed-economy bank run model of Goldstein and Pauzner (2005) into an open economy dynamic stochastic general equilibrium (DSGE) setting. This allows me to obtain unique equilibrium predictions in a dynamic setting with bank runs without having to impose sun spot equilibria.

To summarize, the literature has investigated various combinations of international reserves, endogenizing sovereign debt capacity, and/or twin crises. To the best of my knowledge, this paper is the first attempt at capturing all three phenomena simultaneously.

Layout. The rest of this paper is structured as follows: Section 2 describes the model. This is done in three steps: Section 2.1 provides an overview of the setting, Section 2.2 describes bank runs within the model, and Section 2.3 fills in the remaining details for the dynamic setting. A quantitative analysis of is then conducted in Section 3. Finally, Section 4 concludes.

2 Model

2.1 Overview

My model consists of a representative agent in a small open economy (SOE). The representative agent is composed of a private and a public sector. The private sector consists of households, firms and banks. The public sector consists of a government which we assume to be a benevolent planner. The relationship between the private and public sectors is illustrated in Figure 3. As a further simplification, I abstract from exchange rate concerns so that the economy is entirely real.[9]

Production and liquidation. I think of banks and firms as a joint productive entity. Capital K_t is used to produce goods each period, with production being subject to a liquidation

[9]This is a standard simplifying assumption in models of sovereign debt within the Eaton and Gersovitz (1981) setting. Recent work has begun to relax this assumption (e.g. see Na et al. (2014)).

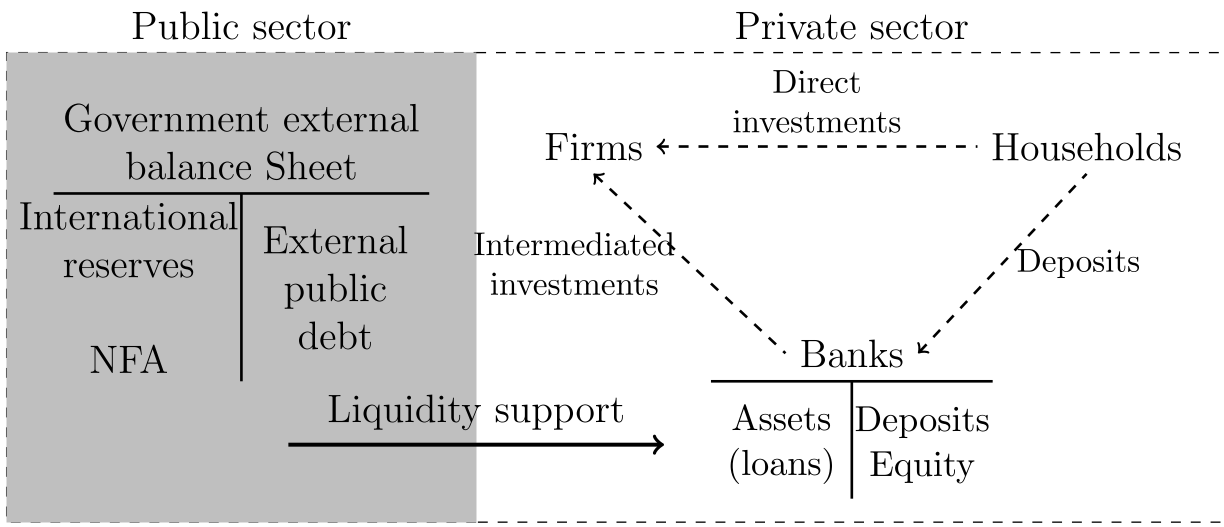

Figure 3: **The representative agent economy.** This figure illustrates the structure of the economy. There is a representative agent consisting of firms, households, banks, and a government. Growth is driven by capital accumulation. Investments are intermediated and partially financed with household deposits. The government accumulates international reserves, issues external sovereign debt, and provides liquidity support for the private sector.

friction which I model following Diamond and Dybvig (1983). I follow the Diamond and Dybvig (1983) setting for its convenience, especially when it comes to studying coordination problems and bank runs.[10] At the start of each period, each unit of capital gives off one unit of seed. It takes a full period for the seed to ripen into fruit which can then be harvested and consumed. All consumption is in terms of "ripened fruit" which also serves as then numeraire. If allowed to mature, each unit of seed produces e^{z_t} units of fruit where z_t is the productivity shock for the period. This total factor productivity (TFP) shock is the only source of uncertainty within our model; it follows an AR(1) process:

$$z_{t+1} = \rho_z z_t + \sigma_z \varepsilon^z_{t+1}. \tag{1}$$

[10] Of course, there are other forms of frictions which can create a similar need for government support and liquidity management; for example, the Holmstrom and Tirole (1998, 2002, 2011) framework readily spring to mind. In addition, the need for government liquidity support can also be driven by solvency concerns following the accumulation of substantial amounts of non-performing loans on bank balance sheets; the recent financial crises of 2008 is a good demonstration of this.

Unfortunately, sometimes banks may have funding needs and not be able to wait until fruits fully ripen before harvesting. These funding needs arise in the event of a bank run (I will describe the banking set up shortly). In the event of an early harvest, each unit of seed will instead only be worth

$$L(z_t) < e^{z_t} \tag{2}$$

units of ripened fruit. The wedge $e^{z_t} - L(z_t) > 0$ is the cost of liquidation, and forms the basis for costs of default. I assume that liquidation losses are larger during good times when productivity is high and the opportunity cost of having idle productive units is also high. That is, the liquidation wedge is increasing in productivity z_t. Total output each period is given by

$$Y_t = \left[\ell_t L(z_t) + (1 - \ell_t) e^{z_t}\right] K_t, \tag{3}$$

where $\ell_t \in [0, 1]$ is the fraction of seeds harvested early (i.e. liquidated).

The timing of production within each period is portrayed in Figure 4 which also illustrates the relative timing of other events occurring within the economy.

Growth. Growth is entirely driven by capital accumulation by firms in the private sector.[11] I think of capital K_t as more than just physical capital, but also include other intangible forms of capital (e.g. human capital) which can be difficult to disinvest. At the aggregate level, the law of motion for capital is given by

$$K_{t+1} = (1 - \delta) K_t + I_t, \ I_t \geq 0, \tag{4}$$

with $\delta > 0$ being the depreciation rate. Note that liquidations apply only to seeds and do not directly pertain to capital K_t. However, investment I_t is still illiquid. This is because an initial capital investment I_t produces a stream of seeds, all of which are subject to liquidation risk.

[11] In the absence of liquidation frictions, the closed economy version of my model reduces to an AK growth model; see Acemoglu (2009) for a textbook treatment of the growth literature. I consider an AK setting for its numerical convenience, as capital K_t can be scaled out.

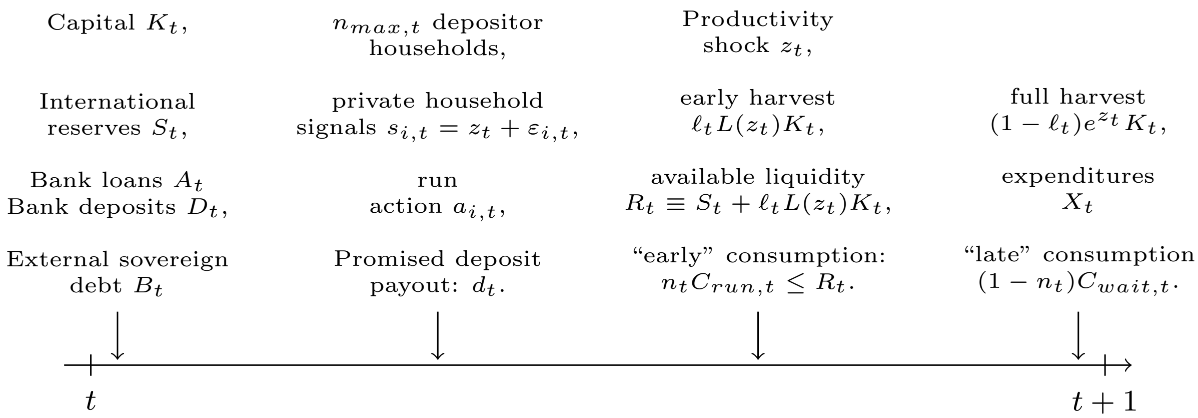

Figure 4: **Within period timing.** This figure illustrates the timing of events within each period t. The small open economy enters each period with capital K_t, international reserves S_t, outstanding bank loans A_t and deposits D_t, as well as some outstanding stock of sovereign debt B_t. In the middle of each period, households observe private signals about productivity $s_{i,t}$. Depositors then choose whether or not to run $a_{i,t}$. Withdrawers are served subject to a "sequential service" constraint. Their total consumption is bounded by the amount of available liquidity, which is given by the sum of international reserves and liquidation proceeds $\ell_t L(z_t) K_t$. Full harvest takes place at the end of each period. Expenditures X_t are then paid. Finally, remaining households consume. Limited liability applies to all households.

In addition, investments are irreversible so that $I_t \geq 0$. Irreversibility captures *technological illiquidity* and gives a meaningful role for accumulating international reserves.

Capital accumulation is subject to convex adjustment costs so that the total cost of increasing capital by I_t units is given by

$$\phi\left(\frac{I_t}{K_t}\right) K_t, \tag{5}$$

where the function $\phi(\cdot)$ captures decreasing returns to scale in capital accumulation. Investment costs form part of expenditures X_t at the end of each period.

Intermediation. I assume that there is a role for banks as intermediaries for investments. There is a representative intermediary which I will refer to as "the bank." I take the intermediation process as given and explore its implications for sovereign debt capacity, international reserves, and twin crises.[12] In particular, a fraction $\chi \in (0,1)$ of total investment costs each period, $\phi\left(\frac{I_t}{K_t}\right)K_t$, is intermediated by banks. Bank loans A_t^{loan} evolve according to

$$A_{t+1}^{loan} = (1-m_A)\,A_t^{loan} + \chi\phi\left(\frac{I_t}{K_t}\right)K_t. \tag{6}$$

Here $\chi\phi\left(\frac{I_t}{K_t}\right)K_t$ is the amount of new investment loans given out by banks in period t. The bank's loan book can be thought of as a collection of small loans, with each outstanding loan randomly maturing with probability $m_A \in (0,1)$ each period. Under this assumption, a fraction m_A of loans comes off the bank's book each period and the average debt maturity of the bank's loan book is $1/m_A$.

Beside financing domestic investments, banks also extend other forms of credit to the domestic economy. Examples of such forms of credit include households loans such as mortgage and auto loans. These loans total

$$A_t^{other} = a_0 K_t, \tag{7}$$

and are proportional to the size of the economy as measured capital stock K_t. The constant of proportionality is given by a_0 and will be set so that the size of the banking sector as measured by its total assets

$$A_t = A_t^{loan} + A_t^{other} \tag{8}$$

is close to its empirical counterpart.

Households. There is a continuum of households with unit mass. The intermediary finances its lending, in part, by taking deposits from households. I assume that the total amount of (demand) deposits is given by

$$D_t = m_L A_t, \tag{9}$$

[12]Generating a need for intermediaries from first principals is beyond the scope of this paper. Such first principals have already been extensively explored in the banking literature (see Freixas and Rochet (2008) for a textbook treatment).

where I have conveniently assumed a constant capital structure for banks[13] with $m_L \in (0,1)$ being the fraction of bank lending financed with demand deposits (or, more generally, short term debt). All depositors have the *option* of withdrawing and consuming their deposits on short notice. Withdrawers are serviced subject to a *sequential service* constraint. The difference between the maturity rate of bank liabilities, m_L, and the maturity rate of bank loans, m_A, captures the amount of maturity mismatch on bank balance sheets. Usually, m_A is less than m_L which corresponds to the empirically relevant case where banks finance long term loans with short term deposits (i.e. banks are doing maturity transformation).

Not all households need be depositors– I denote the fraction of depositor households in period t by $n_{max,t}$. The total stock of deposits, D_t, is evenly split amongst the $n_{max,t}$ depositors according to

$$D_t = d_t \times n_{max,t}, \tag{10}$$

where d_t is the *promised* deposit payout (in terms of fruit). For simplicity, I set the promised payout d_t to be proportional to capital:[14]

$$d_t = d_0 K_t, \tag{11}$$

where d_0 is the constant of proportionality.

[13] This allows me to conveniently keep just one of A_t and D_t as the state variable in the numerical implementation. In general, bank capital structure will be time varying. Incorporating time varying intermediary leverage within my model will generate additional amplification effects over the business cycle; see, for example, Brunnermeier and Sannikov (2014) for a model of time varying banking leverage in a closed economy.

[14] It is possible to determine d_t endogenously by considering an additional household portfolio problem (e.g. see Gertler and Kiyotaki (2010, 2013) for an implementation of this in a dynamic setting). Usually, this involves introducing household preferences for liquidity. As will be remarked upon shortly, I purposefully do away with household liquidity preferences in order to emphasize coordination failure. The key assumption in fixing d_0 is that the contractual terms for banks are sticky and downward rigid so that it is difficult for banks to deleverage on short notice without taking large losses. A possible reason for such downward rigidity is that information asymmetry can become more severe in times of financial distress, which increases the costs of external financing for banks (Myers and Majluf (1984)).

Coordination failure and domestic bank runs: implications for international reserves and sovereign debt capacity. As in Diamond and Dybvig (1983), there is a mismatch between the timing of withdraws and the timing of production. In particular, withdraws occur in the middle of each period when fruits have not yet fully ripened. The total payout to withdrawing depositors is given by

$$n_t C_{run,t}, \tag{12}$$

where n_t is the equilibrium number of households who choose to run, with each such household consuming $C_{run,t}$ on average. The total amount of liquidity available for paying withdrawing households is given by

$$S_t + \ell_t L(z_t) K_t, \tag{13}$$

and is the sum of international reserves and liquidation proceeds. When total liquidity is plentiful, each withdrawing household will be paid the full promised amount and we will have $C_{run,t} = d_t$. Instead, if total liquidity is insufficient, then some withdrawing households may go unserved and end up consuming very little. International reserves act as a buffer against having to liquidate (i.e. being forced to set $\ell_t > 0$) in the event of a bank run. Thus, reserves help stabilize the domestic banking sector.[15]

Total consumption for the remaining $1 - n_t$ households (i.e. depositor households that

[15]Reserves *cannot* be used as collateral for external sovereign debt in my model. In practice, even when sovereign bonds are issued under foreign law, a country's international assets (including international reserves) are still protected under *sovereign immunity*, even in the event of a sovereign default. For example, in the US, central bank assets, including international reserves, are typically immune from attachment proceedings under the Foreign Sovereign Immunities Act (FSIA) of 1976. However, a sovereign can still voluntary waive its sovereign immunity. Despite this, countries are typically hesitant to do so in practice. For example, during the 1990s Argentina included bond clauses declaring that its central bank reserves were unattachable. For more details about legal issues surrounding sovereign defaults, see Chapter 3 of Sturzenneger and Zettelmeyer (2006) as well as Panizza et al. (2009). Given these legal constraints, and the usual commitment problems which countries face, it is difficult for international reserves to *credibly* serve as collateral for securing sovereign debt.

chose not to withdraw, as well as non-depositor households) is given by

$$(1 - n_t) C_{wait,t}, \tag{14}$$

where the average consumption per remaining household, $C_{wait,t}$, is determined by evenly dividing all remaining amounts[16] amongst the remaining households at the end of the period.

Each depositor will decide whether or not to run by comparing the expected utilities from running and obtaining consumption $C_{run,t}$, or waiting and consuming $C_{wait,t}$. As shown in Diamond and Dybvig (1983), coordination problems can result in equilibrium indeterminacy. To overcome this, I use equilibrium selection techniques from the Global Games literature and introduce noisy private household signals

$$s_{i,t} = z_t + \varepsilon_{i,t} \tag{15}$$

of the productivity shock z_t, with the noise component $\varepsilon_{i,t}$ being independent and identically distributed (iid) in the cross-section and over time. The run decision for each depositor will be conditioned on his private signal. I defer details for this Global Games analysis to Section 2.2. For now, I press on with describing the model.

In order to make the results stark, I assume that households do *not* have intrinsic liquidity needs. That is, households do not have liquidity preferences for "early" consumption, so that all runs within my model are the result of coordination failure. This assumption highlights the following: absent coordination failure, my model reduces to a setting without liquidity concerns (i.e. $\ell_t \equiv 0$). In this case, my model will not have any costs of default other than financial exclusion. Furthermore, my model also allows for savings. It is well known that such a setting will have difficulty generating external debt capacity. In this sense, twin crises

[16] This residual amount consists of (1) proceeds from full harvest at the end of the period, (2) any unused liquidity from the middle of the period, and (3) expenditures at the end of the period. Expenditures consist of investments, reserve accumulation, and net (external) debt rollover costs. This will be made clear in Section 2.3

and sovereign debt capacity come hand in hand within our model.[17]

Boom-bust cycles. Implicit in my set up is the assumption that investment booms can lead to banking fragility down the line. Given the assumptions, increased investment I_t during good times leads to increased amounts of bank lending A_t and build-up in domestic bank debt D_t. If the level of international reserves S_t is insufficient when a recession hits, then there may be an inadequate amount of liquidity buffer to prevent bank runs, liquidations and sovereign debt crises from occurring. Empirically, such boom-bust dynamics have been regularly occurring (Reinhart and Rogoff (2009, 2011)). Of course, the government will take into account the possibility of such boom-bust dynamics in formulating the economy's investment, international reserve, and external debt policies.

The government. The government is a benevolent planner with Epstein and Zin (1989) and Weil (1989) preferences who maximizes social welfare V_t given (recursively) by

$$V_t = \left\{ (1-\beta) W_t^{1-\psi^{-1}} + \beta \mathbb{E}_t \left[V_{t+1}^{1-\gamma} \right]^{\frac{1-\psi^{-1}}{1-\gamma}} \right\}^{\frac{1}{1-\psi^{-1}}} \tag{16}$$

where $\psi > 0$ is the elasticity of intertemporal substitution (EIS), $\gamma > 0$ is the planner's relative risk aversion,[18] and W_t reflects the certainty equivalent consumption of the representative household after taking into account the risk of bank runs. Note that in the absence of coordination problems, W_t is just aggregate consumption. Details for the derivation of W_t are deferred to Section 2.2.

The planner oversees domestic investment and external balance sheet adjustments for the small open economy. The planner is, however, constrained by the domestic banking system

[17]This result contains some flavor of Allen and Gale (2004). Twin crises are a natural outcome in my model's constrained efficient setting. It may not be optimal for the planner to always avoid crises as this can be overly costly. For example, the planner can choose to hoard large amounts of international reserves to always prevent crises, but this comes at the cost of forgone investment opportunities.

[18]To the best of my knowledge, the quantitative sovereign debt literature has so far focused on CRRA preferences (i.e. $\psi^{-1} = \gamma$). In my setting where the endogenous costs of default is closely tied to growth, being able to disentangle the EIS ψ and risk aversion γ parameters can be of quantitative importance.

in implementing its policies. That is, the planner takes the role of the banking sector (as described in equations (6) through (10)), as well as household coordination problems as given when it comes to choosing its policies. The possibility of coordination failure and bank runs imply a role for the government to manage liquidity risk at the country level; such liquidity risk management is intimately linked to the government's external balance sheet.

The baseline model studies two instruments for liquidity provision.[19] The first is for the planner to just liquidate projects (i.e. choose $\ell_t > 0$) and incur lost output $e^{z_t} - L(z_t)$, however this is inefficient. The second option is for the planner to accumulate a stock of international reserves S_t which can be thought of as claims on foreign fruit. International reserves are non-state contingent and accumulate at the risk free rate r (e.g. think of an emerging market holding US treasuries). Importantly, international reserves are fully liquid and can be converted into fruit at any time. However, international reserve accumulation is not without its costs: they lack state contingency and may not be as high yielding as domestic investments I_t (especially during domestic booms).

Sovereign default and its costs. The government can also borrow from international creditors by issuing external sovereign debt. This stock of outstanding debt is denoted by B_t. I assume that debt is single period.[20] Debt issuances and repayments are all assumed to take place at the end of the period, and the net proceeds from debt rollover cost form part of the end of period expenditures X_t. Under this timing assumption, sovereign debt issues do not *directly* provide liquidity, nor does it *directly* imply liquidity needs. Rather, its effects are more *indirect*. It can influence liquidity supply by financing international reserve accumulation. Similarly, debt rollover costs will affect the amount of resources available for paying non-withdrawing households at the end of the period, which can potentially alter run incentives for depositors.

I follow the Eaton and Gersovitz (1981) framework and assume that the government is

[19] Some additional liquidity provision instruments are mentioned in Appendix C.

[20] Appendix C discusses long term debt and how it could be combined with externally borrowed liquidity to smooth domestic liquidity shocks.

unable to commit to repaying its external obligations. In particular, the government's default decision is strategic: he is always comparing the value of not defaulting, $V_{ND,t}$, to the value of defaulting, $V_{D,t}$, and default occurs at time t if and only if the latter is higher. That is, default occurs on the set $\{V_{D,t} > V_{ND,t}\}$.

Should the planner default, the economy goes into financial autarky and access to foreign credit is lost. However, the planner is still able to save abroad using international reserves. A defaulted country can regain access to international credit markets with probability $\xi > 0$ each period. All debt is written off when the planner reenters international credit markets.[21]

Furthermore, I assume the domestic banking sector must partake in a larger amount of intermediation after defaulting, so that the fraction of intermediated investments χ (see equation (6)) is larger whilst the small open economy is in default:

$$\chi_D > \chi_{ND}. \tag{17}$$

Assumption (17) is motivated by empirical findings that sovereign defaults trigger losses in private credit. The percentage decrease in external private credit attributable to sovereign default ranges between 20-40% (see Arteta and Hale (2008) and Das et al. (2010)).[22] Note that these numbers do not include decreases in other forms of private credit such as trade credit.[23] Theoretically, this assumption is consistent with a sovereign default having "reputation spill overs" into other parts of the economy (Cole and Kehoe (1998)).

Under assumption (17), sovereign defaults place a higher burden on the domestic banking sector when it comes to financing growth, as the domestic banking sector will have to do more amounts of intermediation just to keep the same level of investments. In the model,

[21] These are standard simplifying assumptions used in the sovereign default literature. Recovery can be incorporated at the cost of additional complexity (for example, see Yue (2010)).

[22] Their results include controls for international competitiveness, investment climate and monetary stability, financial development, macroeconomic fundamentals, political stability, and global capital supply. The average drop in private credit attributable to sovereign default found in Das et al. (2010) is 40%. The magnitude of the drop in private credit found in Arteta and Hale (2008) is smaller. This is due to their expanded definition of a "default event" for their baseline results. They find similar magnitudes once they use a more stringent definition of default.

[23] Trade credit is emphasized in Mendoza and Yue (2012).

maintaining the same investment level after defaulting leads to increased bank domestic lending A_t and debt D_t. This can lead to increased chances of bank runs and liquidation losses. Alternatively, the planner can try to reduce banking fragility by hoarding more reserves after defaulting. In this case, the budget constraint implies that something else must give: either consumption must decrease and/or some investment opportunities must be forgone (this will lead to lower consumption in the future).

Remaining details. The remainder of this section lays out the remaining details: Section 2.2 uses Global Game methods to study equilibrium household run behavior, taking as given the planner's policies; Section 2.3 then considers the planner's problem taking as given the equilibrium run behavior of households, and characterizes the equilibrium.

2.2 Equilibrium Bank Runs

In this section, I focus on the within-period decision of depositors' regarding whether or not to run (cf. Figure 4). The households' run decision takes as given the actions of firms and the government. As is well known from the original analysis of bank runs in Diamond and Dybvig (1983), bank run models are prone to equilibrium indeterminacy. In order to resolve this problem, I use techniques from the global games literature to obtain unique equilibrium predictions;[24] in particular, I build upon the bank run model of Goldstein and Pauzner (2005).[25]

The structure of the economy as well the timing was described in Section 2.1 (also see Figure 4 for a graphical illustration). The analysis here will be static in nature, and so I drop

[24] An alternate approach is to use sunspot equilibria where the modeler can then specify an exogenous equilibrium selection rule (for example, see Ennis and Keister (2003) for an example of this in a closed economy). The global games approach does not allow such flexibility for the modeler. The general idea is to add a little bit of noise to the economy so that in the perturbed economy we no longer have any issues with multiple equilibria, and then take the limiting economy as the noise vanishes. This approach was introduced by Carlsson and van Damme (1993); see Morris and Shin (2003) for a review of the global games literature.

[25] We generalize the Goldstein and Pauzner (2005) model to allow for reserves S as well as end of period expenditures X. These generalizations are important for my setting– they correspond to endogenous quantities from the full DSGE model (see Section 2.3 for details).

all time subscripts. I index households by $i \in [0,1]$. Following Goldstein and Pauzner (2005), the noise component of households' private productivity signals (15), is uniformly distributed

$$\varepsilon_i \sim Unif(-\Delta, \Delta) \tag{18}$$

with support between $-\Delta$ and Δ. Furthermore, the noise components ε_i are independent and identically distributed (iid) in the cross section (and over time).

After observing the private signals, depositor households decide whether or not to withdraw their deposit. This decision will ultimately depend on the payoffs, which we go through now.

Payoffs. Given the realized productivity z and the (still to be determined) equilibrium fraction of households who withdraws early $n \in [0, n_{max}]$, the fraction of seeds subject to early harvest $\ell \in [0,1]$ is given by

$$\ell\left(z, K, d, S, X, n\right) = \begin{cases} 0 & \text{if } nd \leq S \\ \frac{nd-S}{L(z)K} & \text{if } nd \in (S, S+L(z)K) \\ 1 & \text{if } nd \geq S+L(z)K \end{cases} . \tag{19}$$

Here, d is the promise payout to withdrawers (11). The total amount of fruit demanded by withdrawing depositors is given by nd. When international reserves S are sufficient to cover this amount (i.e. $nd \leq S$), there is no need for early harvest. In the intermediate region where withdraws can only be fulfilled by harvesting some of the unripe fruit (i.e. $S < nd < S + L(z)K$), the bank chooses the minimal required amount of early harvest and sets $nd = S + \ell L(z)K$.[26] Finally, when withdraws are too large for the total amount of liquidity available (i.e. $nd \geq S + L(z)K$), the bank is forced to harvest early its entire crop of fruit and pay out all the proceeds to withdrawing households.

[26]This is because early harvest involves a loss relative to late harvest– it is never efficient to harvest early more than is needed and store the proceeds to the end of the period.

The realized consumption from running is given by

$$C_{run}\left(z,K,d,S,X,n\right) \tag{20}$$
$$=\begin{cases} d & \text{if } nd \leq S+L(z)K \\ \begin{cases} d & \text{with probability } p_{withdraw} \\ 0 & \text{with probability } 1-p_{withdraw} \end{cases} & \text{if } nd > S+L(z)K \end{cases}.$$

In the region where the bank has a sufficient supply of liquidity to pay all n withdrawing households (ie. $nd \leq S+L(z)K$), we see that each withdrawing depositor receives the promised amount d. Outside of this region, the bank cannot guarantee payment to all withdrawing households. In this case, households are paid according to the sequential service constraint so that each withdrawing household receives d with probability

$$p_{withdraw}=\min\left\{\frac{S+L(z)K}{nd},1\right\} \tag{21}$$

and zero otherwise.

The corresponding utility obtained by withdrawing households is then given by

$$U_{run}\left(z,K,d,S,X,n\right) \tag{22}$$
$$=\begin{cases} u\left(C_{home}+d\right) & \text{if } nd \leq S+L(z)K \\ p_{withdraw}u\left(C_{home}+d\right)+\left(1-p_{withdraw}\right)u\left(C_{home}\right) & \text{if } nd > S+L(z)K \end{cases}$$

where the probability of a successful withdrawal $p_{withdrawal}$ is given in (21), and the utility function $u(\cdot)$ is assumed to be in CRRA form

$$u(c)=\frac{c^{1-\eta}}{1-\eta} \tag{23}$$

with $\eta > 1$ being the depositors' risk aversion. For numerical convenience, I bound utility from below by introducing negligible amounts of home production (in terms of fruit) given by

$$C_{home} = c_{lb} K, \tag{24}$$

with $0 < c_{lb} \ll 1$ set to be very small.[27] I specify this amount to be proportional to capital in order to preserve homogeneity.

All non-withdrawing households are residual claimants on the economy at the end of the period. Their consumption can be calculated as follows: after paying off withdrawing depositors, the amount of liquidity carried over to the end of the period is given by $(S + \ell L(z)K - nd)_+$ where the amount of early harvest ℓ is given in (19).[28] This amount is then supplemented by proceeds from late harvest (which amounts to $(1-\ell)e^z K$) before the bank finally pays end of period expenditures X. The residual amount $(S + \ell L(z)K - nd)_+ + (1-\ell)e^z K - X$ is then split evenly amongst the remaining $1-n$ households who each obtain consumption

$$C_{wait}(z, K, d, S, X, n) = \frac{\left[(S + \ell L(z)K - nd)_+ + (1-\ell)e^z K - X\right]_+}{1-n}, \tag{25}$$

where $C_{wait} \geq 0$ due to limited liability for households.[29]

These households obtain utility

$$U_{wait}(z, K, d, R, X, n) = u\left(C_{home} + C_{wait}(z, K, d, R, X, n)\right). \tag{26}$$

with $C_{wait}(\cdot)$ given by (25) and home production given by (24).

[27]Think of each household as having a cabbage patch which provides him with a subsistence level of consumption.

[28]The notation $(x)_+$ denotes $\max\{x, 0\}$.

[29]This is only imposed for the determination of bank runs. In the full model of Section 2.3, the presence of the budget constraint means that the limited liability constraint will never bind in equilibrium.

Equilibrium. Based on the noise structure (18), the posterior distribution for productivity conditional on household private signal $s_{i,t}$,

$$z\,|s_i \sim Unif\,(s_i - \Delta, s_i + \Delta)\,, \tag{27}$$

is uniformly distributed between $s_i \pm \Delta$.

After observing his signal, each depositor then chooses whether or not to run according to

$$\max_{a_i \in \{wait, run\}} \mathbb{E}\left[U_{a_i}\left(z, K, d, n_{max}, S, X, n\left(\{a_{-i}\}\right)\right)|s_i\right]. \tag{28}$$

Conditional on their private signals s_i, households decide whether or not to run. Households obtain expected utility $\mathbb{E}\left[U_{run}(\cdot)\,|s_i\right]$ by running or $\mathbb{E}\left[U_{wait}(\cdot)\,|s_i\right]$ by waiting, with the utility from withdrawing (waiting) being given by equation (22) (equation (26)). Note that in formulation (28), each household understands the structure of the coordination problem. In particular, each household i realizes that equilibrium number of runs $n = n\left(\{a_{-i}\}\right)$ depend on the collective actions of other households $\{a_{-i}\}$, each of whom is facing a similar problem.

I focus on a symmetric equilibrium. The equilibrium run decision is given by a threshold policy where an individual depositor runs if and only if his private signal is below some threshold $s^\star_\Delta\left(K, d, n_{max}, S, X\right)$. That is, in equilibrium the solution to (28) is given by

$$a_i = \begin{cases} run & \text{if } s_i \leq s^\star_\Delta\left(K, d, n_{max}, S, X\right) \\ wait & \text{if } s_i > s^\star_\Delta\left(K, d, n_{max}, S, X\right) \end{cases}. \tag{29}$$

It remains to characterize the equilibrium threshold $s^\star_\Delta\left(K, d, n_{max}, S, X\right)$.

Under the conjectured threshold equilibrium (29) and the assumed noise structure for households' signals, (18), we have the following outcome for the run variable n given the realization of the productivity shock z:

$$
\begin{aligned}
n(z, s_\Delta^\star) =& mass\left\{i \in [0, n_{max}] : \; s_i \leq s_\Delta^\star\right\} \\
=& \begin{cases} 0 & \text{if } z \geq s_\Delta^\star + \Delta \\ \frac{s_\Delta^\star - z + \Delta}{2\Delta} n_{max} & \text{if } z \in \left(s_\Delta^\star - \Delta, s_\Delta^\star + \Delta\right) \\ n_{max} & \text{if } z \leq s_\Delta^\star - \Delta \end{cases} .
\end{aligned}
\tag{30}
$$

That is, the mass of depositors who end up running is given by the number of depositors with private signals below the run threshold $s_\Delta^\star$.

As in Goldstein and Pauzner (2005), the run threshold $s_\Delta^\star$ can then be characterized by observing that the hypothetical "threshold depositor" who has signal $s_i = s_\Delta^\star$ is indifferent between running and waiting:

Proposition 1. (Indifference condition for the run threshold). *The run threshold $s_\Delta^\star$ is given by the solution to the following indifference condition:*

$$
\int_0^{n_{max}} U_{run}\left(z\left(n, s_\Delta^\star\right), K, d, S, X, n\right) \, dn = \int_0^{n_{max}} U_{wait}\left(z\left(n, s_\Delta^\star\right), K, d, S, X, n\right) \, dn. \tag{31}
$$

where $z(n, s_\Delta^\star) = s_\Delta^\star + \Delta - \frac{2\Delta n}{n_{max}}$.

Proof. Consider the expectations within the maximization problem (28) for the "threshold depositor." Given posterior distribution (27), and the conjectured number of runs under the threshold policy (30), the threshold depositor will have posterior belief $n\left(z, s_\Delta^\star\right) | s_i = s_\Delta^\star \sim Unif\left(0, n_{max}\right)$. We can then obtain (31) through a change of variable given by $z = z(n, s_\Delta^\star)$. □

The limiting economy without noise. Finally, I take $\Delta \downarrow 0$ and let the noise component vanish[30] so that the limiting economy recovers the structure of original economy. The run

[30]This is standard in the global games literature. Another reason for considering the limiting economy is more subtle. While threshold equilibria are well defined for every value of $\Delta > 0$, some of the parameters (e.g. expenditures X) will be determined by choice variables of the planner in the full model. In the limiting model

threshold in the limiting economy is now in terms of the *productivity* z. The certainty equivalent value of consumption, W_t, appearing in the planner's objective function (16) takes into consideration consumption in the cross-section of households after the equilibrium run outcome. This is summarized below:

Proposition 2. (Equilibrium in the limiting economy.) *In the limiting economy as noise goes to zero (i.e. $\Delta \downarrow 0$), the run threshold is given in terms of* productivity*:*

$$z^{\star}(K, d, n_{max}, S, X) = \lim_{\Delta \downarrow 0} s^{\star}_{\Delta}(K, d, n_{max}, S, X). \tag{32}$$

Furthermore, this run threshold is implicitly determined as the root to the following equation:

$$\int_0^{n_{max}} U_{run}(z^{\star}, K, d, S, X, n)\, dn = \int_0^{n_{max}} U_{wait}(z^{\star}, K, d, S, X, n)\, dn, \tag{33}$$

where $U_{run}(\cdot)$ and $U_{wait}(\cdot)$ are given, respectively, by (22) and (26).

Households have symmetric information (they observe realized productivity z) and run if and only if productivity z is below the run threshold $z^{\star}$. The equilibrium number of runs is given by

$$n_{eq}(z, K, d, n_{max}, S, X) = n_{max} 1_{\{z \leq z^{\star}(K, d, n_{max}, S, X)\}}. \tag{34}$$

The associated certainty equivalent household consumption value appearing in the planner's objective function (16) is given by

$$W(z, K, d, n_{max}, S, X) \equiv u^{-1}\left(\begin{array}{c} n_{eq} U_{run}(z, K, d, S, X, n_{eq}) \\ +(1 - n_{eq}) U_{wait}(z, K, d, S, X, n_{eq}) \end{array}\right). \tag{35}$$

where the equilibrium number of runs $n_{eq} = n_{eq}(z, K, d, S, X)$ is given by (34), and $u(\cdot)$ is

where signals are fully revealing, there is no additional information content contained in choice variables over and above each household's signal. This greatly simplifies the characterization of bank run problem (28). In particular, it avoids feedbacks of the sort studied in Bond and Goldstein (2014) where government policies contain information over and above agents' private signals and act as a public coordination device. Considering policy feedbacks of this sort is beyond the scope of this paper.

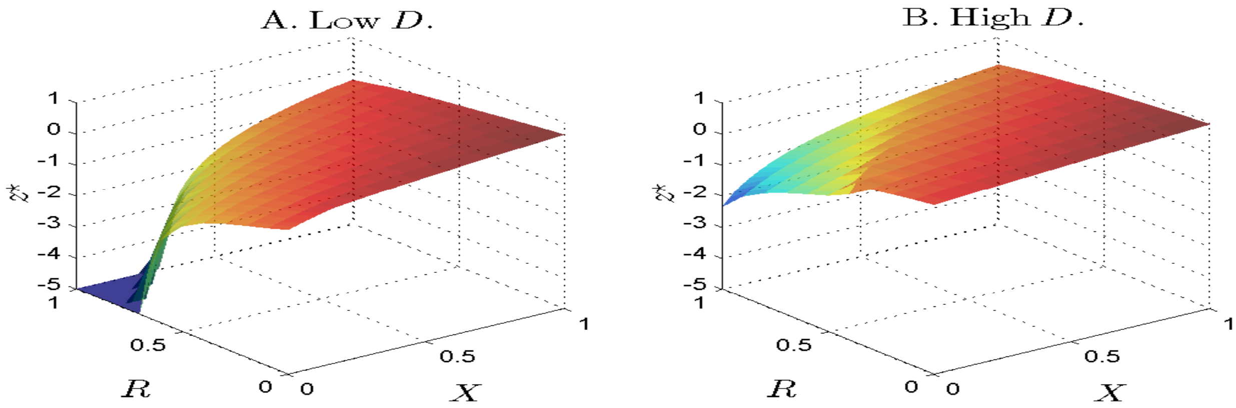

Figure 5: **Run thresholds.** This figure illustrates run thresholds $z^\star$ as a function of international reserves on hand R and end of period expenditures X. Panel A (B) plots the run thresholds for low (high) amounts of (total) deposits $D = d \times n_{max}$. The run threshold is characterized in (32) and is given by the solution to equation (33).

the utility function (23).

Proof. The indifference condition (33) for the limiting economy can be seen by taking the limit $\Delta \downarrow 0$ in (31). The rest follow straight from their definitions. $\square$

The welfare function $W(\cdot)$ defined in (35) serves as a key input for the dynamic stochastic general equilibrium model of Section 2.3. To build intuition for the full DSGE model, I first illustrate some of the key properties of the bank run model.

Run thresholds. I first focus on the endogenous nature of bank runs. In Figure 5, I plot the run threshold $z^\star$ (see (32)). The run thresholds display intuitive comparative statics.

First, we see that the run threshold decreases as the amount of international reserves on hand increases. A higher level of liquidity buffer prevents the liquidation of projects and ensures final output will be high. In this case, the payoff to waiting becomes higher and households' incentives to run are decreased.

Second, run thresholds are increasing in (end of period) expenditures X. Since waiting households are residual claimants on the bank at the end of the period, higher expenditure levels only serve to decrease the consumption levels associated with waiting and so increase households' run incentives. For example, if sovereign debt repayments are prohibitively high, then bank runs are likely unless the sovereign defaults.

Third, comparing panels A and B, we see that run thresholds are higher for higher deposit levels. Higher amounts of short term (domestic) debt create larger coordination problems within the domestic banking sector, and make the banking sector more prone to runs.

Fourth, we also observe substantial amount of non-linearity. When domestic leverage is low (panel A), the run threshold falls quickly when reserve levels increase and/or when expenditures decrease. In contrast, when domestic leverage is high (panel B), the run threshold falls more slowly. We see that when the domestic economy is more highly leveraged, more substantial amounts of liquidity on hand and/or larger cut backs in expenditures are necessary in order to decrease run incentives. Instead, when the domestic economy is not so highly leveraged, a small increase in liquidity and/or a small cut back in expenditures will go a long way to stabilize the domestic banking sector.

Equilibrium runs. The corresponding equilibrium run schedule n_{eq} is plotted in Figure 6. The equilibrium run schedule inherits the properties of run thresholds according to (34): runs are more likely to occur when (i) there is an insufficient amount of international reserves S, (ii) expenditures X are too high, (iii) domestic leverage levels D are high, as well as (iv) when productivity z is low. Furthermore, the transition between a no run and a run equilibrium can be very abrupt. When the economy is slightly over the run threshold, a small amount of additional liquidity will be extremely valuable in preventing bank runs. Similarly, a small reduction in expenditures can also go a long way in preventing bank runs. These forces are important in shaping debt capacity in the full model.

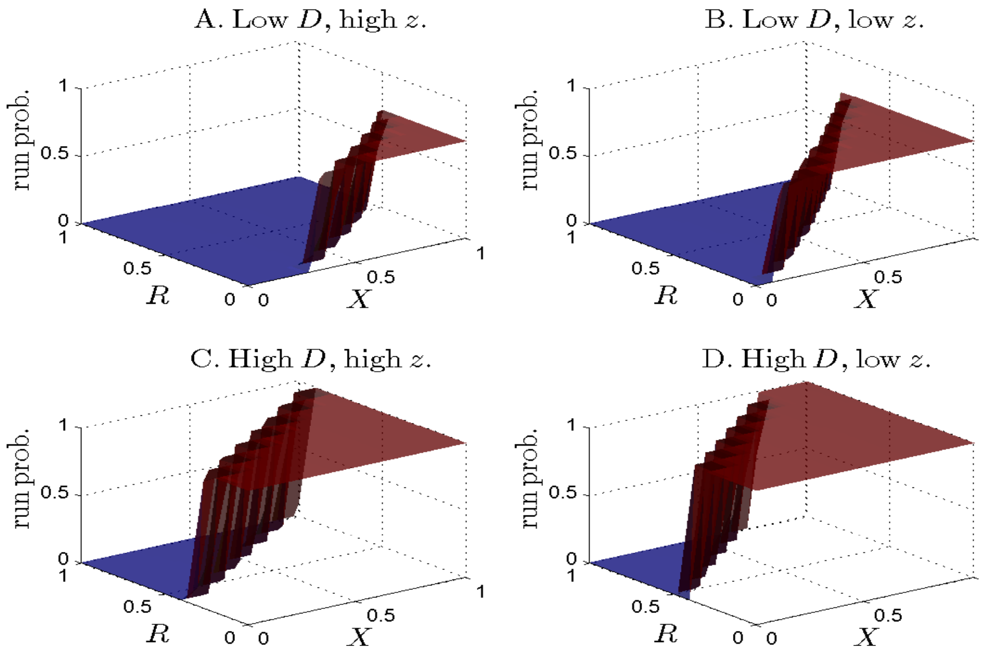

Figure 6: **Bank run probabilities.** This figure plots the equilibrium amount of bank runs n_{eq}, defined in (34), as a function of liquidity on hand R and end of period expenditures X. The top (bottom) panels correspond to low (high) levels of (total) deposits $D = d \times n_{max}$, while the left (right) hand side panels correspond to a high (low) realization of the productivity shock z.

2.3 A Model of Sovereign Debt, International Reserves, and Twin Crises

Having characterized the equilibrium run behavior and obtained the welfare function $W(\cdot)$ in Section 2.2, I now provide the remaining details to the full model which was first described in Section 2.1.

Timing. The timing is illustrated in Figure 7 which supplements Figure 4 with the timing of various choice variables available to the planner. The SOE enters each period with capital stock K_t, international reserves S_t, external debt B_t, banking lending A_t and bank deposits D_t. The productivity shock z_t is then realized. At this stage, the planner chooses whether or not to default. Bank runs and liquidations then occur according to (34) and (19), respectively.

If the planner chooses not to default, debt B_t is repayed and new debt B_{t+1} is issued at per-unit price Q_t which will be determined in equilibrium. In addition, the planner chooses investments I_t and next period's reserves S_{t+1}.

In the event of a default, the planner repudiates on outstanding debt B_t and loses access to international credit markets. Default is also accompanied by a drop in private credit which translates into a higher burden on the domestic banking sector under assumption (17). Whilst in default, the ability to borrow is unavailable and the planner can only choose reserve S_{t+1} and investment I_t policies. The planner can subsequently regain access to international credit markets in the following period with exogenous probability $\xi > 0$. All previous debt is forgiven in the event of a reentry.

Capital accumulates according to (4), while bank balance sheets evolves according to equations (6) through (9). Since I assume a constant capital structure (9), I will keep D_t as the state variable and discard A_t (only one of the two need to be kept).

Budget constraint. Total end of period expenditures are given by

$$X_t = \frac{S_{t+1}}{1+r} + \phi\left(\frac{I_t}{K_t}\right)K_t + 1_{\{V_{ND,t} \geq V_{D,t}\}}\left(B_t - B_{t+1}Q_t\right), \tag{36}$$

and consist of reserve accumulation costs for next period, investment costs, and in the case when default does not take place, the net proceeds from debt rollover. The magnitude of the expenditures will in turn influence the equilibrium run outcome n_t given by (34).

The planner's choices are subject to the following budget constraint:

$$\max\left\{S_t + \ell_t L(z_t)K_t - n_t d_t, 0\right\} + (1-\ell_t)\, e^{z_t}K_t - X_t \geq 0. \tag{37}$$

In the expression above, the amount of expenditures X_t implied by a particular set of choices must be such that the limited liability constraint for households is never violated in equilibrium.[31]

[31] Note that the choice is non-empty: expenditures X_t can always be set to zero by defaulting and then

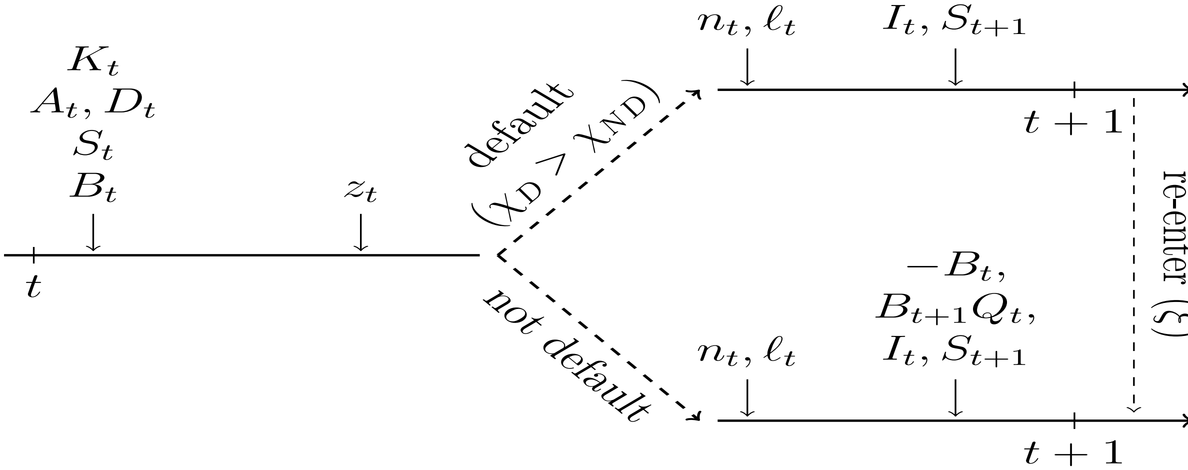

Figure 7: **Within period timing for the full model.** The SOE enters period t with capital K_t, international reserves S_t, outstanding debt B_t, and domestic bank assets and debt A_t and D_t, respectively. The productivity shock z_t is then realized. The planner then chooses whether or not to default. Equilibrium runs n_t and liquidations ℓ_t then occur according to the run model of Section 2.2. If default occurs, then there is a loss of private credit resulting in a greater burden for domestic intermediaries ($\chi_D > \chi_{ND}$), the planner then makes investment and reserve accumulation decisions. The SOE can regain access to international credit markets with probability ξ next period. If default does not occur, then the planner repays outstanding debt B_t, issues new debt with face value B_{t+1} at per unit price Q_t, and makes investment and reserve accumulation decisions.

We can rewrite the budget constraint (37) in the more usual form as follows:

$$\underbrace{\left[\ell_t L(z_t) + (1-\ell_t)e^{z_t}\right] K_t}_{Y_t} = \underbrace{n_t p_t^{withdraw} d_t + (1-n_t) C_{wait,t}}_{C_t} + \underbrace{\phi\left(\frac{I_t}{K_t}\right) K_t}_{I_t} \\ + \underbrace{\frac{S_{t+1}}{1+r} - S_t + 1_{\{V_{ND,t} \geq V_{D,t}\}} (B_t - B_{t+1} Q_t)}_{NX_t}, \tag{38}$$

with $C_{wait,t} \geq 0$, where I have made use of equations (20), (21), (25) and (36) to arrive at the above expression. Note that this is just the usual budget constraint in an open economy setting adjusted for runs and liquidations.

setting investment costs and reserve accumulation costs to zero. Constraint (37) will always hold for $X_t = 0$.

Planner's problem in default. The planner's objective function whilst in default is given by

$$V_D\left(z_t, K_t, D_t, S_t\right) = \max_{I_t, S_{t+1}} \left\{ \begin{array}{c} (1-\beta) W\left(z_t, K_t, D_t, S_t, X_{D,t}\right)^{1-\psi^{-1}} \\ +\beta CE_D\left(z_t, K_{t+1}, D_{t+1}, S_{t+1}\right)^{1-\psi^{-1}} \end{array} \right\}^{\frac{1}{1-\psi^{-1}}} \tag{39}$$

where the choice variables are investment I_t and international reserves S_{t+1} for the next period, and $W(\cdot)$ is given by (35). End of period expenditures during default are given by

$$X_{D,t} = \phi\left(\frac{I_t}{K_t}\right) K_t + \frac{S_{t+1}}{1+r}, \tag{40}$$

which is the sum of investment and international reserve accumulation expenditures. The planner gets the autarky continuation value $CE_D\left(z_t, K_{t+1}, D_{t+1}, S_{t+1}\right)$ in which no borrowing is allowed and external private credit decreases so that bank loans accumulate according to $\chi = \chi_D$ (cf. equation (6)).

Planner's problem when not in default. If the country is not in default, then the planner solves

$$V_{ND}\left(z_t, K_t, D_t, S_t, B_t\right) = \max_{I_t, S_{t+1}, B_{t+1}} \left\{ \begin{array}{c} (1-\beta) W\left(z_t, K_t, D_t, S_t, X_{ND,t}\right)^{1-\psi^{-1}} \\ +\beta CE_{ND}\left(z_t, K_{t+1}, D_{t+1}, S_{t+1}, B_{t+1}\right)^{1-\psi^{-1}} \end{array} \right\}^{\frac{1}{1-\psi^{-1}}} \tag{41}$$

where the choice variables are investment I_t, international reserve policy S_{t+1}, and debt policy B_{t+1}. The net expenditure under the no default regime is given by

$$X_{ND,t} = \phi\left(\frac{I_t}{K_t}\right) K_t + \frac{S_{t+1}}{1+r} + B_t - B_{t+1} Q_t. \tag{42}$$

It consists of investment costs, the cost of accumulating international reserves for tomorrow, and the cost of repaying outstanding debt B_t. Note that additional borrowing can be used to offset expenditures for the current period. Similarly, choices are also subject to the budget constraint (37).

The SOE gets the credit access continuation value $CE_{ND}(z_t, K_{t+1}, D_{t+1}, S_{t+1}, B_{t+1})$ under which access to international credit markets is maintained, and external private credit is available to help decrease the burden on domestic banks so that bank loan accumulation (6) happens with $\chi = \chi_{ND}$.

Note that unlike the planner's problem in default, when international reserves S_t are too low and/or when debt levels B_t are very high, no amount of expenditure cutbacks can satisfy the budget constraint (37), so that defaulting is the only option.

Bond prices and continuation values. Finally I determine bond prices and continuation values. The planner will always choose the greater of the default, $V_{D,t}$, and non-default values $V_{ND,t}$. Default occurs on the set $\{V_{D,t} > V_{ND,t}\}$ when defaulting is more attractive. The credit access continuation value is given by

$$\begin{aligned} & CE_{ND}\left(z_t, K_{t+1}, D_{t+1}, S_{t+1}, B_{t+1}\right) \\ =& \mathbb{E}_{z_{t+1}|z_t}\left[\max\left\{V_{ND}\left(\widehat{z}_{t+1}, K_{t+1}, D_{t+1}, S_{t+1}, B_{t+1}\right), V_D\left(\widehat{z}_{t+1}, K_{t+1}, D_{t+1}, S_{t+1}\right)\right\}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}. \end{aligned} \tag{43}$$

Whilst in default, the continuation value is given by

$$CE_D\left(z_t, K_{t+1}, D_{t+1}, S_{t+1}\right) = \left(\begin{array}{c} (1-\xi)\,\mathbb{E}_{z_{t+1}|z_t}\left[V_D\left(z_{t+1}, K_{t+1}, D_{t+1}, S_{t+1}\right)^{1-\gamma}\right] \\ +\xi CE_{ND}\left(z_t, K_{t+1}, D_{t+1}, S_{t+1}, 0\right)^{1-\gamma} \end{array}\right)^{\frac{1}{1-\gamma}}, \tag{44}$$

where ξ is the probability of reentering credit markets. All outstanding debt is written off when the country reenters credit markets.

Under the small open economy setting, sovereign bonds are priced by international investors who are assumed to be risk neutral.[32] Bond prices are given by

$$\begin{aligned} & Q\left(z_t, K_{t+1}, D_{t+1}, S_{t+1}, B_{t+1}\right) \\ = & \frac{1}{1+r} \mathbb{E}_{z_{t+1} \mid z_t}\left[1\left\{V_{ND}\left(z_{t+1}, K_{t+1}, D_{t+1}, S_{t+1}, B_{t+1}\right) \geq V_D\left(z_{t+1}, K_{t+1}, D_{t+1}, S_{t+1}\right)\right\}\right] \end{aligned} \tag{45}$$

and reflect the probability of repayment next period.

Equilibrium. The equilibrium concept is the standard Markov equilibrium:

Definition (Markov Equilibrium)**.** *A Markov Equilibrium for the economy consists of (i) default and non-default value functions V_D and V_{ND}, (ii) a bond price schedule Q, and (iii) a run schedule n_{eq}, such that*

1. *Given the bond price and run schedules Q and n_{eq}, V_D and V_{ND} are respectively characterized by (39) and (41).*
2. *The bond prices are consistent with the planner's default behavior so that the bond price schedule Q satisfies (45).*
3. *The equilibrium run schedule n_{eq} captures households' run incentives and is given by (34).*

Relation to Eaton and Gersovitz (1981). In relation to Eaton and Gersovitz (1981), our setting contains the following additional elements: (1) reserve accumulation, (2) investments and growth, and (3) an additional "liquidity" constraint modeled through bank runs.

Relation to the Bulow and Rogoff (1989) puzzle. When countries are able to default and then save in complete markets, Bulow and Rogoff (1989) show that sovereign debt cannot exist if exclusion from credit markets is the only form of punishment for defaulting. Their

[32] This can be easily relaxed by incorporating a pricing kernel for international lenders. See Borri and Verdelhan (2011) for an example of this.

striking result is based on a replication argument: the sovereign is always able to save in such a way as to replicate the lending relationship and subsequently achieve a strictly higher stream of consumption after defaulting. Hence, in the absence of further forms of default, a country will always default and so sovereign debt cannot exist in the first place.

More recently, Auclert and Rognlie (2014) show that even in an incomplete-markets setting, a similar replication argument holds within the canonical single period debt setting of Eaton and Gersovitz (1981). In Auclert and Rognlie (2014), the only constraint for the incomplete-markets replication argument is the budget constraint which any replicating portfolio must be subject to.

In my setting, the incomplete-markets replicating portfolio will in general not be feasible. The intuition is as follows: the possibility of bank runs introduces an additional liquidity constraint which the replicating portfolio must be subject to. In general, the replicating portfolio will not be able to satisfy the liquidity constraint along some sequence of shocks, and in this case, this can trigger very costly bank runs. This is exacerbated under assumption (17) in which the higher burden on the domestic banking sector can only make the liquidity constraint more likely to bind.

It must be emphasized that even with assumption (17), coordination problems are still key in generating any form of debt capacity. If domestic households can coordinate on not running, then there will not be any liquidity constraints to hinder the planner from defaulting and subsequently applying a replication strategy, and we know that in this case, it is very difficult for sovereign debt to exist in equilibrium.

Numerical implementation. The problem is homogenous of degree one in capital K_t. This allows me to scale out capital in the numerical implementation. In addition, the problem as formulated will not converge. This is because the equilibrium run schedule (34) is discontinuous at the run threshold. In turn, this makes $W(\cdot)$ and the Bellman equations discontinuous. To achieve numerical convergence, I instead use a smoothed version of the run threshold (34). The state space and choice sets are then discretized, and the model is

preferences, (log) TFP, and credit markets			Growth and banking		
η	depositors' risk aversion	5	$\phi(i)$	total investment cost, $i \geq 0$	$283000 i^{4.5}$
γ	planner's risk aversion	5	δ	depreciation	0.037
ψ	EIS	0.5	m_A	bank loan maturity	0.1225
β	discount rate	0.99	χ_{ND}	bank financing share, not default	0.1776
ρ_z	log TFP autocorrelation	0.93	χ_{ND}	banking financing share, default	0.2336
σ_z	log TFP volatility	0.027	m_L	deposits to loans	0.71
r	interest rate	0.01	d_0	promised deposit payout to capital	0.7
ξ	reentry probability	0.0385	$L(z)$	liquidation proceeds	0.5
			a_0	other bank lending to capital	0.6

Table 1: **Baseline parameters.**

then numerically computed using value function iteration methods. Details for these steps are given in Appendix B.

3 Quantitative Analysis

3.1 Calibration

The calibration is at a quarterly frequency, and is loosely based on Argentina for the period 1993Q1 to 2001Q4. Much focus has been placed on Argentina by the sovereign default literature (e.g. see Arellano (2008) and Chatterjee and Eyigungor (2012)), and the period 1993-2001 is often chosen because Argentina had a fixed exchange rate vis-a-vis the dollar during this period. The parameters used in the baseline calibration are summarized in Table 1.

Productivity. Parameters for the TFP process (1) are estimated after linearly detrending and deseasonalizing real Argentine GDP.[33] The quarterly GDP series is taken from Neumeyer and Perri (2005). The resulting parameters are $\rho_z = 0.93$ and $\sigma_z = 0.027$.

[33]This is the same procedure used in Chatterjee and Eyigungor (2012).

Preferences. I set the elasticity of inter-temporal substitution (EIS) parameter ψ to be 0.5. This is a standard value used in the literature. Both the planner's and the depositors' risk aversion parameters, γ and η respectively, are set to be 5. The subjective discount rate β is chosen to be 0.99. Note that the sovereign default literature usually calibrates this number to be much lower in order to increase the frequency of default.[34] The literature can afford to do so because they usually assume exogenous output costs to generate debt capacity.[35] This is a luxury which my model does not enjoy.

Liquidation. I set liquidation proceeds to be constant $L(z) = 0.5$. Since the productivity process (1) fluctuates around zero, on average full harvest will yield $e^z \approx 1$ units of fruit. This means that on average 50% of potential output is lost for each unit of fruit harvested early. Note, however, that the liquidation loss at the aggregate level is instead given by $\ell_t \left(e^{z_t} - L(z_t)\right)$ which will be much lower on average since not all fruit will be harvested early each period. In addition, fixing liquidation proceeds to be constant also implies that liquidation is more costly during good times when productivity is high.

Growth. I set the capital depreciation rate to be $\delta = 0.037$. The total cost of investment takes the form $\phi\left(\frac{I_t}{K_t}\right) = \phi_0\left(\frac{I_t}{K_t}\right)^{\phi_1}$, a common specification in the literature (e.g. Jermann (1998)). I set $\phi_0 = 283000$ and $\phi_1 = 4.5$, which are chosen, in conjunction with the depreciation rate, so that both growth rates as well as investment-to-output ratios are reasonable. Since the production function is AK and lack curvature, larger amounts of curvature in the investment cost function $\phi(\cdot)$ are required in order to achieve these objectives.

External financing dependence on domestic banks. The parameters χ_D and χ_{ND} are meant to capture domestic firms' external financing dependence on the domestic banking

[34]For example, the subjective discount factor is 0.95 in Arellano (2008) and 0.88 in Mendoza and Yue (2012). A more impatient government will care less about the consequences of default and therefore default more often.

[35]Mendoza and Yue (2012) is an exception. However, they can still set the subjective discount factor to be quite low as they do not consider reserve accumulation after defaulting.

sector. In general, this is very difficult to measure. First, such data is typically not available, and even if it were, the observed data will reflect equilibrium forces of demand and supply for external financing. In an influential study, Rajan and Zingales (1998) measure external financing dependence for US manufacturing firms as the fraction of investments in excess of a firm's free cash flows,[36] and find that external financing dependence for investments is, on average, 32%. They then use their US measures of external financing dependence to proxy for firms around the world.[37] I follow their approach in the baseline calibration. However, for my purposes, I still need to know the split between domestic and foreign financing. Gozzi et al. (2013) find that for developing countries, 45% of the proceeds of bond issues are raised abroad on average. Based on these findings, I set the domestic financing dependence to be $\chi_{ND} = 0.32 \times 0.55 = 0.176$ while the country is not in default.

Empirical studies by Arteta and Hale (2008) and Das et al. (2010) show that external private credit decreases in the event of a sovereign default. I assume a drop of 40% based on findings in Das et al. (2010).[38] Based on these findings, I set domestic financing dependence to be $\chi_D = \chi_{ND} + 0.32 \times 0.45 \times 0.4 = 0.2336$ while the country is in default.[39] The difference between χ_D and χ_{ND} reflects that portion of investments no longer financed with external

[36]Only investments in tangible assets are included in Rajan and Zingales (1998)'s study. In our context where we think of additional forms of intangible investments, there is good reason to believe that external financing dependence will be higher than that assumed in the baseline calibration. In addition, there are substantial amounts of variation in the Rajan and Zingales (1998) measure of external dependence across industries and firm size and age.

[37]They argue that in a perfect capital market, the actual amount of external funds raised by a firm would equal its desired amount. Since US capital markets are amongst the most advanced in the world, their US measures of external financing dependence can then serve as a good proxy for *demand* for external financing around the world.

[38]This amount is the portion attributable to a sovereign default after controlling for a whole set of other variables. Such variables include international competitiveness, investment climate and monetary stability, financial development, macroeconomic fundamentals, political stability, as well as global capital supply. The decrease in private credit found in Arteta and Hale (2008) and is between 20-30%. A reason for this difference is that Das et al. (2010) only focuses episodes of sovereign default to private creditors, which is in line with definitions of sovereign default in other empirical studies (e.g. Reinhart et al. (2003), Tomz and Wright (2007), and Panizza et al. (2009)). In contrast, Arteta and Hale (2008) uses an expanded definition of defaulting so that their sample contains "smaller" defaults on average. It is interesting to note that Arteta and Hale (2008) find similar magnitudes for worst case scenario defaults in their sample.

[39]Note that this "back of the envelope" adjust only reflects changes in quantities. Given that the private costs of borrowing from external sources also increase after a sovereign default (Agca and Celasun (2012)), χ_D is likely to be higher in a price-adjusted calculation.

private credit; in the event of foreign private credit outflows, the domestic intermediary will have to step in and provide credit.

I set $m_A = 0.1225$ so that on average 12.25% (49%) of bank loans come due every quarter (year). This is based on average long term debt shares for Argentinean firms considered in Kirch and Terra (2012). This corresponds to bank loans having an average maturity of approximately 8 quarters or 2 years.

Bank capital structure. I choose $m_L = 0.71$ based on deposit-to-asset ratios for Argentine banks during 1993 to 2001. The data for this is available from the World Bank's World Development Indictors (WDI) database. Recall that total bank lending A_t consists of investment loans A_t^{loan} as well as other forms of credit $A_t^{other} = a_0 K_t$. I set $a_0 = 0.6$, so that total bank assets to (quarterly) GDP will look reasonable. Based on WDI data for Argentina, bank assets to (quarterly) GDP is 112% during 1993-2001, and 81% for the full sample 1961-2011.

For promised deposit payouts, I set $d_0 = 0.7$. To get a feel for this number, note that in a closed economy, consumption to capital for late withdrawing households will be approximately $c_{wait} \approx 1 - \phi\left(\frac{I_t}{K_t}\right)$ on average. The ratio c_{wait}/d_0 can then be thought of as the spread between long term and short term deposits.

International capital markets. The quarterly international interest rate is set to be $r = 0.01$ and is based on US short rates. I follow Chatterjee and Eyigungor (2012) and set the probability of reentry after defaulting to be $\xi = 0.0385$. This is based on exclusion periods after Argentine default, and implies an average exclusion period of 26 quarters or 6.5 years.

3.2 Results

Baseline model moments. Moments from the baseline calibration are shown in Table 2. My model is able to generate (external sovereign) debt to GDP of 20%. While this amount

	model	data
debt to gdp	0.2	0.7
sovereign spread	5bps	$\approx$ 200bps
reserves to gdp	0.76	0.25
gdp growth	0.01	0.007
investment to gdp	0.29	0.2 (tangible)
deposits to gdp	0.7	0.81
bank assets to gdp	0.99	1.12
average withdraws	0.6	
average liquidation	0.0005	

Table 2: **Baseline moments.**

falls short of its empirical counterpart, which I take to be 70%,[40] it is still comparable to debt levels generated in previous studies involving single period debt.[41,42]

International reserves to (quarterly) GDP averages 76%. Based on the World Bank's WDI dataset, the mean international reserve to (quarterly) GDP ratio for Argentina over the period 1993 to 2001 is 25%, which is much lower than the model's counterpart. Figure 8 plots average external sovereign debt to GDP and international reserve to GDP ratios for middle income countries. Since the late 80s, there have been a steady decline in debt levels alongside an increase in reserve levels, with the trend in reserves increasing in a much more pronounced fashion since 2000. Middle income countries have steadily evolved from a high debt-low reserves regime to a high reserves-low debt regime. My model seems to be a better

[40] Actual debt to (quarterly) GDP ratios for Argentina over the period 1993-2001 is 100%. However, because my model does not feature recovery upon default, and since the eventual recovery on debt defaulted in 2001 was around 30%, I follow Chatterjee and Eyigungor (2012) in treating 70 cents out of each dollar as the truly unsecured portion of debt.

[41] In an endowment economy setting with exogenous costs of default, Arellano (2008) generates debt to GDP of 6% in a stationary setting, while the non-stationary models investigated in Aguiar and Gopinath (2006) generate debt to GDP ranging between 18% and 27%. Mendoza and Yue (2012) features trade-related endogenous costs of default and generates debt to GDP of 23%. Gornemann (2013) extends the stationary setting of Mendoza and Yue (2012) by further incorporating endogenous growth, and generates debt to GDP of 12%. The studies of Mendoza and Yue (2012) and Gornemann (2013) do not consider reserve accumulation. To the best of my knowledge, there are no other papers featuring both endogenous costs of default and international reserve accumulation with which to compare results.

[42] Chatterjee and Eyigungor (2012) was successful in fully matching empirical levels of debt to GDP in an endowment economy setting with long term debt and calibrated exogenous output costs after defaulting.

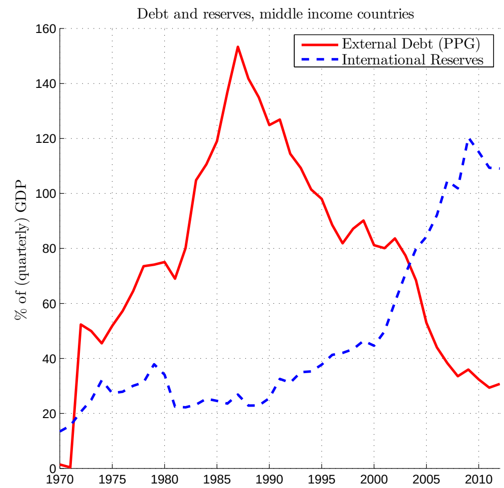

Figure 8: **External public debt and international reserves, middle income countries.** This figure plots the time-series of external public and publicly guaranteed debt to (quarterly) GDP, and international reserves (inclusive of gold) to (quarterly) GDP ratios for middle income countries for the period 1970-2012. For each period, the averages are GDP-weighted. Middle income countries include both lower middle income and upper middle income country groups as per the World Bank's definitions. The data is taken from the World Bank's WDI database.

fit for the more recent period. One possibility for this is that liquidity concerns, the central ingredient in my setting, played a more central role more recently after the onset of various liquidity crises (e.g. the 1998 LTCM crises).

The model generates a negligible sovereign spread of 5 basis points when (quarterly) sovereign spreads are instead around 200 basis points in the data. While this is a common pitfall in sovereign default models with single period debt, it is especially pronounced in my baseline calibration. This is due to my stringent requirement of endogenously generating debt capacity in a setting where reserve accumulation is possible. A common strategy in the sovereign debt literature for generating sovereign spreads is to lower the subjective discount factor (a more impatient country will care less about the consequences of defaulting, and hence borrow more and also default more often), and at the same time increase default costs so as to increase debt capacity. Sovereign default models with exogenously specified default costs have a lot of flexibility when it comes the latter, which is a luxury that I cannot afford in my setting. In addition, the requirement that default costs be robust to (post-default) international reserve accumulation is especially stringent.[43] Given these constraints, my framework will first have to generate additional debt capacity[44] before it can generate more realistic levels of sovereign credit spreads.

Average growth rates in the baseline model is 1% per quarter, which is a bit higher than Argentina's quarterly real growth rate of 0.7%. This growth rate is accompanied by a mean investment-to-GDP ratio of 29% in the model. Average investment, in tangible capital, to GDP is 20% in the data.[45] The difference between our model's investment rate and the data is attributable to the accumulation of intangible capital which can be difficult to measure. Studies indicate that investments in intangible assets can be substantial. For example, Eisfeldt and Papanikolaou (2013) find that for US firms, the size of the organizational capital stock (i.e. human capital within the firm) is on par with physical capital stock, and that the investment rate in organizational capital is 11%. This number is lower in less developed countries but can still be substantial. In a study of Brazil, Dutz et al. (2012) find that investment in organizational capital is around 4% of GDP. Furthermore, there are still other forms of investments, such as research and development, and education, which will ultimately affect the capital stock of a country. Finally, the planner will also care about positive spillover effects associated with various types of investments and include these in his accounting of the total capital stock. These considerations imply that *total* investments (in both tangible and intangible forms of capital) can be substantially higher than 20% of GDP.

The size of the banking sector is, on average, 99% of (quarterly) GDP in terms of bank assets, while deposits to (quarterly) GDP is 70% within the model. These values are below their counterparts in the data of 112% and 81% respectively.[46] While it is possible to further

[43] For example, in Mendoza and Yue (2012), lost trade credit after a sovereign defaulting *directly* translates into losses in trade and output. This is because reserve accumulation is assumed away in their setting so that countries cannot self-finance trade credit whist in default. More broadly, this involves tackling the Bulow and Rogoff (1989) puzzle.

[44] Appendix C considers some extensions pointing in this direction.

[45] I take a longer sample, 1960-2013, for computing average growth rates and investment rates. The data for this is annual and is from the World Bank's WDI dataset.

[46] These are average numbers for Argentina between 1993-2001. There is a lot of variation both in the time series and in the cross-section.

increase debt capacity by increasing the size of the domestic banking sector (e.g. I can increase the a_0 parameter), I have chosen not to do so in the baseline calibration as this will lead to too many withdraws on average. As it stands, we see that withdraws n_t average 60% in the model. Given $d_0 = 0.7$, an average deposit to GDP of 70% implies that on average we have $n_{max,t} \approx 1$. In other words, on average, all households end up being depositors in the baseline model and a substantial fraction of them (60%) choose to take out their deposits and consume the promised amount d_t. Given my timing assumption, investment choice made at the end of the period is very flexible and can be readily adjusted in order to avoid runs. The planner is choosing investments so that there is not too much difference in consumption levels between waiting or running. Note, however, that liquidations are effectively zero on average within the model. This is due to the planner's high choice of international reserves.

Output, investment, and international reserves around default. I now use the baseline model to conduct an event study of key variables around default. Recall the model's key predictions: the increased burden on the domestic banking sector as a result of decreased foreign private lending after a sovereign default means that the planner will have to accumulate more international reserves after defaulting. As a consequence, this leads to lower investment and lost growth opportunities.

The results for this exercise are shown in Figure 9. Qualitatively, the results are consistent with the data (see Figure 2)– after defaulting, reserve to output ratios increase, investment to output ratios decrease, and output falls. The differences in the responses are attributable to my model's simplifying assumptions. Output losses within the model are transitory due to my assumption that liquidations do not destroy capital,[47] whereas output losses contain a permanent component in the data. Also, changes in investment and reserve policies in the model do not feature any transition dynamics, in contrast to the data. This lack of transition dynamics is due to the AK growth setting.[48]

[47] Appendix C.1 considers an extension along these lines.

[48] This is because of the lack of curvature in a linear production function. See Acemoglu (2009) for details. Transition dynamics can be introduced by incorporating curvature into the production function (e.g. by

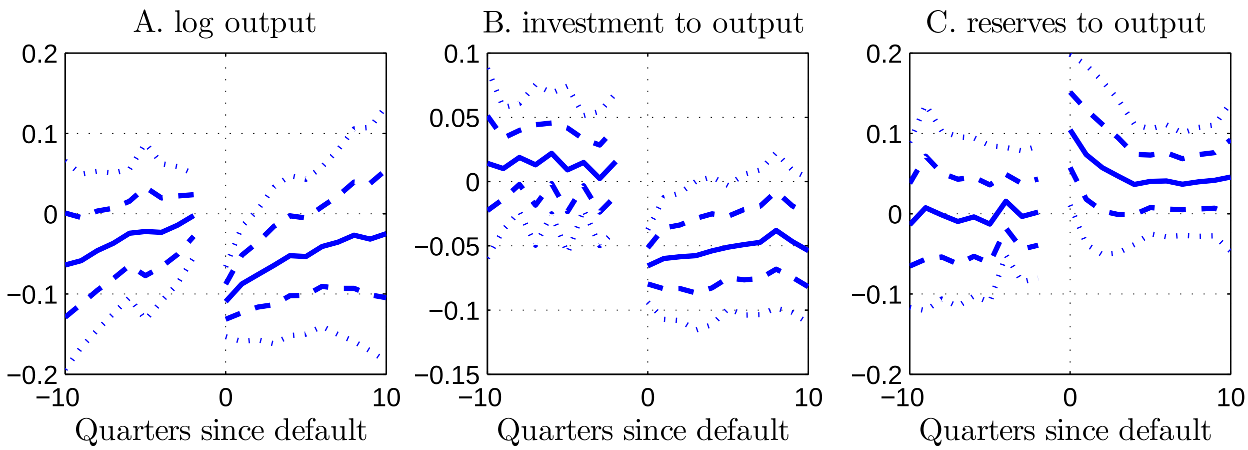

Figure 9: **Output, investment, and international reserves around default: model.** This figure plots (log) output, investment to output, and reserve to output around a default event. Default occurs at time 0. All responses are relative to their respective values right before defaulting. The solid line plots the average response, while the dashed (dotted) lines plots the one (two) standard deviation bounds. This is the model's counterpart to the empirical responses plotted in Figure 2.

Counter-cyclical sovereign default. Even though sovereign credit spreads are low in the baseline model, default nevertheless still occur (but rarely). Panel A of Figure 10 show productivity z_t around a sovereign default. We see that sovereign default is counter-cyclical and occurs when productivity is low. Default in the baseline model occurs after a sudden large drop in productivity– the average drop in productivity which triggers a default is 0.1 which is a 3.7 standard deviation shock. This is why average sovereign spreads are negligible in the model.

Panel B shows that within the model, domestic leverage increases after a sovereign default as the domestic banking sector raises additional domestic deposits in order finance investments (recall that default triggers a flight in foreign private credit in the model).[49]

To summarize, the model is able to capture important qualitative aspects of the data.

using a Cobb-Douglas production function). However, this comes at the expense of additional computational complexity.

[49]My model is missing domestic capital flight. In the data, deposits decrease after a sovereign default. Often, these deposits go overseas, perhaps out of fear of hyperinflation (a feature not present within my setting).

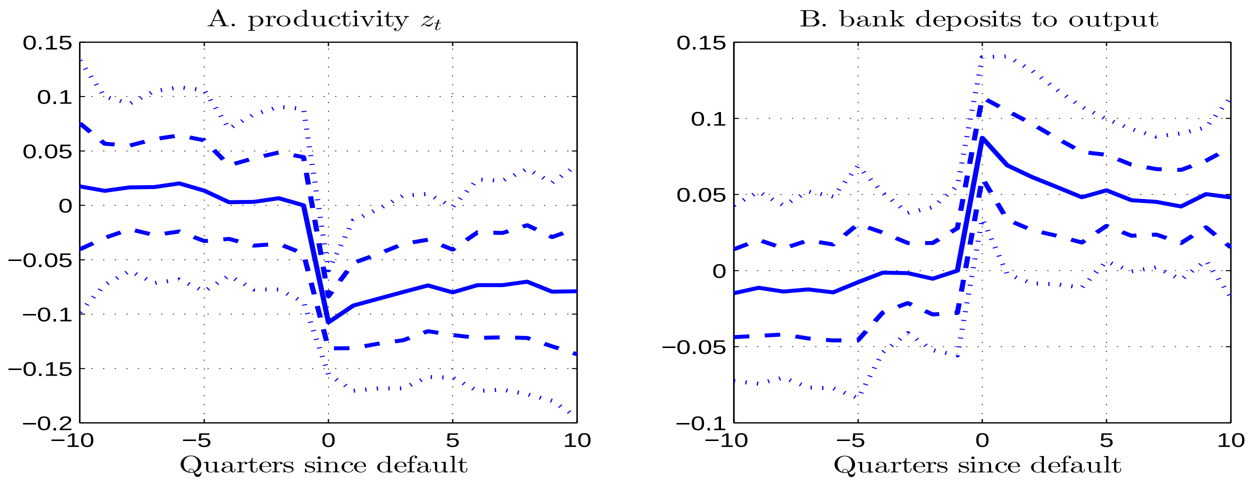

Figure 10: **Productivity and domestic leverage around default: model.** This figure plots productivity and domestic deposit to GDP around a sovereign default. Default occurs at time 0. All responses are relative to their respective values right before defaulting. The solid line plots the average response, while the dashed (dotted) lines plots the one (two) standard deviation bounds.

These include banking related costs of default and sovereign debt capacity in the presence of international reserve accumulation, as well as the dynamics of key macroeconomic quantities (including output, investments, and international reserves) around a sovereign default. Quantitatively, my model still has shortcomings. Possible remedies for this are discussed in Appendix C.

4 Conclusion

I have introduced growth and domestic banking fragility into the canonical Eaton and Gersovitz (1981) sovereign default model. This allows us to tackle some puzzles in international economics. First, I obtain a theory of sovereign debt capacity that can withstand international reserve accumulation. Second, my model generates twin crises in a dynamic setting. Third, international reserves have "war-chest" like property within the model. Thus my model can account for the high levels of international reserves observed in the data. Finally, my model is able to generate reasonable levels of sovereign debt and international reserves in equilibrium.

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Appendix

A Details for the Event Study

I investigate how output, investment, and international reserves respond around a sovereign default. For this, I conduct an event study of the following form:

$$Y_{i,t+k} - Y_{i,t-1} = \alpha_i^k + \gamma_t^k + \beta^k D_{i,t} + e_{i,t}, \tag{A.1}$$

where Y is a response variable of interest, the α's and γ's are, respectively, country and time fixed effects, and the $D_{i,t}$'s are indicators for sovereign defaults which take on a value of one if and only if country i defaults in year t. The coefficients of interest are the β_k's which are the average response of the variable of interest, relative to its value just a year before default, k years since the time of default.

The response variables of interest are motivated by my model and include output in logs, investment-to-output ratios, as well as international reserves-to-output ratios. In order to conduct this study, I make use of macroeconomic time series data from the World Bank's World Development Indicators (WDI) database.[50] In addition, I make use of the crises database of Laeven and Valencia (2008, 2012) to construct the default indicators $D_{i,t}$. The resulting merged sample is for 155 countries and runs from 1970 to 2011.

The results are tabulated in Table A.1 as well as plotted in Figure 2. Following a sovereign default, we see that output drops,[51] investment-to-output ratios decrease, while international reserve-to-output ratios increase. These results are consistent with the mechanism of my model.

B Numerical Algorithm

B.1 Scaled System

The AK setting allows for numerical tractability as capital K_t can be scaled out to reduce the dimension of the dynamic programming problem. All scaled variables will be denoted with a tilde so that $\widetilde{x}_t$ denotes x_t/K_t. I first summarize the scaled system of equations.

Per period welfare after scaling is given by

$$\widetilde{W}\left(z_t, \widetilde{D}_t, \widetilde{S}_t, \widetilde{X}_t\right) = W\left(z_t, K=1, \widetilde{D}_t, \widetilde{S}_t, \widetilde{X}_t\right) \tag{A.2}$$

[50] For output, I use the GNI (current US$) series NY.GNP.MKTP.CD; for international reserves I use the total reserves (includes gold, current US$) series FI.RES.TOTL.CD; finally, for investments I use the gross capital formation (current US$) series NE.GDI.TOTL.CD.

[51] Gornemann (2013) finds that the output drop remains statistically and economically significant when additional controls are included.

k	log gni	inv/gni	res gni	k	log gni	inv/gni	res/gni
-10	0.043	0.028	0.034	1	-0.18	-0.032	0.014
	(0.054)	(0.016)	(0.019)		(0.051)	(0.01)	(0.008)
-9	0.023	0.023	0.039	2	-0.177	-0.027	0.021
	(0.054)	(0.016)	(0.018)		(0.062)	(0.011)	(0.007)
-8	0.006	0.017	0.04	3	-0.184	-0.011	0.021
	(0.055)	(0.016)	(0.016)		(0.063)	(0.013)	(0.008)
-7	0.013	0.026	0.033	4	-0.198	-0.01	0.024
	(0.045)	(0.013)	(0.013)		(0.064)	(0.015)	(0.009)
-6	0.045	0.02	0.03	5	-0.211	-0.009	0.026
	(0.046)	(0.014)	(0.01)		(0.075)	(0.016)	(0.011)
-5	0.06	0.028	0.028	6	-0.22	-0.011	0.024
	(0.043)	(0.013)	(0.007)		(0.08)	(0.017)	(0.011)
-4	0.079	0.027	0.019	7	-0.238	-0.016	0.026
	(0.04)	(0.011)	(0.006)		(0.081)	(0.016)	(0.012)
-3	0.07	0.017	0.016	8	-0.231	-0.008	0.029
	(0.036)	(0.01)	(0.005)		(0.067)	(0.017)	(0.012)
-2	0.061	0.01	0.004	9	-0.244	0.004	0.034
	(0.027)	(0.008)	(0.005)		(0.055)	(0.017)	(0.013)
0	-0.138	-0.016	0.006	10	-0.245	0.02	0.042
	(0.025)	(0.004)	(0.006)		(0.054)	(0.018)	(0.014)

Table A.1: **Output, investment and international reserves around default.** This table gives estimates for the coefficient β^k in the regression (A.1). This is done for the following response variables: (log) output, investment to output, and international reserves to output. The table reports estimates for k ranging from -10 to 10 years since a sovereign default. Standard errors are clustered by country and time, and are shown in brackets. The corresponding plot is shown in Figure 2.

with $W(\cdot)$ being given in (35).

The scaled value function in default is given by

$$\widetilde{V}_D\left(z_t,\widetilde{D}_t,\widetilde{S}_t\right)=\max_{\widetilde{I}_t,\widetilde{S}_{t+1}}\left\{\begin{array}{c}(1-\beta)\,\widetilde{W}\left(z_t,\widetilde{D}_t,\widetilde{S}_t,\widetilde{X}_{D,t}\right)^{1-\psi^{-1}}\\+\beta\left[\left(1-\delta+\widetilde{I}_t\right)\widetilde{CE}_D\left(z_t,\widetilde{D}_{t+1},\widetilde{S}_{t+1}\right)\right]^{1-\psi^{-1}}\end{array}\right\}^{\frac{1}{1-\psi^{-1}}} \tag{A.3}$$

$$\widetilde{X}_{D,t}=\frac{\left(1-\delta+\widetilde{I}_t\right)\widetilde{S}_{t+1}}{1+r}+\phi\left(\widetilde{I}_t\right)$$

$$\widetilde{D}_{t+1}=m_L\left[a_0+\frac{(1-m_A)\left(\frac{1}{m_L}\widetilde{D}_t-a_0\right)+\chi_D\phi\left(\widetilde{I}_t\right)}{1-\delta+\widetilde{I}_t}\right]$$

where the law of motion for $\widetilde{D}_t$ follows from (6), (7), (8), and (9).

The scaled value function when not in default is given by

$$\begin{aligned}&\widetilde{V}_{ND}\left(z_t,\widetilde{D}_t,\widetilde{S}_t,\widetilde{B}_t\right) \\ &=\max_{\widetilde{I}_t,\widetilde{S}_{t+1},\widetilde{B}_{t+1}}\left\{\begin{array}{c}(1-\beta)\,\widetilde{W}\left(z_t,\widetilde{D}_t,\widetilde{S}_t,\widetilde{X}_{ND,t}\right)^{1-\psi^{-1}}\\+\beta\left[\left(1-\delta+\widetilde{I}_t\right)\widetilde{CE}_{ND}\left(z_t,\widetilde{D}_{t+1},\widetilde{S}_{t+1},\widetilde{B}_{t+1}\right)\right]^{1-\psi^{-1}}\end{array}\right\}^{\frac{1}{1-\psi^{-1}}}\end{aligned} \tag{A.4}$$

$$\widetilde{X}_{ND,t}=\frac{\left(1-\delta+\widetilde{I}_t\right)\widetilde{S}_{t+1}}{1+r}+\phi\left(\widetilde{I}_t\right)+\widetilde{B}_t-\left(1-\delta+\widetilde{I}_t\right)\widetilde{B}_tQ\left(z_t,\widetilde{D}_{t+1},\widetilde{S}_{t+1},\widetilde{B}_{t+1}\right)$$

$$\widetilde{D}_{t+1}=m_L\left[a_0+\frac{(1-m_A)\left(\frac{1}{m_L}\widetilde{D}_t-a_0\right)+\chi_{ND}\phi\left(\widetilde{I}_t\right)}{1-\delta+\widetilde{I}_t}\right].$$

Note that only scaled state variables now appear in the bond price schedule $Q(\cdot)$.

Scaled continuation values are given by

$$\widetilde{CE}_{ND}\left(z_t,\widetilde{D}_{t+1},\widetilde{S}_{t+1},\widetilde{B}_{t+1}\right)=\mathbb{E}\left[\max\left\{\begin{array}{c}\widetilde{V}_D\left(z_{t+1},\widetilde{D}_{t+1},\widetilde{S}_{t+1}\right),\\\widetilde{V}_{ND}\left(z_{t+1},\widetilde{D}_{t+1},\widetilde{S}_{t+1},\widetilde{B}_{t+1}\right)\end{array}\right\}^{1-\gamma}|z_t\right]^{\frac{1}{1-\gamma}} \tag{A.5}$$

$$\widetilde{CE}_D\left(z_t,\widetilde{D}_{t+1},\widetilde{S}_{t+1}\right)=\left(\begin{array}{c}(1-\xi)\,\mathbb{E}\left[\widetilde{V}_D\left(z_{t+1},\widetilde{D}_{t+1},\widetilde{S}_{t+1}\right)^{1-\gamma}|z_t\right]\\+\xi\widetilde{CE}_{ND}\left(z_t,\widetilde{D}_{t+1},\widetilde{S}_{t+1},\widetilde{B}_{t+1}=0\right)^{1-\gamma}\end{array}\right)^{\frac{1}{1-\gamma}} \tag{A.6}$$

Finally, the bond price schedule in terms of scaled variables is given by

$$Q\left(z_t, \widetilde{D}_{t+1}, \widetilde{S}_{t+1}, \widetilde{B}_{t+1}\right) = \frac{1}{1+r}\mathbb{E}\left[1\left\{\begin{array}{c} \widetilde{V}_{ND}\left(z_{t+1}, \widetilde{D}_{t+1}, \widetilde{S}_{t+1}, \widetilde{B}_{t+1}\right) \\ \geq \widetilde{V}_{D}\left(z_{t+1}, \widetilde{D}_{t+1}, \widetilde{S}_{t+1}, \widetilde{B}_{t+1}\right) \end{array}\right\} |z_t\right]. \tag{A.7}$$

B.2 Numerical Implementation

Details for implementing the bank run model. In order to ensure that the integrals appearing in the threshold condition (33) are well behaved, I assume

$$n_{max} \leq N_{max} < 1. \tag{A.8}$$

That is, runs are uniformly bounded away from 1. In the numerical implementation, I set $N_{max} = 0.99$.[52]

Since the population is bounded above, conditions (9) and (10) may not simultaneously hold when A_t gets large. To account for this, I will instead be using the following definitions for the numerical implementation:

$$d_t = \begin{cases} d_0 K_t & \text{if } d_0 K_t N_{max} \leq D_t \\ \frac{D_t}{N_{max}} & \text{otherwise} \end{cases} \tag{A.9}$$

$$n_{max,t} = \begin{cases} \frac{D_t}{d_0 K_t} & \text{if } d_0 K_t N_{max} \leq D_t \\ N_{max} & \text{otherwise} \end{cases} \tag{A.10}$$

That is, definitions (9) and (10) will normally apply. However, if banks lend large amounts, then households will have to provide more that just $d_0 K_t$ units of deposits.

The run threshold (32) is found by solving (33) numerically.

Smoothing the run threshold. The equilibrium run scheduled $n(\cdot)$ as defined in (34) is discontinuous. In particular, there is a jump at the run threshold $z^\star$. This implies that $\widetilde{W}(\cdot)$ as well as the Bellman equations for the value functions will also be continuous. Thus, convergence cannot be achieved in general.

To get around this, I instead use a smoothed version of (34):

$$n_{smooth}\left(z, K, D, S, X\right) = n_{max}\Phi\left(\frac{e^{z^\star(K,d,n_{max},S,X)} - e^{z}}{h}\right), \tag{A.11}$$

where $z^\star(\cdot)$ is the run threshold in (32), $\Phi(\cdot)$ is the standard normal cumulative distribution function, and h is a smoothing parameter. In (A.11), I have used a sigmoid function to approximate the step function; my choice of the "link" function is probit. Note that the

[52] Perhaps some fraction of depositors were away on a fishing trip and therefore inattentive to runs.

approximation converges to the step function as $h \downarrow 0$.

Discrete state space value function iteration. I first discretize the TFP process (1) using the Rouwenhorst (1995) method. This method is known to have good approximation properties, especially for highly persistent processes (Kopecky and Suen (2010)). I then discretize the remaining state variables $\widetilde{D}$, $\widetilde{S}$, and $\widetilde{B}$. The scaled value functions are then computed over the resulting tensor grid $\{z_i\} \otimes \left\{\widetilde{D}_j\right\} \otimes \left\{\widetilde{S}_k\right\} \otimes \left\{\widetilde{B}_l\right\}$. The choice variables $\widetilde{I}$, $\widetilde{S}$, and $\widetilde{B}$ are discretized in a similar fashion. The discretized choice grids for $\widetilde{S}$ and $\widetilde{B}$ are the same as their state space counterparts. Note that $\widetilde{D}_{t+1}$ may be off grid, so I use linear interpolation when necessary. In order to speed up numerical convergence, I also pre-compute $\widetilde{W}(\cdot)$ over a discrete set of points $\left\{\widetilde{X}_m\right\}$ using the smoothed run definition (A.11), and subsequently linearly interpolate when necessary. Finally, the scaled system of equations in appendix B.1 is iterated until convergence.

C Extensions

In this section I briefly describe a few extensions of the baseline model, which can potentially boost sovereign debt capacity. These extensions are:

1. Permanent capital destruction as a result of liquidation.
2. The possibility for the sovereign to borrow liquidity internationally.
3. Additionally using long term debt to achieve further inter-temporal smoothing of liquidity shocks.

C.1 Permanent Liquidation Losses

The baseline model only features temporary liquidation losses in the sense that the total capital stock does not decrease when liquidations occur. This can remedied by modifying (4) as follows:

$$K_{t+1} = (1 - \delta(\ell_t)) K_t + I_t \tag{A.12}$$

where the effective depreciation rate $\delta(\cdot)$ is an increasing function of the equilibrium liquidation amount ℓ_t. The resulting model will be an endogenous disaster risk model (with disasters stemming from banking fragility) with permanent capital destruction (similar in spirit to Gourio (2012)).

This modification can potentially achieve the following:

1. Permanent capital destruction will quantitatively generate higher default costs, especially in states with a lot of liquidations. The planner will care a lot about this given his

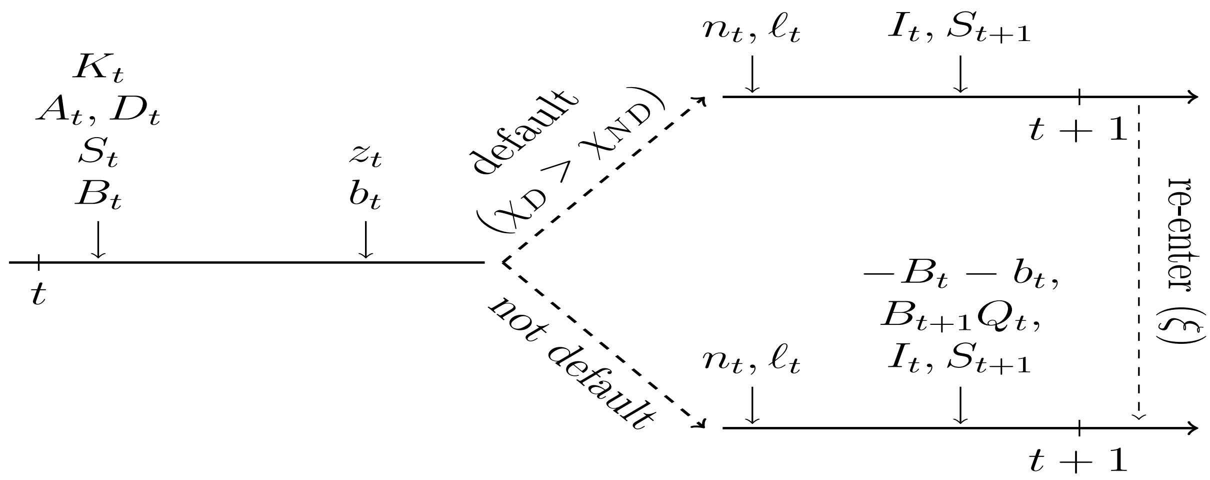

Figure A.1: **Timing period timing for the full model modified for within-period borrowing.** For the most part, the timing in this extended model is the same as that of our baseline model (cf. Figure 7). The only modification is as follows: right after the productivity shock has been realized, the sovereign can borrow liquidity internationally by issuing bonds with face value b_t. These bonds are due at the end of the period and is priced at q_t in equilibrium.

recursive preferences (16). This is likely to increase debt capacity beyond what I currently have. Afterwards, I will have more scope for lowering the subjective discount factor in order to generate higher sovereign spreads.

2. There will be additional feedback loops between runs today, capital destruction and lost capital for tomorrow, sovereign default incentives tomorrow, and sovereign credit spreads and rollover costs today. This can potentially generate more realistic twin crises dynamics.

C.2 External Sources of Liquidity

The baseline model forgoes the possibility of being able to borrow liquidity internationally. I consider this possibility here. I modify the timing in Figure 7 as follows: just after the realization of productivity z_t and before runs and liquidations take place, the planner is able to borrow liquidity internationally. The planner does so by issuing bonds with face value b_t which corresponds to the promised amount to be repaid at the end of the period. This form of "within-period" debt is issued at an endogenously determined price of q_t. This price reflects the planner's inability to commit to repaying b_t at the end of the period. The modified time line is shown in Figure A.1.

In equilibrium, q_t is given by the following fixed point problem:

$$q\left(z_t, K_t, D_t, S_t, B_t, b_t\right) \tag{A.13}$$
$$= \frac{1}{1+r} 1 \left\{ \begin{array}{c} V_{ND}\left(z_t, K_t, D_t, S_t + b_t q\left(z_t, K_t, D_t, S_t, B_t, b_t\right), B_t + b_t\right) \\ \geq V_D\left(z_t, K_t, D_t, S_t + b_t q\left(z_t, K_t, D_t, S_t, B_t, b_t\right)\right) \end{array} \right\}$$

where V_{ND} and V_D are given by (41) and (39) respectively. Bond prices (A.13) reflect the fact that by borrowing liquidity from abroad, the planner will instead have $S_t + b_t q_t$ units of liquidity on hand with which to manage domestic liquidity needs. In addition, the planner will also have additional obligations at the end of the period ($B_t + b_t$ instead of B_t). The pricing equation (A.13) reflects these changes.[53]

The liquidity provision role of b_t. Within-period debt b_t is not present in traditional models in the Eaton and Gersovitz (1981) setting, which does not involve liquidity constraints.[54] With the introduction of b_t, the planner gains an additional tool for liquidity provision. This is illustrated in Figure A.2.

The planner can now directly resources from full harvest to the liquidity constraint by borrowing liquidity internationally (i.e. use b_t). Such an option is not available in the baseline model.

We do observe empirical counterparts to b_t:

- In practice, there is a timing mismatch between government expenditures and revenues. Within period debt b_t can be thought of as short term credit which the government uses to finance its day to day operations. As demonstrated by the recent US government shutdown, we see that such forms of short term financing for the government can have important consequences for the real economy. This is true even for advanced countries such as the US with deep financial markets and credible governments.

- The IMF has created a Short-Term Liquidity Facility (SLF)[55] to provide countries facing temporary liquidity problems with short-term financing. In addition, the IMF also provides additional liquidity provision instruments such as its Flexible Credit Line (FCL)[56] as well as its Rapid Financing Instrument (RFI).[57]

[53] The fixed point in equation (A.13) does create additional challenges for numerical convergence. Randomization methods will have to be used to achieve numerical convergence, even if B_t is single period debt.

[54] Trade credit as modeled in Mendoza and Yue (2012) does have liquidity provision roles. However, the key difference is that Mendoza and Yue (2012) assumes that the country cannot default after trade credit has already been borrowed by firms (their within period timing assumes that default occurs before firms borrow trade credit). Formulation (A.13) assumes that the country cannot commit to repaying b_t. It is possible to model q_t following the timing convention in Mendoza and Yue (2012), however, this would introduce a form of limited commitment for debt repayment.

[55] See `https://www.imf.org/external/np/sec/pr/2008/pr08262.htm`

[56] See `http://www.imf.org/external/np/exr/facts/fcl.htm`.

[57] See `http://www.imf.org/external/np/exr/facts/rfi.htm`.

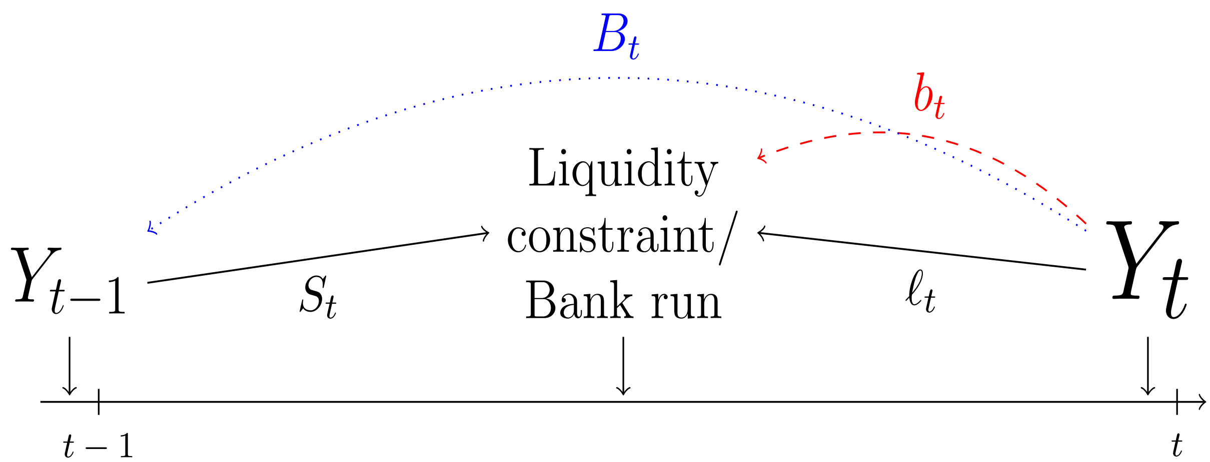

Figure A.2: **Instruments for liquidity provision.** This figure illustrates the various ways in which the planner can move resources from full harvest at the end of each period in order to satisfy liquidity constraints in the middle of each period.

- Some countries have created internal reserve pools to circumvent reliance on IMF funding. For example, BRICS countries have banded together to form shared pools of emergency reserves.[58] These types of reserve pools can be thought of as a combination of non-state contingent international reserves S_t and ex-post usage of within-period debt b_t whenever the need arises.

Introducing such an additional liquidity provision tool can potentially boost debt capacity even further– the loss of access to international liquidity in financial autarky is likely to make defaulting even more costly.

C.3 Long Term Debt and Liquidity Risk Management

The use of long term debt along with within-period debt b_t can be especially useful in generating additional debt capacity. This is because large liquidity shocks can be rolled over in a very smooth manner using a combination of these instruments. This means that the planner can hoard even smaller amounts of reserves while not in default. This is illustrated in Figure A.3. The link between debt maturity, liquidity risk management, and endogenous debt capacity in my setting warrants further study.

[58] See http://uk.reuters.com/article/2014/07/15/uk-brics-summit-deals-idUKKBN0FK28F20140715.

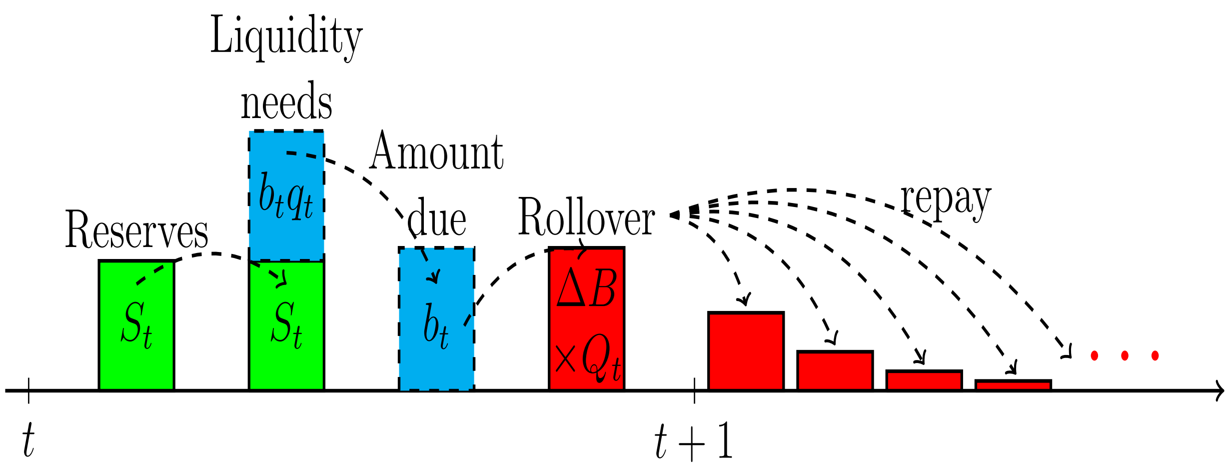

Figure A.3: **Debt maturity and liquidity smoothing.** This figure illustrates how liquidity shocks can be smoothed through a combination of international reserves, externally borrowed liquidity, and rollover into long term debt.